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Momentum of Particles from Time-Of-Flight Measurements

Joseph Best
University of South Carolina - Columbia

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By

Joseph Best

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Approved:

Steffen Strauch
Director of Thesis

Ralf Gothe
Second Reader

Steve Lynn, Dean
For South Carolina Honors College

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Thesis Summary

In order to find the momentum of particles from time of flight measurements, I used a program called Geant4 to simulate experiments. I made a simple two detector setup, and I recreated a real world experiment. I spent a lot of time learning to code in C++ so I could use Geant4 correctly. I simulated these experiments shooting electrons, muons, and pions through the geometry and measured the time at two points in their flight. Subtracting the second time from the first gave me the time of flight distribution for each particle. I used ROOT to draw histograms of the time of flight for each experiment and calculate the mean values. From the time of flight I found the momentum, knowing the mass and path length of each particle. I calculated the ideal times for each experiment from the fixed particle momentum at which I fired. I then compared these calculated times to the experimental times to see the relationship between the particles. This led me to theorize why the pions and muons were so much slower than electrons, and why heavier particles have more energy loss from ionization. I also used my data from the recreation of the real world muon scattering experiment to predict initial momentums of particles based on the measured time of flight.

Abstract

The proton-radius problem of disagreeing experimental results could mean a new understanding of physics is on the horizon. Measuring momentum accurately is integral to muon scattering experiment furthering the proton radius research, so I simulated time of flight experiments to figure out how accurately momentum could be measured. Measuring the time of flight of electrons, muons, and pions, I figured out the momentum, and compared this with the initial momentum of the particles. I figured out the correction needed when measuring momentum in order to find what the initial momentum was. Pions and muons travel at a slower velocity than electrons when the momentum is the same for all three particles. If I were to continue with these experiments, it would be extremely helpful to find a mathematical relationship between the difference in the measured and initial momentum and the detector separation distance so that, for any setup, it would be possible to figure out the initial momentum of any particle.

Chapter 1

Introduction

Recent studies in physics are focusing on a problem that has yet to be solved. There are two different types of experiments currently being conducted for determining the radius of a proton, scattering and spectroscopy. The scattering experiment involves shooting a beam of particles at a proton and recording how the trajectory changes. The particle interacts with the proton through electromagnetic forces, which deflects the particle and changes its flight path. Detectors are used to find the particles' angles of deflection, which are then analyzed. The scattering angles depend on the size of the cross section of the proton, so from the form factor obtained from this data the radius of a proton can be deduced. In spectroscopy, specifically muonic spectroscopy, the Lamb shift of muonic hydrogen (a proton with one muon instead of one electron) is measured. The Lamb shift is the small difference in energy between two energy levels of hydrogen, $^2S_{1/2}$ and $^2P_{1/2}$. Small contributions to the Lamb shift come from finite-size effects due to the overlap of the lepton wave function with the wave function of the proton. Therefore, a measurement of the Lamb shift allows the determination of the proton charge radius. Since muons are over 200 times heavier than electrons, they orbit much closer to the proton, and thus have a higher change in attraction to the proton. This causes the Lamb shift to be more than 10 million times more sensitive (Ouellette).

These experiments do not have agreeing results. According to Jan Bernauer's experiments at the MAMI accelerator in Mainz, Germany, the radius of the proton is 0.879 ± 0.008 fm (Bernauer). Compared with the value of 0.84087 ± 0.00039 fm

(Antognini), "the difference is more than five times the uncertainty in either measurement" (Bernauer and Pohl). Research into this problem is extremely important today, as there seems to be something we still do not understand about the proton, or there is new fundamental physics we have yet to discover. In order to find the answers, muon scattering experiments are being prepared by participants of the MUSE project (Gilman). Similar to electron scattering, a muon beam will be scattered off of a proton in order to determine the proton radius from the scattering angles. In these experiments, time of flight detectors (two separated scintillators) are used to determine a particle's momentum by measuring the time it takes to travel a certain distance. Scintillators detect particles by absorbing their energy and using it to emit light, the light is then detected and the time is recorded. Determining the uncertainty in measuring the momentum of the particles depends on attributes of these detectors and the incident particles, including the trajectory of the beam, the time resolution of the detector, and the knowledge of the flight path. The measured momentum is a function of the particle mass, the distance between scintillators, and the time of flight. The time of flight measurement may be affected by unknown time offsets between the two detectors. It is because of this that I measured time of flight for electrons, muons, and pions, but then used the electron time as a baseline and compared the times for muons and pions relative to electrons.

In order to test these experiments without the physical equipment, I used a simulation program, Geant4 (Agostinelli). The program allows me to build the geometry of an experiment, then test it with beam of any particle. I have control over the size, type, material, and positioning of the detectors, and can control the position, direction, and energy of the particle beam. Using one of Geant4's simple examples, I recreated the experiment using electrons, muons, and pions. Through Geant4 I recorded the times when the particle hits the detectors, then used those times to find the time of flight. I ran 10,000 events and found the mean time of flight using

ROOT. I compared the mean to the theoretical time of flight for the same momentum and figured out the difference between the actual momentum and the momentum measurement.

Chapter 2

Background Physics

Finding Momentum

In order to determine the momentum of a particle, I must derive an equation for momentum that depends on time of flight and path length.

In kinematics, for a particle that travels a distance L in time t, the velocity v is defined by

$$v = \frac{L}{t} \tag{2.1}$$

Velocity can be related to momentum through the equation

$$\beta = \frac{pc}{E} \tag{2.2}$$

where c is the speed of light and β is the ratio v/c. Substituting, Eq. (2.2) becomes

$$v = c^2 \frac{p}{E} \tag{2.3}$$

To find the energy E, we must use the equation

$$E^2 = (pc)^2 + (m_0 c^2)^2 \tag{2.4}$$

Solving for E and plugging it into Eq. (2.3), we get

$$v = c\frac{pc}{\sqrt{(m_0c^2)^2 + (pc)^2}} \tag{2.5}$$

If the particle's rest mass m_0 is much less than its momentum p, then the denominator can be simplified to pc and the fraction becomes 1. Then we simply have $v = c$. This applies for electrons at the momentum I'm working with, 161 MeV/c, but muons and pions are much heavier, so the rest mass must be taken into account.

I want to use the time of flight I measured to find the momentum of electrons, muons, and pions, so I must solve for pc.

Since $v = \frac{L}{t}$, we can substitute that in Eq. (2.5) and solve for pc to get

$$pc = \frac{L(m_0c^2)}{\sqrt{t^2c^2 - L^2}} \tag{2.6}$$

Error Propagation

The equation for momentum depends on time and path length, both of which will have uncertainty in measurement. In order to see how the uncertainty in these measurements will affect the result for momentum, I can use the equation

$$\sigma_p^2 = \left(\frac{\partial p}{\partial t}\right)^2 \sigma_t^2 + \left(\frac{\partial p}{\partial L}\right)^2 \sigma_L^2 \tag{2.7}$$

This will give me the standard deviation of the momentum in terms of the standard deviations of time and length. By taking the partial derivative of Eq. (2.6) with respect to t and L and plugging in, I get that the uncertainty in the momentum is defined by the equation

$$\sigma_p^2 = \frac{t^2c^4(m_oc^2)^2}{(t^2c^2 - L^2)^3}[L^2\sigma_t^2 + t^2\sigma_L^2] \tag{2.8}$$

Rewriting this in terms of relative uncertainties with t expressed in terms of p, I get

$$\frac{\sigma_p^2}{p^2} = \left(\left(\frac{pc}{m_0c^2}\right)^2 + 1\right)^2 \left[\frac{\sigma_t^2}{t^2} + \frac{\sigma_L^2}{L^2}\right] \tag{2.9}$$

Using the experimental data as an example, a muon with path length $L = 167$ cm, time of flight $t = 6.7$ ns, where $\sigma_t = 100$ ps and $\sigma_L = 1$ mm, has an uncertainty in momentum $\sigma_p = 8$ MeV/c. With the electron being fired at an initial momentum of 161 MeV/c, that is a huge uncertainty. We want to shoot for the uncertainty in the momentum to be less than 0.2%, or 0.3 MeV/c. What immediately helps reduce statistical uncertainties is doing multiple events in one run. For every simulation, I did 10,000 events, so the mean time will have a much better uncertainty than each individual time, because

$$\sigma_{\bar{t}} = \frac{1}{\sqrt{N}}\sigma_t \tag{2.10}$$

with N being the number of events. So for a $\sigma_t = 100$ ps, after 10,000 events, $\sigma_{\bar{t}} = 1$ ps, and $\sigma_p = 0.33$ MeV/c.

Chapter 3

Experiments

Setup

Using Geant4, I recreated the MUSE experimental setup. The setup, shown in Fig. 3.1, is placed in a world that is a 2 m $\times$ 2 m $\times$ 4 m box, filled with air. There are two scintillators, a window, and Cherenkov, a plate of silicon dioxide placed at an angle. In this setup, the particle beam comes from just outside the window toward the two scintillators. The window is 1 m from the center of the Cherenkov, which is 125.5 cm from the close face of the small scintillator. That face is 37 cm from the close face of the large scintillator. The particle beam will go from right to left.

When I run the particle beam, the GUI shows the particle paths in different colors according to their charge. For this experiment, all the particles fired from the particle gun have a negative charge, so the primary particle will always be red, with other red or green paths being any neutral particles generated through interactions with air and the detectors, and blue being positive particles.

If there are ever any particles that go through a detector more than once, only the earlier time will be taken. Each particle has its own identification number, so the first time and the second time will always be recorded for the correct particle. Many particles missed the small scintillator, or even both, completely, leading to incorrect time of flight results that I could not analyze. This is all the more reason to run the 10,000 events that I did.

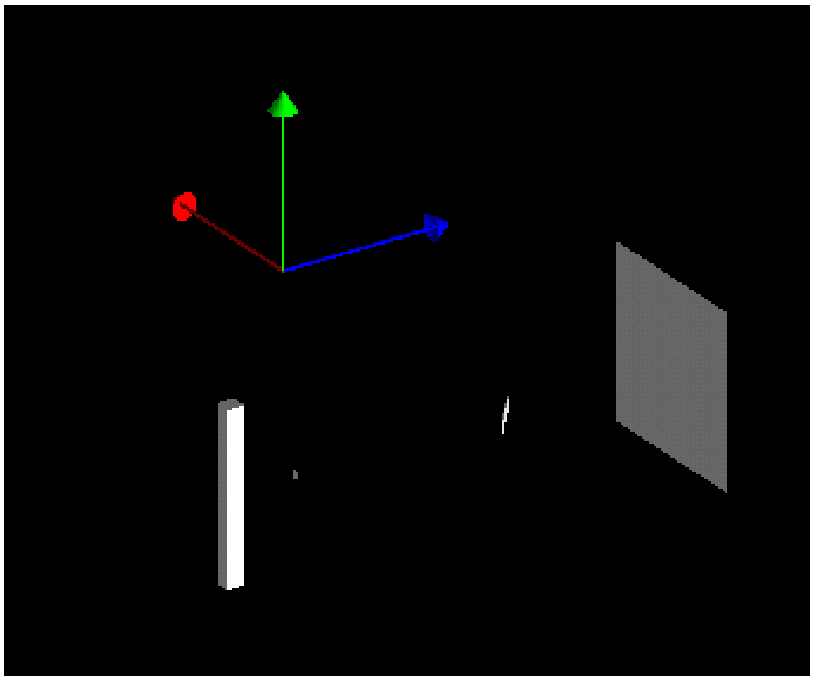

Figure 3.1 From left to right: 5 cm × 50 cm × 5 cm scintillator, 2 cm × 2 cm × .2 mm scintillator, 14 cm × 4 cm × 3.25 mm Cherenkov at a 39° angle with respect to the normal to the beam, 0.2 mm thick window. The x, y, and z axes are shown in red, green, and blue respectively. The detectors are moved 200 cm away from the origin in the x direction for visibility only.

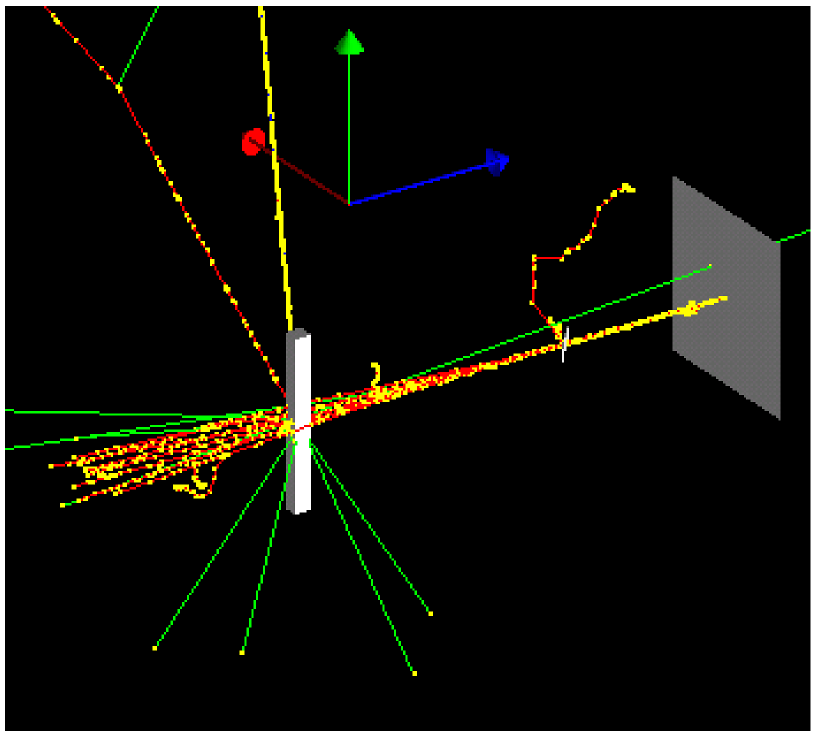

Figure 3.2 Experimental setup with 10 events. Many of these particle paths miss the small scintillator, leading to a lot of unusable time of flight measurements.

Chapter 4

Data

I ran the experiment with electrons, muons, and pions. I made a histogram of the differences between the muon or pion times and the average electron time.

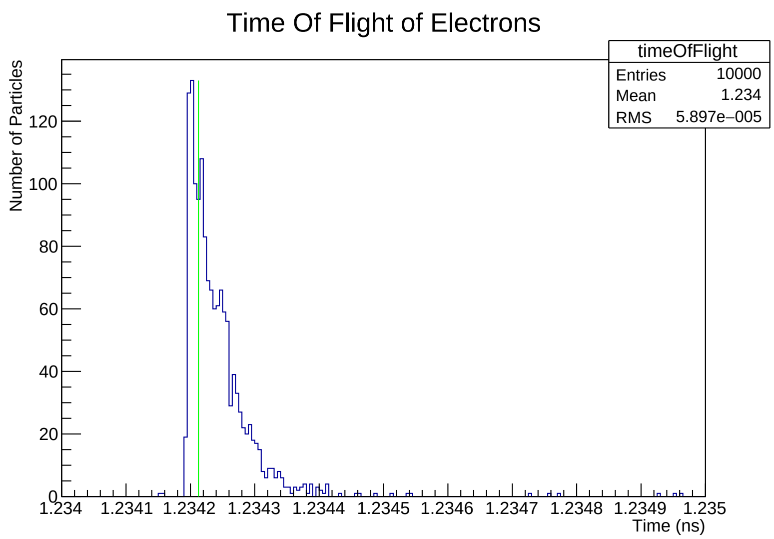

Figure 4.1 The time of flight for electrons at a momentum of 161 MeV/c

I used the mean value of .1234 ns as the baseline for finding the time of flight differences for muons and pions.

In Fig. 4.2, the pion peak is a bit wider than the muons, and it is also farther from the calculated value. The pion curve is more spread out, so the pions have a wider range of times, giving them wider range of measured momenta than muons.

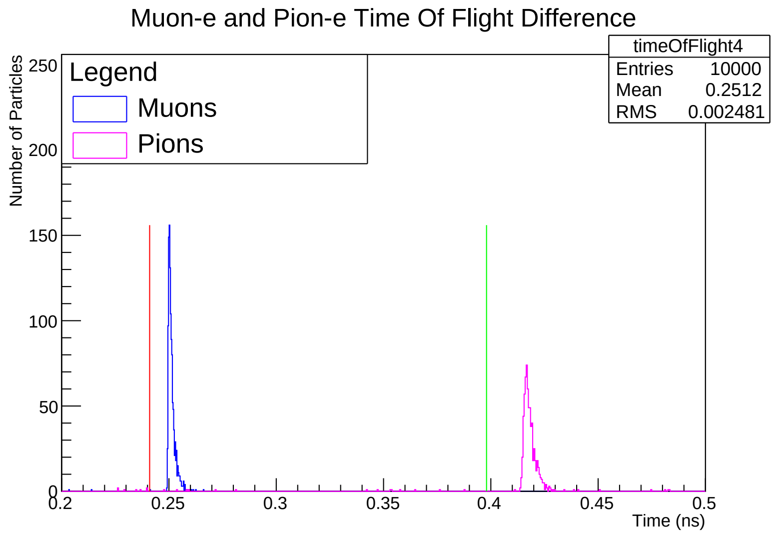

Figure 4.2 Time of flight differences for muons and pions at a momentum of 161 MeV/c.

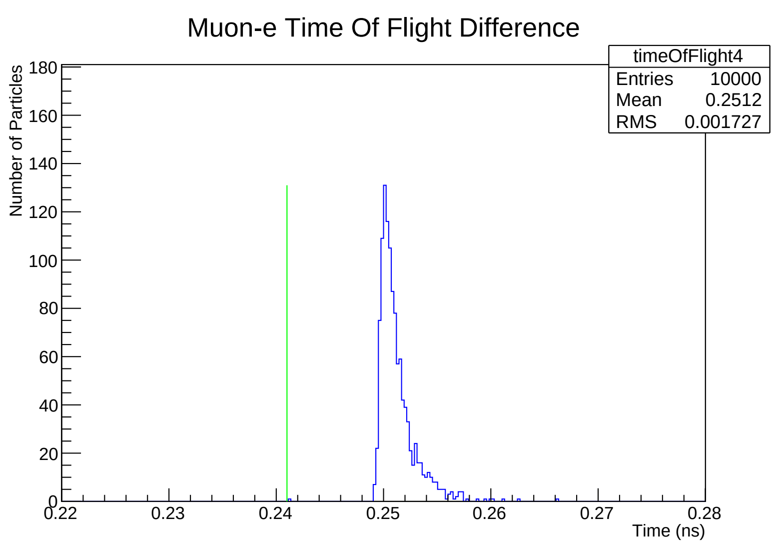

Figure 4.3 The difference between the muon time of flight and the average electron time of flight. The green line indicates the theoretical value.

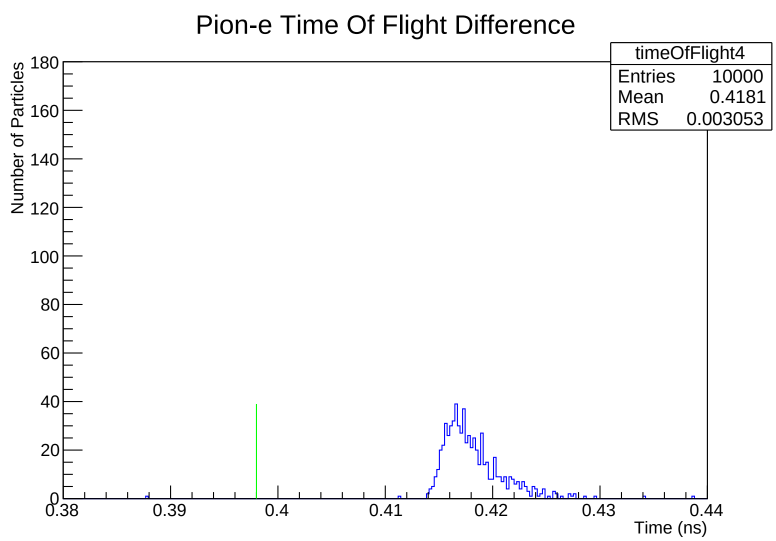

Figure 4.4 The difference between the pion time of flights and the average electron time of flight. The green line indicates the theoretical value.

Chapter 5

Analysis

In general, for a heavier particle at the same momentum of a lighter particle, the heavier particle will travel at a slower velocity. The mean energy loss of charge particles at high velocity depends on the particle's speed. Lower velocity leads to more energy loss and a lower measured momentum.

From the time of flight data, I can calculate the momentum for each particle with Eq. (2.6) from the mean time of flight and the path length.

The simulation setup has a path length of 37 cm, while the experimental setup has a path length of 1.67 m. Both tested muons and pions with an initial momentum of 161 MeV/c. Table 5.1 shows the momentum reconstructed from simulated data, while Table 5.2 shows the experimentally measured momentum. The momentum measured in the simulation is much closer to the initial momentum than the experimentally measured momentum.

A problem with the simulation was that I measured my time of flight from the small 2 cm $\times$ 2 cm scintillator to the big 5 cm $\times$ 50 cm scintillator, while the physical experiment measured from the Cherenkov to the big scintillotor, meaning my simulated flight path was different. This technically would only affect the precision of my momentum measurement, as a larger distance would give a more precise measurement, so the results are still valid.

Particle	Path Length (m)	ToF (ns)	Measured p (MeV/c)
Muon	.37	1.485	158.377
Pion	.37	1.652	157.134

Table 5.1 Time of flight, path length, and measured momentum for each particle type in the simulation.

Particle	Path Length (m)	ToF (ns)	Measured p (MeV/c)
Muon	1.67	6.752	153.9
Pion	1.67	7.561	151.8

Table 5.2 Time of flight, path length, and measured momentum for each particle type in the experiment.

Chapter 6

Conclusion

The simulation yielded a momentum measurement smaller than the actual beam momentum of 161 MeV/c. This is expected, as there is energy loss of the particle when traveling through air, the window, and the detectors. This means the time of flight measurements are for a particle that is gradually decreasing in energy. This simulation helps find the needed corrections to a measurement of the momentum of a particle losing energy.

The experimentally reconstructed momenta are also smaller than the actual beam momentum of 161 MeV/c. The simulation accounts for part of this effect, but the there is still a large difference between the simulated measurements and the experimental measurements. The correction from the simulation is not large enough to account for the rest of the difference. There is something else in the experiment causing a lower momentum measurement. The origin of this discrepancy is still being studied.

Bibliography

1. J.C. Bernauer et al., Phys. Rev. Lett. 105, 242001 (2010).

2. A. Antognini, Science 339, 417 (2013).

3. J.C. Bernauer and R. Pohl, Sci. Am. 310, 18 (2014).

4. R. Gilman et al. (MUSE Collaboration), arXiv:1303.2160 [nucl-ex]

5. Ouellette, Jennifer. "Physicists Tackle the Shrinking Proton Puzzle."Scientific American Global RSS. 18 Apr. 2013.

6. S. Agostinelli et al., Nucl. Instrum. Meth. A506, 250-303 (2003).