10-1-1995

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Publication Info
Published in *Current Research in Social Psychology*, ed. Michael Lovaglia, Volume 1, Issue 2, 1995, pages 8-15.
http://www.uiowa.edu/~grpproc/crisp/crisp.html

CURRENT RESEARCH IN SOCIAL PSYCHOLOGY

Volume 1, Issue 2, October 24, 1995

Submitted September 8, 1995
Resubmitted October 16, 1995
Accepted October 16, 1995

# Assessing Fundamental Power Differences in Exchange Networks: Iterative GPI

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ABSTRACT

Networks have been discovered for which Network Exchange Theory (NET Markovsky, Willer and Patton 1988; Lovaglia, Skvoretz, Willer and Markovsky 1995) fails to provide tenable predictions. Here we elaborate NET to create a more general method. We show not only when and where exchange networks break into simpler substructures, but propose rules to decisively classify networks and substructures as strong, weak, or equal power. In doing so, we advance general heuristics for power development in exchange networks and demonstrate the promise of an approach using reciprocal comparison of general heuristics, formal theory, and computer simulation.

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## INTRODUCTION

Exchange networks must first be classified by power type before an accurate prediction of power distribution in the network can be made (Markovsky, Skvoretz, Willer, Lovaglia and Erger 1993). Here we classify networks as strong, weak, or equal power. In strong power networks, high power actors can use power with impunity. That is, over a series of exchange opportunities, they come to control nearly all available resources. By contrast, in weak power networks, power use by high power actors results in countervailing changes in exchange relations, changes that moderate future power use. Power in these networks reaches a stable equilibrium in which high power actors maintain a reliable, though moderate, advantage. That equilibrium point can be accurately predicted for positions in a wide variety of weak power networks (Skvoretz and Willer 1993; Lovaglia, Skvoretz, Willer and Markovsky 1995). Finally, in networks of equal power, no actor has an exchange advantage; thus, no resource differentiation is predicted in them.

The Graph-theoretical Power Index (GPI) method developed by

Markovsky, Willer and Patton (1988) and Markovsky, Skvoretz, Willer, Lovaglia, and Erger (1993) uses a path-counting algorithm to identify how advantaged one actor is in comparison to another. A position's GPI value is calculated by counting non-intersecting paths of different lengths leading away from it, with odd-length paths adding advantage, even-length paths taking away advantage.

Consider the network in Figure 1 suggested by Noah Friedkin (personal communication). Position A has a single 1-path to B, a 2-path to C (the 2-path to D would intersect with the first 2-path at B and so is not counted), a 3-path through B and C to D, and a 4-path through B, C, D, and ahead to the other C. Adding 1 to the GPI index for the 1-path and 3-path while subtracting 1 for the 2-path and 4-path yields a GPI value of 0 for position A. In contrast, position B has four 1-paths, a 2-path through C to D, and a 3-path through C and D to the other C. Thus its GPI value is 4 - 1 + 1 = 4. (See Markovsky et al. 1988 for details of GPI analysis.)

Figure 1. Friedkin Network and GPI Values

```
          C                  0
         / \                / \
    A - B - D          0 - 4 - 3
         \ /                \ /
          C                  0
```


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When GPI values differ for two positions, one has a strong power advantage over the other. Markovsky et al. (1988, Axiom 2) assume that actors seek exchange with partners whose GPI value is lower than theirs. Or, if all partners have a GPI value equal to or greater than an actor, the actor is assumed to seek exchange with the weakest partner(s) available. However, exchange is possible only when an actor and a partner mutually seek exchange with each other. Hence, if an actor and a partner do not mutually seek each other, that tie is broken. When such broken ties cause networks to break into subnetworks, GPI is iteratively applied to resulting subnetworks.

Using Axiom 2 to analyze the Friedkin network, C actors will seek exchange with D, but not B. The network breaks into an A-B dyad and a C-D-C 3-line network. GPI equals 1 for the positions in the dyad. In the 3-line, D's GPI equals 2, whereas C's equals 0. The exchange seek assumption then applies to these new GPI scores. C actors seek exchange with B, but not with D, thus leaving D isolated from the rest of the network. Iterating GPI again returns the dyad and 3-line. The analysis cycles indefinitely from one iteration to the other. Thus, the Friedkin network cannot be classified as strong power. Nevertheless, simulation using Markovsky's X-Net program shows that B and D are in fact strong power positions (Markovsky in press describes the simulator).

The anomalous Friedkin network has a relatively easy solution, a modification of the exchange seek assumption. Markovsky et al. (1988) assume that C actors will see exchange with D while avoiding B because B is more powerful than D. However, whenever a strong power advantage exists, low power actors eventually lose nearly all available resources. Intuitively, it matters little to a disadvantaged actor whether the difference in GPI scores is large or small. As such, a better specification of the exchange seek assumption is:

Revised Exchange Seek Assumption (Axiom 2)

Actors seek exchange with those less powerful than they are. If no actors with less power are available, actors seek exchange with actors of equal power. If, however, no actors of equal power are available, actors seek exchange with more powerful actors.



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Applying the revised exchange seek assumption to the Friedkin network, C actors seek both B and D and the network does not break into subnetworks. We classify it as a strong power network because GPI values differ for related actors. Although this new axiom thus satisfactorily resolves the anomaly of the Friedkin network, exploring its implications soon revealed other networks that challenged GPI analysis.

## Heuristics in the Construction of Test Networks

It is difficult to find networks that test theoretical advances. GPI, for instance, was in use for six years before the Friedkin network was discovered. Here we develop a general method of iterating GPI by building up complex networks from simpler structures, using heuristics about the way power develops through exchange (Willer and Willer 1995). Then, we use GPI analysis to see whether it indicates strong power. When a discrepancy occurs between GPI and heuristic analysis, we simulate the network using the X-Net program. In all cases thus far, the X-Net simulator and the analyses that employ the following heuristics have agreed.

Heuristic 1: Adding a relation between a low strong-power position and a high strong-power position does not change the type of power of any position in the network.

Heuristic 2: Adding a relation between two high strong-power positions does not change the type of power of any position in the network.

Heuristic 3: Adding a relation between two low strong-power positions creates a weak or equal power structure.

Heuristic 4: Adding a relation between weak or equal power positions cannot create a strong power structure.

Heuristic 5: Breaks occur between high strong-power positions or between high strong-power positions and equal or weak power positions, but not between equal or weak power positions.

(Cf. Willer and Willer 1995 for heuristics 1, 2 and 3.)

Our explorations uncovered many networks for which GPI analysis produced repeating cycles of subnetworks that would not allow for the classification of positions as strong power in any simple way, even though simulation and heuristic analysis suggested that strong power was present. Further analysis of the problem was necessary that, when performed,


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yielded a general method for the GPI analysis of networks. The method decomposes a network into strong power, weak power, and equal power components. The heuristics and computer simulation then serve as checks on the method's results in particular cases.

Iterating GPI

The general method for iterating GPI to decompose complex networks uses the following 7 rules:

1. Iterate GPI using the new exchange seek assumption until a stable solution--wherein GPI values of all positions remain the same in two consecutive iterations--or a repeating cycle of solutions emerges. A stable solution ends analysis.

2. (a) When a stable solution appears, inequalities between connected positions indicate strong power.

   (b) When a repeating cycle of solutions appears, draw the network that includes all relations across iterations in the cycle except when (i) breaks occur in every iteration of a repeating cycle and (ii) a position has a GPI advantage over other connected positions in every iteration of a repeating cycle. In (i), the breaks are considered permanent and are not redrawn. In (ii), the advantaged and disadvantaged positions form a strong power component that breaks off permanently. Then, reiterate GPI on the redrawn network until a stable solution or repeating cycle of solutions appears.

3. Re-apply rule 2 until the redrawn network is identical to the previous application's redrawn network or until a repeating cycle of redrawn networks appears.

Rules 1 - 3 above identify most strong power structures in exchange networks. (A computer program for analyzing networks using these rules is available from John Skvoretz.) However, computer simulation reveals that some structures harbor strong power differences not identified by the first three rules. For example, consider the 7p40 network in Figure 2 below. (We started labeling networks sequentially for each size. 7p40 is the 40th network with 7 positions.)



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Figure 2. 7p40 Network

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     A - B - C - B - A
          \     /
           D - D
```

All positions in the 7p40 network have GPI values of 1. We would not classify this network as strong power, nor would we predict any breaks in negotiations between network positions. However, computer simulations and the heuristics tell a different and convincing story. The B positions in the 5-line, A-B-C-B-A, are high strong power. In addition, each B is connected to one member of the D-D dyad. The heuristics reveal that D actors will initiate a break from the B actors. D actors will prefer to exchange equally with each other rather than exchange at a disadvantage with B actors. In turn, B actors are indifferent to exchange with D actors because B actors have low strong-power alternatives to exploit. X-Net simulation confirms that a break will develop between B and D actors, and that B actors have a strong power advantage over A and C actors.

The following rules correctly decompose networks such as 7p40 that have strong power hidden within them. The rules work by breaking down networks to their core structures to insure that no lurking potential for strong power remains undetected.

4. Look for a "stem-dyad" in networks and subnetworks that have not been identified as strong power. A stem-dyad is understood to be a position of degree 1 (i.e., connected to only one other actor) and the position connected to it. The position connected to the degree 1 position has the potential to be high power. Thus we call it the high power position in the stem-dyad; and, we call the degree 1 position the low power position in the stem-dyad.

5. Remove the stem-dyad from the network and examine the residual network.

6. (a) If the residual is strong power and if the high power position in the stem can reconnect to a low power position in the residual, then the original structure is strong power.

   (b) If the residual is strong power and if the high power position in the stem can reconnect only to high power positions in the residual, then the stem breaks from the residual as an equal power dyad.



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(c) If the residual breaks into strong and weak power components while the high power position in the stem connects to a low strong-power position in the residual, then the network breaks where the residual breaks.

7. For remaining structures not identified as strong power, re-apply steps 4 - 6. Continue until no stem dyads remain attached to a larger structure not yet identified as strong power. Then reconnect all relations among structures not identified as strong power.

Rules 4 - 7 allow networks of any size to be broken down to an easily analyzed core structure. For example, consider a 7-actor line A-B-C-D-E-F-G. First remove the stem-dyad, A-B, leaving a five-actor line. (Note that removing F-G instead has the same result.) If the power type of a five-actor line is unknown, remove the stem-dyad, C-D, from it. The remaining core structure, E-F-G, is a 3-line, the prototypical strong power structure. Hence, we conclude that the 7-Line is a strong power structure. Now try the 7p40 network. Removing the A-B stem-dyad results in a 5-actor T structure, which breaks into a strong power 3-line and a dyad (Markovsky et al. 1988). Therefore, by rule 6c and through symmetry, 7p40 breaks into a 5-actor line and a dyad.

Rules 1 - 7 should identify all strong power structures--at least all those in networks of 7 or fewer positions. (More complex networks may require more subtle analysis.) We have used them to analyze more than 200 6-position and 7-position networks without finding predictions at odds with simulations or other forms of analysis. Having thus identified and broken out strong power structures, remaining networks can be analyzed using the probability tree method of Markovsky et al. (1993) to determine whether weak power exists in them. The method can therefore be said to classify the fundamental power type (strong, weak, or equal) of all positions in all exchange networks. Exact resource point predictions at equilibrium can then be made using the method of Lovaglia et al. (1995).

Pending empirical confirmation, this iterative GPI method appears to solve a fundamental, though narrow, problem in network exchange theory. Moreover, the heuristics developed as tools in the solution have general implications. For example, in exchange networks it seems impossible to gain a strong power advantage by opening channels of exchange to weak or equal power network members. Rather, strong power can only be achieved by cutting off the alternative exchange opportunities of one's partners (oppression), or by establishing new connections to isolated individuals outside the network (colonization).



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More generally, we are developing a procedure for conducting social exchange research. We compare the results of analyses using general heuristics, formal theory, and computer simulation. Although these analyses are related to the extent that they make similar predictions, all begin from quite different foundations. Those networks for which analyses differ thus become the target sites of future research. Hence, one can systematically analyze all networks of a certain size, culling anomalous networks for further study. As such, locating interesting test networks may no longer be a hit-or-miss endeavor requiring years of ancillary study, thus bringing closer the goal of analyzing networks of the size and complexity found in naturally occurring social situations.

In addition, the increasing complications of the GPI have led us to develop an independent method for determining whether networks are strong, weak, or equal power. After examining over two hundred networks, the iterative GPI and the new method make identical predictions, thereby validating GPI and adding a new dimension to our evolving general procedure. Consequently, we will be able to compare the predictions of two independent formal theories with each other and with general heuristics and computer simulations to find interesting exchange networks.

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