

Summer 2023

## **Streamwise and Vertical Dispersal of Tracer Stones From a Continuously Supplied Source**

Amanda Grace Balkus

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STREAMWISE AND VERTICAL DISPERSAL OF TRACER STONES FROM A  
CONTINUOUSLY SUPPLIED SOURCE

by

Amanda Grace Balkus

Bachelor of Science in Engineering  
University of South Carolina, 2022

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Submitted in Partial Fulfillment of the Requirements

For the Degree of Master of Science in

Civil and Environmental Engineering

College of Engineering and Computing

University of South Carolina

2023

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## ACKNOWLEDGEMENTS

The work presented here would not have been possible without the support of my advisor, Enrica Viparelli, who has always been a huge inspiration for me in both my graduate and undergraduate studies. Her resiliency in the face of adversity as well as her dedication to research and support for her students is admirable and motivated me to grow both professionally and personally.

A special thanks to everyone in the Viparelli lab group, including my friends Will Logan, Sydney Sanders, Mahsa Ahmadpoor, and Heather O'Donal. Thanks to Ryan Johnson who spent countless hours with me in the lab and never failed to make me laugh. I would also like to thank all the other graduate students and postdocs I met along the way who introduced me to a variety of topics and fed my desire to continue learning throughout this journey. Lastly, I would like to thank my family for always supporting my endeavors.

## ABSTRACT

Individual movement of sediment grains is oftentimes studied with the use of tracer stones. Studies performed both in the field, in the laboratory and with numerical models involve installing a patch of stones (tracers) on the bed surface and rarely deep in the deposit. Once installed, particle displacement is monitored over time scales varying from one flood to several years. These studies are useful, for example, to study bedload transport dynamics, understand contaminant fate and transport as well as to identify and define stream restoration practices (e.g. quality of fish habitat). Here we use a vertically continuous model to study tracer dispersal when tracers are supplied from upstream at a constant rate. In particular, particle dispersal is examined in both the streamwise and vertical directions. The mathematical formulation is simplified by assuming uniform particle size where particle deposition is modeled in terms of a constant step length in an equilibrium bed. Numerical runs were made to simulate laboratory experiments and investigate the effects of sediment size, sediment transport rate, and bed level irregularities on the evolution toward an equilibrium tracer concentration profile in the alluvial bed. Simulations were performed until an equilibrium condition was reached, meaning the volume of tracers in the streamwise and vertical directions did not significantly change in time. At equilibrium, the volume of tracers present throughout the deposit was smaller in cases with large sediment sizes than in cases with small sediment sizes. In addition, smaller particles were shown to be present deeper in the deposit than coarser sediment sizes. In cases of high sediment transport rate, the average tracer elevation at equilibrium was lower

than in runs with small sediment transport rate, indicating that particles were buried deeper in the deposit. In runs with large bed level irregularities, particles were buried deeper in the deposit than in cases having small irregularities. In addition to this, as more vertical mixing occurred with higher dune heights, these cases took longer to reach an equilibrium state. The observed equilibrium tracer concentrations were the results of the interaction between the magnitude of bed level change around the mean value and the elevation of the maximum probability of particle entrainment in transport.

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# CHAPTER 1

## INTRODUCTION

Bedload transport of individual particles, commonly studied with tracer stones, is crucial in developing a more comprehensive understanding of sediment exchange between the riverbed and sediment transport. Tracer stones are particles that share a unique identifying characteristic, such as color, which is different than that which makes up the already existing bed deposit (Wong et al., 2007), so that they can be easily recovered over time. To simplify tracer recovery, magnetic tags (Hassan et al., 1999, Ferguson et al., 2002) and radio-frequency identification (RFID) tags (Bradley and Tucker, 2012, Brousse et al., 2019) have been used as unique identifiers. Experiments conducted both in the field and the laboratory often start by seeding a finite patch of stones on or near the bed surface, then monitoring particle displacement over time, with temporal intervals ranging from seconds to several years (Hassan et al., 1999, Ferguson et al., 2002, Wong et al., 2007, Bradley and Tucker, 2012, Brousse et al., 2019). These studies have led to the creation of various models to describe individual particle transport (Hassan and Church, 1994, Pelosi et al., 2014, Pelosi et al., 2016, Papangelakis et al., 2022, Viparelli et al., 2022), however, these models only consider the case in which tracers begin seeded on or near the bed surface, failing to account cases in which tracers are supplied to a river for a relatively long time.

Understanding individual particle movement can be useful in, for example, modeling fate and transport associated with contaminant spill (Dietrich et al., 1999, Day et al., 2008) as well as identifying stream restoration practices (Brousse et al., 2019). For

instance, Brousse et al. (2019) conducted a study on the dispersal of a patch of tracer stones to characterize bedload transport upstream of a reservoir, determine how much sediment was transported downstream, and assess the impact of adding gravel to improve the aquatic ecosystem in a regulated river reach. Studying sediment dispersal where particles are supplied from a continuous source can provide novel insight on sediment transport, erosion, and deposition within the river channel and the adjoining floodplain. For example, the dispersal of mine tailings containing copper in Papua New Guinea was monitored to characterize sediment exchange between a river and the floodplain (Day et al., 2008) and assess the response of sand bed rivers to overloading (Dietrich et al., 1999). Further, Vergilio et al. (2020) described ecological impacts following a collapse of a tailings dam in Brazil that caused a rise of heavy metal concentration, which is toxic to different trophic levels both in the aquatic and riparian ecosystems, as the contaminants do not only remain in the riverbed but have also been proven to deposit on the floodplain.

Morphodynamic models that consider the non-uniformity of the sediment (including tracer and non-tracers, mixtures of particles differing in size) typically account for streamwise displacement along the surface but fail to fully capture dispersal in the vertical direction (Wilcock and Crowe, 2003, Orrú et al., 2017, Papangelakis et al., 2022). This limitation occurs as the majority of these models is based on the active layer approximation, where the alluvial bed deposit is divided into an active layer and substrate, or bed deposit underneath the active layer (Parker et al., 2000). Particles in the active layer have an equal and finite probability of being entrained in transport and displaced, meaning the probability of particle entrainment does not vary in the vertical direction. Additionally, this approximation restricts the ability to account for fluxes between bedload particles and

particles in the substrate, as sediment in the substrate lacks the ability to be entrained in bedload transport at short time scales compared to time scales of channel bed aggradation and degradation. To overcome these challenges, Blom and Parker (2004), Pelosi et al. (2014, 2016) and Viparelli et al. (2017, 2022) modeled tracer dispersal and vertical sorting in an equilibrium bed with the vertically continuous rather than active layer-based morphodynamic framework presented in Parker et. al (2000).

Here we use the vertically continuous framework proposed by Parker et al. (2000) to model tracer dispersal from a continuous source in an experimental channel operated in sediment feed mode. As in Viparelli et al. (2022), we consider the case of uniform sediment size and constant particle step length (Einstein, 1950). Tracers are supplied to the upstream portion of the model domain, where only a fraction of what is being fed will contain these stones to resemble a contamination event more closely. The channel is assumed to be in equilibrium, that is where the overall average bed elevation does not change in time, despite local fluctuations caused by the displacement and entrainment of sediment particles. The vertically continuous approach is advantageous as it readily shows the portion of the bed deposit is involved in bedload transport. In performing this study on a laboratory scale, we can improve the understanding of individual particle movement under consistent flow conditions, whereas in the field there are disruptions to the flow that arise from flood events. With these simplifications, this study aims to serve as a basis towards being able to model fate and transport of sediment mixtures from a continuous source without utilizing an active layer.

Upon the introduction of tracers into the channel, we model the displacement and entrainment of sediment temporally and spatially until the system reaches a steady state,

defined as a condition where transport rate of tracers is equal to the tracer supply rate, and tracer composition in the deposit does not change in the vertical and in the flow direction. In laboratory experiments and field studies, where a finite number of stones are installed on the bed surface, steady state corresponds to a bed containing no tracers. Conversely, if tracer particles are constantly being supplied from upstream, the final steady state is characterized by a vertical profile of tracer concentration which is not significantly changing in time. Investigating the evolution towards this steady state and the characteristics of the vertical profile can provide information on bedload transport of uniform material that can be used in future work to account for the non-uniformity of sediment size. In this study, we seek to use tracer stones to determine the morphodynamic response to alterations in sediment conditions on the vertical profile, including sediment supply rate, particle size, and standard deviation of bed elevation changes.

This thesis is organized as follows, I first introduce a model describing tracer dispersal from a continuous source introduced to an equilibrium bed, using the vertically continuous morphodynamic framework presented by Parker et al. (2000). I then present the experimental conditions that were simulated on a laboratory scale, and how the sediment characteristics were varied for each experimental run. Upon reaching an equilibrium state, implications resulting from the changes in sediment characteristics were evaluated by examining the volume of tracers at different elevations along the streamwise direction.

## CHAPTER 2

### MODEL DESCRIPTION

To numerically evaluate streamwise and vertical distances traveled by tracer stones in an equilibrium bed, i.e. in the absence of aggradation and degradation, the equation of mass conservation of tracers at a given elevation within the deposit is solved. A schematic representation of the control volume considered here is shown in Figure 2.1, having a streamwise dimension  $\Delta x$  and a vertical dimension  $\Delta y$ . The vertical position of the control volume is denoted by  $y$  which is as an upward vertical coordinate having an origin located at the local mean bed level. The volume of sediment in the control volume is written as

$$V_{sediment} = -c_b B \int_y^{y+\Delta y} P_s dy \Delta x \quad (1)$$

where  $P_s$  is the exceedance probability of bed elevation,  $c_b$  denotes the sediment concentration in the deposit where  $P_s$  is equal to 1 for all practical purposes, and  $B$  is the channel width. The volume of tracers present in this control volume is equal to

$$V_{tracers} = -c_b B \int_y^{y+\Delta y} f_t P_s dy \Delta x \quad (2)$$

where  $f_t$  denotes the volume fraction content of tracers at streamwise location  $x$ , elevation  $y$ , and time  $t$ .

As shown in Viparelli et al. (2022), the equation of conservation of tracers at elevation  $y$  in an equilibrium bed (when  $P_s$  does not vary in time and space), takes the form

$$c_b P_s \frac{\partial f_t}{\partial t} = E_{ty} - D_{ty} \quad (3)$$

where  $E_{ty}$  and  $D_{ty}$  represent the entrainment and deposition rates per unit bed area of tracers at a given elevation, respectively. Therefore, the right-hand side of this equation denotes the net entrainment of tracers at elevation  $y$ , meaning the deposition rate of tracers is subtracted from the entrainment rate.

The value of  $P_s$  at various elevations was derived from time series of instantaneous bed elevations measured at equilibrium in lower plane bed laboratory experiments in lower plane bed laboratory experiments (Wong et al., 2007). Although there is no net aggradation or degradation at equilibrium, the bed level fluctuates around the local mean bed elevation  $\eta$  due to bedload transport. In particular,  $P_s$  was defined as the fraction of time that a given elevation is expected to contain sediment. This can be visualized by looking at the instantaneous bed elevation at a given location over a period of time for time scales where bed elevation changes are associated with bedload transport, as shown in Figure 2.2 for a time series recorded in experiments of tracer dispersal with gravel (Wong et al., 2007). The value of  $P_s$  at the elevation of the purple line corresponds to the fraction of time that the line falls at or below the bed surface elevation, i.e. in the deposit. A  $P_s$  value of 0 would thus indicate a point high in the water column, whereas a  $P_s$  value of 1 would denote a point deep in the bed deposit that does not interact with the water column. Wong et al. (2007) determined that in lower regime plane bed equilibrium experiments the probability density function of instantaneous bed elevation  $\eta'$  can be represented by a normal distribution. The exceedance probability of bed elevation  $P_s$ , or the probability the  $\eta' > y$  was therefore set equal to

$$P_s = 1 - \frac{1}{\sqrt{2\pi}\sigma_\eta} \int_{-\infty}^y \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_\eta}\right)^2\right] dy \quad (4)$$

where  $\sigma_\eta$  is the standard deviation of bed level fluctuations around the local mean bed elevation.

## 2.1 CALCULATION OF ENTRAINMENT AND DEPOSITION RATES

Particle entrainment and deposition rates are determined with a constant particle step length approach. It is thus assumed that once any particle is entrained in transport, it moves a constant, specified distance each time, which is designated as the step length, shown in Figure 2.3. Particle step length, denoted by  $\Lambda$ , is assumed to linearly scale with the sediment grain size (Einstein, 1950) as  $\Lambda = \lambda d$  where  $\lambda$  is a non-dimensional number equal to 150 based on the step lengths of Wong et al. (2007), and  $d$  is particle grain size, assumed to be uniform.

At equilibrium, the total entrainment rate  $E$  is given as  $E = q_b/\Lambda$ , where  $q_b$  is the volumetric bedload transport rate per unit width (Einstein 1950). In a sediment feed flume at equilibrium,  $q_b$  is equal to the sediment supply rate per unit width that does not change in space and time to ensure mass conservation (Parker, 2004), therefore entrainment and deposition rates do not vary in space and time. The elevation specific rates for tracer entrainment and deposition for bedload transport are computed as follows

$$E_{ty} = f_t E p_{ent} \quad (5)$$

$$D_{ty} = E_t|_{x-\Lambda} p_{dep} \quad (6)$$

where  $p_{ent}$  is the probability of entrainment and  $p_{dep}$  is the probability of deposition, where  $E_t|_{x-\Lambda}$  denotes the entrainment rate of tracers one step length upstream of the cross section



at  $x$ . Because the total deposition rate at  $x$  is equal to the total entrainment rate at  $x-\Lambda$ , we can rewrite the deposition rate of tracers as

$$D_{ty} = EF_t|_{x-\Lambda}p_{ent}$$

where  $F_t$  denotes the fraction of tracers in the bed surface sediment and is defined as (Viparelli et al., 2022)

$$F_t|_{x-\Lambda} = \int_{\text{deep in the deposit}}^{\text{high in the water}} f_t p_{ent} dy \quad (8)$$

As in Viparelli et al. (2022), the probability of entrainment,  $p_{ent}$ , was modeled as

$$p_{ent} = \frac{1}{\sqrt{2\pi}\sigma_\eta} \exp\left[-\frac{1}{2}\left(\frac{y - y_o}{\sigma_\eta}\right)^2\right] \quad (9)$$

where the Gaussian shape of  $p_{ent}$  is the result of the analysis by Ghasemi (2020) of discrete element modeling results for lower regime plane bed. The parameter  $y_o$  is an offset distance from the local mean bed elevation representing the elevation of the maximum probability of entrainment. For lower regime plane bed equilibrium conditions,  $y_o$  is equal to  $n_o\sigma_\eta$ , where  $n_o$  is a function of the Shields number and is shown in Figure 2.4 (Viparelli et al., 2022).

To determine the Shields number, i.e. a non-dimensional bed shear stress, the volumetric bedload transport rate per unit width  $q_b$  was calculated by dividing the imposed feed rate  $G_b$  by the sediment density to get a volumetric feed rate  $Q_b$  which was then divided by the flume width  $B$ . The Einstein number  $q^*$  was then computed as  $\frac{q_b}{\sqrt{RgD}}$  where  $R$  is the submerged specific gravity of the sediment and  $g$  is the acceleration of gravity. To find the Shield number  $\tau^*$ , the Ashida and Michiue (1972) bedload transport relation for mixtures was used. This bedload relation takes the form

$$q^* = 17(\tau^* - \tau_{sc}^*)(\sqrt{\tau^*} - \sqrt{\tau_{sc}^*}) \quad (10)$$

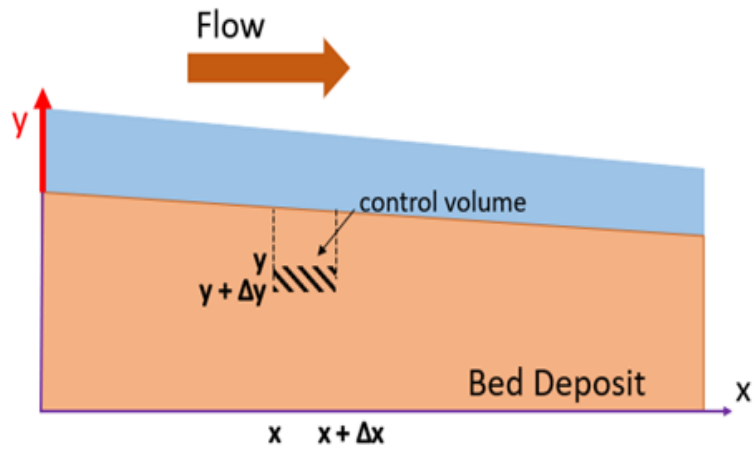
where  $\tau_{sc}^*$  is a critical Shields number for the initiation of significant bedload transport and the value of 0.05 was used in the calculations (Ashida and Michiue, 1972).

## 2.2 MODEL IMPLEMENTATION

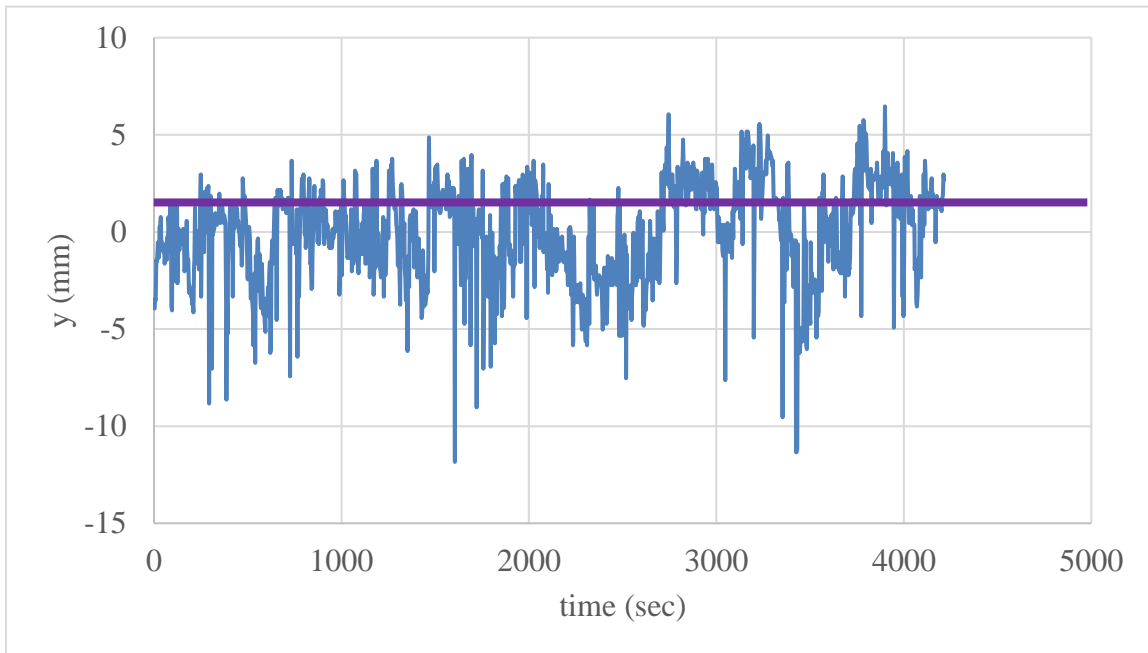
Substituting equations 5 and 6 into equation 3, the final form the elevation-specific equation of conservation of tracer stones is found.

$$\frac{\partial f_t}{\partial t} = \frac{E}{c_b P_s} p_{ent}(F_t|_{x-\Delta} - f_t) \quad (11)$$

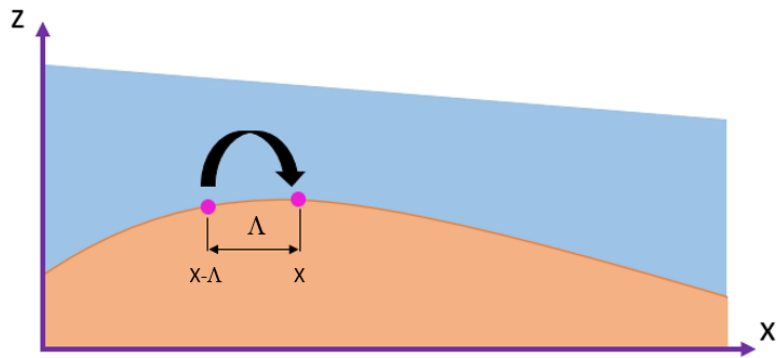
This equation is used to model the streamwise and vertical dispersal of tracer stones in an equilibrium bed when numerically integrated (Viparelli et al. 2022). For this study, it is assumed that initially the bed is at equilibrium and contains no tracers before a specified volumetric rate of tracers starts to be fed at the upstream end. The model in Viparelli et al. (2022) was thus modified to ensure that tracers were being fed into the flume at the proper rate. In particular, a “ghost reach” was added upstream of the model reach. In this ghost reach bed elevation, flow characteristics, and the volume fraction content of tracers  $f_i$  did not vary in space (vertical and streamwise directions) and time and was equal to 10%. Once the model begins to run, tracers are entrained in transport in the ghost reach and transported downstream into the model domain where they start mixing with sediment of the existing bed deposit.



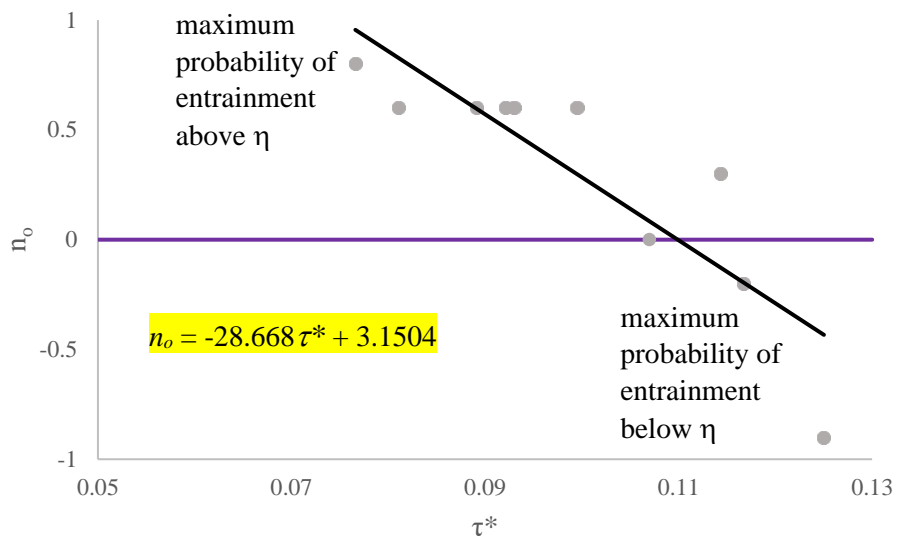
**Figure 2.1:** Schematic representation of the bed deposit where  $x$  is a streamwise coordinate and  $y$  is an upward vertical coordinate measured from the local mean bed elevation  $\eta$ .



**Figure 2.2:** Instantaneous bed elevations for the lower regime plane bed experiments of Wong et al., 2007.



**Figure 2.3:** Schematic representation of particle step length,  $\Lambda$ .



**Figure 2.4:** Relation between  $n_o$  and Shields number  $\tau^*$  determined in Viparelli et al. 2022.

## CHAPTER 3

### OVERVIEW OF NUMERICAL SIMULATIONS

Numerical runs simulate laboratory experiments that take place in a glass flume operated in sediment feed mode with a width of 0.2 m and streamwise length of 20 m. In all cases the volume fraction content of tracer stones is equal to 10% of the sediment supplied to the flume and the rest is the same sediment that makes up the already existing bed deposit. The tracers are introduced to an equilibrium bed, meaning there is no net aggradation or degradation, and to a bed deposit which does not contain any tracers prior to the start of the simulation. The magnitude of fluctuations around the local mean bed level  $\eta$  is controlled by the standard deviation of bed level fluctuations  $\sigma_\eta$ . The higher the value of  $\sigma_\eta$ , the larger the bed level will fluctuate around this mean (see Figure 2.2, equation 9).

Characteristics of the sediment for each experimental run are summarized in Table 3.1, where  $d$  is the uniform grain size,  $G_s$  is the sediment supply rate, and  $\sigma$  is the standard deviation of local bed level change. To understand the result sensitivity to the model parameters, I systematically changed one of these parameters at a time. Run 2 serves as a control run, meaning it uses the base values of all parameters that were investigated in this study. Alterations in short term bed level fluctuations were investigated in run 1 to run 4, where  $s$  changed from 1 cm to 4 cm. The rate at which sediment was fed to the flume was modified in runs 5 to 7, where  $G_s$  ranged from 360 to 720 g/min. In runs 8 to 11, the grain size of the sediment was increased from the original diameter of 0.5 mm up to 1.5 mm.

Each simulation modeled experiments that lasted for at least 100 hours, with printouts of results in 2-hour increments. This total time duration ensured tracer equilibrium was achieved in all experimental runs. Equilibrium was identified as the point in time where the vertical profile of tracers concentrations does not vary significantly in time at the downstream-most cross section of the model reach. In particular, tracer equilibrium was quantitatively defined as the time period when the percent change between the sum of the volume of tracers at the downstream end of the flume (20 m) was less than 0.5% between two 2-hour increments. The time each run arrived at a steady state of tracer volume concentration at the downstream for each run fell between 26 to 46 hours.

**Table 3.1:** Summary of numerical runs  
studied in this modeling exercise.

Run	Grain size $d$ (mm)	Feed rate $G_s$ (g/min)	$\sigma$ (cm)
1	0.5	480	1
2	0.5	480	2
3	0.5	480	3
4	0.5	480	4
5	0.5	360	2
6	0.5	600	2
7	0.5	720	2
8	0.75	480	2
9	1	480	2
10	1.25	480	2
11	1.5	480	2



## CHAPTER 4

### RESULTS

As tracers are continuously supplied to the model domain, an equilibrium profile of tracer concentration is eventually reached everywhere in the bed deposit. This tracer equilibrium, or steady state, was reached in each numerical run no later than 46 hours of simulated time with tracers being supplied from the ghost reach. Section 4.1 analyzes how this steady state equilibrium is approached over time. All figures shown in this section correspond to Run 2, as each model run does not vary qualitatively in how each simulation approaches equilibrium. To investigate the impacts of changing the characteristics of the sediment, section 4.2 discusses vertical profiles for each simulation performed with different grain sizes (Runs 2, 8, 9, 10, and 11). For all figure references in this section, panel “a” refers to the volume of tracers at the downstream end of the model reach, and panel “b” displays the mean tracer elevation across the model reach.

#### 4.1 APPROACHING STEADY STATE EQUILIBRIUM

To monitor tracer dispersal, we look at how the vertically averaged tracer elevation varies along the test reach in time (Figure 4.1). Initially, tracers are able to bury deeper in the upstream section of the model domain than the downstream section (light red lines). As vertical mixing slows down over time due to the entrainment of tracers previously deposited and more tracers get transported downstream, the mean tracer elevation tends to become uniform along the entire test reach. In addition to this, as time passes the average elevation moves deeper in the deposit because the few particles that are buried in the

deposit are hard to be re-entrained in transport. The downward movement of the vertically averaged tracer elevation of tracers slows down in time, indicating that the tracer concentration throughout the deposit tends to become uniform. This vertical slowdown can also be visualized by plotting the local average tracer elevation in time (Figure 4.2). At earlier times of the simulation, at different streamwise locations, average tracer elevations are different along the deposit. Every streamwise location, however, asymptotically approaches the same average equilibrium elevation, with the upstream sections approaching this equilibrium value more quickly. The standard deviation of tracer elevation follows a similar trend where an equilibrium value is asymptotically approached and is approached more quickly in the upstream of the model domain (Figure 4.2). The variance is smaller in the early stages of the simulations because some tracers are buried deep in the deposit, where the probabilities of entrainment and deposition are low.

Another way to visualize how tracer concentration approaches equilibrium (steady state) is to look at the variation of the fraction of sediment in the bed surface sediment, defined in equation 8, over time along the streamwise direction of the test reach. This is displayed in Figure 4.3 which shows the volume fraction content of tracers in the bed surface sediment for Run 2 as it approaches equilibrium. As equilibrium is reached, the fraction of tracers present in the bed surface approaches the fraction content of tracers that is fed from the upstream. Similar to when viewing the mean tracer elevation and standard deviation over the test reach, this steady state fraction of tracers is first reached upstream near where the sediment is being supplied, and then approaches this value more slowly tracers are transported downstream.

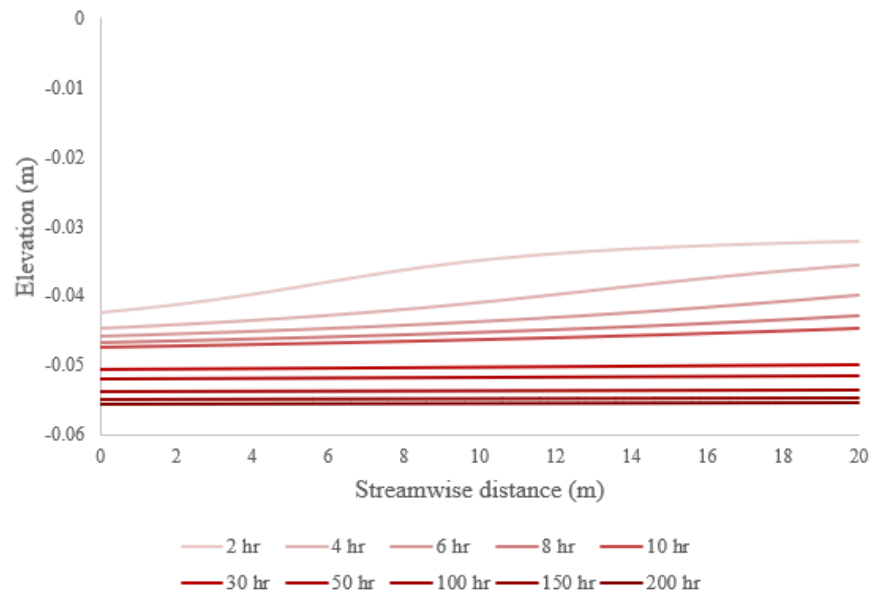
To understand how tracers disperse vertically in the deposit, volume of tracers is plotted along elevation at specified streamwise coordinates. Figure 4.4 shows these vertical profiles at 5 m, 10 m, 15 m, and 20 m downstream of the model domain for Run 2. Elevation 0 m corresponds to the local mean bed elevation  $\eta$ . The presence of tracers above  $\eta$  occurs due to the presence of sediment for short-term bed level fluctuations resulting from bedload transport. Each vertical profile approaches an unchanging volume of tracers throughout the deposit, meaning a steady state is reached. As for the vertically averaged quantities, steady state is reached first upstream and is approached more gradually continuing downstream.

#### 4.2 IMPACT OF MODEL PARAMETERS ON STEADY STATE PROFILES

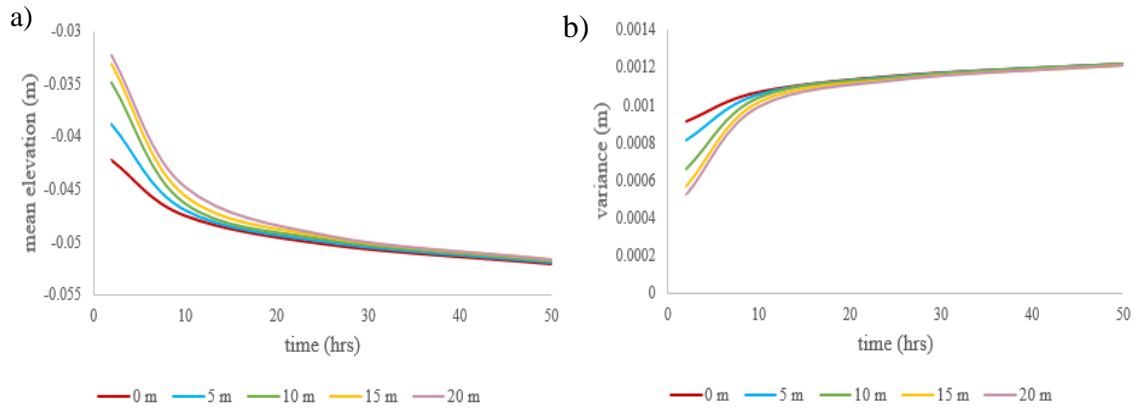
Increasing the standard deviation of bed level fluctuations results in a larger variation of instantaneous bed elevation around the local mean bed level. The larger these fluctuations are, the particles become buried deeper into the alluvial deposit (Figure 4.5). Likewise, the closer the bed is to resembling a plane bed with amplitude of the fluctuations on the order of the sediment grain size (Wong et al., 2007), particles tend to remain closer to the surface as less vertical exchange between the tracers and the existing deposit can occur. The time it took for each of these simulations to reach equilibrium, defined following the procedure outlined explained in section 2, was determined and plotted as a function of the imposed standard deviation of bed level fluctuations,  $\sigma$  (Figure 4.6). It takes a shorter amount of time to approach tracer equilibrium in cases of lower bed level fluctuations since a smaller volume of tracers is required to be exchanged between the existing deposit and the load to reach this steady vertical profile.

As sediment is fed to the flume at higher rates, equilibrium bed shear stress increases, and the elevation of the maximum probability of entrainment becomes deeper in the deposit as shown in Figure 2.4. Because of this, particles can be entrained in transport and deposited deep in the deposit and the average equilibrium tracer concentration occurs deeper in the deposit (Figure 4.7). In addition to this, runs with high sediment transport rates reach steady state more quickly as larger volumes of particles are exchanged between the bedload transport and the bed deposit.

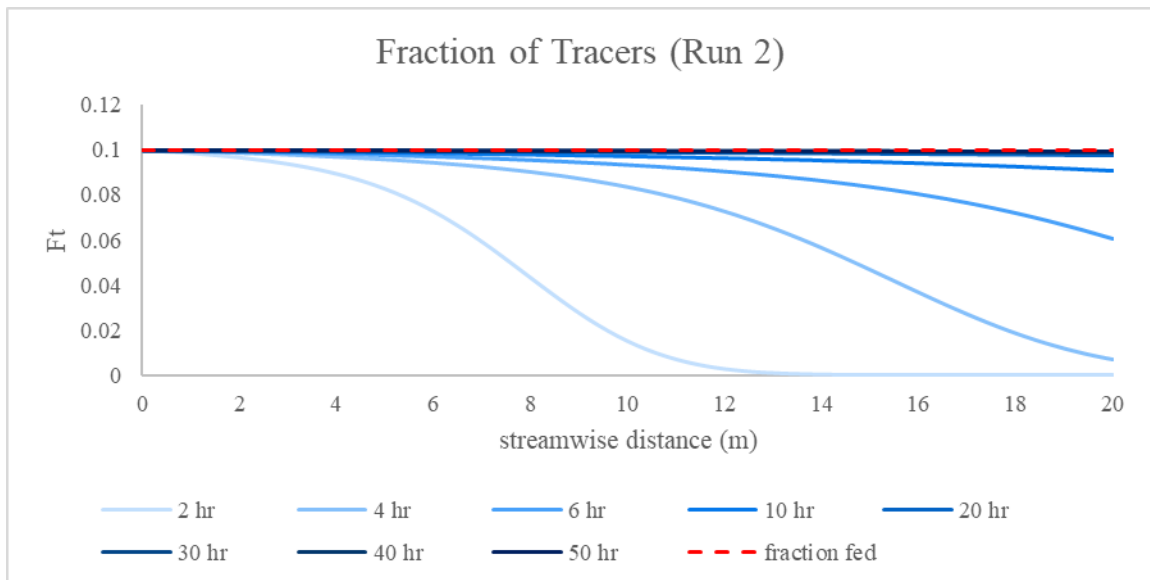
When a bed deposit made of a uniform sediment size is considered, tracers of coarse grain sizes tend to remain closer to the bed surface than finer tracers (Figure 4.8). For the same volumetric sediment load, the equilibrium entrainment rate, equal to  $q_b/\Delta$ , is larger for finer particle sizes than it is for coarser sizes. Because of this, finer particles are transported more easily and therefore are buried deeper in the deposit (Figure 2.4, equation 9). In addition, as fine grains move more readily than coarser grains, cases with finer particle sizes approach steady state more quickly than with larger particle sizes.



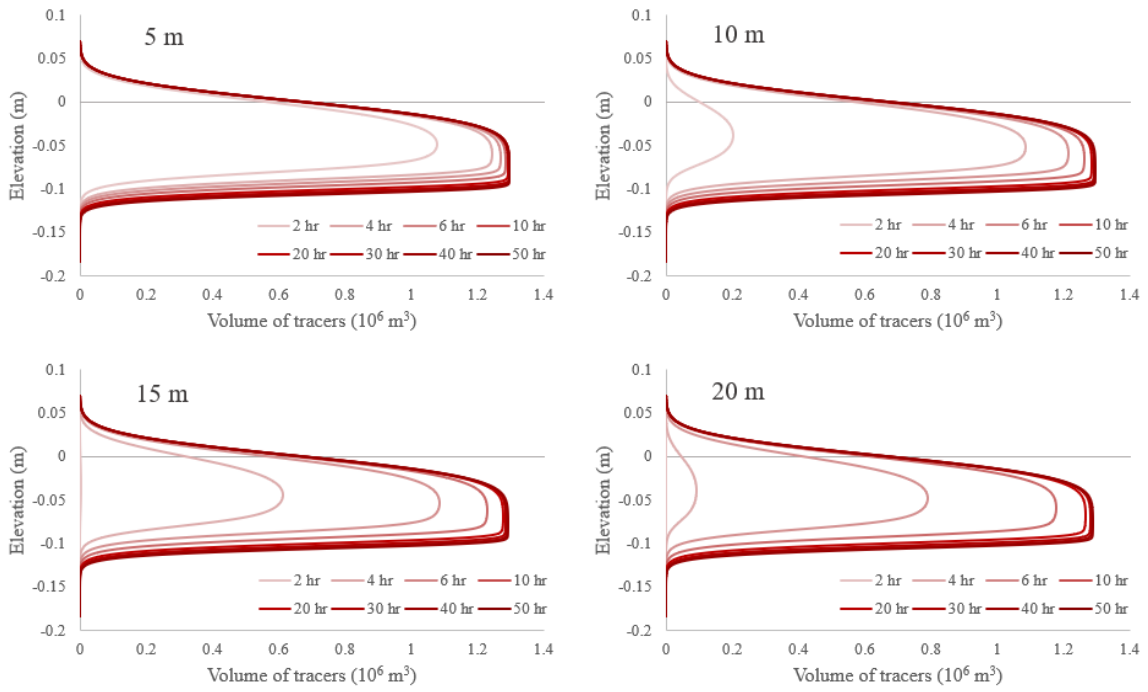
**Figure 4.1:** Temporal change of the average tracer elevation along the entire test reach of 20 m for Run 2.



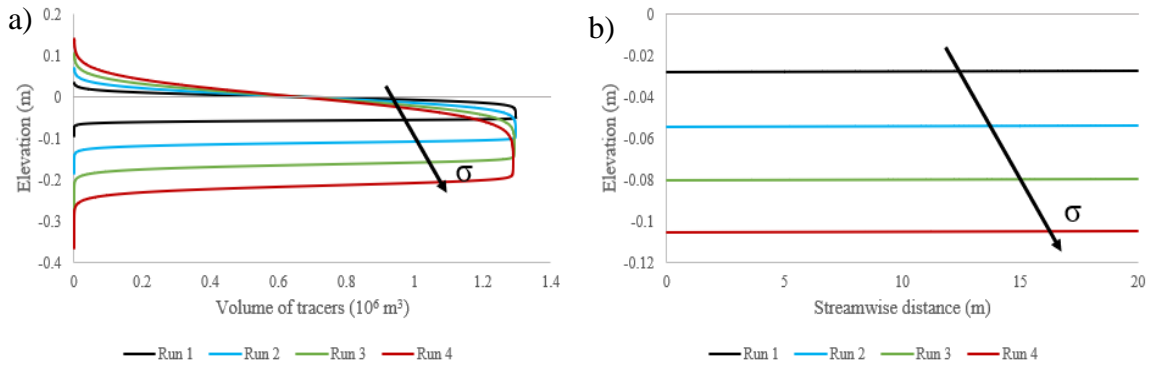
**Figure 4.2:** Comparison of (a) average tracer elevation and (b) variance at different streamwise coordinates for Run 2.



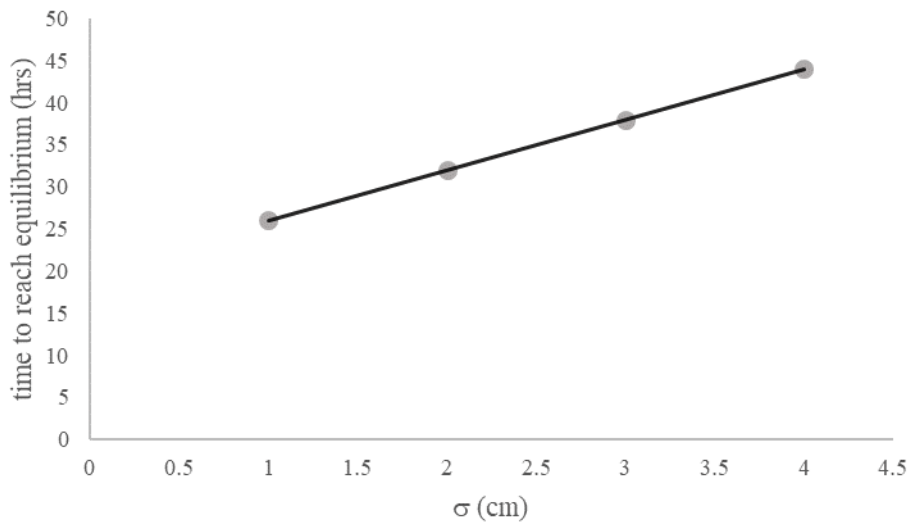
**Figure 4.3:** Time series of the fraction of tracers along the entire model reach for Run 2.



**Figure 4.4:** Comparison of the volume of tracers in the deposit at channel cross sections at 5 m increments along the streamwise direction.

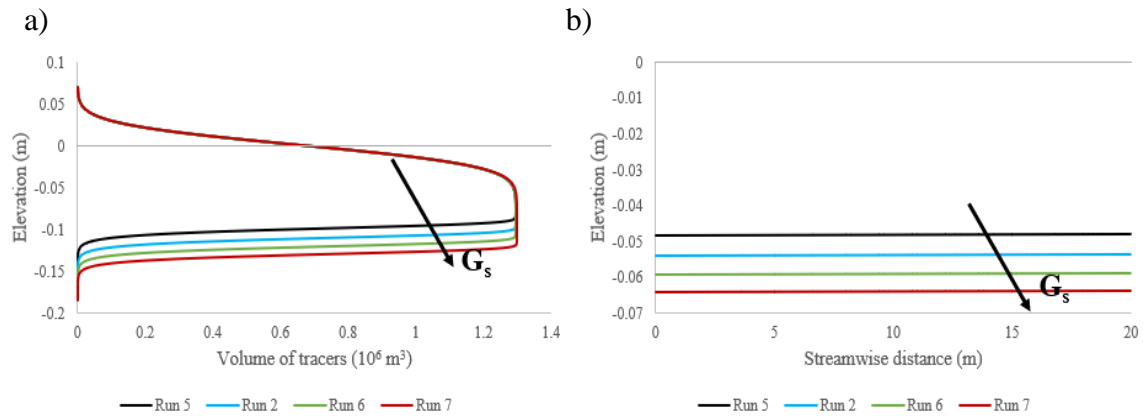


**Figure 4.5:** Comparison between experimental runs in terms of a) volume of tracers and b) mean tracer elevation where the standard deviation of bed level change  $\sigma$  was varied.

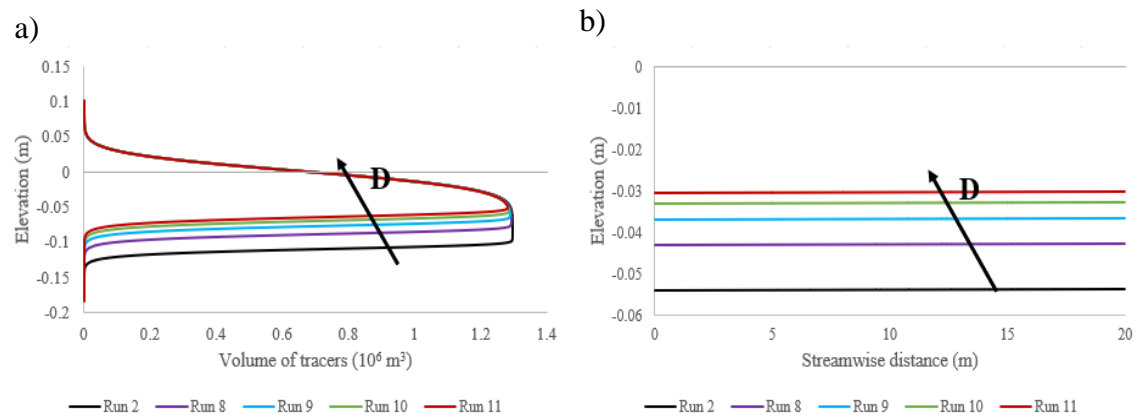


**Figure 4.6:** Relationship between the time it takes to reach equilibrium and the standard deviation of bed level change  $\sigma$ .





**Figure 4.7:** Comparison between experimental runs in terms of a) volume of tracers and b) mean tracer elevation where the sediment supply rate  $G_s$  was varied.



**Figure 4.8:** Comparison between experimental runs in terms of a) volume of tracers and b) mean tracer elevation where the grain diameter  $d$  was varied.

## CHAPTER 5

### DISCUSSION

A vertically continuous model used to study tracer fate and transport in both the streamwise and vertical directions supplied from a continuous source. The model used here could be validated using data from future laboratory experiments conducted in a glass flume. To expand upon this model, field studies from sediment supplied from a continuous source, such as mine tailings, can be conducted and compared against model results. One such area of interest that could serve as a potential model application is for studies done to develop a better understanding of the increased sediment load due to mining, such as what was studied by Day et al (2008) on the Fly River in Papua New Guinea.

To begin the conceptualization of this model, several assumptions were made to simplify the formulation. One such instance of this was that the model assumes a single characteristic grain size. This assumption allows us to overcome the complexities that arise from mix-size sediments which impose certain effects that are difficult to account for with our current mathematical understanding of how to model the vertical sorting and these processes such as armoring with a vertically continuous approach. The model also relies on the assumption that the channel bed is in equilibrium. In an equilibrium bed, the probabilities of entrainment and deposition, exceedance probability of bed elevation, and deviation of bed level fluctuations are unchanging in both spatially and temporally. This assumption is limiting, however little is currently known on how these parameters would adjust as the bed moves from one equilibrium state to another, where flow conditions and

bedform geometries change. The assumption of a constant step length also serves as a good starting point for the model, however it is unrealistic as the distance an individual particle travels is stochastic in nature.

Describing individual particle movement in modeling is oftentimes aided with the implementation of various probability density functions (PDFs) (e.g. Bradley and Tucker, 2012, Pelosi et al., 2014, Ghasemi, 2020, Viparelli et al., 2022). In this study,  $P_s$  was modeled using a Gaussian distribution as Wong et al. (2007) found this to be a fitting distribution from data obtained experimentally. Ghasemi (2020) found that the Gaussian distribution was best to model the probability of entrainment in the case of lower regime plane bed from DEM modeling results. As a result of these findings, the work of Viparelli et al. (2022) and for this study, a normal distribution was used for both  $P_s$  and the  $p_{ent}$ . Pelosi et al. (2014) also applied the vertically continuous framework formulated by Parker et al. (2000) and described the vertical dispersion using the probability that a particle jumps from a given elevation. This was compared using two PDFs, one where the distribution was assumed to be Gaussian, having thin tails, and the other employed an  $\alpha$ -stable PDF for a heavy tail distribution. Using a PDF with heavy tails is useful in modeling  $p_{ent}$  because it offers a method in understanding how the probability a particle entrains decays as a power law rather than exponentially as  $y$  approaches  $\infty$ , while still maintaining the overall shape of a normal distribution. From this comparison, Pelosi et al. (2014) found that both tails represented subdiffusive behavior, however due to higher probabilities of sediment migrating deeper in the bed it was found that heavy tail distributions reflected less subdiffusive behavior than thin tail distributions. Viparelli et al. (2022), on the contrary, found that superdiffusion and subdiffusion can also occur solely from varying the standard

deviation of bed level fluctuations of Gaussian distributions. Implementing other distributions to describe  $p_{ent}$ , particularly with the use of field and laboratory could give more insight as to how having a higher probability of entrainment deeper in the deposit impacts the steady state profiles shown in this study.

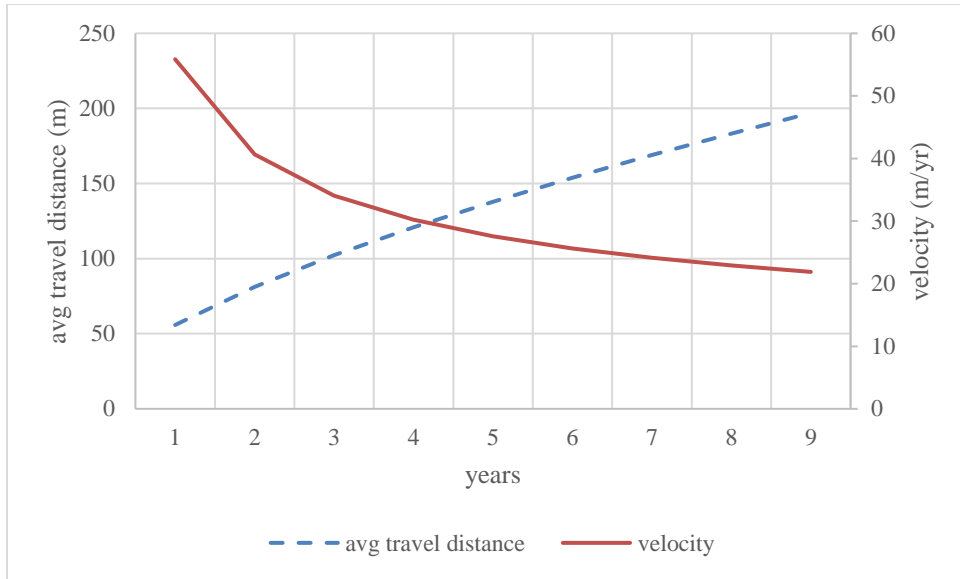
A common observation reported in many tracer studies is tracer slowdown (Hassan and Church, 1994, Ferguson et al., 2002, Wong et al., 2007, Bradley and Tucker, 2012, Pelosi et al., 2014, Pelosi et al., 2016). Over time, as tracers initially move and get buried deep in the deposit, it becomes more difficult for the particles to get re-entrained in transport. As a result, the velocity of tracers tends to decrease as time goes on. To see if modeling dispersal from a continuous source could capture this, the test reach was extended to 1000 m and other model parameters were changed to resemble those of a mountain stream, particularly one in Colorado, Halfmoon Creek (Bradley and Tucker, 2012, Viparelli et al., 2022). Model parameters are summarized in Table 5.1. The simulation was run for a time period of 9 years, and results were analyzed each year. The velocity of the centroid of the streamwise tracer distribution at each year continued to decrease, indicating that the model is capturing tracer slowdown even as tracers are being continuously supplied, as shown in Figure 5.1.

The model presented here is also in agreement with the work of Hassan and Church (1994), where it was found that after many flow events, a uniform profile of tracer concentration occurs in the vertical direction. It was assumed that after many flow events, tracers were able to reach a steady state in the model implemented by Hassan and Church, which was then compared to the model of continuously supplied tracers at equilibrium as well. To apply this, the proportion of tracers at layers of 0.02 m thickness was plotted

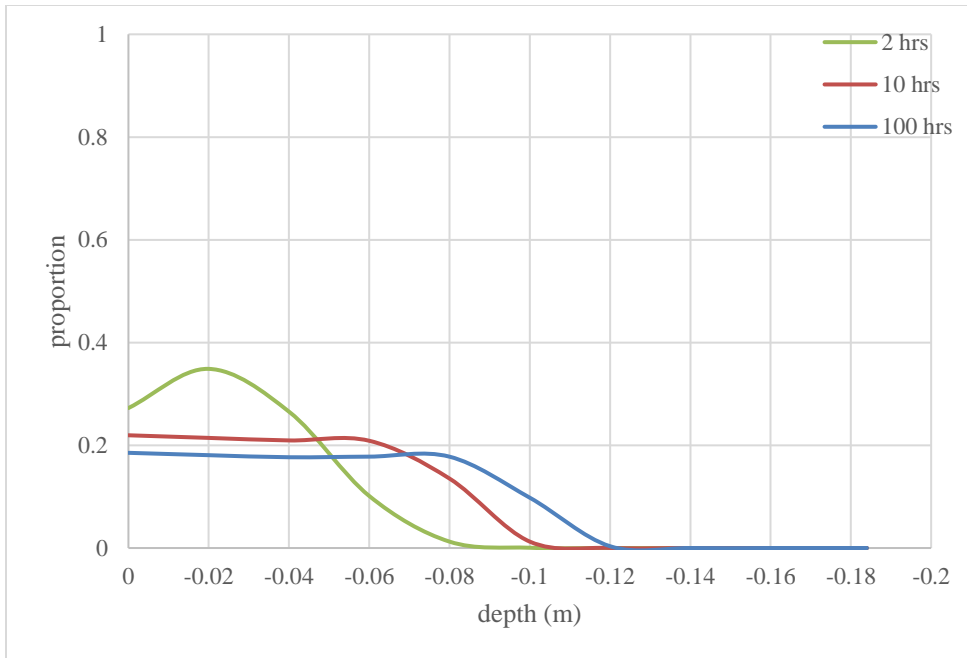
against the depth. This was done at three different time periods to see how the proportions changed as steady state equilibrium was approached, shown in Figure 5.2.

**Table 5.1:** Model parameters for field simulation to Halfmoon Creek where input parameters are from Bradley and Tucker (2012) and Mueller and Pitlick (2005).

<b>Parameter</b>	<b>Value</b>
Reach length $L$	1000 m
Effective channel width $B$	8 m
Bed slope $S$	0.0086
Grain size $d$	55 mm
Feed rate $Q_s$	160 ton/yr
Shields number $\tau^*$	0.046
Flow intermittency $I_f$	0.05
Nondimensional step length $L$	150
Standard deviation of bed level fluctuations $\sigma_n$	0.22 m
Number of simulated years	9 yr
Submerged specific gravity of the sediment $R$	1.65
Sediment concentration in the deposit $c_b$	0.65
Number of computational nodes in the x direction $N_x$	101
Distance between two nodes in the x direction $dx$	10 m
Number of computational nodes in the vertical $N_{ver}$	2500
Temporal increment in the integration of Equation 11	0.005 days



**Figure 5.1:** Travel distance and velocity to show tracer slowdown.



**Figure 5.2:** Temporal variation of proportion of tracers in vertical layers.

## CHAPTER 6

### CONCLUSION

The elevation specific equation for tracer conservation was numerically solved in this study to investigate streamwise and vertical displacement of tracer stones with a vertically continuous model, in a case where tracer stones are consistently fed in the upstream section of the model domain. Eleven model runs were performed with varying sediment conditions in an experimental channel to study the effects of tracer dispersal under consistent flow conditions. Results from this study show that a steady state volumetric concentration of tracers in the deposit at each streamwise point is achieved over time. This steady state is achieved more quickly in the upstream-most part of the model domain, and later in the downstream-most section. When a larger standard deviation of bed level fluctuations is imposed, this steady state covers a larger portion of the bed deposit as the tracers mix deeper as the bedform amplitudes are increased thus more vertical mixing occurs. This numerical exercise also showed that the mean elevation of tracers at steady state is lower when the tracers are supplied at a higher rate as more tracers are available to exchange with the existing bed deposit. In addition, larger grain sizes had a more difficult time burying in the bed deposit, as smaller particles can more easily exchange with each other under a uniform grain size distribution. The accuracy of the model can be determined in future work through experimental validation. The model's complexity can be expanded upon by accounting for sediment mixtures rather than assuming a uniform size, as well as applying it to field studies.



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