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History and Philosophy of Feynman's Electrodynamics: From the Absorber Theory of Radiation to Feynman Diagrams

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Abstract

In this dissertation I individuate and discuss Richard Feynman's overall spacetime view. I argue that the absorber theory of radiation, the path integral formulation of quantum mechanics and quantum electrodynamics, all share the overall spacetime view as a common conceptual framework. Even though this framework changed with the different theories, its most general form can be characterized as the looking at physics phenomena in their spacetime entirety. I show how the absorber theory of radiation is based on the intertwining of past and future within a closed system of absorbers and emitters. I show how the path integral formulation of quantum mechanics considers all the possible configurations within the initial and final states. I address how the overall spacetime view fits with Feynman diagrams and perturbation theory. Such a conceptual framework, I maintain, led Feynman to look at quantum phenomena from a new and revolutionary perspective, and to the formulation of one of our best scientific theories.

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Chapter 1

Introduction

The main objective of this dissertation is to reconstruct the philosophical intuition that led the physicist Richard Feynman to develop the absorber theory of radiation, the path integral formulation of quantum mechanics and the Feynman diagrams. This intuition, in its most general form, corresponds to the idea that the dynamics of some quantum phenomena does not have to be studied by looking at the system's infinitesimal time evolution from initial to final state. Rather, some phenomena are better accounted for if we consider the initial and final states and evaluate whatever happens in-between as happening all-at-once.

Quantum mechanics is an exceptionally well-tested theory, and one of the greatest accomplishment of our scientific enterprise. It has been successfully applied to chemistry and to the understanding of how the elements of the periodic table interact; it has been used to study the sun and how stars produce energy; it was fundamental in developing some world-changing technologies such as lasers, computers and machines for medical diagnosis (such as the MRI). However, despite its successes and precise predictions, the theory remains fundamentally mysterious in the type of reality it describes. It is astoundingly good at making predictions, but it does not tell us how the things it predicts come to happen. As odd as it may be, the interpretation of quantum mechanics is still an unsolved problem, even though it traces back to the very beginning of the theory —for example: Heisenberg's matrix mechanics in 1925 and Schrödinger's wave mechanics in 1926. Yet, philosophers and physicists are still debating (among other things) the measurement problem, the superposition principle, the reality of the wave function and, in general, the foundations of such a successful and yet mysterious theory.

It is against this backdrop that the present dissertation reconstructs the evolution of Feynman's overall spacetime view and, although this is not enough to solve the interpretative problems of quantum mechanics, I believe Feynman's view makes an important step in the right direction. Notably, Feynman did not offer a clear and in-depth analysis of his view, which led to various interpretations that span from instrumentalism to weak forms of realism. It is thus the purpose of this dissertation to present a cohesive narrative of the evolution of Feynman's view, and to discuss its philosophical relevance. What will emerge from the first three chapters of this dissertation —each corresponding to a step in the evolution of the overall spacetime view— is the necessity of revising some of the classical concepts we use to describe the physical world. More specifically, Chapter 2 will discuss the notion of classical radiation, Chapter 3 will question the concept of trajectory and Chapter 4 will addresses how the notion of quantum event diverts from its classical counterpart. Finally, Chapter 5 will address how to interpret the scientific understanding that Feynman gained from the use of the overall spacetime view.

1.1 The Structure of the Dissertation

The dissertation begins with Feynman's attempt to explain the radiative damping of an accelerated electric charge; this attempt resulted in the formulation of the absorber theory of radiation, for which the radiative damping is explained via advanced radiation emitted by the absorbers that received the initial radiation of the source. Radiative phenomena are thus explained by considering the entirety of the physical system (absorber-emitter) and the intertwining of past and future within its boundaries.

After formulating his new theory of classical electrodynamics, Feynman attempted the quantization of the theory and, even though it proved unsuccessful, he ended up with a new formulation of quantum mechanics based on an action principle: the path integrals formulation of quantum mechanics. I will argue that this formulation of non-relativistic quantum mechanics comes with a change in the overall spacetime view: from a view that emphasizes the role of future conditions onto the present, to a view that also considers all the possible configurations of a quantum system. The case discussed in Chapter 3 is that of a quantum particle moving from an initial to a final spacetime point. To calculate the probability amplitude of the system, the path integral method sums over the probability amplitude of all the possible trajectories that the particle can traverse. The chapter will also discuss in what sense the total ensemble of quantum paths is weakly non-reducible to the single possible trajectories. To do so, I will comment on some attempts to reduce the total ensemble of possible trajectories to a single path and show, unsurprisingly, that at best we can reduce it to subsets of possible trajectories. I will then show that not all the trajectories that contribute to the probability amplitude are 'physically possible', since some of them are non-differentiable. The consequence is a form of (weak) non-reductionism, for the physically possible trajectories are necessary but not sufficient to calculate the total probability amplitude of the system.

Chapter 4 reconstructs the development of Feynman's quantum electrodynamics and Feynman diagrams. Roughly speaking, not only ought one to consider all the possible trajectories, but also all the possible interactions that can occur between the initial and final states of the quantum system. More specifically, the chapter focuses on the use of spacetime boundaries — which will later lead to the use of asymptotically free states— that define the limits of the physical events. The second aspect reconstructed in the chapter is the application of one of the pillars of the absorber theory of radiation to the theory of positrons. Feynman, as a matter of fact, modifies the hole-theory by Dirac and posits that positrons are electrons moving backward in time. This changes the narrative of the positron-electron pair creation with that of electrons being scattered backward and forward in time.[1] The intertwining of past and future is a clear heritage from the absorber theory of radiation. Moreover, this reinterpretation of the positron reinforces the way Feynman looks at quantum phenomena: not from a step-by-step perspective, but rather from what he defines as the 'bird's eye view' —in this case, by looking at the total charge of the system. The chapter concludes with the discussion of Feynman diagrams and how each term of the perturbative expansion is associated with a pictorial diagram. The scattering amplitude is thus calculated by taking into account all the terms of the expansion and thereby all the diagrams.

Finally, Chapter 5 of the dissertation discusses the type of understanding that is implicit in the overall spacetime view. To do so, I will begin with (De Regt 2017)'s account, and with the concept of intelligibility of a theory. Intelligibility consists of a cluster of values (for example: visualizability) that makes a given theory more usable to scientists. In De Regt's view a phenomenon is scientifically understood if there is an explanation to that phenomenon that is based on an intelligible theory. However, in the chapter I argue that Feynman tried to understand the phenomena and through that understanding he was able to develop his quantum electrodynamics. The overall spacetime view and the visualization of physical processes contributed to such understanding. I will also emphasize that both the understanding and visualization are only partial, in that they do not constitute a one-to-one representation of physical reality.

In what follows I will summarize some of the philosophical consequences of the overall spacetime view, but to do so, I need to discuss a caveat first. The caveat is that Feynman was neither very detailed nor clear about his philosophical perspectives,

[1] Wheeler, in a letter to Feynman, will push the theory to its limits and suggest that the entire universe could be constituted by a single electron moving backward and forward in time.

especially if we compare him to some founding fathers of quantum mechanics such as Heisenberg and Schrödinger. This makes the reconstruction of the overall spacetime view much more difficult, especially with respect to the question as to what extent Feynman believed that physical reality behaves the way described by the different instances of his view. This is precisely why this dissertation can be characterized as a work in the field of both history and philosophy of science: it reconstructs the development of a philosophical concept (the overall spacetime view) in the theories of classical electrodynamics, path integrals and Feynman diagrams.

The philosophical lesson that can be drawn from the discussion on the overall spacetime view is that similarly to how special and general relativity brought about a revolutionary conception of space and time, quantum mechanics presents us with a revolutionary conception of the dynamics of quantum systems. Feynman calls 'customary view' the idea, typical of classical (Hamiltonian) mechanics, that systems move from an initial to a final state guided by a dynamical equation. The role of the dynamical equation is to evolve the system from its present state to its infinitesimal time subsequent in a recursive manner. With Feynman's quantum mechanics, the customary view is challenged and a new perspective is suggested. To calculate the relevant properties of the quantum system (usually the transition/scattering amplitude) the entire system ought to be taken into consideration, from its initial to its final state, and all its possible evolutions need to be accounted for by the equations. The new perspective (the overall spacetime view) becomes especially evident with the path integral formulation of quantum mechanics (Chapter 3) and with Feynman diagrams (Chapter 4). However, as I will present in the following chapters, the overall spacetime view assumes different forms in different theories.

In sum, the dissertation focuses on the reconstruction of Feynman's quantum electrodynamics starting from the absorber theory of radiation and through the path integral formulation of quantum mechanics. In that, the dissertation emphasizes the evolution of the intuition that accompanied Feynman in developing each of these theories: from the intuition that the future can affect the present, to the idea that all the possible trajectories ought to be considered in the calculation of the probability amplitude, to the view that all possible interactions need to be accounted for within the initial and final states of a quantum electrodynamic system. This intuition, which evolved with the different theories, I call it: 'the overall spacetime view'.[2] In what follows I will spell out the concept in more detail and I will relate it to the different chapters of the present dissertation.

1.2 Overall Spacetime View

The second chapter of the dissertation develops and discusses the concept of overall spacetime view with respect to the absorber theory of radiation, i.e., with respect to the attempt by Feynman (and Wheeler) to develop an alternative formulation of classical electrodynamics. The theory makes use of the time-symmetric solutions to Maxwell equations in order to explain the radiative damping of an accelerated charged particle as the advanced response of the absorbers re-emitting the radiation of the source. Thus, I characterize the first instance of the overall spacetime view (the chapter refers to it as 'overall process view') with the following two clauses:

- The micro-dynamic laws of electromagnetic radiation 'depend' on both the past (retarded radiation) and the future (advanced radiation) of both emitters and absorbers.
- The interaction between absorbers and emitter is essential to the radiative phenomena.

[2]To be precise, Feynman already uses the expression 'overall spacetime view' in his theory of positrons: (Feynman 1949b)

The first clause indicates how Feynman foresaw the possibility that some physical processes do not move forward-in-time only and, even though he does not provide a full-fledged philosophical account of time in physics, this feature will remain in the theory of positrons.

In addition, the chapter discusses the intertwining of past and future in relation to the Newtonian and Lagrangian scheme. The former is characterized by the fine-grained study of how a given physical system evolves in time 'step-by-step' and it is for the most part associated with hyperbolic partial differential equations. Indeed, the solution to those equations is provided by the specification of the initial data of the system (usually, the value of the function at the initial point and its derivative) and by evolving those data into infinitesimal time subsequent states. Therefore, the solution to the equation is constructed in such a way that only the past of the system influences the future.

The Lagrangian scheme, on the other hand, makes use of the action of the system which, in the case of a classical particle moving along a trajectory, is calculated along the entirety of the path: from the initial to the final state. As defined in (Wharton 2014, p. 3):

> One sets up a (reversible) two-way map between physical events and mathematical parameters, partially constrains those parameters on some spacetime boundary at both *the beginning and the end*, and then uses a global rule to find the values of the unconstrained parameters and/or a transition amplitude. This analysis does not proceed via dynamical equations, but rather is enforced on entire regions of spacetime 'all at once'.

The 'all-at-once' of the previous quote is represented by the set of past and future interconnections between particles. As described in (Wheeler and Feynman 1945, p. 181): "past and future of all particles are tied together by a maze of interconnections. The happenings in neither division of time can be considered to be independent of those in the other". The constraints posed by the absorber theory are such that the universe is a complete absorber and there can not be radiation without absorption. This latter constraint, I have translated it as the necessity of having interactions between absorbers and emitters.

With the path integrals formulation of quantum mechanics (Chapter 3), the constraints are now represented by the initial and final states of the quantum system and the maze of interconnections is now given by the summing of the different transition amplitudes of each possible trajectory within the boundaries. There is a substantial change in the nature of these interconnections: in the case of the absorber theory they are still separable and distinguishable, but, in the case of the path integrals, the different possible trajectories are not separable from the whole, and this constitutes a form of bottom-up holism.

I characterize bottom-up holism as that view for which, to determine a given property of the whole, the properties of the parts are not sufficient. With respect to path integrals, this means that the probability amplitude of the system is not solely determined by the probability amplitudes of the single possible trajectories. Or, to be more specific, it is not enough to sum over the probability amplitudes of the physically possible trajectories to obtain the total probability amplitude. One also ought to sum over the probability amplitudes of trajectories that are physically not possible —for example non-smooth and ill-behaved paths. The chapter discusses the problem of treating ill-behaved paths as physical possibilities. I will suggest that we can interpret the non-differentiable paths as genuine 'quantum trajectories' and thus that the very notion of trajectory would become scale-dependent and in need of some conceptual revision.

In the chapter, I also discuss two accounts that attempt to reduce the total ensemble of possible trajectories to subsets of physically possible paths. The first account is advanced by (Wharton 2016) and the main idea is that even though it is not possible to reduce the ensemble to a single real trajectory, it is nonetheless possible to group the various paths in subsets of non-interacting real trajectories —due to the cancellation process of the different single probability amplitudes. I argue that, although the grouping seems possible, some of the ill-behaved paths remain and thus we cannot reduce the ensemble to classical physical possibilities.

The second attempt to reduce the ensemble is suggested by (Gell-Mann and Hartle 2012) and it consists of a modified version of the decoherent history interpretation of quantum mechanics. While this accounts admits, at least in principle, the possibility of fine-graining the ensemble to a single trajectory, it also requires that these trajectories can be assigned a negative probability. Furthermore, the single trajectory would escape testing and observations, for the latter are proper of the coarse-grained histories only —where a coarse grained history is a set of fine-grained histories defined as sets of alternatives at successive times and expressed as sets of ad-hoc projector operators. Because of these two reasons, I argue that the account does not rule out holism.

In Chapter 4, I focus mostly on the historical development of Feynman diagrams and on the last form of the overall spacetime view —i.e., on the necessity of considering not only all the possible trajectories, but also all the possible interactions. I will reconstruct the use by Feynman of a spacetime boundary to limit the quantum event under consideration and show how this will translate into the use of asymptotically free states. It is within these states that the process of scattering happens and all the possible events need to be accounted for to calculate the proper scattering amplitude.

The last chapter of the dissertation (Chapter 5) discusses the type of scientific understanding provided by the overall spacetime —especially as discussed in Chapter 4. Starting from the view on scientific understanding and intelligibility offered in (De Regt 2017), I will argue that Feynman was not trying to understand a theory and its equations, but rather he was using visualization and the overall spacetime view to understand the quantum phenomena. However, I will also maintain that Feynman diagrams can not be considered as a one-to-one representation of quantum processes, and that the understanding provided by visualization and overall spacetime view is only partial. As a consequence, while the overall spacetime view played an important philosophical role in the formulation of Feynman's quantum electrodynamics, it cannot be considered as a truthful representation of the quantum world.

In the conclusions to the dissertation I will briefly look at Feynman's late works and comment on whether his commitment to the overall spacetime view changed with time. The section is relegated to the conclusions, since a deeper analysis would require a new historical work on the development of the diagrams and a closer look at the relationship between Feynman and Dyson.

Chapter 2

The Philosophical Underpinning of the Absorber Theory of Radiation

2.1 Introduction

The chapter advances the idea that the absorber theory of radiation —by (Wheeler and Feynman 1945) and (Wheeler and Feynman 1949)— is based on a different way of looking at electrodynamic phenomena. Generally speaking, this consists of looking at radiative phenomena in their entirety, rather than at the evolution of a given state from one instant of time to its subsequent. The point is not entirely new: for example (Blum 2017) has shed light on a 'paradigm shift' occurred between the 1930s and 1940s during which quantum field theory moved from being a theory of instantaneous quantum states to a theory of scattering. In his historical reconstruction, Blum also points out that the notion of an 'overall process' is a common theme among the various attempts to formulate a relativistic quantum theory:

> What all these formulations had in common was that in some sense they problematized the quantum mechanical notion of an instantaneous state and tended toward replacing it with a focus on overall processes. This stemmed from the relativistic need to treat space and time on the same footing and the consequent tendency of relativity towards a block universe view (Blum 2017, p. 2)

This notion of overall process is thus associated with terms such as "overall spacetime point of view" (Schweber 1986, p. 393) and with the idea that some phenomena divert from what (Wharton 2015) defines as the Newtonian scheme, where things evolve step-by-step and always forward in time. The point of the present chapter is to specify what this notion of overall process amounts to in the context of the absorber theory and to embed it into the philosophical literature.

The general intuition behind the absorber theory is that both retarded and advanced radiations, as well as the presence of the absorbers, ought to be accounted to understand the total radiation. I call this intuition 'overall process view' (for short: overall process) and I define it as:[1]

Overall Process View (of the absorber theory of radiation):

- (i) The microdynamic laws of electromagnetic radiation 'depend' on both the past (retarded radiation) and the future (advanced radiation) of both emitters and absorbers.
- (ii) The interaction between absorbers and emitter is essential to the radiative phenomena.

The first clause refers to the space-time character of the overall process view. Namely, to describe a radiative phenomena one does not 'simply' apply the initial conditions of the system to the dynamics laws. As we will see below, the absorber theory is a theory of action-at-a-distance that does not only demand knowledge of the past of the system (or of its initial conditions), but also of its future.[2] The second clause amounts to the claim that the mutual interaction of source and absorbers constitutes the radiative phenomena. The stronger claim that all radiation emitted by a source ought to be absorbed is debatable and it will be discussed in section 5.

[1] For the sake of clarity: the idea of overall process is what constitutes the philosophical underpinning of the theory.

[2] The point is also made clear in (Blum 2017) when he reconstructs the use of Fokker's action in Wheeler and Feynman's theory.

With respect to the terms ‘retarded’ and ‘advanced’, they refer to the fact that the total electromagnetic field induced by an accelerated charge is represented by the one-half advanced plus one-half retarded Liénard-Wiechert solution of Maxwell’s equations. In other words: according to the mathematical formalism, the equations of wave propagation have a retarded and an advanced solution where the former describes a wave at (r, t) caused by a source which lies in the past of t, and the latter describes a wave at (r, t) whose source lies in the future of t. However, since it is only the retarded solution that is observed in nature, the advanced solution is usually discarded as nonphysical. In the absorber theory of radiation, on the other hand, part of the total radiation of an accelerated source depends on the future response of the absorbers which arrives at the source at the time of the initial acceleration. Thereby, there is no *ad-hoc* discarding of the advanced solution, and the total radiation depends on both the past and the future of both emitter and absorbers. We will discuss the details in section 3.

Furthermore, the absorber theory of radiation falls under the category of ‘direct action theory’. Thereby, it modifies the classical field picture of classical electrodynamics in that fields are not independent entities providing a background for particles interactions. Thus, the use of terms like ‘field’ and ‘radiation’ becomes either ambiguous or seemingly contradictory with the direct action theory. To avoid the confusion, I will resort to the view suggested in (Kastner 2020, pp. 1–2) according to which the absorber theory does not thoroughly dispose of the concept of fields, but rather it makes the existence of fields dependent on the presence of a given particle. “[...] [T]he concept of ‘field’ is taken as still physically applicable in the form of a potential describing the strength of an interaction between sources. However, it does not constitute an independently existing medium [. . .] It differs from the standard notion of ‘field’ in that its existence is contingent on the existence of the charges that are its source."[3]

After having presented the theory, I will suggest that the initial intuition of Wheeler and Feynman played a role in Feynman's subsequent theories, such as, for example, the path integrals formulation of quantum mechanics. Afterward, I will consider a reinterpretation of the absorber theory by (Price 1991), which was originally meant to solve the problem of the time (experimental) asymmetry in Wheeler and Feynman's work. I will then show that the overall process view is at least partly incompatible with Price's reinterpretation of the absorber theory. Finally, in the last section I will introduce a connection between the overall process view and the broader philosophical debate on holism. In a separate appendix, I will briefly address some attempts to solve the time-asymmetry problem.

2.2 The Idea Behind the Theory

The absorber theory of radiation was developed in two famous works: (Wheeler and Feynman 1945) and (Wheeler and Feynman 1949), and its original drive was to explain the so-called radiative damping for electrodynamics. The problem the two physicists tried to address was that: a charge undergoing an acceleration emits a field and such field acts on the space surrounding the particle as well upon the particle itself. As Feynman reconstructed on his Nobel lecture (Feynman 1966, p. 2):

> Well, it seemed to me quite evident that the idea that a particle acts on itself, that the electrical force acts on the same particle that generates it, is not a necessary one —it is sort of a silly one, as a matter of fact. And, so I suggested to myself that electrons cannot act on themselves, they can only act on other electrons.

[3]For example, a different view seems to be held in (Bauer et al. 2014), where the fields in the absorber theory are taken to be only phenomenological descriptions on a macroscopic level and the use of fields-based language is a matter of convenience.

The radiating accelerated particle has a sort of inertia that needs to be accounted once you calculate the emitted radiation. In other words: when an electron is accelerated, it changes its momentum, but not all the 'acceleration energy' goes into the momentum variation. Part of the energy is emitted in form of a radiation. The resistance of the source to the acceleration is called radiation resistance or radiative damping. One can interpret it as a form of inertia of the source to accelerate. As expressed in: (Feynman 1966, p. 3)

> Then I went to graduate school and somewhere along the line I learned what was wrong with the idea that an electron does not act on itself. When you accelerate an electron it radiates energy and you have to do extra work to account for that energy. The extra force against which this work is done is called the force of radiation resistance.

It follows that even though such inertia-type-of-force diverges to infinity for point-like electrons, it cannot simply be discarded due to the principle of conservation of energy.

To solve the problem, Feynman worked with Wheeler and focused on a relational principle for which: given a radiating (accelerated) charged particle, this will affect a second particle that will in turn radiate back to the source. A relevant problem then emerges: if the radiation resistance is caused by the second particle, how can it reach the first particle at the moment of acceleration? From a mathematical and physical point of view, the answer to the question is to use the advanced and retarded solutions of Maxwell's equations. The general idea is represented in Figure 1:

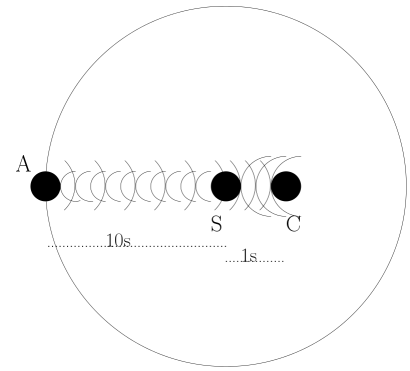

Figure 2.1 Scheme of the retarded and advanced interactions between a source (S), an absorber (A) and a target particle (C)

Imagine a source (S) being surrounded by an absorbing wall ten light-seconds away. Then imagine a test charge one light-second on the right of the source. The idea behind using the retarded and advanced solutions to Maxwell's equations can be summarized by the following conditions:

$$\begin{aligned}
&(S_{(t=0)}A)_{ret} = +10s \\
&(A_{(t=10)}S)_{adv} = 10s - 10s = 0s \\
&(A_{(t=10)}S)_{ret} = 10s + 10s = 20s \\
&(A_{(t=10)}C)_{adv} = +10s - 11s = -1s \equiv (-)(S_{(t=0)}C)_{adv} = -1s \\
&(A_{(t=10)}C)_{ret} = +10s + 11s = 21s \equiv (S_{(t=20)}C)_{ret} = +21s
\end{aligned} \tag{2.1}$$

What (2.1) expresses is the time relations of the advanced and retarded interactions of the source (S). At $t = 0s$ the source is accelerated and emits a radiation that reaches a particle of the wall (A) at time $t = +10s$.[4] The particle of the absorber emits at time $t = +10s$ advanced and retarded waves that reach the source at time $t = 0$ and $t = 20s$ respectively. Then, the advanced interaction of the absorber (A) with the test charge

[4] The notation $10s$ stands for 'ten seconds' and the sign (+) represents retarded (forward in time) radiation and the sign (−) represents advanced (backward) radiation. The formula is meant to give an intuitive idea of how advanced and retarded fields interfere and cancel each other.

(C) happens at the same time as the direct advanced interaction between the source and the test charge (that is at $t = -1$). These two interactions, Feynman argues, are equal and opposite and hence cancel out. Ultimately, the retarded interaction from the absorber and from the source acts on the test charge at the same moment $t = 21s$ (but this time the interactions have the same sign and hence add up).

The formal proof of the theory is divided in four derivations and I intend to provide a more detailed overview in the following sections. By means of an introduction, it is enough to point out that the radiative force acting upon the source is given by the superposition of the advanced interactions of the absorber particles —as expressed by the condition: $(A_{(t=10)}S)_{adv} = 10s - 10s = 0s$ in (2.1). It follows that, to account for the total radiation and hence to replace the traditional "field-picture", one needs to consider advanced and retarded interactions —of both the source and all the absorbers surrounding the source. We then return to the overall process intuition, i.e., if we consider the 'forward-in-time' interactions only, we are not able to account for the electron's radiation.

2.3 Derivations of the Theory

As mentioned above, looking at the derivations of the absorber theory should help understanding its relational character and the consequent necessity of considering both the advanced and the retarded fields. The purpose, as imagined by Feynman, was to provide an empirically equivalent theory of radiation that did not use the concept of fields. In such a way, the 'infamous' self-interaction —which causes mathematical divergences— would fade away, replaced by the backward (advanced) reaction of the absorber to the source's original radiation.

Let's consider the radiation force for an accelerated charged particle as calculated by Abraham and Lorentz:

$$F_{rad} = \frac{2q^2\ddot{\mathbf{v}}}{3c^3} \tag{2.2}$$

which is equivalent to the formulation given in (Wheeler and Feynman 1945, p. 158):

> A charged particle on being accelerated sends an electromagnetic energy and itself loses energy. This loss is interpreted as caused by a force acting on the particle given in magnitude and direction by the expression
>
> $$\frac{2(\text{charge})^2(\text{time rate of change of acceleration})}{3(\text{velocity of light})^3} \tag{2.3}$$

Then, starting from (Tetrode 1922), the two physicists developed a theory for which the absorber plays a fundamental role in the mechanism of radiation. As addressed in (Wheeler and Feynman 1945, p. 160):

> Using the language of the theory of action-at-a-distance, we give the idea the following definite formulation:
>
> 1. An accelerated point charge in otherwise charge-free space does not radiate electromagnetic energy.
> 2. The fields which act on a given particle arise only from other particles
> 3. These fields are represented by one-half the retarded plus one-half the advanced Lienard-Wiechert solutions of Maxwell's equations. This law of force is symmetric with respect to past and future. [...]
> 4. Sufficiently many particles are present to absorb completely the radiation given off by the source.

Definitions (1) and (2) state the relational nature of the absorber theory. For there cannot be any radiation unless there is something to absorb it and given that each field comes from the reaction of another particle, the theory naturally requires the relation between source and absorber. Definition (3) corresponds to the mathematical core of the theory. The necessity of taking into consideration both the advanced and the retarded radiations of a charged particle needs not only to be justified, but also to be proven equivalent to the experimental observations. Definition (4) is needed, especially for the fourth derivation, to reinforce the relational character of the theory and to account for the absence of residual advanced fields that are not measured or observed by the experiments.

Wheeler and Feynman (1945) present four derivations of the theory in which: the first one considers the particles of the absorber to be taken far from each other, derivation II evaluates the field of the absorber in the vicinity of the source to show that the field compensates the advanced field of the source and gives the proper retarded field. Derivation III takes into account arbitrary velocities and, as it does not concern us here, I will not comment on it. The last derivation provides a general approach for deriving the proper radiation —under the assumption expressed by definition (4).

2.3.1 Derivation I

Let's consider a source with charge $+e$ which undergoes an acceleration $\mathfrak{A}$ and emits a retarded radiation which reaches a particle of the absorber e_k at distance r_k and at time r_k/c. The field emitted by the source, due to the disturbance is (Wheeler and Feynman 1945, p. 161):

$$-(e\mathfrak{A}/r_k c^2)\sin(\mathfrak{A}, r_k) \tag{2.4}$$

The field in (2.4) causes a particle of the absorber to undergo an acceleration

$$\mathfrak{A}_k = -\frac{e_k}{m_k}\frac{e\mathfrak{A}}{r_k c^2}\sin(\theta)$$

where θ is the angle between the acceleration $\mathfrak{A}$ of the source and r_k.

The field produced by the acceleration of the absorber particle is half advanced and half retarded. The advanced component will reach the source (e) at the moment of the original acceleration and it will exert a force of magnitude equal to the integral over the distance r of (Wheeler and Feynman 1945, p. 160):

$$(2\mathfrak{A}e^2/3c^3)(2\pi N e_k^2/m_k c)dr_k \tag{2.5}$$

where the factor N considers all the particles of the absorber. However, Wheeler and Feynman pointed out that (2.5) does not accord with experimental data. That is because: (i) the reaction depends on the nature of the absorber (because of m_k); (ii) the radiative force is proportional to the acceleration, rather than to its time derivative and (iii) the integral diverges to infinity as the number of particles of the absorber increases.

However, for the absorber is composed of many absorbing particles, one needs to consider that the radiation is the product of both the proper field of each absorber and also of the interaction between these fields. To account for such an interaction, Wheeler and Feynman add a refractive index $n = 1 - 2\pi N e_k^2/m_k\omega^2$ to the formula (2.5), thereby obtaining the reactive force of the absorber (Wheeler and Feynman 1945, p. 161):

$$(2e^2/3c^3)\mathfrak{A}\int_0^\infty (2\pi N e_k^2/m_k c)dr_k \times \exp\Big\{(-ir_k 2\pi N e_k^2/m_k c\omega)\Big\} \tag{2.6}$$

which, by considering one single Fourier component of the acceleration, reduces to (Wheeler and Feynman 1945, p. 162):

$$\begin{aligned}(\text{total reaction}) &= (2e^2/3c^3)(-i\omega\mathfrak{A}) \\ &= (2e^2/3c^3)(d\mathfrak{A}/dt)\end{aligned} \tag{2.7}$$

The distribution of the various particles of the absorber determines a phase lag that leads to the cancellation of the advanced component of the absorbers reaction and the advanced component of the source's radiation.

The first derivation has shown that from calculating the advanced reaction of the absorber particles it is possible to obtain the original radiative force (2.2). It follows that the radiative damping of the source comes from the advanced reaction of all the particles of the absorber. Thus, the study of the phenomenon of radiation (at least in this formulation) needs to consider not only the immediate present of the source, but also the advanced reaction of the absorber. In other words, the derivation hinges on the fact that both accelerated sources and absorbers emit a half-advanced and half-retarded radiation. This particular time-symmetry, I addressed it as the first leg of the overall process view: *(i) the microdynamic laws of radiation depend on both the past and the future of emitters and absorbers respectively.* Thus, applied to the radiative damping, it reads: *the radiative damping of an a accelerated source (in the absorber theory of radiation) depends on half-advanced (future) and half-retarded (past) radiation of both the source and the absorbers.*

I conclude the comments on the first derivation with a quote from (Wheeler and Feynman 1945, p. 162) that emphasizes the importance of (2.7):

> We conclude that the force of radiative reaction arises, not from the direct action of a particle upon itself, but from the advanced action upon this charge caused by the future motion of the particles of the absorber.

2.3.2 Derivation II

The second derivation shows that the advanced field produced by the absorber, when summed with the radiation emitted by the source (which is half advanced and half retarded), gives rise to the total disturbance (or total retarded field) measured by the experiments

$$F = -\frac{2e\dot{\mathbf{v}}}{3c^2 r} + \frac{2e^2\ddot{\mathbf{v}}}{3c^3} \tag{2.8}$$

where the second term is the radiative damping and the first is equivalent to (2.4).

For the second derivation, let's consider a source located at the center of a spherical cavity of radius R and let's evaluate the strength of the radiation at some distances

$r << R_{cav}$.[5] The particles of the absorber are now not necessarily free and therefore one needs to account for the dispersion of the radiation in terms of a complex function $p(\omega)$ which tends to 1 when the binding between the particles is weak.

The equation of motion of a particle of the absorber medium is (Kamat 1970, p. 477):

$$m_k \ddot{\mathbf{r}}_{\mathbf{k}} = e_k p(\omega) \mathbf{E} \tag{2.9}$$

Now, the advanced field of the absorber reduces at the source to:

$$(2e/3c^3)(d\mathfrak{A}/dt) \tag{2.10}$$

which, when multiplied by the charge of the source, gives the radiative damping.

let's now consider the Fourier component of the acceleration (assumed to be periodic) such that:

$$\mathfrak{A} = \mathfrak{A}_0 \exp(-i\omega t) \tag{2.11}$$

At some wavelengths of distance ($> r$), equation (2.10) reduces to (Kamat 1970, p. 479):[6]

$$\left[-\frac{e\mathfrak{A}_0}{2rc^2} \exp\left\{ -i\omega t + \frac{i\omega(n-ik)r}{c} \right\} + \frac{e\mathfrak{A}_0}{2rc^2} \exp\left\{ -i\omega t - \frac{i\omega(n-ik)r}{c} \right\} \right] \sin^2 \chi \tag{2.12}$$

where χ is the angle between $\mathfrak{A}$ and $\mathbf{r}$, ω is the frequency and $(n-ik)$ is the refraction index. Equation (2.12) accounts for the total advanced radiation emitted by the absorbers which takes the form of half retarded minus half advanced the radiation emitted by the source. The formula (2.12) is presented by (Wheeler and Feynman

[5]To be precise: the term r is the distance between the point of evaluation of the field and the source

[6]I have adjusted the notation to be consistent with that of Wheeler and Feynman

1945, p. 164) in the form:[7]

$$- 1/2(e\mathfrak{A}_0/2rc^2)exp(i\omega/c - i\omega t) \tag{2.13}$$

$$+ 1/2(e\mathfrak{A}_0/2rc^2)exp(-i\omega/c - i\omega t) \tag{2.14}$$

The superposition of the advanced radiation emitted by all the absorbers will look like a spherical wave collapsing onto the source. The concept is expressed by the phrasing in (Wheeler and Feynman 1945, p. 165):

$$\begin{pmatrix} \text{total disturbance} \\ \text{converging on} \\ \text{the source} \end{pmatrix} = \begin{pmatrix} \text{proper advanced} \\ \text{field of source} \\ \text{itself} \end{pmatrix} + \begin{pmatrix} \text{field apparently converging on source} \\ \text{actually composed of parts convergent} \\ \text{on individual absorber particles} \end{pmatrix} \tag{2.15}$$

For the second and the third term in (2.15) have opposite signs —to be more precise they are out of phase— they interfere destructively. Consequently, the advanced components of the radiations (emitted by either the absorber or the source) get cancelled with each other. Once the advanced fields coming from the absorber have passed over the point charge, they will start looking like a retarded field coming from the source, although they actually are fields converging on the single particles of the absorber. These ‘apparently-diverging-fields’ will add up (as the direction is the same) with the half retarded radiation emitted by the source, producing the full retarded radiation. As (Wheeler and Feynman 1945, p. 166) phrased it:

$$\begin{pmatrix} \text{total disturbance} \\ \text{diverging from} \\ \text{source} \end{pmatrix} = \begin{pmatrix} \text{proper retarded} \\ \text{field of source} \\ \text{itself} \end{pmatrix} + \begin{pmatrix} \text{field apparently diverging from source} \\ \text{actually composed of parts converging} \\ \text{on individual absorber particles} \end{pmatrix} \tag{2.16}$$

The third term refers to the part of the advanced field of the absorber which is now diverging from the source.[8]

The second derivation has reconstructed —starting from the results of the first one— the total retarded disturbance measured by the experiments. The important aspect, at least from a philosophical point of view, is the relational character that the

[7] In the original work, Wheeler and Feynman derive the factor 1/2 on a separate paragraph.

[8] The first to notice that the phrasing was somehow ambiguous was (Leeds 1994).

theory takes. The total retarded radiation emitted by the source is only apparently such. As a matter of fact, one needs to consider the radiative component which is apparently coming from the source but which is actually converging on the absorber particles. The total radiation diverging from the source is then constituted by the (constructive) interference between the proper retarded field of the source and the response of the absorbers converging onto the source. Therefore, the theory accounts for the total radiation by making the role of both absorbers and source fundamental —this is in accord with the first point of the theory of action-at-a-distance for which an accelerated charge does not radiate in a free-charge space.

While derivation one has pointed out the necessity of considering the time-symmetry of both absorbers and emitter, the second derivation has emphasized the relational character of the absorber theory. The latter is what I have addressed as the second point in the definition of the overall process view: *the interaction between absorbers and emitters is a constitutive one*, where the interaction is represented here by the interference between the proper radiation of the source and the response of the absorbers.

I will not discuss the third derivation as it generalizes the argument to fast-moving particles and hence it does not concern us here. The fourth derivation, on the other hand, is the most general one and also the most discussed one among the philosophers of science (see, among others: (Ridderbos 1997), (Price 1991), (Frisch 2000), (Davies 1977)).

2.3.3 Derivation IV

Let's now consider a charge a surrounded by a complete absorber (for which each particle is labeled k), which means that it is not possible to have a radiation that is not absorbed.

$$\sum_k F_{ret} + \sum_k F_{adv} = 0 \quad \text{outside of the absorber} \tag{2.17}$$

As it is not possible for a retarded component of a field to destructively interfere with the advanced component of the same field, it follows that each component of the previous formula has to be zero separately:

$$\sum_k F_{ret} = 0 \tag{2.18}$$

$$\sum_k F_{adv} = 0 \tag{2.19}$$

Also, as we are under the assumption that the absorber is a completely absorbing medium, the following also holds:

$$\sum_k (F_{ret} - F_{adv}) = 0 \quad \text{everywhere} \tag{2.20}$$

The consequence of the complete absorbing assumption is that the field exerted on the ath particle (the source) by the absorbing particles is given by (Wheeler and Feynman 1945, p. 169):

$$\sum_{k \neq a} \left(\frac{1}{2} F^k_{ret} + \frac{1}{2} F^k_{adv} \right) \tag{2.21}$$

The formula can be broken into three components:

$$\sum_{k \neq a} \left(\frac{1}{2} F^k_{ret} + \frac{1}{2} F^k_{adv} \right) = \sum_{k \neq a} F^k_{ret} + \left(\frac{1}{2} F^a_{ret} - \frac{1}{2} F^a_{adv} \right) - \sum_k \frac{1}{2} \left(F^k_{ret} - F^k_{adv} \right) \tag{2.22}$$

The second term on the right corresponds to the radiative damping which, in turn, can be expressed as coming from the advanced response of the absorber (see derivation I):

$$e_a \left(\frac{1}{2} E^a_{ret} - \frac{1}{2} E^a_{adv} \right) = \left(\frac{2e_a^2}{3c^3} \frac{d\mathfrak{A}}{dt} \right) \tag{2.23}$$

Because of the complete absorber assumption, the third term of equation (2.22) is equal to zero: $(1/2) \sum_k (F^k_{ret} - F^k_{adv}) = 0$. The first term $\sum_{k \neq a} F^k_{ret}$ corresponds to $-(2e\dot{\mathbf{v}})/(3c^2\mathbf{r})$ which is the usual retarded field, due to the force accelerating the emitter. Thereby, the relational aspect of the theory becomes evident because the sum is over all the particles of the absorber. Furthermore, the relational aspect yields that there is no self-action of the source. What follows is that to account for the phenomenon of radiation, both the emitter and the absorbers are necessary while the self-action is ruled out (which was Wheeler and Feynman's original point).

To sum it up: I have argued that the four derivations employ the overall process intuition, namely, *(i) the microdynamic laws of radiation depend on both the past and the future of both absorbers and emitters, (ii) the interaction between absorbers and emitters is needed to account for radiative phenomena.* The two points were made evident in the first and the second derivation respectively, while the fourth derivation encompasses them both. As a matter of fact, in equation (2.22), both absorbers and emitter are centered on half-advanced and half-retarded radiations (corresponding to (i)). Furthermore, because the total retarded radiation depends on the sum over all absorbers' particles, the relation between absorber and emitter becomes constitutive of radiative phenomena (which corresponds to (ii)).

What about the advanced components? The advanced component of the field gets cancelled by the advanced component of the radiation emitted by the source, for they have opposite signs. This is because the superposition of the advanced components of the absorber response will form a spherical wave converging on the single absorber particles, thereby destructively interfering with the advanced component of the source which will form a spherical wave converging on the source. Wheeler and Feynman (1945, pp. 169–170) conclude that: "[...]we have shown that the half-advanced, half-retarded fields of the theory of action at a distance lead to a satisfactory account of the mechanism of radiative reaction and to a description of the action of one particle on another in which no evidence of the advanced fields is apparent."

2.4 The Philosophical Underpinning

The main debate around the absorber theory of radiation involved the temporal symmetry endorsed by the Maxwell's equations and the phenomenological asymmetry of radiation measured by the experiments. In other words, the main issue that authors such as (Ridderbos 1997), (Price 1991), (Frisch 2000), (Davies 1977) tried to address is the 'symmetry breaking' happening from the laws of micro-dynamics to the empirical data.

This sections provides: first an analogy between the philosophical underpinning of the theory and the so-called Lagrangian schema proposed by (Wharton 2015). The purpose of the analogy is to further flesh out the intuition of overall process and to suggest that a similar intuition was applied by Feynman in his path integrals interpretation of quantum mechanics. Second, I will briefly consider how the overall process intuition can relate to the philosophical debate about holism.

2.4.1 Analogy Between Overall Process and Lagrangian Schema

I have anticipated earlier that the philosophical underpinning of the absorber theory of radiation —what I have called overall process view— is twofold. First, the presence of the absorber is essential if we are accounting for phenomena of radiation. As presented by (Pegg 1975, p. 173): "The presence of the absorber is essential for the calculation to work. For example, it will not work in an empty universe surrounding the electric charge." It is then fundamental that some interaction occurs between absorbers and emitter. Such an interaction is represented (at the level of electrodynamics) by the mutual interference of retarded and advanced radiation in the way explained in the previous sections. This brings us to the second aspect of the philosophical underpinning: the time-symmetry of both absorbers and emitters. As it was made clear in the rehearsal of Wheeler and Feynman derivations, the emitter is centered on a radiation that is half advanced and half retarded and the response of the absorber is likewise half advanced and half retarded. Having to consider both the past and the future of both absorbers and emitters is what constitutes the second aspect of the philosophical underpinning. To clarify these aspects further, let's consider an analogy with the Lagrangian schema as opposed to the Newtonian one, proposed in (Wharton 2015).

In his recent work, (Wharton 2015) ascribes Newtonian mechanics to a general scheme (Newtonian schema) of computing solutions to dynamics equations. According to such a schema, the universe is taken to be a computational mechanism which inputs a given initial state and outputs another state (of a given system). A similar concept was expressed in (Smolin 2009, p. 23):

> The separation of scientific explanation into laws and initial conditions leads to one of the most universal and powerful notions in physics —the notion of *configuration space*. This is the space of all possible configurations, or states, of the system. In classical and quantum physics we assume that this space exists *a priori* and outside of time, and that it can be studied independently of the laws of motion. These laws then specify the rules for how the point that describes the initial conditions in configuration space evolves in time. We call this the *Newtonian schema for explanation*.

Wharton (2015, pp. 1–2) further clarifies the concept of Newtonian schema as represented by three steps: "1) map the physical world onto a mathematical state, 2) mathematically evolve that state into a new state, 3) map the new mathematical state onto the physical world". The main feature I wish to emphasize here is the step-by-step character of the schema. In classical physics, we would take the initial velocity and position of a particle, we would apply a force on that particle and by means of Newton's law we can construct the trajectory the particle traverses instant-by-instant (or as I have called it: step-by-step). As addressed in Feynman (2017, pp. 51–52): "Newton's law tells us what happens at one time in terms of what happens at another instant. It gives from instant to instant how to work it out, but in space leaps from place to place."

However, the development of quantum mechanics, argues Wharton, poses a new set of challenges to the Newtonian schema. Heisemberg's uncertainty principle, the probabilistic nature of the theory, the measurement problem (and so on), they all make the use of Newton's laws 'inadequate'. Therefore, another schema is suggested, one that differs "in the philosophy and qualitative ideas involved" (Feynman 2017, p. 52). Instead of taking the initial values of (say) position and velocity and construct the classical trajectory step-by-step, we can take the initial and final position and study how the average of the difference between kinetic and potential energy varies. We call the latter quantity *Lagrangian* and its integral *action* (S). The classical trajectory, constructed by means of Newton's law, is the one for which the action is an extremum, i.e.: $\delta S/\delta x(t) = 0$.[9] We thus have a different way of solving physical problems, one that is called by (Wharton 2015): Lagrangian schema.

The main (philosophical) difference with respect to the Newtonian schema is the 'all-at-once' way of looking at the evolution of the given system. For the quantity action is assigned to the overall trajectory, that is from its initial to final point, it means that the future path of the particle 'influences' the path already traversed (and *vice-versa*). In other words: the difference between past and future 'fades away': "[a]nd because of the time-symmetric way in which the constraints are imposed, there's no longer any mathematical difference between the past and the future; both constraints directly map to the real world, without further manipulation" (Wharton 2015, p. 6).[10]

What is the connection, one might ask, with the absorber theory of radiation? The 'all-at-once' analysis of a phenomenon represented by the Lagrangian schema shows

[9] The one stated above is known as the principle of least action. See, for example: (Feynman 1948c).

[10] A (almost) straightforward way of applying this new schema to quantum mechanics is by means of Feynman's path integrals (Feynman and Brown 2005) where: the probability amplitude of a particle going from an initial to a final point is calculated by summing over all the possible trajectories within the boundary conditions and where to each trajectory a classical Lagrangian is assigned. More on this: (Feynman 1966), (Kent 2013) and (Wharton 2016).

some similarities with the type of intuition at play in the absorber theory of radiation. Roughly speaking, if we were to apply the Newtonian schema to radiative phenomena, we would consider the source as emitting a radiation and then the absorber (if any) would radiate it back. In this picture, the radiative damping would have to come from either self-interaction —which is what Wheeler and Feynman tried to avoid— or if it was caused by the response of the absorber, it would occur at a time later than the original acceleration of the source. From the Lagrangian schema perspective, on the other hand, the phenomenon of radiation would be considered 'all-at-once', that is: we would set some boundary conditions and study the phenomenon in such a way that the response of the absorber has some influence on the initial emission of radiation by the source. The similarity is that the radiative damping in the absorber theory of radiation comes from the advanced response of the absorber. Thus, the influence of the future on the past —grounded on the time-symmetry of both emitter and absorbers— as well as the relational character of radiation —in terms of the interference between the radiation emitted by the source and absorbers— yield an analogous fading of the distinction between past and future pointed out by Wharton.[11]

The purpose of presenting the derivations of the theory in the previous sections, and of the analogy presented above, was to emphasize the character of the theory both in terms of the time-symmetry of both absorbers and emitters and in terms of their respective interactions. The use of advanced fields to account for the radiative damping conflicts with the picture of nature represented by the Newtonian schema —for which the future does not influence the past. The following quote further

[11] There is tension between the terms 'all-at-once' and 'overall process' in that a theory that is time symmetric and posits future boundary conditions can endorse a static ontology. In this latter case, usually known as block worldview, it becomes problematic to justify any dynamical process. More on this in (for example): (Kastner 2017). The analogy with the Lagrangian schema clarifies that the dynamism implied by the 'process' language is only ostensible and it does not necessarily refer to an 'unfolding dynamical process'. Nonetheless, whether the absorber theory endorses a block worldview of the universe and how to reconcile it with the asymmetry of radiation goes beyond the scope of the present chapter.

highlights the point: (Wheeler and Feynman 1945, p. 181):

> Pre-acceleration and the force of radiative reaction which calls it forth are both departures from that view of nature for which one once hoped, in which the movement of a particle at a given instant would be completely determined by the motions of all other particles at earlier moments. [...] past and future of all particles are tied together by a maze of interconnections. The happenings in neither division of time can be considered to be independent of those in the other.

The view of nature mentioned in the first part of the quote has the same features as the Newtonian schema addressed by (Wharton 2015). The second part of the quote, instead, strongly emphasizes the relational nature of the new theory as well as the need to consider both future and past interactions among the particles. This was made evident, for instance, in the first derivation when the radiative reaction was calculated by using the advanced reaction of the absorbers. A similar intuition is what characterizes the Lagrangian schema: to analyze the evolution of a system by calculating the average of the difference between the kinetic and potential energy of the whole (overall) classical trajectory.

Is it possible to further characterize the similarity between the Lagrangian schema and the overall process view of the absorber theory? Although it is not the main purpose of the present chapter, I wish to give a suggestion toward a possible answer. This, I believe, comes from the type of boundary conditions at play in the two views. To be noted that there seems to be not much agreement on the role that boundary conditions play in science. For example, (Paksi 2014) argues that they are arbitrary conventions and instrumental tools that help finding some physical laws. Wilson (1990, p. 566), on the other hand, argues that they represent claims about "how a certain portion of the universe interacts with its surrounding along their natural boundaries", and thus they are not fully 'freely-choosable'. Lindsay (1929) focuses on the role of boundary conditions in physics and divides them into two classes: initial and general. The former corresponds to the initial conditions of the system in terms of, for example, specific positions in space and time or past history of the system. An example would be the solution to Newton's second law for which to calculate the dynamics of a moving particle the initial position and momentum are needed. General boundary conditions, on the other hand, correspond to "fundamental restrictions on the type of behavior of a physical system, expressible by a mathematical relation among a set of physical quantities characteristic of this system" (Lindsay 1929, p. 466). An example of the latter type would be to set the behavior of a moving fluid colliding with a surface (continuity condition). Another example would consist of setting the condition that within a boundary S no radiation is propagated outside of S into the future (complete absorber condition). Independently of the philosophical account we prefer, with respect to the role of boundary conditions in science —nonetheless granted that they play an important role— we can draw a similarity between the ones used by the Lagarangian schema and the absorber theory.

I have mentioned that to study the dynamics of a system by means of Newton's second law, one needs two constants, i.e., the initial position and momentum of the system at a given time. To 'choose' these quantities allows us to calculate the position of the system at a subsequent time (this corresponds to the 'step-by-step' dynamics mentioned above). To solve problems with the Lagrangian schema, on the other hand, we need to set the initial state of the system (position and momentum at initial time) as well as the final state. With the Lagrangian schema we calculate the classical trajectory of a particle by finding the path that minimizes the quantity action. As summarized in (Wharton 2015, p. 4):

> To summarize the Lagrangian Schema, one sets up a (reversible) two-way map between physical events and mathematical parameters, partially constrains those parameters on some spacetime boundary *at both the be-*

ginning and the end, and then uses a global rule to find the values of the unconstrained parameters. These calculated parameters can then be mapped back to physical reality.

In this case, the parameters are the values of the action and the global rule is the stationary value that the action takes within the boundaries. By means of the global rule, it is then possible to calculate the evolution of the function (say) $q(t)$ which can be mapped back to the physical trajectory of the classical particle. The important aspect is that the Lagrangian schema sets the boundaries on both the initial position and time of the said particle and also on its future. Similarly, the absorber theory sets a boundary condition with respect to the future radiation outside of the boundary. This was made evident in derivation IV when we have assumed the complete absorption of future radiation. With respect to the overall process view, the complete absorption amounts to a requirement similar to the second clause of the overall process view.[12]

The Lagrangian schema and the overall process view of the absorber theory of radiation both use some 'future-type' boundary conditions which are imposed on the system. This is also what makes them different from the Newtonian schema —for which it is enough to set the initial conditions of momentum and position at a given time. It would be interesting to see if this suggestion can be made more precise by, for example, looking at the mathematical form of the differential equations involved. This seems a promising direction for a broader account of the overall process view, one that includes electromagnetism in general, thermodynamics, diffusion phenomena and so on.[13] However, I will leave these considerations to later works.

Before I move to the next section, two (intertwined) points seem to be relevant:

[12]To be more precise, the absorber condition is a stronger requirement than the presence of interaction between absorbers and emitters. More on this in Section 5.

[13]I will say more on the boundary conditions of the absorber theory in the appendix.

First, that the philosophical underpinning of the absorber theory of radiation and the Lagrangian schema introduced by Wharton, although similar, are not identical. Second, that the philosophical debate on Lagrangian mechanics is not limited to (Wharton 2015). With respect to the latter point, both variational principles, the principle of least action and the path integral formulation of quantum mechanics, have been frequently discussed by scientists and philosophers, among others: (Terekhovich 2018), (Lanczos 2012), (Yourgrau and Mandelstam 1979). Also motivated by the historical contiguity, I will thus consider Feynman's path integrals and suggest that an intuition similar to the one of the absorber theory was applied to path integrals as well.

With respect to the first point: although similar, the overall process view considered in the absorber theory of radiation is not the same as the Lagrangian schema. The similarity rests on the idea of looking at physical phenomena from a 'bird-eye' point of view or from the use of a 'future-type' boundary condition. However, the theory of radiation proposed by Wheeler and Feynman does not use the classical Lagrangian. Historically, the classical Lagrangian was employed by Feynman in his subsequent work on path integrals. As I will briefly reconstruct below, the philosophical intuition is a similar one and, I suggest, it is what guided Feynman in building his interpretation of quantum mechanics.

Nonetheless, although the quantization of the absorber theory of radiation does not directly lead to path integrals, others tried to extend it to quantum mechanics. While in the next section I will focus on path integrals —because of its continuity with the absorber theory— I will say more on other extensions to quantum mechanics in the appendix dedicated to the asymmetry problem.

2.4.2 Form Absorber Theory to Later Works

After reformulating classical electrodynamics with his absorber theory, Feynman moved to quantum mechanics —or to be precise, he attempted to quantize the absorber theory so as to derive quantum mechanics. From (Feynman 1966, p. 5):

> We also found that we could reformulate this thing [the absorber theory] in another way, and that is by a principle of least action. Since my original plan was to describe everything directly in terms of particle motions, it was my desire to represent this new theory without saying anything about fields. It turned out that we found a form for an action directly involving the motions of the charges only, which upon variation would give the equations of motions of these charges.

The principle of least action mentioned by Feynman is one which describes paths of particles through space-time.[14] The description of these paths by means of such an action was opposed to what the physicist calls the 'customary view':

> In the customary view, things are discussed as a function of time in very great detail. For example, you have the field at this moment, a differential equation gives you the field at the next moment and so on; a method, which I shall call the Hamiltonian method, the time differential method (Feynman 1966, p. 7).

The customary view mirrors (for field theory) the Newtonian schema we considered above: the 'discussion' of a function in great detail mirrors the 'step-by-step' perspective that was emphasized by (Wharton 2015).

In general, the work on the absorber theory left the physicist with different possible formulations of classical electrodynamics and with "the overall space-time point

[14] The principle is Fokker's action principle, see: (Wheeler and Feynman 1945).

of view" (Feynman 1966, p. 8).[15] Nonetheless, quantum mechanics in those days was formulated in terms of Hamiltonian operators, thus in terms of a 'step-by-step' dynamics of quantized fields.[16] As recalled by (Wüthrich 2010, p. 52):

> The standard procedure for quantizing a classical theory was to interpret the classical Hamiltonian function as an operator in a Hilbert space of state vectors. This operator would then determine the time evolution of the quantized system described by a certain state vector. The problem with quantizing the Wheeler-Feynman theory of electrodynamics was that it could not be formulated by specifying a Hamiltonian function. Therefore, a method was needed to quantize physical systems, the classical description of which could not be given by a Hamiltonian function.

The use of Fokker's action principle allowed Feynman to have a theory that: (i) explained the radiative reaction, (ii) used a combination of both advanced and retarded interactions and (iii) was also relativistically invariant. As reconstructed by (Blum 2017, p. 21), a classical theory can be expressed in terms of least action where the action is the integral over the Lagrangian. From the Lagrangian, one can build an Hamiltonian and therefrom apply canonical quantization. Fokker's action, though, is different in that:

> [T]he integrations are carried out from $-\infty$ to ∞, instead of from an initial time t_0 to a final time t_1... For an action formulated in terms of a Lagrangian, the time integration range can be taken infinitesimally small.

[15] It is worth mentioning that, in Feynman's reconstruction, the latter quote came after he was given the idea (by Wheeler) that positrons could be electrons traveling backward in time.

[16] To be precise: the absorber theory is a direct-action theory and thus fields are no longer 'ontologically independent'. As emphasized in (Kastner 2015): "[...] in a direct action theory (DAT) the field interactions are not mediated by quantized fields considered as independent degrees of freedom, but instead by a direct, 'nonlocal' interaction between sources of the field." Therefore, the theory does not need a quantization in the usual sense, which makes the use of the Hamiltonian unsuitable for the expansion of the theory to quantum mechanics.

> The action is then minimized in each infinitesimal step and thus contact is made with the Hamiltonian formulation of differential time evolution of instantaneous states. This is not possible for the Fokker action: Since the interaction is retarded (and advanced), one always needs to take take into account the entire trajectories.

The solution, as reconstructed in (Feynman 1966) and (Schweber 1986), came from Herbert Jehle, who told Feynman about a paper by Dirac (1933) in which the connection between quantum mechanics and Lagrangian was made explicit. Starting from the work of Dirac, Feynman completed his PhD thesis giving birth to the path integrals formulation of (non-relativistic) quantum mechanics. He was able to reconstruct the probability amplitude in terms of an infinite series of integrals, in which each path is weighted by a phase factor: $\exp(iS/\hbar)$ and where S is the action of the given path.[17]

It remains that Feynman did not quantize the absorber theory: he had obtained a reformulation of classical electrodynamics and a new formulation of non relativistic quantum mechanics. I have argued that the former had a philosophical underpinning in what I have defined as the overall process view and the latter seems to have retained an at least similar view —as emphasized by the use of the Lagrangian.

I leave the task to provide a more precise definition of 'overall process' in the context of path integrals to subsequent works. Nonetheless, I have already mentioned in the introduction that (Blum 2017) has (among other things) reconstructed the attempt by Feynman to merge the absorber theory with path integrals and the developments of the use of path integrals in quantum electrodynamics. What distinguishes his perspective from the works of (Schweber 1986) and (Wüthrich 2010) is the focus on the 'paradigm shift' which characterized the transition from early quantum

[17] A more thorough and accurate historical reconstruction can be found, for instance, in (Schweber 1986) and (Wüthrich 2010).

field theory (based on quantum states) to that of Feynman and Dyson (based on scattering). This strengthens the idea of a common notion of overall process among Feynman's works. What is left out is the specification of such notion with respect to the other theories.

To sum it up: path integrals use the Lagrangian to calculate the probability amplitude and the use of the Lagrangian entails a view of certain physical phenomena in terms of the Lagrangian schema proposed by Wharton. Therefore, as long as the overall process view shows some similarities with the Lagrangian schema —as we have argued above— it seems legit to suggest that path integrals are based on an intuition similar to the one that was adopted for the absorber theory.[18]

The next section will present the reinterpretation of the absorber theory by Price and some of the replies by ((Ridderbos 1997), (Leeds 1994), (Frisch 2000)). These authors have shed light on some of the problems of Price's arguments, mainly with respect to the physical content of the theory. What I will emphasize is that, not only the reinterpretation changes the physical content, but also the philosophical underpinning we have addressed above.

2.5 Reinterpretation and Objections

According to the absorber theory, an oscillating charge in a complete absorbing medium radiates and hence loses energy. This would not be a problem if it weren't for the fact that electrodynamics is symmetrical with respect to time. As a matter of fact, it can be shown that by reversing time, one can get a fully advanced radiation contra to experience.[19] Wheeler and Feynman —to reconcile the time symmetry of the theory with the time asymmetry of experience— suggest that the asymmetry

[18]The point is even stronger if we consider the chronology of Feynman's theories. As we have pointed out: the absorber theory of radiation comes before Feynman's thesis on path integrals.

[19]The full argument is fleshed out in (Wheeler and Feynman 1945) and pointed out also by (Ridderbos 1997) and (Price 1991)

of radiation comes from statistical mechanics, (an important role is played by the refractive index in derivation I) and from the complete absorber conditions (as in derivation IV). In this way, their theory remains time-symmetric with respect to the micro-dynamics laws while the (experimental) asymmetry emerges from the statistical behavior of the system.[20]

Contra the explanation advanced by Wheeler and Feynman and in general against the statistical argument, Price (1991) argues that: (i) Wheeler and Feynman's derivation of the time-asymmetry is fallacious and (ii) that the mathematical core of the theory needs to be reinterpreted so as to establish that radiation is fully symmetric. Though the second point does not necessarily conflict with the original theory, Price's reformulation leads to a change in the type of symmetry at play in the phenomenon of radiative reaction. His starting point is that the absorber theory of radiation endorses two types of symmetries:

1. Emitters (i.e., those entities normally thought of as emitters) are associated equally with advanced and retarded wavefronts, in either the statistical or the individual case (Price 1991, p. 963).

2. Emitters and absorbers are both centered on coherent wavefronts, *these being half outgoing and half incoming in both cases* (Price 1991, p. 962).

Then, Price argues, two main problems arise. The first one is concerned with the possibility of distinguishing the different constructive interfering components from either the source or the absorber. The second one is related to the 'reversibility argument' —the fact that by reversing time (due to the time-symmetry of Maxwell's equations) one can obtain a fully advanced field rather than a retarded radiation.

[20]Bauer et al. (2014) formulate another explanation for the macroscopic time asymmetry which is based on the statistical argument but not on the complete absorber condition.

In what follows, I will consider the first problem, for it is the more relevant with respect to the absorber theory of radiation and to the overall process view. Because the second problem is only tangential to the overall process view, I will briefly address it in a separate appendix.

2.5.1 The Distinguishability Argument

Price maintains that it is not possible to distinguish the different components of radiation and thus they can be conceived of as one and the same. The argument goes as such: the component waves (retarded and advanced) are indistinguishable and thus can be conceived of as one and the same. This leads to a new type of symmetry for which the absorbers emit advanced radiation and the source emits retarded radiation only. In this section I will both present Price's argument and then raise two main objections. First, the advanced and retarded components are not one and the same in the absorber theory —as it was made evident in the derivations. Second, the modified symmetry does not leave room to account for the radiative damping (which was Wheeler and Feynman's original purpose).

Change of Symmetry

Price (1991, p. 368) asks: "Do Wheeler and Feynman have any justification for the claim that these component waves are actually distinct?" Possibly, argues Price, the justification comes from the fact that the waves originate from different sources. However, he continues, such a justification would lead to further troubles. For if we start with half retarded radiation from the source, the retarded component of the advanced response of the absorber —which is also half advanced and half retarded— will ultimately be only one fourth of the original retarded wave. This means that the retarded wave of the the source will ultimately be $1/2 + 1/4$, therefore leaving a 25% off. Price continues by arguing that the issue is not problematic: "as long as we are prepared to allow that waves needn't have a unique source —i.e., to allow that it is simply a matter of our temporal perspective whether we say that the given wave originates at i [the source] or originates at the absorber particles" (Price 1991, p. 968). Therefore, he suggests to give up on the uniqueness of the source so as to avoid the distinction between the components, to the consequence that there are no different components anymore —i.e., they are *one and the same wave*. However, by interpreting the two waves as the same, Price also determines a modification of the type of symmetry that was originally employed by Wheeler and Feynman. Such a modification is well represented by (Frisch 2000, p. 391) in the following:

$$F^a_{ret} = \sum_{k \neq a} F^k_{adv} \tag{2.24}$$

Equation (2.24) implies a different symmetry, one for which all emitters produce retarded rather than advanced radiation. As further emphasized in: (Price 1997, p. 74)

> [The] basis of the proposed reinterpretation of the Wheeler-Feynman argument is the recognition that radiative symmetry does not require radiative emitters be *individually* symmetric in time. [...] Symmetry would also be secured if the class of emitters of retarded radiation turned out to be "mirrored" by a class of advanced radiation. As reinterpreted, the Wheeler-Feynman argument shows that this latter kind of symmetry is mathematically consistent.

Thus, at the micro-level, both absorbers and emitters are centered on coherent waves, these being ingoing in the first case and outgoing in the second. This, argues Price, permits an explanation of the (radiative) asymmetry based on the non-existence of large absorbers and the existence of only large emitters —rather than on statistical basis as argued by Wheeler and Feynman. He concludes:

> On this view the radiative asymmetry in the real world simply involves a [cosmological] imbalance between sources and sinks: large coherent sources of radiation are common, but large coherent sinks are unknown (Price 1991, p. 970).

In what follows, we present the first objection to Price's argument and we show that the converging and diverging components are not one and the same.

They are not *One-and-the-Same*

An objection to Price's statement —for which the wavefronts converging on the absorber are equal to those diverging from the source— was already pointed out by Ridderbos (1997). There, he argues that in the absorber theory the wavefronts are constituted by a superposition of plane waves, each of which having its own source in one of the particles of the absorber. The diverging wavefronts, on the other hand, have their source in the originally accelerated charge:

> In other words, although this superposition of plane waves appears as a spherical wavefront imploding on i and then exploding outward again, each of the nearly plane waves has a definite source, viz. the absorber particle j onto which it converges. The primary field on the other hand of course has the particle i as its source (Ridderbos 1997, p. 480).

Although it might be impossible to distinguish the source of each converging plane wave, it does not follow that the converging and the diverging waves are indistinguishable. If the radiations were one and the same, then there would be no explanation for the difference between (2.10) and (2.12). If the advanced and retarded radiations are one and the same, then it is not clear how they can be different in different regions of space. As promptly emphasized in (Frisch 2000, p. 395):

> But one way in which one might show that Wheeler and Feynman's original interpretation is correct while that of Price is mistaken is by showing that the two representations agree only within a limited region of spacetime. If the two representations do not agree everywhere (which in fact they do not), then they cannot be representations of the one and the same field.

If the advanced and retarded radiations are not *one and the same*, then there is no ground for the new type of symmetry introduced by Price. Nonetheless, one could grant that the new symmetry is needed because it provides a better explanation of the radiative damping and total radiation. In what follows, we will see that to accept Price's new symmetry yields some problems with respect to the radiative damping and the overall process view.

Radiative Damping and the Complete-Incomplete Absorption

An important problem in accepting Price's reformulation is pointed out by (Frisch 2000) who argues that if we accept equation (2.24), either we admit some form of self-interaction —which was the original problem that Wheeler and Feynman tried to solve— or there seem to be no space for the radiative damping. As a matter of fact, if the retarded radiation comes completely from the source, it also means that the radiative damping comes from the self interaction of the source with its own field. Alternatively, we can interpret the total radiation of the source (proper retarded radiation and radiative damping) as a sum of converging waves onto the absorber (corresponding to the advanced field of the absorbers). But then it would not be clear how the absorbers started radiating in the first place, while in Wheeler and Feynman's theory this is explained in terms of the response of the absorbers to the proper field of the initial source. Furthermore, even granted that the advanced fields were already accelerated, it would not be possible to distinguish between the proper retarded field of the source and its radiative damping, for the sum in equation (2.24) makes no distinctions between terms. We could then apply Price's argument and maintain that the two terms in equation (2.8) are the same, which is evidently not the case.

The relevant aspect of having a time-symmetry such that the source and the absorbers emit a half-advanced and half-retarded radiation is that the radiative damping comes from the response of the absorbers and adds-up to the initial retarded radiation of the source —as it was expressed in (2.16). In the original theory then, the total radiation stemmed from the mutual interaction of the half-advanced and half-retarded fields. Therefore, one needs the interaction of these two fields to account for the radiative damping (and the total radiation). By changing the relevant time-symmetry, Price modifies this mechanism and thus undermines the main thrust of the absorber theory. However, this does not imply that the new theory departs from the initial philosophical underpinning of the original formulation.

It could be that Price retains the overall process view (or some form of it), to build a different interpretation of classical electrodynamics. Consider (Price 1997, p. 74):

> [T]he crucial difference is that the usual retarded wave is no longer taken to *need* two (finite) sources, one in the past and one in the future; the claim is simply that insofar as such a wave does have two such sources, their contributions are entirely consistent. [...] In its new form the argument shows that an electromagnetic radiation field may be taken to be determined either by the past source or by its future sinks (absorbers); the two representations give equivalent results.

The relevant point is that the presence of the absorbers (one of the two finite sources) is not needed —against the second clause of the overall process view which states that the relation between absorbers and emitter is necessary in accounting for radiative phenomena. If they are present though, then it is legit to assume they play a role in determining the total radiation. We can then distinguish two cases: one in which the presence of the absorbers determines a complete absorption and one in which either the absorbers are absent or they are not enough to guarantee the complete absorption of the radiation.

Let's start with the case of complete absorption. This means that there are enough future absorbers such that there is no infinite propagation of the retarded field emitted by the source. If this is the case, then we have a similar situation to the one of the absorber theory of radiation. This suggests the theories by Price and Wheeler-Feynman might have a similar philosophical foundation. One way to see this is to question whether Price's theory still retains the analogy with the Lagrangian schema, despite its different time symmetry.

As we have seen in section 4, the Lagrangian schema rests on setting the parameters for both the initial and final space-time positions. The 'influence' of the future boundary on the initial (boundary) condition is what characterizes the 'all-at-once' approach. A system —for example a classical particle moving along a trajectory— is constrained by the initial and final positions in such a way that the trajectory is determined by both the boundaries and the variational principles. If we then take Price's reinterpretation for the case of complete absorption, it does seem that the analogy with the Lagrangian schema partially holds. As a matter of fact, Price's formulation depends on the presence of the future absorber to determine the radiation emitted by the source, similarly to the Lagrangian schema which depends on the final point of a particle to determine its trajectory.

In this sense, Price (1997, p. 71) does not depart from the overall process view of the absorber theory, for he also takes into account the past and the future of the radiative system: "In other words, I think the real lesson of the Wheeler and Feynman argument is that the same radiation field may be described equivalently either as a coherent wave front diverging from i [the source], or as the sum of coherent wave fronts converging on the absorber particles".[21]

With respect to the second clause of the overall process view —that being the necessity of the interaction between absorbers and source— there seem to be no problems with Price's theory. Because we are under the assumption of complete absorption, the absorbers (and their relation with the emitter) are necessary to guarantee that no radiation is propagated infinitely into the future.

Thus far, the main differences between the absorber theory and Price's theory under the condition of complete absorption are: the complete absorption condition being a consequence of the cosmological (im)balance between absorbers and emitters, and the presence of interference between advanced and retarded radiation —caused by the different types of time symmetries.[22]

What about the case of incomplete absorption? In this case the difference with the absorber theory and the overall process view becomes more relevant. Not only the problem of the radiative damping remains, but also the second clause about the necessity of the absorbers turns false in Price's view. As a matter of fact, if the source can emit a radiation that is not completely absorbed, the response of the absorber will not be enough to interfere with the advanced radiation and to provide for the radiative damping (i.e., the response of the absorber would not correspond to equation (2.7)). Even further, if the absorbers are not present at all, then the role of the absorbers becomes unnecessary, contra the second clause of the overall process view. Furthermore, the latter case also breaks the analogy with the Lagrangian schema: since the future boundary concurs in determining the trajectory, once we

[21] One can also read Price more 'literally' and emphasize that his use of past and future (diverging and converging wavefronts) is only a matter of description and not a full commitment to the necessity of using both future and past radiations of absorbers and emitters to account for the total radiation.

[22] These differences plays an important role in the explanation of the time asymmetry at macroscopic level, for example as illustrated in (Price 2006). There, Price argues that the experimental time asymmetry of classical radiation comes from the cosmological imbalance between large emitters and small absorbers, rather than being a boundary condition that applies to the system.

remove the boundary the 'all-at-onceness' disappears. In Price's theory this happens as the radiation can propagate infinitely into the future.

One can reply that (Bauer et al. 2014) developed an account that does not require the complete absorption condition and that perhaps the presence of an absorber is not needed. However, while it is true that one can justify the radiative reaction even in the case of incomplete absorption, this does not mean that the absorbers are not needed. As a matter of fact, in (Bauer et al. 2014) the radiative damping comes from the advanced response of the absorbers to the retarded radiation of the source. Furthermore, the account rests on the low-entropy of the early universe and on a statistical argument —the latter being the point that Price criticized in Wheeler and Feynman's original theory. Given that the second clause of the overall process view does not imply complete absorption and that Bauer's account dispenses such a requirement while preserving the importance of the absorbers-emitter interaction, it follows that Bauer's theory can also be characterized by the second clause of the overall process view. With respect to the first clause, this is also respected because of the explicit use of advanced and retarded radiation of both source and absorbers.

To sum up: we have seen that Price's reinterpretation changes the absorber theory in two respects: (i) it does not account for the radiative damping —unless one resorts to the self-interaction— and (ii) it only partly fits with the original philosophical underpinning. With respect to the latter point, in this last section we have fine-grained Price's view and distinguished between complete and partial absorption. On the one hand, in the case of complete absorption, Price's reinterpretation fits with the second clause of the overall process view (despite it still does not account for the radiative damping). On the other hand, in the case of incomplete absorption, we have shown that Price's view and the second clause diverge.

2.6 Holism

Thus far I have suggested that (part of) the intuition behind the absorber theory was not to consider absorbers and emitters separately, but rather as partaking to a whole process that involves advanced and retarded radiation. It is thus evident that something needs to be said about (non)separability and eventually holism. For we are considering overall processes, one might question whether the system absorbers-emitter can be reduced to its parts. In other words, it seems legit to ask whether the system involved in the overall process is a holistic one, i.e: whether the whole has properties that are not reducible to the properties of the parts. Though, the question whether the absorber theory can be interpreted in terms of holism begs another question first: one concerned with the type of holism under consideration.

One of the most relevant works on holism (within the context of quantum mechanics) can be found in (Healey 2016) where holism is divided into three different types: (i) metaphysical holism, (ii) relational holism and (iii) methodological holism. Although the work by Healey is mainly focused on quantum mechanics and non-separability, we will investigate whether any of these categories apply to the absorber theory of radiation.

With respect to (i), (Healey 2016) distinguishes three different types: ontological, property and nomological, though the common idea remains that: "the nature of some wholes is not determined by that of their parts" (Healey 2016, p. 3). If we consider the whole to be the overall system (S) of absorbers and emitter, then the parts would be the single absorbers as well as the single source. To argue that the nature of the system is not determined by that of its parts (in this case) is a tall order. I have just argued, against Price, that emitter and absorbers are different and that they are both necessary in the absorber theory. To account for the radiative reaction, Wheeler and Feynman did not use any property of the system S that was not derived by the properties of either the absorbers or the source. Therefore, it seems amiss to address the system S in terms of metaphysical holism.

The second type of holism, relational holism (or property holism) is mainly advocated by (Teller 1986) and the central idea is that the properties of the whole are not strongly determined by the properties of the parts. To make the statement non trivial, Teller (1989) specifies that relational holism is concerned only with some properties, thus raising the question as to what properties one ought to consider. A good starting point is to distinguish between relational and non-relational properties: the former depend on the existence or state of other entities, and the latter are independent from the existence or state of other entities. For the present case, let's consider as the relational property of the system absorbers-emitter the total retarded radiation, and as the non-relational properties of the parts the radiations emitted by the absorbers and emitter respectively. If we then consider equation (2.22) we can conclude that the total retarded radiation is indeed fully determined by the radiation of the component parts of system S. It seems that property holism does not hold for the case at hand. However, as Wheeler and Feynman argued, the retarded radiation of the source comes from the advanced radiation of the absorbers. It follows that although they can be distinguished, it might be the case that they cannot be separated, where we take as a separability principle the thesis that:

> Separability Principle: The states of any spatio-temporally separated subsystems $S_1, S_2 \ldots, S_n$ of a compound system S are individually well defined and the states of the compound system are wholly and completely determined by them and their physical interactions including their spatio-temporal relations (Karakostas 2004, pp. 2–3)

By including the spatio-temporal relations between the parts, we can then conclude that the system absorbers-emitter is indeed a separable one. For the radiative reaction is caused by the advance response of the absorbers, one ought to consider the relation between the two and yet without violating the separability principle.

The third type of holism individuated by (Healey 2016, p. 2) is named methodological holism, for which: "The best way to study the behavior of a complex system is to treat it as a whole and not merely to analyze the structure and behavior of its component parts". Methodological holism is a much weaker thesis than (i) and (ii) as it does not say anything with respect to the nature of the system under consideration. If we again consider the case of the absorber theory which accounts for the phenomenon of radiation, one ought to consider both the diverging field of the source and the fields converging on the absorber particles (as expressed in (2.16)). There is therefore a sense in which one methodologically considers the whole system S, but that is because the total retarded radiation ultimately stems from the relation between absorbers and source (as well as from the time-symmetry). Therefore, for there are no instances of non-separability in the absorber theory, methodological holism is what might approximate the idea of overall processes we have addressed here.[23]

However, if methodological holism is cast in terms of having to consider the spatio-temporal relations of the parts to account for the whole, then we might simply call it relationism —namely the intuition for which both the interaction between absorbers and emitter, as well as the temporal relations between advanced and retarded radiations, are fundamental to account for the phenomenon of radiation (and radiative damping).[24] As a matter of fact, relations are necessary for the overall process view and the point was made evident in the previous quote by (Wheeler and Feynman 1945, p. 181): "Those phenomena [...] require us to recognize the complete interdependence of past and future in nature". The term 'interdependence' does not entail non-separability here, but rather the necessity of looking at both advanced and

[23] A slightly different view would be to consider the intuition of the absorber theory as an 'early-stage' form of holism, which will develop into a more robust form in Feynman's later works.

[24] The term 'relationism' is a loaded one in philosophy and philosophy of science. For the present purposes, I will not refer to the debate about space-time substantivalism and/or relationsim (Huggett and Hoefer 2018), or to relationist interpretations of quantum mechanics (Laudisa and Rovelli 2013). By 'relationism' I focus on the relational character of the absorber theory that was advanced here.

retarded radiations of both source and absorbers.

2.7 Conclusions

The main point of this chapter was to emphasize the philosophical underpinning of Wheeler and Feynman's original work. I have been suggesting that: Wheeler and Feynman had an intuition with respect to electrodynamics phenomena. This intuition led them to build the absorber theory of radiation. I have specified the intuition and I have called it: the overall process view. By rehearsing the derivations of the theory, I have shown how the concept of overall process is at play in the theory and the possible connections with Feynman's path integrals and with the Lagrangian schema.

I presented Price's modified version of the absorber theory, where he suggested a different time-symmetry which led to the possibility of expressing the total retarded field of a source in terms of a sum of the advanced radiation emitted by the absorbers. However, I have shown how this reinterpretation bears some problems, physical and philosophical. The problems with the physics are mainly related to the difficulty of justifying the radiative reaction —which was Wheeler and Feynman's initial concern. I have also questioned whether the new theory maintains the philosophical underpinning of the absorber theory. It turned out that if we dismiss the problem of radiative damping, Price's reinterpretation seems to endorse an underpinning similar to the overall process view, although such similarity applies only to the case of complete absorption. In such a scenario, Price's theory advocates for the mutual interaction of past and future, preserving the 'all-at-once' narrative that was shared between the Lagrangian schema and the overall process view.

However, when we consider the case of incomplete absorption, Price's view changes the philosophical underpinning. If the absorbers are not present, or they are not enough to guarantee the complete absorption, the retarded radiation will propagate infinitely into the future. This weakens the second clause of the overall process view which deems the presence of the absorbers and its relation with the source as constitutive of the radiative phenomena. The weakened importance of the future boundary condition (the presence of the absorbers) also weakens the analogy with the Lagrangian schema for which a classical trajectory is calculated through the use of variational principles within a given boundary.

A different reading of Price might focus on the representational character of the total field as either a retarded field centered on the source, or as a sum of advanced fields centered on the absorbers —this would be supported by the 'one and the same' argument. To argue that having a sum of advanced fields or only one retarded radiation is a matter or representation, it undermines the importance of the relation between absorbers and emitters in the sense emphasized by Wheeler and Feynman. In the absorber theory, the different fields (half-advanced and half-retarded) interfere with each other to give rise to the total radiation. If, on the other hand, we deem the distinction of the fields to be a matter of representation, we are providing a different narrative about the radiative phenomena. If this is the case, then the influence of future on the past would be if not dismissed, at least weakened. By leaning toward this interpretation of Price's argument, one could probably make a case that even the case of complete absorption would not fit with the overall process view.

Chapter 3

Path Integrals and Holism

3.1 Introduction

This chapter considers the path integral formulation of quantum mechanics (Feynman 1948c) and argues that it suggests a form of holism for which the whole has properties that are not reducible to the parts. More specifically, we will characterize holism in contraposition to reductionism and maintain that a system is holistic if it does not reduce to its parts.

The overall line of argument of the chapter can be summarized as follows: Holism means that a compound (or its properties) is not reducible to its parts. Consider, for example, the least action principle: the total ensemble is constituted by the possible paths and the principle reduces the possibilities to one actual trajectory. With respect to path path integrals: the total ensemble is not reducible to the single trajectories, even though weak reductionism is at least possible. However, this only means that a complex system is composed of its parts, which is not an instance of holism yet. We will then use an analogy with entangled states to discuss the difference between physical possibilities and non-physical ones. Finally, we show how path integrals rely on the sum over some physical possible trajectories and some mathematical artifacts. In this sense, the ensemble of physically possible trajectories does not reduce to the individual physically possible paths.

The chapter is structured as follows: section 2 provides a brief overview of Feynman's path integrals formulation and the reconstruction of the classical limit. Section

3 considers three different interpretations of path integrals to show that: (i) it is not possible to reduce the ensemble of trajectories to a single real path, and (ii) some of the paths in the total ensemble are non differentiable.

Section 4 starts from the conclusions of the previous section and argues that upon a fine-graining of the definition of trajectory, the total ensemble is weakly non-reducible to the physically possible trajectories and their probability amplitudes. For a trajectory to be characterized as 'physically possible' it means that it has to be continuous and everywhere differentiable —granted that there are no physical obstacles between the boundaries. The fine-graining of the concept is necessary given that some of the possible paths are non differentiable. While it is legitimate to incorporate the physically possible trajectories in the total ensemble as physical possibilities (similarly to the case of the least action principle), the presence of the ill-behaved paths becomes hard to justify. It is then argued that we can consider them as mathematical entities and thus conclude that the total ensemble is non-reducible to the physically possible trajectories and their individual amplitudes. This non-reductionism is only weak, for the differentiable paths are necessary but not sufficient to calculate the total amplitude.

3.2 Path integrals

Let's approach path integrals —which is an equivalent formulation of quantum mechanics— starting from the well-known double slit experiment, where an amplitude is associated to the event of a particle going from a source S to a detector D (we call this amplitude ϕ). Since in the double slit experiment there are two slits A and B, that constrain the possible paths of the particle, what is obtained is a sum over the possible paths that the particle can take, that is, $\phi_{tot} = \phi_A + \phi_B$. Now, suppose we fill up the space between the source and the detector by adding n many screens each of them with n holes. Then, if we let n go to infinity, the probability becomes a sum over an infinite number of terms —representing the possibility for the particle to take every possible path between the source and the detector.

Let's derive the path integral for the transition amplitude of a system from initial time t_i to a final time t_f. We consider the Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}\left|\psi\right\rangle = \hat{H}\left|\psi\right\rangle \tag{3.1}$$

and its solution:

$$\left|\psi(t)\right\rangle = e^{-\frac{i}{\hbar}\hat{H}(t-t_i)}\left|\psi_i\right\rangle \tag{3.2}$$

where $\hat{H}$ is the Hamiltonian operator. We can then write the transition amplitude as:

$$\langle\psi_f|e^{\frac{-i}{\hbar}\hat{H}(t_f-t_i)}|\psi_i\rangle \tag{3.3}$$

and we can express it in terms of position eigenstates:

$$A = \langle x_f|e^{\frac{-i}{\hbar}\hat{H}(t_f-t_i)}|x_i\rangle \tag{3.4}$$

We can now 'time-slice' the transition amplitude, that is we divide a single trajectory in infinitely many (N) time-steps where $\Delta t = (t_f - t_i))$:

$$A = \langle x_f|e^{\frac{-i}{\hbar}\hat{H}\Delta t}|x_i\rangle = \langle x_f|\underbrace{e^{\frac{-i\Delta t}{N\hbar}\hat{H}}e^{\frac{-i\Delta t}{N\hbar}\hat{H}}\dots e^{\frac{-i\Delta t}{N\hbar}\hat{H}}}_{N-times}|x_i\rangle \tag{3.5}$$

We have obtained the propagator for a single trajectory as shown in Fig.1

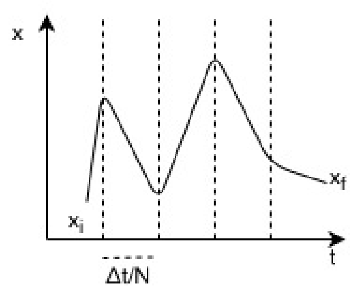

Figure 3.1 Time slice of a trajectory

We then insert a resolution of the identity for each term, that is, we use the completeness relation:

$$\mathbb{1} = \int \left|x\right\rangle\left\langle x\right| dx \quad \langle x|x'\rangle = \delta(x-x') \tag{3.6}$$

So that each x_k varies independently, generating all the possible trajectories as it advances through the time-slices.

$$A = \langle x_f | e^{\frac{-i}{\hbar}\hat{H}\Delta t} | x_i \rangle = \langle x_f | e^{\frac{-i\Delta t}{\hbar N}\hat{H}} \mathbb{1} e^{\frac{-i\Delta t}{\hbar N}\hat{H}} \mathbb{1} \ldots \mathbb{1} e^{\frac{-i\Delta t}{\hbar N}\hat{H}} | x_i \rangle \tag{3.7}$$
$$= \prod_{k=1}^{N} \left[\int dx_k \right] \prod_{k=1}^{N} \langle x_k | e^{\frac{-i\Delta t}{\hbar N}\hat{H}} | x_{k-1} \rangle$$

Now, each step is defined as:

$$A_k = \langle x_k | \exp\left\{ \left(\frac{-i\epsilon}{\hbar} \hat{H} \right) \right\} | x \rangle \tag{3.8}$$
$$= \langle x_k | \exp\left\{ \left(\frac{-i\epsilon}{\hbar} \left[\frac{\hat{p}^2}{2m} + \hat{V}(x_{k-1}) \right] \right) \right\} | x_{k-1} \rangle$$

where we have used the equivalence $\epsilon \equiv \Delta t/N$, we have considered the simple case: $\hat{H} = \hat{T} + \hat{V}$ which are kinetic and potential energy respectively and where $\hat{T} = \hat{p}^2/2m$. We can use the Trotter formula for which: $e^{-it(A+B)} = s - \lim_{N\to\infty}(e^{-itA/N}e^{-itB/N})^N$ and evaluate the potential and kinetic operators separately. We evaluate the action of the potential operator and replace it with its own eigenvalues: $e^{-i\hat{V}(x)} |x_n\rangle = |x_n\rangle e^{-iV(x_n)}$. However, in evaluating the momentum operator, the position state is not an eigenvector and hence we insert a resolution of the momentum operator given that: $\int dp/2\pi\hbar \, |p\rangle \langle p| = \mathbb{1}$, to obtain:

$$A_k = \int \frac{dp}{2\pi\hbar} e^{\frac{-ip^2\epsilon}{2m} + ip(x_k - x_{k-1})} e^{\frac{-i\epsilon}{\hbar}V(x_{k-1})} \tag{3.9}$$

which is a Gaussian integral of the form (Lancaster and Blundell 2014, p. 214): $\int_{\infty}^{\infty} dx e^{-\frac{ax^2}{2}+bx} = \sqrt{\frac{2\pi}{a}} e^{\frac{b^2}{2a}}$ where: $a = i\epsilon/m$ and $b = i(x_k - x_{k-1})$. We thus obtain the time-slice propagator:

$$A_k = \left(\frac{im}{2\pi\hbar\epsilon} \right)^{1/2} e^{i\epsilon/\hbar \left[\frac{m}{2} \left(\frac{x_k - x_{k-1}}{\epsilon} \right)^2 - V(x_{k-1}) \right]} \tag{3.10}$$

We then apply (3.10) to all the various time-sliced terms in (3.7) and if we take $N \to \infty$ and $\Delta t \to 0$ we have the path integral:

$$A = \int \mathcal{D}[x(t)] e^{\frac{i}{\hbar} \int \frac{m\dot{x}^2}{2} - V(x)} = \int \mathcal{D}[x(t)] e^{iS/\hbar} \tag{3.11}$$

where the term $\mathcal{D}x$ indicates that we are integrating over all the trajectories connecting the initial and final point.[1]

3.2.1 The Classical Limit

Let's consider the classical limit $\hbar \to 0$, for which the main contributor to the total amplitude is the path characterized by a stationary phase[2]. All the other paths get canceled out, due to the rapid oscillation of the phases and to the periodicity of the exponential $e^{ix} = \cos x + i \sin x$. As Feynman claimed:

> These small changes in path will, generally, make enormous changes in phase, and the cosine or sine will oscillate exceedingly rapidly between plus and minus values. The total contribution will then add to zero; for if one path makes a positive contribution, another infinitesimally close (on a classical scale) makes an equal negative contribution, so that no net contribution arises (Feynman, Hibbs, and Styer 2010, pg29).

However, in the quantum case, for which the ratio $S/\hbar$ is smaller, more paths will contribute to the total amplitude.

To clarify this point, let's consider a numerical example provided by Townsend (2000) for both the classical and the quantum case respectively. Consider the action of a particle moving at constant speed from $x = 0$, $t = 0$ to $x' = 1cm$, $t' = 1s$, with

[1] For a more comprehensive mathematical treatment see, among others: (Townsend 2000), (Grosche and Steiner 1998), (Chaichian and Demichev 2001)

[2] To be more precise, in the classical limit $\hbar \to 0$, the main constructive interfering paths are those in the infinitesimal neighborhood (in the order of the quantum of action) of the stationary paths.

no forces acting on it and with $m = 1g$. The path is described by $x = (x'/t')t$ and its action is given by (Townsend 2000, p. 226):

$$S_{cl} = S[x_{cl}(t)] = \int_0^{t'} dt \frac{m}{2}\left(\frac{x'^2}{t'^2}\right) = \frac{mx'^2}{2t'} \tag{3.12}$$

The path associated with the action in (3.12) is the path of least action which is the only path for classical systems[3]. Now, Townsend considers a path describing uniform acceleration: $(x = x't^2)/t'^2$ with action $S(x't^2) = (2mx'^2)/3t'$. The phase difference between the two paths is given by the difference between the actions of the paths:

$$\frac{\Delta S}{\hbar} = \frac{S[x't^2] - S[(x'/t')t]}{\hbar} = \frac{mx'^2}{6t'\hbar} \tag{3.13}$$

With respect to the least action path, the phase difference is about $(1/6)10^{27}$ radians. The phases are clearly non-coherent, which means that a small change in action, producing a large change in the phase, will ultimately cause the cancellation of the paths that are not the classical path. Thus, paths whose actions are 'slightly' different from the least-action one, are non coherent (out-of-phase) and hence cancel with each other; thereby the least action trajectory is the dominant path.

However, in the quantum case, when we consider a particle having mass in the order of $10^{-27}g$, the difference between actions is only 1/6 radians. Thus, in the quantum case the two paths are coherent and they both contribute to the total amplitude. Within the range of periodicity of the exponential, more paths positively contribute to the total amplitude, yielding the fact that not only the stationary phase matters, but rather all the possible paths need to be considered. One might argue that, from the fact that more paths count, it does not follow that all paths count. We will see this argument in the next sections when we will consider (Wharton 2016), a form of decoherent history account (Gell-Mann and Hartle 2012) and (Kent 2013).

[3]It can be shown by proving that it is the path that minimizes the action.

3.3 Holism

Let us begin this section by characterizing holism in contraposition to reductionism: a system is holistic if and only if it is not possible to reduce the whole (or its properties) to its component parts (or to the properties of the component parts). In what follows we will argue that holism applies to path integrals, and that the total ensemble (and corresponding probability amplitude) does not reduce to the single trajectories and their respective amplitudes. In the next section we will argue that the form of non-reductionism is only weak and that some fine-graining of the notion of trajectory is needed. With respect to the latter, although path integrals rely on possible trajectories, the ill-behaved ones have no physical interpretation. Consequently, the total amplitude of the total ensemble is calculated based on the physical possibilities of the system and some mathematical entities that are necessary and yet not physical. The possibility of reinterpreting the ill-behaved trajectories as physical possibilities is also considered, but not fully explored.

To be noted that the characterization of holism at the beginning of this section remains vague, in particular, it leaves an open flank to two possible interpretations. We can call them: 'bottom-up holism' and 'top-down holism'. The former says that to determine a given property of the whole the properties of the parts are not sufficient. Clearly, the claim is limited to specific properties of the parts and whole, and on the way we decide to parse the latter. It is trivial that the properties of a whole depend on all the possible properties of all the possible components. This is why we are limiting our case to the probability amplitude of an ensemble of trajectories and to the single probability amplitudes of the single trajectories —and not, for example, to all the possible properties of each spacetime point within the boundary.[4] An example of this 'bottom-up' holism is presented in (Karakostas 2004) where he argues that the

[4] The point was also raised in (Healey 1991).

spin properties of an entangled state (in his case, a *singlet state*) are not determined by the spin properties of the separate individual parts.

A 'top-down' holism, on the other hand, focuses on how the properties (and relations) of the parts are characterized by the whole. In this view, it is the whole that determines some of the properties of the components, which cannot be determined by the properties of the parts alone. One example of this view is defended in (Esfeld 2004, p. 9) where it is argued that: "there are global observables of the whole that have a definite numerical value in the state in question [entangled state] and that can be considered as intrinsic properties of the whole. These properties of the whole indicate the way in which the parts are related with respect to their state-dependent properties, for they contain correlations between the probability distributions of the respective state-dependent properties of the parts, although these state-dependent properties cannot be attributed to each of the parts". In this sense, the whole dictates the relational properties of the parts, even though the parts do not have intrinsic state-dependent properties such as momentum or position. Therefrom, the characterization of what we have called 'top-down' holism.

With respect to path integrals, we will show that the whole —that is, the total ensemble of possible trajectories which expresses the total probability amplitude— is not completely determined by the single trajectories connecting the initial and final point and their respective individual probability amplitudes. This amounts to a bottom-up holism as characterized above.

One might already notice that a potential problem is right around the corner. Should we interpret the ensemble of possible trajectories as a physical system displaying a form of holism? The answer is no. An ensemble of trajectories is neither a physical system nor a physical object. Even further: a trajectory is neither a physical system nor a physical object. Nonetheless, nothing prevents us from discussing whether a trajectory is determined by the collection of all of its spacetime points.

Similarly, nothing prevents us from discussing whether the probability amplitude obtained from an ensemble of possible trajectories reduces to the individual amplitudes of the individual trajectories composing the ensemble.

Consider, for example, the least action principle which individuates one of the possible trajectories of the ensemble as the actual path traversed by an object by considering the action of all the physically possible alternatives. This means that the total ensemble of trajectories is constituted by physical possibilities and that among those possibilities there is one that is actualized. Even though we shall not characterize trajectories and ensembles as physical systems, we still conclude that the ensemble of possible paths is determined by the individual trajectories and that the least action principle individuates the one that is traversed by the classical object. However, path integrals show an important difference if compared to the least action principle. To understand such a a difference, we need a distinction between 'potential possibility' and 'actual reality' as addressed in (Terekhovich 2019, pp. 3–4):

> The variational principles, as well as the path integral formalism use, in the strict mathematical sense, two fundamental philosophical concepts — *potential possibility* and *actual reality* [...] In a potential mode of existence, the real system is in all possible alternative motions at once and interacts with other systems in all possible ways. In the actual mode of existence, the system is in one of the possible alternative motions.

What we will show is that path integrals not only do not reduce to an actual trajectory, but they do not reduce to a possible trajectory either. The paths of the total ensemble in the path integral count as potential possibilities and some of these possibilities are not physically possible in the traditional sense of classical mechanics.

In what follows, we will consider three different accounts that attempted to reduce the total quantum ensemble to either a set of possible paths or even to a single real history. The point is to show that: (i) it is not possible to reduce the total ensemble to a single real trajectory, which is what distinguishes (among other things) path integrals from the principle of least action. This non-reducibility also sets us in the right direction to holism: if the total ensemble was reducible to a single trajectory, the possibility of a holistic picture would be in jeopardy. (ii) Some trajectories of path integrals are ill-behaved and nonetheless ought to be considered as part of the total ensemble.

3.3.1 Parsing the Ensemble

Wharton (2016) suggests a possible parsing of the total ensemble of possible trajectories. Thereby, he can defend a reductionist approach that leads to a real subset of non-interfering paths. Wharton's idea is that: although it is not possible to dismiss all-but-one trajectories that we take as unreal (those, for instance, with negative probabilities), it is nonetheless possible to group them into real sets by exploiting the cancellation process.

Because of the impossibility to reduce to the total ensemble to a single trajectory, we can distinguish between two types of reductionism:

- **Strong reductionism**: It is possible to reduce the total ensemble of possible trajectories to a single real path undertaken by a particle.
- **Mild reductionism**: The total ensemble can be reduced to smaller sets of possible trajectories.

Wharton leans toward the latter option and in doing so he draws a parallel between path integrals and statistical mechanics. In statistical mechanics, he argues, the probability is epistemic in nature, for it describes a state of knowledge "underpinned by one real microstate that exists in ordinary three-dimensional space" (Wharton 2016, p. 3). It follows that, although he admits the impossibility of a strong reductionism for path integrals, such impossibility is pinned on epistemic ground, and thus he recasts the probabilistic nature of path in terms of classical ignorance or subjective probability. To do so, Wharton divides the ensemble into sets of only contributing trajectories. Let $\mathbf{R}$ be the set of the possible trajectories and A and B be two single trajectories, then Wharton constructs a coupling of the different paths so that the total sum is non-negative (Wharton 2016, p. 4):

$$P(x_0, x_1) = \sum_{(A,B)\subset\mathbf{R}} cos\frac{S_A - S_B}{\hbar} \tag{3.14}$$

However, Wharton recognizes that the single terms in (3.14) can be negative, and hence they cannot be interpreted as classical probabilities. He then suggests another parsing according to which: $P(x_0, x_1) = \sum_i F_i + G_i$ where $F_i > 0$ and $G_i = 0$. In other words, he divides (3.14) into two different subsets, such that the probability for the subset G_i is equal to zero, and strictly positive for the subset F_i. The total probability is found by coupling the trajectories A and B into subsets $a_i \in \mathbf{R}$ and summing over these subsets (Wharton 2016, p. 5):

$$P(x_0, x_1) = \sum_i \sum_{(A,B)\in a_i} \cos\frac{S_A - S_B}{\hbar} \tag{3.15}$$

Because there are no more negative terms, he obtains a realistic interpretation of the probability amplitude, but at the expense of “having the fundamental possibility space consist of *many* particle trajectory-pairs, not merely one” (Wharton 2016, p. 5).

To avoid the problem, Wharton further restricts the parsing of (3.14) so as to have different sets c_i of single paths without having the same path appearing in more than one set. Such approach results in the following formula for the calculation of the probability (Wharton 2016, p. 5):

$$P(x_0, x_1) = \sum_i \left| \sum_{A\in C_i} exp(iS_A/\hbar) \right|^2 \tag{3.16}$$

The probability, according to (3.16), is given by the probability of each path A belonging to a group of paths c_i, summed over all the possible groupings. Each group of paths c_i is a positive term, and hence it does not contain redundant paths. In other words: the difference between (3.15) and (3.16) is that the second imposes a restriction on the sets c_i such that a different trajectory does not appear in different sets.[5] To be noted that the phenomenon of interference will occur within the set c_i but not between different sets (Wharton 2016, p. 6):

> Interference is still evidently possible between paths *within* a given set. And because of the appearance of many-path interference in well-established phenomena (say, triple-slit experiment), there is no escaping the conclusion that each set c_i must contain many paths. There is no way to reduce c_i down to a single path or two, in agreement the earlier analysis.

Again, a classical statistical interpretation is available: one set of paths c_i will correspond to the one containing the trajectory taken by the particle, even though it remains impossible to determine which one. Since we still have sets of paths, rather than single trajectories, Wharton's analysis clearly fits into the category of 'mild reductionism'.

What are, one might ask, the implications of Wharton's mild reductionism for the holistic view? In Feynman's formulation, the main contributions to the quantum probability amplitude come from non-differentiable paths. For example, the transition amplitude for a particle of mass m which moves under a potential $V(x)$ from an initial state $\psi(x_k, t)$ to a close subsequent point $\psi(x_{k+1}, t + \epsilon)$ is:

$$\psi(x_{k+1}, t+\epsilon) = \int exp\left[\frac{i\epsilon}{\hbar}\left\{\frac{m}{2}\left(\frac{x_{k+1}-x_k}{\epsilon}\right)^2 - V(x_{k+1})\right\}\right]\psi(x_k, t)dx_k/A \qquad (3.17)$$

where A is a normalization factor and $S(x_{k+1}, x_k) = \frac{m\epsilon}{2}(\frac{x_{k+1}-x_k}{\epsilon}) - \epsilon V(x_{k+1})$. Feynman (1948c, p. 17) argues that:

> Most of the contribution to $\psi(x_{k+1}, t+\epsilon)$ comes from values of x_k in $\psi(x_k, t)$ which are quite close to x_{k+1} (distant of order $\epsilon^{1/2}$) so that

[5] For the mathematical details see: (Wharton 2016, p. 5).

> the integral equation (3.17) [*(23) in the original*] can, in the limit, be replaced by a differential equation. The "velocities", $(x_{k+1} - x_k)/\epsilon$ which are important are very high, being of order $(\hbar/m\epsilon)^{1/2}$ which diverges as $\epsilon \to 0$. The paths involved are, therefore, continuous but possess no derivative.

There are two important consequences for the present argument. First, the most contributing paths are difficult to be pictured in terms of classical trajectories and the reason is that the average velocity of a path diverges once the time interval between two subsequent points tends to zero:

$$\left(\frac{x_{k+1} - x_k}{\epsilon}\right)^2 = -\frac{\hbar}{im\epsilon} \tag{3.18}$$

Therefore, Wharton's idea of having trajectories being grouped into classes and to assign them a classical probability, does not account for the non-classical nature of the trajectories in the first place.

Second, the original formulation of path integrals already entails a distinction between contributing and non-contributing paths: continuous non-differentiable paths contribute to the quantum mechanical probability amplitude and, in the classical limit $\hbar \to 0$, the main contributions come from paths in the neighborhood $\pi\hbar$ of classical (differential) trajectories —as it has been shown in the section on the classical limit. It follows that the difference between the original formulation and the one suggested by Wharton is that the latter eliminates the non-contributing paths before the process of summation.

The final result, if one preserves the empirical adequacy, is the same: the mild reductionism simply anticipates the action of the cancellation process. Hence, the ensemble —although reduced— still plays a fundamental role in the calculation of the final amplitude.

Before moving to Wharton's physical interpretation of the reduced sets c_i, there is one more concern that is worth of our attention. Wharton seems to cast the probability of the real set in terms of subjective probability: "The preceding analysis proves that it is possible to assign a classical-ignorance interpretation to the path integral, essentially no different than the framework of classical statistical mechanics" (Wharton 2016, p. 6). In other words, he compares the probability of path integrals with the one in statistical mechanics, that being grounded on the existence of a real microstate unknown to us. One then might argue that casting the probability in terms of classical ignorance can be justified in terms of the existence of some sort of hidden variables which would prevent the experimenter from individuating the one real set. Alternatively, one might justify the 'classical ignorance' in terms of computational difficulties due to the large amount of variables at play.[6] Both alternatives are ruled out if we consider the nature of the trajectories composing the set c_i —which are the contributing paths to the probability amplitude. Starting from the relation derived in (Feynman 1948c, p. 26):[7]

$$m\left(\frac{x_{k+1}-x_k}{\epsilon}\right)x_k - m\left(\frac{x_k-x_{k-1}}{\epsilon}\right)x_k \leftrightarrow_S \frac{\hbar}{i} \tag{3.19}$$

where the symbol $\leftrightarrow_S$ means that the equivalence is valid under the same action, one can reverse the second term on the left and translate the equation in operator notation, thus obtaining the commutation relation and the uncertainty principle:

$$\hat{p}\hat{x} - \hat{x}\hat{p} = \frac{\hbar}{i} \quad \Rightarrow \quad (\Delta x)(\Delta p) \geq \frac{1}{2}\hbar \tag{3.20}$$

Also, from equation (3.19), Feynman shows the equivalence with equation (3.18) emphasizing the connection between uncertainty and non-differentiability.[8] This con-

[6] One can conceive of this last case as the difficulty of calculating the exact outcome of a dice-tossing because of the presence of variables such as the friction between the dice and the surface, the exact strength of the tossing, the imperfections of the shape of the dice and so on.

[7] The mathematical details do not concern us here, but can be found in: (Feynman, Hibbs, and Styer 2010) and (Feynman 1948c).

[8] For the details of the proof, see: Feynman, Hibbs, and Styer (2010, p. 176)

nection emphasizes that the probabilistic nature of path integral arises from the uncertainty principle affecting each ϵ-*interval* of each path. The probabilistic nature of path integrals is then entrenched into the formalism, making the theory 'objectively probabilistic'. The concept is well expressed by Feynman et al. (1951, p. 533):

> The new theory [i.e., quantum mechanics] asserts that there are experiments for which the exact outcome is fundamentally unpredictable, and that in these cases one has to be satisfied with computing probabilities of various outcomes.

Wharton's account advanced the idea that the construction of the probability amplitude via path integrals is based on classical ignorance. On the contrary, we have argued that probabilities are genuinely entrenched into the formalism, but how to interpret them and why that is the case, it would be a different project.

It seems that there is no much room there for either hidden variables, or for computational complexity. The only other solution seems to be a reinterpretation of the physical nature of the sets c_i, treating them as something different than a collection of trajectories. Wharton seems to opt for the aforementioned solution, for he suggests to consider the sets c_i as an extended field: "the task is then to map the relevant parameters of the set c_i onto a continuous, spacetime-valued field" (Wharton 2016, p. 9). Wharton also gives a hint on how to possibly construct such a map: "One template for how this might work (albeit in reverse) is the connection between the standard complex wave function and the collection of possible paths that particle might take in Bohmian mechanics" (Wharton 2016, p. 9). However, the attempt would require for c_i to have a non-intersecting constraint for the paths of the set. Since the parsing of the ensemble is not fixed, it might be possible to overcome the problem. However, though the parsing of the ensemble might lead to a map onto a spacetime valued field, the not unique parsing could easily rise the objection of being undetermined by other possible sets.

A possible answer, to the problem of the non-uniqueness of the grouping, is proposed in (Wharton 2016, p. 9):

> For example, in a double-slit experiment, a position measurement immediately after the slits (before interference could occur) would only have sets c_i in which every path passed through the same slit. But delaying the time of the position measurement (after potential interference) would lead to sets c_i where the paths passed through both slits before converging onto the measurement point. If each c_i is interpreted as a field, the former case would yield a realistic field history that really did pass through only one slit, while the latter case would yield a realistic field history that really did pass through both slits.

The influence of the future over the past —that is, the fact that a future measurement of position can determine through which slit the particle has passed— is not problematic, as the Lagrangian considers the whole trajectory 'all-at-once'. Wharton (2014, p. 4) calls this framework: 'the Lagrangian Schema':

> One sets up a (reversible) two-way map between physical events and mathematical parameters, partially constrains those parameters on some spacetime boundary at both *the beginning and the end*, and then uses a global rule to find the values of the unconstrained parameters and/or a transition amplitude. This analysis does not proceed via dynamical equations, but rather is enforced on entire regions of spacetime "all at once".

However, we ask: what quantity would be assigned to each point of space-time? If, for example, the field is taken to be an action-valued field (or a phase-valued field) which assigns a value of action to each space-time point, then restricting the phenomenon of interference to paths within each set c_i would not be enough. As previously argued, the paths within the set are non differentiable and might span over the whole space within the boundary conditions. To provide a map from sets of paths to an action-field, one would need to impose a non-interfering condition also on the paths within the sets c_i. Otherwise, same spacetime points would have different action-values due to the superposition rule. But to impose such a restriction seems at best problematic. The interference of the paths within the real set is what guarantees the interference pattern well-verified by the experiments. One might try to avoid this problem by calculating the action-field for ϵ-time-sliced trajectories, rather than for the whole path. The 'step-by-step' calculation, though, might undermine the Lagrangian Schema and the basis for justifying the influence of future measurements on the past. Therefore, Wharton's account remains unclear about what quantity the spacetime points (or trajectories) would represent in a field-based picture.

To sum up: Wharton has shown how a mild-reduction of the total ensemble is possible. However, we have also seen that the trajectories that survive to the cancellation process —the latter being performed before or after the integration— are far from being classical. In the next sections we will see how this latter point plays an important role for holism.

3.3.2 The Decoherent History Approach

A different parsing of the path integral total ensemble is suggested by a reformulation of the decoherent history interpretation (DH) proposed by Gell-Mann and Hartle (2012). This subsection argues that such a reformulation provides a parsing of the ensemble which yet does not undermine a possibility for a holistic interpretation. As a starting point, let's consider the decoherent histories interpretation, originally by Griffiths (1984), which assigns probabilities not to single quantum states, but rather to histories of closed quantum systems.[9] Such histories become the main object of the interpretation; they are sequences of alternatives at successive times described by an exhaustive set of projection operators $\left\{P^k_{\alpha_k}\right\}$ such that $\sum_{\alpha_k} P^k_{\alpha_k} = 1$. Where: "$k$ denotes the set of alternatives at time t_k (e.g. a set of position ranges) and α_k the particular alternative" (Gell-Mann and Hartle 1990). In general, the decoherent histories account coarse-grains the total ensemble of possible paths into different classes α: (Hartle 2005, p. 8)

$$\langle x''|C_\alpha|x'\rangle = \int_\alpha \delta x e^{iS[x(t)]/\hbar} \tag{3.21}$$

The coarse graining in (3.21) is represented by the fact that we integrate over α which corresponds to a region of configuration space Δ_{α_k} where $\alpha_k = 1, 2, \ldots$ at a given sequence of time $t_1, \ldots t_n$.

DH distinguishes between fine-grained and coarse grained histories where: the former represent the most refined description of a system (the example is the motion of a particle of a gas described 'step-by-step' by Newton's laws) and the latter consist of partitions of the ensemble of the possible histories, i.e., sums of complete sets of projector operators. The coarse-graining 'mirrors' the strategy we have seen in the previous subsection. Consider for instance: (Gell-Mann and Hartle 1990, p. 7)

> Feynman's sum-over-histories begins by specifying the amplitude for a completely fine-grained history in a particular basis of generalized coordinates $Q^i(t)$, say all fundamental field variables at all points in space. This amplitude is proportional to $exp(iS(Q^i(t)))/\hbar \ldots$ (However), completely fine grained histories in the coordinate basis cannot be assigned probabilities; only suited coarse-grained histories can.

[9] For a better and more detailed exposition of such interpretation see: (Halliwell 1995), (Gell-Mann and Hartle 1990) and (Hartle 1993).

Gell-Mann and Hartle (2012) suggest a modification to the DH account in order to have a formulation that admits the existence of one real fine-grained history. Such a new account, named 'Extended Probability Ensemble'(EPE), is based on the original DH formulation and on four further ingredients. First, the one real fine-grained history is assumed to be embedded in an ensemble of alternative histories. These fine-grained histories are described in a preferred set of variables, i.e., those of the sum-over-histories of Feynman's interpretation. These preferred variables are denoted by q^i for a particle and, by evolving in time, they define a unique path $q(t)$ in $\mathcal{C}$, where $\mathcal{C}$ is the configuration space spanned by q^i.

Second: an extended notion of probability that goes outside of the range $[0, 1]$, which means that the probabilities for a fine grained history can be negative. Granted that probabilities are in general understood as expressions of our ignorance toward the happening of a given event, there is the implicit assumption that it is always possible to settle whether one of the possible alternatives occurs. Gell-Mann and Hartle point out that in quantum mechanics, to determine such alternative is not only difficult in practice, but it is genuinely impossible. They provide the example of the double-slit experiment for which, unless one disturbs the setting, to determine whether a particle has passed through one of the slits (or both) is a non-settleable bet.[10] The extended probabilities represent a new form of ignorance and: "quantum history ensemble will consist of alternative fine-grained histories assigned extended probability" (Gell-Mann and Hartle 2012, p. 3).

The third ingredient is a fundamental distribution $w[q(t)]$ which assigns an extended probability to each history $q(t)$ (Gell-Mann and Hartle 2012, p. 4):

$$w[q(t)] \equiv Re[\hat{\psi}^*(q_f, t_f) exp\{iS(q(t))/\hbar\} \, \hat{\psi}^*(q_0, t_0)] \tag{3.22}$$

[10]Such an idea of probability fits more the objective probability expressed by the quote from Feynman we have introduced in the previous subsection. As such, it is also different from the epistemic probability advocated by Wharton.

To see how (3.22) assigns an extended probability to a set of fine-grained histories, Gell-Mann and Hartle suppose $w[q_i(t)] \in [0,1]$ and then vary the fine-grained history $q_i(t)$ in such a way that $q_i(t) \sim q_{i*}(t)$ leaving the initial and final conditions (t_i, t_f) unchanged. Then, $q_{i*}(t)$ can contribute to the action in (3.22) and thus might change the sign of $w[q(t)]$ in such a way that $w[q(t)] \notin [0,1]$. Because of the negative probabilities: "the set of fine-grained histories is not the basis for a settleable bet on what the real fine-grained history is like" (Gell-Mann and Hartle 2012, p. 4). Given the non settleable bet, the set of fine-grained alternatives $q_i(t)$ can be coarse-grained in various classes C_α in such a way that the extended probability of each class is given by the sum of the extended probabilities of its members:

$$P(\alpha) = \int w[q(t)]dq \tag{3.23}$$

The coarse-graining into exclusive classes C_α and the classical probability expressed in (3.23) constitute the fourth ingredient.

The reformulation is then three-layered: (1) there are multiple fine-grain histories to which an extended probability is assigned, one of these histories is real although we do not (and cannot) know which one. These histories are expressed in terms of Feynman's sum over histories. (2) The fine-grained histories are coarse-grained into classes C_α in such a way that the probabilities for the latter is strictly positive. (3) The exhaustive set of all the classes C_α for which the probability is equal to: $\int w[q(t)]\delta q = 1$.

The approach suggested by Gell-Mann and Hartle does provide a partitioning of the ensemble, but (unlike Wharton's) it does not limit the reduced ensemble to only possible real trajectories. The probability amplitude is obtained by summing over non-settleable fine-grained histories —to which an extended probability is assigned— embedded in coarse-grained histories. Furthermore, the EPE account does assume the existence of one real fine-grained trajectory and this, one could argue, can undermine a genuinely holistic picture. If the single real trajectory does exist, then the spectre of strong reductionism would pose a threat to the holistic account. However, the measurement of the fine-grained history is deemed impossible, as emphasized in: (Gell-Mann and Hartle 2012, p. 7)

> By measurement and other observation we acquire data D on what the real coarse-grained history is in any realm[11] in which D is valid in some histories but not in others. With these data we also acquire coarse-grained information on what the real fine-grained history is. As we make further observations we learn more and more about the real fine-grained history. However, this process of progressive discovery of reality can never be carried to the completely fine-grained level.

Even though a fine-grained history exists, the inaccessibility to tests and observations is what determines the impossibility of narrowing the ensemble to a single real paths. In support of this argument, one can resort to the mathematical separability of the various trajectories of the ensemble. The single trajectories of the set $T = \left\{q_1(t)e^{iS_1/\hbar}, ..., q_i(t)e^{iS_i/\hbar}, ..., q_n(t)e^{iS_n/\hbar}\right\}$ are mathematically separable in the sense that it is possible to individuate each single path by, for instance, changing the value of the displacement of the action for a given path.

The mathematical separability and the existence (although un-detected) of a fine-grained history in the EPE account suggest a reductionist view that would disprove holism. However, given that the possible fine-grained histories can be bestowed with negative probabilities they have to combine together into a coarse-grained history to yield positive probabilities. It is thus the class C_α that does all the work in determining the total probability amplitude. The point is made clear in the following quote:

[11] A realm is a set of decoherent coarse-grained alternative histories.

> There is a central idea in CSM [classical statistical mechanics] and EPE: they are both concerned with systems with one real fine-grained history about which we have little information from observation either in practice (CSM) or in principle (EPE). The coarse-grained regularities that are accessible to observation and test cannot therefore be predicted from a fine-grained starting point. Rather, both theories use the ensemble method to describe coarse-grained regularities (Gell-Mann and Hartle 2012, p. 10).

That the coarse-grained regularities cannot be predicted from the fine-grained histories means that the probability amplitude cannot be derived starting from the single fine-grained histories —which is precisely the instance of holism advocated in this chapter.

3.3.3 A Reductionist Toy Model

The third reductionist attempt is proposed by (Kent 2013), who starts from the recognition that: "[even] if we had a mathematical rigorous path integral for some preferred choice of variables, we could not use it to explain why macroscopic objects approximately follow classical trajectories" (Kent 2013, p. 1). He then provides a modification of the path integral formulation by means of introducing a new postulate to the purpose of calculating the probability of single paths. His aim is to give a representation that does not count on the (as he defines them) 'pathological' paths —those which are more openly conflicting with the intuitive picture of the physical system. In other words he addresses some paths as 'pathological' given that they deviate from the paths that would resemble a classical trajectory (or those that deviate from the classical least action trajectory). In doing so, he attempts to recover an 'intuitive' idea of a single real trajectory that is probabilistically chosen. Following up from the above sections, we argue that even if one gets rid of the 'pathological paths', the best he obtains is a new parsing of the total ensemble, thus leaving the problem of the individuation of a single actual trajectory still open.

What initially moves Kent is the difficulty of addressing the emergence of the quasiclassical world (to be intended as the classical limit $\hbar \to 0$) starting from the quantum path integral.

To further examine the problem, Kent builds up a toy model (M_1) for a particle moving from A to B and involving a large finite number of possible trajectories. He then imposes that the paths have phases (± 1) and an order such that: "the amplitudes alternate in pairs before and after the quasiclassical paths" (Kent 2013, p. 3), that is:

$$+1, -1, ... + 1, -1, \underbrace{+1, +1, ..., +1}_{quasiclassical\ paths}, -1, +1, ..., -1, +1 \tag{3.24}$$

If we then calculate the total amplitude of the system taking into consideration all the possible paths, or if we calculate it by only summing over the quasiclassical paths, the result does not change.

$$\sum_{i}^{N} A(P_i) = \sum_{i=M}^{M+K} A(P_i) \tag{3.25}$$

where the paths $P_M, ..., P_M + K$ correspond to the quasiclassical trajectories. However, Kent points out that it is not possible to conclude that quasiclassical paths are special —as different parsings of the ensemble are fully legit. He concludes then:

> The standard treatment of the quantum path integral only defines a transition probability from A to B. It does not supply for any rule that tells us that the system actually follows any path. In particular, it gives no rule that ensures the system will follow one path from among a set of adjacent paths with similar phases and amplitudes, or even that we can make some more coarse-grained statement about its behavior characterized by that set. (Kent 2013, p. 4).

To solve the conceptual problem raised by the face-valued interpretation of path integrals, Kent suggests to add a postulate to the purpose of assigning a probability to single paths amidst an ensemble. The new postulate (3.26) assigns a probability to a path P to be actually followed by a given particle (Kent 2013, p. 5):

$$Prob(P) = C \left| \int dQ \exp(-iS(Q)) exp(-d(P,Q)) \right|^2 \left(\int dQ \exp(-d(P,Q)) \right)^{-1} \quad (3.26)$$

where $\hbar = 1$. The integrals in (3.26) are taken to be over all paths Q with the same initial and enpoint of P, that is from A to B. The postulate hinges on a new quantity $d(P,Q)$ which expresses the distance between paths P and Q or, in other terms, it expresses the separation of the various paths. For the postulate to work as an improvement of the original path integral formulation, the distance function needs to be chosen so that it respects the predictions of quantum mechanics. That being said, paths undergoing interference should be microsopically separated $d(P,Q) \approx 0$ and paths that can be distinguished by observation and followed by classical objects, should be macroscopically separated $d(P,Q) \gg 1$.

Such requirement, which is fundamental to making the theory empirically consistent with the predictions of quantum mechanics, has some relevant consequences. When the distance of paths is approximately zero, we obtain phenomena that are proper of quantum mechanics (such as interference), while once we have a distance such that the different paths are observationally distinguishable, classical mechanics emerges. What this seems to suggest is that it is not the probability of a single path that does the work, but rather it is the distance of the various paths that determines the various probabilities. If we 'look' at the trajectory of a quantum particle, the density of paths per unit of space is such that within the range of cancellation of the paths (determined by the ratio $S/\hbar$) there are many possible trajectories. If, on the other hand, we look at the trajectory of a classical particle, we would have a broader picture but with 'less resolution'. We would be looking at the set of paths that survived the cancellation process from a 'bird's eye view' and we would have the impression that only one trajectory exists.

Let's comment on the example of the single particle beam to see how the postulate and toy model work: "We again suppose some set of adjacent paths $P_M, ..., P_{M+K}$ lie in a region in path space where the path space is essentially constant, while for the remaining paths the path space oscillate" (Kent 2013, p. 7). For $1 < M < M+K < N$ where $K \gg N$, Kent provides an ordering of the possible trajectories like in (3.24) and also gives a definition of distance function: (Kent 2013, p. 8)

$$d(P_i, P_j) = \exp(|i - j|/D) \tag{3.27}$$

where

$$\begin{aligned} d(P_i, P_j) =& 0 \; if \; |i - j| < D \\ d(P_i, P_j) =& \infty \; if \; |i - j| > D \\ d(P_i, P_j) =& \log(1/2) \; if \; |i - j| = D \end{aligned} \tag{3.28}$$

Having a distance function, we can apply (3.26) and obtain the formula to calculate the probability for a particle to undergo a given path.

One point is worth mentioning here: the choice of the distance function is somehow underdetermined, i.e., not unique. Different specifications of a distance function will lead to different probability assignments, given that the postulate (3.26) hinges on such a function. As Kent argues: "It seems then, even for thinking about real path quantum theory in the context of single particle interferometry, that we either need some new compelling theoretical reason for picking out some particular distance function, or empirical guidance" (Kent 2013, p. 14). However, we do not have (yet) an empirical guidance that would suggest us a specific distance function to plug into (3.26). Moreover, the theoretical guidance seems to rely on the search for a reductionist picture of the total ensemble to a single real trajectory. However, if that is the case, we would be looking for a function starting from the assumption that a real trajectory exists. Nothing (in quantum mechanics) seems to suggest the existence of such a trajectory. If the real trajectory does not exist because of the way quantum mechanics is, there would be no need for a distance function to plug into (3.26) and the underdetermination would be solved.

Kent also proves that the trajectories with non-zero probability are the ones closer to the area of constant phase.[12] Such area is approximately defined by the term K in the model: "Our parameter K here models [...] the size of the set of paths around the stationary path for which $(S/\hbar)$ is approximately constant in standard path integral quantum theory" (Kent 2013, p. 9). Nevertheless, the conclusion that there is a single real trajectory the particles takes to go from point A to B does not follow. Kent's work is remarkable as it provides a new postulate that allows to reduce the ensemble of possible paths relevant for the total amplitude. However, it still delivers a set — that is an ensemble— of trajectories. In the model, the physically relevant paths are those within the stationary phase region and those d-distant from that region. While the former quantity is defined by the parameter K, the second one (expressed by a parameter D) depends on the chosen definition of the distance function. Therefore, these two quantities are what the theory hinges on. The postulate (3.26) applied to the toy model M1 delivers a parsing of the total ensemble of possible trajectories into a smaller ensemble of physically relevant possible trajectories. We then have a result not too dissimilar from that of either Wharton or EPA: instead of taking into account the total ensemble, we reduce the sum to a subset (or set of subclasses).

What about the case where interference emerges? Consider a new toy model M2 where a second region of constant phase is added. What changes is that instead of looking at the distance between paths within a beam, now we also have to evaluate the distance between different paths in different beams. We thus have a ordered list

[12] I am leaving aside the mathematical details as not relevant for our purposes here.

of amplitudes from A_1 to A_n:

$$1,-1,\ldots,1,-1,\underbrace{\exp\{(-i\theta_0)\},\exp\{(-i\theta_0)\},\ldots\exp\{(-i\theta_0)\}}_{M_0+K_0},1,-1,\ldots \tag{3.29}$$
$$\ldots,1,-1,\underbrace{\exp\{(-i\theta_1)\},\exp\{(-i\theta_1)\},\ldots\exp\{(-i\theta_1)\}}_{M_1+K_1},1,-1,\ldots$$

Although Kent presents all the various possibilities, we are interested in the cases where interference emerges. For instance, when $i+D>M_1+K_1$ and $i-D<M_0$ we have: (Kent 2013, p. 10)

$$Prob(P_i)\approx C\left|((K_0+1)\exp\{(-i\theta_0)\}+(K_1+1)\exp\{(-i\theta_1))\}\right|^2(2D)^{-1} \tag{3.30}$$

where the interference is given by the factor $((K_0{+}1)\exp\{(-i\theta_0)\})((K_1{+}1)\exp\{(-i\theta_1)\})$. On the other hand, if we have d-separated beams such as (for instance): $i-D<M_0$ and $i+D>M_0+K_0$, the interference factor does not appear: (Kent 2013, p. 10)

$$Prob(P_i)\approx C(K_0+1)^2(2D)^{-1} \tag{3.31}$$

Kent concludes that: if the beams are close then the quantum interference emerges. If the beams are widely separated, the paths within K_i+1 collectively represent the beam and do not display interference. In both cases then, the distance function allows one to define a set (or many) of neighboring paths which contribute to the total amplitude of the quantum system.

Models M1 and M2 both provide a way of ordering the total ensemble of possible paths in Feynman's formulation in such a way that some 'pathological' paths get canceled. The philosophical drive for such an attempt seems that of recovering an 'intuitive' idea of a single (or set of) well-defined trajectory(ies). But such an attempt led to a construction based on the choice of the distance function and the combination of other parameters (such as beam separation). How to choose these parameters and functions remains unclear and the adoption of one of them over another might lead to important differences: "A distance function sensitive to spatial separation would mean that a pathological path that travels far from the stationary path makes little contribution to the probability of the latter being realized. Distance functions sensitive to first or higher derivatives can also suppress the contributions of rapidly varying or undifferentiable paths" (Kent 2013, p. 17).

Kent's argument leads us to an ontology of semi-real paths 'living' in the neighborhood of the stationary path. I call the paths 'semi-real', for the model cancels out the pathological paths but ultimately does not individuate a single real trajectory. Again, one at best obtains a reduced set of possible paths, possibly traversed by the particle. The nature of these paths remains unclear insofar as we maintain a 'quasiclassical' ontology, that is, an ontology that takes the paths (one or a defined set) as the real paths traversed by the particle. Furthermore, Kent's model is a simplification and as he claims: "A full path integral description would include infinitely many paths with phases θ_i in the neighborhood of each P_i, and infinitely many more exotic paths that are not piecewise linear and have rapidly fluctuating phases" (Kent 2013, p. 11). It seems, but a deeper mathematical analysis would be required, that by increasing the complexity of the model and hence getting closer to 'real' quantum systems, the hope for a reductionist (real) path ontology fades away.

Even granting that the new ordering based on the distance function allows for the elimination of the pathological paths, the ontological weight still rests upon the shoulders of a (reduced) ensemble. Kent assumes the existence of one real trajectory, and yet he ultimately formulates the theory in terms of ensemble of possible (though non-pathological) trajectories.

Furthermore, but this would require a further mathematical analysis of Kent's model, the pathological paths —those which conflict with classical intuition and in Kent's model are grouped together to give probability zero— might very well be the non-differentiable paths that positively contribute to the total amplitude in Feynman's formulation. The risk is that Kent's account might not consider some of the paths that contribute to the probability amplitude, thus making the model not fully consistent with empirical data.

To sum up: in the previous sections we have looked into three attempts to reduce the ensemble of possible trajectories to either a subset or to a single trajectory in the context of path integrals. As a result, we concluded that strong reductionism is not a viable possibility. However, because some forms of mild-reductionism seemed feasible, we have not proven yet that the total ensemble is holistic with respect to the individual possible trajectories. In the next section, we will draw an analogy between holism and path integrals, and non-supervenience and entangled states. The analogy will serve the purpose of further discussing the ill-behaved trajectories and their contribution in the calculation of the total amplitude and, upon a discussion of what to count as physical possibilities, we will argue for a holistic interpretation of path integrals.

3.4 Non-Physical Trajectories and Holism

In the last section we have argued that the ensemble is not strongly reducible and that the contributing trajectories are not smooth. However, none of these claims proves that the total ensemble is holistic with respect to the individual trajectories.

A tempting argument would be that the origin of the holistic character of the total ensemble comes from the necessary relation among the single paths, mediated by the various phase factors. In other words, because the trajectories get canceled with each other, one could envision this process of cancellation as what makes the probability amplitude of the total ensemble holistic

However, such a tempting argument is problematic. That the interference between the various trajectories is what proves holism would be equivalent (or at least very similar) to saying that the electromagnetic field generated by two charged particles is not reducible to the electromagnetic field at each spacetime point separately. If this were the case, than any linear combination or interference would be a case of holism, which is too strong of a claim. As a matter of fact, if we consider a simple enough system, it is not difficult to reconstruct the destructive and constructive interference and thus reduce the total electromagnetic field to the combination of the two separate fields. We could assign an electromagnetic potential to each spacetime point surrounding the two charges and thus it would be difficult to argue that the total field does not reduce to the values of the potential at each point (even though the determination of these values is a combination of the two fields).

To defend the thesis of holism, we can consider an analogy with the discussion on supervenient and non-supervenient properties and entangled states —see, among others: (Esfeld 2004), (Cleland 1984), (Belousek 2003), (Karakostas 2004), and (French 1989). Given that the concept of supervenience is subject to variation in the literature and since a more accurate discussion of such variations would lead us astray from the our present purposes, I will take the definition offered in (French 1989) at face value. A relation R is said to be strongly non-supervenient: "upon a determinable non relational attribute if the appearance of this relation is neither dependent nor determined by its relata bearing the non-relation concerned" (French 1989, p. 10). Conversely, the relation R is weakly non-supervenient "upon a non-relational attribute if the appearance of this relation is dependent upon the instantiation of the non relation in the sense that the relation could not possibly be exemplified in the absence of each of its relata separately bearing the determinable non-relation in question, *but* there exist no determinable non-relational attributes whose manifestation is sufficient for the appearance of the relation".

To clarify the definitions, we shall briefly address the case of strong non-supervenience as presented in (French 1989) with respect to quantum entanglement. Afterward, we will discuss the analogy with the case of path integrals. Let us consider two systems S_1 and S_2 and the combined system S_{12}, then let's consider two particles and their corresponding properties P_1, P_2 and their combination P_{12}. Each property is independent from the other and it can be exemplified even if in the absence of the other property or system. A given system possesses a certain property, defined by an Hermitian operator $\hat{O}$, if the system is represented by a quantum state, that is, by an eigenvector of the given operator where the property is represented by the corresponding eigenvalue. The operator $\hat{O}$ for our two-particles system has two eigenvalues u and w such that: $\hat{O}\,|u\rangle = u\,|u\rangle$ and $\hat{O}\,|w\rangle = w\,|w\rangle$. French considers three possible states for the composite system $\hat{O}_{12} = \hat{O}_1 \otimes \hat{O}_2$:

i. $|u\rangle_1\,|u\rangle_2$

ii. $|w\rangle_1\,|w\rangle_2$

iii. $1/\sqrt{2}(|u\rangle_1\,|w\rangle_2 \pm |w\rangle_1\,|u\rangle_2)$

where the eigenvalues of the eigenvectors (a), (b) and (c) of $\hat{O}_{12}$ are u^2, w^2 and uw respectively. (French 1989) and (Belousek 2003) argue that if the system is (a) or (b) then each particle has an independent quantum state and a corresponding non-relational property. The sates (a) and (b) —in the sense of the relation between the two particles— supervene upon the non-relational properties of each particle. On the other hand: "if the two-particle system were prepared in quantum state (c) [(iii) in the original], then neither of the particles would have a single-particle quantum state represented by an eigenvector of $\hat{O}$ [O in the original]" (Belousek 2003, p. 796).

In state (a), particle one and particle two separately posses property u and the composite state possesses property u^2. The same goes for state (b) and property w. However, in state (c), the single particles do not posses a non-relational property expressed as an eigenvalue of the observable $\hat{O}$, but the composite system possesses the property $\hat{O}_{12} = uw$. "In other words, since the state function as represented by (c) [*'(1)' in the original*] is not the product of the separate state functions of the particles, one cannot from a knowledge of (c) [*'(1)' in the original*] ascribe to each particle an individual state function" (French 1989, p. 11). Therefore, because state (c) is not determined by the two subsystems having the non-relational properties u or w, state (c) is strongly non-supervenient upon its parts.

That the total probability amplitude in path integrals is dependent on the single complex probabilities of the single trajectories is trivial and emphasized by the formalism. This rules out the possibility of strong non-supervenience and marks a difference with the case of entanglement. However, that the single trajectories are also sufficient to determine the total amplitude is less obvious.

Weak non-supervenience for path integrals would correspond to the statement that: the total amplitude is weakly non-supervenient upon the amplitudes of the individual trajectories because the total amplitude depends upon the individual ones in that it could not be exemplified in the absence of each of the individual paths. But, there exist no determinate non-relational attributes whose manifestation is sufficient to the total amplitude. Therefore, to prove weak non-supervenience we ought to show that the single trajectories (and their probability amplitudes) are necessary and yet not sufficient to explain the total amplitude calculated for the total ensemble.[13]

The core issue hinges on what we take the trajectories to be. Ideally, we want them to be 'classical', that is, smooth and with probability equal to or less than one. This is because, even though the trajectories are not physical objects, it would be odd to consider as possible trajectories some paths that are not 'physically possible'. With respect to negative probabilities, we have seen that we can group the various paths in such a way that the negative ones get canceled out with each other. However, neither Wharton nor EPE, nor Kent were able to provide a parsing that suppressed the non differentiable paths. It follows that the classical notion of trajectory is not

[13]One might promptly note that if a reduction of the ensemble is possible, then the total amplitude does not depend on each single trajectory. One could consider this as a further weakening of the non-supervenience or, alternatively, simply take the ensemble to be the smallest set of trajectories necessary to obtain the appropriate total probability amplitude.

enough to account for the sum-over-paths of Feynman's interpretation. This calls for a fine-grained distinction of the concept of possible trajectories. On the one hand we have physical possibilities in the sense of smooth trajectories that could be actually traversed by a particle in a classical sense. On the other hand, we have possible trajectories whose physical reality is at best blurred: the non differentiable ones. It is in this sense that we ought to distinguish between 'possibilities' and 'physically possible trajectories'. The non-differentiable paths are possible because allowed by the mathematical formalism, but they are physically impossible because of their non-differentiability.

Therefore, there seem to be two alternatives here. The first one is to accept that the computation of the total amplitude comes from the probability amplitude of some trajectories that count as physical possibilities —in the sense of possibly being traversed by a particle— and some non differential paths that count as non-physical. Alternatively, we can expand the concept of trajectories to encompass the contributing ill-behaved paths.[14]

With respect to the first case: if we have a distinction between classical (smooth) and non-differentiable paths, and we deny that the latter are physical, then the physically possible trajectories would be not sufficient to account for the total amplitude. In other words, the total amplitude would be weakly non-supervenient upon the the classical possible trajectories within the boundary. Outside of the analogy with entangled states and non-supervenience: the total amplitude is (weakly) non-reducible to the sum of the individual amplitudes of the physically possible trajectories.[15]

On the other hand, if we maintain that the non-differentiable trajectories are part

[14]A third possibility would be to discuss whether trajectories can be non-local. The argument would require a longer investigation on the non-local nature of quantum mechanics. Nonetheless, it is opinion of the author that this would lead to a case of holism by analogy with entanglement. We leave the issue to further investigations.

[15]We say 'weakly non-reducible' because we have seen how the total ensemble can be reduced to a subset.

of our physical system, then we are moving to the second possibility. This amounts to considering as 'paths' both classical and non-smooth trajectories, thus committing to a much weaker notion of trajectory, something of the form: a trajectory is any set of points connecting the initial and final positions. To each of these sets is assigned a probability amplitude. On this basis, a reduction of the probability amplitude of the whole ensemble to its parts (the various sets of points) seems possible and that is because the new notion of trajectory would encompass the non differentiable ones as well. However, insofar as the latter scenario has changed the notion of trajectory, some problems emerge at the classical limit.

If, by setting $\hbar \to 0$ we can obtain the classical trajectory traversed by a particle as calculated by the principle of least action, it means that the concept of a single trajectory traversed by a classical particle emerges from the ensemble of possible trajectories in quantum mechanics, which is never reducible to a single path. How can we justify that in quantum mechanics we 'accept' the existence of ill-behaved trajectories that do not fit with the notion of classical trajectories in classical mechanics? In other words, to have two types of trajectories in quantum mechanics (classical and quantum) and only one type in classical mechanics begs the question as to how and why we move from one case to the other. A possible answer is that the notion of trajectory is scale-dependent and thus the classical trajectory of the least action principle emerges from the ensemble of trajectories at the level of quantum mechanics. Such a scale-dependency can be seen in the variation of large actions as opposed to actions that have quantum magnitudes. Although the issue would require a longer analysis, it seems to suggest that the notion of single real trajectory is emergent from that of ensemble of possible paths. Whether this counts as holism and why, we leave it to later works.

3.5 Conclusion

What was advanced in this chapter is the idea that the path integral formulation of quantum mechanics suggests a holistic interpretation.

We have argued that the whole is to be taken as the set of possible trajectories while the parts are the single paths, then we have taken into account three attempts to reduce the total ensemble to either a real trajectory or to subsets of possible trajectories. What emerged from the analysis of Wharton, Gell-Mann and Kent is that: (i) it is not possible to calculate the probability amplitude starting from the single independent trajectories, one always ought to sum over an ensemble of paths. (ii) Some of the possible trajectories, under closer scrutiny, are not classical trajectories in the sense of differentiable continuous curves. It thus becomes difficult to provide a physical understanding of these paths although they actively contribute to the total amplitude. This last point was especially relevant, for it forces us to either accept these quantum paths as mathematical objects, or to accept them as part of physical reality.

By considering the quantum paths as 'merely mathematical' we are admitting that the classical paths of the ensemble are not enough to justify the total probability amplitude calculated starting from the whole ensemble. This 'non-sufficiency' is what characterizes the total ensemble as (weakly) non-reducible upon the single (classical) trajectories. We can thus conclude that path integrals suggest a form of holism. It is worth of notice that the top-down holism does not fit because it is not the total ensemble that determines the properties of the single trajectories. As we have seen, we can calculate the probability amplitude of a single path starting from the initial and final position and through the assignment of a specific value of the action.

Although the argument proves the (weak) non-reductionism, a major problem still remains. If the quantum trajectories are not physical, how do they 'form' the total ensemble? The problem is that the ontology connected to the dynamics cannot be derived from a non-real (or semi-real) set of entities. One possibility could be to interpret these entities in terms of mathematical possibilities and thus follow the line of argument presented here, but such view might lean toward some form of mathematical realism. Alternatively, one needs to ensure the existence of the ensemble based on a different ground, that being a different theory (such as: quantum field theory or emergent space-time in quantum gravity) or a different ontology. We leave such discussion to subsequent works.

The other possibility was to consider the non-differentiable paths as physical possibilities, together with the other possible trajectories. If this is the case, then how to recover the notion of classical trajectory at the level of quantum mechanics becomes most relevant. We have not investigated this last concern here, we have only mentioned that a possibility is to consider the notion of a single classical differentiable trajectory (proper of classical mechanics) as emergent from the ensemble of possible paths proper of quantum mechanics.

Chapter 4

The Space-Time View in Feynman's Electrodynamics

4.1 Introduction

Traditionally, the evolution of a quantum system is described by the Schrödinger equation: a differential equation that calculates the evolution of the wave function in time. In his Nobel lecture, Feynman addresses this method as 'the customary view':

> In the customary view, things are discussed as a function of time in very great detail. For example, you have the field at this moment, a differential equation gives you the field at the next moment and so on; a method, which I shall call the Hamiltonian method, the time differential method (Feynman 1966, p. 7).

The present chapter aims at reconstructing how Feynman challenged such customary view, leading to viewing quantum phenomena in their entirety (the overall spacetime view).

What differentiates the current chapter from the detailed and thorough works by, among others, Schweber (1986), Mehra (1994), Kaiser (2009) and Wüthrich (2010), is the focus on the evolution of Feynman's philosophical thinking from his early works with Wheeler to the formulation of his quantum electrodynamics. Kaiser (2009), for example, provides a compelling analysis of how the diagrams spread around the various research groups and how they have been used to solve problems in theoretical physics. Wüthrich (2010), on the other hand, focuses on the evolution of the pictorial aspects of the diagrams, as it becomes evident in his attentive reconstruction of Feynman's theory of the quivering electron. In addition, Wüthrich (2013) emphasizes Feynman's tendency to provide a physical understanding of the mathematical equations as well as the role that modularity has played in combination with the visualization (and thus understanding) of quantum phenomena. Notably, that modularity has played a crucial role in the development of Feynman's electrodynamics is originally advocated in (Galison 1998) where modularity refers to a theoretical culture of favoring the visual and qualitative understanding rather than the mathematical and purely formal niceties.[1] Nonetheless, while a comprehensive and detailed historical study of Feynman's works is provided in (Schweber 1986) and (Mehra 1994), not enough attention has been paid to the evolution of Feynman's overall spacetime view from the absorber theory to quantum electrodynamics.

It is worth noting that the importance of the overall spacetime view to the development of quantum electrodynamics has been emphasized also in (Blum 2017, p. 2):

> What all these formulations had in common was that in some sense they problematized the quantum mechanical notion of an instantaneous state and tended toward replacing it with a focus on overall processes. This stemmed from the relativistic need to treat space and time on the same footing and the consequent tendency of relativity towards a block universe view.

However, while in Blum's work the overall spacetime view is only tangential, the present chapter focuses on how it tweaked and changed along the way to Feynman Diagrams. More specifically, I will point out how, from calculating a transition am-

[1] Such a culture, it is argued in (Galison 1998), especially developed during the years of the war and Los Alamos and became crucial in Feynman's postwar works.

plitude by integrating over all possible trajectories, Feynman expanded his view to all possible interactions in the theory of positron and Feynman diagrams.

More specifically, unlike path integrals —which can be considered as the embodiment of the overall spacetime view for non-relativistic quantum mechanics[2]— quantum electrodynamics requires the use of perturbation theory which seem to constitute a drift from the overall spacetime view. This is because perturbation theory amounts to a finer specification of the possible interactions and events happening within the boundaries of the process under consideration. If path integrals sum over all the possible trajectories that a particle can take from an initial to a final spacetime point, perturbation theory calculates how the amplitude of the system changes if, along one of these trajectories, the particle interacts with either a field or another particle. By means of an analogy: perturbation theory allows us to look into the quantum process through a magnifying glass, but to calculate the final amplitude we need to sum over all the possible events that we can 'see' through the lens (up to a certain cutoff). Feynman's great contribution was to realize that each term in the perturbative expansion could be associated to a diagrammatic representation of a quantum event, i.e., to a Feynman Diagram.

I will divide the present work into two main sections. The first one is meant to provide a general overview of the overall spacetime view in the context of both the absorber theory of radiation ((Wheeler and Feynman 1945) and (Wheeler and Feynman 1949)) and path integrals ((Brown 2005), (Feynman 1948c)). The second part will reconstruct the use of the overall spacetime view applied to positron theory and Feynman diagrams (Feynman 1949b), (Feynman 1949a).

[2] More on this in: (Forgione 2020a)

4.2 Historical Overview: from the Absorber Theory to Path Integrals

In the 1930s, a well known problem in classical electrodynamics (as well as in the first attempts to develop a quantum electrodynamics) was the removal of the divergence terms from the theory. In this context, the young graduate student Richard P. Feynman, together with his supervisor John A. Wheeler, worked on the formulation of a divergence-free classical electrodynamics. The so-called 'absorber theory of radiation', developed in: (Wheeler and Feynman 1945) and (Wheeler and Feynman 1949), is the product of their joint efforts. In general, the theory derives the radiative damping of the accelerated particle not from the self-action of the particle with its own field, but from the response of the absorbers to the original radiation of the source. The relevant aspect of the theory, at least for the present purposes, is that in order to have the radiative damping happening at the same time as the original acceleration, Wheeler and Feynman used both advanced and retarded solutions to Maxwell's equations. The general principles of the absorber theory are presented in (Wheeler and Feynman 1945, p. 160) in the language of the theory of action-at-a-distance:

i. An accelerated point charge in otherwise charge-free space does not radiate electromagnetic energy.

ii. The fields which act on a given particle arise only from other particles

iii. These fields are represented by one-half the retarded plus one-half the advanced Lienard-Wiechert solutions of Maxwell's equations. This law of force is symmetric with respect to past and future. [...]

iv. Sufficiently many particles are present to absorb completely the radiation given off by the source.

The third point is further developed in Wheeler and Feynman's theory in such a way that the radiative damping, constituted by the advance response of the absorber, arrives at the position of the source at a time that is equal to that of the initial acceleration.

In general, the total radiation emitted by the source is thus calculated by summing the proper retarded field of the accelerated source with the response of the absorber, where the latter constitutes the radiative damping. As phrased in: (Wheeler and Feynman 1945, p. 166) and recalled in (Forgione 2020b):

$$\begin{pmatrix} \text{total disturbance} \\ \text{diverging from} \\ \text{source} \end{pmatrix} = \begin{pmatrix} \text{proper retarded} \\ \text{field of source} \\ \text{itself} \end{pmatrix} + \begin{pmatrix} \text{field apparently diverging from source} \\ \text{actually composed of parts converging} \\ \text{on individual absorber particles} \end{pmatrix} \tag{4.1}$$

The second term on the r.h.s of the equality specifies the radiative damping as an apparent diverging (retarded) field from the source, but truly being an individually converging (advanced) radiation coming from the absorbers. The consequence is that in the absorber theory of radiation the future response of the absorbers influences the present total radiation emitted by the source. As reconstructed in (Blum 2017, p. 20): "The radiative reaction emerged as the reaction of the emitting electron to the advanced back-reaction of all the other electrons in the universe, if one assumed there were were sufficiently many of these to ensure that all emitted radiation would eventually be absorbed (hence, Wheeler-Feynman electrodynamics is also known as absorber theory)".

Radiation, in the absorber theory, is not anymore an only forward-in-time process which has a linear causal history from past to present to future. Rather, the future affects the past for it constitutes the radiative damping of the original accelerated source.[3] Thus, as we will see below, the use of both the advanced and retarded

[3]Such a reading of the absorber theory can be contentious as it raises a number of questions

radiation is fundamental to the 'overall space-time view' of the absorber theory.[4] Because of the intertwining of past and future, the time evolution of the theory was not an instantaneous state evolution anymore and thus it was not possible to formulate an appropriate Hamiltonian.

A solution had already been suggested by Fokker (1929) who formulated a non-Hamiltonian theory of electrodynamics based on the combination of retarded and advanced interactions and on a principle of least action (Fokker's action). As (Feynman 1966, p. 7) reconstructs in his Nobel lecture:

> The behavior of nature is determined by saying her whole spacetime path has a certain character. For an action like (1) [Fokker's action] the equations obtained by variation (of $X^i_\mu(\alpha_i)$) [the vector position of the i^{th} particle where α is a parameter] are no longer at all easy to get back into Hamiltonian form. If you wish to use as variables only the coordinates of particles, then you can talk about the property of the paths —but the path of one particle at a given time is affected by the path of another at a different time. If you try to describe, therefore, things differentially, telling what the present conditions of the particles are, and how these present conditions will affect the future you see, it is impossible with particles alone, because something the particle did in the past is going to affect the future.

Thus, the new theory by Wheeler and Feynman which is based on a least action principle keeps track of the positions of the various particles using fields variables

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about, for example, the justification of the asymmetry of time for macroscopic phenomena. As it is not the main point of the current chapter, I simply take at face value the reading of the absorber theory suggested in (Forgione 2020b).

[4] To be more precise, Feynman had originally formulated the theory in terms of retarded radiation only. It was thanks to the contribution of Wheeler that Feynman implemented the use of the advanced radiation as well.

as bookkeepers: "From the overall space-time view of the least action principle, the field disappears as nothing but bookkeeping variables insisted on by the Hamiltonian method" (Feynman 1966, p. 7).

It is thus Feynman that in the previous quote firstly addresses the overall space-time view for the absorber theory: in a theory that makes explicit use of advanced and retarded interactions and of the action principle, the behavior of the system is studied by looking at the entirety of the path of the given particle(s). Feynman emphasizes the point further when he lists what he had obtained during his work on the absorber theory:

> To summarize, when I was done with this [the absorber theory], as a physicist I had gained two things. One, I knew many different ways of formulating classical electrodynamics, whith many different mathematical forms. I got to know how to express the subject every which way. Second, I had a point of view —the overall space-time point of view— and a disrespect for the Hamiltonian method of describing physics (Feynman 1966, p. 8).

However, this is not the end of the story. Since the absorber theory remains a classical theory, it still needs to be quantized. Fokker's main result was to build a theory that was manifestly relativistic invariant and since the problems of the divergences of QED were not fully manifest yet, he did not pursue quantization. But, since at the time of the absorber theory the divergences in QED were much better known, Feynman actively tried to quantize his new classical electrodynamics. However, the task proved to be difficult, as for example emphasized by the anecdote reported in (Schweber 1986, p. 457):[5]

[5]Schweber takes the anecdote from (Feynman, Interview with Weiner, March, Center for history and Philosophy of Physics).

> Wheeler had been scheduled to give a lecture on how to quantize their action-at-a-distance theory at the next meeting of the colloquium. After Feynman's lecture [on the non-quantized absorber theory], while walking back from Fine Hall to Palmer Labs with Feynman, Pauli asked him what Wheeler was going to say. Feynman replied he did not know. "Oh" said Pauli, "the professor doesn't tell his assistant how he has it worked out? Maybe the professor hasn't got it worked out!" As it turned out, Wheeler had in fact overestimated his results, and he canceled the lecture.

One of the main difficulties was that canonical quantization dictates how to quantize a classical theory starting from the similarity in the structure of the Hamiltonian function in classical and quantum mechanics. The main gist is to promote the position and momentum variables (expressed in terms of the Hamiltonian: $p = -\partial H/\partial q$ and $q = \partial H/\partial p$) to Hermitian operators. As recalled in (Wüthrich 2010, p. 52):

> The standard procedure for quantizing a classical theory was to interpret the classical Hamiltonian function as an operator in a Hilbert space of state vectors. This operator would then determine the time evolution of the quantized system described by a certain state vector. The problem with quantizing the Wheeler-Feynman theory of electrodynamics was that it could not be formulated by specifying a Hamiltonian function. Therefore, a method was needed to quantize physical systems, the classical description of which could not be given by a Hamiltonian function.

Because the absorber theory was formulated in terms of an action principle, the problem with its quantization was the absence of an Hamiltonian. Normally, if the action is the classical action $S = \int L dt$, expressed as the integral over the Lagrangian function, one could use Legendre's transformation and obtain a Hamiltonian and once the Hamiltonian is available one can apply the recipe for the canonical quantization.

However, the form of Fokker's action is not that of a Lagrangian and thereby of a classical action. As pointed out in (Blum 2017, p. 21), the two actions differ in two main aspects: "The first is that the integrations are over the proper times of all the individual particles instead of over some universal time coordinate. The second is that the integrations are carried out from $-\infty$ to ∞, instead of from an initial time t_0 to a final time t_1".

The solution to the difficulty of deriving an action for quantum mechanics came from the visiting scholar Herbert Jehle, who advised Feynman to look into a paper from Dirac (1933), where a Lagrangian formulation of quantum mechanics was indeed proposed. But to have a Lagrangian formulation of quantum mechanics was not sufficient, and that is because the absorber theory was not formulated in terms of a classical action. Nonetheless, what interested Feynman the most about Dirac's paper was the relation between the transformation function $(q_t|q_T)$ and the quantity $e^{iS/\hbar}$, where S is the classical action.

Feynman, upon reading the work by Dirac, was able to derive the relation between the wave function $\psi(q_i, t_i)$ and its infinitesimal-time subsequent $\psi(q_{i+1}, t_{i+1})$:

$$\psi(q_{t+\delta t}, t+\delta t) = \int \exp\left\{ \frac{i\delta t}{\hbar} L\left(\frac{q_{t+\delta t} - q_t}{\delta t}, q_{t+\delta t}\right) \psi(q_t, t) \right\} \frac{\sqrt{2} dq_t}{A(\delta t)} \tag{4.2}$$

where $A(\delta t)$ is a normalization constant. The relation instantiated by equation (4.2) is equivalent to how Schrödinger's equation evolves the wave function for an infinitesimal time interval δt. However, Feynman still needs to generalize the result to non infinitesimal time-intervals and to do so, it is enough to inductively reiterate the infinitesimal time evolution along the whole non-infinitesimal time interval. Then, by gluing together the various infinitesimal transition amplitudes, one obtains:

$$\begin{aligned}\psi(q_{m+1}, t_{m+1}) = \int\int\cdots\int \exp\left\{ \frac{i}{\hbar} \sum_{i=-m'}^{m} \left[L\left(\frac{q_{i+1} - q_i}{t_{i+1} - t_i}, q_{i+1}\right)(t_{i+1} - t_i)\right]\right\} \\ \times \psi(q_0, t_0) \frac{\sqrt{g_0} dq_0 \ldots \sqrt{g_n} dq_m}{A(t_1 - t_0) \ldots A(T - t_m)}\end{aligned} \tag{4.3}$$

4.2.1 The Generalization to any Action and the Boundary Conditions

Because Feynman intended to quantize his absorber theory, he discussed, in his dissertation, a system that looked similar to the one he was trying to quantize. As recalled in (Darrigol 2019, p. 356): "he [Feynman] considered the simpler similar theory obtained by eliminating the oscillator's coordinates in a system of two particles coupled through a harmonic oscillator. This system is analogous to two particles interacting through an electromagnetic field because this field, by Fourier analysis, can be regarded as a superposition of harmonic oscillators at various frequencies". Path integrals will provide a way to the quantization of the coupled system and to the subsequent attempt of eliminating fields in electrodynamics. The elimination of the fields should then give way to the theory of direct action at a distance.[6] [7]

By considering a toy model composed of two atoms A and B, both interacting with a harmonic oscillator, the question is whether one can find an action $\mathcal{A}$ such that the system is described by a principle of least action that involves only A and B. Consider an action integral (Brown 2005):

$$\int \left[L_y + L_z + (\frac{m\dot{x}^2}{2} - \frac{m\omega^2 x^2}{2}) + (I_y + I_z)x \right] \tag{4.4}$$

where L_y and L_z are the Lagrangians for A and B, the term $(\frac{m\dot{x}^2 - m\omega^2 x^2}{2})$ is the Lagrangian of the harmonic oscillator and the last term amounts to the interaction between the elements of the system. Feynman argues that the oscillator (in the toy model) has only one degree of freedom and thus velocity and positions are enough to specify its state and to uniquely determine the motion of A and B. To solve the equation of motion of either particle, one needs to solve the equation of the harmonic oscillator.

[6] A reconstruction of the elimination of fields in Feynman's electrodinamics can be found in, for example: (Darrigol 2019).

[7] To be noted that the idea of treating a field as a collection of harmonic oscillators was not new, as it was originally suggested in (Fermi 1932). The use of Fermi's intuition is for example made explicit in: (Feynman 1950).

Not surprisingly, the action function depends on the parameters one chooses for the harmonic oscillator. What is surprising though, argues Feynman, is that the choice of the parameters determines whether the motion of two particles is expressible in terms of a least action principle. To show his point, Feynman lists the possible ways to express the solutions $x(t)$ to the equation of the harmonic oscillator: (Brown 2005, p. 18):

$$x(t) = x(0)\cos\omega t + x(0)\frac{\sin\omega t}{\omega} + \frac{1}{\omega m}\int_0^t \gamma(s)\sin\omega(t-s)ds \tag{4.5}$$

where to solve the equation one needs the additional data $x(0)$ and $\dot{x}(0)$. Alternatively, $x(t)$ can be expressed as:

$$\begin{aligned} x(t) = & \frac{1}{\sin\omega T}\left[R_T \sin\omega t + R_0 \sin\omega(T-t)\right] + \\ & + \frac{1}{2m\omega}\int_0^t \sin\omega(t-s)\gamma(s)ds - \frac{1}{2m\omega}\int_0^T \sin\omega(t-s)\gamma(s)ds \end{aligned} \tag{4.6}$$

where

$$R_0 = 1/2\left[x(0) + x(T)\cos\omega T - \dot{x}(T)\frac{\sin\omega T}{\omega}\right] \tag{4.7}$$

$$R_T = 1/2\left[x(T) + x(0)\cos\omega T - \dot{x}(0)\frac{\sin\omega T}{\omega}\right] \tag{4.8}$$

The additional information are now R_0 and R_T which are combinations of $x(0)$, $\dot{x}(0)$ and $x(T)$ and $\dot{x}(T)$. Their physical meaning is recalled in (Brown 2005, p. 19):

> It is seen that R_T is the mean of the coordinate of the oscillator at time T and what that coordinate would have been at this time if the oscillator had been free and started with its actual initial conditions. Similarly, R_0 is the mean of the initial coordinate and what that coordinate would have had to be, were the oscillator free, to produce the actual final conditions at time T. Outside the time range 0 to T the oscillator is, of course, simply a free oscillator.

The third expression for $x(t)$ is:

$$x(t) = \frac{\sin \omega(T-t)}{\sin \omega T}\left[x(0) - \frac{1}{m\omega}\int_0^t \sin \omega s \gamma(s) ds\right] + \\ + \frac{\sin \omega t}{\sin \omega T}\left[x(T) - \frac{1}{m\omega}\int_t^T \sin \omega (T-s)\gamma(s) ds\right] \quad (4.9)$$

where to solve this last equation, the additional data are $x(0)$ and $x(T)$.

The various expressions for $x(t)$ are thus used to calculate the equation of motion for y and z, but different expressions of the solution to the harmonic oscillator lead to different expressions for the equation of motion of the particles. What differentiates the various expressions for the harmonic oscillator are the conditions needed to compute $x(t)$. Feynman discovered that if one is to impose Cauchy data (i.e., $[x(0)]$ and $[\dot{x}(0)]$) on the equation, it is not possible to obtain an action function that satisfies the action principle in the form: $\frac{\delta \mathcal{A}}{\delta y(t)} = 0$ and $\frac{\delta \mathcal{A}}{\delta z(t)} = 0$. To satisfy the action principle one needs to either specify the initial and final position of the oscillator (i.e., Dirichlet's conditions $x(0)$ and $x(T)$) or to specify Robin's condition (i.e., by fixing the values of R_0 and R_T). For example, the case in which $R_0 = R_T = 0$ is a special one that in electrodynamics "leads to the half advanced plus half retard interaction used in the action at a distance theory" (Brown 2005, p. 22).

The connection between boundary conditions and the absorber theory is also recalled in (Mehra 1994, p. 134):

> Feynman showed that it is possible to find such a new action functional only if one chooses a definitely determined solution of the oscillator equation, a symmetric one which included one-half advanced and one-half retarded interaction between the atoms A and B.

While the action functional mentioned in the quote is the one obtained by Feynman by taking the solution (4.6) to $x(t)$ and the special case $R_0 = R_T = 0$, (Blum 2017, p. 25) notes that Mehra's statement is too strong since Feynman does not claim that the condition $R_0 = R_T = 0$ is the only one that leads to a possible action functional. It is nonetheless true that such special case leads to connection with the action at a distance theory. But, and this seems important with respect to the overall spacetime view, it is the specification of boundary rather than initial conditions that allows the derivation of an action functional that respects the minimum principle.

The fact that an action principle was not obtainable starting from the use of Cauchy data suggests an interesting consideration. The use of boundary conditions restricts the occurrence of the event within an initial and final spacetime points. Since the equations represent the motion of the harmonic oscillator, which is a toy model for the behavior of the electromagnetic field, is there a difference in what boundary conditions we use other than the formal consequence of deriving the action principle? The enclosing of the event within the initial and final space-time points not only is compatible with the description of the event that is extended in time (i.e., not instantaneous), but it is also compatible with the overall spacetime view implied by Feynman in his absorber theory.

However, as well emphasized in (Blum 2017, p. 25): "[...] there are additional boundary terms, which can only be made to vanish by assuming that the interaction is adiabatically turned off in the distant past and future". Nonetheless, the 'turning-off' of a given event (interaction) will promptly come back in the theory of positron and Feynman diagrams as the notion of asymptotic states. This will further emphasize the aspect of enclosing the physical process within a 'space-time boundary' as well as the use of the overall specetime view.

An immediate objection would be that such a physical interpretation is too far fetched. This is because the harmonic oscillator is but a simplified representation of the field: a toy model. However, Feynman seemed to believe in his approach and tried to generalize it to the quantum case:

> Drawing on the classical analogue we shall expect that the system

with the oscillator is not equivalent to the system without the oscillator for all possible motions of the oscillator, but only for those for which some property (e.g., the initial and final position) of the oscillator is fixed. These properties, in the cases discussed, are not properties of the system at just one time, so we will not expect to find the equivalence simply by specifying the state of the oscillator at a certain time, by means of a particular wave function. It is for this reason that the ordinary methods of quantum mechanics do not suffice to solve this problem. (Brown 2005, p. 62)

In the last sections of the dissertation, Feynman generalizes the classical case to the quantum case, thereby showing that the system cannot be expressed by a quantum mechanical principle of least action if one is to hold constant the initial position and velocity of the oscillator.

It is worth mentioning that, before Feynman, Fermi had already developed a form of the equations of classical electrodynamics suitable for quantization:

> We must now write in Hamiltonian form the equations that describe the motion of the particles and the variation of the electromagnetic field. For this we simply write the Hamilton function and then verify that the canonical equations that can be derived by it actually represent the motion of the particles and the Maxwell equations. (Fermi 1932, p. 128)

Fermi's derivation of the probability amplitude and Hamiltonian eliminates both scalar and vector potentials, but the operator that multiplies the differential equation for the probability amplitude shows an extra term $\frac{1}{2}\sum_{ij}\frac{e_i e_j}{r_{ij}}$ which amounts to an instantaneous Coulomb interaction. The latter term is problematic in Feynman's picture because it does not leave room for the delayed interaction. Furthermore, the term is problematic in general for it leads to self actions and divergencies:

> In conclusion we may therefore say that practically all the problems in radiation theory which do not involve the structure of the electron have their satisfactory explanation; while the problems connected with the internal properties of the electron are still very far from their solution. (Fermi 1932, p. 131)

In other words, Fermi's quantization of the electromagnetic field (which was considered the traditional one, see: (Heitler 1984)) had to face the problem of the infamous divergencies (emerging from the internal properties of the electron), which was a main thrust in Feynman's work. Furthermore, Feynman could only partly rely of Fermi's formulation since the latter displayed an instantaneous rather than delayed Coulomb interaction. To be more precise, it is not that Fermi's formulation commits to instantaneous interactions, for that would be a violation of special relativity. The point is made clear in (Heitler 1984, p. 50): "It would seem as if in this gauge [Coulomb gauge] the interaction of two particles were only the instantaneous Coulomb interaction and not the retarded interaction. However, this is not the case. The effect of retardation is contained in the part of the Hamiltonian which depends on the transverse waves". Fermi's delayed interaction involves retarded radiation only, thereby leaving no space to the advanced components of Feynman's formulation.[8]

We can now return to the generalization of the classical case to the quantum case, as advanced in the last sections of Feynman's dissertation. The starting point is to consider the form of a general action of a particle in a potential $V(x)$ interacting with a mirror:

$$\mathcal{A} = \int_{-\infty}^{+\infty} \frac{m\dot{x}(t)^2}{2} - V(x) + k^2 \dot{x}(t)\dot{x}(t+T_0)dt \tag{4.10}$$

The first difficulty is that the integral over a Lagrangian —as calculated starting from equation (4.2) and (4.3)— is taken over two finite times and because the action $\mathcal{A}$

[8] I wish to thank Alexander Blum for the reference to Heitler's book and the clarification on this point.

might depend on values that are external to such time interval, the action integral is meaningless.

> This difficulty may be circumvented by altering our mechanical problem. We may assume that at a certain very large positive time T_2, and at a large negative time T_1, all of the interactions (e.g., the charges) have gone to zero and the particles are just a set of free particles (or at least their motion is describable by a Lagrangian). We may then put wave functions, χ and ψ, for these times, when the particles are free (Brown 2005, p. 41)

In other words: Feynman stipulates a 'space-time boundary' within which the event (e.g., an interaction) takes place and then postulates that outside this 'space-time boundary' the evolution of the system is described by the free Lagrangian. To be more precise, Feynman calculates that for two wave functions χ and ψ —representing a state at time T_2 and T_1 respectively— the matrix elements of a general operator F are:

$$\langle\chi|F|\psi\rangle = \int \chi^*(q_{T_2}) \exp\Big\{\frac{i}{\hbar}\mathcal{A}(q_{T_2}...q_{T_1})\Big\} F(..., q_1, q_0, ...)\psi(q_{T_1})\sqrt{g}dq/A \qquad (4.11)$$

where the action is expressed in equation (4.10).

Feynman then discusses the role played by the wave function in equation (4.11) addressing it as an useful but not strictly necessary tool:

> We can take the viewpoint, then, that the wave function is just a mathematical construction, useful under certain particular conditions to analyze the problem presented by the more general mechanical equations (4.11) [*(68) in the original*] [...], but not generally applicable [...] Quantum mechanics can be worked entirely without a wave function, by speaking of matrices and expectation values only. (Brown 2005, p. 45)

The convenience of the wave function is that one can assume that outside the interval $[T_1, T_2]$ the action has a Lagrangian form and thus, having 'fixed' the initial and final wave functions, Feynman can start looking into the evolution of the system within the time interval.

Starting from the transition amplitude $\langle\chi|1|\psi\rangle$, calculated under the action $\mathcal{A}$, it is possible to alter the action within the time interval such that: $\langle\chi|1|\psi\rangle_{\mathcal{A}+\epsilon\mathcal{F}} = \langle\chi|e^{\frac{i\epsilon}{\iota}\mathcal{F}}|\psi\rangle_{\mathcal{A}}$ where the latter can be expanded in powers of ϵ (perturbation theory). Perturbation theory warrants the possibility of expressing "the average of a function for one action in terms of averages of other functionals for a slightly different action" (Brown 2005, p. 47). Such use of the wave functions foreruns the use of asymptotic states in the positron theory and Feynman diagrams.

However, Feynman is initially dissatisfied with the use of the wave functions to represent the states of the system —as emphasized in the previous quote (and more widely in (Blum 2017)). To overcome the dissatisfaction, Feynman attempts to replace the matrix elements and transition amplitudes with the expectations values of the relevant quantities directly.[9]

In pursuing quantization via expectation values, and in analogy with the classical case, Feynman investigates a system of two atoms A and B interacting through an harmonic oscillator O and asks (Brown 2005, p. 61): "to which extent can the motion of the the oscillator be disregarded and the atoms be considered as interacting directly?" However, the problem is not only to show that one can eliminate the oscillator in favor of direct interaction and expectation values of a given functional: Feynman wants such functional to be an expression of an action principle of the particles alone. In other words: The point is to calculate under what conditions the

[9]Feynman also admits that the project is only tentative and not complete: "An alternative possibility is to avoid the mention of wave functions altogether, and use, as the fundamental physical concept, the expectation value of a quantity, rather than a transition probability. The work done in this connection, which is presented in this section, is admittedly very incomplete and the results tentative" (Brown 2005, p. 50).

trace of the functional $\mathcal{F}$ for atoms and oscillator is equivalent to the trace of the functional $\mathcal{F}$ for an action principle —granted that the initial and final states of the oscillator are fixed: $x(0) = \alpha$ and $x(T) = \beta$. Formally, (Brown 2005, p. 62):

$$\mathrm{Tr}\left\langle \mathcal{F} \cdot \delta\left(\frac{x_T + x'_T}{2} - \beta\right) \cdot \delta\left(\frac{x_0 + x'_0}{2} - \alpha\right)\right\rangle_{A,B,\&O} = const \cdot \mathrm{Tr}\left\langle \mathcal{F} \right\rangle_{A,B} \qquad (4.12)$$

To simplify the calculations, Feynman sets the action of the system without the oscillator to be $\mathcal{A}_0 + \mathcal{I}$ which depends on the coordinates (Q and Q') of the two atoms only and where the first term is the action of the particles and the second term is the action of the interaction. Feynman calculates that the action of interaction $\mathcal{I}$ is analogous to the action calculated in the classical case where the solution of the oscillator is taken to be (4.9). However, this does not prove that the systems with and without oscillator are equivalent, but only that, similarly to the classical case, under some conditions it is possible to replace the intermediate oscillator with an action functional:

> Thus, the particles with the action of interaction $\mathcal{I}$, may be replaced by a system with an intermediate oscillator, provided that, in calculating the expectation of any functional of the particles, it is calculated under the condition that the oscillator's initial position is known to be α and the final position is known to be β. It is to be noted that we have no proved that, in general, the system with the oscillator is equivalent to one without, for that is not true. The equivalence only holds if the oscillator is known to satisfy certain condition. (Brown 2005, p. 65)

In the reminder of the section (the last few pages of the dissertation) Feynman proves that: (1) one can obtain an action functional from Robin conditions and (2) that no action exists if one is to specify the Cauchy data for the functional $\mathcal{F}$, i.e.: "no action exists in case the initial position and velocity are held constant" (Brown 2005, p. 67).

However, Feynman ultimately decided not to publish these results in his condensed version of the dissertation. The use of the trace to calculate the expectation values was tainted by an unsolved difficulty: "The trace of an arbitrary functional is not always a real number!" (Brown 2005, p. 52). The difficulty with the complex valued quantities is also emphasized in (Blum 2017, p. 24): "This was especially problematic concerning the expectation value of the energy, where a complex expectation value implied the loss of unitarity." This makes the last pages of the dissertation —where it is shown how it is possible to obtain a direct action theory if the oscillator has similar boundary and Robin conditions as the classical case— a formal result with no physics interpretation. The conclusions of the dissertation report Feynman's dissatisfaction:

> The interpretation of the formulas from the physical point of view is rather unsatisfactory. The interpretation in terms of the concept of transition probability requires our altering the mechanical system, and our speaking of states of the system at times very far from the present. The interpretation in terms of expectations, which avoids this difficulty, is incomplete, since the criterion that a functional represent a real physical observable is lacking (Brown 2005, p. 68)

The conclusions make clear that the inclusion of the system into a spacetime boundary led Feynman to dive into the use of expectation values. Ultimately, this led to the derivation of an action functional, but only for those cases in which the boundary conditions of the oscillator are known (and are the same as those of the classical case).

Thus far, Feynman's work amounts 'only' to a reformulation of quantum mechanics by means of the Lagrangian function. There is still no application of the new method to quantum electrodynamics, which would have stemmed from the proper quantization of the absorber theory:

> The results of the application of these methods to quantum electrodynamics is not included in this thesis, but will be reserved for a future time when they shall have been more completely worked out. [...] All the analysis will apply to non-relativistic systems. The generalization to the relativistic case is not at present known (Brown 2005, pp. 5–6)

Before moving on, it is worth taking a detour and briefly comment on the condensed and revised version of Feynman's doctoral thesis. The article emphasizes the physical interpretation behind the path integral formulation and at the very end it also provides a first derivation of an action functional for relativistically moving particles.

4.2.2 The Paper from 1948

In his post-war revised and condensed version of the thesis, Feynman (1948c) begins the derivation of his path integral formulation by remarking one of the fundamental differences between classical and quantum mechanics. In classical mechanics three measurements (say) A, B and C can give as a result the values a, b, c respectively. One can thus express the probability of the event P_{abc} as $P_{ab} \times P_{bc}$ granted that the events are independent. If we have a set of mutual exclusive possibilities for the values of B, then we can express the probability P_{ac} as a sum over the alternatives of b, i.e.: $P_{ac} = \sum_b P_{ab} P_{bc}$. The same happens in quantum mechanics with the difference that to obtain a real probability one needs to square the probability amplitude: $P_{ab} = |\varphi_{ab}|^2$ and $P_{bc} = |\varphi_{bc}|^2$. Feynman then considers a particle that can take many values of coordinate x in one dimension and by making many successive measurements of the particle's position at ϵ time intervals, then the function $x(t)$ defines a path for the particle along the x coordinate. The probability that the particle undergoes a path in a region R is defined as:

$$\int_R P(\ldots, x_i, x_{i+1}, \ldots) \ldots dx_i, dx_{i+1} \ldots \tag{4.13}$$

With respect to quantum mechanics, Feynman considers an ideal experiment which does not disturb the system and only determines whether the particle lies somewhere within R. The probability is thus given by $|\varphi(R)|^2$ where:

$$\varphi(R) = \lim_{\epsilon \to 0} \int_R \phi(\ldots, x_i, x_{i+1,\ldots}) \ldots, dx_1 dx_{i+1} \ldots \tag{4.14}$$

The integration over R amounts to integrating over the entire spacetime region within which the particle can wiggle and where the complex function $\phi(\ldots, x_i, x_{i+1,\ldots})$ defines a path. Similarly to having summed over the different values of b to the quantum probability $P_{ac} = \sum_b \varphi_{ab}\varphi_{bc}$, equation (4.14) adds all the intermediate spacetime points as possible values that the particle can take: $P_{an} = |\sum_b \sum_c \ldots \varphi_{ab}\varphi_{bc} \ldots \varphi_{mn}|^2$.

The physical interpretation is then made evident in the two postulates as expressed in (Feynman 1948c, p. 8):

> Postulate I. If an ideal measurement is performed, to determine whether a particle has a path lying in a region of space-time, then the probability that the result will be affirmative is the absolute square value of a sum of complex contributions, one for each path.

The second postulate specifies how to properly calculate the probability amplitude, i.e., it specifies how to determine the contribution to the probability amplitude of each path:

> Postulate II. The paths contribute equally in magnitude. [B]ut the phase of their contribution is the classical action (in units of $\hbar$); i.e., the time integral of the Lagrangian taken along the path.

This means that the contribution of a given path such as $\phi(\ldots, x_i, x_{i+1} \ldots)$ is proportional to $e^{iS/\hbar}$, where the action principle $S(x_{i+1}, x_i) = \min \int_{t_i}^{t_{i+1}} L(\dot{x}, x) dt$ is applied to the action $S = \sum_i S(x_{i+1}, x_i)$. Because the sum in the action is infinite, upon restricting the time interval to an arbitrarily long finite interval, Feynman obtains that (Feynman 1948c, p. 10):

$$\varphi(R) = \lim_{\epsilon \to 0} \int_R \times \ \exp\left[\frac{i}{\hbar}\sum_i S(x_{i+1}, x_i)\right] \ldots \frac{dx_{i+1}}{A}\frac{dx_i}{A} \ldots \tag{4.15}$$

This new formulation of non-relativistic quantum mechanics calculates the transition amplitude (and therefrom the probability amplitude) by taking into account the ensemble of all possible paths where each of these path is 'weighted' by a phase factor $e^{iS/\hbar}$. The ensemble is defined by the space of mathematical possibilities (granted that the paths are continuous) and since there is no such a thing as a 'real trajectory' —in the sense of a path actually being traversed by the quantum particle— we are left with a new and deeply-non classical understanding of quantum phenomena:[10]

> One characteristic of the present formulation is that it can give one a sort of bird's-eye view of the space-time relationships in a given situation. (Feynman 1948c, p. 32)

What has changed with respect to the absorber theory of radiation is the use of the entire space of possibilities to calculate a probability amplitude. On the other hand, what has remained the same is that in both the absorber theory and path integrals the dynamics of the system is calculated by taking into account two different times: the initial and final spacetime positions in path integrals (to be precise: the entirety of a given trajectory) and the advanced and retarded radiation in the absorber theory.

4.3 The Road to Feynman Diagrams and QED

4.3.1 To Understand and To Explain

What path integrals amount to is a reformulation of quantum mechanics, one that does not involve relativistic phenomena. It was thus a yet incomplete work, espe-

[10]More on this in: (Forgione 2020a)

cially in light of Feynman's attempt to quantize the absorber theory of radiation.[11] Nonetheless, the last section of (Feynman 1948c) is a tentative generalization of path integrals to relativistically moving particles and the consequent derivation of an action functional for the relativistic Dirac equation. However, despite obtaining the correct result, Feynman remains ultimately dissatisfied with the too formal nature of the derivation, one which does not provide the 'understanding' of the quantum phenomena he had been seeking: "These results for spin and relativity are purely formal and add nothing to the understanding of these equations. There are other ways of obtaining the Dirac equation which offer some promises of giving a clearer physical interpretation to that important and beautiful equation" (Feynman 1948c, p. 36).

The first attempt to derive the Dirac equation based on a stronger 'visual' component —that is a derivation that had its basis on a clearer physical understanding— resulted in the so-called theory of the quivering electron. The theory is thoroughly reconstructed in (Wüthrich 2010) by means of some of Feynman's original manuscripts. For our purposes, it is enough to say that the theory consists of an initially one-dimensional model of the Dirac equation where the electron is taken to move either to the left or to the right on a lattice and the probability amplitude is thus calculated based on the counting of those turns.

> Through the model of an electron zigzagging through an infinitesimally fine space-time lattice, Feynman can now *explain* the time evolution of a relativistic electron, though only in one dimension. And, unlike in the final section of (Feynman 1948c) [RMP48 in the original], Feynman can now justify the action function, since he has derived it from the a description of the zigzagging electron. (Wüthrich 2010, p. 77)

[11] The most thorough discussion of the quantization of the absorber theory via path integrals is provided in (Feynman 1950). For a general reconstruction of the mathematically dense argument, see: (Darrigol 2019).

The term ‘explain’ in the previous quote has the same significance of the term ‘understanding’ used in Feynman’s quote. They both refer to a ‘visual understanding‘ that was absent in the study of spin and relativity in (Feynman 1948c) and that was ‘recovered’ by the image of an electron zigzagging on a lattice in the theory of the quivering electron.[12]

Perhaps, the scientific attitude represented by the search for an ‘understanding’, as intended by Feynman, is best understood when contrasted with the almost antithetical approach pursued by another peak expert in quantum mechanics during those years: (Dirac 1981, p. vi)

> The classical tradition has been to consider the world to be an association of observable objects (particles, fluids, fields, etc.) moving about according to definite laws of force, so that one could form a mental picture in space and time of the whole scheme. This led to a physics whose aim was to make assumptions about the mechanism and forces connecting these observable objects, to account for their behavior in the simplest possible way. It has become increasingly evident in recent times, however, that nature works on a different plan. Her fundamental laws do not govern the world as it appears in our mental picture in any very direct way, but instead they control a substratum of which we cannot form a mental picture without introducing irrelevancies.

Thus, Dirac’s take on the possibility of understanding quantum mechanics (in general) seems to rest on the contrast between ‘old classical physics’ and the ‘new quantum mechanics’ in terms of the possibility of visualizing the phenomena that the theory describes. Dirac maintains that once the ‘visualizability’ of phenomena is of little

[12]I am here taking the two terms ‘understanding’ and ‘explain’ as both referring to having a world picture, a *scientific Weltanschauung*. On the other hand, for example, (Salmon 2006, p. 182) distinguishes the two terms in that while ‘explain’ refers to systematized knowledge, ‘understanding’ involves having a scientific world picture.

(if any) help, then one ought to resort to 'mathematical ideas'. As, for example, reconstructed in (Crease and Mann 1996, p. 77):

> In one of his last addresses, Dirac explained his credo: "[O]ne should allow oneself to be led in the direction which the mathematics suggest ... [o]ne must follow up [a] mathematical idea and see what its consequences are, even though one gets led to a domain which is completely foreign to what one started with ... Mathematics can lead us in a direction we would not take if we only followed up physical ideas by themselves."

But, contra to a view that is mostly based on 'purely mathematical reasoning', Feynman seems to rely on the value of visualization independently of how unintuitive the result is. Such visualizations, which amount to a general 'understanding' of the physics phenomena, have already played a relevant role in Feynman's early theories: the absorber theory and the path integrals formulation. As we have seen, Feynman imagined the self-action of the electron to be caused by the advance response of the absorbers and that a quantum particle moves following every possible trajectory. It is in this sense that his scientific attitude is almost antithetical to that of Dirac:

> I dislike all this talk of there not being a picture possible but we only need know how to go about calculating any *phenomena*. True we only *need* calculate. But a picture is certainly a *convenience* & one is not doing anything wrong in making one up. It might be completely haywire while the equations are nearly right —yet for a while it helps ... I want to go back & try to understand them [the equations]. What do I mean by understanding? Nothing deep or accurate —just to be able to see some of the qualitative consequences of the equations by some method other than solving them in detail.[13]

[13]Quoted in (Wüthrich 2010, p. 94): *Dirac Equation a, folio 12 (page 11).*

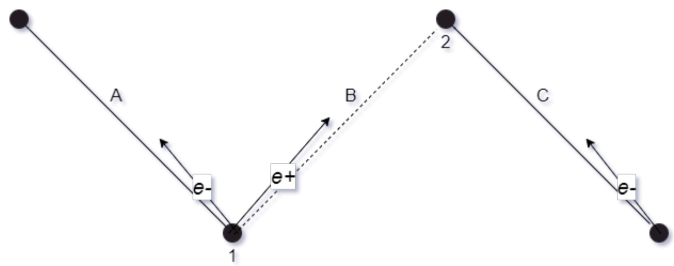


Figure 4.1 Representation of the Dirac electron-positron pair creation.

What happened, though, to the theory of the quivering electron? Did it provide for the visual understanding that Feynman sought? The generalization from the one-dimensional model to two and three dimensions led to an unsatisfactory conclusion, akin to the last section of (Feynman 1948c). Feynman had obtained a generalization to three dimensions, but it was again too formal and devoid of the 'visual understanding' he required.[14]

4.3.2 The theory of Positrons

The seminal article (Feynman 1949b) provides the best presentation of the overall spacetime view and, together with (Feynman 1949a), it formulates a theory of electron and positron interaction with both an external potential and among each other. More specifically, the article on the theory of positrons presents a method for calculating (and understanding) the motion of electrons and positrons in an external potential.

The initial intuition is not to focus on the creation and annihilation operators —which are needed in the traditional (Dirac) view where pairs of particles and anti-particles are created and destroyed— but rather to look at the total charge that remains conserved throughout the electrodynamic event.

Let's first consider the conventional (Dirac) way of representing an electron positron pair creation as represented in Figure 4.1. In terms of hole theory, the figure represents

[14] I will not report the details of the argument, as they are already extensively presented in (Wüthrich 2010, Chapter 4).

an electron traveling along world-line C while, at the same time, an electron-positron pair is created (world-lines A and B) at spacetime point 1. Following the time evolution of the system, at spacetime point 2 the electron from world-line C and the positron from world-line B get annihilated, thereby leaving on electron on world-line A traveling forward in time.[15] This picture follows the evolution of the system as time unfolds unidirectionally from past to future and it is thus comparable to the 'customary view' that Feynman had rejected in his Nobel lecture.

What happens if, as suggested by Feynman, we look at the conservation of the charge rather than at the number of particles?

> Following the charge rather than the particles corresponds to considering the continuous world line as a whole rather than breaking it up into its pieces. It is as though a bombardier flying low over a road suddenly sees three roads and it is only when the two of them come together and disappear again that he realizes that he has simply passed over a long switchback in a single road. This over-all space-time point of view leads to considerable simplification in many problems. One can take into account at the same time processes which ordinarily would have to be considered separately. (Feynman 1949b, p. 749)

Instead of looking at the time evolution of the process described in Fig.1, Feynman suggests to consider the entirety of the time interval within which the event happens, and to look at it from the bird's eye point of view. This leads to the realization that what is represented in the figure does not amount to a time-ordered series of distinct processes (electron C coming-in while positron-electron pair is created, then positron electron get annihilated while electron A propagates outwards), but rather, it amounts to a single continuous world-line in which the electron moves forward-in-

[15] The representation in the picture of subatomic particles traveling along a fixed trajectory is only visual, standing for a representation of the direction of propagation.

time and the positron moves backward-in-time. The manuscript from 1947 provides a straightforward description of Feynman's view, (which was shortened in the published article from 1949).

> In common experience the future appears to us to develop out of conditions of the present (and past). The laws of physics have usually been expressed in this form. (Technically, in the form of differential equations, or 'Hamiltonian Form'.) The formulae tell what is to be expected to happen if given conditions prevail at a certain time. The author has found that the relations are often very much more simply analyzed if the entire time history be considered as one pattern: The entire phenomena is considered as all laid out in the four dimensions of time and space, and that we come upon the successive events. This is applied to simplify the description of the phenomena of pair production in the present paper. A bombardier watching a single road through the bombsight of a low flying plane suddenly sees three roads, the confusion only resolving itself when two of them move together and disappear and he realizes he has only passed over a long reverse switchback of a single road. The reversed section represents the positron in analogy, which is first created along with an electron and then moves about and annihilates another electron. The relation of time in physics to that of gross experience has suffered many changes in the history of physics. The obvious difference of past and future does not appear in physical time for microscopic events (the connection of the laws of Newton and of statistical mechanics). (Schweber 1986, p. 488)[16]

A further example (also provided in the manuscript) considers a rope being im-

[16]From: Feynman, R. P., handwritten notes entitled 'Theory of Positrons', fall, 1947. RPF, CIT 13.1.

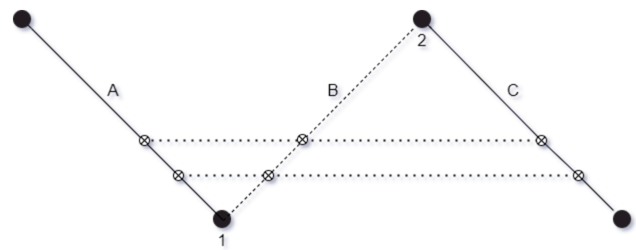


Figure 4.2 Doubling-back of a rope immersed in collodion.

mersed in a hardened cube of collodion (see: Figure 4.2). The rope is not completely stretched from top to bottom, but rather, it doubles back at the points 2 and 1. Now, suppose we slice the cube into thin layers and each of them will display a black dot which originally belonged to the rope.

If we look at the various layers from top to bottom (see for example the horizontal dotted lines in Figure 4.2), we notice that at the points where the rope doubles back each layer will have 3 black dots. From a 'layer-by-layer' perspective, an immediate explanation would be that at point 2 a dot-pair was created and then the dot from segment A will be annihilated by the dot from segment B, while the dot on segment C will continue to the bottom of the cube of collodion, analogously to the case of positron-electron pair creation. Feynman's view, on the other hand, is to look at the entirety of the cube and to interpret the various dots as a doubling back rope where, outside of the analogy, the segment B amounts to the electron traveling backward in time, i.e., to a positron.

How does the analogy of the rope relate to the overall spacetime view with respect to quantum electrodynamics? To see this we need to look more closely into the details of how such change of perspective leads to the calculation of the probability amplitude in the context of quantum electrodynamics.

The starting point is the use of Green functions to express probability amplitudes of the system under consideration.[17] Feynman expresses the solution to Schrodinger's

[17]Informally, a Green function turns the operator into a delta function, thereby making possible

equation $\psi(x_2, t_2)$ in terms of an initial state times the Green function (which will act as a propagator for the initial wave function):[18]

$$\psi(x_2, t_2) = \int K(2,1)\psi(x_1, t_1)d^3x_1$$

Because $\psi(x_2, t_2)$ can also be expressed as: $\psi(x_2, t_2) = \exp(-iH(t_2 - t_1))\psi(x_2, t_1)$ and $\psi(x_2, t_1)$ can be expressed as a superposition of eigenfunctions ϕ_n of the Hamiltonian, Feynman obtains that for $t_2 > t_1$:

$$K(2,1) = \sum_n \phi_n^*(x_1)\phi_n(x_2)\exp(-iE_n(t_2 - t_1)) \tag{4.16}$$

As emphasized in (Feynman 1949b, p. 750), $K(2,1)$ is "the total amplitude for arrival at x_2, t_2 starting from x_1, t_1" and it "results from adding an amplitude, $\exp\{iS\}$, for each space time path between these [x_1, t_1 and x_2, t_2] points, where S is the action along the path". Furthermore, Feynman sets that for $t_2 < t_1$ the propagator $K(2,1) = 0$ in such a way that the expression for the wave function $\psi(2) = \int K(2,1)\psi(1)d^3x$ is not valid.

To familiarize with the Green function and the method thus far suggested, Feynman discusses the case of a particle in a potential $U(x,t)$ and assumes the potential to be different than zero only in the interval Δt_3 such that: $(t_1 < t_3 + \Delta t_3 < t_2)$. Because of perturbation theory, it is possible to expand the Green function K as: $K(2,1) = K_0(2,1) + K^{(1)}(2,1) + K^{(2)}(2,1) + ...$, where $K_0(2,1)$ is the free propagator from the initial to final spacetime point expressed as a path integral:

$$K_0(2,1) = \int \mathcal{D}[x(t)]e^{\frac{i}{\hbar}S_0} \tag{4.17}$$

The subsequent terms in the expansion correspond to the multiple interactions that the particle has with the the potential when the potential is different than zero.

to find the solution to the associated differential equation. Feynman became familiar with the use of Green functions during his time at Los Alamos.

[18] Equation (2) in (Feynman 1949b, p. 750).

Feynman derives that for the extended time Δt_3, if the particle interacts once with the potential U, then the amplitude of the particle to undergo that single interaction is (Feynman 1949b, p. 750):

$$K^{(1)}(2,1) = -i \int K_0(2,3)U(3)K_0(3,1)d\tau_3 \tag{4.18}$$

where $d\tau_3 = d^3x_3dt_3$. The 'physical meaning' of the equation is that a particle freely propagates from (x_1, t_1), then it undergoes a scattering (that is, it interacts with the potential) which is represented by the term $U(3)$ and it ultimately freely propagates to the final spacetime point (x_2, t_1). Furthermore, the initial and final states define a 'spacetime boundary' within which we need perturbation theory to properly calculate the interaction of the particle with the potential.

What does 'properly calculate' mean though? The probability amplitude calculated by means of free-propagator only is not 'precise' enough. One needs to calculate the first order expansion of K and, to gain even further precision, one resorts to higher and higher order expansions of the propagator, where each new order corresponds to a new scattering of the particle.[19] For instance, if the particle is scattered a second time, then one needs to expand the propagator to its second order (Feynman 1949b, p. 750):

$$K^{(2)}(2,1) = (-i)^2 \int\int K_0(2,4)U(4)K_0(4,3)U(3)K_0(3,1)d\tau_3 d\tau_4 \tag{4.19}$$

thereby obtaining that the particle interacts with the potential at two distinct points and times —in this case: $U(4)$ and $U(3)$ as represented in Figure 3(b).

To clarify the matter further, we imagine the spacetime boundary defined by the initial and final states as the walls of a black box inside of which there is a potential. We only know that when the particle passes through the first wall it gets out from the

[19]It is in this sense that perturbation theory looks like a fine-grained analysis of the possible interactions that might occur between the particle and the potential when the potential is different than zero.

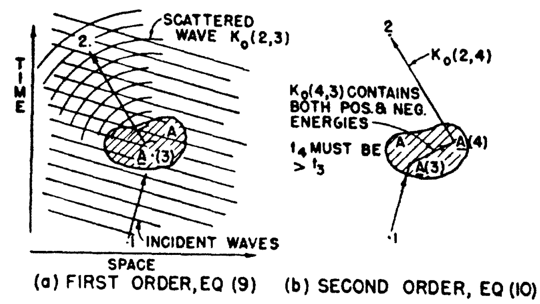


Figure 4.3 First (a) and second (b) order solutions to Schrodinger (and Dirac) equation can be visualized as single (a) and double (b) scattering of a particle in a potential. The figure is taken from: (Feynman 1949b, p. 751).

second wall, but whatever happens inside the box is unbeknown to us. Perturbation theory allows us to switch on a torchlight on a precise point inside the box, so that we can see the particle interacting with the potential. If we then turn on the light a second time and then a third a fourth etc., we will also be able to see the particle a second, a third and a fourth time. Whenever we switch the light on, that corresponds to a further expansion of $K(2,1)$. It is in this sense that further expansions of $K(2,1)$ amount to giving a 'close look' at what happens to the particle interacting with the potential. However, each expansion provides a probability amplitude that adds up to the final amplitude, which is then calculated by gluing together all the times we have switched on our torchlight (i.e., by gluing together all the probability amplitudes of further interactions with the potential).

The use of perturbation theory to derive the complete transition amplitude in the case of interacting systems is similar, at least conceptually-wise, to the inductive way of constructing the transition amplitude in the case of non-relativistic quantum mechanics as presented in Feynman's thesis. There, after deriving the transition amplitude for infinitesimal time intervals, the total transition amplitude is calculated by dividing the time interval into many infinitesimal time intervals and by integrating over all of their transition amplitudes. Similarly, perturbation theory fine-grains the possible interactions that either an electron or a photon undergo within the spacetime boundary defined by the initial and final states. The difference is that, instead of having now a particle freely propagating in spacetime, one needs to account for the interactions that this particle undergoes. Each order in the expansion, as understood in the theory of positron, corresponds to a different interaction between (say) the electron and the field, thus leading to the gluing of all the various terms in the expansion (the various possible interactions) to obtain the total transition amplitude for the interacting system. This same strategy is then further generalized to the case of Feynman diagrams where electrons and photons not only interact with a potential, but they interact with each other.

4.3.3 Relativity and Dirac

Before moving on, let's take stock of this initial treatment of perturbation theory and the use of Green function. To calculate the amplitude of a free-moving particle one needs to take into account all the possible trajectories the particle might undertake. Then, once we add an interaction with a potential, to calculate the the total amplitude we divide the problem in three different steps. The first one is the free-propagation of the particle approaching the interval, the second one is the particle interacting with the potential for a given time interval and, ultimately, the particle freely propagates away from the potential. To calculate the amplitude of the interaction phenomena, Feynman uses perturbation theory to expand the propagator (Green function) of the particle, thereby obtaining a series of possible events —each with an assigned probability amplitude— corresponding to the different terms of the expansion. It is to obtain a more precise result for the amplitude that one calculates higher order of the expansion, but the final amplitude is ultimately calculated by summing over all these possible events. In this sense, while perturbation theory alone amounts to a further specification of the possible events within the time interval of the interaction, the total amplitude is obtained by summing all these possible events.

We have then an evolution of the overall spacetime view: from ‘all the possible trajectories’ to ‘all the possible interactions of a particle with an external potential’. However, thus far all these possible events proceed forward in time. This changes with the treatment of the Dirac equation.

The treatment of the Dirac equation is conceptually analogous to that of the Schrodinger equation, i.e., it can also be visualized as describing the scattering of waves by a potential with the potential with U now being replaced by the scalar and vector potential times the electric charge (the potential is now denoted with $\mathbf{A}$).

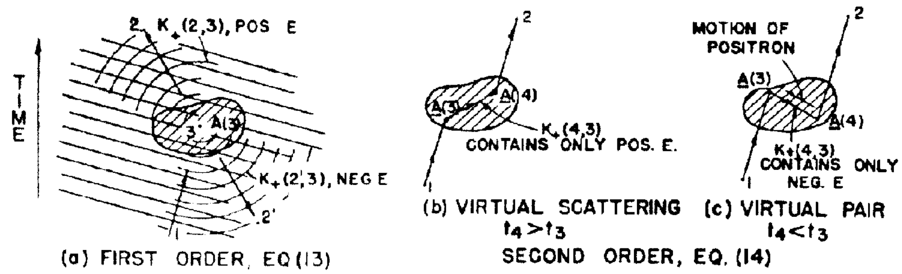


Figure 4.4 The Dirac equation admits negative energies solutions of a scattered wave (a). The second order processes are represented in (b) and (c) where the former represents a double scattering and the latter an electron-positron pair creation. Feynman interprets (c) as double scattering similar to (b) but with the electron traveling backward in time. The figure is taken from: (Feynman 1949b, p. 752).

However, the main difference is that the Dirac equation admits negative energy solutions (as also visualized in Figure 4(a)) coming from the fact that a free particle has energy $E = \pm\sqrt{(p^2 - \mu^2)}$. The negative solutions are not problematic in the classical case, as pointed out in (Heitler 1984, p. 110): “because one can define energy to be the positive square root, and then it does not change in time”. On the other hand, the quantum case is trickier and that is because of the possible transitions of the particle from a positive to a negative energy state.

> The wave equation [...] refers equally well to an electron with charge e as to one with charge $-e$. [...] One gets over the difficulty on the classical

> theory by arbitrarily excluding those solutions that have a negative W [term of energy which is replaced by $i\hbar\partial/\partial t$ in Klein Gordon equation]. One cannot do this on the quantum theory, since in general a perturbation will cause transitions from states with W positive to states with W negative. Such a transition would appear experimentally as the electron suddenly changing its charge from $-e$ to $+e$, a phenomenon which has not been observed. The true relativity wave equation should thus be such that its solutions split up into two non-combining sets, referring respectively to the charge $-e$ and the charge $+e$ (Dirac 1928, p. 612).[20]

To account for the extra solution of the equation, (Dirac 1930) suggests that the positive energy solutions of the wave equation amount to particle-behaving holes in a sea of negative energies:

> Let us assume there are so many electrons in the world that all the most stable states are occupied, or, more accurately, that all the states of negative energy are occupied except perhaps a few of small velocity. Any electrons with positive energy will now have very little chance of jumping into negative-energy states and will therefore behave like electrons are observed to behave in the laboratory. We shall have an infinite number of electrons in negative-energy states, and indeed an infinite number per unit volume all over the world, but if their distribution is exactly uniform we should expect them to be completely unobservable. Only the small departures from exact uniformity, brought about by some of the negative-energy states being unoccupied, can we hope to observe. (Dirac 1930, p. 362)

[20]The double solution fits well with the description of the spin components by means of $4x4$ matrices. An accurate historical overview of the development of the Dirac equation can be found in: (Valente 2020), (Pais 1986) and (Kragh 1990).

In the same work Dirac also suggests —it was actually an idea by Weyl— that the positive energy particle ought to be a proton and thus the wave equation describes the dynamics of both protons and electrons. However. as reconstructed in (Pais 1986), Dirac was never fully convinced of interpreting the positive energy solutions as representing a proton, especially because of the problem of the charge and mass conservation. In 1928, though, that seemed to be the only answer because protons and electron were the only known subatomic particles. Finally, it is in a letter to Bohr that Dirac introduces the double transition interpretation which is now (after the discovery of the positron) known as the electron-positron creation which is represented in figure 4(c).[21]

Thus, the second order processes, as represented in Figure 4(b, c) can be interpreted in Dirac's theory as a double scattering of an electron wave traveling forward in time and as an electron-positron pair creation at $A(4)$:

> A pair could be created by the potential $A(4)$ at 4, the electron of which is that found later at 2. The positron (or rather, the hole) proceeds to 3 where it annihilates the electron which has arrived there from 1 (Feynman 1949b, p. 752).

The novelty of Feynman's approach was to reject the pair-creation mechanism in favor of taking the positron as an electron moving backward in time. He first determined the propagation of a free particle from the Dirac equation as: $(i\nabla_{21} - m)K_{+}(2,1) = i\delta(2,1)$ where $K_{+}(2,1)$ plays the role of the free propagator and its first and second order corrections are, by analogy with (4.18) and (4.19) (Feynman 1949b, p. 752):

$$K_{+}^{(1)}(2,1) = -i \int K_{+}(2,3)\mathbf{A}(3)K_{+}(3,1)d\tau_3 \qquad (4.20)$$

[21]For more on this, see: (Pais 1986) and (Heitler 1984)

and

$$K_+^{(2)}(2,1) = -\int\int K_+(2,4)\mathbf{A}(4)K_+(4,3)\mathbf{A}(3)K_+(3,1)d\tau_3 d\tau_4 \tag{4.21}$$

Then, while it would be easy to take $K_0(2,1)$ as equal to $K_+(2,1)$, Feynman recognized that it was not a viable possibility anymore, for the negative energy solutions need now to be accounted for in the theory. He thus re-interpreted the Green function from (4.16) into:

$$K_+(2,1) = \sum_{n^+} \phi_{n^+}(2)\phi^*_{n^+}(1) \times \exp(-iE_n(t_2 - t_1)) \tag{4.22}$$

for $t_2 > t_1$, and

$$K_+(2,1) = -\sum_{n^-} \phi_{n^-}(2)\phi^*_{n^-}(1) \times \exp(-iE_n(t_2 - t_1)) \tag{4.23}$$

for $t_2 < t_1$. While the negative energy solution —which amounts to the negative sum in equation (4.23)— was interpreted in the Dirac hole theory as the negative energy particle (a hole) moving along with the electron, Feynman reinterpreted it as the same electron represented by equation (4.22) but moving backward in time. Both negative and positive solutions are thus accounted for by the integral in equation (4.21):

> The expressions such as (4.21) [(14) in the original] can still be described as a passage of the electron from 1 to 3 ($K_+(3,1)$), scattering at 3 by $\mathbf{A}(4)$, arriving finally at 2. The scattering may, however, be toward both future and past times, an electron propagating backwards in time being recognized as positron (Feynman 1949b, p. 753)

The original manuscript provides, as emphasized by the metaphor of the collodion cube, a more philosophically oriented explanation which thereby emphasizes Feynman's physical understanding of quantum electrodynamics phenomena:

> The author [Feynman] has found that the relations are often very much more simply analyzed if the entire time history be considered as

> one pattern: The entire phenomena is considered as all laid out in the four dimensions of time and space, and that we come upon the successive events.[22]

One could question whether Feynman first wished for a theory with electrons traveling backward in time and then derived the new propagator, or rather, whether he first derived the new propagator from the comparison with the Dirac hole theory and then interpreted the negative energies as backward electrons.[23] For example, (Mehra 1994), maintains that Feynman was tweaking with the propagator and found out that the use of the minus sign gave the correct result. But then, because of the dissatisfaction with the negative energy term, Feynman reinterpreted it as an electron moving backward in time. This is similar to the reconstruction by (Schweber 1986) where it is emphasized (similarly to Mehra) that the comparison with the hole theory by Dirac is what led Feynman to the new propagator and therefrom to the new interpretation. However, Schweber also points out that in the manuscript of the 1949 paper: 'The Theory of Positron', Feynman already presents his doubts about the complexity of hole theory:

> One of the disadvantages of this [hole] theory is that even the simplest processes become quite complicated in its analysis. One must take into account besides the limited number of real particles, the infinite number of electrons in the sea. The present work results from a reinterpretation of the Dirac equation so that this complexity is not required.[24]

Wuthrich, on his part, focuses on equation (4.21) and emphasizes that in hole

[22] Quoted in: (Schweber 1986, p. 489) from: Feynman, 1947, unpublished manuscript.

[23] Feynman was already aware of the possibility of the electron traveling backward in time because Wheeler had pitched him the idea that there could be only one electron in the universe moving back and forth in time, see: (Schweber 1986, p. 460).

[24] Cited in: (Schweber 1986, p. 488).

theory: "[...] there are two physical processes that contribute quantitatively to such a second-order correction: the electron may be scattered twice by the potential, or an electron-positron pair may be created in an intermediate state of the process" (Wüthrich 2010, p. 121). On the other hand, equation (4.21) includes both processes in one single second-order correction term, and thus: "Feynman proposes an alternative interpretation. An interpretation, that is, in which only one, not two, physical processes corresponds to the one quantitative expression. If a positron is viewed not as a "hole" but as an electron moving backwards in time, the creation of an electron-positron pair and the subsequent annihilation of the positron can be described as the sequence of propagations of a single electron [both backward and forward in time] (Wüthrich 2010, p. 122)".

Alternatively, (Blum 2017) seems to suggest that Feynman tweaked the form of the propagator to obtain a term for waves propagating backward in time: "His new method was to modify the Green Function obtained from the Dirac equation taken as a one particle equation (4.16) [Eq.(38) in the original], so that the intermediate negative energy states would propagate backwards in time".

Notably, assessing the chronological order of ideas is somewhat problematic because the order of publications of the various papers does not fully represent the order of Feynman's thinking process. This is evident if we consider that, for example, Feynman had already presented his theory of electrodynamics at the Pocono conference in April 1948. But, due to the lackluster reception of his talk, Feynman decided to publish his work on papers.[25] As a matter of fact, shortly after the conference, Feynman publishes in order: (Feynman 1948a), (Feynman 1948b), (Feynman 1949b) and (Feynman 1949a).

[25]Feynman recalls: "It's very simple, I'll have to publish this and so on, let them read it and study it, because it's right." As reported in: Interview of Richard Feynman by Charles Weiner on 1966 June 27, Niels Bohr Library & Archives, American Institute of Physics, College Park, MD USA, www.aip.org/history-programs/niels-bohr-library/oral-histories/5020-3. A detailed commentary of the Pocono conference can be found also in: (Schweber 1986).

Nonetheless, what remains is that the new physical interpretation, as opposed to the Dirac pair-creation, brings back the original idea of the absorber theory for which radiation can travel backward in time and thereby shows a consistency with the overall spacetime view. While the absorber theory of radiation was characterized by advanced radiation influencing the strength of the field of the accelerated particle in the present, path integrals, on the other hand, were representative of a view for which a quantum particle seems to undergo all the possible trajectories. Finally, the theory of positron expands the idea of 'all the possible paths' to 'all the possible interactions' that could occur under the effect of a potential where such interactions evolve both forward and backward in time.

4.3.4 Feynman Diagrams

The article (Feynman 1949a) can be taken as an expansion of the arguments in the theory of positron to interacting electrons, positrons and photons.[26] The basic idea is still to use Green functions and perturbation expansions to represent physical processes corresponding to higher order terms:

> Furthermore, each term in the expansion can be written down and understood directly from a physical point of view, similar to the space-time view (Feynman 1949b) [I in the original] (Feynman 1949a, p. 769)

Similarly to his work on the theory of positron, Feynman begins discussing the interaction picture by considering the solutions of Schódinger equation for particles

[26]Mehra (1994, p. 284) seems to remark a philosophical shift in Feynman's paper on quantum electrodynamics: "Here one can see an important change in the evolution of Feynman's belief about the two forms of electrodynamics. He did not insist anymore on the action-at-a-distance theory as the only right form. Instead of this he tried to use both the forms in a most practical way." But, as I pointed out earlier, such a change is already made explicit by Feynman in his work on the theory of positron.

interacting instantaneously. He will then proceed to generalize his account to delayed interactions and relativistic particles.[27]

The first step is to define the free propagator for two particles a and b to move from (x_a, x_b) to (x'_a, x'_b) as:

$$\begin{aligned} K(x_a, x_b, t; x'_a, x'_b, t') &= K_{0a}(x_a, t; x'_a, t')K_{0b}(x_b, t; x'_b, t') \\ &\equiv K_0(3, 4; 1, 2) = K_{0a}(3, 1)K_{0b}(4, 2) \end{aligned} \tag{4.24}$$

Feynman stipulates that at $t_1 = t_2$ and $t_3 = t_4$ the two particles are well separated, i.e., at the initial and final spacetime positions the interaction is 'turned off' similarly to the case of the potential in the theory of positron. Feynman, in analogy with equation (4.18), calculates that if the interaction between two particles is mediated by a Coulomb potential during an infinitesimal time, the first order correction to (4.24) is (Feynman 1949a, p. 772):

$$K^{(1)}(3, 4; 1, 2) = -i\epsilon^2 \int\int K_0(3, 5)K_0(4, 6)r^{-1}_{5,6}\delta(t_{56})K_0(5, 1)K_0(6, 2)d\tau_5 d\tau_6 \tag{4.25}$$

where the delta function makes the integrand non-zero if the interaction is instantaneous ($t_5 = t_6$). But, since the potential which is acting between the two separate particles cannot be instantaneous (because of relativity), Feynman's initial solution is to replace $r^{-1}\delta(t_{56})$ in equation (4.25) with $r^{-1}\delta(t_{56} - r_{56})$, where: $t_{56} \equiv (t_5 - t_6)$ and r_{56} is the time the interaction needs to travel from one particle to the other.[28]

But, fields are represented as quantum harmonic oscillators, as recalled in (Wüthrich 2010, p. 134):

> While working on the cut-off papers [...], Feynman becomes used to representing a classical electromagnetic potential as an assembly of harmonic oscillators and, in the quantum case, to conceiving of the interaction

[27] Feynman explains the difference between the overall spacetime view and the Hamiltonian in view in that the former, which employs delayed interactions, is bets suited to account for virtual phenomena, i.e., close interactions.

[28] The actual term should be r_{56}/c, but Feynman had set $c = 1$ for simplicity.

as being brought about by emissions and absorptions of of the quanta of these oscillators —the photons.

Therefore, since the Fourier transform of the delta function $r^{-1}\delta(t_{56}-r_{56})$ had positive and negative frequencies, Feynman had to restrain the result to positive frequencies only: $r_{56}^{-1}\delta_+(s_{56}^2)$.[29] The new function results from the combination of $r_{56}^{-1}\delta_+(t_{56}-r_{56})$ for $t_5 > t_6$ and $r_{56}^{-1}\delta_+(-t_{56}-r_{56})$ for $t_5 < t_6$ which amounts to particle a receiving a photon and particle b emitting it, and *vice versa*. The final result is that $r^{-1}\delta(s_{56})$ is "replaced by $\delta_+(s_{56}^2)$ where $s_{56}^2 = t_{56}^2 - r_{56}^2$ is the square of the relativistically invariant interval between points 5 and 6" (Feynman 1949a, p. 772).[30]

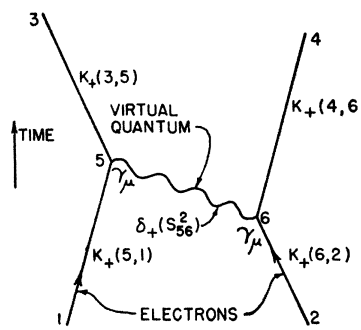


Figure 4.5 (Feynman 1949a, p. 772)

Having defined the appropriate delta function for the delayed interaction, the first-order correction to the interaction represented in figure 4.5 is thus:

$$K^{(1)}(3,4;1,2) = -ie^2 \int\int K_{+a}(3,5)K_{+b}\gamma_{a\mu}\gamma_{b\mu}\delta_+(s_{56}^2)K_{+a}(5,1)K_{+b}(6,2)d\tau_5 d\tau_6 \tag{4.26}$$

Equation (4.26) describes the fundamental interaction in Feynman's quantum electrodynamics: the exchange of one photon (quantum) between two electrons. Such basic interaction marks a departure from one of the premises of Feynman and Wheeler's

[29]The restriction to positive frequencies only amounts to having an electrodynamics of retarded interaction only. The point is also emphasized in (Mehra 1994, p. 231).

[30]Cf. figure 4.5.

absorber theory: the future response of the absorber as part of the mechanism of radiation. Furthermore, Feynman also notices that the premise of the direct action theory is incompatible with the interpretation of the positron being an electron traveling backward in time. It is in a letter that Feynman writes to Wheeler in 1951 that the physicist questions and shows the demise of the premises of his absorber theory and the incompatibility with his positron theory:[31]

> I wanted to know what your opinion was about our old theory of action at a distance. It was based on two assumptions:
> (1) Electrons act only on other electrons
> (2) They do so with the mean of retarded and advanced potentials
> The second proposition may be correct but I wish to deny the correctness of the first. The evidence is twofold. First there is the Lamb shift in the hydrogen which is supposedly due to the self-action of the electrons. [...] The second argument involves the idea that positrons are electrons going backward in time. If this were the case, an electron and positron which are destined to annihilate one another would not interact according to proposition one, since they are actually the same charge.[32]

However, the advance radiation in the absorber theory had the purpose of justifying the self interaction of the electron with its own field. By abandoning the premise of radiation going backward in time, Feynman explained the self-interaction of the electron by using his new quantum electrodynamics interaction, as shown in 4.6(a) below:

The self action of the electron is now represented by the particle freely propagating to spacetime point 3 and to then emitting a virtual photon that gets reabsorbed at a

[31] The point is also made in (Sauer 2008) and (Schweber 1986).

[32] R. P. Feynman to J. A. Wheeler, 4 May 1951, Feynman Papers, Caltech, folder 3.10. The complete quote can be found in: (Schweber 1986, p. 503).

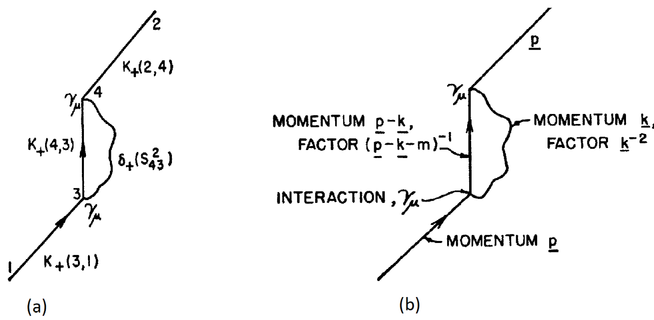


Figure 4.6 (a):(Feynman 1949a, p. 773). (b):(Feynman 1949a, p. 774).

later spacetime point 4. The self action thus consists of a spontaneous emission and subsequent absorption of a photon. The transition amplitude for such self interaction process assumes the form (Feynman 1949a, p. 774):

$$k^{(1)}(2,1) = -i\sigma^2 \int\int K_+(2,4)\gamma_\mu k_+(4,3)\gamma_\mu K_+(3,1)d\tau_3 d\tau_4 \delta_+(s_{43}^2) \tag{4.27}$$

From equation (4.27) Feynman derives the change of energy brought about by the self interaction (the self energy):

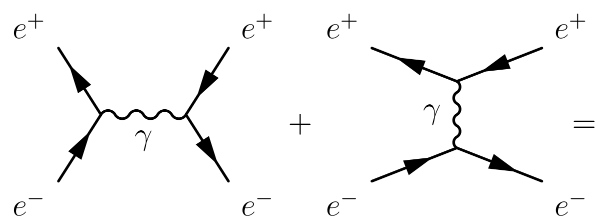
$$\Delta E = e^2 \int\int (\bar{u}\gamma_\mu K_+(4,3)\gamma_\mu u)\exp\{(ip\cdot x_{43})\}\delta_+(s_{43}^2)d\tau_4 \tag{4.28}$$

However, for such a process to happen —and the ones corresponding to the increasing orders in the perturbation theory— the electron must lose some energy which is thus 'bestowed' to the photon. Wouldn't this borrowing of energy cause the 'collapse' of the electron? This might be even more concerning if we consider subsequent orders in the expansion, where the electron already deprived of the energy for the first photon would have to borrow energy to another photon etc., thereby enforcing the worry about the disappearance of the initial electron. Although it is not my purpose to investigate the role of virtual and real particles here, the reply to the worry is 'simply' that because of the short interval in which these interactions happen, the energy-momentum relation does not need to be respected.

Similarly to the case of the positron theory, increasing precision in the calculation of the transition amplitude of a given quantum electrodynamics process involves the calculation of increasing orders in perturbation theory, where each subsequent order will consist of combinations of the fundamental interaction between electrons and the self action of the electron with itself. Since Feynman associated a specific expression to every graphic element of his diagrams, more complicated calculations became possible by iteration of the fundamental processes and corresponding expressions. As Feynman will recall many years later:

> The diagrams were intended to represent physical processes and the mathematical expressions used to describe them. Each diagram signified a mathematical expression. In these diagrams I was seeing things that happened in space and time. (Mehra 1994, p. 290):

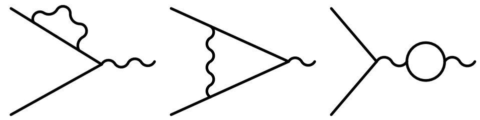
We can consider, as an example, the amplitude of the leading order of the positron-electron scattering which is calculated by considering both diagrams:



$$= g_e^2\mathcal{M}_1 + g_e^2\mathcal{M}_2 \tag{4.29}$$

where g_e^2 is related to the coupling constant by: $g_e^2 = 4\pi\alpha$ and $\mathcal{M}_1, \mathcal{M}_2$ are the amplitudes calculated on the basis of the diagrams above.[33] By increasing the perturbation order and thus by accounting for more complex processes one obtains a more precise transition amplitude, which also means that the vertexes of the diagrams become more numerous. For example, as shown in (4.30), one can consider the self interaction of a fermion with itself via production of a virtual particle, or one can calculate an additional interaction between the two fermions or, in the last diagram, one can have a virtual photon splitting into a electron-positron pair (diagram that is usually associated with phenomena of vacuum polarization).

[33] I have used the term 'consider' because usually the amplitudes of the diagrams in the same order are summed, but that depends also on the calculated sign of each amplitude.



(4.30)

All these diagrams add-up (for example) to those in (4.30) in such a way that the amplitudes of the higher order processes also contribute to the calculation of the total final amplitude. Thus, for example, the total amplitude of the positron-electron scattering is best characterized not only by the two second-order processes, but also by the subsequent orders of the perturbation expansion.[34] It is in this sense that Feynman Diagrams constitute the final version of the overall spacetime view: one that considers all the possible events within the spacetime boundary defined by the initial and final states.[35]

4.4 Conclusion

In this chapter I have reconstructed how the notion of overall spacetime view that was originally employed by Feynman in his absorber theory of radiation has changed. Starting from a view that involved radiation traveling backward in time, Feynman attempted to quantize the theory incurring though into a number of difficulties such that a final quantization remains nowadays incomplete. Nonetheless, during such attempts, Feynman ended up developing a new form of quantization (and thus a new formulation of quantum mechanics): the path integrals formulation. The main difference with respect to the absorber theory, more specifically in terms of the overall spacetime view, is that to calculate the transition amplitude of a particle moving from an initial to a final spacetime point one ought to sum over all the possible trajectories

[34]It does not concern us here that the integrals above a certain order will diverge. However, the probability amplitude for those processes is negligible.

[35]I have not discussed vacuum polarization, to which Feynman dedicates a whole section of (Feynman 1949a). Ultimately, the problem will be solved with the implementation of the renormalization techniques.

that the particle might undertake. This amounts to a generalization of the overall spacetime view: from the idea that some radiation can travel backward in time to the idea that the propagation of a quantum particle happens along all the possible paths.

While the two mentioned theories are of great relevance to Feynman's work, the present chapter has focused mostly on the theory of positron and quantum electrodynamics. Following Feynman's heuristic, as addressed in terms of the search for a physical understanding of the theories, I have addressed how the use of perturbation theory has contributed to the change of the overall spacetime view. Now, with the theory of positrons, it is not only that the electron (or a quantum particle) undergoes all the possible trajectories, but rather the electron undergoes all the possible interactions with a given potential. Furthermore, it is in the theory of positron that a more specific understanding of the overall spacetime view is addressed, one that comes with the rejection of the intuition of studying the dynamics of a quantum system interacting with a potential by following the total charge of the system, rather than the Hamiltonian of each particle.

Finally, the paper on quantum electrodynamics constitutes the extension of positron theory to interaction phenomena between electrons and positrons. Notably, the theory drops the hypothesis of the single electron and introduces two fundamental interaction from which all the others are derived. These interactions (positron-electron scattering and self-interaction) are the basis for the more complex phenomena of quantum electrodynamics.

Chapter 5

Visualization and Understanding of Quantum Phenomena

5.1 Introduction

The present chapter aims at clarifying an important implicit assumption that I have left lingering around in the previous chapters of this dissertation. Such assumption involves the epistemological character of the overall spacetime view and the type of scientific understanding of quantum phenomena that Feynman had when he was developing his famous diagrams. I will argue that such understanding came from the physicist's capacity of forming a partial visualization of the phenomena and that this visualization helped him write the appropriate equations for the calculation of the scattering amplitude.

Historically, the importance of partial visualization in physical theories has already been discussed. For example, Boltzmann (1974) believed that theories should provide pictures of the physical world that guide scientific thought and experiment. These pictures, though, should not be considered as faithful representations (one-to-one) of physical phenomena. Similarly, Schrödinger believed that spacetime visualizability contributes to the making of good scientific theories —even though the theories are not representative of the real world. How do these considerations apply to the case of Feynman diagrams? One possible answer is provided in De Regt (2017) where an extensive analysis of the relation between visualizability, intelligibility and scientific understanding is laid out.

The starting point of De Regt's analysis is that: "Scientists seek explanations that fit the phenomenon to be explained into a theoretical framework and connect it with relevant background knowledge" (De Regt 2017, p. 36). The connection between the theoretical knowledge, the background knowledge, and the phenomenon is based on the construction of appropriate models which ultimately provide the explanation to the phenomenon. What are the characteristics that the theories should have to be able to facilitate the construction of such phenomena-explaining models? The suggestion is that: "scientists prefer theories with properties that facilitate the construction of models for explaining phenomena, and that is the case if their skills are attuned to these properties" (De Regt 2017, p. 39). This means that scientists search for theoretical virtues (for example: simplicity and visualizability) also in relation to their own scientific skills. When such a combination between theoretical virtues and scientific skills is met, De Regt argues that we have a pragmatic understanding.

It is at this point that De Regt introduces his newly forged definition of intelligibility, to be intended as the value assigned by a scientist (or group of scientists) to the qualities of a scientific theory that make the theory more usable. The usability of a theory is then referred to as the capacity of scientists to build explanatory models for a given phenomenon starting from that theory. In other words, in De Regt's view, (i) the understanding of a phenomenon comes from an explanation which is provided by a model. Then, (ii) the model is constructed starting from the pragmatic understanding of a theory and (iii) the pragmatic understanding of a theory is to be understood in terms of intelligibility —to wit, we have an understanding of a given theory when we can use that theory to build models that can provide an explanation to some phenomena. Visualizability, which is a property I will discuss in this chapter in relation to Feynman's works is, according to De Regt, a quality that makes a theory (QED) intelligible.

With respect to Feynman diagrams, (De Regt 2017, p. 252) maintains that: "visual Feynman diagrams functioned as conceptual tools that made quantum field theory more intelligible for most theoretical physicists". While I can agree on the sociological effect that the diagrams had in the scientific community (cf: (Kaiser 2009)), the claim seems at odds with how Feynman developed the diagrams, especially in light of the overall spacetime view.

In what follows, I will maintain that Feynman did not develop the diagrams to make quantum electrodynamics more intelligible, but rather, that the visualization of quantum phenomena and the consequent writing down of the diagrams were fundamental to the development of quantum electrodynamics in the first place. More specifically, I will argue that the explanations of a given phenomenon, that is, the writing of mathematical equations attuned with the empirical data, come from a form of partial understanding of that phenomenon. The understanding is only partial since it is not backed by a 'one-to-one' representation with physical reality, and it is intended as providing an answer to plausible explanatory why-questions. For example, we can interpret such an understanding as the capacity of providing a narrative of how a given phenomenon comes to be, where, for example, the phenomenon is the shift in the $2p_{1/2}$ and $2s_{1/2}$ spectral lines of the Hydrogen atom (Lamb shift). In the present chapter, though, I will not provide an assessment of what such narrative corresponds to, as this would require some precise account of representation of quantum processes (the topic is still being discussed in the literature, cf.: (Dorato and Rossanese 2018), (Brown 2018), (Meynell 2018), and others). What I will argue is that the form of partial understanding came, in the case of Feynman diagrams, from the capacity of visualizing a physical interpretation of the phenomenon under consideration and, more in general, from the overall spacetime view.

In support of this reading, we can consider some historical examples: (1) Feynman's attempt to understand the Dirac equation is based on his previous works on path integrals, but it is also guided by the intention of visualizing a physical system that satisfies the equations, i.e., the quivering electron and the path counting. The example shows that Feynman's method of searching for a physical, partially visualizable system, traces back to his previous theories. Notably, this should also clarify that the visualization of phenomena, that I argue helped Feynman developing the diagrams, was not the product of an isolated mental reasoning. Feynman's background was the overall spacetime view, as I have analyzed in this dissertation, but also those theories and equations that proved to be empirically adequate. In this sense, for example, Feynman 'reinvented' the physical interpretation of the Dirac equation via visualization and overall spacetime view, but the Dirac equation remained essential to Feynman's thinking process and necessary to his theory. (2) Feynman's presentation at the Pocono conference was ill-received because it made use of the diagrammatic simplifications to avoid some mathematical complexities and the audience was not used to such visual-based thinking. The lackluster reception of his talk forced Feynman to publish his works in a mathematically more rigorous way. What emerges from examples (1) and (2) is that the use of visualization techniques was fundamental to Feynman's scientific method and theory building, and that it was fundamental to the development of the actual theory, and not not just to its understanding.

In what follows I will briefly address De Regt's view and its philosophical background (Section 2), then I will argue that Feynman's overall spacetime view aimed at understanding the phenomena, rather than the theory (Section 3). In Section 3 and Section 4 I will discuss the role payed by visualizability in Feynman's theorizing and, more specifically, I will emphasize its use in the theory of the quivering electron.

5.2 Boltzmann and De Regt

Before diving deeper into De Regt's account of scientific understanding, it is perhaps worth taking a detour and look at Boltzmann's philosophy of science and theory of picture (*Bildtheorie*). Since the latter directly influenced De Regt's account of understanding for Feynman diagrams, this detour shall give us some historical and conceptual context, together with the tools to better capture Feynman's understanding of quantum phenomena.

Generally speaking: Boltzmann suggests that we should find pictures that represent phenomena as accurately as possible, and not the absolute truthful theory. With this, he remarks his difference from a naive realist conception of scientific theories and thus rejects the one-to-one correspondence between theory and physical reality.

> (Boltzmann 1974, p. 33):"Task of theory consists in constructing a picture of the external world that exists purely internally and must be our guiding star in all thought and experiment."
> (Boltzmann 1974, pp. 90–91): "No theory can be objective, actually coinciding with nature, but rather, [...] each theory is only a mental picture of phenomena, related to them as sign is to designatum."

While Boltzmann's *Bildtheorie* (theory of picture) has been mostly viewed in the context of epistemological questions —that is, mainly about the relation between physical reality and scientific theory— De Regt (1999) investigates how the *Bildtheorie* relates to the explanatory purpose of science, i.e.: how pictures "contribute to the scientific understanding of natural phenomena" (De Regt 1999, p. 114).[1]

For example: a question about the kinetic theory of gasses is whether picturing the atoms offers a better understanding of the behavior of the gas. Boltzmann answers in the affirmative, even though there is not a one-to-one correspondence between the picture (*Bild*) and the phenomena. As addressed in De Regt and Dieks (2005, p. 223): "Kinetic theory of gasses does not have a literally true representation of reality but provides a picture [*Bild*] that possesses a certain similarity with unobservable reality. In general, theories should not pretend to give true representations of states of affairs

[1]More on the general aspects of Boltzmann's theory of picture in the context of philosophy of science: (Hiebert 1980), (De Regt 1996).

and processes in nature, but only to describe mechanisms that have a strong analogy with natural phenomena."

However, modern physics made evident the limits of a view of scientific understanding based on mechanical models; the same Boltzmann acknowledged those limits for theories such as classical electrodynamics. How should we understand the reference to 'mechanisms' then? De Regt (1999, p. 122) reports Boltzmann's answer: "What, then, is meant by having perfectly correct understanding of a mechanism? Everybody knows that the practical criterion for this consists in being able to handle it correctly. However, I go further and assert that this is the only tenable definition of understanding a mechanism (Boltzmann, 1974, 150)". It is from this latter point that De Regt builds a more structured theory of scientific understanding, one that starts from "the idea that scientific understanding of a phenomenon is achieved if one possesses a theory of it that is both empirically adequate and intelligible", and a theory is intelligible if "one is able to recognize at least qualitatively its consequences without performing exact calculations" (De Regt 1999, pp. 122–123). Thus, while intelligibility assumes a pragmatic sense, visualizability —which is grounded on mechanical explanation— is a useful tool, but not a necessary condition for the intelligibility of theories in physics.[2]

5.3 Feynman's Look at the Phenomena

How do such considerations about 'understanding' relate to this dissertation? I intend to pry into De Regt's new tool (intelligibility) and show that the application of his theory of understanding is misleading when applied to Feynman's works, even

[2]De Regt mentions the distinction between two different interpretations of mechanical picture (as originally presented by Boltzmann) for which, on the one hand, we have a physical theory that can be regarded as a picture and, on the other hand, "one may employ specific mechanical analogies in order to obtain visualization (*Versinnlichung*) of the consequences of a theory" (De Regt 1999, p. 116). The distinction serves the purpose of keeping separated the use of visualizability and pictures as forms of representation, and their use for scientific understanding.

though some of the concepts and references can and do play an important role in understanding Feynman's understanding of quantum phenomena.[3]

The starting point is the pivotal (and new) concept of De Regt's view, which is the concept of intelligibility defined as:

> **Intelligibility**: Value that scientists attribute to the cluster of qualities that facilitate the use of the theory (De Regt 2017, p. 12)

The definition implies that, among the different qualities a theory might have, there are some that concern the usability of such theory for building explanatory models, and that intelligibility is a value attributed to a theory and assigned by one (or more) group of scientists. Consequently (as De Regt will clarify), insofar as different scientists are members of different research contexts, and different values can be attributed to the same qualities, intelligibility will be a contextual concept. In addition, and this is more relevant for my purposes here, the definition emphasizes how the concept of intelligibility applies to theories and not to phenomena. Moreover, because the concept of intelligibility directly refers to the attribution of a qualitative value by some scientists, it suggests (at least in its definition) a component of subjectivism. If intelligibility (as we will see below) is fundamental to De Regt's view of scientific understanding and the concept implies subjectivism, then scientific understanding will also be subjective. But, De Regt argues that there is more to scientific understanding than the division between objective understanding (i.e., explanation) and the subjective understanding which is dependent on psychological factors. In fact, he proposes a more sophisticated account which is based on the distinction between:

- Phenomenology of Understanding (PU): It is the feeling of understanding that may accompany an explanation (e.g., an *aha!* or *eureka* experience).

[3] What I will not do, however, is to fight against the general project advocated by De Regt, as it is not my intention to defend or propose a specific account of scientific understanding.

- Understanding a Theory (UT): Corresponds to being able to use the theory. It is a pragmatic understanding that depends on the scientist or on the scientific community.

- Understanding a Phenomenon (UP): Corresponds to having an adequate explanation of the phenomenon. It is associated with Hempel's account of scientific understanding and with his Deductive Nomological (DN) model.[4] (De Regt 2017, p. 23)

Intelligibility pertains to UT since it is concerned with the practical skills of the individual scientists (or the scientific community) and with the capacity of producing effective explanatory models. These practical skills, argues De Regt, are in general a tacit knowledge that can be acquired in social contexts. An example from my experience: it is not enough to teach students the rule of *reductio ad absurdum* in classical logic, one also needs to teach them strategies to individuate the contradiction needed to continue the derivation. Different strategies for finding such a contradiction (most relevantly in difficult derivations) constitute practical skills that are shared by a given scientific community, which is, in this case, a class of students.

In general, the relation between UT and UP marks the first difference between the view that De Regt is proposing about scientific understanding and what I think is implicit in the evolution of the overall spacetime view in Feynman's works. As it emerged from the discussion of both the absorber theory of radiation, path integrals and quantum electrodynamics, Feynman did not learn the overall spacetime view from a scientific context, but rather, his approach brought about a drastic change in the practices of the scientific communities of his time. Perhaps, the strongest evidence in favor of such a reading can be found in the reports of the Pocono conference

[4] De Regt specifies that in his account every explanation is an argument and that the DN model is thus one specific articulation.

(Pennsylvania) in 1948 in which Feynman presented his new approach to quantum electrodynamics:

> At each step he was asked to justify his procedure; instead he offered to work on physical example to demonstrate the correct results it produced. But the audience objected to the time this would require and the hair involved, even though these had been drastically reduced by his methods. The culmination of his audience's feeling that Feynman was running amok without being rigorous came when Niels Bohr stood up, objected to Feynman's use of trajectories for small particles, and started reminding him about Heisenberg's uncertainty principle. Here Feynman gave up in despair, realizing that he couldn't communicate the fact that his analysis was justified by its correct results (Schweber 1986, p. 491).

What the quote emphasizes is that the quantum mechanics community originally rejected Feynman's new approach, and part of the reason is that Feynman's presentation was too physical and lacked mathematical rigor:

> Feynman was prepared to present "this whole thing backward . . . not formally . . . with all physical ideas starting from path integrals" (Schweber 1986, p. 491)[5]

It is only in the subsequent years that the diagrams became accepted and started to spread among different scientific communities (see: (Kaiser 2009)). In addition, the diagrams did not spread uniformly across the different countries and communities, thereby testifying the contextual aspect of the scientific understanding advocated by De Regt. The reason being that the diagrams did not come with a clear and unambiguous 'user's manual' and thus different scientists applied them to different

[5]Quoted from Feynman's interview with Schweber in 1980 Nov. 1st

set of problems and under different assumptions. But again, this view seems to apply to the historical phase of the diffusion of the diagrams and not to the phase of their development from Feynman's intuition.

Returning to the distinction between UP and UT, De Regt's theory rests on the idea that "scientists need intelligible theories in order to achieve scientific understanding of phenomena". Early quantum mechanics is an exemplar case, as pointed out in De Regt (2017, p. 91): "the fact that the theory of matrix mechanics appeared unintelligible to many physicists hampered the construction of explanations to understand phenomena by mean of this theory".

But, as I tried to emphasize earlier, Feynman's approach was to start from the problem of an accepted theory (i.e., the divergence of the strength of the field produced by point charge at $r \to 0$ in classical electrodynamics) and then develop a 'new theory' based on the understanding of the physical phenomena. The understanding of the physical phenomena, in this case, means the formulation of a narrative of how the phenomenon under consideration (radiative damping and self action in classical electrodynamics) might occur. Such a narrative, I argued, came from the application of the overall spacetime view in contraposition to the accepted theory of classical electrodynamics. The point, I believe, is especially evident in the brief reconstruction of the attempts to quantize the absorber theory of radiation that Feynman discusses in his doctoral thesis, as I emphasized in the first and last chapters of this dissertation.

With the work on the absorber theory of radiation, Feynman obtains a new way to formulate classical electrodynamics and, as he mentions in his Nobel lecture, a new view on these phenomena:

> I had a point of view —the overall space-time point of view— and a disrespect for the Hamiltonian method of describing physics (Feynman 1966, p. 8).

As a consequence of this 'disrespect' and motivated by the new view, Feynman moves to quantize the absorber theory of radiation which has no Hamiltonian—this will lead to the formulation of the path integrals for non-relativistic quantum mechanics. The relevant point is that in those years the method of quantization for a classical theory was fairly well established and it was based on the promotion of the position and momentum variables to non-commuting operators (expressed in terms of the Hamiltonian of the system). But the absorber theory of radiation was not formulated around a Hamiltonian and thus Feynman had to provide a new method for quantizing the theory. Thus, motivated by the new overall space-time view and moved by the necessity of quantizing the absorber theory of radiation, Feynman searched for a suitable action functional for a theory of direct-action-at-a-distance with half advanced and half retarded components and that could be successfully quantized. The solution came partly from the work by (Dirac 1933), who developed a quantization of the classical action functional expressed as the integral over the classical Lagrangian, and from the attempts to eliminate the electromagnetic oscillators. This led to the necessity of embedding the system into a spacetime boundary outside of which the system is in free initial and final states.

This reconstruction, that I have now presented only briefly, suggests that Feynman was not trying to understand the theory and then, through the theory, to understand the phenomena. Rather, it seems that Feynman had a new understanding of the physical phenomena —represented by the overall spacetime view— and that he built a theory around such a view.

We can now return to De Regt and to his theory of scientific understanding which, he argues, is based on the following idea:

> **CUP**: A phenomenon P is understood scientifically if and only if there is an explanation of P that is based on the intelligible theory T and conforms to the basic epistemic values of empirical adequacy and internal consistency (De Regt 2017, p. 92).

Intelligibility is thus a necessary and yet not sufficient condition for scientific understanding, for empirical adequacy ad internal consistency are also generally deemed necessary. If we apply this view to the analysis I have offered in this dissertation, we run into some troubles. One could argue that the absorber theory is internally consistent (under the condition that the universe is a complete absorber), but there is no evidence of advanced radiation in classical electrodynamics. However, Wheeler and Feynman argue that the advanced response of the absorber is what constitutes the radiative reaction of the emitter and thus, although we do not 'see' the radiation moving backward in time, the theory remains consistent. The issue is not entirely solved because unless we assume that the universe is a complete absorber there is no justification for the asymmetry between advanced and retarded radiation and for the asymmetry of time-direction of the electromagnetic radiation.

With respect to path integrals: the empirical adequacy is preserved, for one can calculate the right probability amplitude of a system. But, one might argue that the internal consistency is threatened by the fact that some trajectories are 'physically impossible' (although this would not really constitute a contradiction in the theory). These trajectories, as discussed in chapter 3, are non-differentiable and thus they are hard to interpret physically. The solution I have suggested in the chapter is that since the very notion of trajectory is problematic in quantum mechanics (as made evident by Heisenberg's principle), one needs to resort to a holistic view. If we consider the ill-behaved non-differentiable trajectories as 'non-physical' then it is not possible to reduce the probability amplitude of the system to the sum of the physically possible trajectories. Thereby, the holistic character of the probability amplitude is determined by the (weak) non-supervenience of the whole (the ensemble of possible trajectories) upon its parts (the physically possible trajectories).

With respect to Feynman diagrams, the empirical adequacy is guaranteed by the fact that quantum electrodynamics is considered as one of the best empirically verified theories of contemporary physics. As for the internal consistency: first, the divergence problem is something that caused quite a headache to the physicists at the time. The problem was tamed by the introduction of renormalization. Second, Feynman's idea of depicting the physical processes by means of some diagrams and the fact that these processes can be infinite in number is still there, and this has raised and still raises questions about the type of representation that the diagrams offer. However, even though I deem the topic to be of great interest, I will not discuss it in this dissertation.

As a matter of fact, here is a caveat: I do not suggest, neither here nor in the other chapters, that the overall spacetime view, as codified in the various theories, provides a complete and entirely trustworthy representation of the phenomena under consideration. Rather, I have implicitly suggested that the overall spacetime view can offer a partial (visual) description of the phenomena and that such a description provided Feynman with a form of (partial) understanding. As a matter of fact, Feynman never maintained that his new view corresponded to a complete and truthful understanding of the phenomena described by his theories. For example, when drawing diagrams at the Pocono conference in 1948, he was aware of Heisenberg's principle and that a definite trajectory is a concept unavailable in quantum mechanics. It is likely that Feynman implicitly thought of an approximate understanding of the physics phenomena for which an answer to an *explanatory why-question* is acceptable as long as we do not precisify the phenomenon too much.[6] What 'too much' corresponds to and whether we can provide a complete understanding of phenomena described by quantum electrodynamics are questions that far exceed the scope of this dissertation. Consider that whether we have a complete understanding of non relativistic quantum mechanics or whether such an understanding is in principle achievable is already

[6] An *explanatory seeking why-question* is a question that asks for an explanation about why a given phenomenon has occurred. See: (Hempel et al. 1965).

a very tall question, as proven by the extensive literature on the interpretations of quantum mechanics.

5.3.1 Visualizability

One more aspect I would like to briefly discuss is the relation between visualizability and intelligibility and, consequently, the relation between visualizability and understanding. With respect to visualization and Feynman diagrams, De Regt (2017, pp. 251–252) maintains that the diagrams provide: "a visualization of interaction processes, albeit one that cannot be taken as one-to-one representation of actual occurrences in nature. [...] The visual Feynman diagrams functioned as conceptual tools that made quantum field theory more intelligible for most theoretical physicists. [...] Rather than realistic representations of physical processes, Feynman diagrams are tools for solving problems and making calculations".

From the previous quotation, De Regt makes clear that the diagrams do not have a relation with physical reality and thereby they are not representations (in any sense) of the physical processes they depict. Such an instrumentalist view is not new in the literature. As a matter of fact, the works by (Brown 2018) and (Dorato and Rossanese 2018) move in a similar direction. However, I believe this view might be too much cut-and-dried. In what follows, I will suggest that Feynman diagrams can be taken to be a form of weak-representation, even though I will leave the task of specifying what is that they represent and what is a weak-representation to later works.

First of all, without stirring up a hornet's nest, I want to distinguish between visualization and representation. The latter pertains to the epistemological question about theory and reality, while the former refers to the capacity of more or less accurately describing a given phenomena.[7] Now, that Feynman diagrams are not

[7] I deliberately leave the term 'description' vague, as a fine graining of what constitutes a description of a phenomenon would lead me astray from the purposes of this chapter. Intuitively: the overall spacetime view in the case of Feynman diagrams is a form of visualization. A single diagram

a one-to-one representation of physical processes is not a mystery, as, for example, they represent the motion of fundamental particles in straight-lined trajectories and definite trajectories are mostly incompatible with quantum theories. However, the fact that they are not a one-to-one representation does not rule out the possibility for them to be at least a partial description of an interaction process.

Clearly, how to characterize the expression 'partial description' is but a simple task and I haven't found much clarification in Feynman's writings. However, something can be speculated, especially if we compare Feynman's view to that of Dirac. Dirac emphasizes how modern physics is becoming less and less visualizable and how the attempt of forming a mental picture of some physical processes requires the introduction of some irrelevancies:

> The methods of progress in theoretical physics have undergone a vast change during the present century. The classical tradition has been to consider the world to be an association of observable objects (particles, fluids, fields, etc.) moving about according to definite laws of force, so that one could form a mental picture in space and time of the whole scheme. This led to a physics whose aim was to make assumptions about the mechanism and forces connecting these observable objects, to account for their behavior in the simplest possible way. It has become increasingly evident in recent times, however, that nature works on a different plan. Her fundamental laws do not govern the world as it appears in our mental picture in any very direct way, but instead they control a substratum of which we cannot form a mental picture without introducing irrelevancies. (Dirac 1981, p. vii)

is also a form of visualization but, instead of visualizing the entire process, it describes only one of the contributing factors.

On the other hand, Feynman emphasizes the importance that visualization plays in understanding the equations, seemingly placing the two physicists on opposite positions:

> I dislike all this talk of there not being a picture possible but we only need know how to go about calculating any *phenomena*. True we only *need* calculate. But a picture is certainly a *convenience* & one is not doing anything wrong in making one up. It might be completely haywire while the equations are nearly right —yet for a while it helps . . . I want to go back & try to understand them [the equations]. What do I mean by understanding? Nothing deep or accurate —just to be able to see some of the qualitative consequences of the equations by some method other than solving them in detail.[8]

However, a more attentive reading can lead to a partial reconciliation. First, Dirac does not claim that the visualization of phenomena described by modern physics is impossible, but rather that one ought to add some irrelevancies. This resonates with Boltzmann's *Bildtheorie* in that there is not a one-to-one correspondence between theory and physical reality. Second, while Feynman emphasizes the importance of forming a picture of a physical phenomenon, he is also clear in saying that the picture might be "completely haywire". This, again, resonates with Boltzmann's position (Feynman, here, goes even further and weakens the link between the visualization and the physical reality even more). What Feynman adds is that the picture can help the physicist to understand the equations, where, here, 'to understand' is the capacity of foreseeing the qualitative consequences of the equations. This comes very close to what is defended by De Regt, especially the practical role played by visualization. What the quotation does not show is that Feynman's use of visualization will often

[8]Quoted in (Wüthrich 2010, p. 94): *Dirac Equation a, folio 12 (page 11).*

be directed toward the phenomena and that it will be a fundamental aspect of his theorizing.

5.3.2 Quivering Electron

One example of the importance of visualizability for Feynman is in the work on the Dirac equation after the paper on the path integral formulation of quantum mechanics. The last section of (Feynman 1948c) attempts to generalize the path integrals to relativistically moving particles, and to derive an action functional for the relativistic Dirac equation. Even though Feynman derives the proper equation, he remains dissatisfied with the too formal nature of the derivation.

> These results for spin and relativity are purely formal and add nothing to the understanding of these equations. There are other ways of obtaining the Dirac equation which offer some promises of giving a clearer physical interpretation to that important and beautiful equation (Feynman 1948c, p. 387).

One could argue that what emerges from the previous quote is that Feynman was trying to provide a physical picture (based on path integrals) of the Dirac equation and thus he was not trying to understand the phenomena directly, contrary to what I have been arguing thus far. A possible response to the objection is that, indeed, Feynman tried to understand the Dirac equation, but it is also true that he was coming from a new formulation of quantum mechanics and from his overall spacetime view. Because the Dirac equation is the dynamical equation for relativistically moving particles in quantum mechanics, it is only natural that Feynman tried to derive it starting from his new theoretical framework. In addition, his attempts ultimately failed and forced the physicist to undertake a different strategy. This led to the so-called theory of the quivering electron: an initially one-dimensional model of the Dirac equation where the electron is taken to move either to the left or to the right on a lattice; and the probability amplitude is calculated based on the counting of those turns.[9] As argued in (Wüthrich 2010, p. 65): "[...] what Feynman means by "understanding" the Dirac equation is not the study of the mathematical properties of the Dirac equation or the search for an ingenious method of solution required by the application of the equation to complex problems. Rather, Feynman is looking for a physical system, the appropriate description of which would satisfy the Dirac equation." But, the description of the system (e.g., the diagrams) needs not be a truthful representation of the physical reality and, in this respect, Feynman comes close to the *Bildtheorie* by Boltzmann that I reviewed earlier in the previous sections.

Another clue suggesting that visualization was fundamental for Feynman is reported in (Schweber 1986, p. 466):

> One delightful example that I really got big pleasure out of, is the liquid helium problem . . . So the whole thing was worked out first, in fact, was published first as a descriptive thing... which doesn't carry much weight, but to me was the real answer. I really understood it and I was trying to explain it.

Based on the last quotation, De Regt distinguishes between understanding and explaining where understanding is provided by the visualization and explanation is provided by the mathematical derivation: "First one needs intelligibility (i.e., understanding of the theory), which may be provided by visualization, and subsequently one can construct explanations, which may consist of mathematical derivations. Thus, visualization contributes to the intelligibility required for developing explanations. The success of Feynman's diagrammatic method indicates that most physicists prefer visualization as a tool for making theories intelligible" (De Regt 2017, p. 255).

[9]That an electron as described by the Dirac equation oscillates around a mean trajectory was already suggested by (Breit 1928) and (Schrodinger 1930).

However, thus far I have argued that the story is almost entirely flipped over: it is the visualization of the phenomena as overall processes that led Feynman to an understanding of those phenomena and, through that understanding, to the formulation of Feynman diagrams. Consider the excerpt of Feynman's interview in (Schweber 1986, p. 465):

> But visualization in some form or other is a vital part of my thinking and it isn't necessary I make a diagram like that. The diagram is really, in a certain sense, the picture that comes from trying to clarify visualization, which is a half-assed kind of vague, mixed with symbols. It is very difficult to explain, because it is not clear. [...] It is hard to believe it, but I see these things not as mathematical expressions but a mixture of a mathematical expression wrapped into and around, in a vague way, around the object. [...] Ordinarily I try to get the pictures clearer but in the end, the mathematics can take over and can be more efficient in communicating the idea than the picture. In certain particular problems that I have done it was necessary to continue the development of the picture as the method before the mathematics could really be found.

It is clear that Feynman does not have a precise philosophical account of visualization and/or understanding, but he also makes clear that his visualization and understanding is directed toward the phenomenon and not toward the theory. The formal and mathematical theory comes after the visual understanding of the phenomena, even though the mathematically well-formed theory can be more efficient in communicating such understanding. Allegedly, this is because scientific communities tend to receive formal mathematical arguments better than physical reasoning based on diagrammatic representations —see the reception of Feynman's quantum electrodynamics at the Pocono conference as I recalled earlier.

5.4 Conclusion

The objective of this last short chapter was to clarify some implicit assumptions of the philosophical analysis I have offered in this dissertation. Such assumptions are related to the type of scientific understanding that I believe the overall spacetime view offered to Feynman in the development of the absorber theory, path integrals and quantum electrodynamics.

While a full-fledged theory of scientific understanding is never spelled out in Feynman's works, I have relied on the recent work by (De Regt 2017) to emphasize the direction of Feynman's understanding and the role played by visualization. Notably, I have discussed these topics in light of Feynman's works and by keeping in mind the philosophical analysis that I have offered in this dissertation. As a consequence, I have not tried to discuss the issue as to whether and to what extent Feynman diagrams represent real physical processes.[10] Rather, I have offered an account of scientific understanding and visualization that fits Feynman's personal and peculiar way of doing physics. The overall spacetime view provided Feynman with a partial understanding of the phenomena under consideration. Such an understanding was not characterized by a one-to-one correspondence with physical reality, but it allowed the physicist to develop the theory of quantum electrodynamics and the well-known Feynman diagrams. In addition, I have argued that The visualization of the physical processes, even without a pretense of realism, was a fundamental aspect of Feynman's scientific understanding and theorizing. The final result was a theory capable of calculating the scattering amplitude of quantum processes to an extremely high-degree of precision and this is achieved through the use of diagrams that are only partly representative of the actual process. I leave the issue of better refining the concept of 'partial representation' to future and more specific investigations.

[10]Most recently, the issue has been discussed by (Brown 2018), (Stöltzner 2018), (Meynell 2018) and (Dorato and Rossanese 2018).

Chapter 6

Conclusions

In this dissertation I have identified Feynman's overall spacetime view in the absorber theory of radiation (Chapter 2), then I have analyzed how the view changed with the path integrals formulation of quantum mechanics (Chapter 3) and, finally, with Feynman diagrams (Chapter 4).

More specifically, in Chapter 2 I have presented Wheeler and Feynman's absorber theory of radiation which had the purpose of solving the problem of radiative reaction and thus the problem of the interaction of an electric accelerated charge with its own field. The solution proposed by Wheeler and Feynman makes use of the advanced solutions to Maxwell's equations and the assumption that the universe is a complete absorber. I have then argued that the overall spacetime view in the context of the absorber theory of radiation consists of the intertwining of past and future, together with the necessary relation between absorbers and emitters. Absorbers and emitters constitute a 'closed-system' within which past and future are intertwined. As a consequence, the system ought to be studied in its spacetime entirety.

The failed attempt to quantize the absorber theory of radiation led Feynman to the development of the path integrals formulation of quantum mechanics. In Chapter 3, I have presented Feynman's new interpretation and argued that the closed system is now constituted by the initial and final states. Within such states, all possible trajectories need to be accounted for to calculate the probability amplitude. I have thus compared the path integrals to the Lagrangian schema addressed in (Wharton 2016), and I have compared Feynman's idea of looking at quantum phenomena from the bird's eye view with the Newtonian schema (the hyperbolic partial differential method). In addition, the chapter analyzed the structure of the ensemble of possible trajectories and showed how the total ensemble is not strongly reducible to the single individual paths. The consequence is that the very notion of well-defined trajectory loses its meaning in quantum mechanics —in accordance with the results of Heisenberg's uncertainty principle.

In Chapter 4, I have addressed the overall spacetime view in the context of the theory of positrons and Feynman diagrams. There, I have described how from calculating transition amplitudes by integrating over all the possible configurations of the system, Feynman obtained a view that accounts for all possible interactions within the spacetime boundary. More specifically, I discussed how the use of perturbation theory and Feynman's interpretation of the terms in the perturbative series led the physicist to consider as possibilities not only the various spatial configurations of the system, but also all the possible interaction within the spacetime boundary.

In Chapter 5, I have addressed what type of understanding the physicist gained from the application of his view to the realm of quantum field theory, and the result is that Feynman diagrams, together with the overall spacetime view, provide a partial understanding *via* visualization of quantum processes. However, the last chapter deliberately avoided the question as to whether the diagrams constitute a form of representation of physical reality. I leave the question to later works, since a thorough answer would require diving into troubled waters.

Finally, the next section will conclude this dissertation with a brief discussion about Feynman's commitment to his own spacetime view, especially with respect to his later works.

6.1 I Want to Believe

The last chapter of this dissertation argued that Feynman, through the absorber theory of radiation, path integrals, and Feynman diagrams, tried to understand the physical phenomena interested by the respective theories. The overall spacetime view is what provided Feynman with a partial understanding, characterized by a strong visual component. I wish to conclude the dissertation with some considerations on whether Feynman believed in the physical picture offered by the overall spacetime view and quantum electrodynamics.

Such considerations can only be tentative, since again Feynman was not clear about assessing his philosophical standings. Furthermore, I think there is room for arguing that the extent to which he believed in his theories and their philosophical underpinning has changed over time. As a matter of fact, if one is to accept the reconstruction I have offered in this work, it should be clear that the overall spacetime view was not merely a conceptual framework used by Feynman for building quantum theories. One example in support of this thesis can be found in Chapter 4, where I presented how Feynman interprets the phenomena of electron-positron pair creation. Another example comes from how (Kaiser 2009, p. 175) characterizes the different attitudes toward the diagrams by Feynman and Dyson:

> To Feynman, his new diagrams provided pictures of actual physical processes, and hence added an intuitive dimension beyond furnishing a simple mnemonic calculational device.

Dyson, on the other hand, takes the diagrams to be graphical representations of combinatorial possibilities, i.e., useful tools to manipulate the mathematical terms of the perturbative series.

However, it is also clear that the extent to which Feynman believed that his theories represented actual physical phenomena has changed. For example, (Schweber

1986, p. 503) reports a letter written by Feynman to Wheeler in which the former rejects one of the assumptions of the absorber theory of radiation:

> I wanted to know what your opinion was about our old theory of action at a distance. It was based on two assumptions:
>
> i. Electrons act only on other electrons
>
> ii. They do so with the mean of retarded and advanced potentials
>
> The second proposition may be correct but I wish to deny the correctness of the first.

Schweber (1986) describes the letter as the evidence that a chapter in Feynman's intellectual life had come to a conclusion. While I agree with Schweber, I also believe that the letter conveys more than just that. First, it testifies that Feynman believed in the assumptions of the absorber theory of radiation, which were the basis and first instance of the overall spacetime view. Second, it shows that Feynman had progressively questioned the reality of the overall spacetime view —and the consequent truthfulness of his theories.

With respect to the first point, consider how Feynman recalls the search for a general action for his absorber theory of radiation:

> When the action has a delay, as it now had, and involved more than one time, I had to lose the idea of a wave function. That is, I could no longer describe the program as: given the amplitude for all positions at a certain time, compute the amplitude at another time. However, that didn't cause very much trouble. It just meant developing a new idea. Instead of wave functions we could talk about this: that if a source of a certain kind emits a particle, and a detector is there to receive it, we can give the amplitude that the source will emit and the detector receive. We do this without specifying the exact instant that the source emits or

> the exact instant that any detector receives, without trying to specify the state of anything at any particular time in between, but by just finding the amplitude for the complete experiment. (Feynman 1966, pp. 10–11)

The second point, on the other hand, is emphasized when Feynman recollects his calculation efforts about the Lamb shift experiment. There, he clearly characterizes his theory as a calculation tool: "The rest of my work was simply to improve the techniques then available for calculations, making diagrams to help analyze perturbation theory quicker" (Feynman 1966, p. 14). Nonetheless, Feynman retains the physical approach and in recalling the development of his theory of positrons he comments: "[b]ut one step of importance that was physically new was involved with the negative energy sea of Dirac, which caused me so much logical difficulty. I got so confused that I remember Wheeler's old idea about the positron being, maybe, the electron going backward in time" (Feynman 1966, p. 15).

It is hard to precisely pin down the moment when Feynman changed his attitude toward the overall spacetime view, and yet this change of attitude remains quite evident. As a further evidence, consider the way he recalls the work on the calculation of the interaction of the electron with the neutron:

> That was a thrilling moment for me, like receiving the Nobel Prize, because that convinced me, at last, I did have some kind of method and technique and understood how to do something that other people did not know how to do. That was my moment of triumph in which I realized I really had succeeded in working out something worthwhile. (Feynman 1966, p. 16)

The quote remarks how Feynman, in the end, and after having formulated his version of quantum electrodynamics, came to the conclusion that what he had developed was a calculation method rather than a new theory of quantum electrodynamics. This at least suggests that he stopped believing that Feynman diagrams, and perhaps the overall spacetime view, were a (partial) representation of the physical world. Another remark that seems to confirm this conclusion is quoted in (Mehra 1994, p. 453) where it is reported Feynman's enthusiasm about his results on weak interactions in 1957:[1]

> As I thought about it, as I beheld it in my mind's eye, the goddamn thing was sparkling, it was shining bright! As I looked at it, I felt that it was the first time, and only time, in my scientific career that I knew a law of nature that no one else knew. Now it wasn't as beautiful a law as Dirac's [discovery of the relativistic equation for the electron] or Maxwell's [equation of the electromagnetic field], but my equation for beta decay was a bit like that. It was the first time that I discovered a new law, rather than a more efficient method of calculating from someone else's theory (as I had done with the path integrals method for Schrödinger's equation and the diagram technique in quantum electrodynamics) [...] This discovery was completely new, although, of course, I learned later that others had thought of it the same time or a little before, but that did not make any difference.

I believe that Feynman here is being uncharitable to his own results (path integrals and Feynman diagrams) and I am in good company thinking it, for example: "[. . .] Richard, with his great talent for working out, sometimes in dramatically new ways, the consequences of known laws, was unnecessarily sensitive on the subject of discovering new ones. [. . .] Thus it would have pleased Richard to know (and perhaps he did know, without my being aware of it) that there are now some indications that his PhD dissertation may have involved a really basic advance in physical theory and not just formal development. " (Gell-Mann 1986, p. 51)

[1] R.P. Feynman, Interviews and conversations with Jagdish Mehra, in Pasadena, California, January 1988.

While the history of Feynman's works on weak interactions does not concern us here, the quotation is relevant in that it shows what Feynman thought (years later) about his path integral formulation and diagrammatic methods. Neither of them can be interpreted as a theory that describes the reality of quantum phenomena, and this suggests that even the overall spacetime view will be in the end taken by Feynman to be but a useful aid to develop different formulations of quantum theories. There is thus no commitment to the reality of such a view and, as it will become even more clear in the closing part of the Nobel Lecture, the view was useful for guessing the equations, rather than being a viable representation of the phenomena:

> This completes the story of the development of the space-time view of quantum electrodynamics. I wonder if anything can be learned from it. I doubt it. It is most striking that most of the ideas developed in the course of this research were not ultimately used in the final result. For example, the half-advanced and half-retarded potential was not finally used, the action expression (1) [*the action for the absorber theory of radiation*] was not used, the idea that charges do not act on themselves was abandoned. The path-integral formulation of quantum mechanics was useful for guessing at final expressions and at formulating the general theory of electrodynamics in new ways —although, strictly it was not absolutely necessary. The same goes for the idea of the positron being a backward moving electron, it was very convenient, but not strictly necessary for the theory because it is exactly equivalent to the negative energy sea point of view. (Feynman 1966, p. 17)

What remains of the physical reasoning and intuition behind the overall spacetime view? That Feynman started with the idea of formulating a new theory with the intention of solving the divergences problem remains a fact: "Therefore, a new theory was sought, not just a modification of the old." (Feynman 1966, p. 18). And yet, it seems that in later years Feynman grew skeptical about the purchase on physical reality of his view, especially in favor of a greater enthusiasm toward mathematics. At the end of the Nobel lecture, he characterizes the physical reasoning as helpful to some people, but the only true physical description comes from the mathematics describing the experimental observations:

> The only true physical description is that describing the experimental meaning of the quantities in the equations —or better, the way the equations are to be used in describing experimental observations. This being the case perhaps the best way to proceed is to try to guess equations, and disregard physical models or descriptions. For example [...] Dirac obtained his equation for the description of the electron by an almost purely mathematical proposition. A simple physical view by which all the contents of this equation can be seen is still lacking. (Feynman 1966, p. 18).

Independently of how much Feynman believed in his overall spacetime view and representational character of his diagrams, it remains that they played a crucial role in the development of modern quantum field theory, one of our most successful scientific enterprises. His fight against the customary view and the original way of thinking about quantum processes are surely worth studying, and perhaps even developing further, into new interpretations and theories.

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Appendix A

Time asymmetry and quantized absorber theory

A.1 The reversibility Argument

In what follows, I wish to briefly address the second problem that Price mentioned in his critics to the absorber theory of radiation. That is: the problem of justifying the emergence of the macroscopic time-asymmetry of radiation from time-symmetric laws. Far from being a complete discussion of the issue at hand, I will simply emphasize how a viable answer can be found in some works on quantum electrodynamics, rather than on the cosmological imbalance suggested by Price.

Simply stated: if the micro-laws of electrodynamics are time-symmetric, why is it the case that we do not experience advanced radiation? Wheeler and Feynman deemed the irreversibility of the radiation phenomena to be: "a phenomenon of statistical mechanics connected with the asymmetry of the initial conditions with respect to time" (Wheeler and Feynman 1945, p. 170).[1] In this sense, Wheeler and Feynman ground the asymmetry of radiative phenomena on the statistical behavior of the particles in the absorber. As neatly emphasized by Davies (1977, p. 144):

> This absorption is clearly an irreversible thermodynamical damping effect; the entropy of the absorbing medium increases. The thermodynamic asymmetry in the absorber imposes an asymmetry on the electromagnetic

[1] In this, Wheeler and Feynman seem to side with Einstein with respect to the Ritz-Einstein debate (1909). Ritz thought that the asymmetry of radiation was a fundamental character of thermodynamics while Einstein advocated for a probabilistic explanation. More on this: (Pegg 1975), (Frisch and Pietsch 2016).

> radiation, by permitting the transport of energy from the source at the center of the cavity to the cavity wall, but not the other way around. The advanced self-consistent solution, which is allowed on purely electrodynamic grounds, is thus ruled out as overwhelmingly improbable, because it would require the cooperative ”anti-damping” of all the particles in the cavity wall. [...]
> In the absorber theory of radiation the close relationship between electrodynamic temporal asymmetry is fully exploited. The existence of retarded ‘radiation’ is assured by the thermodynamic properties of the absorbing medium. The time direction of electromagnetic radiation is determined by the time direction of entropy increase in the universe.

The explanation provided by Davies fits the original formulation of the theory, as it accounts for the term $p(\omega)$ in equation (2.9) and hence endows to statistical mechanics the explanation of the macroscopic asymmetry. Price, on the other hand, counter-argues that such explanation is tainted by a ‘double standard fallacy’, for which the time-symmetric argument is applied for one time direction only: “so if a statistical argument rules out the advanced solution, an exactly parallel argument rules out the usual retarded solution” (Price 1991, p. 966).

The solution advanced by Price is that the asymmetry of radiation is not grounded on the thermodynamic asymmetry, but rather, that they share a ‘common-cause’. As he explains in: (Price 1994, p. 1024) “[. . .] we have a reduction of the issue of the (strictly macroscopic) asymmetry of radiation not to that of the thermodynamic asymmetry as such, but to cosmological issues of the same kind as those to which the thermodynamic arrow leads us.” By ‘cosmological issues’, he means a cosmological imbalance between large absorbers and large emitters. However, to support this view, he needs the modification of the mathematical core of the absorber theory expressed in equation (2.24) and thus he needs to change the type of symmetry of the overall process view. This change, as I have already emphasized, bears some problems that seem unresolved in Price's account.

The statistical argument advanced by Wheeler and Feynman is still tainted by the double-standard fallacy pointed out by Price. Therefore, unless we accept the change of time-symmetry —and thus modify (or reject) the overall process view— we ought to provide a possible explanation to the time-asymmetry.

I will now look at a possible extension of the absorber theory to quantum mechanics and show how it explains the imbalance between advanced and retarded radiation. It will be shown that at the level of quantum mechanics the time-symmetry is 'naturally broken', thus providing an explanation for how the phenomena of radiation is not symmetric. The difference, with respect to the solution proposed by Wheeler and Feynman, is that instead of imposing an *a priori* statistical argument that rules out the advanced fields, we have a derivation of the time-asymmetric operator starting from the interactions between absorbers and emitters.

A.2 Extension to Quantum Mechanics

The works by (Davies 1970), (Davies 1971), (Davies 1972) extend the theory of Wheeler and Feynman to the domain of quantum electrodynamics (QED) by means of S-matrix (scattering matrix) and the use of some boundary conditions.[2]

Davies (1971, p. 840) demonstrates that the action propagator for current-field interaction

$$\sum_i \int j^{\mu}_{(i)}(x) A_{\mu}(x) dx^4 \tag{A.1}$$

where A_μ is the quantized electromagnetic field and the $j^{\mu}_{(i)}(x)$ is an emitting current,

[2] A different route was taken by (Hoyle and Narlikar 1995) who instead of using the S-matrix adopted the path integrals formulation based on the works by (Dirac 1933) and (Feynman 1948c).

can be replaced with an operator for current-current interaction:[3]

$$\sum_i \sum_j \frac{1}{2} \int \int j^{\mu}_{(i)}(x) D_F(x-y) j^{\mu}_{(j)}(y) dx^4 dy^4 \tag{A.2}$$

Then (Davies 1971) and (Davies 1972) prove that one can decompose Feynman's time-asymmetric photons propagator $D_F(x-y)$ into: $D_F = \bar{D} + D_1$; where $\bar{D} = 1/2(D^{ret} + D^{adv})$ is the time-symmetric propagator for virtual photons and $D_1 = 1/2(D^{ret} - D^{adv})$ describes the free field and thus real photons. The situation, argues (Davies 1972, p. 1027), is similar to the one in classical electrodynamics: "[...]the virtual photons (time symmetric bound field) give rise to the near field, because of the finite lifetime of the virtual photons, while the real photons can escape to infinity as the far field". The analogy is that we can consider a point charged particle in arbitrary motion and construct two fields: $\bar{A} = 1/2\,(A_{ret} + A_{adv})$ and $A = 1/2\,(A_{ret} - A_{adv})$. The first one is 'near' $(1/R^2)$ and it is a solution to the inhomogeneous equation and the second is a 'far' $(1/R)$ source-free field.

When $a^{\mu} = 0$ (i.e., for a uniform motion) the far field vanishes and thus what remains is the time symmetric velocity field at the source without the radiative damping. At the quantum level, we only have virtual photons for which the order of emission and absorption can't be determined:

> To see this, we appreciate that when no energy source is available the time ΔT required for the emission process is related to the frequency of the photon by $\Delta T \sim 1/\omega$. That is, in the wave zone of the source, we are completely unsure of the order of emission and absorption. This order is only well defined in the far zone, but the far field vanishes here (Davies 1972, p. 1028).

What happens, though, if the system is accelerated and hence $a^{\mu} \neq 0$? At the classical level, accelerated particles emit fields to which the universe (absorber) responds.

[3] The second current being $j^{\mu}_{(j)}(y)$.

In other words, the presence of the free field is due to the response of the universe which applies to the originally emitting charge. That this radiation is completely absorbed is the light-tight box condition for the classical absorber theory:[4]

$$\sum_j \frac{1}{2}(A_j^{ret} - A_j^{adv}) = 0 \tag{A.3}$$

In the quantum case, Davies distinguishes real and virtual photons based on having infinite and finite lifespan respectively. The quantum equivalent of the complete absorber condition is that there are not real photons propagating infinitely into the future. Under this condition, though, the presence of the term D_1 is paradoxical. Davies' solution is that the presence of propagating real photons comes from the interaction with the virtual ones:

> The paradox [of having infinite lifespan photons] can be resolved by appealing to the classical theory. Just as we can never separate the A and $\bar{A}$ fields, and both of them carry away radiation when $a^\mu \neq 0$, so the virtual photons continually interfere with the real photons when we have the quantum analogue of acceleration (ie energy available for the transition) (Davies 1972, p. 1027).

As well summarized in (Kastner and Cramer 2017):

> In the Davies theory, the usual quantum electromagnetic field $A(x)$ (suppressing components indices for simplicity, i.e. $A = A^\mu$ and $x = x^\mu$) is replaced by the direct current-to-current interaction as above. Thus, the field at a point x (on a charged current i) arising from its interaction with responses of all currents j_j is given by $A(x) = \sum_j \int D_F(x-y) j_j(y) d^4y$ where the quantum analog of the 'light-tight-box' condition is imposed: namely, that for the totality of all currents j_j, there are no initial or final

[4] Equation (A.3) corresponds to equation (2.20) in Derivation IV.

states with real photons —i.e., no genuine photon 'external lines'. (In the Davies theory, this amounts to the requirement that the existence of a real photon requires both an emitter and an absorber.)

While Davies' theory represents a step toward a full quantum treatment of the original absorber theory of radiation, Kastner (2020, p. 3) argues that it remains a semi-classical theory: "insofar as it tacitly identified radiation with continuous fields, and assumed that a real photon could be unilaterally emitted, which is not the case at the quantum level". She also points out that a fully quantum direct action theory needs no cosmological conditions in the sense of a completely absorbing universe.[5] What is really needed, from the mathematical point of view, is the equivalence between equations (A.1) and (A.2) which is obtained by setting the electromagnetic field operator $\hat{A}_\mu$ to zero in the scattering matrix (Kastner 2020, p. 20):

$$S = \exp\left(-\frac{i}{2}\int j^\mu(x) D_F(x-y) j_\mu(y) d^4x d^4y\right) \times \exp\left(i\int j_\mu(x)\hat{A}^\mu(x) d^4x\right) \qquad \text{(A.4)}$$

Kastner argues that to set the quantum operator to zero corresponds to the classical light-tight-box condition in equation (A.3).[6] The latter, however, is interpreted not as the condition that all radiation is ultimately absorbed, but rather as the condition that there are no free (sourceless) fields:

> While selective cancellation of fields does occur among charges to produce the effective radiation field, the absence of an unsourced radiation field is the primary physical content of the 'LTB' [light-tight-box] condition for the quantum form of the DAT [direct action theory.]

[5]Whether the universe is a complete absorber in the sense expressed by the absorber theory is investigated in: (Hoyle and Narlikar 1995).

[6]Setting the electromagnetic field operator to zero fits with the original attempt by Wheeler and Feynman to build a theory without an independent classical electromagnetic field.

A fundamental point in the argument is that real and virtual photons ought not to be distinguished based on finite and infinite lifespan, because real photons can be absorbed and emitted as well: "[...] the response of the absorbers is what gives rise to the 'free field' that in the quantum domain is considered a 'real photon'. So the 'realness' of the photon is defined in the transactional picture not by an infinite lifetime —which, in reality, is practically never obtained— but rather by the presence of an absorber response" (Kastner 2014, p. 5)[7]. In other words: the near field —which is mediated by virtual photons (off the mass shell)— does not involve the absorber response and thus virtual photons are only direct time-symmetric connections between currents. On the other hand, the far field —which is mediated by real (on the shell) photons— exists conditioned to an appropriate response of the absorber which is the only field that ought to be absorbed (according to the light-tight-box condition). Some confusion might arise from the use of the term 'absorber response', which is misleading because it is conducive of a temporally oriented mechanism for which a source emits a field and the absorber responds. This is not the case in Kastner's reinterpretation: "So rather than 'absorber response', this quantum relativistic process is really a *mutual* agreement to generate an on-shell field" (Kastner 2020, p. 2).

The technical details of how the presence of this mutual interaction between absorbers and emitters determines the presence of D_1 and thus the emission of real photons under quantum acceleration, do not concern us here; they are specified in (for example): (Kastner 2020) and (Kastner and Cramer 2017). What is of most interest to us is that both Kastner and Davies advance a theory that strongly relies on the absorber theory of radiation by Wheeler and Feynman.

Where does the time asymmetry come from? In Davies' theory, the imposition

[7]The expression 'transactional picture' in the quote referes to the Transactional Interpretation of Quantum Mechanics by (Cramer 1986) and (Kastner 2013)

of complete absorption guarantees the response of the absorbers which causes the radiative damping and the cancellation of the advanced fields. The theory is thus time symmetric and justifies the time asymmetry at the level of boundary conditions.

However, we have seen that Davies' account bears some problems and questionable assumptions —namely, that the universe is a complete absorber and real photons have an infinite lifetime. That being the case, we have briefly presented Kastner's solution which is based on a different definition of virtual-real photons and light-tight-box condition. The temporal asymmetry and the phenomena of radiation are derived from the mutual interactions between absorbers and emitters, where the presence of the former dictates the presence of radiation. It remains that Kastner's account does not diverge from the philosophical underpinning of the original absorber theory. As a matter of fact, it makes the interaction between absorbers and emitters even more fundamental in accounting for the phenomena of radiation, and this is because it does not need a cosmological boundary.[8]

To be more precise, Kastner replaces the cosmological boundary condition with the quantum completeness condition which makes explicit the role of the mutual interaction between absorbers and emitter and leaves us with no need for the cosmological boundary:

> Physically, this means that absorbers corresponding to each possible value of k must respond; or, more accurately at the relativistic level, that the emitter and absorbers must engage in a mutual interaction, above and beyond the off-shell time-symmetric field $\bar{D}$, to generate an on-shell

[8] One can still object that the use of Feynman's propagator, for which positive energy solutions propagate forward in time, is a form of double standard. For example, one could use Dyson's propagator and thus have negative energy solutions propagating forward in time. However, this does not constitute a double standard because an observer in a 'Dyson Universe' would not see any difference from an observer in a 'Feynman Universe'. Both observers would 'see' a unidirectional flow of energy. 'Positive' or 'negative' energies are, in this sense, conventional. The point is made evident in (Kastner 2020).

> field that can be factorized, corresponding to the quantum completeness condition (Kastner 2020, p. 13).

Although the overall process view I have presented here was meant to refer to the absorber theory only (thus, classical electrodynamics), the mutual interaction of absorbers and emitter in the quantum completeness condition sets the ground for a generalization to quantum electrodynamics —given that it mirrors the second clause of the overall process view.

In general, neither of the theories considered here required the application of a statistical argument to one time direction only. They both constructed the asymmetric operator starting from interactions between real-virtual photons and absorbers and emitters. It is in this sense that the double-standard argument is at least quite weakened. The asymmetry of time emerges from the very formalism of the theories, despite the time symmetry of the laws.