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Geminal Model Chemistry II. Perturbative Corrections

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Vitaly A. Rassolov, Feng Xu, and Sophya Garashchuk

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A geminal model chemistry
I. INTRODUCTION

We seek to formulate and investigate a model chemistry that is applicable to multiconf igurational systems, computationally inexpensive, and chemically accurate. It is natural to base such a model on two-electron functions, or geminals. In the framework of single-electron basis sets geminals can be represented as linear combinations of two-electron determinants. The chemical models based on geminals were introduced into chemistry by Hurley, Lennard-Jones, and Pople, along with strong orthogonality approximation in order to make the model practical. The simplest variant of such a model represents wavefunctions as a single antisymmetrized product of strongly orthogonal geminals (APSG). Sometimes this model is referred to as separated electron pair (SEP) theory. Most prior studies focused on the total amount of correlation energy recovered by the product of strongly orthogonal geminals. It was generally concluded that the model has its limitations, primarily attributable to either lack of intergeminal correlation, strong orthogonality approximation, or both.

In Ref. 15 one of us has introduced the antisymmetrized product of singlet-type strongly orthogonal geminal model (SSG), which is a well-defined and size-consistent version of APSG. It is important for a chemical model to be both size consistent (the energy must be an extensive property and well defined) and size-consistent (free from adjustable parameters specific to the system under investigation). In the case of APSG the size-consistency requirement imposes a constraint on the quality of each geminal: The number of orbital pairs in each geminal must be the same in a molecule and in constituent atoms. The SSG model satisfies this constraint by optimization of the orbital pairs in each geminal. The geminals that describe fully broken bonds optimize into single determinants made of spin-unrestricted orbitals, ensuring size consistency. 

SSG is implemented in atomic orbital basis, with formal computational scaling of each optimization iteration similar to that of the Hartree–Fock model. The application of SSG was focused on modeling of chemical properties, using various diatomic molecules as test studies. It was found that the SSG model describes covalent bonds surprisingly well, and is deficient when bonds are formed between elements with extreme electronegativity. The primary reason for this deficiency was attributed to the absence of the intergeminal dispersion interaction in the APSG formulation. The SSG model recovers only 20%–30% of the correlation energy for many chemical bonds. To improve the accuracy of a geminal model, it is necessary to include missing electron–electron interactions. An active work in this direction is pursued by Surjan’s group focused on the development of general multiconfiguration perturbation theories, with APSG being a special case. Because of this, Surjan and collaborators use the projection operator techniques in the multideterminant space to define the reference Hamiltonian. Our goal is different. We analyze the strengths and shortcomings of the SSG model in terms of physical interactions that are accounted for or missing in the model. Therefore, in current work we define a reference Hamiltonian by including some two-electron interactions in addition to a one-particle mean-field Hamiltonian of the single reference theory.

Let us put the SSG model into perspective. The SSG reference wave function is similar to the generalized valence bond model with perfect pairing (GVB-PP). The main difference between the two models is that in the SSG the number of orbital pairs in each geminal, or “bond,” is not restricted and is determined variationally. In GVB it is limited to two orbital pairs for each valence electron pair and a single orbital pair for each pair of core electrons. Another, less significant distinction is that the SSG is formulated in either spin-restricted or spin-unrestricted forms, while the GVB is usually spin restricted.

In the context of multiconfigurational theories, the PP approximation assigns a pair of virtual orbitals to a single pair of occupied ones. This approximation is assumed to be
the most restrictive in the context of the GVB formalism. The full relaxation of the PP approximation leads to the expensive complete active space self-consistent field (CASSCF) model. The main theoretical developments had been focused on some form of intermediate approximations, such as GVB-RP (restricted pairing). A very promising alternative was developed by Van Voorhis and Head-Gordon, who use the coupled-cluster formalism to simplify the configuration space. They have formulated the GVB-RCC (restricted coupled cluster) model, and even less restrictive imperfect pairing model (IP).

The main drawback of these models is that for weakly correlated systems they are inferior to a simple Møller–Plesset perturbation theory (Møller–Plesset-PBS). These studies were pioneered by Pulay and Saebø in the context of Møller–Plesset perturbation theory, and further developed using the coupled-cluster formalism. The alternative localization schemes based on assignments of basis functions to individual nuclei were developed by Head-Gordon et al. The SSG theory bridges these two directions of research. The grouping of all orbitals into geminal subspaces is performed variationally, and is based on the strength of electron interactions between electrons on different orbitals. Arguably, this is the best way to subdivide the orbital space into classes. All electron interactions within each group are treated without approximations in the reference model. An important question that is addressed in the current work is the quality of such subdivision, and the nature of missing effects. In particular, it is important to compare the role of strong orthogonality (somewhat similar to perfect pairing in GVB) with the dispersion interactions that do not change identity of individual electron pairs.

II. THEORY

The SSG reference wave function is defined (assuming $n_{\alpha} \geq n_{\beta}$) as

$$\Psi_{SSG} = \hat{A}\left[ \psi_1(r_1, r_2) \cdots \psi_{n_{\beta}}(r_{2n_{\beta}-1}, r_{2n_{\beta}}) \right. $$

$$\left. \times \phi_1(r_{2n_{\beta}}) \cdots \phi_{n_{\beta}}(r_{2n_{\beta}+n_{\alpha}-1}, r_{2n_{\beta}}) \right],$$

$$\psi_\alpha(r_1, r_2) = \sum_{k \in A} \frac{D_k}{\sqrt{2}} \left[ \phi_\alpha(r_1) \tilde{\phi}_\beta(r_2) - \phi_\beta(r_1) \tilde{\phi}_\alpha(r_2) \right],$$

$$\phi_\alpha(r_1) = \sum_\lambda C_{\lambda \alpha} \chi_\lambda(r_1),$$

$$\tilde{\phi}_\alpha(r_1) = \sum_\lambda \tilde{C}_{\lambda \alpha} \chi_\lambda(r_1),$$

where $\hat{A}$ is the antisymmetrization operator. Molecular orbital (MO) coefficients $C_\alpha$, geminal expansion coefficients $D_\alpha$, and subspaces $A$ are variationally optimized to minimize the energy

$$E_{SSG} = \langle \Psi_{SSG}, \hat{H} | \Psi_{SSG} \rangle / \langle \Psi_{SSG} | \Psi_{SSG} \rangle,$$

evaluated with exact Hamiltonian $\hat{H}$. The optimization of the geminal subspace is the optimization of a number of MOs assigned to a given geminal. A further constraint $\tilde{C}_{\lambda \alpha}^\dagger = C_{\lambda \alpha}^\dagger$ for all MO coefficients yields a spin-restricted version of SSG, or RSSG. The spin-unrestricted form is labeled USSG.

For every geminal $A$ we define four one-electron density matrices $P_{\alpha A}$, $P_{\beta A}$, $P_{\alpha A}^T$, and $P_{\beta A}^T$ with elements

$$P_{\lambda,\alpha}^{A} = \sum_{\beta \in A} (D_{\alpha})^2 C_{\lambda \alpha}^\dagger C_{\alpha \beta},$$

$$P_{\lambda,\beta}^{A} = \sum_{\alpha \in A} (D_{\beta})^2 C_{\lambda \beta}^\dagger C_{\alpha \beta},$$

$$P_{\lambda,\alpha}^{T A} = \sum_{\alpha \in A} D_{\alpha} C_{\lambda \alpha} C_{\alpha \beta},$$

Note that the last cross-spin matrix $P_{\beta A}^T$ is not symmetric in the USSG case. Open-shell orbitals $\phi_\lambda$ form an additional density matrix

$$P_{\lambda,\alpha}^{open shell} = \sum_{\iota} C_{\lambda \alpha}^\dagger C_{\alpha \iota}.$$

We also define density matrices for the whole system $P_{\alpha A} = \sum_A P_{\alpha A} + P_{\alpha A}^T$, $P_{\beta A} = \sum_A P_{\beta A} + P_{\beta A}^T$, and $P_{\alpha A}^T + P_{\beta A}^T$. The two-electron integrals are contracted using these matrices as

$$J_{\lambda,\alpha}^{A} = \sum_{\mu \nu} P_{\mu \nu}^{A} (\lambda \alpha | \mu \nu),$$

$$K_{\lambda,\alpha}^{A} = \sum_{\mu \nu} P_{\mu \nu}^{A} (\lambda \alpha | \mu \nu),$$

$$L_{\lambda,\alpha}^{A} = \sum_{\mu \nu} P_{\mu \nu}^{A} (\lambda \alpha | \mu \nu).$$

Expressions for $J_{\lambda,\alpha}^{T}$, $K_{\lambda,\alpha}^{T}$, $K_{\beta,\alpha}^{T}$, $K_{\beta,\alpha}^{T}$ can be obtained by substituting the corresponding density matrices into the above equations. Note that the intergeminal matrix $L_{\lambda,\alpha}^{A}$ is not symmetric in the USSG case. A global Fock matrix is defined as $F = \mathbf{H} + J + K$. The Fock matrices for each geminal are $F_{\alpha A} = F_{\alpha} + J_{\alpha} + K_{\alpha A}$ (with similar expressions for $\beta$-spin matrices). The geminal Fock matrices describe all one-electron interactions plus mean-field (i.e., Coulomb and exchange) interactions between electrons in a given geminal and all other electrons in the system.

The first step in the definition of perturbation expansion is to establish a zeroth-order reference Hamiltonian, $H_0$. We define it as close as possible to the one-electron Fock operator used in Møller–Plesset perturbation theory (MPPT). In order to ensure that the SSG reference wave function is the eigenfunction of the reference Hamiltonian, two-electron
terms must be included in \( H_0 \). In the context of APSG formulation it is natural to include all intrageminal interactions into \( H_0 \), as was done by Rosta and Surján. 17 This choice, however, complicates solutions for excited states of \( H_0 \) with three or four electrons in a geminal. To simplify computation of such excited states we define an auxiliary Hamiltonian \( \tilde{H}_0 \) as

\[
\tilde{H}_0 = \sum_i F_i a_i^\dagger a_i - \sum_{i,j=1}^{N} (i,j)(\delta_{i,j} - \delta_{i,j}) [a_i^\dagger a_j^\dagger a_j a_i] + (1 - \delta_{i,j}) a_i^\dagger a_i^\dagger a_j a_j a_j a_i, \tag{6}
\]

where bars indicate beta spin-orbitals, and \( F_i \) are diagonal matrix elements of a Fock matrix of a geminal containing orbital \( i \), \( \delta_{i,j} \) is Kronecker delta function, and \( (i,j) \) = \( \int \phi_i(\mathbf{r}_i) \phi_j(\mathbf{r}_j) (1/\mathbf{r}_{ij}) \phi_i(\mathbf{r}_i) \phi_j(\mathbf{r}_j) d\mathbf{r}_i d\mathbf{r}_j \). The two-electron intrageminal electron repulsion part contains two terms. The first term couples doubly occupied orbital pairs to each other and is required to make the reference function an eigenfunction of \( \tilde{H}_0 \). The second term couples spins in open-shell orbitals within each geminal. It is included in \( \tilde{H}_0 \) for two reasons. First, it preserves the symmetry of Coulomb operator with respect to interchange of same-electron orbitals. Second, it describes relatively strong interactions between excited states that can be nearly degenerate, thus having a potentially significant effect on the rate of convergence of perturbation expansion.

An important and undesirable feature of \( \tilde{H}_0 \) is that it contains only diagonal elements of a generalized Fock matrix. The issue here is the invariance of zeroth-order Hamiltonian with respect to transformations that do not change the reference wave function. In contrast to the single determinant case, any transformation among APSG orbitals changes the wave function. The exception to this rule are the transformations among orbitals that are fully occupied in SSG wave function, such as those in uncorrelated geminals and open-shell orbitals. Therefore, one has to choose a particular set of these transformable orbitals to be used in the definition of \( \tilde{H}_0 \). We choose to diagonalize the local Fock matrix \( F \) in the subspace of fully occupied orbitals. This choice is well defined and consistent with MPPT. In future studies of open-shell systems we plan to investigate the generalization of Edmiston–Ruedenberg localization 32 of such orbitals.

The perturbative expansion of exact wave function and its energy around small perturbation \( V = H - \tilde{H}_0 \) leads to familiar expressions for leading corrections to wave function

\[
|\Psi^1\rangle = \sum_{k=1}^{N} |\Psi_k\rangle \frac{\langle \Psi_0 | V | \Psi_k \rangle}{\langle \Psi_0 | H_0 | \Psi_0 \rangle - \langle \Psi_k | H_0 | \Psi_k \rangle}, \tag{7}
\]

and energy

\[
E^2 = \sum_{k=1}^{N} \frac{\langle \Psi_0 | V | \Psi_k \rangle \langle \Psi_k | V | \Psi_0 \rangle}{\langle \Psi_0 | H_0 | \Psi_0 \rangle - \langle \Psi_k | H_0 | \Psi_k \rangle}, \tag{8}
\]

where the summation runs over all excited states \( \Psi_k \) of \( \tilde{H}_0 \).

### Epstein–Nesbet form of perturbation expansion

An alternative formulation of perturbative expansion includes all diagonal terms \( \langle \Psi_k | H | \Psi_k \rangle \) in the reference Hamiltonian

\[
H_0 = \sum_{k=0}^{N} |\Psi_k\rangle \langle \Psi_k | H | \Psi_k \rangle |\Psi_k\rangle, \tag{9}
\]

where the summation runs over all eigenstates of \( \tilde{H}_0 \). This perturbation expansion, known as Epstein–Nesbet (EN) PT, 33,34 has expressions for the leading corrections to wave function and energy similar to Eqs. (7) and (8)

\[
|\Psi^1\rangle = \sum_{k=1}^{N} |\Psi_k\rangle \frac{\langle \Psi_0 | V | \Psi_k \rangle}{\langle \Psi_0 | H_0 | \Psi_0 \rangle - \langle \Psi_k | H_0 | \Psi_k \rangle}, \tag{10}
\]

The only difference between Epstein–Nesbet and Möller–Plesset perturbation equations taken to the leading order is that ENPT denominators contain matrix elements with exact Hamiltonian, as opposed to the reference Hamiltonian in MPPT.

The main formal advantage of ENPT is that its perturbation is much weaker than in MPPT, at least in the manifold of excited states. MPPT uses ground-state mean-field potential in the expression for excited state energies. This is known to be a poor approximation, 35 as can be seen by comparing CI excitation energies with those obtained from orbital energy differences. This deficiency prompted a number of researchers to study ENPT with single-reference and multi-reference 37 wave functions. The ENPT was found deficient for three reasons. First, it exhibits slow and oscillatory convergence for some open-shell systems. 35 Second, it is not invariant under unitary transformation of the degenerate orbitals, leading to serious artifacts in intermolecular interaction energies. 36 Third, it is very sensitive to transformations of virtual orbitals, even in the multireference case. 37

We argue that ENPT is free from these deficiencies when used with the SSG reference wave function. It is easy to see that orbital rotation problems are irrelevant, because the SSG wave function is, in general, not invariant under any orbital transformation. Even for fully occupied orbitals the orbital rotation invariance is formally lost if the reference Hamiltonian is defined by Eq. (6) due to the presence of explicit two-electron interactions between fully occupied orbitals of the same geminal. It is important to note that the reference SSG wave function is invariant under such a rotation, and this invariance is lost by a particular choice of the reference Hamiltonian \( \tilde{H}_0 \). Optimized SSG orbitals are localized in space, and ENPT works better with localized orbitals. 36

The oscillatory and slow convergence problem requires a more careful analysis. It is apparent that the source of convergence problem in the single reference case is nearly vanishing energy denominators. In order to understand the difference between excited state energies in single determinant and SSG cases, let us consider a geminal calculation of helium atom with 6-31G(2p) basis set as an example. In particular, let us focus on \( p \) orbitals. There are two of them for
shows CCSD angle of 103.2°. We show SSG and CCSD−Davidson. 35 The N–H bond distance is 2.026 Å, with HNH2p-quality of SSG reference wave function. The Epstein–Nesbet corrected geminal model, and on the investigation of the capabilities are extended to open-shell systems. In this paper be studied further. We plan to pursue this study when our reference state for a wide variety of chemical systems has to yielding the result in close agreement with CCSD model. ing term recovers 89% of remaining correlation energy, such shifts may lead to smoother, but slower convergence. This is so because the lead-

terms. The big difference in SSE–D radical, studied by Murray and interactions in the reference Hamiltonian. As an illustration of perturbation convergence we look at semidissociated NH2 radical, studied by Murray and Davidson. 35 The N–H bond distance is 2.026 Å, with HNH angle of 103.2°. We show SSG and CCSD(T) results along with their single reference UMPPT and ENPT results in Table I. Our actual calculation was performed by adding an extra hydrogen supporting a single STO-3G basis function 10 Å away from nitrogen, as described in the next section. While we have computed only the leading term in the EN perturbation expansion as applied to spin-unrestricted SSG reference, it is apparent that this model is free from problems seen in the single reference case. This is so because the leading term recovers 89% of remaining correlation energy, yielding the result in close agreement with CCSD model.

The comparison of ENPT and MPPT applied to SSG reference state for a wide variety of chemical systems has to be studied further. We plan to pursue this study when our capabilities are extended to open-shell systems. In this paper we focus on overall chemical performance of perturbatively corrected geminal model, and on the investigation of the quality of SSG reference wave function. The Epstein–Nesbet version of perturbation theory is better suited for the latter task, as discussed in Sec. V. Therefore, we choose H0 given by Eq. (9) as the reference Hamiltonian, with many-electron basis functions Ψk taken as eigenfunctions of the Hamiltonian given by Eq. (6). We designate this theory as SSG(EN). In this paper we use leading corrections of SSG(EN) and designate them as SSG(EN2).

The formal computational bottleneck of SSG(EN2) perturbation expansion is the full O(N5) integral transformation to the molecular orbital basis.

III. TECHNICAL DETAILS

One of the biggest challenges that we have faced in this work is computer code debugging. A relatively complicated structure of excited states that differs by the number of electrons and numbers of unpaired orbitals in each geminal requires that many separate cases need to be coded. For testing purposes we have implemented SSG(EN2) theory in two different ways. One implementation examines each individual excited state and computes all relevant matrix elements for it. The second implementation precomputes all excited states of each geminal, assembles them into multigeminal excited states, and computes each type of matrix elements for the whole list of these states. In addition, we have used GAMESS-US38 to assemble MCSCF wave functions that resemble the reference SSG wave function. Then, we add individual excited states to the MCSCF and examine the relevant matrix elements. We are reasonably confident that the data presented in this work are free from errors.

So far, the theory has been implemented only for closed-shell systems. The open-shell case does not present any additional conceptual challenges. Its implementation, however, would require a large amount of additional computer code, and will further complicate debugging. Therefore, we limit current studies to molecules in singlet spin states. In the meantime, atomic energies that are required to investigate size consistency were obtained by performing spin-unrestricted calculations on the dissociated XHn molecules with hydrogen atoms removed by at least 10 Å from the atom under study. The hydrogen atoms supported a single basis function each, eliminating a possibility of dispersion interaction between the ghost hydrogen(s) and a heavy atom. In the course of wave function optimization each hydrogen would support a single spin-unrestricted uncorrelated geminal, with orbital of one spin type localized on hydrogen, and the other spin orbital localized on heavy atom. In the subspace of a heavy atom such wave function is equivalent to atomic USSG.

An additional technical issue that must be discussed is the efficiency of the proposed perturbative treatment. The reference Hamiltonian that defines the spectrum Ψk is separable into parts associated with individual geminals. Therefore, each Ψk can be represented as antisymmetrized product of geminal wave functions. In general, each geminal can contain an arbitrary number of electrons in excited states, up to a total number of orbitals in the geminal. In practice, the leading terms of perturbation expansion do not involve terms beyond four electrons in a geminal. Anything above that does not couple to the ground state, which contains two elec-

tron in each geminal. It is convenient to classify all excited states by the number of electrons in excited geminals. The overall classification scheme and formal properties of excited states are discussed by Rosta and Surjan in Ref. 19. We label the excited state types by the number of electrons in excited geminals. Thus, \( \Psi (1,3) \) contains two excited geminals, one geminal with one electron and the other geminal with three electrons. All other geminals remain in their ground states. It is easy to see that leading perturbative terms require electrons. All other geminals remain in their ground states. It

Now, we must classify the excitations within each geminal. Because of the two types of two-electron interactions in \( \widetilde{H}_0 \), all excited states can be separated with respect to these terms. One type of excitation couples occupied orbital pairs. The other type couples unpaired orbitals in configurations that can be obtained by spin permutations among these orbitals. For instance, three-electron geminals can be divided into two types

\[
\psi_a(r_1, r_2, r_3) = \sum_{i \neq k} D^i_{r_1} \phi_i(r_1) \phi_i(r_2) \phi_i(r_3),
\]

with coefficients \( D^a \) determined by the first two-electron term in Eq. (6), and coefficients \( D^b \) determined by the second two-electron term. For a geminal made up of \( N_a \) orbitals, there are \( N_a(N_a-1) \) states of the first type for a given spin state, and \( 3N_a(N_a-1)(N_a-2) \) of the second type. In general, both of these types of excitations will couple to the ground state via two-electron perturbation. However, only the first type contributes to the most numerous four-geminal \( \Psi (1,1,3,3) \) excited states. Overall, for all excitations of four given geminals with \( N_{a_1}, N_{b_1} \) orbitals in one-electron geminals, and \( N_{a_3}, N_{b_3} \) orbitals in three-electron geminals, there are \( 6N_{a_1}N_{b_1}N_{a_3}N_{b_3}(N_a-1)N_a(N_a-2) \) excited states coupled to the ground state. The numerical factor 6 comes from all possible spin permutations. Evaluation of matrix elements between all these excitations and the ground state is a computational bottleneck of this perturbative scheme. To alleviate this, we use the approximation in which all states of three-electron geminal \( \phi_a \) are further divided into two groups. One group consists of states formed by creation operators \( a_k^\dagger \) acting on the ground-state geminals, and the second group is formed by states that are eigenfunctions of \( \widetilde{H}_0 \) subject to orthogonality to the first group. Only the first of these contributes nonzero matrix elements to \( \Psi (1,1,3,3) \) excited states. This reduces the total number of excited states in the example above to \( 6N_{a_1}N_{b_1}N_{a_3}N_{b_3}(N_a-1)N_a(N_a-2) \), leading to significant computational savings. This approximation is not expected to affect the computed properties of chemical systems, including those with strong multireference character. For the systems that were studied in the present work, this approximation changes absolute energies by fractions of millihartrees, while changes in relative energies are below microhartree.

All reported calculations are performed with a modified version of the q-CHEM program. Efficiency of computer code was sacrificed in favor of code simplicity in order to simplify the debugging process. For instance, the two-electron integrals are retrieved from scratch space on hard drive one by one as needed in the evaluation of matrix elements. This has a major impact on timing of calculations. Nevertheless, the computation of perturbative corrections for FOX-7 molecule with 158 basis functions took 45 wall clock hours on a modern Linux workstation, compared to 321 hours for CCSD calculation. We are confident that better I/O management will reduce the reported computational time of SSG(EN2) by factor of 5 or more.

### IV. DESCRIPTION OF CHEMICAL BONDS

The original investigation of the SSG model was based on a study of all diatomic molecules from the G2/97 test set. This set includes molecules with diverse types of chemical bonds. Currently we have implemented perturbative corrections only for singlet states. This reduces the test set to 16 molecules, and includes single and multiple covalent bonds, ionic bonds, and bonds with strong dispersion contributions. We believe this to be a sufficiently diverse set for studying the general quality of SSG(EN2) model.

First, we optimized bond distances using SSG(EN2) theory. The results are summarized in Table II. The root-mean-square deviations from experimental geometries are summarized in Table III. Overall, the agreement with experimental values is very good when sufficiently large G3MP2Large basis set is used. It is somewhat disappointing that for a popular 6-31G* basis set the calculated bond distances are inferior to MP2. With G3MP2Large basis [which is very close to 6-311+G(2df,2p)], the SSG(EN2) geometries dramatically improve single reference MP2 values.
Next, we analyze bond energies. The atomization energies are taken from Ref. 43. They are essentially deduced from experimental enthalpies of formation (0 K) in the JANAF thermochemical tables.44 In order to compare these data to the computed bond energies, we use atomic spin–orbit interaction corrections45 and zero-point vibration energy, as computed in G3 theory.46 The molecular and atomic energies are summarized in Table IV. These values are then used to compute atomization energies, shown in Tables V and VI. For smaller 6-31G* basis, the SSG model yields results in close agreement with CCSD calculation. When the bond is stretched, the single reference wave function is no longer adequate. This is reflected in a significant deterioration of MP2 energy. Interestingly, when this happens the SSG(EN2) model no longer follows MP2, and yields results in close agreement with CCSD calculation.

V. QUALITY OF SSG REFERENCE WAVE FUNCTION

One of the principal goals of the present work is the assessment of suitability of strongly orthogonal geminals to model chemical phenomena. It has been argued that strong orthogonality is too severe of an approximation.4,13,47 We think that some deficiencies associated with strongly orthogonal geminals were due to the incomplete optimization of the geminal wave function in earlier studies.15

The best way to evaluate the quality of approximations used in the SSG model is, in our view, to examine all perturbative corrections. The EN2 perturbation expansion is the best choice for such a study, because the EN perturbation coefficients of wave function expansion are equal to those in configuration interaction in the limit of weak correlation. Essentially, each EN2 coefficient is a result of a perturbative diagonalization of a 2x2 matrix in a subspace formed by the ground- and a single excited configurations.

We divide all perturbative corrections to the SSG reference wave function into three types. A dispersion-type correction (D) involves simultaneous excitation of two electrons, each within its geminal. We have denoted such excitations as Ψ(2, 2). These corrections give rise to dispersion interactions missing in the mean-field description. If such interactions had significant contributions to the perturbed wave function, this would indicate that mean-field description of intergeminal interactions is too restrictive for a given system. Mathematically, the mean-field approximation is manifested in the description of a wave function as an antisymmetrized product of geminals. It is independent of the strong orthogonality approximation.
A strong orthogonality correction (S) involves simultaneous transfer of two electrons between geminals. They include \( \Psi(0, 3, 3) \), \( \Psi(1, 1, 3, 3) \), \( \Psi(1, 1, 4) \), \( \Psi(0, 4) \), and part of \( \Psi(1, 3) \) type that keeps three-electron geminal in its ground state. All these excitations break strong orthogonality approximation, and their large role would indicate that strong orthogonality approximation is too restrictive for a given system. The third type of correction (DS) is the mixture of these two types. It includes the remaining part of \( \Psi(1, 3) \) and \( \Psi(1, 2, 3) \) types. They describe simultaneous breaking of strong orthogonality and mean-field approximations.

The significance of perturbative contributions does, naturally, depend on the target accuracy of the method. In the case of single reference states it is sometimes assumed that the configurations with the expansion coefficients higher than 0.1 in magnitude in the CI calculations indicate that single reference may be deficient. A similar criterion is often used to examine doubles amplitude in the CC expansion. We will use the value of 0.1 in the magnitude of the perturbative correction by individual configuration as a guide to the quality of approximations.

First, let us examine our set of diatomic molecules. Table VII shows the magnitudes of largest perturbative corrections of each type, computed with the 6-31G* basis set at the SSG(EN2) equilibrium bond distances. The ground state of all SSG wave functions is spin restricted. These are com-

<table>
<thead>
<tr>
<th>Molecule</th>
<th>( D_0(\text{expt}) )</th>
<th>MP2</th>
<th>CCSD</th>
<th>SSG</th>
<th>SSG(EN2)</th>
</tr>
</thead>
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<tr>
<td>LiH</td>
<td>56</td>
<td>1.81</td>
<td>39.9</td>
<td>44.2</td>
<td>44.1</td>
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<td>129.2</td>
<td>122.4</td>
<td>97.4</td>
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<tr>
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<td>256.2</td>
<td>3.43</td>
<td>253.7</td>
<td>235.6</td>
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<tr>
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</table>

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<tr>
<th>Molecule</th>
<th>( D_0(\text{expt}) )</th>
<th>MP2</th>
<th>CCSD</th>
<th>SSG</th>
<th>SSG(EN2)</th>
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<td>91.7</td>
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<td>99.6</td>
</tr>
<tr>
<td>SiO</td>
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<td>197.0</td>
<td>176.3</td>
<td>134.1</td>
<td>195.8</td>
</tr>
<tr>
<td>SC</td>
<td>169.5</td>
<td>171.8</td>
<td>154.1</td>
<td>104.9</td>
<td>178.3</td>
</tr>
<tr>
<td>FCl</td>
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<td>60.8</td>
<td>49.9</td>
<td>14.4</td>
<td>54.9</td>
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<tr>
<td>H(_2)</td>
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<td>96.5</td>
<td>101.5</td>
<td>101.5</td>
<td>101.5</td>
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<td>4.9</td>
<td>10.6</td>
<td>42.5</td>
<td>6.3</td>
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</table>
pared with leading amplitudes of CCSD single reference wave functions, which are spin unrestricted in the Li$_2$ case, and spin restricted in all other cases.

It is obvious that the approximation of antisymmetric product is more restrictive than that of strong orthogonality. In all cases the perturbations that break strong orthogonality approximation have amplitudes less than or around 0.05. The important corrections come from dispersion, with the largest amplitude of 0.1318 in the case of interactions between $\pi$ bonds in N$_2$ molecule. Somewhat smaller amplitudes in the cases of CO, P$_2$, and SC molecules all arise from $\pi$-bond dispersion interactions.

The use of a larger basis set does not alter this conclusion. For instance, the leading perturbative amplitudes for CO molecule wave function with G3MP2Large basis are 0.1191 for “D” type, 0.0282 for “S” type, and 0.0275 for “DS” type. With exception of the mixed DS type of excitation.

### Table VII. Amplitudes of the leading perturbative corrections for diatomic molecules with 6-31G* basis set. “D” labels dispersion corrections, “S” labels corrections that break strong orthogonality approximation, “DS” labels mixed corrections, and “CC” labels leading amplitudes in coupled-cluster CCSD wave functions.

<table>
<thead>
<tr>
<th>Molecule</th>
<th>D</th>
<th>S</th>
<th>DS</th>
<th>CC</th>
</tr>
</thead>
<tbody>
<tr>
<td>LiH</td>
<td>0.0032</td>
<td>0.0028</td>
<td>0.0013</td>
<td>0.0565</td>
</tr>
<tr>
<td>FH</td>
<td>0.0582</td>
<td>0.0120</td>
<td>0.0135</td>
<td>0.0463</td>
</tr>
<tr>
<td>HCl</td>
<td>0.0295</td>
<td>0.0251</td>
<td>0.0219</td>
<td>0.0708</td>
</tr>
<tr>
<td>Li$_2$</td>
<td>0.0042</td>
<td>0.0024</td>
<td>0.0011</td>
<td>0.2421</td>
</tr>
<tr>
<td>LiF</td>
<td>0.0660</td>
<td>0.0126</td>
<td>0.0143</td>
<td>0.0364</td>
</tr>
<tr>
<td>CO</td>
<td>0.1211</td>
<td>0.0297</td>
<td>0.0193</td>
<td>0.0788</td>
</tr>
<tr>
<td>N$_2$</td>
<td>0.1318</td>
<td>0.0341</td>
<td>0.0206</td>
<td>0.1067</td>
</tr>
<tr>
<td>F$_2$</td>
<td>0.0466</td>
<td>0.0193</td>
<td>0.0120</td>
<td>0.1881</td>
</tr>
<tr>
<td>Na$_2$</td>
<td>0.0058</td>
<td>0.0043</td>
<td>0.0029</td>
<td>0.1356</td>
</tr>
<tr>
<td>P$_2$</td>
<td>0.1287</td>
<td>0.0366</td>
<td>0.0302</td>
<td>0.1361</td>
</tr>
<tr>
<td>Cl$_2$</td>
<td>0.0320</td>
<td>0.0242</td>
<td>0.0257</td>
<td>0.1163</td>
</tr>
<tr>
<td>NaCl</td>
<td>0.0325</td>
<td>0.0255</td>
<td>0.0263</td>
<td>0.0364</td>
</tr>
<tr>
<td>SiO</td>
<td>0.0837</td>
<td>0.0330</td>
<td>0.0193</td>
<td>0.0716</td>
</tr>
<tr>
<td>SC</td>
<td>0.1278</td>
<td>0.0525</td>
<td>0.0367</td>
<td>0.0993</td>
</tr>
<tr>
<td>FCl</td>
<td>0.0584</td>
<td>0.0263</td>
<td>0.0208</td>
<td>0.1126</td>
</tr>
</tbody>
</table>

The multireference character of these wave functions can be deduced by looking at the degree of intrageminal correlation in the reference wave functions. The $\pi$-bond geminals of oxygen molecule dication have geminal expansion coefficients of second orbital pair of $-0.235$. The multireference character of O$_3$ is even more pronounced: The geminal that corresponds to HOMO is localized on peripheral oxygen nuclei, with its second orbital pair coefficient of $-0.477$. We find that it is a common feature of molecular fragments that contain two oxygen atoms bound to the same nuclei. Such fragments usually have a geminal that is localized on both oxygens, with a high degree of correlation in it. The FOX-7 compound is an exception: Its most correlated geminals describe N–O bonds, with second orbital pair expansion coefficient of $-0.199$. The SSG wave function of singlet VH is strongly correlated. The $d$-electron geminal on vanadium has a second coefficient of $-0.424$. In benzene the most correlated geminal has its second coefficient equal to $-0.150$. This geminal is located on a pair of adjacent carbon atoms. It reflects the general feature that geminal wave functions often break spatial symmetry. Geminals tend to favor orbital localization at the expense of overall wave function...
TABLE VIII. Amplitudes of the leading perturbative corrections for molecules with various types of chemical bonding, using 6-31G* basis set. All calculations are spin restricted. “D” labels dispersion corrections, “S” labels corrections that break strong orthogonality approximation, “DS” labels mixed corrections, and “CC” labels leading amplitudes in coupled-cluster CCSD wave functions.

<table>
<thead>
<tr>
<th>Molecule</th>
<th>D</th>
<th>S</th>
<th>DS</th>
<th>CC</th>
</tr>
</thead>
<tbody>
<tr>
<td>O₂⁺</td>
<td>0.1343</td>
<td>0.0343</td>
<td>0.0195</td>
<td>0.1425</td>
</tr>
<tr>
<td>O₂</td>
<td>0.0438</td>
<td>0.0218</td>
<td>0.0486</td>
<td>0.2133</td>
</tr>
<tr>
<td>C₂H₆</td>
<td>0.0710</td>
<td>0.0195</td>
<td>0.0246</td>
<td>0.0934</td>
</tr>
<tr>
<td>VH</td>
<td>0.0428</td>
<td>0.0461</td>
<td>0.0336</td>
<td>0.1672</td>
</tr>
<tr>
<td>(NH₂)₂C₂(NO₂)₂</td>
<td>0.0886</td>
<td>0.0200</td>
<td>0.0457</td>
<td>0.0595</td>
</tr>
</tbody>
</table>

VI. CONCLUSIONS

The first paper on geminal model chemistry has introduced a computationally inexpensive model which is applicable to multireference systems, variational, and size consistent. The model is based on a single antisymmetrized product of strongly orthogonal geminals, or SSG. Application of the SSG to studies of various diatomics revealed that the model describes covalent bonds well, and is deficient for bond between atoms with extreme electronegativities. We have attributed the deficiency to dispersion interactions between geminals that is omitted in the SSG. In the present paper we correct this by the inclusion of perturbative corrections to the SSG reference state. Our analysis and computation of test systems revealed that the Epstein–Nesbet form of perturbation theory is well suited for geminal models.

Application of a new model to equilibrium bond distances and bond energies of various diatomics demonstrated the accuracy of this model. Overall, the accuracy is comparable to the CCSD model for systems that are well described by single reference wave functions. The study of multireference cases is in progress. The computation of potential energy surface of CO molecule confirms size consistency and accuracy of the SSG model.

A detailed analysis of various perturbative corrections provides an important insight into the nature and quality of the SSG model. There are two independent approximations that are used in the SSG theory. The strong orthogonality approximation describes each correlated electron pair (or geminal) in its own orbital subspace. The antisymmetrized product approximation assumes only mean-field interactions between geminals. It is often assumed that for chemical systems the strong orthogonality is a more restrictive approximation of the two.

We compare relative importance of these approximations by investigating leading perturbative corrections to the SSG wave function in a diverse set of chemical systems. In all cases we find that the dominant correction comes from dispersion interactions. The strong orthogonality approximation is less restrictive and can be improved perturbatively, judging by small values of expansion coefficients of perturbed wave functions.

There are three main practical conclusions that we draw from the present study. First, it may not be advantageous to seek the improvement of geminal reference wave functions by introducing an explicit dependence in the framework of the antisymmetrized product approximation. This would significantly complicate the wave function without addressing the main deficiency of a reference model. Second, the most important perturbative correction terms are of the dispersion type. This is true even for the systems with strong multireference character. Formally, they are the easiest to treat, because they do not involve charge transfer between the geminals. Third, the Epstein–Nesbet form of perturbation theory taken to the leading order performs well when applied to the SSG reference wave function. The SSG(EN2) model appears to be a promising candidate for the description of single- and multireference wavefunctions with comparable levels of accuracy. The SSG(EN2) model may be effective for studying chemistry of transition metal elements.
ACKNOWLEDGMENT

This research is supported by the Chemistry Division of the NSF.