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Recoil effects and CP violation in neutron scattering

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The problem of the imitation of CP violation in neutron scattering is discussed. The thermal motion of nuclei cannot contribute to symmetry-violating effects. The influence of the nuclear depolarization due to neutron scattering is estimated.

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I. INTRODUCTION

After the observation of the abnormally large P-violating difference of the total cross sections $\Delta \sigma_P$ for the transmission of neutrons with opposite helicities through an unpolarized target in the vicinity of $p$-wave compound resonances [1] (see also Refs. [2–4]), it was suggested that one should look for $P$- and $T$-violating effects [5,6], which should also be enhanced [7]. According to the CPT theorem, the $P$ and $T$ violation in these cases is caused by the $CP$-noninvariant interaction of nucleons. The neutron-induced reactions make possible detailed investigation of the problem of $CP$ violation in low-energy physics [8]. The study of the $T$-violating $P$-odd effects in neutron scattering in the vicinity of $p$-wave compound resonances can give not only a large enhancement to help find $CP$ violation in nuclei but also information about the $CP$-violating mechanism. In other words, we have an additional possibility besides the neutron electric dipole moment and the $K^0$ and $B^0$ decays. It should be noted that the different models of $CP$ violation display different effects in these processes. For example, $CP$ violation which is caused by the $\theta$ term in QCD leads to $CP$-violating effects in neutron scattering but cannot produce $CP$ violation in $K^0$-meson decay.

Let us consider the $T$-odd and $P$-odd correlation $(\sigma[k \times I])$, where $\sigma$ and $I$ are neutron and target spins, and $k$ is the neutron momentum. This correlation leads [5,6] to the difference of the total cross sections for the transmission of neutrons, polarized parallel and antiparallel to the axis $[k \times I]$, through the polarized target

$$\Delta \sigma_{CP} = \frac{4\pi}{k} \text{Im}(f^0_{1,1} - f^0_{1,1}) .$$

(1.1)

Here $f^0_{1,1}$ are the zero-angle scattering amplitudes on the polarized nuclei for neutrons polarized parallel and antiparallel to the $[k \times I]$ axis, respectively.

It has been shown [7,9] that the quantity $\Delta \sigma_{CP}$ is proportional to $\Delta \sigma_P$, the corresponding $P$-violating difference of cross sections caused by the $T$-even $P$-odd correlation $(\sigma k)$, and therefore they have the same enhancement factors, which lead to their increase by a factor of $10^5$–$10^6$. For example, the relation between the corresponding total cross-section differences is [10]

$$\Delta \sigma_{CP} = \lambda \Delta \sigma_P ,$$

(1.2)

where $\lambda$ is the ratio of the $CP$-violating nucleon-nucleon coupling constant to the $P$-violating one. From Eq. (1.2), one can see that the measurement of the $CP$-odd and $P$-odd effects at the same $p$-wave compound resonance (where the values reach the maximum) leads to the possibility of extracting the ratio $\lambda$. According to the proposals to test for $CP$-odd effects in neutron scattering [11,12], there is the experimental possibility of obtaining an upper limit on $\lambda$ of less than $10^{-3}$ (or potentially about $3 \times 10^{-5}$ [13]), which could provide new constraints on some models of $CP$ violation.

One of the more important points of the $CP$-violation problem in neutron-induced reactions is the following: $T$-odd correlations in elastic scattering at zero angle cannot be imitated by a final-state interaction. Therefore, there are in principle no restrictions on the measurement accuracy for testing $CP$ violation [8]. However, to use this conclusion one should guarantee that the process is truly elastic. This problem for the case of an infinitely heavy target has been studied carefully in many papers. To provide background information, we will recall the main points of this case in Sec. II. The purpose of this paper is to evaluate the nuclear recoil effects. Section III is devoted to the consideration of the influence of nuclear thermal motion of the $T$-violating effects. The problem of target depolarization is discussed in Sec. IV.

II. NEUTRON PROPAGATION THROUGH AN INFINITELY HEAVY TARGET

First, let us recall that the time-reversal operator, unlike the parity operator, has no eigenstates and eigenvalues, but that it leads to a relation between two different processes. Let us consider the binary process $a + A \rightarrow b + B$ and also the reversed process. Through the $T$-invariance condition, these two different processes with inverted momenta and spins $(k_f \rightarrow -k_f, s \rightarrow -s)$ are related to each other:

$$\langle k_f, m_b, m_B| 4 \hat{P} | k_i, m_a, m_A \rangle = (-1)^{\lambda} \langle -k_f, -m_b, -m_B| 4 \hat{P} | -k_i, m_a, m_A \rangle .$$

(2.1)
Here \( \mathbf{k}, \mathbf{f} \) is the initial- (or final-) state momentum; 
\( v = \sum s_i - m_i \), where \( s_i \) and \( m_i \) are the spin of \( i \)th particle and its projection; and \( \hat{T} \) is a reaction matrix. In other words, the \( T \) invariance leads to no restrictions on the reaction amplitude.

However, if it is possible to describe some process in the first Born approximation (e.g., in the first power of the electromagnetic interaction), then from the unitarity property of the scattering matrix

\[
\hat{T}^\dagger = \hat{T} = i\hat{T} \hat{T}^\dagger ,
\]

one can obtain the condition of Hermicity for the reaction matrix

\[
\langle i | T | f \rangle = \langle f | T^\dagger | i \rangle ,
\]

Here \( i \) and \( f \) are the quantum numbers of the initial and final states. Using the time-reversal invariance condition (2.1) and Eq. (2.3), we obtain the expression

\[
\langle f | T | i \rangle = \langle -f | T^\dagger | -i \rangle^*,
\]

or

\[
|\langle f | T | i \rangle|^2 = |\langle -f | T^\dagger | -i \rangle|^2 ,
\]

where the minus sign indicates opposite signs for the particle spins and momenta in the corresponding states. From Eq. (2.5) we can see that the probability of this process is an even function of the time. In other words, when (and only in this case) the process is described in the first Born approximation, any \( T \)-odd correlation is connected with the \( T \)-violating interaction. In this case, the picture is similar to the well-known one for the \( P \)-odd quantities. This similarity is correct only up to the level of the \( T \)-odd correlation at which the other Born terms become significant. These terms (which are known as a final-state interaction) can produce the \( T \)-odd correlation without a \( T \)-violating interaction. An example of such a \( T \)-odd correlation being produced by a strong \( T \)-invariant interaction is the well-known right-left asymmetry in neutron reactions: \( \langle \sigma | \mathbf{k} \times \mathbf{k} \rangle \), where \( \sigma \) is the neutron spin, and \( \mathbf{k} \) and \( \mathbf{k} \) are the neutron and final particle momenta.

For an elastic-scattering process, the initial and final states coincide. Therefore, we can obtain Eq. (2.5) without the Hermicity condition from Eq. (2.3). Due to this, for elastic scattering, the \( T \)-odd correlation is always connected with a \( T \)-violating interaction. It should be noted that this conclusion is valid for the case of an infinitely heavy target.

However, it was shown [14] that it is impossible to measure the \( T \)-odd and \( P \)-odd difference of total cross sections (1.1) directly. The \( \Delta \sigma_{CP} \) is not a true elastic-scattering effect because we do not control a final state. Therefore, an interference effect may imitate the \( T \)-odd correlation in the way that was suggested in Refs. [5,6]. Indeed, in the majority of polarized targets, there exist strong magnetic fields \( (H) \) causing neutron spin precession with Larmor frequency \( \omega_L = (2\mu H)/\hbar \) (here \( \mu \) is the neutron magnetic momentum). As a result of this precession, the magnitude of the effect is reduced by a factor [14] \( v/(\omega_L I) \), where \( v \) is the neutron velocity and \( I \) is the sample length. Besides the magnetic fields, one should also consider the nucleon pseudomagnetism phenomenon [15], which consists of neutron spin precession around the target axis caused by the nuclear spin-spin interaction \( (\sigma_1) \). According to Refs. [15,16], the frequency \( \omega_p \) of this precession can be expressed in terms of zero-angle scattering amplitudes \( (f_+ \) and \( f_-) \) for neutrons with spin parallel and antiparallel to the target spin direction

\[
\omega_p = \frac{4\pi N\hbar I}{m} \frac{1}{2I+1} \text{Re}(f_+ - f_-) ,
\]

where \( m \) is the neutron mass. The sign and magnitude of \( \omega_p \) varies with energy and different target nuclei, having the characteristic value \( \omega_p \sim 10^8 \text{s}^{-1} \). As was mentioned, the \( P \)-violating and \( T \)-violating effects are proportional to each other and have the same energy dependence. When the Larmor and pseudomagnetic precessions cause the appearance of nonzero helicity, this helicity leads to \( P \)-violating effects which are several orders of magnitude larger than the \( P \)- and \( T \)-violating ones and completely camouflage them. Moreover, each slight inaccuracy in neutron spin orientation from the plane, which is orthogonal to the target spin, should cause a total cross-section difference arising from the strong spin-spin interaction.

However, as was shown in Refs. [14,17], it is possible in principle to reduce all the interference effects in measuring the \( P \)- and \( T \)-odd correlation (see also Refs. [18–20]). For completeness, we give a short discussion of the main points of such a reduction.

Due to the neutron spin precession in the target, the \( T \)-odd (and \( P \)-odd) correlation value will be decreased by the ratio

\[
\rho = \frac{\omega_p}{\omega_L} ,
\]

where \( \omega \) is the total frequency of the neutron spin precession. The reason for the appearance of this reduction factor is connected with the spin averaging of the \( T \)-odd correlation: a length of the target where the neutron spin has rotated through \( 2\pi \) produces a zero contribution to the effect. Therefore, a very important problem is the reduction of large spin rotation. Due to the coherent nature of neutron scattering at zero angle, one can obtain the expression for the frequency of the spin rotation as

\[
\omega = \omega_L + \sum c_i \omega_p .
\]

Here \( c_i \) is the relative concentration of the \( i \)th isotope in the sample, and \( \omega_p \) is the pseudomagnetic precession frequency for the isotope. Since \( \omega_p \), for different isotopes, differs both in magnitude and in sign (and is dependent on the neutron energy), one can in principle always obtain a crude cancellation of the pseudomagnetic and magnetic precession frequencies by the choice of the sample.

Thus, by a fine regulation of the magnetic field, one can obtain the necessary value \( \omega \) (with \( \rho \sim 1 \)) and control it by
a direct measurement of the angle of rotation of the neutron polarization. (For a proposal of such an experiment see, e.g., Ref. [11].)

As was mentioned above, the strong spin-spin ($\sigma I$) interaction causes large differences in total neutron cross section for different values of the projection of $\sigma$ on $I$. However, it is possible to control this interference effect using the nearest $s$-wave resonance (if necessary by introducing a slight admixture with strong nearby $s$-wave resonance into the target). The strong ($\sigma I$) interaction is enhanced in the vicinity of the $s$-wave resonance. Therefore, one can easily achieve the proper choice of the experimental conditions by reducing the relative contribution of this ($\sigma I$) interference in the vicinity of the chosen $s$ resonance to within the required limit. This guarantees the same relative reduction of the strong spin-spin effect in the energy region of interest to us.

Now we can consider one of the possible experimental procedures for the measurement of the $T$-odd correlation. First of all, one should reduce as much as possible the total precession frequency. To control this, one might measure the value of $\Delta\sigma_F$ on the unpolarized target and then vary the magnetic field of the polarized target to obtain almost the same value for $\Delta\sigma_F$. Next one should control the final neutron polarization by independent measurements (detailed discussion of the different possibilities for measuring the $T$-violating effects is given in Refs. [14,17,18]). Therefore, we can exclude the final-state interaction in the approximation of the infinitely heavy nuclei. Unfortunately, the nuclei of the target are permanently moving due to thermal motion. Moreover, the strong spin-spin interaction, which depolarizes the neutrons, also leads to the depolarization of the nuclei.

III. THE MOTION OF NUCLEI

The motion of nuclei in the target can lead to interference effects due to the influence of the Doppler effect on the form of the neutron resonances. It is obvious that the transverse (with respect to the direction of the neutron momentum) motion of nuclei cannot give any interference for a target with a cylindrical symmetry due to averaging over the azimuthal angle. We will show that the contribution from the Doppler effect for symmetry-violating correlations in neutron scattering is equal to zero if the temperature of the target is low.

Let us consider the influence of the Doppler effect on the total cross section. It is well known that, due to the thermal motion, the averaged neutron cross section is

$$\bar{\sigma}(E) = \int w(E,E')\sigma(E')dE', $$

(3.1)

where $\sigma(E')$ is the cross section for rest target and $E$ is the neutron energy. For simplicity, we will consider one resonance, the Breit-Wigner cross section. The relative energy distribution function (corresponding to the Maxwell distribution) is

$$w(E,E') = \frac{1}{\Delta \sqrt{\pi}} \exp \left\{ -\frac{(E'-E)^2}{\Delta^2} \right\},$$

(3.2)

where the Doppler width

$$\Delta = 2 \left( \frac{mET}{M+m} \right)^{1/2}. $$

(3.3)

Here $m$ and $M$ are the neutron and target masses, and $T$ is the target temperature. A low temperature for the target ($T \approx 10 \text{ K}$) leads to the value $\Delta \approx 2 \times 10^{-3} \text{ eV}$, which is less than the characteristic value of the total resonance width $\Gamma \approx 10^{-1}$. (Here the parameters of the $^{139}\text{La}$ $p$-wave resonance are used: $E = 0.74 \text{ eV}$ and $M = 139$.)

Further, the low temperature will mean the condition $\Delta/\Gamma < 1$ is satisfied. Using the expansion for the small parameter $\Delta/\Gamma$, one has in the vicinity of the resonance

$$\bar{\sigma}(E) = \sigma(E) \left[ 1 - 2 \frac{\Delta}{\Gamma} + O \left( \frac{\Delta^2}{\Gamma^2} \right) \right].$$

(3.4)

It should be noted that there are no terms in the expansion which are proportional to odd powers of the parameter $\Delta/\Gamma$ because the Doppler width arises from averaging the cross section but not the neutron scattering amplitude.

In the same way, one can obtain the value of the total cross section for the other symmetry-violating interactions, whether $P$ violating, $CP$ violating, or $T$ violating $P$ conserving. Let us consider the total cross sections $\sigma_+$ or $\sigma_-$ (which are dependent on the neutron spin orientation along a fixed axis) in the vicinity of a $p$-wave compound resonance. Defining $P = \sigma_+ / \sigma_-$, where $\sigma_{\text{viol}}$ is the part of the cross section due to the symmetry violation, one has the corresponding averaged cross section

$$\bar{\sigma}_\pm(E) = \sigma_\pm(E) \left[ 1 \pm P - 2 \frac{\Delta}{\Gamma} \right].$$

(3.5)

Taking into account that in Eq. (3.5) the symmetry-violating part changes sign with the change of the neutron spin orientation (because $\sigma_{\text{viol}}$ is proportional to one of the correlations $\sigma \cdot k$, $\sigma \cdot [k \times I]$, or $\sigma \cdot [k \times I] \cdot k \cdot I$ for the $P$-odd, $P$-odd and $T$-odd, or $P$-even $T$-odd effects, respectively) but that the symmetry-conserving part and the contribution from the Doppler effect do not change sign, we obtain for the difference of the total cross sections,

$$\sigma_+ - \sigma_- = \sigma(2P).$$

(3.6)

It should be noted that Eq. (3.6) is the correct result to any power in the parameter $\Delta/\Gamma$ because the Doppler effect gives the same contribution for $\sigma_+$ and $\sigma_-$ for all powers of the expansion if $\Delta/\Gamma < 1$. Therefore, if the target temperature is low enough, the thermal motion of nuclei cannot change the difference of the total cross sections for any of the symmetry violations under discussion. In other words, the difference of the total cross sections is independent of the Doppler effect.

IV. NUCLEAR DEPOLARIZATION

Let us consider the nuclear depolarization due to the strong spin-spin interaction of the neutrons and target nuclei. This phenomena is similar to the pseudomagnetic rotation of the neutron spin discussed in Sec. II; the spin
of the nucleus is rotated in the pseudomagnetic field which is produced by the polarized neutron beam. To calculate the magnitude of this spin precession, we use the expression for the difference $\Delta n$ of indices of refraction for the coherent neutron propagation through the target with opposite directions of neutron spin with respect to the nuclear spin [15]

$$\Delta n = \frac{4\pi N}{k^2} \frac{I}{2I+1} \text{Re}(f_+ - f_-), \quad (4.1)$$

where $k$ is the neutron momentum and other parameters are as in Eq. (2.6). Taking into account the Lorentz invariance of the ratio $f_+/k$, it is easy to evaluate the corresponding difference of the refractive indices for the coherent nucleus scattering on the neutrons in the neutron beam rest frame as

$$\Delta n_N = \frac{4\pi N}{k^2} \frac{I}{2I+1} \text{Re} \left( \frac{f_+ - f_-}{k} \right). \quad (4.2)$$

Here $N_N$ is the neutron density, and $k_N$ is the momentum of the nucleus. Then, the frequency of the nuclear spin precession is

$$\Omega_p = \frac{4\pi N}{m} \frac{I}{2I+1} \text{Re}(f_+ - f_-), \quad (4.3)$$

where $f_\pm$ are the neutron-scattering amplitudes in the nucleus rest frame.

The comparison of Eqs. (2.6) and (4.3) leads to the relation between the precession frequencies of neutrons and nuclei,

$$\Omega_p = \frac{N}{N_N} \Omega_p. \quad (4.4)$$

To estimate the characteristic value of $\Omega_p$, we calculate the neutron density as

$$N_n = \frac{\Phi c}{mc^2} \left( \frac{mc^2}{2E} \right)^{1/2} \approx 10^4. \quad (4.5)$$

Here we assume the neutron flux $\Phi = 10^{10}$ neutrons/cm$^2$s for the neutron energy $E = 1$ eV. Taking into account the characteristic values $\omega_p \approx 10^8$ s$^{-1}$ and $N \approx 10^{22}$, we obtain $\Omega_p \approx 10^{-10}$ s$^{-1}$. This value corresponds to the nuclear spin precession in the magnetic field $H \approx 10^{-14}$ G. Therefore, an experimental cycle of about $\tau \approx 10$ s provides an angle of nuclear spin rotation of about $\sim 10^{-9}$, which leads to a contribution to the relative CP-violating difference of the total cross sections due to the strong interaction at the level of $10^{-9}$. This contribution is negligible for the planned experiments [13,11] with the accuracy $\sim 10^{-3}$ to $10^{-5}$. However, it may be important for very large neutron flux.

V. SUMMARY

The analysis given shows that the contribution of thermal motion for symmetry-violating effects is equal to zero for targets with low temperature. The imitation of CP-odd effects due to the depolarization of nuclei from the strong spin-spin interaction can be negligible for the planned experiments if the neutron flux is not large ($\Phi \ll 10^{14}$ neutrons/cm$^2$s). If the flux is large, special efforts to prevent the depolarization and to control the contribution from the final-state interaction are needed.

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