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Patrick J. McGowan
University of Southern California

Robert M. Rood
University of South Carolina - Columbia, rood@mailbox.sc.edu

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Alliance Behavior in Balance of Power Systems: Applying a Poisson Model to Nineteenth-Century Europe*

PATRICII J. McGOWAN
University of Southern California

ROBERT M. ROOD
University of South Carolina

Alliances and Balance of Power Systems

In this paper we partially test Professor Morton A. Kaplan's "theory" of the Balance of Power System first proposed in his well known System and Process in International Politics.1 Given that nineteenth-century Europe comprised a balance of power international system and given that Kaplan has specified the "essential rules" of such a system, we ask the question: Did European alliance behavior in this period conform to Kaplan's theoretical expectations of a random alliance process?

The outbreak of World War I in 1914 traditionally marks the end of the nineteenth-century historical period. Conventional diplomatic wisdom suggests that war occurred because the balance of power system collapsed owing to a "hardening of alliances" some time after 1900. If a balance of power system looses its flexibility because the participants have abandoned systemic alliance formation rules, Kaplan's theory predicts such a result.2 We therefore also ask the question: Why did a general European war break out in 1914?

We attempt to answer these related questions by applying probability theory, specifically a Poisson model, to the analysis of new data on fifty-five alliances among the five major European powers during the period 1814-1914. Because our research questions are so large-scale, our conclusions cannot be regarded as definitive. We do find, however, that the data examined very strongly support our hypotheses.

Definitions. No two concepts are more central to the analysis of international politics than "alliance" and "balance of power." Theory in the field is so underdeveloped, however, that these concepts are usually vaguely defined, mixing description, prescription, and evaluation, so that an agreed-upon meaning for these terms does not at present exist.

Since we intend to test aspects of Morton Kaplan's theory of the behavior of balance of power systems, we shall use his definition of this concept. The pattern of interactions between two or more state actors, which composes a system of action, will be called a balance of power system if the following three conditions hold: (1) the system is without a political subsystem that authoritatively regulates the behavior of system members, such as a fully effective United Nations, (2) there are at least five essential state actors, and (3) the six rules of actor behavior specified by Kaplan are followed.4 Kaplan regards nineteenth-century European international politics as one such balance of power system.5

Our conception of alliances is that they are a subclass of alignments wherein at least two states make a military commitment against at least one other state to fight or to remain neutral. Alliances share with alignments the following features: interstate cooperation or coordination over a prob-

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1 New York: Wiley Science Editions, 1964. First published in 1957. All quotations of Kaplan in this paper are from the 1964 paperback reprint unless otherwise noted.


3 Representative discussions may be found in: George Liska, Nations in Alliance: The Limits of Interde-
lem; combination of state capabilities; pursuit of state interests; and mutual assistance. Alliances are
distinct from alignments because: they have
an actual or anticipated enemy (in Liska’s terms—
“Alliances are against, and only derivatively for,
someone or something”9); they contemplate mili-
tary engagement and the risk of war; and they are
based upon mutual interest in the international
status quo or its revision.7 While alliances are often
embodied in formal treaties, as part of the process
of international politics, they can be informal and
highly dynamic.

Most authorities recognize that alliances are
basic to the operation of a balance of power sys-
tem.8 Kaplan argues that the six essential rules of
the balance of power system operate to produce
fluid alliances that “regulate” the system and pre-
vent any one power or coalition of powers from
achieving hegemony.9 Indeed, the literature sugg-
ests that alliances are the primary means avail-
able to the foreign-policy maker in a balance of
power situation; they enable leaders to maintain
the pluralism of the system and to ensure the con-
tinued existence of its essential actors, albeit at
the cost of solidarity. In a balance of power sys-
tem, alliances are able to deter hegemonial ambi-
tions by being fluid and flexible. Alliances are
made without regard to ideology, cultural affin-
ities, relations among monarchs and other elites;
and in particular, each alliance is independent of
past alliances and alignment patterns. Each is based
solely upon present state interest and current
threats to the balance of power.10

Behind this model of alliance process in balance
of power systems is an assumption of actor ra-
tionality.11 First, it is assumed that national inter-
ests have been ordered into a scale of preferences
and, second, that decisions on when and with
whom to form an alliance are based upon carefully
considered cost-benefit analyses.12 Thus, the pro-
cess of alliance combination and recombination
in a balance of power system resembles an in-
finite, ongoing, n-person, non-zero-sum game
where the payoff remains indefinitely in the pot.13

Theory. In his study of System and Process in
International Politics, Professor Kaplan produced
a “heuristic” theory of six international systems
that was meant to be explanatory, predictive, and
prescriptive.14 Kaplan’s theory is a rational theory,
for it only “predicts what state behavior will be if
statesmen are rational, completely informed, and
politically free to make external decisions on the
basis of considerations of external rationality.”15
The theory is “heuristic” or general16 for two rea-
sons; first, the paucity of comparable and coded
historical data made precise predictions “incau-
tious” in 1957,17 and second, a theory of inter-
national politics should not be expected to predict
individual events. It should, however, be able to
predict characteristic or modal behavior patterns.18
Because in many respects Kaplan’s theory repre-
sents a verbal formalization of traditional theoriz-
ing on international political behavior, his theory
of the operation of a balance of power system is
more specific than that of any of the other five
types of system he analyzes. Moreover, Kaplan
makes it explicit how his theory of balance of
power politics can be tested: “... a high correla-
tion between the pattern of national behavior and
the essential rules of the international system

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8 Liska, Nations in Alliance, p. 12.
7 Julian Friedman, “Alliance in International Politi-
cs,” in Alliance in International Politics, ed. J. Fried-
man et al., pp. 4-5 and George Modelski, “The Study
of Alliances: A Review,” Journal of Conflict Resolu-
tion, 7 (December, 1963), 769-776 provide discus-
sions of the salient characteristics of alliances.
8 Representative discussions are: Kaplan, System and
Process, pp. 35, 66, 115; Hans J. Morgenthau, Politics
Among Nations: The Struggle for Power and Peace,
3rd ed. (New York: Knopf, 1963), pp. 167-223; and
Friedman, “Alliance in International Politics,” p. 23.
9 Kaplan, System and Process, p. 125. See also Fried-
man, pp. 21-22 for the “functions” of alliances in in-
ternational systems.
10 These aspects of the role of alliances as a tool of
foreign policy are further discussed in: Herbert Diner-
stein, “The Transformation of Alliance Systems,”
American Political Science Review, 59 (September
1965), 589-601. See also: Liska, Nations in Alliance,
p. 16; Kaplan, System and Process, p. 66; J. G.
Cross, “Some Theoretic Characteristics of Economic
and Political Coalitions,” Journal of Conflict Resolu-
tion, 11 (June, 1967), 187; and Roger Masters, “A
Multibloc Model of the International System,” Amer-
ican Political Science Review, 55 (December, 1961),
788.
11 This “model” is clearly prescriptive, and hypo-
theically it is an accurate description and prediction
of alliance politics in any balance of power system.
Like other rational models, it mixes description, ex-
planation and possible prescription, a point well made
by Otto Davis in “Notes on Strategy and Methodology
for a Scientific Political Science,” in Mathematical
Applications in Political Science IV, ed. Joseph K. Bernd
(Charlottesville: The University Press of Virginia,
12 Friedman, “Alliance in International Politics,”
p. 23.
13 Arthur Lee Burns, “From Balance to Deterrence:
A Theoretical Analysis,” World Politics, 9 (July, 1957),
495.
14 Kaplan, System and Process, preface to the 1964
15 Ibid.
16 Heuristic, of course, does not mean general; it
means serving to discover. Professor Kaplan is not
alone, however, in apparently equating the two terms.
18 Ibid., p. 24, and Morton A. Kaplan, “Some Prob-
lems in International Systems Research,” in Interna-
tional Political Communities: An Anthology (Garden
would represent a confirmation of the predictions of the theory." One does not have to share Professor Kaplan's positivism to be intrigued by the question of whether or not historical data fit his predictions about modal alliance behavior in a balance of power system.

In such a system, Kaplan states, alliances are fluid and flexible, they are made for instrumental and not ideological reasons, and actors are indifferent about whom their alliance partners are. These characteristics flow from the operation of the six essential rules of such a system, and they are similar to the views of many other authors on this topic. Kaplan's contribution lies in his prediction that, when viewed from the perspective of the system, alliances are equiprobable and time-independent: "the 'balance of power' system postulates that any alignment is as probable as any other alignment prior to a consideration of the specific interests which divide nations. Moreover, any particular alignment should not predispose the same nations to align themselves with each other at the next opportunity." We read Professor Kaplan as saying that the alliance formation process in a balance of power system is a stochastic process. That is, in a balance of power system alliances occur from time to time, and these events over time are subject to probability laws because the past behavior of the alliance process has no influence on future behavior. If we have a process in which "the future is independent of the past" we have a purely random or stochastic process. Kaplan specifically indicates that he is thinking in such probabilistic terms when he uses the metaphor of the behavior of molecules in a tank of gas to characterize balance of power alliance politics.

Hypotheses. Now, there are many probability models of stochastic processes, but of all processes that generate random variables, this property of independence of past history is unique to the Poisson process. Poisson processes are also stationary stochastic processes, meaning that the origin of time is irrelevant. This accords with Kaplan's theory because it implies that at whatever point in time new decision makers or new states enter the alliance process, they too are free of the past. Finally, in a balance of power system, alliances must be formed from time to time but it is predicted that they must not be so infrequent as hardly ever to occur or so frequent as to make rational calculations of state interest impossible. Thus, alliances are relatively rare events, and it is a Poisson process that generates the distribution of such rare events over time. We therefore hypothesize, following Kaplan, that

H1: in a balance of power international system, the occurrence of alliances will be stochastically distributed (the number of alliances formed per unit of time is a Poisson random variable), and

H2: in a balance of power international system, the time intervals between alliances are randomly distributed (the distribution of interalliance intervals is a negative exponential random variable).

Because nineteenth-century Europe represents an

temporal

versal language of science" (System and Process, p. 25).

Professor Richard E. Hayes of C.A.C.I., Inc. disagrees with our reading of Kaplan's example, arguing in a personal communication that the metaphor refers to the limited predictability of social systems in general. Our disagreement illustrates the difficulties involved in deriving falsifiable hypotheses from verbal "theories" and "models" such as Kaplan's.


Kaplan, System and Process, p. 35.

empirical instance of a balance of power system, data on alliance behavior between 1814 and 1914 permit a direct empirical test of these two hypotheses inferred from Kaplan’s theory.29

These two hypotheses about alliance behavior in balance of power systems can be thought of as characterizing the system while it is in an equilibrium state; that is, when it is operating according to Kaplan’s six rules. Kaplan’s theory is also concerned with the conditions under which one type of international system changes into another type. Conventionally, we think of the nineteenth-century European balance of power system as having broken down in World War I and as having been replaced by a new worldwide system featuring the League of Nations. Kaplan tries to identify factors that lead to changes in balance of power systems.

He identifies a number of “parameters” whose values, if they change, can lead to changes in system structure.30 A key parameter is the “activity rate of a system,” which is a cause of system flexibility.31 An instance of this parameter in an empirical balance of power system would be the rate of alliance formation among essential system actors as measured over time for the system as a whole. The six essential rules of a balance of power system imply a pattern of fluid and moderately frequent alliances.32 If these rules are violated, either by a rigid enmity, such as existed between France and Germany after the annexation of Alsace Lorraine, or by a decline in the systemic alliance formation rate,33 then a loss of system flexibility will result and system-changing events are likely. We have set aside Kaplan’s rigidity hypothesis for subsequent research and in this paper examine his prediction that

H3: in a balance of power international system, a decline in the systemic rate of alliance formation precedes system changing events, such as general war.

This hypothesis agrees with considerable speculation by historians about the origins of World War I and it can also be tested against data on nineteenth-century European alliance politics.

Previous Research. Published studies that empirically and systematically test propositions deduced from Professor Kaplan’s heuristic theory are extremely rare. Several studies have used his concepts to describe historical interstate systems

—Chi on the Chinese warlord system and Franke on the Italian city-state system34—but description is not hypothesis testing. One recent study by Healy and Stein does test Kaplan’s rule 4 and part of his rule 6 against historical data on balance of power politics in the short interval 1870–1881. They find no support for Kaplan’s rules, however.35

The study of historical alliance patterns has been conceptual and analytical, with few hypothesis tests.36 Singer and Small, however, have made major contributions to our existential and correlational knowledge of historical alliances in their “Correlates of War” project.37 The possibilities of theoretically informed research on alliances are illustrated in the recent volume by Holsti, Hopmann, and Sullivan38 and in the most recent work of the “Correlates of War” project.39

There have been a few applications of stochastic models to international political phenomena, but only four that apply such models to alliance behavior. The first application of a Poisson model to international politics was that of Lewis Fry Richardson, which was extended by J. R. Moyal.40 As discussed by Richardson, both authors demon-

31 Ibid., p. 74.
32 Ibid., pp. 35, 125.
33 Ibid., pp. 27–29, 74.
36 See Julian Friedman et al., eds., Alliances in International Politics, for a survey of this literature.
strated that Richardson's data on wars between 1820 and 1929 and Wright's data on wars between 1500 and 1931 were Poisson-distributed over time.\textsuperscript{41} Much more recently, Singer and Small have demonstrated that the intervals between the outbreaks of all international wars and all interstate wars between 1816 and 1965 fit the negative exponential Poisson density function.\textsuperscript{42} A further application of the Poisson model is Midlarsky's demonstration that coups in Latin America (1935–64) and Africa (1963–67) were Poisson-distributed over time.\textsuperscript{43} Finally, another stochastic process model, Markov chains, has been applied to foreign conflict behavior 1955–1960,\textsuperscript{44} and to the outbreak of World War I.\textsuperscript{45}

The first application of a probability model to alliance behavior was by Horvath and Foster, who showed that Richardson's data on the size of wartime alliances fit a Yule distribution. This is consistent with the hypothesis that nations join alliances at a rate proportional to the number of nations in alliances of that size and that such alliances dissolve whenever a single member leaves.\textsuperscript{46} Rood, who used the same data that we present in this paper, found that choice of alliance partner in nineteenth-century Europe was approximated by a probability model of random choice in voting bodies developed by Brams and O'Leary.\textsuperscript{47} Brian Job has tested the hypothesis that alliance formation is a random process with a constant formation rate by applying a Poisson model to Singer and Small's data on 178 formal international alliances between 1815 and 1965. Despite imaginative treatment of the data, Job was unable to confirm his hypothesis except for all alliances, ententes and defense pacts between 1871 and 1914, and for all alliances and defense pacts from 1914 to 1939.\textsuperscript{48} Finally, Siverson and Duncan have applied three different stochastic models (the Poisson, the contagious Poisson, and the Yule-Greenwood heterogeneity model) to the same Singer and Small alliance data set. Unlike Job's paper and this article, Siverson and Duncan do not present hypothesis tests because their objective was simply to examine long-run patterns in the initiation of alliance activity. They find that for the 1815–1914 period both the Poisson process model ($X^2 = .76$, df = 1, $p = .40$) and the contagious model ($X^2 = .00$, df = 1, $p > .99$) fit the data, a finding that contradicts Job.\textsuperscript{49}

To our knowledge, these are the only applications of stochastic models, including the Poisson, to historical international political behavior. It is encouraging that certain central phenomena such as wars and the size of wartime coalitions can be described by stochastic process models. This suggests that other basic international behavior, such as alliances, may also be generated by such processes. It is discouraging to note the conflicting findings of Job and of Siverson and Duncan and the fact that except for the studies by Job, Rood, and Horvath and Foster, applications of stochastic models have not been related to hypothesis tests. Even in these three exceptions, the hypotheses tested were not inferred from an explicit theory of international politics. We therefore believe that this paper can break new ground in research on alliance behavior in balance of power systems and that it can make a contribution to positive theory building in international politics by testing three hypotheses inferred from Professor Kaplan's well known heuristic theory of system and process in international politics.

**Data Making**

In order to evaluate our hypotheses, a data set on nineteenth-century European alliance behavior was created. The procedure used was Poisson sampling, which "consists of observing the process over a predetermined amount of time, length or other dimension, and counting the number of events which occur. . . ."\textsuperscript{50} In Poisson sampling the time dimension ($t$) is predetermined,


\textsuperscript{46} W. J. Horvath and C. C. Foster, "Stochastic Models of War Alliances," *Journal of Conflict Resolution*, 7 (June, 1963), 110–116.


in the present case \( t \) represents the one hundred and one year period from January 1814 to August 1914. On the other hand, in Poisson sampling the number of events \((x)\) that occur is left to chance. In the present study \( x \) represents an alliance between two or more core European powers and \( \bar{x} \) is the distribution of these fifty-five alliances across the \( t \) dimension. Raiffa and Schlaifer prove that if the number of years preceding the \( x \)th alliance is \( t \) or less, the conditional distribution of \( \bar{x} \) given \( t \) and the intensity of the alliance formation process, \( m \), is provided by the Poisson function:  
\[
P(\bar{x} | t, m) = f_p(x | m) = \frac{e^{-m}(m)^x}{x!},
\]
(1)  
\[x = 0, 1, 2, \ldots, \]
\[m > 0.\]

Thus, in undertaking a Poisson sampling data collection strategy, we have made it possible to analyze the distributional characteristics of our data by application of the well-known theoretical Poisson distribution and its variants.  

Our data-making strategy was similar to the events data approach, but as applied to diplomatic histories rather than current events chronologies. A similar data-making strategy is being used by Rosecrance in his Situational Analysis Project at Cornell.  

extracting and coding rules, we read nine authoritative and representative diplomatic histories and recorded each instance of alliance behavior reported by at least one historian to have occurred between January 1814 and August 1914.  

Operationally, the events \((x)\) we call alliances are defined as all commitments, formal and informal, for the use or nonuse of military force \((a)\) that were made between 1 January 1814 and 29 July 1914 by Great Britain, France, Prussia-Germany, Austria-Hungary, or Russia and at least one other power including those mentioned and \((b)\) that had as their target the behavior in Europe of Great Britain, France, Prussia-Germany, Austria-Hungary, Russia or any grouping of these five states. Our sources usually reported formal commitments by citing a treaty, such as the 1834 Quadruple Alliance of Great Britain, France, Spain, and Portugal against Prussia, Russia, and Austria, who were joined by the Munchengratz Agreement of 1833. Informal alliances were most often embodied in the exchange of diplomatic notes or even verbal agreements by ambassadors to temporary joint fleet demonstrations such as that made in October, 1849, by Britain and France against Russia in support of Turkey. Following this definition and our coding rules, fifty-five alliances were observed during the 1814–1914 time period. Twenty-eight of these alliances are of the formal type and are also part of the Singer and Small alliance data set. The remaining twenty-seven are informal alliances and are unique to the present study. A complete listing of all fifty-five alliances is given by Rood.  

The reliability of our data collection operation  


As described in Singer and Small, “Formal Alliances, 1815–1939,” and “Formal Alliances, 1816–1965.”  

cannot be assessed by the usual means of an inter

coder reliability coefficient because all coding de-

cisions were made by one author. Our data can be
easily replicated, however, by consulting the

methodological appendices of Rood’s disserta-
tion.60 The use of multiple sources was our strat-
egy for achieving an acceptable level of reliability
in the alliance counting process. Most of the fifty-
five alliances, 74 per cent, are cited by two or
more authors. The average number of refer-
ces per alliance is nearly three. Only fourteen
alliances were mentioned by just one diplomatic
historian. Finally, the nine histories we used were
taken from the most frequently cited authorities
used by the “Correlates of War” Project.61 We
therefore doubt that we have included in our data
set an alliance that in fact did not occur in

nineteenth-century Europe. We may have failed,
however, to include some alliances, particularly
the informal type, that did in fact take place be-

tween 1814 and 1914. This possibility can only be
discounted after our data set has been inde-

pendently replicated.

The validity of our data set for the purposes
of this paper is enhanced by the fact that we ask only
two simple questions of the data: (1) did an alli-
ance commitment consistent with our operational
definition occur? and (2) in what year did it oc-

cur? In this paper we do not engage in the difficult
task of attempting to measure such things as the
scope, level of commitment, or duration of alli-
ances. The validity issue as it concerns this paper
thus reduces to the question of whether or not our
operational definition of alliance is suitable for
the study of alliance behavior in balance of power
systems.

Our assertion that nineteenth-century Europe
was a balance of power international system is
supported by the fact that thirteen out of fourteen
scholars surveyed, including Morton A. Kaplan,
claim that it was.62 Only Rosecrance does not ac-


cord the term balance of power to the nine-

teenth-century European state system.63 Our de-
cision to study only the alliance formation be-

havior of the five greatest European powers is
validated by the theoretical argument presented
earlier that it is the essential or great powers that
operate a balance of power system and that these
five states were the major European powers of
the nineteenth century.64 Our focus on alliances di-

rected at the European behavior of one or more of
these five states derives from the fact that the
balance of power system we are studying is the
system of Europe from 1814 to 1914 and not the
incipient worldwide system that emerged as a re-

sult of World War I. According to Singer and
Small, extra-European powers achieved great-

power status only very late in the 19th century—

Japan in 1895 and the United States in 1899.65

Finally, we have included informal as well as

formal alliances because a focus on written com-

mitments would exclude much of the nineteenth-

century “balancing” behavior of Great Britain, a

power regarded by many authorities as central to
the operation of the European balance of power
system,66 and because there is nothing in Kaplan’s
theory to suggest otherwise.

Figure 1 presents our alliance formation data
arrayed by five-year intervals for the entire nine-

teenth century. The first five years, which included
the Congress of Vienna, saw the greatest amount
of alliance formation by the five great powers.
These relations held for the next period, when no
new alliances were formed. In 1825 there began a
fifty-year period of moderate levels of alliance
formation that included the Crimean War and the
unifications of Italy and Germany. The Bismarck
era then saw a high level of alliance behavior that
continued until 1909, the year of our last observed
alliances. The five years preceding the outbreak of
World War I in August 1914 saw no new alliances
among the five great European powers. The mean
rate of alliance formation by five-year periods is

2.75 with a moderate standard deviation of 1.94.
As the figure suggests, formal and informal alli-

ances co-occur. The product moment correlation

coefficient (r) between the two types of alliances
by five year period is 0.43 (p ≤ .05 in a two-tailed
test).

The Poisson Distribution

We shall use the Poisson distribution in two

distinct fashions in this paper. First, we shall com-

pare the fit of our historical data to Poisson-

derived distributions that predict the number of
alliances formed over time and the intervals be-

tween alliances. Second, we shall use the Poisson

Small, “National Alliance Commitments and War In-

volvement, 1815–1945,” in International Politics and

Foreign Policy, ed. J. N. Rosenau, 2nd ed. (New York:

64 Singer and Small, “National Alliance Commit-
mments . . . .” p. 515 in the Rosenau reader.

65 Albrecht-Carrié, A Diplomatic History of Europe,

pp. 9–17; Taylor, The Struggle for Mastery in Europe,
p. 54; Seton-Watson, Britain in Europe, p. 312; and

Ludwig Dehio, The Precarious Balance: Four Centuries
of the European Power Struggle (New York: Random
distribution to compute rates of alliance formation in nineteenth-century Europe.

The Poisson distribution, one of “a few distributions of great universality which occur in a surprisingly great variety of problems,” is discussed in most texts on probability and mathematical statistics. A comprehensive treatment of the Poisson distribution is given by Haight, including many applications, and values of the distribution for given means \( m \) are tabulated by Kitagawa. The applicability of the Poisson distribution and its variants to social science questions is extensively discussed with numerous examples by Coleman.

Following Lindley, the aspects of Poisson process theory relevant to our paper can be briefly outlined. If in a period of time \((0, t)\) events occur and in a period \((t, t+\Delta t)\) events occur and \(A \) and \(B\) are independent so that \( p(B | A) = p(B) \), this not depending upon \( t \), the process is said to be a purely random stationary process, or a Poisson process. In a Poisson process the following theorems hold:

1. the probability of no events in a fixed interval of length \( t \) is—
   \[ p_0(t) = e^{-m \Delta t}, \text{ where } m > 0 \]  

2. the density of time between any two events \((x_1 \text{ and } x_2)\) is given by—
   \[ f_1(x) = me^{-mx}, \text{ when } x \geq 0 \]  

3. if \( t \) is any fixed number, the probability of \( x \) events in a fixed interval of length \( t \) is—
   \[ p_x(t) = \frac{e^{-m \Delta t}(m \Delta t)^x}{x!} \]  

4. the expected number of events \((x)\) in a time interval of length \( t \) is \( m \)

5. the expectation of time up to the \( x \)-th event, \( t_x \), is \( x/m \)

The key parameter in a Poisson process is seen to be \( m \), which equals \( \lambda t \) when \( t \) is one. The proper physical interpretation of \( m \), given theorem 4, is that it is the expected number of alliances per unit of time, i.e., the rate of formation. Conversely, from theorem 5, when \( x = 1 \), the expected or average time between successive alliances is \( 1/m \). Theorem 2 states that the interalliance intervals are independently distributed negative exponential random variables. This theorem will be used to test H2: that in a balance of power system, the time interval between alliances is randomly dis-

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**Figure 1. Frequency Histogram of Alliance Formation in 19th Century Europe by Five-Year Period**

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**References:**

1. Feller, *An Introduction to Probability Theory*, p. 156, who argues that the three principal distributions are the binomial, the normal, and the Poisson, a point agreed to by Yule and Kendall. *An Introduction to the Theory of Statistics*, p. 169.
distributed. Theorem 3 states that the number of alliances per unit of time is a Poisson random variable. This theorem will be used to test H1: that in a balance of power system, the occurrence of alliances will be randomly distributed.

The standard statistical test for comparisons between Poisson-based predicted distributions and empirical alliance distributions is the Chi-square goodness of fit test as described by Pearson and Hartly and by Yule and Kendall. The null hypothesis is that the observed distribution does not significantly depart from the expected Poisson-based distribution. A second test of goodness of fit is based on the fact that in a Poisson distribution, \( \sigma^2 = \mu \), the variance equals the mean, a unique feature of Poisson distributed data.

In a recent article, Hayes has discussed applications of Poisson theory to the measurement of changes in the frequency of international political phenomena. He presents a Poisson-based comparison technique that permits one to say, for example, how likely it is that eleven alliances occurred in the period \((t, t+h)\), given that just seven happened between 0 and \( t \). This technique can be directly applied to test H3: that, in a balance of power system, a decline in the systemic rate of alliance formation precedes system-changing events, such as general war. In this application of Hayes's measurement routine to our data, one calculates the rate of alliance formation by specified time intervals over the century between 1814-1914 and then calculates the probability of observing \( x \) alliances in each interval under the assumption that the observed \( x \) represents a decline from the average rate \( m \) (H1: \( x < m \)). These probabilities then become one's measures of balance of power system flexibility over time.

We conclude this discussion of the Poisson distribution by noting Coleman's belief that it is particularly appropriate for the analysis of sociopolitical phenomena because the Poisson distribution does not assume continuous level measurement; because a Poisson process occurs continuously over time rather than at discrete "trials" and thus is readily applied to naturally occurring events like alliances; and because the Poisson process is a rational model whose assumptions can mirror our assumptions about actual phenomena. Moreover, when its assumptions are met, a Poisson distribution is not an approximation to data; "it is the exact distribution and any other becomes an approximation." We believe our research problem and our data fit the assumptions of the Poisson model, especially in that alliance formation in a balance of power international system is said by Kaplan to be based on rational calculations of costs and benefits.

**Empirical Results**

Let us assume that in a balance of power system that is in equilibrium the decision makers of each essential actor have a propensity to form alliances with and against other essential actors that is rationally designed to keep the system in equilibrium. Let this alliance propensity be denoted by \( \alpha \) and let us additionally assume that each essential actor has about the same propensity. Then, if there are \( N \) actors in the system, the systemic propensity to form alliances during a fixed period of time, \( t \), will be \( \alpha N \). When our data on alliance formation frequencies among the five great European powers between 1814 and 1914 are arrayed as in Table 1, \( \alpha N = m = 0.545 \). That is, one alliance was formed about every two years, or 0.545 alliances per year; this is the average rate of alliance formation in Europe in the nineteenth century when \( t \) equals one year. Therefore, \( \alpha = 0.545/5 = 0.109 \), is our estimated actor propensity to form alliances. In applying the Poisson distribution to our data we therefore assume that \( m = 0.545/5 \) and \( \alpha = 0.109 \), is our estimated actor propensity to form alliances. In applying the Poisson distribution to our data we therefore assume that \( m \) is constant throughout the nineteenth century, and by implication since \( N \) is constant, \( \alpha \) was constant as well.

Table 1 represents our test of H1: that in a balance of power international system, the occurrence of alliances will be stochastically distributed over time. The evidence in the table strongly supports this hypothesis, for all alliances and for both formal and informal alliances. The Poisson expected frequencies are very close to observed frequencies; the Chi-square values indicate that in all three experiments more than 50 per cent to 90 per cent of comparable observations would show worse agreement; and the means and variances of the three empirical distributions are remarkably close. Table 1 indicates that formal and informal alliances are homogeneous, that they are part of the same alliance formation process. Moreover, since the alliances are Poisson distributed, we may conclude that they were generated by a Poisson process because "not only does

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94 That is, only when \( x^2 \) is large, so that for a given degree of freedom its probability is less than .10 or .05, is the null hypothesis rejected and the inference made that the observed distribution was not generated by a Poisson process. This is not a very conservative procedure. It unfortunately leaves open the possibility of Type II error, i.e., the inference that the null hypothesis prevails when in fact the alternate hypothesis is correct.


Table 1. The Formation of Alliances in Europe, 1814–1914, as a Poisson Distribution

<table>
<thead>
<tr>
<th>Experiment*</th>
<th>$x = 0$</th>
<th>$x = 1$</th>
<th>$x = 2$</th>
<th>$x \geq 3$</th>
<th>$x^{2b}$</th>
<th>$p$</th>
<th>$m$</th>
<th>$s_e^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Alliances:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observed $N_x$</td>
<td>61</td>
<td>31</td>
<td>6</td>
<td>3</td>
<td></td>
<td>.545</td>
<td>.501</td>
<td></td>
</tr>
<tr>
<td>$Np(x; 0.545)$</td>
<td>58.6</td>
<td>31.9</td>
<td>8.7</td>
<td>1.8</td>
<td>.338</td>
<td>&gt; .50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>“Formal” Alliances:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observed $N_x$</td>
<td>77</td>
<td>21</td>
<td>2</td>
<td>1</td>
<td></td>
<td>.277</td>
<td>.299</td>
<td></td>
</tr>
<tr>
<td>$Np(x; 0.277)$</td>
<td>76.6</td>
<td>21.2</td>
<td>2.9</td>
<td>0.3</td>
<td>.019</td>
<td>&gt; .80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>“Informal” Alliances:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observed $N_x$</td>
<td>78</td>
<td>21</td>
<td>3</td>
<td>0</td>
<td></td>
<td>.267</td>
<td>.256</td>
<td></td>
</tr>
<tr>
<td>$Np(x; 0.267)$</td>
<td>77.3</td>
<td>20.6</td>
<td>2.8</td>
<td>0.3</td>
<td>.012</td>
<td>&gt; .90</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


b The degree of freedom for Chi Square in each test is one.

The process yields the distribution but the distribution, with mean proportional to the length of the interval, can only arise from the process.79

Table 2 presents our test of $H_2$: that in a balance of power international system, the time intervals between alliances are randomly distributed. As the table indicates, the negative exponential Poisson distribution provides a good fit to the data when intervals of six months are the time units. The $x^2$ value of 4.74 does not permit us to reject the null hypothesis that the two distributions are similar. More than 30 per cent of comparable observations would be expected to show a worse fit. Hypothesis $2$ is therefore confirmed. We note, however, that the observed frequencies of less than six months and between thirty-six and forty-two months are somewhat higher than expected. This tendency for alliances in nineteenth-century Europe to be formed quickly upon one another or with a lag of about three and one-half years deserves further study.

In order to test $H_3$: that in a balance of power international system a decline in the systemic rate of alliance formation precedes system changing events, such as general war, we recoded our data into five-year periods as given in Figure 1. When so recoded, the number of five-year periods in which $x$ alliances occurred ($x \geq 0$), were also found to be Poisson distributed with a $x^2$ value of 1.11 with one degree of freedom, thus giving a $p > .20$. The closeness of fit between theory and observation is illustrated in Figure 2. This finding

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Table 2. Interalliance Intervals in Europe, 1814–1914, as a Poisson Distribution

<table>
<thead>
<tr>
<th>Length of Time Between Alliances in Six Month Units</th>
<th>Observed Relative Frequency</th>
<th>Expected Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.315</td>
<td>.222</td>
</tr>
<tr>
<td>2</td>
<td>.148</td>
<td>.174</td>
</tr>
<tr>
<td>3</td>
<td>.093</td>
<td>.135</td>
</tr>
<tr>
<td>4</td>
<td>.074</td>
<td>.105</td>
</tr>
<tr>
<td>5</td>
<td>.074</td>
<td>.082</td>
</tr>
<tr>
<td>6</td>
<td>.056</td>
<td>.064</td>
</tr>
<tr>
<td>7</td>
<td>.167</td>
<td>.050</td>
</tr>
<tr>
<td>8</td>
<td>.000</td>
<td>.038</td>
</tr>
<tr>
<td>9</td>
<td>.018</td>
<td>.030</td>
</tr>
<tr>
<td>10</td>
<td>.000</td>
<td>.023</td>
</tr>
<tr>
<td>11</td>
<td>.018</td>
<td>.018</td>
</tr>
<tr>
<td>12</td>
<td>.000</td>
<td>.014</td>
</tr>
<tr>
<td>13</td>
<td>.000</td>
<td>.011</td>
</tr>
<tr>
<td>14</td>
<td>.018</td>
<td>.009</td>
</tr>
<tr>
<td>15</td>
<td>.000</td>
<td>.007</td>
</tr>
<tr>
<td>16</td>
<td>.000</td>
<td>.006</td>
</tr>
<tr>
<td>17</td>
<td>.000</td>
<td>.004</td>
</tr>
<tr>
<td>18</td>
<td>.018</td>
<td>.004</td>
</tr>
<tr>
<td>Totals</td>
<td>.999</td>
<td>.996</td>
</tr>
</tbody>
</table>

$x^2 = 4.74$, d.f. = 4, $p > .30$

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Figure 2. Theoretical and Observed Distribution of Fifty-Five 19th Century European Alliances by Five-Year Period Under the Assumption that Alliance Formation is a Poisson Process ($m = 2.75$)
lends additional support to our confirmation of the first hypothesis.

Given that the five-year period average rate of alliance formation is \( m = 2.75 \), we can calculate the cumulative probability that the observed number of alliances formed in each period, \( x \), is less than \( m \). Taking our probabilities from Kitagawa, we have plotted and joined them in Figure 3. The vertical axis of the figure represents the cumulative probability that \( x < m \); as a probabilistic measure of changes in the rate of alliance formation it obviously ranges from zero to unity. Two horizontal axes are given in Figure 3. The lower one represents twenty periods of five-years length each. The upper time dimension divides the century into five diplomatic periods quite similar to Rosecrance’s five nineteenth-century periods of multipolar concert (1814–22), quasi-polar truncated concert (1822–48), multipolar concert (1848–71), unipolar concert and alliances (1871–90), and bipolar concert and alliances (1890–1918) but in fact based on Frank Denton’s hostility cycles which indicate that war intensity was highest at each demarcation point. Thus, the vertical dashed lines that separate the five periods represent the approximate points in time at which historical evidence suggests that the system changed to a new equilibrium state (Rosecrance) and at which quantitative data suggest war was most intense (Denton).

Figure 3 supports hypothesis 3, for the cumulative probability of observing no alliances being formed in the period 1910–1914, given that the century-long average activity rate of the system was 2.75, is only .064. This is the lowest measured alliance formation rate of the century except for 1820–1824, the quiet period after the burst of diplomatic activity that concluded the Napoleonic Wars. If we take the two diplomatic periods of 1868–1894 and 1895–1914 and calculate their five-year period activity rate, which is \( m = 3.3 \), the cumulative probability that \( x < m \) in 1910–1914 is just .037. Whichever way we look at it, a clear-cut decline in system flexibility occurred after 1909, and this period immediately preceded an event that destroyed the European balance of power, perhaps forever.

While we would not want to push it too far, there appears to be a serendipitous lining of some theoretical interest in Figure 3. Note that in four out of five instances of change from one diplomatic period to another a decline in the alliance formation rate occurred. Only in the change from 1846/67 to 1868/94 is the change point not crossed by a negatively sloped line. These five points represent the times at which war was most intense (Denton) and when diplomatic historians

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80 Kitagawa, Tables of Poisson Distribution, p. 65.
81 Rosecrance, Action and Reaction in World Politics, pp. 232–266.
point to changes in the structure of the system (Rosecrance). This finding would appear to lend further credence to our third hypothesis and it certainly merits further research.83

Interpretations and Conclusions

Our data strongly support our three hypotheses, and if these hypotheses have been reasonably inferred from Professor Kaplan's theory of the process of balance of power systems, we conclude that his theory has greater credibility than heretofore. The principal limitations of our study center

83 This research is under way. In a recent paper we correlated our alliance flexibility scores with the Singer and Small interstate war data for the same period of time and for our five actors only. We found strong and statistically significant evidence for the hypotheses that alliance formation (hence balance of power system flexibility) is negatively associated with the occurrence of war and war magnitude, severity and intensity (R. M. Rood and P. J. McGowan, "Flexibility in Balance of Power Alliance Systems and International War," a paper delivered at the Third Annual Conference of the Southern Section of the Peace Science Society [International], Durham: Duke University, April 4-5, 1974). Our findings represent an independent replication of the well-known results of Singer and Small that alliance aggregation in the nineteenth century was negatively related to warfare; see Singer and Small, "National Alliance Commitments and War Involvement, 1815-1945," and "Alliance Aggregation and the Onset of War, 1815-1945."

on the fewness of the testable propositions we were able to derive from Kaplan, their high level of generality, and the simplicity of the data with which they were tested. Our research cannot be regarded as definitive on any of the questions it asks, but we would argue that our evidence is decisive with respect to our three hypotheses and that our paper overall presents a potentially fruitful research strategy for work on alliance behavior and international systems research.

Further research topics immediately suggest themselves. Obviously, our study should be replicated on other data sets on nineteenth-century Europe and extended to alliance behavior in other historical balance of power systems. The implications of Figure 3 about systemic change and alliance behavior and the clustering of interalliance intervals in Table 2 should be looked at. If it is true that alliances are generated by a Poisson-type process, then theoretical models that account for other Poisson-type processes, such as subatomic behavior and telephone exchange performance, might well be adapted to the study of international systems. For not only are alliances Poisson-distributed, so are wars.84

84 Richardson, Statistics of Deadly Quarrels, pp. 128-142; Singer and Small, The Wages of War, pp. 205-206.