Numerical and Experimental Investigations of Dam and Levee Failure

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Numerical and experimental investigations of dam and levee failure

by

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DEDICATION

To "ALLAH", the only true Lord of the heavens and the earth.
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Both numerical and experimental investigations are conducted to study the failure of earthen dams and levees.

Boussinesq equations describing one-dimensional unsteady flow including non-hydrostatic pressure distribution are solved numerically along with Exner equation for the sediment mass conservation to include the effects of the streamline curvature on the failure of non-cohesive earthen embankments. In addition, the effects of the steep bottom slope on the flow variables during the failure are studied by solving the Saint-Venant equations modified for steep bed slope along with the Exner equation to simulate non-cohesive earthen embankment failure due to overtopping. The Boussinesq equations are solved by using the two-four finite-difference scheme which is second order accurate in time and fourth order accurate in space, while the modified Saint-Venant equations are solved by using the MacCormack finite-difference scheme which is second order accurate in time and space. The performance of three sediment transport formulae, namely Ashida and Michiue, Meyer-Peter and Müller, and Modified Meyer-Peter and Müller for steep slope is compared. The comparison of the numerical results with the experimental results shows that: (1) The improvement in the prediction of the breach evolution and the downstream hydrograph by including the Boussinesq terms is minimal; (2) The predicted results by using the modified Saint-Venant equations are slightly better than those calculated by using the classical Saint-Venant equations; (3) Ashida and Michiue transport equation overestimates the erosion rate which results in an overestimation of peak discharge but predicts the time to peak fairly well; (4) Meyer-Peter and Müller and Modified Meyer-Peter and
Müller equations give almost the same results for the top elevation of the eroded dam and the water surface levels, and the peak value and time to peak of the downstream hydrograph. A number of non-dimensional equations are presented to relate different model variables to the peak discharge of the downstream hydrograph. A sensitivity analysis of different model parameters to determine the most dominant factor affecting the peak downstream discharge indicates that the most dominant factor affecting the peak discharge is the upstream reservoir volume, while the sediment grain size shows very little effect on the peak discharge.

The lateral outflow through a levee breach may be computed as an outflow over a broad-crested side weir. The lateral outflow has been computed previously by assuming the flow in the main channel to be one-dimensional, and most of the equations for computing the outflow are based on the local flow variables near the breach. These are unknown and difficult to measure during a flood. In this study, a numerical model is developed to solve the two-dimensional, shallow water equations using MacCormack finite-difference scheme. The model is applied to a breach in a rectangular channel. A comparison of the numerical results with the experimental measurements shows satisfactory agreement. Different cases are simulated by varying the breach width, the bottom level of the breach, and the discharge in the main channel. Comparison between the breach outflows obtained using the numerical model with the results of a simple one-dimensional approach indicates that the breach outflows are overestimated by the latter. A new discharge correction factor is introduced for the lateral breach outflow predicted using the simple one-dimensional approach. The correction factor is a function of the submergence ratio of the breach, breach width, and inlet Froude number.

An accurate prediction of the evolution of the breach resulting from overtopping of a non-cohesive earthen levee is important for flood mitigation studies. Laboratory experiments are conducted with various inlet discharges, and downstream water
depths. The breach shape is recorded using a sliding rods technique. A sequence of discrete mass failure of the sides of the breach due to slope instability is observed during the failure process. A simple geostatic failure mechanism is suggested to calculate the lateral sediment load due to the mass failure. This mechanism is implemented in a two-dimensional numerical model which solves the shallow water equations along with the sediment mass conservation. In order to assess the effect of the lateral sediment load resulting from slope instability on the failure process, the predicted breach shape evolution and breach hydrograph with and without the slope failure mechanism are compared with the experimental results. The predicted breach shape is improved by adding the lateral sediment inflow due to slope instability especially in terms of the maximum depth of erosion. Also, the peak discharge of the breach hydrograph is captured more accurately by adding the slope failure mechanism.
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Chapter 1

Introduction

1.1 General

Dams and levees, also known as embankments, are structures used for water storage, water level regulation in streams, recreation activities, power generation, and for the protection of surrounding areas against floods. Failure of these structures may result in catastrophic property damage and loss of life. In the last few decades, the climate has changed significantly due to Global Warming which leads to ice melt, and sea level rise. This has increased the intensity and frequency of heavy storms such as hurricanes Katrina (2005), Sandy (2012), and Joaquin (2015). Embankments may be classified according to the material of construction as non-erodible, such as concrete, and erodible, such as earth and rock fill. The latter type is more preferable for construction due to the availability of material and the low construction costs. Failure of concrete dams usually happens suddenly when part of the structure is overstressed. However the failure of an earthen embankment is gradual, and the simulation of the flow field is more complex. This requires an appropriate understating of the interaction between the water and the soil. Almost 80 percent of the dams in the United States are constructed from erodible material (U.S. Committee on Large Dams, 1975). The major causes of the failure of earthen embankment are: overtopping, structural defects, and piping. The present study only focuses on the overtopping failure of non-cohesive embankments.
1.2 Objectives and scope of the study

The main objectives of this research are to develop one- and two-dimensional numerical models to accurately predict the breaching process of earthen embankments and the resulting hydrograph. In order to develop these numerical models, the following tasks are proposed:

1- Conduct small scale experiments on earthen embankment failure due to plane surface erosion, and use the measured results for numerical model validation.
2- Develop a one-dimensional numerical model based on the classical, shallow-water equations with sediment mass conservation.
3- Investigate the effect of steep slope on the pressure, gravity, and friction terms of the shallow-water equations.
4- Investigate the effect of non-hydrostatic pressure assumption on the shallow-water equations.
5- Develop and validate a two-dimensional numerical model to simulate the steady state flow field due to levee breach in rectangular and trapezoidal channels.
6- Develop a two-dimensional numerical model including slope failure mechanism to simulate the gradual failure of a non-cohesive earthen levee.

In the present study, the literature is reviewed in Chapter 2. Different governing equations with various sediment transport formulations to simulate one-dimensional plane embankment failure are presented in Chapter 3. Chapter 4 discusses the estimation of the steady outflow discharge through levee breach of a rectangular channel using one- and two-dimensional numerical models. Two-dimensional numerical modeling of gradual failure of earthen levee including slope failure mechanism is presented in Chapter 5. Finally, the summary and conclusions of these studies are presented in Chapter 6.
Chapter 2

Literature review

2.1 Earthen dam failure

Dams are barriers constructed across streams for water storage, irrigation, navigation, flood control, power generation, and recreation activities. About 58 percent of the dams in the world are earthen embankments and the remainder are made of concrete (Costa, 1985). This may be due to their low construction costs, availability of construction materials, and no special foundation requirements. Earthen embankments, the oldest type of dams, are generally the most economical and have a spillway to control the upstream water level. According to the Fourth Inter-governmental Climate Change Assessment (IPCC, 2013), the intensity and the frequency of occurrence of heavy storms has increased in the last two decades, which may lead to flood events that exceed the design capacity of embankment spillways, thereby causing overtopping and failure. In the United States about 57,000 dams have the potential for overtopping (Ralston, 1987). An accurate prediction of peak discharge and time to peak in case there is an embankment breach is needed for preparing emergency action plans and risk assessments.

The major causes of earthen embankment failure may be classified into three categories (ASCE/EWRI Task Committee on Dam/Levee Breaching, 2011): Overtopping (34%), piping (28%) and structural failure (30%). If a dam is overtopped, the water flowing over the downstream slope of the embankment starts to erode the embank-
ment surface when the hydraulic shear stress exceeds the critical shear stress of the soil. Piping or internal erosion occurs when fine materials between upstream and downstream slope are washed by seepage and a pipe is formed inside the embankment. The diameter of this pipe increases with time due to an increase in the internal discharge until the embankment collapses internally. Structural failure includes slope instability, differential settlement, and sliding. The present work focuses only on the overtopping failure.

The past research may be classified into two major categories: (1) Experimental investigations and (2) Numerical modeling. These are discussed in the following paragraphs.

**Experimental investigations**

Tinney and Hsu (1961) conducted experiments on the mechanics of washout of fuse plugs utilizing the sediment transport equations. Simmler and Samet (1982) investigated the evolution of embankment erosion due to overtopping. They found that the erosion process is affected by the dam material, dam geometry, reservoir volume, and the location of impervious element. They also provide a relationship for the breach flow as a function of the breach volume. Coleman et al. (2002) studied the failure of non-cohesive homogenous embankments due to overtopping along the crest of the embankment. They derived expressions to predict the development of the breach shape, eroded volume, and breach flows. Dupont et al. (2007) carried out a scale-model study to validate a numerical model for the time evolution of dam breach and the outflow hydrograph. Schmocker and Hager (2009) carried out a series of experiments of plane embankment breach of loose, uniform, non-cohesive sediment to study the test repeatability, side-wall, and scale effects. Pickert et al. (2011) conducted laboratory experiments of non-cohesive embankments to obtain data about
breach discharge, breach profile, erosion rates, and pore-water pressure within the embankment.

**Numerical modeling**

Wang and Bowles (2006a,b) presented a slope stability model with a finite-difference scheme to simulate the dam-breach outflow by solving the two-dimensional Saint-Venant equations. They applied their numerical model to an arbitrary breach in a non-cohesive earthen embankment due to overtopping failure. Pontillo et al. (2010) applied a two-phase, one-dimensional numerical model based on the shallow water equations coupled with a sediment erosion model for the case of loose, non-cohesive, homogenous, earthen embankment. Their numerical results reasonably match the measured temporal free surface and bed evolution of the experimental data. They indicate some inaccuracies in the numerical results due to streamline curvature. Van Emelen et al. (2011) developed two numerical models; the first model is based on the classical two-dimensional Saint-Venant equations coupled with the Exner equation with a Meyer-Peter and Müller formula for calculating the sediment transport rate; and the second model includes a two-dimensional bank failure mechanism into the classical, two-dimensional shallow-water model. Both models assume hydrostatic pressure and neglect the effects of streamline curvature. Van Emelen et al. (2012) applied two numerical approaches involving different sediment transport formulae for solving the breach in a channel with a loose, non-cohesive sand embankment due to overtopping. The first model utilized coupled Saint-Venant-Exner equations, and the second model assumes a mixture layer of water and sediment. They concluded that the first model involving the Meyer-Peter and Müller formula gives the best results.
2.2 Steady flow through breached levees

A levee is a structure, constructed by humans or formed naturally along the course of a river to regulate its water level and or to protect its floodplain. Due to climate change, the risk has increased of large scale disasters caused by levee breach (Kakinuma and Shimizu, 2014). The assessment of this risk requires the identification of various geometric and hydraulic parameters causing this failure.

Several levee breaches occurred in the last decade: New Orleans, Louisiana 2005, Fernley, Nevada burst 2008, Munster, Indiana 2008, Southern Taiwan 2009, Vendée, France 2010, and Poplar Bluff, Missouri 2011. The death and property damage caused by these events was greater than the failure of any other man-made structure, especially because most of these levees were constructed near urban areas.

Levee breach has been studied by several researchers. Jaffe and Sanders (2001) developed a model for flood mitigation by designing an engineered levee breach which creates depression waves that interact with the flood wave to reduce the flood stages at certain locations of the channel where severe damage from flooding would be expected. Their model is based on the solution of the two-dimensional shallow water equations by finite-volume method. They derived a non-dimensional relation to calculate the optimal flood plain area of the engineered levee breach. Sattar et al. (2008) tested a 1:50 scale model of the 17th street canal breach in New Orleans based on Froude similitude relationships. They investigated different procedures for breach closure.

Han et al. (1998) developed a combined, one- and two-dimensional hydrodynamic model. The one-dimensional model is based on the dynamic wave equation and utilizes Preissmann scheme which is an implicit, finite-difference method. The two-dimensional model solves a diffusion wave equation by integrated finite differences. They applied their model to an actual levee-break in the downstream reaches of the Han river. They validated the simulated results with the observed data, including inundated depth, flood arrival time, and the inundated areas. Testa et al. (2007)
conducted a physical model of an urban district and provided water depth history of
different configurations of the model city for the purpose of mathematical modeling.
Soares-Frazão et al. (2008) introduced a two-dimensional, shallow-water model with
porosity for urban flood modeling and solved the governing equations by finite volume
method. They compared their results with experimental measurements. Their model
reproduced the mean characteristics of the flow inside and around the urban area
with low computational cost compared to the classical shallow-water model.

Roger et al. (2009) solved the shallow-water equations by using a total variation
diminishing, Runge-Kutta discontinuous Galerkin finite-element method, and by us-
ing a finite-volume scheme involving a flux vector splitting approach. Both models
had satisfactory agreement with the experimental results. They also found that the
simulated results are less sensitive to the bed and wall roughness and to the turbu-
ulence modeling due to the advective nature of the levee-breach flow which is governed
mainly by the pressure gradient and the advective terms in the momentum equations
rather than the diffusion. Kesserwani et al. (2010) presented new governing equations
for predicting flow divisions at open-channel T-junctions. The new approach predicts
the lateral-to-upstream discharge for different flow regime satisfactorily. Van Emelen
et al. (2012) developed a finite-volume model to solve the two-dimensional, shallow-
water equations. The model was applied to an idealized case study of 17th street
canal breach in New Orleans, Louisiana. The steady-state numerical results satisfac-
torily agreed with the experimental results. They concluded that the depth-averaged
models give an acceptable flow prediction in the complex domains at reasonable com-
umputational cost.

2.3 Gradual failure of earthen levees

According to the U.S. Committee on Large Dams (1975), about 80 percent of the
large dams in the United States were built using earth. An accurate prediction of the
earthen levee failure process (i.e., breach shape evolution, breach hydrograph, and the resulting flow field) is quite complex, requiring deep understanding of the interaction between the water flow, sediment transport, and the corresponding geomorphological changes.

The breaching process in dams and levees differs due to the direction of the momentum flux (ASCE/EWRI Task Committee on Dam/Levee Breaching, 2011). A variety of research was conducted in the past concerning earthen embankment breaching, however few of that research focused on the earthen levee failure.

Faeh (2007) developed a two-dimensional numerical model, 2bMb, which uses a finite-volume approach to solve the shallow-water equations and a sediment transport module for bed and suspended load for an arbitrary number of grain size classes. The model was validated with the results of experiments of dam breaks due to erosion (Bechteler and Kulisch, 1998), and field data of one of the dike breaches on the Elbe River following a flood in Sachsen (Germany) 2002. He found that, the most sensitive parameter of an earthen dike failure, is the breach side-slope which affects the lateral erosion. Kakinuma and Shimizu (2014) conducted large-scale experiments on riverine levee breach. They categorized the levee failure process into four stages: (1) Downstream slope erosion; (2) The erosion advances to the top of the upstream slope, and the breach at the crest starts to widen; (3) The breach widening advances in the downstream direction and the breach outflow peaks; (4) The breach outflow becomes constant and the rate of breach widening is decreased. They proposed a breach rate equation similar to the sediment transport formula by Meyer-Peter and Müller (1948) using their experimental measurements. The proposed equation substituted the sediment transport formula for the levee cells in a two-dimensional numerical model which solves the shallow-water equations. The numerical model was validated using the experimental measurements and the breach widening stage was reproduced successfully. However, the model did not capture the early stages of the
failure.

2.4 Conclusions

Earthen dam failure

To the best of the author’s knowledge, all cited studies used the classical Saint-Venant equations with the hydrostatic pressure assumption and neglected the steep slope and streamline curvature effects on the erosion process. The objectives of the present work are to study the effects of steep slope and streamline curvature on the failure of non-cohesive earthen embankments by modifying the the governing equations for the flow, assess the performance of various sediment transport formulas, and determine the most dominant factor affecting the peak discharge of the downstream hydrograph.

Steady flow through breached levees

Most of the cited studies are for special cases of levee breach. In the present work, a generalized approach is used to study levee breach by considering the lateral outflow through a levee breach as lateral outflow over a broad-crested side weir. Most of the previous studies predicted the flow over a side weir by applying the energy principle in the longitudinal direction and assume the flow in the main channel is one-dimensional (Chow, 1959; Subramanya and Awasthy, 1972; Hager, 1987; Singh et al., 1994; Swamee et al., 1994; Borghei et al., 1999). The objectives of the present study are to develop a numerical model based on two-dimensional depth-averaged equations to simulate the flow field resulting from a generalized levee breach of a rectangular channel and to develop a non-dimensional equation to calculate the discharge correction factor for the breach discharge estimated using the one-dimensional models.
Gradual failure of earthen levees

The effects of the lateral sediment inflow on the failure shape evolution and on the flood hydrograph of an earthen levees have not been studied. This lateral sediment inflow is caused by the slope instability of breach sides. The objective of the present study is to include this sediment inflow in a two-dimensional depth-averaged model and to compare the results of this numerical model with the laboratory measurements.
Chapter 3

Numerical and Experimental Investigations on Earthen Embankment Breach due to Overtopping

3.1 Introduction

The one-dimensional earthen embankment breach has been studied extensively during the last few decades. However, most of these studies used the classical Saint-Venant equations with the hydrostatic pressure assumption and neglected the effects of steep slope and streamline curvature on the erosion process. The objectives of the present work are to study the steep slope and streamline curvature effects on the failure of non-cohesive earthen embankments by modifying the governing equations for the flow, assess the performance of various sediment transport formulae, and determine the most dominant factor affecting the peak discharge of the downstream hydrograph.

3.2 Experimental setup

To record the gradual failure of the earthen embankment (Fig. 3.1), the experiments are conducted in the Hydraulics Laboratory of the University of South Carolina in a flume comprising a wooden part and a plexiglas part. A grid is marked on the sides of the plexiglas at 0.05 m interval. The flume is 6.1 m long, 0.2 m wide, 0.25 m deep, and has a horizontal bottom. The toe of the upstream slope of the embankment is located 3.6 m from the water intake. The flume has a wave suppressor, followed by
a honeycomb, to reduce the water surface fluctuations at the entrance. The water is pumped from a steel tank at the upstream end of the flume through a control valve to regulate the flow from the pump to the flume. The pump discharge used for all experiments is 0.47 l/s.

The embankment height is 0.15 m, top width 0.1 m, with the upstream and downstream slope 2:1 (H:V) as shown in Fig. 3.1. The embankment is constructed from loose medium sand with uniform grain size diameter \( D \) of 0.6 mm in three layers, each layer with a thickness of 0.05 m. First, the soil is mixed with the optimum water content of 5.2 % from the Proctor compaction test. Second, each layer is poured separately and the surface of each layer is leveled horizontally. Third, the soil is trimmed to the final dimension of the embankment.

The upstream reservoir of the earthen embankment is filled rapidly to a certain level to minimize seepage into the embankment and then the water is continuously pumped. Also, the upstream slope is covered with a layer of low-permeability clay (Dupont et al., 2007). The gradual failure of the earthen embankment is recorded at 60 frames/s using a high-definition camera (HDR-XR160), facing perpendicular to the side of the flume. The video is split into individual frames and the images are digitized to determine the embankment level and the water surface level. The discharge is measured at the end of the flume by collecting the water in buckets per unit time. Then the downstream hydrograph is created using these discharges. The test run is set to be 5 minutes to ensure the flow is approaching steady state. Before overtopping, the area downstream of the embankment is dry; therefore the embankment overflow is free flow. The test is repeated three times to ensure repeatability.
3.3 Numerical model description

Modified Saint-Venant Exner model (MSV)

Hydrodynamic equations

Based on the governing equations for two-dimensional flow over a steep bed slope (Chaudhry, 2008), the one-dimensional form may be written as:

\[
\frac{\partial \tilde{h}}{\partial t} + \frac{\partial \tilde{q}}{\partial \tilde{x}} = 0 \tag{3.1}
\]

\[
\frac{\partial \tilde{q}}{\partial t} + \frac{\partial}{\partial \tilde{x}} \left( \tilde{q}^2 \right) + \frac{\partial}{\partial x} \left( g \tilde{h}^2 \right) \cos^4 \alpha + g \tilde{h} \cos \alpha \frac{\partial z}{\partial \tilde{x}} + g \tilde{h} \tilde{S}_f = 0 \tag{3.2}
\]

where \( t = \text{time}, \tilde{x} = \text{horizontal coordinate}, \tilde{h} = \text{vertical flow depth}, \tilde{q} = \text{flow discharge per unit width in the } \tilde{x} \text{ direction}, \alpha = \text{angle between the bed and the horizontal}, z = \text{bed elevation}, \text{ and } \tilde{S}_f = \text{friction slope. Using the Manning equation for a wide rectangular channel, the friction slope } \tilde{S}_f \text{ may be written as}

\[
\tilde{S}_f = \frac{n^2 \tilde{q}^2}{\tilde{h}^{1/3} \cos^2 \alpha} \tag{3.3}
\]

where \( n = \text{Manning roughness coefficient} \)

Sediment equation

Exner equation (mass conservation of bed sediment) may be written as

\[
(1 - \lambda_p) \frac{\partial z}{\partial t} = - \frac{\partial q_b}{\partial \tilde{x}} \tag{3.4}
\]

where \( \lambda_p = \text{bed porosity is equal to 0.43, and } q_b = \text{volumetric bedload transport rate per unit channel width.} \)
Boussinesq-Exner model

Hydrodynamic equations

The governing equations for flow with non-hydrostatic pressure assumption are given as (Gharangik and Chaudhry, 1991; Mohapatra and Chaudhry, 2004; Chaudhry, 2008):

\[
\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{3.5}
\]

\[
\frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{q^2}{h} \right) + \frac{\partial}{\partial x} \left( \frac{gh^2}{2} \right) - \frac{\partial}{\partial x} \left( \frac{h^3}{3} E \right) + gh \frac{\partial h}{\partial x} + ghS_f = 0 \tag{3.6}
\]

where \( x \) = longitudinal coordinate, \( h \) = flow depth, \( q \) = discharge per unit width, and \( S_f \) = friction slope. By using the Manning equation for a wide, rectangular channel, the friction slope \( S_f \) may be written as

\[
S_f = n^2 \frac{q^2}{h^{1.5}} \tag{3.7}
\]

The E term accounts for the non-hydrostatic pressure and can be expressed as

\[
E = \left[ \frac{\partial^2 (q/h)}{\partial x \partial t} + \frac{q}{h} \frac{\partial^2 (q/h)}{\partial x^2} - \left( \frac{\partial (q/h)}{\partial x} \right)^2 \right] \tag{3.8}
\]

Sediment equation

Exner equation for mass conservation of bed sediment is the same as Eq. (3.4).

Sediment transport formulations

Several empirical formulae are available to calculate the sediment transport capacity. These formulae are divided into two categories: The first category does not include the effect of slope, e.g., Ashida and Michiue (1972) (AM), and Meyer-Peter
and Müller (1948) (MPM) shown in Eqs. 3.9 and 3.10 respectively; and the second category includes the effect of bottom slope, such as modified Meyer-Peter and Müller (MMPM) (Eq. 3.13) which was developed by Wu (2004) by adding the effect of the streamwise component of the gravity force to the bed shear stress $\tau^*$ without modifying $\tau^*_c$. These formulae are listed below:

\[ q_b^* = 17.0(\tau^* - \tau^*_c)(\sqrt{\tau^*} - \sqrt{\tau^*_c}) \]  
\[ q_b^* = \alpha_t(\tau^* - \tau^*_c)^{nt} \]  

In Eq. (3.9) and Eq. (3.10), $\tau^*$ is expressed as:

\[ \tau^* = \frac{\tau}{\rho gRD} \]  
\[ \tau = \begin{cases} \rho gh \cos \alpha S_f & \text{for MSV} \\ \rho ghS_f & \text{for the Boussinesq model} \end{cases} \]  

\[ q_b^* = \alpha_t(\tau^*_{eff} - \tau^*_c)^{nt} \]  

\[ \tau^*_{eff} = \frac{\tau_{eff}}{\rho R g D} \]  

\[ \tau_{eff} = \tau + \lambda_o(\tau^*_c \rho g RD)^{\frac{\sin \alpha}{\sin \phi}} \]  

\[ \lambda_o = \begin{cases} 1 & \text{if } \alpha \leq 0 \\ 1 + 0.22 \left( \frac{\tau_{grain}}{\tau^*_c \rho g RD} \right)^{0.15} e^{2 \sin \alpha / \sin \phi} & \text{if } \alpha > 0 \end{cases} \]
\[ \tau_{\text{grain}} = \left( \frac{n_{\text{grain}}}{n} \right)^{\frac{3}{2}} \tau \]  

(3.17)

\[ n_{\text{grain}} = \frac{D^{(\frac{1}{2})}}{20} \]  

(3.18)

\[ q_b = \sqrt{RgDDq_b^*} \]  

(3.19)

where, \( q_b^* \) = dimensionless bedload transport rate, \( \tau^* \) = dimensionless hydraulic shear stress, \( \tau_c^* \) = dimensionless critical shear stress (equal to 0.047 for MPM and MMPM and is equal to 0.05 for AM), \( \alpha_t \) = coefficient in MPM and MMPM relations (equal to 8), \( n_t \) = exponent in MPM and MMPM formulae (equal to 1.5), \( \tau \) = hydraulic shear stress, \( \rho \) = density of water, \( g \) = gravitational acceleration, \( R \) = submerged specific gravity of sediment (equal to 1.65), \( D \) = grain diameter, \( \tau_{\text{eff}}^* \) = dimensionless effective shear stress, \( \tau_{\text{eff}} \) = effective shear stress, \( \lambda_o \) = coefficient related to the bed slope and sediment condition, \( \phi \) = repose angle, \( \tau_{\text{grain}} \) = grain shear stress, \( n_{\text{grain}} \) = Manning’s coefficient corresponding to grain roughness, and \( q_b \) = bedload transport rate.

**Numerical scheme**

The governing equations of MSV are solved numerically by using an explicit finite-difference approach. The solution consists of a two-step, predictor-corrector scheme (MacCormack, 1969).

The Boussinesq equations are also solved using the explicit, finite-difference approach. However, since the momentum equation (Eq. 3.6) has higher-order derivative terms, it is necessary that the numerical scheme is at least third-order accurate in space to reduce the truncation errors (Abbott, 1979; Gharangik and Chaudhry, 1991). The finite-difference scheme developed by Gottlieb and Turkel (1976) is used in the
The present study for this purpose. It is a dissipative, two-four scheme and is a two-step generalization of the Lax-Wendroff method. It is fourth-order accurate in space and second-order accurate in time.

Solution procedure

The computational domain is divided by equally spaced nodes. The governing partial differential equations are represented by the corresponding finite-difference equations. The computational time step is selected to satisfy the Courant-Friedrichs-Lewy stability condition. The computations are repeated until steady conditions are obtained.

Discretization

Only the discretization of Boussinesq model is presented. The mixed derivative in the E term of Eq. 3.8 is solved after the predictor and corrector step (Mohapatra and Chaudhry, 2004). The primitive model variables after each step are calculated as follows:

Predictor step (forward finite difference)

\[
h_i^* = h_i^k + \frac{\Delta t}{6\Delta x} \left( q_{i+2}^k - 8q_{i+1}^k + 7q_i^k \right)
\]

(3.20)

\[
q_i^* = q_i^k + \frac{\Delta t}{6\Delta x} \left( \left( \frac{q_i^2}{h_i} \right)^k_{i+2} + \frac{1}{2} g \left( h_i^k \right)^2 - \frac{1}{3} \left( h_i^k \right)^E_{i+2} \right) \right.

- 8 \left( \left( \frac{q_i^2}{h_i} \right)^k_{i+1} + \frac{1}{2} g \left( h_i^k \right)^2 - \frac{1}{3} \left( h_i^k \right)^E_{i+1} \right) \right.

+ 7 \left( \left( \frac{q_i^2}{h_i} \right)^k_i + \frac{1}{2} g \left( h_i^k \right)^2 - \frac{1}{3} \left( h_i^k \right)^E_i \right) \]

\[- \Delta t \rho g h_i^k \left( \frac{z_{i+1}^k - z_i^k}{\Delta x} \right) - \Delta tgh_i^k \left( \frac{n_i}{\left( h_i^k \right)} \right)^2 \]

(3.21)
\[ z^*_i = z^k_i - \frac{\Delta t}{\Delta x} \left( \frac{q^k_{i+1} - q^k_{i}}{1 - \lambda_p} \right) \] (3.22)

in which

\[
E^k_i = \left( \frac{q}{h} \right)_i \left( \frac{(q/h)^k_{i+2} - 2(q/h)^k_{i+1} + (q/h)^k_i}{\Delta x^2} \right) - \left( \frac{(q/h)^k_{i+1} - (q/h)^k_i}{\Delta x} \right)^2 \] (3.23)

Corrector step (backward finite difference)

\[
h^*_i = \frac{1}{2} \left( h^*_i + h^k_i \right) + \frac{\Delta t}{12\Delta x} \left( -7q^*_i + 8q^*_{i-1} - q^*_{i-2} \right) \] (3.24)

\[
q^*_i = \frac{1}{2} \left( q^*_i + q^k_i \right) + \frac{\Delta t}{12\Delta x} \left( \left[ \left( -\frac{1}{2} \left( \frac{q^2}{h} \right)_i \right) + \frac{1}{2} g \left( h^*_i \right)^2 - \frac{1}{3} \left( h^*_i \right) E^*_i \right] \right.
\]

\[
+ 8 \left[ \left( \frac{q^2}{h} \right)_{i-1} + \frac{1}{2} g \left( h^*_{i-1} \right)^2 - \frac{1}{3} \left( h^*_{i-1} \right) E^*_{i-1} \right] \right.
\]

\[
- \left[ \left( \frac{q^2}{h} \right)_{i-2} + \frac{1}{2} g \left( h^*_{i-2} \right)^2 - \frac{1}{3} \left( h^*_{i-2} \right) E^*_{i-2} \right] \right)
\]

\[
- 0.5\Delta tgh^*_i \frac{z^*_i - z^*_{i-1}}{\Delta x} - 0.5\Delta tgh^*_i \frac{n^2(q^2)^*_i}{(h^\frac{11}{2})^*_i} \] (3.25)

\[ z^*_i = z^k_i - \frac{\Delta t}{\Delta x} \left( \frac{q^*_i - q^*_{i-1}}{1 - \lambda_p} \right) \] (3.26)

in which

\[
E^*_i = \left( \frac{q}{h} \right)_i \left( \frac{(q/h)^*_i - 2(q/h)^*_{i-1} + (q/h)^*_{i-2}}{\Delta x^2} \right) - \left( \frac{(q/h)^*_i - (q/h)^*_{i-1}}{\Delta x} \right)^2 \] (3.27)

The intermediate flow variables are:

\[ h^k_{i+1} = h^*_i \] (3.28)
\[ q_i^{k+1} = q_i^{*} \quad (3.29) \]

\[ z_i^{k+1} = \frac{z_i^{*} + z_i^{**}}{2} \quad (3.30) \]

The intermediate unit discharge is modified by solving the following equation to include the mixed derivative term:

\[
\frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left( -h^3 \frac{\partial^2 (q/h)}{\partial x \partial t} \right) = 0 \quad (3.31)
\]

\[ q_i^{final} = q_i^{k+1} - \frac{B_{i+1}^k - B_{i-1}^k}{2\Delta x} \Delta t \quad (3.32) \]

in which

\[ B_i^k = -\frac{(h_i^{k+1})^3}{6\Delta x \Delta t} \left( \frac{q_{i+1}^{k+1}}{h_{i+1}^{k+1}} - \frac{q_{i-1}^{k+1}}{h_{i-1}^{k+1}} - \frac{q_{i+1}^k}{h_{i+1}^k} + \frac{q_{i-1}^k}{h_{i-1}^k} \right) \quad (3.33) \]

In Eqs. 3.20 to 3.33, subscript \( i \) indicates the node number and superscripts \( k \), \( * \), and \( k + 1 \) indicate the values of the variable at the known time step, predictor step, and unknown time step, respectively.

**Boundary conditions**

The discretization scheme discussed previously is applied to the interior nodes, i.e., for \( i = 5 \) through \( i_{end} - 4 \), where \( i_{end} \) is the last node in the computational domain. The flow regime of the upstream boundary is subcritical, thus \( h \) and \( z \) are extrapolated from the interior nodes and a constant value for \( q \) is specified. The computation of the downstream boundary depends on the flow regime. For subcritical flow, \( h \) is specified equal to the critical depth, and \( q \) and \( z \) are extrapolated from the interior nodes. For supercritical flow, however, \( h \), \( q \), and \( z \) are extrapolated from the interior nodes. The model variables at nodes 2 to 4 and \( i_{end} - 3 \) to \( i_{end} - 1 \) are determined by solving the Saint Venant equations using the MacCormack scheme.
Stability conditions

For stability, the Courant-Friedrichs-Lewy stability condition is applied,

$$\Delta t = \frac{C_n \Delta x}{\left| \frac{q}{h} \right| + \sqrt{gh}} \quad (3.34)$$

In the present work, $C_n \leq 0.65$ is used for the two-four scheme and $C_n \leq 1.0$ is used for the MacCormack scheme.

Artificial viscosity

The governing equations are nonlinear and are solved numerically. During the overtopping failure of an earthen embankment, bores may form (Stoker, 1957) thereby increasing the possibility of having discontinuous solutions. In the present work, the second-order accurate, MacCormack scheme and fourth-order accurate, Gottlieb and Turkel scheme are used. These produce spurious oscillations around the discontinuities. For accurate results, it is necessary to capture these discontinuities without generating spurious oscillations. To eliminate these oscillations, artificial viscosity (Jameson et al., 1981) is added to smooth the regions where the solution has large, free-surface gradients while leaving the smooth areas relatively undisturbed (Chaudhry, 2008). Thus, the values of the variables $q$ and $h$ (represented by symbol $f$ in the following equation) are modified after the end of each time step as follows:

$$f_{i+1}^{k+1} = f_i^{k+1} + \nu_{i+\frac{1}{2}} \left( f_{i+1}^{k+1} - f_i^{k+1} \right) - \nu_{i-\frac{1}{2}} \left( f_i^{k+1} - f_{i-1}^{k+1} \right) \quad (3.35)$$

in which

$$\nu_{i+\frac{1}{2}} = \kappa \ max(\nu_{i+1}, \nu_i) \quad (3.36)$$

$$\nu_i = \frac{|h_{i+1} - 2h_i + h_{i-1}|}{|h_{i+1}| + 2|h_i| + |h_{i-1}|} \quad (3.37)$$
3.4 Model application

Grid-independent results are obtained using mesh size, $\Delta x = 0.02$ m. The Manning roughness coefficient $n$ is estimated using the experimental results and it is found that $n = 0.021$ gives the best results. In this section, the effects of steep slope and the non-hydrostatic pressure distribution are studied by comparing the simulated results with the measurements in terms of the prediction of failure evolution and downstream hydrograph. Then, a comparison between using various empirical bed-load transport formulae is discussed and a parametric study for the effects of different model variables on the downstream peak discharge is presented.

**Effect of steep slope**

Two numerical models are developed to investigate the effect of steep slope on the erosion process and on the downstream hydrograph of a non-cohesive embankment. The first model utilizes the Saint-Venant equations (SV) and the second model utilizes modified Saint-Venant equations (MSV). MacCormack scheme which is second-order accurate in space and time is used to solve these equations. Figures 3.2 and 3.3 compare dam and water surface profiles predicted by using both models with the measured data at various times. The results from both models compare well with the measured data. It is observed that, at $t= 10$ and 15 seconds, MSV predicts the dam and water surface profiles slightly better than SV. However, after that the results are almost the same because the slope of the downstream face of the dam decreases with time. Also, there is no difference in the peak of the downstream hydrograph and the time to peak (Fig. 3.4). Generally, the effect of steep slope is marginal which may be due to the maximum depth on the downstream face of the dam being small (approximately 0.035 m).
Effect of non-hydrostatic pressure distribution

A two-four scheme is used to solve the Boussinesq equations and the SV equations to investigate the effect of Boussinesq terms on the failure evolution and the downstream hydrograph. Figures 3.2 and 3.3 show that the predicted dam and water surface levels using Boussinesq and SV equations are exactly the same except for the first 15 seconds of the failure when the crest curvature is high. For the first 60 seconds of the dam failure, the average magnitude of Boussinesq terms is about 8.6% of the other spatial derivative terms.

Effect of sediment transport formula

The performance of three different empirical sediment transport formulae (AM, MPM, and MMPM) are investigated using (SV) solved by two-four scheme (Figs. 3.5 and 3.6). The first two formulae, AM and MPM, do not include correction for the surface erosion over steep slope while MMPM does. As shown in Figs. 3.5 and 3.6, the erosion rate is overestimated by AM which results in the underestimation of the dam and water surface levels, while the erosion rate predicted by the other formulae is closer to the measured dam and water surface levels. Figure 3.7 compares the downstream hydrograph using these formulae. It shows that the peak value is overestimated by 50 percent using AM while the time to peak is predicted well. Both MPM and MMPM were in good agreement with the measured hydrograph in terms of peak value and the time to peak.

Parametric study for different model variables

In order to study the effect of model variables on the peak unit discharge ($q_{peak}$), a parametric study is performed by changing one variable at a time. Figure 3.8 shows the relation between the embankment top width ($w$) and $q_{peak}$. $w$ is normalized by
the dam height ($H_{\text{dam}}$) and $q_{\text{peak}}$ is normalized by the inlet discharge ($q_{\text{inlet}}$). The normalized $q_{\text{peak}}$ is inversely proportional to the normalized $w$. A trendline is fitted to the data with coefficient of determination ($R^2$) equal to 0.99 and an empirical equation is presented. Figure 3.9 shows the relationship between the reservoir volume ($V_{\text{res.}}$) and $q_{\text{peak}}$, with $V_{\text{res.}}$ normalized using the initial volume of the dam ($V_{\text{dam}}$). The normalized $q_{\text{peak}}$ is directly proportional to the normalized $V_{\text{res.}}$. A trendline is fitted with $R^2$ equal to 0.99 is achieved and an empirical equation is presented. Figure 3.10 shows the relationship between the inlet discharge ($q_{\text{inlet}}$) and $q_{\text{peak}}$, both variables are normalized by $\sqrt{gH_{\text{dam}}^3}$. The normalized $q_{\text{peak}}$ is directly proportional to the normalized $q_{\text{inlet}}$. Best curve with $R^2$ equal to 1.0 and an empirical equation are presented. In order to measure the sensitivity of the $q_{\text{peak}}$ to different model variables, a percentage change of $q_{\text{peak}}$ is plotted against percentage change of each variable (Fig. 3.11). It is clear that $q_{\text{peak}}$ is more sensitive to $V_{\text{res.}}$ as compared to $q_{\text{inlet}}$, and is more sensitive to $w$ as compared to grain size.

3.5 Summary and conclusions

Boussinesq equations and Saint-Venant equations modified for steep bed slope are numerically solved to investigate the effects of non-hydrostatic pressure distribution and steep slope on the breach evolution and the downstream hydrograph of a non-cohesive earthen embankment. The Exner equation for sediment mass conservation is solved using both sets of equations. Boussinesq equations are solved by the Gottlieb and Turkel explicit, finite-difference scheme. The Saint-Venant equations modified for steep bed slope are solved by the MacCormack explicit finite-difference. Artificial viscosity technique is used to smooth the spurious oscillations around the bores for both models. Three different sediment transport equations are tested: Ashida and Michiue, Meyer-Peter and Müller, and Modified Meyer-Peter and Müller for steep slopes. The results of a parametric study are presented to show the effect of various
model variables on the peak discharge of the downstream hydrograph.

A comparison of the numerical and measured results shows that: (1) The Boussinesq terms have little effect on predicting the temporal failure evolution and the downstream hydrograph; (2) The correction of Saint-Venant equation for steep bed slope has a marginal effect on the dam and water surface levels for small scale dam experiments; (3) Ashida and Michiue formula overestimates the erosion rate which results in an overestimation of peak discharge but predicts the time to peak fairly well, however Meyer-Peter and Müller and Modified Meyer-Peter and Müller formulae almost give the same results of dam, and water surface levels, and the peak discharge value and the time to peak; (4) A number of non-dimensional equations are presented to relate different model variables to the peak discharge of downstream hydrograph. A sensitivity analysis of various model parameters indicates that the most dominant factor affecting the peak discharge is the upstream reservoir volume, while the sediment grain size shows very little effect on the peak discharge.
Figure 3.1: Experimental setup (all dimensions are in meters)
Figure 3.2: Dam surface at various times using different numerical models

(a) $t=10$ s

(b) $t=15$ s

(c) $t=25$ s
Figure 3.2: Dam surface at various times using different numerical models (continued)
Figure 3.3: Water surface at various times using different numerical models

(a) t=10 s

(b) t=15 s

(c) t=25 s
Figure 3.3: Water surface at various times using different numerical models (continued)
Figure 3.4: Downstream hydrograph using different numerical models
Figure 3.5: Dam surface at various times using different sediment transport formulae
Figure 3.5: Dam surface at various times using different sediment transport formulae (continued)
Figure 3.6: Water surface at various times using different sediment transport formulae.
Figure 3.6: Water surface at various times using different sediment transport formulae (continued)
Figure 3.7: Downstream hydrograph using different sediment transport formulae
Figure 3.8: Effect of embankment top width on the peak discharge

Figure 3.9: Effect of reservoir volume on the peak discharge

Figure 3.10: Effect of inlet discharge on the peak discharge
Figure 3.11: Effect of percentage change of the variable on the peak discharge
CHAPTER 4

ESTIMATION OF OUTFLOW DISCHARGE THROUGH
DIFFERENT LEVEE BREACH SIZES USING ONE- AND
TWO-DIMENSIONAL MODELS

4.1 INTRODUCTION

The unsteady part of a levee breach is usually short, and most of the damage occurs during the steady-state part. In the present work, a generalized approach is used to study the steady-state levee breach flow by considering the lateral outflow through a levee breach as lateral outflow over a broad-crested side weir. Most of the previous studies predicted the flow over side weirs by applying the energy principle in the longitudinal direction of the main channel and most of the developed equations are in terms of the local flow variables near the side weir. In this section, a numerical model is developed based on the two-dimensional, depth-averaged equations to simulate the flow field resulting from a levee breach in a rectangular channel. The numerical results are compared with the experimental measurements by Riahi-Nezhad (2013). A parametric study is conducted by varying the breach dimensions, inlet discharge, and downstream submergence. Then, the numerical results are compared with the results of a simple one-dimensional approach (Hager, 1987). A non-dimensional equation is developed for the discharge correction factor so that one-dimensional model results are of similar accuracy as the two-dimensional models.
4.2 Experimental setup

The experiments were conducted in a rectangular flume with a side breach (Riahi-Nezhad, 2013). The experimental setup consisted of a wooden rectangular flume, 11.9 m long, 0.3 m deep, and 0.61 m wide, and horizontal bottom. The breach width is 0.61 m, and the flood plain is 2.96 m wide by 6.1 m long and connected to the left bank of the main channel, as shown in Fig. 4.1. The main channel, the breach area, and the flood plain are built on a raised platform to allow a free fall from the flooded area. The water is supplied to the main channel from an axial pump with a constant discharge of \( Q = 0.06 \text{ m}^3/\text{s} \). The discharge is measured by an electromagnetic flow meter on the delivery side of the pump. The turbulence at the channel inlet is reduced by using a honey comb, followed by a flow straightener. The water surface fluctuations are also reduced by using a wave suppressor. The downstream water depth is kept constant at 0.15 m by a sluice gate during the experiment. A measurement grid is marked on the bottom of the model, with a grid spacing of 0.152 m in each direction. The depths and flow velocities were recorded from Y1 to Y3 and from X16 to X27, as shown in Fig. 4.1.

4.3 Numerical model description and solution procedure

**Governing equations**

The two-dimensional flow equations may be written in the vector form as (Chaudhry, 2008)

\[
U_t + E_x + F_y + S = 0
\]  
(4.1)

in which
\[
U = \begin{pmatrix} h \\ q_x \\ q_y \end{pmatrix}; \quad E = \begin{pmatrix} q_x \\ \frac{q_x^2}{h} + \frac{1}{2}gh^2 \\ \frac{q_x q_y}{h} \end{pmatrix} \\
F = \begin{pmatrix} q_y \\ \frac{q_x q_y}{h} \\ \frac{q_y^2}{h} + \frac{1}{2}gh^2 \end{pmatrix}; \quad S = \begin{pmatrix} 0 \\ -gh(S_{ox} - S_{fx}) \\ -gh(S_{oy} - S_{fy}) \end{pmatrix}
\]

where \( t = \) time, \( x = \) streamwise coordinate, \( y = \) transverse coordinate, \( h = \) flow depth, \( q_x = \) discharge per unit width in the streamwise direction, \( q_y = \) discharge per unit width in the transverse direction, \( g = \) gravitational acceleration, \( S_{ox} = \) bed slope in the streamwise direction, \( S_{oy} = \) bed slope in the transverse direction, \( S_{fx} = \) friction slope in the streamwise direction, and \( S_{fy} = \) friction slope in the transverse direction.

By using the Manning equation for a wide, rectangular channel, the friction slope \( S_{fx} \) and \( S_{fy} \) may be written as

\[
S_{fx} = \frac{n^2 q_x \sqrt{q_x^2 + q_y^2}}{h^{10/3}} \quad (4.2)
\]
\[
S_{fy} = \frac{n^2 q_y \sqrt{q_x^2 + q_y^2}}{h^{10/3}} \quad (4.3)
\]

**Numerical scheme**

The two-dimensional, depth-averaged flow equations are solved using an explicit, finite-difference scheme comprising of a two-step predictor-corrector steps (MacCormack, 1969; Chaudhry, 2008). The scheme is second-order accurate in time and space. The computational domain is divided into equally spaced nodes. The governing partial differential equations are represented by the corresponding finite-difference
equations. The computational time step is selected to satisfy the Courant-Friedrichs-Lewy stability condition as Sanders et al. (2008).

**Discretization**

The discretization of the partial derivative terms for its equivalent finite-difference approximations are:

Predictor step (forward finite difference)

\[
\frac{\partial F}{\partial t}_{i,j} = \frac{F^*_{i,j} - F^k_{i,j}}{\Delta t} \tag{4.4}
\]

\[
\frac{\partial F}{\partial x}_{i,j} = \frac{F^k_{i+1,j} - F^k_{i,j}}{\Delta x} \tag{4.5}
\]

\[
\frac{\partial F}{\partial y}_{i,j} = \frac{F^k_{i,j+1} - F^k_{i,j}}{\Delta y} \tag{4.6}
\]

\[
F|_{i,j} = F^k_{i,j} \tag{4.7}
\]

Corrector step (backward finite difference)

\[
\frac{\partial F}{\partial t}_{i,j} = \frac{F^{**}_{i,j} - F^k_{i,j}}{\Delta t} \tag{4.8}
\]

\[
\frac{\partial F}{\partial x}_{i,j} = \frac{F^*_i - F^*_i_{-1,j}}{\Delta x} \tag{4.9}
\]

\[
\frac{\partial F}{\partial y}_{i,j} = \frac{F^*_i_{j} - F^*_i_{j-1}}{\Delta y} \tag{4.10}
\]

\[
F|_{i,j} = F^*_i_{j} \tag{4.11}
\]
Final value

\[
F_{i,j}^{k+1} = \frac{F_{i,j}^* + F_{i,j}^{**}}{2}
\]

(4.12)

where \( F \) represents a flow variable, i.e., \( h \), \( q_x \), and \( q_y \). Subscripts \( i \) and \( j \) indicate the node number, and superscripts \( k \), \( * \), and \( k + 1 \) indicate the values of the variable at the known time step, predictor step, and unknown time step, respectively.

**Boundary conditions**

The discretization scheme discussed previously is applied to the interior nodes, i.e., for \( i = 2 \) through \( i_{\text{end}x} - 1 \), and for \( j = 2 \) through \( i_{\text{end}y} - 1 \), where \( i_{\text{end}x} \), and \( i_{\text{end}y} \) are the last nodes in the computational domain in the x and y directions, respectively. The boundary conditions may be divided into two types (Kassem, 1996): open boundaries, such as upstream, downstream of the main channel, and the floodplain outlets; and solid boundaries, such as the main channel walls. For the open boundaries, since the flow at the upstream boundary is subcritical, \( h \) is extrapolated from the interior nodes, and \( q_x \) and \( q_y \) are specified equal to constant values. The flow at the downstream boundary of the main channel is subcritical, thus \( h \) is specified, and \( q_x \) and \( q_y \) are extrapolated from the interior nodes. The boundary condition at the flow outlets of the floodplain is free outflow. Thus it is governed by the type of flow: For subcritical flow, \( h \) is specified equal to the critical depth and \( q_x \), and \( q_y \) are extrapolated from the interior nodes; for supercritical flow, however, \( h \), \( q_x \), and \( q_y \) are all extrapolated from the interior nodes. For the solid boundary, a free-slip boundary condition is imposed, \( h \) is extrapolated from the adjacent nodes and the flux adjacent to the wall is extrapolated from the interior nodes while the flux normal to the wall is set equal to zero.
Stability conditions

The Courant-Friedrichs-Lewy stability condition is following (Sanders et al., 2008)

$$\Delta t = \frac{C_n}{\left| \frac{q_x}{h} + \sqrt{gh} \right| \Delta x + \left| \frac{q_y}{h} + \sqrt{gh} \right| \Delta y}$$  \hfill (4.13)

In the present work, $C_n \leq 1.0$ is used.

Artificial viscosity

It has the same definition as the previous chapter, however the two-dimensional version is adopted here as follow:

$$U_{i,j}^{k+1} = U_{i,j}^{k+1} + D_x U_{i,j}^{k+1} + D_y U_{i,j}^{k+1}$$  \hfill (4.14)

in which

$$D_x U_{i,j} = [\epsilon_{x_{i+\frac{1}{2},j}} (U_{i+1,j} - U_{i,j}) - \epsilon_{x_{i-\frac{1}{2},j}} (U_{i,j} - U_{i-1,j})]$$  \hfill (4.15)

$$\epsilon_{x_{i-\frac{1}{2},j}} = \frac{\Delta t}{\Delta x} \max(v_{x_{i-1,j}}, v_{x_{i,j}})$$  \hfill (4.16)

$$v_{x_{i,j}} = \frac{|h_{i+1,j} - 2h_{i,j} + h_{i-1,j}|}{|h_{i+1,j}| + |2h_{i,j}| + |h_{i-1,j}|}$$  \hfill (4.17)

$$D_y U_{i,j} = [\epsilon_{y_{i,j+\frac{1}{2}}} (U_{i,j+1} - U_{i,j}) - \epsilon_{y_{i,j-\frac{1}{2}}} (U_{i,j} - U_{i,j-1})]$$  \hfill (4.18)

$$\epsilon_{y_{i,j-\frac{1}{2}}} = \frac{\Delta t}{\Delta y} \max(v_{y_{i,j-1}}, v_{y_{i,j}})$$  \hfill (4.19)

$$v_{y_{i,j}} = \frac{|h_{i,j+1} - 2h_{i,j} + h_{i,j-1}|}{|h_{i,j+1}| + |2h_{i,j}| + |h_{i,j-1}|}$$  \hfill (4.20)
where subscripts $i$ and $j$ indicate the node number, and superscripts $k$ and $k + 1$ indicate the values of the variable at the known time step and unknown time step, respectively. $D_x$ and $D_y$ = dissipative operator in the $x$ and $y$ directions, respectively, $\epsilon_{x_{i+\frac{1}{2},j}}$ and $\epsilon_{y_{i,j+\frac{1}{2}}}$ = normalized parameter of water depth in the $x$ and $y$ direction respectively, and $v_{x_{i,j}}$ and $v_{y_{i,j}}$ = gradient of the water depth in the $x$ and $y$ directions, respectively.

4.4 Model application

The numerical solution of the depth-averaged flow equations is applied to the experimental model. Grid independent results are achieved using a square orthogonal grid with a mesh size equal to 0.01 m. The initial conditions are set by specifying a constant water depth at rest in the main channel and a dry floodplain. Only the steady state results are compared, the numerical model is executed for 300 s to ensure that the solution converged to the steady state.

Comparison between the numerical and the experimental results

The numerical and the experimental results are compared in Fig. 4.2. The breach location is represented by dotted vertical lines. The main trend of the water surface is captured satisfactorily by the numerical model (Figs. 4.2a, b and c) with an average RMSE = 0.005. The simulated depth-averaged streamwise velocity is in good agreement with the experimental values (Figs. 4.2d, e and f) with little overestimation at the end of section Y1 (Fig. 4.2 d). This is due to the reflection from the downstream sluice gate. The transverse velocity is predicted well by the numerical
model (Figs. 4.2g, h and i) with an average RMSE = 0.05. Overall, the numerical and the experimental results compare satisfactorily.

**Parametric study**

The lateral flow through a levee breach is approximated as a flow over a rectangular, broad-crested side weir. Since the maximum depth within the breach is only about 0.15 m, it is not sufficient to test a variety of breach depths. Accordingly, the parametric study is conducted on a 1:5 scale model of the experimental set-up following Froude similitude. The cases tested with the numerical model are shown in Table 4.1. The downstream submergence ratio \( S_r \) is defined as

\[
S_r = \frac{h_{d.s} - h_b}{h_{d.s}}
\]  

where \( h_{d.s} \) is the downstream water depth which is constant for all cases and is equal to 0.75 m, and \( h_b \) is the breach crest height as indicated in Table 4.1.

Figures 4.3 and 4.4 show different flow variables for different values of inlet discharge \( Q_i \) for two different configurations of the levee breach. The zero value on the abscissa represents the inner side of the right wall of the main channel. Contrary to the previous studies which dealt with flow through a side weir by assuming a unidirectional flow in the main channel, the figures show a strong gradient of the flow variables which emphasize the need to include the two-dimensional effect when calculating the lateral discharge through a levee breach.
Comparison between the one- and two-dimensional models results

The one-dimensional model is developed to solve the energy equation. The solution is divided into two parts: The first part is adjacent to the breach, and the second part starts from the end of the breach and extends to the downstream section of the main channel. The grid spacing for both parts is set equal to 0.001 m. For the first part, the spatially varied flow with decreasing discharge (S.V.F) (Chow, 1959) (Eq. 4.22) is solved

\[
\frac{dh}{dx} = \frac{S_0 - S_f - Qq_b / gA^2}{1 - Q^2 / gA^2D} \tag{4.22}
\]

where \( dh/dx \) is the water depth gradient along the breach, \( S_0 \) is the bed slope and is equal to zero for a horizontal bed, \( S_f \) is the friction slope and is evaluated using Manning’s formula, \( Q \) is the main channel discharge which is decreasing along the breach, \( q_b \) is the lateral outflow intensity per unit width through the breach, \( A \) is the area across the breach, and \( D \) is the hydraulic mean depth across the breach. \( q_b \) is evaluated using a one-dimensional approach as Hager (1987)

\[
q_b = \frac{3}{5} n^* c \sqrt{gH^3(y - W)^{3/2}} \left[ \frac{1 - W}{3 - 2y - W} \right]^{1/2} \tag{4.23}
\]

\[
c = 1 - \frac{2}{9(1 + \zeta_b^4)} \tag{4.24}
\]

\[
\zeta_b = \frac{H - w}{L} \tag{4.25}
\]

where \( n^* \) is the number of outflow sides and is equal to 1, \( c \) is the breach shape factor, \( \zeta_b \) is the relative breach length, \( H \) is the energy head, \( w \) is the breach height, \( L \) is the crest thickness, \( y \) is the relative flow depth and is equal to \( h/H \), and \( W \) is the relative breach height and is equal to \( w/H \).
For the second part, the gradually varied flow (G.V.F) (Chow, 1959) (Eq. 4.26) is solved which is a special case of (Eq. 4.22) by substituting $q_b = 0$.

\[
\frac{dh}{dx} = \frac{S_0 - S_f}{1 - Q^2 / gA^2 D}
\]  

(4.26)

Figure 4.5(a) shows the effect of $S_r$ on the relative flow depth ($h/h_{d,s}$) along the breach using the one- and two-dimensional models. The predicted $h/h_{d,s}$ using the one-dimensional model is almost linear along the breach, although the two-dimensional model could capture the curvature of the water surface at the inner side of the breach which results from the wake zone developed at the upstream side of the breach. This curvature increases as $S_r$ increases. Figure 4.5(b) shows the effect of $S_r$ on the ratio of the unit discharge to the unit inlet discharge ($q_b/q_i$) along the breach using the one- and two-dimensional models, and it is noticed that $q_b/q_i$ is overestimated using the one-dimensional model since the model is not capable of capturing the water surface curvature in the inner side of the breach. Generally, Figure 4.5 shows that the two-dimensional effect increases with increasing $S_r$. This has been confirmed by Hager (1987) where the lateral outflow angle increases with decreasing depth at the outflow side. Also, as shown from the figure, $q_b/q_i$ decreases with decreasing $S_r$, and theoretically it vanishes for $S_r = 0.0$.

Figure 4.6 compares the flow variables within the breach from the one- and two-dimensional models for different ratios of the breach width to the channel width ($b_r$) which equals $b_b/b_c$. Three different values of $b_b$ are chosen for that purpose (0.2 m, 0.6 m, and 0.8 m). It is clear that for small values of $b_b$ the distribution of $q_b$ within the breach tends to be almost uniform since the water depth gradient within the breach is small, although as $b_r$ increases the curvature of the water surface in the inner side of the breach increases. Comparison of Figs 4.6(a) and 4.5(a) shows that the effect of $S_r$ on $h/h_{d,s}$ is more than that of $b_r$. 

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Figure 4.7 shows non-dimensional plots of the ratio of the total breach discharge to the inlet discharge \( (Q_b / Q_i) \) versus \( b_r \) for different \( S_r \) and different \( Q_i \). It is found that as \( Q_i \) increases, \( Q_b \) decreases. This may be due to the increase of inertia of the water to bypass the breached section.

Figure 4.8 shows the discharge correction factor, \( c_r \), which is equal the ratio of \( Q_b \) predicted using the two-dimensional model to that predicted using the one-dimensional model. As indicated before, \( c_r \) decreases as \( S_r \) increases, indicating more overestimation by the one-dimensional model as compared to the two-dimensional model. Note that, for Froude number approaching unity, the one-dimensional model fails to capture the water surface along the breach because of the formation of a hydraulic jump along the breach. A non-dimensional equation (Eq. 4.27) is derived using regression analysis for the estimation of \( c_r \) as

\[
c_r = 0.626 \quad S_r^{-0.115} \quad b_r^{0.081} \quad F_i^{-0.371}
\]

(4.27)

where, \( F_i \) is the inlet Froude number significantly upstream of the breached area. Figure 4.9 shows a good correlation of \( R^2 = 0.93 \) between the fitted Eq. 4.27 and \( c_r \) calculated from the simulation of the one- and the two-dimensional models. From the equation, it is noted that \( c_r \) is more dependent on the Froude number and submergence ratio than on the relative breach width.

4.5 Summary and conclusions

A generalized case of levee breach on a rectangular channel is studied. A numerical model is developed to solve the two-dimensional shallow-water equations using finite-difference schemes. The numerical results are compared with the experimental
results. Different cases are simulated for different breach width, breach level, and inlet discharge. The submergence ratio has a major effect in changing the water surface elevation along the breach. The total outflow discharge through the breach decreases as the inlet discharge increases. The simulated lateral outflows through the breach are compared with a one-dimensional model. The one-dimensional model overestimates the breach outflows. A new non-dimensional equation is developed to calculate the discharge correction factor for the breach discharge estimated using the one-dimensional model.
Table 4.1: Range of model variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Number of cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Breach width (m) ( (b_b) )</td>
<td>0.20, 0.40, ......., 2.80, 3.05</td>
<td>15</td>
</tr>
<tr>
<td>• D.S submergence ratio ( (S_r) )</td>
<td>1.00, 0.87, 0.73, 0.60, 0.47</td>
<td>5</td>
</tr>
<tr>
<td>or Breach crest height ( (h_b) )(m)</td>
<td>0.0, 0.1, 0.2, 0.3, 0.4</td>
<td></td>
</tr>
<tr>
<td>• ( Q_i ) (m(^3)/s)</td>
<td>3.20, 3.35, .......,3.80, 3.95</td>
<td>6</td>
</tr>
<tr>
<td>• Approaching Froude number at the center of the main channel ( (F_r) )</td>
<td>0.48-0.94</td>
<td></td>
</tr>
<tr>
<td>• Total number of runs</td>
<td></td>
<td>450</td>
</tr>
</tbody>
</table>
Figure 4.1: Experimental model of a levee breach of a rectangular channel (Riahi-Nezhad, 2013) (not to scale, all dimensions in meters)
Figure 4.2: Comparison between numerical (dashed line) and measured (dots) results
Figure 4.2: Comparison between numerical (dashed line) and measured (dots) results (continued)
Figure 4.2: Comparison between numerical (dashed line) and measured (dots) results (continued)
Figure 4.3: Flow variables across the main channel at the breach upstream section for $S_r$ and $b_b = 0.73, 1.6$ m, respectively
Figure 4.4: Flow variables across the main channel at the breach upstream section for $S_r$ and $b_b = 1.0, 3.05$ m, respectively
Figure 4.5: Effect of $S_r$ on $h/h_{d,s}$ and $q_b/q_i$ for ($Q_i = 3.2$ $m^3/s$ and $b_b = 1.6$ m)
Figure 4.6: Effect of $b_r$ on $h/h_{d,s}$ and $q_b/q_i$ for ($Q_i = 3.2 \, m^3/s$ and $S_r = 0.6$)
Figure 4.7: Normalized $Q_b$ for different cases

(a) $Q_i=3.20 \text{ m}^3/\text{s}$

(b) $Q_i=3.35 \text{ m}^3/\text{s}$

(c) $Q_i=3.50 \text{ m}^3/\text{s}$
Figure 4.7: Normalized $Q_b$ for different cases
(continued)
Figure 4.8: $c_r$ for different cases
Figure 4.8: $c_r$ for different cases (continued)
Figure 4.9: $c_r$ (fitted equation) vs $c_r$ (simulated)
5.1 Introduction

An accurate prediction of the evolution of the breach resulting from an overtopping failure of non-cohesive earthen levees is important because it may be helpful in flood-mitigation studies. Laboratory experiments are conducted with various inlet discharges and downstream water depth. The breach shape evolution was recorded using sliding rods technique (Tabrizi et al., 2015). A sequence of discrete mass failure of the breach sides due to slope instability are observed during the failure process. A simple geostatic failure mechanism is suggested to calculate the lateral load due to this mass failure. This mechanism is implemented in a two-dimensional numerical model which solves the shallow water equations along with the sediment mass conservation. In order to assess the effect of the lateral load resulting from slope instability on the failure process, the predicted breach shape evolution, and breach hydrograph with and without the slope failure mechanism are compared with the experimental results.
5.2 Experimental setup

A number of experiments are conducted in a channel with a half-trapezoidal cross section in the Hydraulics Laboratory of the University of South Carolina. Figure 5.1 shows a schematic diagram of the plan view of the physical model test facility comprising a flume and a floodplain adjacent to the left side of the flume. The flume is constructed with vertical plywood walls on the right side and levee-shaped wall on the left side, except for a 0.70 m long centrally located section where the breach is located. The levee cross section has a trapezoidal shape with 0.20 m height, 2:1 (H:V) side slopes, and 0.10 m crest width. A flow straightener, a honeycomb, and a wave suppressor are used at the intake section of the flume to reduce the inflow turbulence. The water is supplied to the main channel from an axial pump with a constant discharge. The downstream water depth is adjusted using a sharp-crested weir. Two sets of experiments are conducted on the same setup. The first set is used to test the ability of the numerical model to predict the flow field of a fully-developed levee breach in a trapezoidal channel. The pump discharge is kept constant at 0.047 m$^3$/s. The downstream water depth is kept constant at 0.10 m. The water depth near the breach is measured using a Baumer ultrasonic distance measuring sensor with a scanning range between 0.06 and 0.4 m. The Baumer is mounted on a movable bridge. The Baumer can pass over the water surface of the channel, starts from the right wall of the main channel and ends at the end of the breach. The measurements of the water depth start 1 m upstream of the breach and repeated every 0.152 m and ended at 1 m downstream of the breach as shown in Fig. 5.1. The water surface velocity is measured using Particle Image Velocimetry. High definition video recording is performed from the top, and a large number of small floating particles are added to the upstream section of the flume. The PIVLab Matlab tool (Thielicke, 2014; Thielicke, W. and Stamhuis, E.J., 2014a,b) is used for post-test analysis.
The second set of experiments (Tabrizi et al., 2015) is conducted while the breach initially filled with a medium sand of uniform grain size 0.6 mm except a pilot channel 10 cm width and 5 cm depth located at the first third of crest of the earthen levee to initiate the overtopping, as shown in Fig. 5.2. The breach shape evolution is recorded using sliding rod technique (Tabrizi et al., 2015), and breach hydrograph is calculated by subtracting the downstream hydrograph from the constant inflow discharge. The downstream discharge is measured using calibrated weir, and the depth is recorded using a fixed Baumer.

5.3 Numerical model description

Governing equations

The governing equations are divided into two major parts:

Hydrodynamic equations

The hydrodynamic equation are the same as Eq. 4.1.

Sediment equation

The two-dimensional form of Exner equation for bedload only may be written as

\[(1 - \lambda_p) \frac{\partial z}{\partial t} = - \frac{\partial q_{bx}}{\partial x} - \frac{\partial q_{by}}{\partial y} - \frac{\partial q_{sf}}{\partial x} \]

where \(\lambda_p\) = bed porosity (equal to 0.43), \(q_{bx}\) and \(q_{by}\) are the sediment transport capacity in the \(x\) and \(y\) directions, respectively, and \(q_{sf}\) = lateral sediment inflow due to slope failure which will be discussed in the next section. Meyer-Peter and Müller formula (MPM) is used to evaluate \(q_b\) in the \(x\) and \(y\) directions as

\[q_b^* = \alpha_b(\tau^* - \tau_c^*)^{n_t}\]
Geostatic failure mechanism

Spinewine et al. (2002) proposed a simple failure criterion for discrete slumps of bank material into a stream resulting from dam-break flow. In the present study, this mechanism is adapted to simulate the slope failure occurring due to water flow through the breach. The approach mainly depends on the slope stability of the breach sides. The soil used in the present experiment is uniform medium sand with a diameter $D = 0.6$ mm. Spinewine et al. (2002) differentiated between the repose angle values of submerged (under the water surface) and emerged cells (above the water surface). They stated that usually the repose angle of emerged cells is steeper than the repose angle of the submerged cells due to the apparent cohesion for the emerged cells.

Wu et al. (2009) developed a two-dimensional, depth-averaged model to simulate the non-cohesive earthen dam breach due to overtopping. They used a soil with median diameter of 0.25 mm, and the repose angle of the submerged cells, $\phi_s = 33^\circ$ and of the emerged cells, $\phi_e = 79^\circ$. Also as reported by Spinewine et al. (2002) and Swartenbroekx et al. (2010), for a coarse sand with uniform diameter of 1.8 mm, $\phi_s$ is equal to 35$^\circ$ and for the emerged cells $\phi_e$ is 87$^\circ$. Therefore assuming a linear interpolation, we can estimate $\phi_s$ and $\phi_e$ for the soil used in this study. This gives $\phi_s$ equal to 33.5$^\circ$ and $\phi_e$ equal to 81$^\circ$.

Figure 5.3 shows a cross section of the breach having three slopes in the submerged zone and one slope above the water with. Then $q_{sf}$ for each reach can be calculated from

\[
q_{sf} = \frac{\Delta A (1 - e^{-\Delta t / T_b})}{\Delta t}
\]

where $\Delta t$ is the computational time step selected to satisfy the Courant-Friedrichs-Lewy stability condition (Eq. 4.13) (Sanders et al., 2008), and $T_b$ is the adaptation time which reflects the time the unstable slope takes to slump downward. $T_b = 2.0$ s
gave the best agreement with the experimental measurements.

**Numerical scheme**

The numerical scheme presented in chapter 4 is used to discretize the hydrodynamic equations. Only the discretization of Exner equation (Eq. 5.1) is described in this section.

**Predictor step (Forward finite difference)**

\[
\frac{\partial z}{\partial t} \bigg|_{i,j} = \frac{z_{i,j}^* - z_{i,j}^k}{\Delta t} \tag{5.4}
\]

**Corrector step (Backward finite difference)**

\[
\frac{\partial z}{\partial t} \bigg|_{i,j} = \frac{z_{i,j}^{**} - z_{i,j}^k}{\Delta t} \tag{5.5}
\]

**Final value**

\[
z_{i,j}^{k+1} = \frac{z_{i,j}^* + z_{i,j}^{**}}{2} \tag{5.6}
\]

**Boundary conditions**

The boundary conditions are treated in the same manner as in the previous chapter. Figure 5.4 shows different computational blocks of the model, and the boundary condition of each block. All the blocks are considered non-erodible when the model is tested for steady flow through a fully developed levee breach, while the blocks of the breach and floodplain zone are considered erodible when the model is tested for a gradual failure of the earthen levee.
Artificial viscosity

The artificial viscosity technique is added at the end of each time step in the same manner as in the previous chapter.

Stability conditions

The numerical stability is achieved by applying Courant-Friedrichs-Lewy stability condition as in the previous chapter.

5.4 Model application

Steady flow through fully developed levee breach in a trapezoidal channel

Only the hydrodynamic equations are modeled to validate the model with the experimental setup (Fig. 5.1). The sections from (Y1 to Y5) are in the main channel area, while sections from (Y6 to Y10) are in the breach area as shown in Fig. 5.1. The simulated and measured water levels at different cross-sections from (Y1 to Y10) compare satisfactorily, as shown in Fig. 5.5. The average RMSE for the sections (Y1-Y5) is 0.005, however the average RMSE for the sections (Y6-Y10) is 0.009. The simulated depth-averaged velocity is compared with the measured surface velocity, as shown in Fig. 5.6. The simulated velocity within the upstream area of the breach compares well with the measured velocity indicating that the average velocity is within 0.75 m/s, while in the downstream area of the breach, a recirculation zone adjacent to the right wall is observed but not captured by the numerical model. This recirculation zone may be present due to reflection from the downstream boundary. In the breach area, the simulated velocity agrees well with the measurements, and the average velocity value is 1.15 m/s. In the floodplain, the numerical model underestimates the
velocity and this is due to the nature of the flow in this area which has shallow depths with high velocity gradients which requires three-dimensional modeling.

**Unsteady gradual failure of earthen levee**

The hydrodynamic equations are solved along with the Exner conservation equation in order to capture the gradual failure of the earthen levee. The flow is trigged to overtop the dam by creating a pilot channel in the first third of the crest, as shown in Fig. 5.2. The breach shape evolution is measured using a sliding rod technique (Tabrizi et al., 2015). The predicted breach shape are compared with the experimental results. The main parameters affecting the failure evolution, and the breach hydrograph are the coefficients of MPM equation ($\alpha_{bx}$ and $\alpha_{by}$), and $q_{sf}$. Table 5.1 shows five different scenarios for these parameters to assess their effects on the levee failure process. Figures 5.7 and 5.8 show the comparison between the five different scenarios with the measured levels during the failure. Figure 5.9 shows the maximum eroded depth within the breach for the five different scenarios. It is observed that, scenario 5 gives better agreement between the predicted and the measured results in terms of breach width and depth. Figure 5.10 shows the breach hydrograph for the five different scenarios. It is noticed that, scenario 5 captures the early stages of the failure better than other scenarios.

5.5 **Summary and conclusions**

A two-dimensional depth-averaged model is developed to study the levee breach. The model is first validated against a fully developed levee breach on a fixed bed. The water depth in the main channel and within the breach are in a good agreement with the measured results, however the velocity in the flood plain is underestimated due to the nature of the flow which tends to be three-dimensional. Also, the hydrodynamic
equations are solved along with the Exner equation to simulate the gradual failure of the earthen levee. The lateral sediment load resulting from the mass failure of the breach sides is added to the Exner equation. By adding this load, the prediction of the breach shape evolution and the breach hydrograph is improved.
Table 5.1: Failure process parameters

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\alpha_{bx}$</th>
<th>$\alpha_{by}$</th>
<th>$q_{sf}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
<td>8.0</td>
<td>8.0</td>
<td>Excluded</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>18.0</td>
<td>8.0</td>
<td>Excluded</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>8.0</td>
<td>18.0</td>
<td>Excluded</td>
</tr>
<tr>
<td>Scenario 4</td>
<td>8.0</td>
<td>18.0</td>
<td>Included</td>
</tr>
<tr>
<td>Scenario 5</td>
<td>0.0</td>
<td>18.0</td>
<td>Included</td>
</tr>
</tbody>
</table>
Figure 5.1: Experimental model of fully developed levee breach (not to scale, all dimensions in meters)
Figure 5.2: Experimental model with earthen levee (not to scale, all dimensions in meters)
Figure 5.3: Geostatic failure mechanism
22 boundaries
w wall
i inlet
o outlet
f free outfall

12 blocks
erosion & deposition permitted

Figure 5.4: Solution procedure
Figure 5.5: Comparison between numerical and measured water depth
Figure 5.5: Comparison between numerical and measured water depth (continued)
Figure 5.5: Comparison between numerical and measured water (continued)
Figure 5.5: Comparison between numerical and measured water depth
(continued)
Figure 5.6: Comparison between numerical and measured surface velocity
Figure 5.6: Comparison between numerical and measured surface velocity (continued)
Figure 5.7: Comparison between numerical and measured breach shape (scenarios 1,2,3)
Figure 5.7: Comparison between numerical and measured breach shape (scenarios 1,2,3) (continued)
Figure 5.7: Comparison between numerical and measured breach shape (scenarios 1, 2, 3) (continued)

(g) t=35 s
Figure 5.8: Comparison between numerical and measured breach shape (scenarios 4, 5)
Figure 5.8: Comparison between numerical and measured breach shape (scenarios 4,5) (continued)
Figure 5.8: Comparison between numerical and measured breach shape (scenarios 4, 5)
(continued)

Figure 5.9: Maximum breach depth with time (scenarios 1, 2, 3, 4, 5)
Figure 5.10: Breach hydrograph(scenarios 1,2,3,4,5)
Chapter 6

Summary and Conclusions

6.1 Summary and conclusions

Boussinesq equations and Saint-Venant equations modified for steep bed slope are numerically solved to investigate the effects of non-hydrostatic pressure distribution and steep slope on the breach evolution and the downstream hydrograph of a breach in a non-cohesive earthen embankment. The Exner equation for the sediment mass conservation is solved using both sets of equations. Boussinesq equations are solved by the Gottlieb and Turkel explicit, finite-difference scheme which is second order accurate in time and fourth order accurate in space. The Saint-Venant equations modified for steep bed slope are solved by the MacCormack explicit finite-difference scheme which is second order accurate in time and space. An artificial viscosity technique is used to smooth the spurious oscillations around the bores for both models. Three different sediment transport equations are tested: Ashida and Michiue, Meyer-Peter and Müller, and Modified Meyer-Peter and Müller for steep slopes. The results of a parametric study are presented to show the effect of various model variables on the peak discharge of the downstream hydrograph.

A comparison of the numerical and measured results shows that: (1) The Boussinesq terms have little effect on the prediction of the temporal failure evolution and the downstream hydrograph; (2) The correction of Saint-Venant equation for steep bed slope has a marginal effect on the dam and water surface levels for small scale dam experiments; (3) Ashida and Michiue transport equation overestimates the ero-
sion rate which results in an overestimation of peak discharge but predicts the time
to peak fairly well, however Meyer-Peter and Müller and Modified Meyer-Peter and
Müller equations give almost the same results of dam and water surface levels, and
the peak discharge, and time to peak of the downstream hydrograph; (4) A number
of non-dimensional equations are presented to relate different model variables to the
peak discharge of the downstream hydrograph. A sensitivity analysis of various model
parameters indicates that the most dominant factor affecting the peak discharge is
the upstream reservoir volume, while the sediment grain size shows very little effect
on the peak discharge.

A generalized case of levee breach through a rectangular channel is studied. A
numerical model is developed to solve the two-dimensional shallow-water equations
using finite difference approximation. The numerical results are compared with exper-
imental results. Different cases are simulated by varying breach width, breach level,
and inlet discharge. The submergence ratio has a major effect in changing the water
surface elevation along the breach. The total outflow discharge through the breach
decreases as the inlet discharge increases. The simulated lateral outflows through the
breach are compared with a previous one-dimensional model. The one-dimensional
model overestimates the breach outflows. A new non-dimensional equation is devel-
oped to calculate the discharge correction factor which corrects the breach discharge
estimated using the one-dimensional model.

The two-dimensional depth-averaged model successfully simulates the flow field
in the case of a fully developed levee breach on a fixed bed of a trapezoidal channel.
The water depth in the main channel and within the breach is captured satisfactorily
as compared to experimental measurements. Also, the simulated velocity is in good
agreement with the measured surface velocity, except in the flood plain which the flow
tends to be three-dimensional. The depth-averaged flow equations are solved along with the Exner equation to simulate the gradual failure of a non-cohesive earthen levee. The maximum depth of erosion within the breach is overestimated. The predicted maximum depth of erosion is improved when the lateral sediment inflow due to slope failure is included in the Exner equation.

6.2 RECOMMENDATIONS FOR FUTURE INVESTIGATIONS

The following future investigations are recommended:

- The effects of Boussinesq terms and the steep-slope correction may be tested on large scale experiments with higher inlet discharges and larger flow depths to assess their effects.

- For the earthen embankment failure, the results of the depth-averaged flow equations and the Navier-Stokes equations should be compared.

- For the levee breach, experiments may be conducted to obtain more data for different soil types to help determine the erodibility coefficients which are used in the numerical models.
BIBLIOGRAPHY


