On the Influence of Afp-Induced Defects on Mechanical Behavior of Composite Laminates Under Fatigue Loading

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ON THE INFLUENCE OF AFP-INDUCED DEFECTS ON MECHANICAL BEHAVIOR OF COMPOSITE LAMINATES UNDER FATIGUE LOADING

by

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Thank you.
The objective of the work presented in this thesis is to propose and validate a numerical solution to quantify the influence of Automated Fiber Placement (AFP) induced defects on the mechanical behavior of conventional composite laminates in fatigue. The work presented here will be divided in six sections. At first, a description of the Automated Fiber Placement process and its associated defects is proposed. In association, two tools developed for a rapid analysis and a progressive damage analysis respectively are discussed and used as an introduction to the work conducted in this thesis. In the second section, an extensive literature review on AFP defects, their influence on the quasi-static behavior of conventional composite laminates, and as well as their influence in fatigue is conducted. Then, the third section of this document mainly focuses on AFP induced defects and two potential modeling approaches: a "simple" model versus an "advanced" modeling solution. The modeling approaches presented in the previous section are put to the test within the following two sections of this document. Section four of this thesis focuses on the modeling and simulation of the influence of AFP induced defects on quasi-static tensile test of Open Hole specimens to identify critical configurations to be used for the analysis in fatigue. The following portion of this document is dedicated to the presentation of the modeling approach and the results of fatigue analysis on configurations identified in the quasi-static portion of the work conducted. Finally, the last section presents the implementation of a new capability to incorporate a stochastic representation of the material properties in a progressive damage analysis tool. This allows for a propagation of uncertainty from the material level to the number of cycles to failure.
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Chapter 1

Introduction

To answer the industrial need expressed in the composite manufacturing field, the Automated Fiber Placement (AFP) process was developed in the early 1970s to offer an alternative to hand layup and Automated Tape Placement (ATP). Therefore, providing a faster and more reliable solution to manufacture more complex shapes than ever before at a higher pace. However, AFP manufacturing is not exempt of flaws. As a matter of fact, depending on the process parameters selected and the shape complexity of the part manufactured, a vast variety of manufacturing defects can occur during the layup process and persist even after the curing process. The identification of these defects is already well documented in the literature as will be presented in Chapter 2. Their influence on the mechanical behavior of composite laminates under quasi-static loading and, even more, under fatigue loading is however, a lot less understood or documented. The existence of defects and the lack of global understanding of their influence on the mechanical behavior of the manufactured parts has consequences on the certification process of aeronautical parts and, consequently, engender a cost increase for industrials. The work presented in this document is an attempt to offer a broad view of the influence of defects on quasi-static and fatigue behavior of composite laminates. It is however, by no means trying to propose an exhaustive and parametric analysis of the different defects and their associated parameters which will be presented in more details in Chapters 2.2. In fact, the work presented here, as it will be explained in the Chapters 4 and 5, intends to narrow down the selection of defect types and composite related parameters to a handful of
configurations that could realistically occur during manufacturing. This is motivated by the wide variety of AFP-induced defects which will be presented in the following subsection. Finally, Chapter 6 is an attempt to incorporate, in the numerical models, the variability of material properties observed in experimental approaches to propose a propagation of uncertainty from the material properties to the number of cycles to failure.

1.1 AFP-induced defects

As mentioned in the introduction, the AFP manufacturing process can sometimes be afflicted by the creation of defects. The defects mentioned in the following sections of this document refer to situations during which a perturbation affects the placement of a ribbon of material called a tow, leading to a tow positioning that does not match with the nominal configuration expected. This type of imperfection can take several forms. Most commonly, the tow can be shifted along its transverse direction which leads to an opening or gap in the layer and potentially two tows overlapping each other on the opposite side. Another common possibility is for the geometry of the tow to be affected. A tow can indeed fold onto itself therefore creating a double thickness region and a gap simultaneously. A tow can also be twisted in a “bow-tie”-like pattern as presented in figure 1.1. A more detailed review of the identified defect types is presented in the literature review (Chapter 2). The work presented in the following section aims to select a realistic configuration. Realistic in the sense that it can both appear during the manufacturing process and at the same time slip through the net of a visual inspection conducted by the machine operator at the end of each ply layup. Therefore, based on the manufacturing experience gathered in the lab and through the publications reviewed in the process of writing the literature review, the majority of the configurations studied include a combination of a gap and an overlap created by the shift of a tow from its nominal position. Regarding the width of the defect to
study, since the current quality control checks are conducted via a visual inspection and that defects smaller than half a tow width are commonly left unrepaired, this dimension of half a tow width or 1/8” (3.175mm) is used in the models presented in sections 4 and 5.

Figure 1.1: Picture of a twist

1.2 Quasi-static Response of Laminates Containing Defects

As it was already emphasized on several times in the introduction (Chapter 1) and will be emphasized again in the following sections, the potential configurations in which defects can occur during an AFP manufacturing process are numerous. Variables of interest include, but are not limited to, the defects’ type, location, and frequency of occurrence but also, as will be shown in the literature review presented in Chapter 2, the stacking sequence and therefore orientation of said defects. Consequently, in order to restrain the scope of interest of the present dissertation, a selection of both the geometrical characteristics of the laminate and the defect are presented in Chapter 4 where the Progressive Failure Analysis (PFA) of Open Hole Tension (OHT) specimen in quasi-static is conducted to establish a good understanding of the influence of the selected defect type before moving to the study of the same configuration loaded in Tensile-Tensile fatigue in Chapter 5. The studied configurations illustrated in Figure 1.2 also attempt, as explained in Chapter 4, to quantify the relative position of a defect to that of a stress raiser such as a hole in the center of the laminate using a Continuum Damage Mechanics (CDM) approach based on the work presented by F.Leone[1] and implemented in an Abaqus Explicit user-subroutine named Comp-
Dam [2]. Using a CDM approach allows to capture the influence of microscopic matrix crack developments on the macroscopic level with the use of reduction factors in the stiffness matrix $C$. This comes with the cost of the need of a fine mesh preferably oriented with the fiber orientation as explained in Chapter 4 through B.Justusson et al’s work [3]. In the work presented in this document, a focus is given to ten defect configurations in an PFA of OHT specimens. More specifically, the relative location of a combination of gap and an overlap with respect to the hole in the specimen is studied. The considered defects are in a combination of a $\frac{1}{8}$” (3.175mm) wide overlap and a $\frac{1}{8}$” (3.175mm) wide gap created by shifting a single tow by half of its width in its transverse direction as illustrated for the three main configurations Figure 1.2.

![Figure 1.2: Main defect configurations for OHT specimens](image)

1.3 Fatigue Response of Laminates Containing Defects

The natural transition from the work presented in Chapter 4 on the quasi-static tension behavior of composite laminates containing defects is to move on to the fatigue behavior of the same configurations. This is the work presented in Chapter 5. The same FEM are adjusted to include an additional step in fatigue still using the CDM approach offered by CompDam. During this new step, the maximum load
applied during the first step, the quasi-static loading of the specimen, is held constant and equal to the maximum load and an additional degradation coefficient is introduced in the cohesive component of the Deformation Gradient Decomposition (DGD) ([1]) to capture the damage generated in the material by the application of load cycles. This new coefficient accounts for the progressive degradation of the material properties as simulation time elapses. This allows to link the number of cycles per increment to the time elapsed in the simulation and the material properties degradation necessary for the cohesive law to capture the elements degradation as presented in Dávila et al’s work [4].

1.4 Stochastic Distribution of Material Properties

Finally, the models and results presented in Chapters 4 and 5 are based on average material properties obtained by following the recommendation of American Society for Testing and Materials (ASTM) standards. However, a more accurate representation of the actual distribution of the material properties across a laminate in the form of a stochastic distribution of the stiffnesses and strengths of the composite is proposed in Chapter 6. Introducing a probabilistic representation of the material properties allows for the proposed model to offer a new capability in the form of the propagation of uncertainty from the material properties all the way to the number of cycles to failure in a specimen under tension-tension fatigue test. An additional interest can be seen for pristine laminates for which the absence of defect, which can also be associated with the absence of stress raiser, render the simulation more susceptible to spurious damage initiation at the edges of the laminate. Introducing local modifications to the properties distribution allows for the creation of initiation points within the laminate which in the end, after several simulations of different configurations, allows
for the evaluation of a range of cycles to failure instead of a single data point. As a result, confidence intervals on the numerically generated S-N curves can also be provided therefore allowing for the pristine configuration to be used as a reference when compared to configurations containing defects.
CHAPTER 2

LITERATURE REVIEW

In this section, a literature review on the influence of AFP induced defects on the quasi-static and fatigue behavior of composite laminate is presented. This work was published as a survey paper in the International Journal of Fatigue [5] and is presented in the following pages. As so, the reader must be warned that it may be found that some sections presented in this review are overlapping with the work presented in the other sections.

2.1 INTRODUCTION

Numerical and experimental studies have been conducted to try to quantify the influence of Automated Fiber Placement (AFP) induced manufacturing defects on the quasi-static behavior of composite parts ranging, on the building block approach, from coupons to sub-structural parts. In more recent studies, researchers started to conduct the same investigations for fatigue load cases with the purpose of determining the influence of defects on damage initiation and propagation mechanisms. This document will present a review of both experimental and numerical studies conducted so far on the influence of AFP defects on laminates response and failure characteristics during fatigue loading. In the first section, a brief introduction of the types of AFP manufacturing defects identified will be presented before focusing on publications related to the influence of the aforementioned defects on the quasi-static behavior of composite laminates. Following, in the second section, we propose to detail the numerical investigations on the fatigue behavior of composite laminates. Then, the
third section will present a combined experimental and numerical investigation of the influence of the manufacturing defects presented in the first section on the fatigue behavior studied in detail in the second section. Finally, the fourth section is dedicated to a comparison of the time cost associated with numerical and experimental approach.

2.2 Influence of AFP Induced Defects on Quasi-static Response of Composite Laminates

In AFP manufacturing, both the shape complexity and manufacturing processing parameters have a significant influence on defects occurrence rate and severity. Based on manufacturing experience, most commonly encountered defect types have been identified and studied by Harik et al. [6] using four different perspectives: Anticipation, Existence, Significance, and Progression of defects. In the context of Ref. [6], Anticipation refers to the capacity of predicting defects occurrence based on process parameters values. The Existence perspective focuses on describing the detectable aspect of defects from an inspection point of view and provide guidance on inspection protocol needed based on the defect type: visual, semi-automated or automated. The Significance perspective investigates the consequence of a defect on part performance. Finally, the Progression perspective aims at describing the potential evolution of the defect under service loads. This allows for a better understanding of the origin of defects depending on their nature and also gives a first estimation of what can be expected for their influence on a laminate’s response. We can, however, highlight the need of a fifth category: Disposition. This category would, once the consequences of the defect on the part performance are understood, deal with further treatment of the defect or the part; i.e., continue using the part, cosmetic repair, temporary repair, structural repair, or rejection (remove and replace defective part). The researchers
identified fourteen types of defects: Gap/Overlap, Pucker, Wrinkle, Bridging, Boundary Coverage, Angle Deviation, Fold, Twist, Wandering Tow, Loose Tow, Missing Tow, Splice, Position Error and Foreign Object. A schematic for each defect type is presented in Figure 2.1.

![Defect Schematics](image)

Figure 2.1: Defects’ schematics from [6]

Recently, following the increased use of AFP manufacturing, extensive quasi-static testing on composite laminates containing defects was conducted [7–16]. Mainly three types of the previously identified defect types were tested: gap, overlap and wrinkle. Two main reasons can explain the authors’ choice to focus on these specific types. First, those three types of defects are the most commonly encountered types with AFP manufacturing. Gaps and overlaps frequently occur between tows and courses in conventional laminates and on the part boundaries. They also are unavoidable in steered-laminates [17–20]. Wrinkles, on the other hand, generally occur in variable-stiffness laminates due to the difference of length between the outer and the inner
tows path [21]. They can also be observed in conventional laminates with complex geometries. Secondly, the above-mentioned defect types can also be encountered within other manufacturing techniques. The formation of defects such as distortion, fiber waviness, and non-uniform fiber volume fraction were observed and studied for Resin Transfer Molding (RTM) techniques [22, 23]. This justifies the interest in understanding their influence on composite laminates response and failure characteristics.

Another important factor when it comes to understanding the influence of defects on a laminate response is the type of loading applied. Researchers are, to the greatest extent, focusing on tensile failure [8, 11, 17, 24] and compressive failure [7, 12, 15, 25] of composites.

The influence of several AFP induced defect types on the quasi-static response of laminates has been experimentally observed by Croft et al. [8]. They include lamina level (tension, compression and in-plane shear) tests on unidirectional laminates, and laminate level tests (Open Hole Tension (OHT) and Open Hole Compression (OHC)) on quasi-isotropic laminates. The results are summarized qualitatively in Figure 2.2. It appears that the simplest defect types (one gap or one overlap) have a little to no influence on the strength of unidirectional tensile specimens. This can be explained by a better consolidation of defective plies in a unidirectional laminate during the

Table 1

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<th>Overlap</th>
<th>Half Gap/Overlap</th>
<th>Twisted Tow</th>
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<td>Compression</td>
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<td>Length</td>
<td>Width</td>
<td>Half Gap/Overlap</td>
<td>Twisted Tow</td>
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<tr>
<td>OHC</td>
<td>Length</td>
<td>Width</td>
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Figure 2.2: Quasi-static test results from [8]
cure cycle. A similar observation was made for compressive specimens where the only
defect of influence seems to be the overlap with an improvement of the compressive
strength by 7%. However, this is mainly due to thickness change of the laminate.
OHT specimens showed that the influence of the stress concentration generated by the
hole was overshadowing any influence the defect might have had. On the other hand,
OHC specimens displayed a high sensitivity to the introduction of a defect either
within the length or the width of the laminate. Defects along the specimen’s length
improved the global compressive strength by facilitating the creation of delamination
around the hole which alleviates the local stress concentration. However, defects along
the laminate width, in the stress concentration area, lead to a strength reduction.

To summarize, it is observed that defects have a higher influence on what the Croft
et al. [8] call laminate level test (OHT and OHC) than they do on what they qualify
as lamina level test (tension, compression and in-plane shear coupons). However, that
observation is based on a comparison of results between unidirectional laminate used
for lamina level test coupons and quasi-isotropic laminates used for laminate level
tests. Hence, the conclusion comparing the test results from different scale levels
may not be appropriate. This sensitivity of OHT and OHC laminates to defects is
probably due to different phenomena interacting between plies of different orientations.
Additionally, out-of-plane ply waviness generated by a defect on its adjacent layers
have a significant influence on shear and compressive strengths. Yet, as can be seen
on micrographs of defective specimens, out-of-plane layers waviness in unidirectional
laminates (see Figure 2.3a) are less severe than the one observed in quasi-isotropic
laminates (see Figure 2.3b). A situation that, when it affects 0° layers, can lead to a
significant reduction a compressive strength by facilitating the creation of kink-band
zones. This conclusion is in agreement with Sawicki and Minguet’s finding [7]. The
authors indeed found that by introducing a combination of a gap and an overlap
in all 90° layers, the out-of-plane ply waviness generated in adjacent 0° layers lead
to a premature failure in compression in both un-notched laminates and OHC. It is also worth mentioning that a defect width threshold was identified. Defects wider than 0.03 in had no meaningful additional influence of the material properties [7]. A similar conclusion was drawn regarding the insensitivity of the laminate properties with respect to the number of defects introduced in the laminate. No significant degradation of the material properties was observed when more than one defect was introduced in the laminate. Those two observations combined emphasize the fact that defects are mainly acting as local stress concentrations. Therefore, they have a more significant influence on load level of damage initiation than on damage propagation as they tend to trigger delamination earlier in the vicinity of stress concentrations.

OHC test results from Sawicki and Minguet [7] are presented in Figure 2.3c. The earlier mentioned threshold value above which the defect width has no additional influence on the laminate failure correspond to 0.03in as the test results illustrate. Laminates containing 0.1in Lap/Gap are indeed showing a slightly higher failure strain to Baseline failure strain ratio than laminates containing 0.03in defects.

In both previously mentioned publications, overlaps are generated by introducing a 90° strip of material in the laminate. This allows for a thickness build-up without considerably modifying the laminate stiffness. This is one of the three techniques identified and studied by Wang et al. [16]. The strip technique was employed by Mukhopadhyay et al. [11, 12] to generate out-of-plane ply waviness, or wrinkle, in the adjacent layers. The severity of the created defect can therefore be controlled by the thickness and the position of the added strips. However, this technique presents the inconvenience of slightly modifying the defect type in the sense that it requires introduction of a piece of material where a natural defect would not have one. A post-cure fiber waviness generated by this technique is presented in the micrograph in Figure 2.4. Similarly to Sawicki and Minguet [7], the authors noticed that not only does the defect influence the laminate tensile and compressive strength but,
depending on its severity, it can also modify the failure mechanism. Tensile and compressive failure strength for different wrinkle severities are presented in Figure 2.5a and Figure 2.5b, respectively. As previously stated, a severity threshold above which the defect severity has no meaningful influence on the material properties was observed in compressive test. Such phenomenon was not observed with tensile tests for which the strength was reduced by 4% in the less severe configuration and up to 23% in the worst case. Additionally, tensile specimens containing the two most serious defects also demonstrated a small load drop long before final failure of the laminate. That drop was associated with the initiation of matrix cracks and delamination between 45° and 90° layers in the vicinity of the waviness. That observation comes as a confirmation of the previous conclusion that defects mainly act as damage initiation zones.

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**Table 5. Open Hole Compression Test Results**

<table>
<thead>
<tr>
<th>Laminate ID</th>
<th>Environment</th>
<th>Nominal Lap/Gap Width [in]</th>
<th>No. of Tests</th>
<th>Mean Compression Failure Strain [µε]</th>
<th>Coefficient of Variation [%]</th>
<th>Percent of Mean Baseline Strain [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>OL1</td>
<td>RTD</td>
<td>0.00</td>
<td>5</td>
<td>5488</td>
<td>5.32</td>
<td>100.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.03</td>
<td>5</td>
<td>4851</td>
<td>3.19</td>
<td>88.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.10</td>
<td>5</td>
<td>4966</td>
<td>5.79</td>
<td>90.5</td>
</tr>
<tr>
<td>OL1</td>
<td>180°F/Wet</td>
<td>0.00</td>
<td>5</td>
<td>4737</td>
<td>4.84</td>
<td>100.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.03</td>
<td>5</td>
<td>4040</td>
<td>3.08</td>
<td>85.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.10</td>
<td>5</td>
<td>3459</td>
<td>3.00</td>
<td>73.0</td>
</tr>
</tbody>
</table>

Figure 2.3: OHC and compression specimens with gaps
Figure 2.4: Out-of-plane fiber waviness from [11]

(a) Influence of wrinkle severity on Tensile failure strength [11]  
(b) Influence of wrinkle severity on Compressive failure strength [12]

Figure 2.5: Influence of wrinkles on tensile and compressive strengths

The experimental work presented in the previously mentioned articles illustrate the large variety of parameters, such as defect type, location, their size and number, fiber orientation of the layer with a defect, the stacking sequence of the laminate, and local stress state associated with the applied loads, which can influence the effect of defect in composite laminates. Therefore, the number of possible combinations can be very large and performing experiments to address the effect of defects for all the parameters would be prohibitively expensive. One can choose from two options: creating a well thought-through Design of Experiment or developing a generic computational tool capable to represent as many configurations as possible. Both can help reduce the number of experimental testing necessary to understand the influence of defect on a
broader scale. That observation is the reason that has led authors to develop finite element models for gaps/overlaps [13, 14, 26] or other defect types such as wrinkles [11, 12]. In addition, switching from an experimentally governed study of the influence of AFP induced defects towards a numerical modelling solution opens the possibility to scale-up towards test on elements, details, sub-component or components of the building block approach presented in Figure 2.6.

![Building block approach](image)

Figure 2.6: Building block approach [27]

The following will present the most commonly used geometric representation of AFP induced defect using the finite element method. In most cases, it is observed that the authors decided to base their model on a semi-empirical representation of the defect geometry. A parametrized model is generally designed from observation of micrographs of composite laminates containing representative defects. For example, geometry and dimensions of the model developed by Mukhopadhyay et al. [11, 12] are based on measurements made on micrographs of defective laminates containing wrinkles generated by the introduction of a 90° strip of material between two adjacent layers. A cosine function was selected to approximate the shape of the wavy ply. That approximation was combined with a linearly decreasing variable that is used to
model the subsequent layer’s waviness magnitude by progressively reducing it until it reaches zero at the laminate top surface. It must be noted that a flat top surface implicitly implies the use of hard tooling as identified by Lan et al. [9]. In order to model the geometry computationally, through the thickness locations of nodes in a 3D finite element mesh of a pristine specimen are then modified to match with the analytical model by using a MATLAB script. The script also introduces cohesive elements at the plies’ interface to capture delamination. An interesting note on that modelling approach is the fact that it allows for an easy classification of defects severity through the misalignment angle of the first wavy ply. A numerical value of the defect severity was obtained with the model developed in the form of a through-the-thickness deviation angle which was then compared to measurement made on micrographs as shown in Figure 2.4.

A similar modelling procedure was proposed by Li et al. [14] on gaps and overlaps. At first, a generic representation of gaps and overlaps was characterized. This skeleton of defects’ model was respectively divided into four and two different regions for gaps and overlaps, respectively, as shown in Figure 2.7. For gaps, a region of minimum thickness, constituted only of resin, forms the center of the defect. Going towards the edge of the original gap, a resin pocket of length $R_{gap}$ serves as a transition between the thin resin rich area and a somewhat thicker region formed by the resin flow from the thickest section of the ply. Overlaps, on the other hand, only exhibit two distinct regions. One being the overlapping region of constant thickness and the second being the transition area. In both cases the thickness transition between the nominal and the defective regions is approximated by a cosine function. The parameters of the defect skeleton are measured from micrographs of reference specimens containing 2mm long gaps and overlaps. Representing the thickness variation with a cosine function is a common approximation shared by Mukhopadhyay [11, 12], Li et al. [14] as well as Davidson and Waas [28] who used that shape as a resin-insert to generate
out-of-plane waviness in thick laminates tested in compression. In a second step, Li et al. [14] created a tool which takes the stacking sequence and a list of defect types and their location as an input to generate a 3D finite element model with cohesive elements at the plies interfaces. In addition, to address the issue observed with structural mesh elements, a special unit cell mesh was developed in order to capture damage propagation in off-axis plies. The cell includes areas that can potentially experience transverse splitting along the fiber directions of the laminate stacking sequence. Additionally, in the case of unconventional laminates, the unit cell’s aspect ratio can be modified to capture elements splitting in directions others than the conventional zero, ninety- and forty-five-degree angles.

Once the geometric aspect of the defective laminate has been covered, the remaining part of the model consists of representing the influence of the defect on the material properties and their degradation. As it was the case for the geometrical representation, modelling of material property degradation has significantly overlapping features from one publication to the other. However, several degrees of approximation can be identified. In their work, Marouene et al. [13] studied the influence of gaps and overlaps on OHC tests by changing the defect location and orientation relatively to the hole (see Figure 2.8). In this case, a first order approximation was proposed by the authors in which gaps were modeled by changing the material properties to

Figure 2.7: Skeleton of gaps/overlaps models [14]
an isotropic material representing the mechanical properties of resin. It was assumed that the laminate thickness was not locally modified by the defect. Overlaps, on the other hand, were modelled by considering only a local thickness variation due to the introduction of an additional tow but the material properties were kept similar to the nominal ply. On a more detailed level, Li et al. [14] proposed to locally modify the Young’s modulus in the fibers direction \((E_{11})\) based on the fiber volume fraction variation caused by the flow of resin from high to low pressure zones. In other words, based on the resin flow from bigger laminate thickness (overlaps) to smaller laminate thickness areas (gaps). Ferreira et al. [29] proposed an intermediate solution where the global geometry of the laminate remains straight, which allows the use of straight finite elements. The fiber orientation was, however, modified through the definition of material properties. The geometric aspect of the defect presented in [11, 12, 14] is therefore neglected but this approach allows for a faster tool that can more easily be parametrized.

Figure 2.8: Various configurations of gaps/overlaps in OHC specimens [14]
As mentioned in the introduction of this section, numerical models can be used to study large scale components. Wilden et al. [30] conducted an extensive manufacturing program to assess the costs and risks of two manufacturing solutions for large subcomponents. Such as 7’ by 10’ panels with windows close-outs among others. Even though closely monitored, evaluation of the influence of band and tow gap/overlap (presented in Figure 2.9) on the part performance were not conducted even though FE models had been developed [31]. Due to the financial and time costs of manufacturing large scale subcomponents, experimental tests are usually limited in number. However, a parametrized numerical representation of the tested configuration offers the possibility to convert the recurrent costs of building parts into a non-recurrent building cost of developing the model. Validation data is still necessary to ensure the model accuracy is acceptable and a more in-depth investigation can thereafter be conducted.

![Figure 2.9: Band and tow gap/overlap definition [30]](image)

At a lower scale than in NASA’s reports presented above, Cairns et al. [32] studied the influence of inhomogeneities on the strain field and the damage progression in notched specimens. A local representation of the notch and inhomogeneities is presented in Figure 2.10. A global/local finite element model was developed. The global model was used to capture the influence of the notch and the displacement field at the boundary with the local model was used as a boundary condition. Similarly to Marouene [33], local representations of inhomogeneities such as gaps and overlaps.
represented by the darkened areas in Figure 2.10 were modeled by a resin region or a double thickness region, respectively. A moderate influence on the strain field was observed, even for a critical case of a grid distribution combined by a severe modification of material properties as described previously. However, a potentially high influence on damage progression was observed as larger damage zones were identified through dye-enhanced radiographies. These damage regions were associated with splitting and delaminations.

Figure 2.10: Notched specimen containing inhomogeneities [32]

To conclude the discussion on the influence of AFP induced defect on composite laminates’ quasi-static behavior, the following main aspects can be noted. The first logical step of unidirectional coupon configurations in tension has shown little to no influence on composite laminates static strength of simple defects such as gaps and overlaps. This observation comes with no surprise since it only consists of adding or removing a really small portion of the cross-sectional area. In addition, the consolidation during the curing process of unidirectional laminates is not affected as much as it is in other laminates. Defects in a non-unidirectional laminate can indeed result in the creation of out-of-plane waviness of load carrying plies as shown in the work conducted by Sawicki and Minguet [7]. On the other hand, specimens tested in compression displayed a much higher sensitivity to defects. The most plausible cause being that compressive tests are known for being highly affected by
geometrical imperfections. As micrographs of cross-sections of defective laminates showed, defects in a layer act as a source of out-of-plane geometrical imperfections. This contributes to the formation of kink-bands during compression tests. Hence the substantial difference observed on compressive static strength. Finally, from the previously mentioned articles, it can be observed that no global conclusion can be drawn on the influence of defects on notched laminates. It is indeed observed that, depending on the defect type and its relative position with respect to the stress concentration, a reduction or an amplification of the static strength may be expected. However, it is worth noticing that the variations in strength were significantly higher on notched laminates under compression than on unnotched specimens.

2.3 Modelling Fatigue Behavior of Composite Laminates

The next logical step after studying the influence of defects on the quasi-static response of laminate is to focus on fatigue. However, before presenting the work conducted on the influence of AFP defect on the fatigue behavior, a review of experimental and numerical investigations on pristine composite laminates’ fatigue life is beneficial. Composite materials, due to their inhomogeneity and anisotropy, combine transverse matrix cracks, delamination, fiber-matrix interface failure and other types of damages that can grow at different rates and eventually coalesce. Therefore, damage mechanisms in composite material subject to cyclic loading are considerably more complex than in isotropic material where a single crack progression occurs. The vast amount of publications focusing on metals fatigue behavior demonstrate the complexity to be expected for composite material fatigue modelling. To fully capture the fatigue behavior of composite laminates, various researchers have developed several modelling techniques either based on experimental results or understanding of failure
mechanisms to implement appropriate failure criteria. The modelling solutions can be classified into three categories [34]: fatigue life models, residual strength/stiffness models, and progressive fatigue damage models. Each of them will be presented in detail in the following.

2.3.1 Fatigue Life Models

One of the three modelling techniques developed for composite laminates fatigue behavior is the fatigue life model. This technique relies on S-N curves or Goodman diagrams to identify the ultimate strength of a laminate as a function of the number of cycles. It is one of the simplest models as it does not require a micromechanical understanding of failure modes. The model relies on an extensive amount of experimental testing since S-N curves depend on a large number of parameters such as the laminate stacking sequence, the stress ratio R, the load condition (tension, compression, shear, etc.) and the material properties. Therefore, the models falling in that category are focusing on a specific subset of configurations to reduce the amount of testing needed.

In their work, Hashin and Rotem [35] identified two major failure mechanism from the observation of quasi-static tests conducted on unidirectional laminates: fiber failure and matrix failure for which a crack propagates in the matrix in the direction of fibers. This information was used later on to design a quadratic failure criterion presented in equation 2.3.1a where $\sigma_A$, $\sigma_T$, $\tau$, $\sigma^U_A$, $\sigma^U_T$, and $\tau^U$ are the axial transverse and shear stress and the axial, transverse and shear ultimate strengths, respectively, determined in a previous step

\[ \sigma_A = \sigma^U_A \quad \text{(2.3.1a)} \]
\[
\left( \frac{\sigma^U_A}{\sigma_T} \right)^2 + \left( \frac{\tau}{\tau^U} \right)^2 = 1 \quad \text{(2.3.1b)}
\]
Where, equation 2.3.1a is used as the fiber failure criterion and equation 2.3.1b is the matrix failure criterion. The model provided results in good agreement with experimental results. However, it must be emphasized that the fatigue life models, as they rely on extensive experimental results, have to be restricted to a specific subset of configurations of cycle stress ratio (R), frequency and stacking sequence. As a matter of fact, even the authors emphasize the fact that one cannot expect the failure criteria presented above to be used as a way to down select a stacking sequence based on desired fatigue properties. It must rather be seen as a tool to control the fatigue characteristics once the stacking sequence has been selected.

Ellyin and El-Kadi [35] also proposed a fatigue life model based on experimental data. However, their failure criterion was based on the strain energy density used as a damage function. In a previous work [36], Ellyin came to the conclusion that the total energy input can be related to the laminate fatigue life by a power of the form presented in equation 2.3.2 where $\kappa$, $\alpha$, and $C$ are materials constants obtained experimentally, and $\Delta W^t$ and $N_f$ are the total energy and the number of cycles to failure, respectively.

$$\Delta W^t = \kappa N_f^\alpha + C \quad (2.3.2)$$

Since the authors were focusing on in-plane stress state, an analytical expression of the strain energy in the case of uniaxial loading can be obtained in the following form, equation 2.3.3:

$$\Delta W = \bar{S}_{11} \left[ \frac{\Delta \sigma_x^2}{2(1 - R_x)^2} \right] \quad (2.3.3)$$

where $\bar{S}_{11}$ is the first term of the compliance matrix and $R_x$ is the stress ratio for a unidirectional loading in the longitudinal direction $x$ defined as:

$$R_x = \frac{\sigma_x^{min}}{\sigma_x^{max}} \quad (2.3.4)$$
By combining equations 2.3.2 and 2.3.3 and by assuming a value of zero for C, a relation between the laminate fatigue life and the applied stress state can be expressed as follows (equation 2.3.5):

$$\bar{S}_{11} \left[ \frac{\Delta \sigma_x^2}{2(1 - R_x)^2} \right] = \kappa N_f^\alpha$$

(2.3.5)

A comparison between the model presented above and experimental results from Hashin and Rotem [35] was then conducted by the authors and a good match with experimental data was observed as can be seen on Figure 2.11. Even if the experimental and numerical results are in good agreement, this model is inconvenient, as it was already the case for the one presented by Hashin and Rotem [35], since it dependent on experimental data. Indeed, in this model updated values of $\kappa$, $\alpha$, and $C$ are needed depending on the material used, the fibers orientation and the stress ratio.

Figure 2.11: Correlation of the strain energy density $\Delta W$ calculated with the experimental data

In their work, Wu and Yao [37] proposed a damage model based on the observation that the evolution of damage during fatigue testing of composite laminates is non-linear. The damage evolution history is presented on Figure 2.12 as identified by Reifsnider et al. [38]. Initially, the damage occurrence rate is relatively high since it
only consists of non-interacting matrix cracks occurring in off-axis plies. Past a certain threshold of matrix crack density, or critical crack density, the damage phenomenon will transition towards delamination and some fiber breakage will occur at the tip of matrix cracks due to the stress concentration they cause. During those two steps, a stress redistribution occurs where the load that can no longer be supported by the damaged plies and is progressively transferred to the other plies. Eventually, the combination of the statistical distribution of fibers strength and the critical crack density state lead to breakage of the weakest fibers as the load carrying capability of the fibers can no longer be transferred by the matrix around them. This leads to a stress redistribution across the laminate until final failure is reached. Generally, both the first and last step of the process present a high damage evolution rate in comparison to in-between cycling period.

![Fatigue damage evolution in composite laminates from [38]](image)

Figure 2.12: Fatigue damage evolution in composite laminates from [38]

To take that observation in consideration, Wu and Yao [37] proposed a non-linear expression of the damage variable in the form presented in equation 2.3.6:

$$D(n) = \frac{E_0 - E(n)}{E_0 - E_f s} = 1 - \left(1 - \left(\frac{n}{N_f}\right)^B\right)^A \quad (2.3.6)$$
In this expression, \( E_0 \) is the initial Young’s modulus, \( E_{fs} \) is the Young’s modulus at failure, \( E(n) \) is the residual Young’s modulus after \( n \) cycles, \( N_f \) is the laminate fatigue life, and \( A \) and \( B \) are experimental parameters. The authors then decided to make the assumption that the damage evolution rates for any given normalized life are the same which allows for a simplified expression of the constants \( A \) and \( B \) as shown in equation 2.3.7 in which \( p \), \( q \) and \( k \) are constants, and \( \sigma_{ult} \) is the ultimate strength in tension,

\[
A = pB + q \tag{2.3.7a}
\]

\[
B = k \frac{\log (N_f)}{(1 - R) \frac{\sigma_{max}}{\sigma_{ult}}} \tag{2.3.7b}
\]

The above model is then verified against experimental data gathered from other publications where constant amplitude loading tests were conducted on laminate with different stacking sequences and under a range of stress ratios \( R \). To evaluate the damage evolution with the above model, the authors had to model the S-N curve with the following equation 2.3.8, where \( a \), \( b \) and \( m \) are experimental parameters,

\[
S = 1 + m \left( e^{-\left( \frac{\log N_f}{b} \right)^a} - 1 \right) \tag{2.3.8}
\]

A good agreement of the model with experimental results was observed. However, even if the results were satisfying, this work emphasizes once more the fact that fatigue life models are relying on a significant number of experimental tests and make use of several parameters, typically obtained through fitting experimental data. In this particular case [37], no less than thirteen sets of experimental data were necessary to demonstrate the model validity in various configuration of stacking sequences, load configuration and stress ratio \( R \).
2.3.2 Residual Strength/Stiffness Models

As mentioned previously, damages in composite laminates under fatigue loading are complex and therefore a phenomenological understanding of damages’ initiation and propagation may be overlooked. However, it is a well-known fact that damages, as the number of applied cycles increases, will influence the material properties of the laminate. Keeping track of these material property degradations can provide a good estimation of the fatigue state of the laminate. Several authors [39–46] proposed models based either on stiffness or strength degradation. The latter having the advantage of explicitly influencing the failure criteria since residual strength-based models use a slightly modified conventional quasi-static failure criteria. The slight modification being that the material strength is no longer equal to the static strength but is dependent on the number of cycles already applied to the laminate. In other words, the material properties are progressively degraded as the number of cycles increases. During the loading process of multidirectional laminates, one can indeed observe the creation of matrix cracks oriented within the fiber direction in transverse and off-axis plies. The laminate can therefore be considered as a laminate with reduced material properties in every cycle for which a modified failure criterion can be applied. Consequently, this can be seen as considering a different failure criterion for every cycle of the laminate fatigue life. This way, the state of the structure can be related directly with the most important quantity, the strength. This also makes for a simple failure condition or criterion: In an S-N curve scenario, when the applied stress equals the residual strength, failure occurs. Although it is relatively easy to understand and implement, residual strength models present two major drawbacks. The first being that the tested specimen will be destroyed in the process when industrial applications are more likely to be looking for a non-destructive testing technique. Secondly, this method requires a significant amount of experimental testing. The residual strength is indeed dependent on the number of cycles applied but also on the laminate stacking
sequence, the stress ratio \( R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} \) or the loading spectrum. In addition, it is not straightforward to evaluate the residual strength of a specimen directly. This can significantly increase the cost of testing in the case of complex design or for specimens in high categories of the building block approach. A potential solution is to monitor a critical parameter whose evolution can be related to that of the residual strength through coupon test. Another potential way around the difficulty with models based on residual strength was proposed in the form of residual stiffness models, which are based on the degradation of a material property that can easily be measured without damaging the specimen. The stiffness degradation as a function of the number of cycles applied is present in Figure 2.13 from the work of Hwang and Han [45]. The laminate residual stiffness or fatigue modulus is equal to the slope of the line joining the origin 0 to the point \( n' \) where \( n' \) is the number of cycles applied.

![Figure 2.13: Fatigue modulus degradation as a function of n from [45]](image)

The above representation of the fatigue modulus is the starting point of Hwang and Han’s model definition [45]. The authors first assumed a linear relationship between the applied stress and the resultant strain through the fatigue modulus. Then they assumed that the fatigue modulus degradation rate follows a power law as presented in equation 2.3.9a, where \( A \) and \( c \) are material constants and \( F_0 \) and \( F_f \) are the initial
fatigue modulus and the fatigue modulus at failure, respectively. The initial fatigue modulus is assumed to be the static Young’s modulus $E_0$. Finally by applying a strain based failure criterion for which the laminate final failure is assumed to happen when the fatigue resultant strain equates the static failure strain, the authors obtain an expression for the laminate fatigue life as presented in equation 2.3.9b, where $B = \frac{F_0}{A}$,

$$r = \frac{\sigma_u}{\sigma_a}$$

with $\sigma_u$ is the ultimate strength and $\sigma_a$ is the applied stress;

$$F(n) = F(0) - An^c$$ \hspace{1cm} (2.3.9a)

$$N_f = [B (1 - r)]^{\left(\frac{1}{r}\right)}$$ \hspace{1cm} (2.3.9b)

The use of degradation of material properties such as stiffness and strength based on damage occurrences in composite laminate was also developed and applied to Finite Element Analysis (FEA), as it is presented by Lian and Yao [44]. In their work, the authors proposed to use a set of three failure criteria modified to take the residual strength in place of nominal strength as it is the case in criteria used for quasi-static tests. Each criterion is used to capture the occurrence of one of the three identified modes of failure: longitudinal fatigue failure (fiber breakage), matrix fatigue failure and in-plane shear fatigue failure. A stiffness degradation rule is then proposed. This rule is to be applied to each of the three principal axes and is presented in equation 2.3.10. In the following, $E(n)$ is the residual stiffness after n cycles, $E_0$ is the initial stiffness, $E_{fs}$ is the stiffness at failure or, in other words, a critical stiffness and $u, v$ and $a$ are experimental parameters,

$$\frac{E(n)}{E_0} = 1 - \left(1 - \frac{E_{fs}}{E_0}\right) \left(\left(\frac{n}{N_f}\right)^u + a \left(\frac{n}{N_f}\right)^v\right)$$ \hspace{1cm} (2.3.10)
As they also intended to introduce a strength reduction rule, the authors proposed a way around the non-repeatability of the process used to identify the residual strength. It is indeed necessary to break the specimen during the test, which precludes from getting the properties reduction in the two remaining principal axes. However, the observation that both stiffness and strength degradation are caused by the same manifestation of damage led the authors to propose to relate the two damage function by a power law presented in equation 2.3.11a, where $w$ is an experimental parameter. That relation, when combined with equation 2.3.10 provides an expression for the residual strength as presented in equation 2.3.11b, where $S_0$ is the initial strength and $S_f$ is the strength at failure:

$$D_S = D_E^w$$  \hspace{2cm} (2.3.11a)

$$S(n) = S_0 - (S_0 - S_f) \left( \frac{(n/N_f)^w + a (n/N_f)^v}{1 + a} \right)^w$$  \hspace{2cm} (2.3.11b)

The rules of gradual degradation of material properties presented above intend to model the degradation of the laminate that occurs during the loading cycles before any failure arises. That progressive degradation of the material is depicted in the aforementioned failure criteria. After a certain amount of cycles, some finite elements will activate those failure criteria but that does not necessarily mean the final failure of the laminate. Therefore, in a similar manner that it is done for progressive damage analyses, a strength and stiffness discount method are proposed by the authors. It consists of locally and suddenly reducing the material properties depending on the type of failure mechanism triggered. A summary of the sudden degradations is presented in Table 2.1 where a superscript $r$ denotes a residual variable and a superscript 0
refers to initial properties. Given the difficulty to obtain experimental values for \( \nu_{23} \) and \( G_{23} \), the authors proposed to use Christensen’s work [47] to evaluate their value (equations 2.3.12a and 2.3.12b), in which \( c \) is a discount factor taken equal to the small value of 0.05 since it cannot be equal to zero to avoid computational issues.

Table 2.1: Sudden properties degradation rules from [44]

<table>
<thead>
<tr>
<th>Failure mode</th>
<th>( E_{11} )</th>
<th>( E_{22} )</th>
<th>( E_{33} )</th>
<th>( \nu_{12} )</th>
<th>( \nu_{13} )</th>
<th>( G_{12} )</th>
<th>( G_{13} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber failure</td>
<td>( cE_{11}^0 )</td>
<td>( cE_{22}^0 )</td>
<td>( cE_{33}^0 )</td>
<td>( c\nu_{12}^0 )</td>
<td>( c\nu_{13}^0 )</td>
<td>( cG_{12}^0 )</td>
<td>( cG_{13}^0 )</td>
</tr>
<tr>
<td>Matrix failure</td>
<td>( E_{11}^r )</td>
<td>( cE_{22}^0 )</td>
<td>( E_{33}^r )</td>
<td>( \nu_{12}^0 )</td>
<td>( \nu_{13}^0 )</td>
<td>0.6( G_{12}^0 )</td>
<td>( G_{13}^0 )</td>
</tr>
<tr>
<td>In-plane shear failure</td>
<td>( E_{11}^r )</td>
<td>( cE_{22}^0 )</td>
<td>( E_{33}^r )</td>
<td>( \nu_{12}^0 )</td>
<td>( \nu_{13}^0 )</td>
<td>( cG_{12}^0 )</td>
<td>( G_{13}^0 )</td>
</tr>
</tbody>
</table>

\[
\nu_{23} = \frac{\nu_{12}(1 - \nu_{12} E_{22})}{(1 - \nu_{12})} \quad (2.3.12a)
\]

\[
G_{23} = \frac{E_{22}}{(2(1 + \nu_{23}))} \quad (2.3.12b)
\]

The coefficients not mentioned in Table 2.1 are evaluated by assuming that the material still follows Maxwell-Betti’s reciprocity relations after it fails in one direction [equations 2.3.13a to 2.3.13c]:

\[
\frac{\nu_{12}}{E_{11}^r} = \frac{\nu_{21}}{E_{22}} \quad (2.3.13a)
\]

\[
\frac{\nu_{13}}{E_{11}^r} = \frac{\nu_{31}}{E_{33}} \quad (2.3.13b)
\]

\[
\frac{\nu_{23}}{E_{22}^r} = \frac{\nu_{32}}{E_{33}} \quad (2.3.13c)
\]
The abovementioned model was then compared with experimental results for six different stacking sequences subject to a tension-tension fatigue stress with a stress ratio $R = 0$. The six stacking sequence were: $[90]_s$, $[0]_s$, $[0/90/90/0]_s$, $[45/-45/45/-45]_s$, $[45/0_2/-45]_s$ and $[45/90/−45/0]_s$. With the exception of the quasi-isotropic laminate, all the simulation data match well with the experimental data. The mean fatigue life estimations were in good agreement with the observation made during testing. The only major difference observed between the model and the experiments was that, by introducing a random material properties distribution to model an “as-manufactured” laminate containing imperfections, a larger scatter in numerical results was observed. As for the quasi-isotropic laminate, the model proposed only matched with the reality of experiments for high stress ratio where the fatigue life is relatively short. For low stress ratio, delaminations were observed in the early stage of experimentation but the model does not include delamination. Hence the discrepancy between simulation and experimental results.

Even though stiffness based models are attractive because it is easy to monitor the stiffness of a structure or specimen and estimate the remaining life, they suffer from the fact that, over most of the fatigue life, except at the very beginning and very end, the stiffness degradation is very gradual. This is illustrated by the work conducted by Simonds et al. [48] who identified three stages in the residual stiffness degradation of composite laminates under fatigue as shown in Figure 2.14. A degradation of 5 to 15% during both stage I and stage II were observed but stage I is limited to roughly 15% of the fatigue life when stage II spans over 60 to 70% of the specimen fatigue life. Thus, it is not known if a measured stiffness reduction is related to fatigue or to the relatively large experimental scatter fatigue tests have.
In his work, Kassapoglou [41] proposed a strength reduction model based on the initial assumption that the residual strength is the solution of a first order linear differential equation. This equation was solved with the following boundary conditions: the residual strength is equal to the static strength when no cycles have been applied, the residual strength at the end of the penultimate cycle is equal to the maximum applied stress in the cycle, and the residual strength converges to the endurance limit. This results in the expression of the residual strength presented in equation 2.3.14. In these equations $S(n)$ is the residual strength after $n$ cycles, $S_{fs}$ is the static strength and $\sigma_E$ is the endurance limit. The endurance limit is the applied stress level below which no fatigue failure is observed regardless of the number of cycles applied.

$$S(n) = \sigma_E + (S_{fs} - \sigma_E) \left( \frac{\sigma - \sigma_E}{S_{fs} - \sigma_E} \right)^{N_f-1} \quad (2.3.14)$$

By assuming that the endurance limit to be equal to zero, that only one type of damage occurs during the fatigue life of the laminate, and that the static strength can be modeled by a two-parameter Weibull distribution, the author demonstrated that the residual strength also follows a two-parameters Weibull distribution with parameters that are related to those of the static strength. From that distribution, an expression of the number of cycles-to-failure is determined based on equation 2.3.15,
\[ N_f = \frac{-1}{\ln(1 - p)} \] (2.3.15)

In equation 2.3.16 \( N_f \) is the number of cycles-to-failure for a stress \( \sigma \), and \( p \) is the probability that the strength of the given specimen is less than \( \sigma \). This value has been proven by the author to be independent from the number of cycle \( n \) under the assumption of a unique type of damage. This means that \( p \) is equal to its value for the static strength population. By assuming that \( p \) takes a sufficiently small value, an expression of the stress required for the laminate to fail at \( N_f \) cycles is obtained with equation 2.3.16.

\[ \sigma = \frac{\beta}{N_f^\alpha} \] (2.3.16)

Equation 2.3.16 is a function of the two Weibull parameters of the static strength distribution: \( \alpha \) the shape parameter and \( \beta \) the scale parameter. While equation 2.3.16 describes the entire population of fatigue specimens, in order to apply it to individual specimens, beta must be changed to the static strength of the specimen in question. Or, referring to the “mean” specimen, beta is replaced by the mean of the static strength distribution. A corrected expression for the S-N curve is presented in equation 2.3.17, where \( X_m \) is the mean of the static strength distribution.

\[ \sigma = \frac{X_m}{N_f^\alpha} \] (2.3.17)

Equations 2.3.15-2.3.17 are valid for a stress ratio \( R=0 \). For any other stress ratio, they are modified [41] to account for the effect of load cycle not returning to zero and for differences between tension and compression part of the cycle. Contrary to all the models presented in this section, the work proposed by Kassapoglou [41] presents the significant advantage of not requiring expensive and time consuming data from fatigue tests since the expression of the S-N curve presented above only
relies on the Weibull distribution parameters of the static strength. “The comparison of the analytical predictions of the model to test results taken from the literature showed that the accuracy of the model ranged from poor to excellent depending on the loading case” [41]. The main explanation being the initial assumption that only one type of damage occurs during the laminate fatigue life. This is not an issue for relatively simple stacking sequences and loading conditions but can be of significant influence when a more complex situation is studied. An improved technique capable of taking into consideration several damage types is proposed in the last part of the author’s work. The promising results of the initial model and the possibility to evaluate both the residual strength and laminates fatigue life only from data on static strength are encouraging signs that the proposed framework of improvement will provide an efficient tool. Alternatively, Kassapoglou [49] proposed and compared to the previously presented non-linear residual strength degradation law, a linear model based on the assumption that the residual strength takes the functional form presented in equation 2.3.18:

$$S(n) = S_{fs} + g(\sigma, n, N_f)$$

(2.3.18)

The expressions of the residual strength after \(n_1\) and \(n_2\) cycles for a constant applied stress \(\sigma\) lead to the conclusion that \(g\) is a solution of Cauchy’s functional equation (equation 2.3.19a). That observation combined with the boundary condition mentioned previously yield a linear relation between the residual strength and the number of cycle \(n\) as presented in equation 2.3.19b:

$$g(n_2) - g(n_1) = g(n_2 - n_1),$$

(2.3.19a)

$$\frac{S(n)}{S_{fs}} = 1 - \left(1 - \frac{\sigma}{S_{fs}}\right) \cdot \frac{n}{(N_f - 1)}$$

(2.3.19b)
The author then compared the results of the linear model to those of the non-linear model (continuous line and dashed line respectively in Figure 2.15a and Figure 2.15b) for different load ratios R. It can be observed that the higher the load ratio is, the more the two models agree. In fact, the author points out that for load ratios higher than 65% it is virtually impossible to distinguish the two curves. This is particularly interesting as, from a designer point of view, aiming for a higher load ratio is a must-go to ensure an efficient weight to load carrying capability. In addition, the linear model can provide an estimation of the laminate residual strength degradation which, in an engineering sense, can be satisfactory, allowing for the definition of a maintenance inspection schedule.

(a) $\sigma/\sigma_{fs} = 0.2$ [8]  
(b) $\sigma/\sigma_{fs} = 0.5$ [8]

Figure 2.15: Residual strength prediction comparison

2.3.3 Progressive Fatigue Damage Models

As underlined in the previous sections on fatigue life and residual strength/stiffness models, modelling capabilities used in these methods heavily rely on empirical data. However, it is the authors’ belief that modelling capabilities should serve as a cost reduction (monetary and time) tool in providing numerical, parametric, and reliable simulation capabilities for structures, components, and sub-components of the testing building blocks. The design space for the selection of elementary coupons of the building block approach is already significant and going higher up the building block towards structural parts significantly increases the dimension of the design spaces.
available during the selection process. This observation combined with the need of experimental data is a serious limiting factor to the use of fatigue life and residual strength/stiffness models which has led to the development of Progressive Damage Analysis (PDA) models by numerous authors [50–62]. PDA models were initially designed for capturing the initiation and propagation of damages in parts tested under quasi-static loading. They rely on three governing components: stress analysis, failure analysis, and material properties degradation laws. For more information on the state-of-the-art PDA modelling techniques, the interested reader should refer to Dávila et al.’s extensive review of analysis methods for progressive damage in composite structures [63].

In their work, Shokrieh and Lessard [61, 62] proposed an adaptation of the above-mentioned PDA model towards what they called a Progressive Fatigue Damage Analysis (PFDA) model. A similar concept was proposed by Eliopoulos et al. [58, 59]. Shokrieh and Lessard proposed a comparison of a traditional progressive damage method’s flowchart to the flowchart of a progressive fatigue damage model as presented in Figure 2.16a and Figure 2.16b, respectively. As shown, the three-governing components of a PDA model mentioned above are still represented, and the main differences resides in the material properties degradation laws. In PFDA a distinction is made between sudden material properties degradation, which is associated with the failure modes captured through the failure analysis and the selected failure criterion, and a gradual material properties degradation law, which correlates strength and stiffness degradation to the number of load cycle applied.

The above mentioned residual strength and stiffness degradation law developed by Shokrieh and Lessard [61, 62] are presented in equations 2.3.20a and 2.3.20b respectively. The fatigue life model is presented in 2.3.20c. Where \( S(n,\sigma,\kappa) \) and \( E(n,\sigma,\kappa) \) are the residual strength and stiffness respectively. \( S_{fs} \) and \( E_{fs} \) are the static strength and stiffness, respectively, and \( n \) is the number of cycles applied, \( \sigma \) is
(a) Flowchart of a progressive damage model [55]  
(b) Flowchart of a progressive fatigue damage model [55]

Figure 2.16: Flowcharts of (a) PDM and (b) PFDM

maximum amplitude of the applied stress, $\epsilon_f$ is the average strain to failure, $N_f$ is the fatigue life, and $\kappa = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}}$ is the load ratio. Finally, $\alpha, \beta, \lambda, \gamma, \mu$ and $u$ are experimental curve fitting parameters obtained by testing unidirectional ply under uniaxial fatigue and quasi-static load as presented by the authors in [64, 65].

$$S(n, \sigma, \kappa) = \left(1 - \left(\frac{\log(n) - \log(0.25)}{\log(N_f) - \log(0.25)}\right)^{\frac{1}{\alpha}}\right) \cdot (S_{fs} - \sigma) + \sigma \quad (2.3.20a)$$

$$E(n, \sigma, \kappa) = \left(1 - \left(\frac{\log(n) - \log(0.25)}{\log(N_f) - \log(0.25)}\right)^{\frac{1}{\gamma}}\right) \cdot (E_{fs} - \frac{\sigma}{\epsilon_f}) + \frac{\sigma}{\epsilon_f} \quad (2.3.20b)$$

$$u = \frac{\ln(a/f)}{\ln((1-q) \cdot (c+q))} = A + B \cdot \log(N_f) \quad (2.3.20c)$$

A comparison of experimental results to numerical simulation for fatigue test of a pin and bolt loaded cross-ply $[0/90]_s$, angle-ply $[+45/ - 45]_s$ and quasi-isotropic $[0/\pm 45/90]_s$ laminates were conducted by the authors [62]. The PFDA model results for bolt and pin loaded cross-ply laminates were in good agreement with experiments.
They slightly over-estimated the laminate fatigue life but were still within the scatter range of the experimental results. A similar observation was made for angle-ply laminates. The comparison with the quasi-isotropic laminates was conducted based on experimental data extracted from the publication of other authors. The pin loaded configuration yielded satisfactory results, but the bolt loaded model highly over-estimated the fatigue life. The main explanation is provided by Shokrieh and Lessard as being a disagreement in the definition of the final failure criterion. It must, however, be noted that, even if experimental and numerical results generally are in good agreement, equations 2.3.20a to 2.3.20c showing the expressions of the residual strength, the residual stiffness and the fatigue life $N_f$ are still relying on an experimental program that, even though limited to unidirectional laminates, remains nonetheless extensive if not deterrent.

Other proposed models such as the Progressive failure simulation developed by Iarve et al. [56] or the mixed-mode cohesive model presented by Dávila [60] are introducing a cohesive zone model as the fatigue simulation algorithm flowchart presented in Figure 2.17 indicates. Dávila’s proposed law for fatigue cohesive elements [60] is presented in Figure 2.18. The initial base of the fatigue model is similar to the bilinear law used in quasi-static models. The first portion, from O to E, corresponds to an un-damaged state for which the maximum applied stress $\sigma_{\text{max}}$ remains below the critical stress $\sigma_c$ that would lead to the degradation of material properties in the cohesive zone. The second portion, from E to T, is the progressive degradation of the cohesive element until fracture at the point T. In the case of fatigue, if $\sigma_{\text{max}} < \sigma_c$, without modification of the cohesive law, no damage will ever be captured as the AEF portion in Figure 2.18 (a) is never reached. A fatigue damage factor $d^f$ is therefore included to progressively reduce the stiffness of the cohesive bond up until the element reaches failure. Failure in the fatigue case correspond to a displacement $\lambda^f$ (at point F).
2.3.4 Theory versus Experiments

Alternative ways of determining the residual strength exist. For instance, Whitney [66] proposed a statistical model for the residual strength for which the static strength is assumed to follow a two-parameter log-normal distribution and the residual strength to follow a three-parameter power law. Parameters that must be determined experimentally through tension-tension and tension-compression fatigue tests. S-N curves generated with this model for constant amplitude fatigue testing agreed well with experimental data. However, since delamination was not taken into account, the residual strength distribution is less encouraging. Whitney’s intuition that not taking delamination into account to estimate the residual strength of a laminate tested
under fatigue is the reason for that lack of accuracy is supported by Highsmith and Reifsnider’s [67] investigation on the load distribution that follows damage occurrence during fatigue loading. Whitney’s and Highsmith and Reifsnider’s investigations were guided by the belief that strength and stiffness reduction can be attributed to two main phenomena: the strength/stiffness reduction of each individual components and the load redistribution that follows a damage occurrence. The latter being considered by Highsmith and Reifsnider to be the main driving factor. To capture that load redistribution, perturbations in interferometric fringes obtained with a Moiré interferometer are observed. The authors identified, for a cross-ply laminate \([0/90_2]_s\), a first stage in the development of the Characteristic Damage State (CDS), as mentioned earlier in this section, which consists of transverse cracks in the center plies. This was followed by splitting in the zero-degree plies. Then, at the intersection of transverse cracks and longitudinal splitting, internal delamination occurred and grew. According to the authors’ findings, both longitudinal splitting and delamination were associated with a noticeable drop in the laminate stiffness. These observations of the damage chronology are consistent with the detailed classification of damage development under three phases as presented by Reifsnider et al. [68]. Even though material and stacking sequence dependent, the results reported by the authors give a broad idea of the damage phenomena, and their interactions, to be expected under tension-tension fatigue testing. And, to some extent, for compression-compression loading tests. The following was identified for a tension-tension loading case. The first phase mainly consists of matrix cracks development in off-axis plies resulting in local load redistribution in adjacent plies. This was associated, for the laminates studied by the authors, to an average 10% loss of stiffness. A high development of damage site was observed while the damage occurrence rate progressively reduced before transitioning to the second phase which presents a more constant damage occurrence rate. During the second phase, described as “a period of coupling and
growth” by the authors, local delamination and debonding occur due to the development of interlaminar tensile normal stress and interlaminar shear stress at cracks tip. The creation of local delamination implies that, locally, no lateral motion constraint between off-axis plies and zero-degree plies exist. Therefore, the zero-degree plies are locally returning to a uniaxial loading condition. Finally, during phase three, a sharp increase in the damage occurrence rate is noticed near the end of life of the laminate.

The main damage phenomenon is the development of the delaminations mentioned earlier, the connection of cracks in a stair-step like pattern through fiber debonding zones and fiber fractures. Fiber fractures were already observed during phase two, but the occurrence rate drastically increases. They initially develop near the specimen surfaces before moving in the laminate in the vicinity of matrix crack regions. Under compression-compression, the main damage mode identified is delamination which initiated from the exterior surfaces before moving in the laminate at locations of high interlaminar stresses. Interestingly, in their study on thick and notched laminates under compression-compression fatigue test, Black and Stinchcomb [69] noticed a wear-in phenomenon resulting in an increase of the residual strength of laminates subjected to low to medium load ratios. The strength increase was attributed to the development of damage in the stress concentration area that helped mitigate the stress concentration. An observation that is in line with the situations observed in tension-tension fatigue test of notched laminates ([70–72]) and was also identified by Marouene [13] on the quasi-static response of open-hole specimens under compression. A review of the theories developed for evaluating the residual strength of composite laminates after fatigue loading can be found in the work conducted by Philippidis et al. [73].
In section 2.2, a summary of the studies conducted on the influence of AFP induced defects on the quasi-static behavior of composite laminates was presented. It was concluded that, depending on the defect type, the load configuration, the stacking sequence of the laminate, and the relative position of the defects with respect to stress concentration inducers such as a hole, defects can provoke a reduction or an increase in strength. In addition, the need for a generic modelling tool capable of representing the broad variety of configurations was highlighted. In section 2.3, the focus was shifted on the fatigue response of laminates. The three main FE modelling techniques, fatigue life, residual strength/stiffness, and progressive fatigue damage models, were presented along with their drawbacks and advantages. In this section, research efforts conducted to understand and model the combined influence of manufacturing defects and cyclic loading will be presented. At first, results from experimental oriented publications will be presented, followed by description of numerical modelling solutions.

2.4.1 Experimental Investigations

As it was the case for the study of the influence of manufacturing defects on the quasi-static behavior of composite laminates, the influence of the aforementioned defects on the fatigue life and chronology of damage initiation and propagation in composite laminate was investigated experimentally in [74–83]. It is the author’s belief that some of the conclusions drawn from studies from non-AFP defects that are geometrically similar to those encountered with AFP operations can provide insightful information of damage initiation and propagation. Therefore, some of the references introduced in this section will not directly relate to AFP but to other manufacturing processes such as hand layup and resin infusion. A significant research effort was directed to understanding the influence of out-of-plane fiber waviness, or
wrinkle on fatigue response [75, 78, 79, 83]. This emphasis on wrinkle is motivated by several factors. First, as shown by Sawicki and Minguet [7], out-of-plane waviness has the potential of significantly affecting the quasi-static mechanical behavior of a laminate and therefore can potentially be of high importance in fatigue as well. The second reason is that out-of-plane waviness is, a defect type shared between different manufacturing techniques such as AFP [84], vacuum infusion [79], or Resin Transfer Molding (RTM) [78]. As a consequence, there are as many manufacturing techniques and specimens’ configurations as studies. For instance, Wang et al.’s approach [75] relies on two ply drops surrounding two middle plies as presented in Figure 2.19. The defect severity $\delta/\lambda$, where $\delta$ is the amplitude and $\lambda$ is the wavelength, can be controlled by adjusting the length of the gap created between the two discontinued plies. Another common approach consists of including a circular object such as a rod [79], wires combined with strips of unidirectional material [83], or polymer rods [78] to induce a waviness in the surrounding layers.

![Figure 2.19: Out-of-plane waviness using ply drops [75]](image)

Each method presents advantages and drawbacks. For instance, the use of ply drops proposed by Wang et al. [75] allows for an easy, repeatable, and consistent control of the defect severity by simply modifying the gap length. However, for the configuration presented on the left in Figure 2.19, the ply drop implies the abrupt termination of load carrying plies. Such ply-drops are not a concern when polymer rods are used as presented by Hörrmann et al. [78] in Figure 2.20. The defect
severity is then controlled through the distance $c$ between the two rods in the fiber direction and the radius $r$ of the introduced rods. The authors [78], however, warn the reader about the fact that “the distance $c$ between the locations of the outer layers undulations is not constant for the high pressure RTM process”. It must also be noted that the polymer rods introduced have, according to the numerical model for quasi-static testing created by the authors, different mechanical properties. Such a difference, as small as it might be, could be the source of an imperfect bonding process between the rod and the resin rich region surrounding it, leading to premature crack initiations. The use of copper wires and unidirectional strips of material as proposed by Adams [83] and presented in Figure 2.21a and Figure 2.21b offer a similar solution for thermoplastic material as the control parameters for the defect severity are the wires radii and their relative placement.

Figure 2.20: Use of polymer rods to create wrinkles. (a) schematic, (b) micrograph [75]

(a) Copper wires to create waviness [83]  (b) Insertion of unidirectional strips [83]

Figure 2.21: Artificial out-of-plane waviness creation methods
Regardless of the manufacturing technique employed to generate the defect, the authors first proceeded by quantifying the influence of the imperfection on the quasi-static strength of the laminate or structure. Given the geometry of the defect, all authors focused on the quasi-static compressive strength and on Compression-Compression or Tension-Compression fatigue. Hörmann et al. [78] determined a quadratic correlation between the defect severity and the static compression strength as illustrated in Figure 2.22. A reduced compressive strength of 33% to 50% of the pristine configuration was identified with a critical value obtained for $\alpha = 45^\circ$ for unidirectional laminates of six layers. For this configuration, the defect severity can be evaluated at $\delta/\lambda = 0.22$ where $\lambda$ is the wavelength and $\delta$ is the amplitude of the defect. In comparison, Mukhopadhyay et al. [12] observed a 28% reduction of the compression strength of quasi-isotropic laminates containing multiple wrinkle of a severity $\delta/\lambda = 0.06$. Finally, and even if the configuration studied is slightly different from coupons, the work conducted by Leong et al. [79] on sandwich laminates with a wrinkle on the face sheet confirms the significant influence of wrinkles on the static compression strength.

![Figure 2.22: Normalized static compression strength as a function of the deviation angle [78]](image)

Figure 2.22: Normalized static compression strength as a function of the deviation angle [78]
One of the main interests of conducting quasi-static tests on specimens containing wrinkles is to compare the failure modes observed in static and fatigue loading. If a similar trend of damage type, initiation, and propagation is observed this could help reduce the need for expensive and time-consuming experimental testing. All the authors of the abovementioned publications have concluded that the same trend of failure can be observed in specimens tested under Tension-Compression, Compression-Compression and static compression. It also appears that, independently from the defect severity, the fracture lines follow a preferred path that goes through the inflection points of the wrinkle as presented in Figure 2.23 by Adams et al. [83].

![Figure 2.23: Angle of damage propagation in specimen containing a wrinkle [78]](image)

In addition, Hörmann et al. [78] identified two ranges of defect severity that result in different chronologies of damage initiation and propagation. For defect with a low value of the angle $\alpha$ presented in Figure 2.20 ($\alpha \in [0^\circ, 20^\circ]$), there is no sign of damage propagation and the final failure is limited to two fracture lines connecting the point of highest misalignment and the resin rich regions as presented in Figure 2.24 (a). On the other hand, wrinkles of high severity ($\alpha \in [20^\circ, 65^\circ]$) first exhibited early signs of damage in the form of matrix cracks in the resin rich regions, which then led to delamination between the top layer and the four inner plies. Local buckling of the outer layers followed the delamination presented in Figure 2.24 (b). The final failure mechanism occurred in the form of kink bands in the inner layers, Figure 2.24 (c).
Finally, to come to a full understanding of the influence of wrinkles on the fatigue behavior of composite laminates, the influence of such defects on the fatigue life and the endurance limit were examined. Similarly to what was identified of the influence of wrinkles on the static compression strength, Hörmann et al. \cite{78} established a quadratic correlation, equation 2.4.1a, between the fatigue life of the specimens and the angle of deviation $\alpha$ presented in Figure 2.20. In equation 2.4.1a $c_1$ and $c_2$ are constants obtained by fitting the experimental data using a least squares method. In addition, the relation between the number of cycles and the applied cyclic load is assumed to follow a Basquin law as presented in equation 2.4.1b, where $N_e$, $\sigma_{max}$, $\sigma_{max,e}$, and $k$ are the endurance life, the maximum load applied in the cycle, the maximum load for which $N_e$ cycles can be withstand, and a constant, respectively.

It is important to emphasize here that, unlike some metal alloys, the existence of an endurance limit in composites has not been established and there are indications that for some layups and loading situations it may not exist \cite{85}.

\[
\sigma_{max,e} = c_1 + c_2 \cdot (\alpha - 45^\circ)^2 \quad (2.4.1a)
\]
Regardless of the load ratio and the frequency of loading employed by the authors, experimental results from Hörmann et al. [78], Leong et al. [79], and Adams et al. [83] all highlight the high influence of wrinkles on the fatigue life and the fatigue endurance of the laminates with defects. A reduction of nearly 50% of the endurance load for laminate with an angle deviation of $\alpha = 45^\circ$ is illustrated in the work of Hörmann in Figure 2.25 (b). In addition, the experimental fatigue life data points were projected with a slope $k$ on the vertical line representing the endurance life in Figure 2.25 (a). The presented data was obtained for specimens tested in Tension-Compression (T-C) with a load ratio $R=-1$ and a frequency $f=10$ Hz. The same work is presented for Compression-Compression with a load ratio of $R=100$ in Figure 2.25 (c) and Figure 2.25 (d) for the projected endurance limit and the influence of the deviation angle on the endurance load respectively. In the four sub-figures of Figure 2.25, $\bar{\sigma}$ refers to the normalized stress. It must also be noted that the stress levels in Figure 2.25 are all normalized by the static compression strength. Similarly, Adams et al. reported a reduction of a decade and half in fatigue life of specimens containing what the authors qualified as a moderate wrinkle. The experimental results for a load ratio of $R=10$ (Compression-Compression) and a frequency of $f=5$ Hz are reported in Figure 2.26. A reduction of the endurance limit from 75% of the static strength to 45% of the static strength was identified. It is, however, interesting to note the highest scatter of the results for specimens containing a defect due to the disparity in the geometry of the defects post-cure. In addition, the location of the final failure in the specimen was reported and all but one specimen from the control group failed in the grip section while all but one of the specimens containing a moderate wrinkle failed next to the waviness. Despite the absence of standard test method for specimens containing
manufacturing defects, ASTM D6484 on Open Hole Compressive Strength of Polymer Matrix Composite Laminates [86] sets a reference for coupons containing a stress concentration such as defects. Even though the ASTM standards recommendation would indicate that the failure mode observed for reference specimens is not acceptable, one can at least notice that the introduction of a moderate waviness was sufficient to force the final failure back into the gauge section, near to the wrinkle. Finally, Leong et al. [79] also concluded on a detrimental influence of wrinkle on the fatigue strength of laminate. The observed fatigue strength of laminates with defects was only of 33% of the reference elements as presented in Figure 2.27. It must, however, also be mentioned that the data for the reference specimen correspond to Tension-Tension test conducted at a load ratio of $R=0.1$ as the results scatter for the pristine beams under a fully reversed loading ($R=-1$) was too high. This nonetheless provides good indication of the influence of wrinkles on fatigue life as, according to Leong et al. [79], this “indicate[s] that the S–N relations for GFRP material loaded at $R = -1$ and $R = 0.1$ are almost identical for $N > 10^5$”.

Wrinkles are not the only type of defect for which the influence on the fatigue behavior of composite laminates was studied. Gaps [74], voids [76], folds [80, 81], or delamination [82] were also considered. As for the publications presented above, the influence of the defect of interest is first assessed under static loading. For instance, in the work conducted by Colombo et al. [82], the creation of a delamination by introducing a Teflon tape in a non-crimp glass-fiber laminate produced by vacuum infusion lead to no reduction of the ultimate tensile strength of a $[\pm 45]_{10}$ laminate. However, the Tension-Tension fatigue behavior, as presented in Figure 2.28 is significantly influenced as the slope of the linear interpolation of the S-N curve presents a reduction of 36%. It must, however, be noted that the confidence interval of 95% for laminates containing a delamination is significantly wider than for pristine specimens. Interestingly, the authors employed an Infrared (IR) camera on static and stepwise
Figure 2.25: (a) Calculated endurance limit for T-C, (b) correlation of T-C endurance limit and $\alpha$, (c) Calculated endurance limit for C-C, and (d) correlation of C-C endurance limit and $\alpha$ [78]

Figure 2.26: S-N curves for C-C fatigue test ($R=10$, $f=5$Hz) [83]

loading to capture the variation of temperature on the surface layer of S specimens (pristine) and T specimens (with delamination) as the change in the slope of the temperature increase could be associated with the fatigue limit. A similar use of IR camera was proposed by Elsherbini et al. [74]. The objective was to determine the
stress level below which the introduction of a gap in a unidirectional $[0]_9$, a cross-ply $[90/0/90/0/0]_s$, or a four-angles laminate $[90/45/-45/0/0]_s$ does not influence the fatigue life. This was attained by determining the stress level at which the slope of the linear approximation of the variation of temperature as a function of the maximum stress level during the fatigue test displayed an abrupt change. In the previously mentioned stacking sequences, the plies with a bar are not repeated through the symmetry.

Figure 2.28: Influence of delamination on Tension-Tension fatigue (S=Pristine specimens, T=Specimen with defect)
The results obtained by Elsherbini et al. [74] for cross-ply and four-angles laminates are presented in Figure 2.29. CP-R and CP-D refer to reference and defective cross-ply specimens respectively when FA-R and FA-D refer to reference and defective four-angles ply specimens. By extrapolating the trend of the linear approximation of the data presented in Figure 2.29, it can be observed that, in contradiction to most of the results presented in the previous publication, the static tensile strength of laminates containing a gap is significantly reduced compared to the static strength of pristine laminates. This behavior could be linked to the fact that the defect introduced in the laminates by Elsherbin et al. [74] is actually a cut-and-restart of fibers in the 0° plies. This is a more severe configuration than the natural gaps occurring during AFP manufacturing as the fibers in the load carrying plies present a severe discontinuity, leading to a stress redistribution potentially more severe at high stress levels than at low fatigue stress level. The precision achieved by using IR camera is extremely promising as the fatigue limit estimated with this method for the three most common types of stacking sequence yielded a result with less than 10% error with an average reduction of the required experimental time of 99.8% as can be observed in Figure 2.30.

<table>
<thead>
<tr>
<th>Laminate configuration</th>
<th>Wohler (MPa)</th>
<th>IR (MPa)</th>
<th>Error (%)</th>
<th>Testing time (Wohler) (hrs)</th>
<th>Testing time (IR) (hrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unidirectional</td>
<td>$1740 \pm 15$</td>
<td>$1730 \pm 4.15$</td>
<td>$-0.58$</td>
<td>219.7</td>
<td>0.39</td>
</tr>
<tr>
<td>Four-angle</td>
<td>$570 \pm 25$</td>
<td>$615 \pm 11.37$</td>
<td>$7.89$</td>
<td>334.4</td>
<td>0.44</td>
</tr>
<tr>
<td>Cross-ply</td>
<td>$990 \pm 30$</td>
<td>$935 \pm 10.33$</td>
<td>$-5.6$</td>
<td>189.2</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Figure 2.30: Comparison of fatigue limit estimation with Wohler and IR methods [74]
The conclusion observed by Elsherbini et al. [74] that introducing a defect has a higher influence on low-cycle fatigue life than it does on high-cycle has also been observed in other configurations. In the case of Vacuum Assisted Resin Transfer Molding (VARTM) of glass-fiber \([0/90]_s\) and \([0/45/0/-45]_s\) laminates with modified process parameters to cut down on the production time, voids are created. The influence of the voids density on the life to crack initiation, as shown in Figure 2.31, display a similar trend to the influence of the gap studied by Elsherbini. The higher the void area fraction \(A_v\) and higher the applied loads were, the higher the reduction in life to crack initiation was. The crack growth rate, the crack density evolution, and the stiffness degradation were all negatively affected by the introduction of voids.

![Influence of void content on the life to crack initiation of \([0/45/0/-45]_s\) \([76]\)](image)

![Influence of void content on the life to crack initiation of \([0/90]_s\) \([76]\)](image)

Figure 2.31: Influence of void content on the life to crack initiation

### 2.4.2 Numerical Studies

The experimental work presented in the previous paragraphs illustrates that the multiplicity of defect configurations already highlighted in section 2.2 is now combined with the fatigue load related parameters such as the load frequency \(f\), the load ratio \(R = \sigma_{\text{min}}/\sigma_{\text{max}}\), and the ratio between the maximum cyclic load \(\sigma_{\text{max}}\) and the ultimate static strength. The need for a comprehensive numerical and parametric FE representation as developed by the authors of [84, 87–89] is therefore heightened. However, due to the complexity and the computational cost of such models, a limited amount of
research has been conducted in that field. The following section will present the Progressive Fatigue Damage Models applied to the study of the influence of defects. The work conducted by Elsherbini et al. [33] on the development of a Fatigue Progressive Damage Model (FPDM) for the gap configuration presented in the previous section on experimental work relies on the flowchart presented in Figure 2.33a. The model was validated against experimental results by comparing S-N curves for a unidirectional [0]₉ laminate as presented in Figure 2.33. This model falls in the category of residual strength/stiffness presented in section 2.3. It relies on a sudden degradation captured by Hashin’s criterion combined with the maximum stress criterion. In addition, a gradual degradation scheme that associates the number of cycles that are applied with the residual strength and stiffness was used. The gradual degradation model used by Elsherbini et al. [33] is based on a model of Shokrieh et al. [55] presented in section 2.3. The combination of failure index was selected because of the high degree of conservativeness of the fatigue life estimation when solely based on Hashin’s criterion. Depending on the failure mode identified, a set of material properties was reduced by following a conventional ply discount method as presented in Figure 2.32 to reduce the load carrying capability in a specific direction. The proposed model was a good match with the experimental data even though a slight over prediction of the fatigue life was observed. The results of the model confirm that the introduction of a defect has an influence on the fatigue life at any applied level of stress. It can, however, be observed that the reduction in fatigue life is higher in the case of low-cycle fatigue. Hence a higher sensitivity of composite laminate to voids when tested in low-cycle fatigue.

<table>
<thead>
<tr>
<th>Failure Mode</th>
<th>Fiber Failure</th>
<th>Matrix Failure</th>
<th>Fiber/matrix debonding</th>
<th>Delamination failure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon_{11}=\varepsilon_{22}=\varepsilon_{33}=\nu_{12}=\nu_{23}=$</td>
<td>$\varepsilon_{22}=\nu_{12}=0$</td>
<td>$\nu_{12}=G_{13}=0$</td>
<td>$\varepsilon_x=G_{xy}=G_{xz}=\nu_{yx}=\nu_{xz}=0$</td>
</tr>
</tbody>
</table>

Figure 2.32: Sudden degradation rules [33]
Similarly, the model proposed by Mukhopadhyay et al. [84] proposed to capture the initiation and progression of damage in a laminate tested in Tension-Tension fatigue with a load ratio $R = 0.1$ and a frequency $f = 3$Hz. This was accomplished by introducing two sets of cohesive elements. The cohesive elements employed followed a conventional bilinear traction-separation law which was combined with an additional damage variable associated with cyclic degradation. A first set of cohesive elements is introduced between each layer to capture the delamination and a second set is introduced within each layer, with the exception of zero-degree plies, to capture matrix cracks. In addition, two common issues with cohesive elements were treated: the difficulty to capture damage initiation in the case of low load level in the absence of stress concentration, as it is the case in pristine specimens, and the computational cost of using tie constraints between the conventional layers and the cohesive layers. The former was addressed by introducing a strength reduction model which relies on
the number on elapsed cycles $n$ and is presented in equation 2.4.2. $\sigma_{I,fat}^{\max}$, $\sigma_{II,fat}^{\max}$, $a_{SN_I}$, and $a_{SN_{II}}$ are the modified mode I and mode II strengths and their corresponding slopes, respectively. This proposed model for the strength degradation is based on the previous work of May and Hallet [90] for a load ratio $R=0.1$. The strength reduction scheme was only employed to compensate for the absence of static damage in specimens tested under low load or specimens which did not exhibit stress concentration that would have served as initiation points. Looking at the fatigue cohesive law employed in Figure 2.18, this assumption equates to the fact that the maximum applied stress will never be equal to the combined modes load threshold and therefore, no cohesive element will ever reach the second part of bi-linear law during which the damage develops. Applying the strength reduction scheme based on the number of applied cycles results in an interfacial strength reducing with the number of applied cycles. However, since the critical energy release rate is a material property that remains unchanged and is equal to the area under the curve of the bilinear law, the displacement to failure increases accordingly. This is reflected in the bilinear cohesive law presented in Figure 2.34b.

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Figure 2.34: Fatigue cohesive laws
The second inconvenience of cohesive elements regarding the need to use fiber-oriented mesh which in turn implies the use of tie constraints between the cohesive layer and the conventional plies was tackled by introducing a new mesh pattern capable of capturing matrix cracks in all four conventional directions. An example of such element’s geometry is presented in Figure 2.35. As a side note, it is also interesting to note that by applying a non-unitarian aspect ratio, this unit cell can be used for non-conventional ply orientations.

Figure 2.35: Unit cell for capturing cracks in every conventional direction [84]

The final tool employed to reduce the computational cost of running a fatigue model is the use of a “cycle-jump” technique which consists of an initialization phase during which the load is progressively increased from zero to the maximum load of the cycle. Then the load is held constant at this maximum value and the elapsed time is correlated with a fixed number of cycles. This is summarized in Figure 2.36. This technique allows to define a testing frequency sufficiently high to speed-up the analysis process therefore reducing the computational cost.

Experimental results in the form of CT-scans of failed pristine (Figure 2.37a) and wrinkle (Figure 2.37b) specimens were compared with results provided by the experimental model on the global location and type of damage and delamination. Overall, it can be noted that introducing the wrinkle has led the main damage region to shift from the free edges of the pristine specimens towards the region with a
defect in the wrinkle specimens. This is also reflected in the fatigue life estimation presented in Figure 2.38a and Figure 2.38b where a reduction approximating one order of magnitude for a given stress level was observed between the pristine and wrinkle specimens. A result that is coherent with the works presented before on the influence of wrinkles. It is interesting to note that the model, even though solutions were implemented to tackle traditional issues encountered by pristine specimens, presents a better correlation with the experimental data for specimens containing a stress concentration (wrinkle) compared to the pristine ones.

Figure 2.36: Cycle-jump approach [84]

(a) Comparison of CT-scan (top) and numerical results (bottom) for a pristine specimen [84]
(b) Comparison of CT-scan (top) and numerical results (bottom) for a wrinkle specimen [84]

Figure 2.37: Comparison of CT scans and numerical simulations for pristine and wrinkle specimens
2.4.3 Conclusion

In this chapter a summary of the work conducted on the influence of AFP-induced defects on the quasi-static behavior of composite laminates was presented. At first, a list of potential defect types was introduced. Then the focus was shifted towards experimental testing and numerical analysis conducted in order to understand to which extent and how manufacturing defects are influencing the damage mechanisms and the failure load of specimens under various loading configurations such as tension, compression and in-plane shear. The main take away was that manufacturing defects can occur in many configurations, with various shapes and dimensions, and that there is no general rule to describe their influence. However, it was globally observed that defects have a limited to non-existent influence on simple tension, compression and in-plane shear test of un-notched unidirectional laminates. Whereas tests on notched specimens, such Open Hole coupons and tests on more industrially representative stacking sequences containing multiple orientations, seem to be slightly more affected by defects. In a second part of the document, a summary of modelling techniques for the fatigue behavior of composite laminates was presented. The contributions presented were divided into three categories: fatigue life models, residual strength/s-tiffness, and progressive damage models. The first group relies heavily on experimental data in the form of S-N curves to determine fitting parameters but presents the ad-
vantage of not requiring any knowledge of the damage mechanism at play during the test. The last category of modelling approach on the contrary requires to identify and select potential modes of failure to implement appropriate failure criteria. The main advantage of progressive damage models is that they do not necessitate a lot of experimental data, while their main drawback is the high computational cost associated with a single analysis. The residual strength/stiffness models are, in a way, an intermediate between the two other techniques presented. Finally, the third section of this document has focused on the combination of the two first by presenting the experimental and numerical work conducted on the influence of manufacturing defects on the fatigue response of composite laminates. The conclusion is that the number of configurations identified in section 2.2 is now inflated by the fatigue related parameters and this is reflected in the variety of experiments conducted. Regarding the influence of manufacturing defects, the reduction observed in fatigue life and endurance limit tend to indicate that they have a more consistent influence in fatigue than they do in quasi-static loading. Even so, study on the influence of manufacturing defects on the fatigue behavior of composite laminates is still relatively limited as, on the experimental side, the cost of conducting a sufficient amount of tests to obtain statistically significant results can be prohibitive. On the computational side, the difficulty of developing a model capable of providing sufficiently accurate results to serve as a design tool without requiring an unreasonable amount of computational time appears to be challenging. However, promising approaches such as the cycles-jump technique can help reduce the cost. Even though it was not the authors’ initial intention to identify the most promising modeling technique or the most critical defect, the reader of this review can find useful guidance to tackle their own specific problem. Indeed, as the broad variety of geometrical configurations associated with the defects and the
laminate, combined with the vast number of loading types and combinations possible imply that the study of the influence of AFP-induced defects on the fatigue behavior of composite laminates as a whole has not been fully conducted yet. Therefore, the most critical configuration is yet to be found.

The survey study presented above allowed to identify several aspects of the influence of defects which are yet to be studied or need a more detailed work. Regarding the quasi-static behavior of laminates containing defects, a substantial amount of work has already been conducted in both the experimental and the numerical domains. Therefore, the work presented in the following section (section 3) will attempt to propose two defect modeling approaches to answer either the need for a fast simulation with a simple representation or a more accurate representation with the additional overall computational cost.
Chapter 3

Defect modeling

As presented in details in the beginning of Chapter 2, at least fourteen different defect configurations have been identified for manufacturing processes associated with Automated Fiber Placement (AFP) machines. It would be unreasonable to attempt to experimentally study the influence of every defect type given the number of other parameters to take into consideration. To be exhaustive, anyone trying to tackle that task would have to consider: the loading type (tension, compression, shear, ...), the loading scenario (quasi-static or fatigue), the type of material, the stacking sequence, the number, dimensions, and locations of introduced defects. In addition, the potential interaction between different types of defect would have to be analyzed as well. And that is without mentioning the environmental conditions such as temperature and moisture. Therefore, it appears necessary to develop a numerical tool generic enough to be capable of capturing the influence of manufacturing defects on a composite laminate by limiting the effort to a few well defined defect types.

The present chapter presents two modeling approaches to tackle the problem. The first modeling approach is the Material Modification model and is referred to as the Simple Model which entirely relies on local modification of material properties but disregards any geometrical perturbation associated with local AFP lay-down process. The second model, the Material Modification with Geometric Perturbations model, also referred to as the Advanced Model combines the modification of material
properties with the through-the-thickness geometrical perturbation induced by the introduction of a defect via the use of forming simulation that mimics the AFP lay-down process. Then, in the subsequent subsection, the results generated using both modeling approaches are compared and validated against experimental data.

3.1 Modeling approaches

3.1.1 Simple model

In the work presented by Harik et al. [6], fourteen different types of AFP induced defects have been identified. Potential causes for each of them have been determined and first order estimations of the potential influence on a part’s mechanical behavior were assessed. In light of this information, and based on manufacturing experience, it was decided to limit the simple model capabilities to five of the most significant and most frequent defect types. First, because of their high recurrence rate, three defect types were selected to be part of the modeling tool capabilities, namely Gap, Overlap, and Misalignment. A profilometry image for each of them is presented in Figures 3.1a, 3.1b, and 3.1c, for a gap, an overlap, and a misalignment, respectively. Two additional categories of defects were included in the modeling capabilities: Fold and Twist. The last two are presented in Figures 3.1d and 3.1e, respectively. Fold and Twist were selected because of their potentially significant influence on the mechanical behavior of a part. Folds are a combination of a gap and an overlap, which leads to a local abrupt change of thickness as was shown in previous work (such as Sawicki and Minguet [7]), and generates potentially significant strength and stiffness reduction factors. Similarly, twists were selected for they present a singular point and a severe local re-orientation and relocation of the fibers.
Figure 3.1: Profilometric images of defects

The profilometric pictures of defects that are shown in Figure 3.1, presented in more detail in the context of Machine Learning applications for the identification process of AFP defects [91–93], were represented in the *Simple Models* by partitioning the regions of defects into three levels of thickness and material definition: a gap or resin rich region, an overlap or double thickness area, and a nominal tow thickness section. The proposed top-view schematic for each defect type is presented in Figure 3.2. The following color scheme is used in the *Simple Models*: resin rich regions (gap) in blue, double thickness (overlap) regions in yellow, and conventional or nominal tow thickness in grey.

Twist is one of the more complicated defect types and in Figure 3.3 the partitions for a twist are presented with a color scheme indicating the material properties associated with each portion. As opposed to the other defect types shown in Figures 3.2a-3.2d that have multiple tows, the schematic for the twist defect in Figure 3.2e and Figure 3.3 is for a single tow. The defect’s dimensions are used to create an array of points, in Figure 3.3, corresponding to the corners of the geometric partition of the defect. The corners of each portion is represented by red dots in Figure 3.3 with
the dashed lines showing the connectivities. The corners are delimiting three types of regions. In grey (parts P1 and P2) are the conventional regions for which the elastic properties of material IM7-8552 are applied in the FE model. In blue (parts P3 and P4) are the gap regions modeled with the isotropic material representation of a resin-rich region using the properties of the pure resin 8552. Finally, the yellow part P5 is an overlap region for which the conventional material definition are used but the thickness is locally doubled.
3.1.2 Advanced model

The simple representation of the defects illustrated above present the advantage of limiting the modifications to the material properties without affecting the geometry of the FE model as no through-the-thickness modifications are made. The simplicity of the representation allows, for instance, for the creation of vast databases of defect configurations in shell element representation to be conducted. This first approach allows for the identification of the potentially most interesting configurations but lacks details to be implemented in a PFA model. In this section, an advanced defect representation is presented to include the through-the-thickness influence of introducing a defect in a laminate. To do so, the added objective of the new model when compared to the Simple Model is to add a simulation of the manufacturing process that leads to the creation of a defect. The manufacturing process associated with the AFP manufacturing of a composite layup can be decomposed into two steps: the layup process and the curing process. During the layup process, collections of tows are repetitively deposited by the machine head (presented in Figure 3.4a) to create one layer. As the tows are deposited onto the substrate, they also are pressed onto the surface by the attached compaction roller also visible in Figure 3.4a. This process is repeated as many times as needed to create every layer in the desired stacking sequence. The following steps constitute the laminate curing process. First, a caul plate is placed on top of the laminate and this assembly is placed in an autoclave (presented in Figure 3.4b) following the cure cycle recommended by the material manufacturer (an example for IM7-8552 is presented in Figure 3.5). The curing process, by introducing temperatures higher than the glass transition temperature, serves two purposes as it allows, first, for the resin to reach a liquid state and flow within the laminate and towards the extremities of manufactured specimen, and secondly it causes chemical cross-linking of the resin to provide the laminate with its final mechanical properties.
To account for the influence of a defect on the geometry of not only the layer it is included in but also on the adjacent layers, a step-by-step approach described below was designed. This modeling technique can be seen as a first order representation of the AFP manufacturing process. A collection of tows belonging to a single layer is added one at a time by the AFP machine head applying pressure via the compaction roller. Abaqus [94] offers a convenient decomposition of the loading history at significant phases of the studied model. Elements of this decomposition are named steps. The proposed simulation is based on the following principle: each step introduces a
new layer of elements to compact on top of the already existing stacking sequence. Therefore, the FE model will be composed of as many steps as there are layers above the layer containing a defect. This phase represents the first step of the manufacturing process described in the previous paragraph. A pressure field is applied to each layer to mimic the pressure introduced by the compaction roller. In order to represent the process as accurately as possible, a representation of the compaction roller during the process of placing tows under compression, presented in Figure 3.6 from the work of Bakhshi et al. [95], is used as a reference. It was also mentioned in the previous paragraph that the manufacturing process includes a curing phase which leads to a flow of the resin due to the temperature reaching the glass transition temperature of the polymer. However, in an attempt to keep the proposed model with a reasonable scope of capabilities, the approach proposed for the simulation of the tow placement process in this document will exclude the simulation of the resin flow associated with the cure process. This is done in order to provide a compromise between the simple solution presented before and a full representation of the manufacturing process which would encompass too vast of a field. Instead, gap regions left between layers after the compaction phase are considered as resin rich regions with full unhindered flow of the resin and are therefore attributed the material properties associated with the resin.

Figure 3.6: Compaction Roller’s width
In order to fully define the dimensions of the layers in the FE model as well as the
duration of each step, and the amplitude of the pressure field, a detailed understanding
of the compaction phase and the deformation of the associated compaction roller are
necessary. As presented in Figure 3.6, the roller is, before compaction, a cylinder
of radius $R$ and length $l_{cr}$. It will deform under the compaction pressure per unit
width $P$ as shown with the red line with the flat portion of finite length pushing the
tow firmly against the substrate. The contact width $w$ of the flattened portion of
the compaction roller is a function of several of the roller’s mechanical properties as
shown in Equation 3.1.1 from [95]. In equation 3.1.1, $E_{cr}$ is the elastic modulus of
the compacted roller, $\nu_{cr}$ is the roller’s Poisson’s ratio, and $R$ is the radius of the
compaction roller. The former is given as a function of the Shore A hardness $S_{cr}$ in
Equation 3.1.2 and $\nu_{cr}$.

$$w = 4 \cdot \sqrt{\frac{P \cdot R_{cr} \cdot (1 - \nu_{cr}^2)}{\pi \cdot E_{cr}}}$$ \hspace{1cm} (3.1.1)

$$E_{cr} = 2.35 \cdot 10^{-2} \cdot S_{cr} - 0.6403$$ \hspace{1cm} (3.1.2)

Contact problems in FE simulation tend to be computationally expensive as they
require, for each time increment, to check for the potential interaction, or contact, of
an element with the substrate. In order to limit the duration of the simulation, the
dimensions of the model can be limited to that of the flat section of the compaction
roller. Therefore, limiting the dimension of each layer to a rectangle of size $w$ by
the length of the roller. In addition, since the objective is to simulate the placement
of a single layer per step in the FE simulation, the duration $t$ of each step can be
evaluated based on the averaged translational speed $V$ of the AFP machine head and
Table 3.1: Inputs for the compaction model

<table>
<thead>
<tr>
<th>Input</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force : $F$</td>
<td>445 N</td>
</tr>
<tr>
<td>Poisson’s ratio : $\nu_{cr}$</td>
<td>0.49</td>
</tr>
<tr>
<td>Shore A hardness : $S_{cr}$</td>
<td>60</td>
</tr>
<tr>
<td>Speed : $V$</td>
<td>20 000 mm/min</td>
</tr>
<tr>
<td>Radius : $R_{cr}$</td>
<td>25.4 mm</td>
</tr>
<tr>
<td>Length : $l_{cr}$</td>
<td>50.8 mm</td>
</tr>
</tbody>
</table>

The surface covered the compaction roller. The said surface is equal to the roller’s contact width of flattened section $w$ multiplied by its length $l_{cr}$. This is expressed in Equation 3.1.3 which provides the duration of each step since the FE model only represent the portion of the laminate directly under the compacted roller.

$$t = \frac{w}{V}$$  \hspace{1cm} (3.1.3)

Assumptions were made regarding the numerical values necessary for the evaluation of the contact width $w$ of the compacted roller and the duration $t$ of each step. The compaction force $F$ (used to evaluate the pressure per unit width by dividing the force by the roller’s length) is estimated to be equal to 445N on average on a AFP Lynx machine. This leads to a pressure per unit width of $P = 8.76 N/mm$. For the same machine, it is estimated that an appropriate mean translational speed for a conventional laminate can be approximated by $V = 20000$ mm/min. Additionally, from the work of Bakhshi et al. [95] mentioned previously, the roller’s Poisson’s ratio is taken equal to $\nu_{cr} = 0.49$ and the Shore A hardness is equal to $S_{cr} = 60$. These assumptions are summarized in table 3.1 and the pressure per unit width, the compacted roller’s contact width, and the step duration calculated with the aforementioned equations are summarized in table 3.2.
Table 3.2: Estimations of compaction model characteristics

<table>
<thead>
<tr>
<th>Output</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure per unit width : P</td>
<td>8.7598 N/mm</td>
</tr>
<tr>
<td>Width : w</td>
<td>19.2 mm</td>
</tr>
<tr>
<td>Step duration : t</td>
<td>0.058s</td>
</tr>
</tbody>
</table>

In the following paragraphs, an example of the compaction model applied to the creation of an advanced defect representation is presented for a half tow wide overlap (i.e. 3.175 mm). This example consists of a unidirectional laminate of $[0]_5$ with the overlap in the middle of the bottom layer. The loading conditions used to simulate the compaction include two types of loads: gravity and a pressure field. At the beginning of every new step, the force of gravity is applied to the newly introduced layer and will stay enabled for the subsequent steps up until the end of the analysis. A pressure field, to mimic the compaction force of the roller, is also applied on the newly introduced layer and held for two steps. The force is introduced in the first step with the layer to represent the compaction and is then held for the second step as it prevents the layer to un-stick from the substrate when the subsequent layer is introduced. More details are provided in the following paragraph when more information about the laws used to simulate the contact between layers are presented. This process is repeated until all the layers of the stacking sequence provided have been introduced in the model.

A summary of the loading conditions for each step is presented in Figure 3.7 and the deformed shape at the end of each step is presented in Figure 3.8. The final shape presented in Figure 3.8f illustrates the vertical displacement caused by the overlap in the middle of the laminate and the white wedges at the bottom of the laminate indicate the resin rich regions mentioned before.
As mentioned previously, the simulation of the compaction process implies the necessity to understand and be able to capture how different layers contact each other. Contact between layers is also referred to as an interaction. To satisfy the aforementioned objectives, the interaction of layers during the compaction process can be considered one of two ways given the options given in Abaqus: the general contact formulation or the contact-pair formulation. Contact formulations are tightly related to instances. Instances are created in the assembly module of Abaqus and can be seen as copies of the original part. This is similar to the classic instantiation process with an Object Oriented (OO) program where the part can be seen as a class and the instance is an object of type part. The general contact formulation allows to simply define a contact domain containing a set of instances of interest and therefore allowing to capture the interaction of any two instances included in the domain, including self-interaction for complex shapes. Alternatively, the contact pair formulation requires from the user to define individually every interaction to be monitored by specifying pairs of node/surface or surface/surface. The manufacturing process simulated is well known in the sense that the order in which each layer is introduced is known and a precise list of which layer will come in contact with which other layer is defined from.

Figure 3.7: Boundary conditions on a compaction model example
the beginning of the analysis. Additionally, no element deletion due to damage is to be expected. Therefore, no new element sets are to be taken into consideration for potential interaction with other layers. All of those observations combined justify to select the less computationally expensive option of a contact pair contact formulation. It is also worth mentioning that the penalty method, by modifying the stiffness of the element, can have an influence on the stable time increment. Choosing the kinematic approach allows to avoid this situation which could become computationally-expensive for simulations of laminates with many layers. This choice also reinforces the selection of the contact pair formulation as the kinematic method is not supported with the general contact approach.
Selection of the contact formulation requires one of the two sub-choices for the contact algorithm: either a kinematic formulation or a penalty approach. The former being more stringent on not allowing the penetration of the nodes into another at any instance. Since the simulation is trying to represent the compaction of layers on top of each other, the kinematic method was favored to the penalty solution as it ensures a more accurate physical representation of the contact problem for which it is physically impossible for a composite layer to penetrate another layer. Meshed surfaces selected for the interactions are commonly referred to as a master and a slave surface and their sets of nodes are referred to as master nodes and slave nodes, respectively. The interaction between a meshed master surface and a meshed slave surface is presented in Figure 3.9. For each increment in the simulation, Abaqus starts by determining the displacement of each element without taking the contact into consideration, the kinematic state advances. Then, for each node of the slave surface, Abaqus determines if it penetrated under the surface of the master. If it is the case, a resisting force to the penetration is evaluated based on several variables such as the depth of penetration of the slave node below the master’s surface, the weight associated with the node, and the time increment. It is important to mention however that, as presented in Figure 3.9, it is still possible to experience penetration of the master nodes into the slave surface. Fortunately, since both surfaces entering in contact are meshed with the same type of deformable elements, the default setting of a balanced master-slave contact is used by Abaqus. This entails that both surfaces of interest are treated as both a master and a slave surface thus ensuring a minimized penetration of the two surfaces and consequently a more accurate representation of the contact between two layers. In combination with the balanced master-slave approach, the default pressure-overclosure hard-contact formulation presented in Figure 3.10 is used where the overclosure refers to a negative clearance. The hard-contact formulation is equivalent to determining the reaction force associated with the slave node entering exactly in contact with the
master’s surface. This formulation is defined to ensure that no pressure is introduced on the slave surface before the nodes enter in contact with the master surface, which is represented by the horizontal line for any clearance value smaller than zero on the graph in Figure 3.10. After the contact has been initiated, any value of the contact pressure is possible and the clearance remains equal to zero as represented by a vertical line at a clearance of zero on the graph in Figure 3.10.

Figure 3.9: Master-Slave surface contact example from Abaqus’ documentation [94]

![Diagram of master-slave surface contact](image1)

Figure 3.10: Pressure-Overclosure Hard contact from Abaqus’ documentation [94]

![Diagram of pressure-overclosure hard contact](image2)

When defining a pair of master/slave surfaces, it is generally advised to select the geometrically larger surface of the two as the master. Indeed the algorithm used to determine the clearance, or distance between the slave and the master, is evaluated for each node of the slave surface by measuring the distance from the slave node to the master surface by looking in every direction from the slave node. Selecting the larger surface as the master reduces the number of nodes for which the clearance
must be evaluated and also greatly reduces the numerical difficulty of finding the other surface. One must note that, as the size difference between the two surfaces increases, so does the significance of selecting the proper surface as the master. This is because the number of nodes, and therefore of distance evaluation, needed for a surface is correlated to its size for a given mesh density. Another guidance to identify which surface to use as a master or as a slave surface is that since slave nodes cannot penetrate the master surface, it is advised to use the stiffer/less deformable instance as the master. Finally, and for the same overclosure/node penetration related reason as for the stiffness argument, it is recommended to use surfaces with a coarser mesh as the master since it implies less potential for penetration of the master nodes into the slave surface as the mesh density is lower. However, in the compaction model presented here, all the surfaces are assigned the same material properties and therefore exhibit the same stiffness, are meshed the same way, and are of the same dimension with the exception of the first layer containing the defect which is split into two instances. All the factors of interest for the selection of the role of master/slave surface, that is to say the dimension, stiffness, and mesh size are, if not equal, at least in the same order of magnitude. The previously mentioned option of a balanced master-slave interaction therefore appears most adequate as it entails that both surfaces are treated simultaneously as a master and a slave surface. With the balanced master-slave interaction, a linear combination of the reaction forces needed to prevent the penetration of elements of one surface into the other surface are based on the surface weighing factors of acceleration corrections and are evaluated as described in the previous paragraph on pressure-overclosure. The first correction factor is determined using one surface as the master and the second surface as the slave surface. Then the roles are reversed to evaluated the second correction factor. A linear combination of
the two correction factors is then used to determine the contact pressure to be applied to the nodes based on the weight associated with each surface. By default, and it is the setting used with the present model, the corrections are weighted equally. As a result, the final shape of the compacted layers does not include any penetration.

Commonly, contact problems tend to be computationally expensive. In the eventuality a user wants to speed up the analysis, Abaqus offers the option to parallelize the analysis. One way of doing so is to divide the model into domains. Domains are sets of finite elements whose union is equal to the whole model and whose intersection is empty. This allows us to run the analysis of each domain on a different CPU and exchange necessary information between the domains at the end of each time increment. However, in the case of contact problems, elements appearing in a common definition of a master-slave interaction must belong to the same domain. Otherwise, the simulation will not be able to capture the contact between the two surfaces. Consequently, the proposed simulation of the manufacturing process cannot be parallelized but was dimensioned to be used as an Representative Volume Element (RVE) therefore limiting the computational cost as much as possible.

Figure 3.11: Example of layers after compaction (overlap)
The scope of the model presented in this section is limited to the representation of the compaction and its influence on the deformation of layers on top of a pre-existing defect in the substrate. As mentioned previously, the material properties of the uncured resin-epoxy system are used and the compaction force of the roller is taken to be equal to the average pressure used for a system of IM7-8552. As a result, the present model only reproduces the tow positioning phase and therefore excludes the actual curing process. No simulation of the resin flow due to the temperature and pressure increase in the autoclave, nor any representation of the pressure introduced by caul plates are taken into account in the current configuration. The pressure field, once introduced on a layer, is maintained constant and activated throughout two consecutive steps of the simulation to prevent the balanced master-slave contact constraint to push back up the layers compacted in the previous steps. An alternative would be to introduce a cohesive interaction property between the layers once the two surfaces are in contact. The proposed cohesive interaction would be initially de-activated. Once two surfaces get in contact the cohesive law is activated, creating a bond between the two elements which can only yield if the traction-separation law reaches the critical displacement. This option is offered by Abaqus and would be a more accurate representation of the tow positioning as it would add a representation of the material tackiness. However, this option remained out of the scope of the work conducted for this document as it would require to experimentally characterize the tackiness of the un-cured material.

Since the main objective of the advanced defect model is to be able to modify the geometry of a FEM containing a defect by locally modifying the coordinates of nodes based on the deformation observed in the compaction phase, a Python script was developed to extract the nodal data. For each individual ply in the compaction model, a text file containing the coordinates \((x, y, z)\) and displacement fields \((u, v, w)\) of each node is extracted from Abaqus' Output DataBase (ODB). The objective being
Figure 3.12: Flowchart of the advanced defect model implementation

to facilitate the manipulation and use of the data without having to work in Abaqus’ Python environment allowing for the use of Python 3.6, also allowing to work without necessitating an Abaqus license, and allows to used additional Python packages and modules more easily. Generally, in order to obtain satisfactory results, the mesh in compaction models must be relatively dense which would point to large text files and a computationally-expensive extraction phase of the nodal data. To circumvent this issue, and assuming an homogeneous repetition of the deformation along the length of the defect, the extraction is only performed in a plane perpendicular to the defect and passing through the mid-length of the modeled plies. The added benefit is that the nodal displacements extracted are obtained far from the free-edges of the laminate which could be affected by the abrupt substrate termination of the layers underneath the current layer. However, one must point out that this assumption of repeatability of the deformation fields along the length of the defect can only be applied to defect
configurations that are already initially geometrically self-repeating and therefore only applies to a handful of defect types such as gaps, overlaps, and folds. This approach therefore excludes misalignments and twists for which it would be necessary to extract the nodal information from the whole compaction model.

Finally, the last step consists of including the deformations of the laminate observed in the compaction model and due to the defects into the Global Finite Element Model (GFEM). However, since it is unlikely that the nodal coordinates in the compaction model match with that of the GFEM, to include the deformation associated with the defect an additional Python script is used to generate cubic spline interpolation functions of the nodal displacement extracted from the compaction model. An individual function is created for each ply in the compaction model. Finally, to apply, to each node of each ply in the GFEM, the proper displacement caused by the introduction of defects, the interpolation functions created beforehand are used to evaluate the displacement at the location of interest. The whole process presented here regarding the last step of the advanced defect model is presented in the flowchart in Figure 3.12. Common parameters such as the specimen dimensions, the stacking sequence, and defect related information are defined beforehand and shared between the GFEM and the compaction model as shown in Figure 3.12. The GFEM is first generated without the advanced representation of the defect and the compaction model is analyzed to capture the deformations generated by the introduced defects. The nodal displacement field is then extracted from the post compaction model and used to create the aformentioned interpolation functions. Then, using each node’s coordinates, the through-the-thickness location of each node is updated in the GFEM based on the value provided by the interpolation function. Therefore allowing to create the advanced defect representation as illustrated at the bottom of Figure 3.12.
3.2 Application to OHT model

In this subsection, a direct application of the creation process of an advanced defect representation applied to the configurations studied in the following chapters 4 and 5 is presented. The configuration of interest consists of a double overlap defect introduced in the vicinity of a hole in an Open Hole Tension (OHT) specimen. The two defects are stacked above each other in two consecutive plies, and in the middle of the laminate through the thickness. The choice of combining two overlaps in the same laminate is justified by the necessity to keep the laminate balanced and prevent the coupling of in-plane stress and bending. The defects run along the whole length of the laminate and the stacking sequence $[45_2/90_2/-45_2/0_2/45_2/90_2/-45_2/0/0]_s$ is used. The two middle plies containing the overlap defects are denoted $\bar{0}$. A cross-sectional view
of the FEM used for the compaction simulation in presented in Figure 3.13a and since the defects are in the two middle plies, the midplane in this figure does not match with the bottom of the presented finite elements but instead, the midplane is located on the top surface of the first overlap. Looking from the bottom-up in Figure 3.13a, a fine line can be observed at the very bottom of the stack, it is a layer of rigid body elements used to represent the undeformable substrate which can either represent previous layers or the tool surface on which the tows are deposited. Going up, above the rigid body surface, the first two layers with a defect are introduced. Each layer containing a defect is split into two half layers overlapping by half a tow width (i.e., 3.175mm). Then, the subsequent layers of the stack are full length layers added on top of the defects with a color scheme following the legend introduced in Figure 3.13a. The material orientation assigned to the elements of each layer is based on the stacking sequence mentioned previously. The orientation of the top layer of the compaction model matches with the outermost 45 degree layer in the stacking sequence and the bottom ply of the compaction model matches with the bottommost layer of the stacking sequence containing a defect. A cut view, in the x = 0 plane, of the deformed shape of the laminate with two half-tow-width overlaps stacked above each over is presented in Figure 3.13b. In this figure, the vertical displacement field \( w \) is shown and is then extracted, as explained above, to generate interpolation functions for each ply based on the nodal data available.

The nodal displacement field extracted from the 16-ply, double-defect, configuration in the x = 0 plane is used to create the interpolation functions shown in Figure 3.14. The view presented is that of a cross-section cut of the laminate at x = 0. In addition, since the GFEM generally has a denser mesh and that we are trying to reduce the computational-cost, the displacement due to the introduction of the defect will only be evaluated for a limited number of nodes on each side of the defect’s center at \( y = 0 \). Therefore, the interpolation functions presented in Figure 3.14 are only
Figure 3.14: Shape of interpolation functions for double defect configuration

evaluated for a limited range of y coordinates comprised between -3/2 times the defect width and 3/2 times the defect width. The range of y was narrowed down to the previously mentioned range as it allows for the ply elevation to reach a stable value far from the transient region. In this figure, the double defect region is identified with a gray overlay and the red vertical dashed lines delimit the defect’s boundaries. Each curve represents the elevation of the top surface of one ply and the red horizontal dashed lines indicate the elevation of a nominal ply not containing a defect. The next and final step to include the defects in the GFEM is to, for each node of each ply in the reduced range of y identified earlier, evaluate the displacement $w$ to apply to its z-coordinate based on its y-coordinates and assume that all nodes of the same ply and on an iso-y-value are subject to same out-of-plane displacement. The process described in this section results in a FEM such as the one presented in Figure 3.15.
Figure 3.15: Isometric view of advanced defect in OHT
Chapter 4

Modeling the Influence of Defects in Quasi-Static

As illustrated in the literature review presented in Chapter 2, the influence of AFP induced defects on the quasi-static (subsection 2.2) behavior of composite laminate under tension has been studied to a certain extent for various stacking sequences and density of defects. The density of defects referred to here includes the variability in the number and the location of defects in a given laminate. Depending on the type of defects introduced in the laminate, their locations, dimensions, and their numbers as well as the type of test conducted on the laminate, reductions as well as increases, at times, were observed in laminates strengths. A wide range of variability was observed going from a nearly negligible influence of gaps on the tensile strength of a unidirectional laminate in Croft et al’s work [8] to a significant 5 to 27% reduction in compression strength of quasi-isotropic laminates in Sawicki and Minguet’s work [7].

The general conclusion regarding the influence of defects on the tensile behavior, with regards to the ultimate strength, the longitudinal stiffness, and the failure mechanism is that it is mostly negligible. The catastrophic nature of the failure of specimens subjected to tensile load provides an explanation for how little influence the defects seem to have. By breaking so abruptly, the specimens cannot exhibit a significant enough damage and delamination propagation phase. This progressive failure phase remains in a relatively limited range, which does not allow for the defects to cause any significant changes in the failure mechanism compared to that of a pristine specimen.
On the contrary, the limited number of studies conducted on specimens containing both a defect and a stress concentration such as a hole [7, 13, 96] tend to indicate a more significant sensitivity to the presence of a manufacturing defect. In light of these observations on the stress concentration, it was decided to observe the influence of AFP defects on Open Hole Tension (OHT) specimens.

The present study described below relies on the use of a Abaqus user-subroutine, CompDam, to capture the progressive failure behavior and the changes associated with the introduction of a defect in the aforementioned OHT specimens. First, a presentation of the configurations modeled and studied are presented in this chapter. Then, in a second subsection, a detailed presentation of the subroutine CompDam is provided. Finally, the numerical results are shown and compared with data available in the literature.

4.1 Description of the Studied Configurations

As previously mentioned, the present study focuses on OHT specimens and the influence of inserting defects in the laminate in the vicinity of holes. Since the objective is to dispose of the broadest possible understanding of the influence of defects, without limiting the study to laminates which are not useful industrially speaking, quasi-isotropic stacking sequences appear more adequate than the unidirectional counterparts frequently analyzed. Quasi-isotropic laminates indeed satisfy commonly used rules for the stacking sequence such as the "10% rule" imposing that each orientation included in the laminate to represent at least 10% of the total number of plies. They are also more interesting for the analysis aspect as they allow to combine several phenomenons such as damage initiation in off-axis plies, delamination between plies of different orientations, and stress redistribution post-damage initiation. The remaining parameters defining the geometry of the laminate, namely the length, the width, and the thickness of a laminate, were based on recommendations from
the ASTM standards D5766 [97] and D7615 [98] for the quasi-static and fatigue OHT laminates, respectively. Based on these recommendations, it was decided to study specimens 300mm long, 36mm wide, at least 4mm thick, and with a 6mm hole in the center. The dimensions of the laminates are summarized in Figure 4.1 in the metric system (approximate value in the imperial system is also given in parentheses). The material used is Hexcel’s IM7/8552 with a nominal ply thickness of 0.183mm which combined with the choice of a quasi-isotropic stacking sequence led to the following choice: \([45_2/90_2/-45_2/0_2/45_2/90_2/-45_2/0_2]_s\). The selected stacking sequence presents the advantages of being quasi-isotropic, limiting the ply orientation jump between consecutive layers but still contain blocks of the same orientation which is of interest to observe delamination, and having a total thickness of 5.856mm which satisfies the ASTM standards recommendations.

![Figure 4.1: Geometry of OHT specimens](image)

Still from the literature review, the identification of the defect types and their frequency of occurrence during manufacturing processes presented in subsection 2.2 justify paying more attention to gaps and overlaps given their high rate of creation and potential influence. As a result, it was decided to include both these defect types in the laminate mentioned above. However, as opposed to what seems to be a common practice in the studies already published in the literature, an attempt was made to represent a more realistic defect configuration which is a single tow shift that
combines the two defect types into one as shown in Figure 4.2. In this figure, the left image shows a group of four tows in their nominal position and the red arrow pointing towards the left indicates the direction in which one of the tows is to be 'inadvertently' shifted by half of its width. By doing so, as shown in the two images on the right, the shifted tow creates a gap on the right side of its original position and an overlap on the left side both respectively as wide as half a tow. This is the type of defect that will be thereafter in this document be referred to as a gap-overlap and is introduced in the OHT models in three different configurations and their associated sub-configuration presented hereafter.

Figure 4.2: Creation of a gap-overlap

The present study focuses on OHT specimens which include a hole through the laminate at the middle in both the longitudinal and transverse direction as shown in Figure 4.1. Including a defect in such a specimen requires to select two parameters that are essential to position it in the laminate: the first one dictates the layer (or layers) in which the defect is contained, which consequently defines the orientation the defect is in since it is aligned with the fibers’ direction. The second parameter is the relative location of the defect around the hole. In order to keep a balanced and symmetric laminate, the defects are introduced in the OHT specimens in the two middle zero-degree plies denoted 0 in the stacking sequence \([45_2/-45_2/0_2/45_2/90_2/-45_2/0/0]_s\). The decision to include the defects in these layers was motivated by several advantages.
Placing the gap-overlap in the zero-degree oriented plies introduces a weak spot in the form of a gap in the load carrying plies which should help increase the influence of the defect on the tensile behavior of the laminate by affecting the chronology and spatial distribution of the damage, and potentially the ultimate failure strength. Additionally, the selected location for the defects is sandwiched in between two other zero-degree oriented plies which allows for the creation of the out-of-plane waviness identified by Sawicki and Minguet [7] as the main source of reduction in strength for the Open Hole (OH) coupons.

When it comes to selecting the relative position of defect with respect of the hole it is actually important to go back to the actual procedure that will be followed during a standard manufacturing process. In practice, a panel, containing defects will be manufactured with an AFP machine and then cured in the autoclave. Only after the curing process comes the operation of removing material to create the hole. It is therefore more logical to consider the location of the hole with respect to the already existing defects than the opposite way around which would consist of considering the position of the defects with respect to the hole. With that distinction in mind, three positions were identified as relevant and selected for this study. Since the main interest of introducing a hole in the coupons was to create a stress concentration area where to place the defects to magnify their potential influence on the mechanical behavior by causing the two stress field concentrators to interact, two first positions come to mind. Considering the gap/overlap to be achieved by moving a single tow by half a tow-width either to the left or to the right from its original position which is perfectly flushed between two adjacent tows, the hole can be positioned either completely overlapping the gap region (flushed to the overlap region) as presented in Configuration 1 in Figure 4.3 or completely overlapping the overlap region (flush to the gap region) as presented in Configuration 3. This, therefore, places the respective defects in the stress concentration area of the hole. The last option consists of
partially placing both defect types in the stress concentration area on each side of the hole by placing the center of the hole in the middle of the nominal tow region as presented in Configuration 2 in Figure 4.3. A staggered disposition that can be used for manufacturing the three configurations from a unique panel is presented in Appendix B. There are two additional configurations for which the gap or the overlap is centered with respect to hole. However, neither of these two additional configurations are studied as they involve placing one of the two defects far from the hole and the stress concentration associated to it. In addition to the three main configurations aforementioned, sub-configurations with a single defect type were also studied. In the following parts of this document, they are denominated Configuration-$i$-gap and Configuration-$i$-overlap for $i$ in $[1, 3]$ for a coupon exclusively containing a gap or an overlap respectively.

![Figure 4.3: Three main OHT configurations](image)

The Finite Element Model (FEM) presented below in this section are generated using Abaqus 2018 [94], and the modeling approach employed to capture the damage initiation and propagation in the laminate relies on an approach that falls in the third category of models presented in the literature review, namely the progressive damage simulations (chapter 2 on Progressive Failure Model). CompDam [1] is an implementation, in the form an Abaqus/Explicit User Subroutine V-User-Material
(VUMAT), of a Continuum Damage Mechanics (CDM) model. *C*omp*D*am was used to capture the initiation and propagation of matrix cracks within the laminates presented above. As underlined in the literature review in Chapter 2, CDM models present the advantage of not necessitating for the mesh to be adjusted during the analysis as matrix damage occurs and grows. Indeed, as with all CDM models, the matrix cracks are not explicitly represented but are rather taken into consideration by applying reduction factors to the coefficients of the stiffness matrix $C$. The degradation of the stiffness terms are determined so that, for a same strain level $\epsilon$, the stress level $\sigma$ observed in the laminate without cracks but with degraded stiffnesses match that of the cracked specimen. It is however important to emphasize at this point that, to be able to accurately capture the path of the matrix cracks, a mesh aligned with the fiber orientation (aligned-mesh) is highly recommended. This recommendation is based on the work conducted by Justusson et al [3] on characterizing the influence on the apparent fracture toughness of a misalignment angle between the fiber direction and FE orientation. It was found that the predicted failure of elements with respect to the misalignment angle followed a near parabolic curve with a maximum with a misorientation of 45°. Therefore, the FEM used for this study are meshed with fiber aligned mesh as shown in Figure 4.4 with an example of a 45° ply. *C*omp*D*am, as presented in more details in Frank Leone’s work [1], consist of a consistent approach to capture both inter- and intra-ply matrix cracks by combining the use of cohesive layers in between plies and the embedment of a cohesive behavior in the solid elements. This is obtained by decomposing the deformation gradient of the continuum model into a bulk part and a cohesive part. Allowing, therefore, to decompose the deformation vector $\mathbf{x}$ into the deformation of the bulk part ($x_B$) and the crack opening of the cohesive part ($\delta$). By not physically representing the cracks in the model, this approach
comes with the main benefit, when used with FEM, of not requiring for a new
mesh conforming with the location of the cracks to be generated at each step of the
progression of the cracks. This comes however at the cost of necessitating a rather
dense mesh to be able to capture accurately the location of the cracks and their paths.

![Fiber aligned mesh in 45° ply](image1)

Figure 4.4: Fiber aligned mesh in 45° ply

![Isometric view of mesh](image2)

Figure 4.5: Isometric view of mesh

In view of the number of different configurations to be studied presented in the
previous paragraphs, and given the need of consistency between the models, a con-
struction script created by Imran Hyder [99] to generate FEM for OHT is used. An
isometric view of the thirty-two plies laminate is presented in Figure 4.5 and a top
view of generic spatial decomposition of an OHT model is presented in Figure 4.6.
The central part of the coupons, called the *damage region* of the specimen, consist of three-dimensional, eight-node solid elements with reduced integration C3D8R of a relatively small dimension as this region will capture the initiation and the propagation of damage. However, since a PFA model can be computationally expensive, to reduce the overall number of degrees of freedom in the FEM, both the left and right extremities of the specimen, namely the *far-regions*, are represented with a coarser mesh composed of shell elements for which the capture of damage initiation and propagation by the user subroutine (*CompDam*) has been disabled. In the *far-regions*, a single element through the thickness is used as opposed to one element per ply in the *damage region*. This thereby reduces the number of elements in the model even further. Finally, the *near regions*, serves as a transition area at the interface between the *damage region* and the *far-regions*. In the *near regions* the damage is still disabled but brick elements are used.

![Figure 4.6: Regions in Open Hole models](image)

The bond at the interface between the brick elements and the shell elements is ensured by a tie constraint. This type of constraint available in Abaqus allows to prevent any type of relative motion between two surfaces, in this case the cross sections of the *far-regions* and the cross sections of the *damage region*, even if the meshes are dissimilar. This allows to use a different mesh density for the region away
from the damage. Finally, to represent the displacement limitations imposed by the grips on a testing machine, the out-of-plane displacement $w$ of the top surface nodes in the far-regions is constraint to zero. Finally, since as mentioned earlier $\text{CompDam}$ can be used to capture the inter-ply cracks, or delaminations, zero-thickness cohesive layers are embedded in between ply blocks of different orientation as shown with the red highlights in Figure 4.7. The formulation used to capture the delamination in cohesive elements implemented in the user-subroutine $\text{CompDam}$ is based on the work of Turon et al. [100]. The authors proposed solution relies on a novel constitutive equation to capture the onset and propagation of delamination with a bilinear law capable of thermodynamically account for changes in the mode mixity in the loading mode.

![Figure 4.7: Cohesive layers in Open Hole models](image)

The mesh displayed above serves as the baseline of the nine configurations introduced in the previous paragraphs as it defines the configuration to use for the pristine model. Depending on the type of defect representation used, based on the work presented in Chapter 3, the modifications to conduct on the pristine model can be relatively limited. Whichever of the two options, simple or advance defect model, is selected, the modifications to be conducted onto the pristine model will at least include the creation of element sets at the location where the defects are physically present in the laminate so that the material properties definitions can be locally modi-
fied. A direct application for configuration 1 is shown in Figure 4.8 with the elements in the gap region highlighted in yellow, those in the overlap region are in blue and the orange and black strips represent the rows of elements for which the matrix damage is locally disabled in the respective defects. The region in grey corresponds to the **near-region** and the green is the **damage region**. In the case of a simple defect model this is the only modification conducted on the pristine model. However, when the advanced defect model is used, the procedure described in Chapter 3 must be followed to include the out-of-plane displacement caused by the introduction of the defect as reflected in Figure 3.15.

![Figure 4.8: Defect location in Configuration 1](image)

4.2 Results and Conclusion

In this section, the results of the OHT numerical analysis will be presented into different subsections based on the type of model approach used for the defect: simple or advanced. Then a comparison between the two solutions is presented. Before presenting results for both types of defect representation, let us summarize the FE model settings shared by both approaches. This includes the loading conditions, the material properties for the IM7-8552 as well as the cohesive elements properties.
Figure 4.9: Bi-linear cohesive law [101]

In order to be able to compare the behavior of the different laminate configurations presented in section 4.1, a consistent definition of all the non-defect related parameters of the FEM must be established. Starting with the material properties. For the work presented in this document, the material properties were obtained from the reconciled material properties from the following publications [102–109] and are summarized in Tables 4.1 and 4.2 for the stiffness and strength properties of IM7-8552 used in the numerical models respectively. The bi-linear cohesive law presented in Figure 4.9 from the work of Davila et al. [101] is obtained by superposing two linear cohesive laws for modes I and II by defining two ratios and ensuring that both laws are peaking for the same displacement jump $\delta$. To be defined and implemented in the subroutine, several material properties (reported in Table 4.1) are necessary. The first ratio relating the critical load $\sigma_c$ and the critical loads in modes I and II, $\sigma_{c1}$ and $\sigma_{c2}$ respectively, as presented in equation 4.2.1 and referred to as the longitudinal tensile strength ratio $f_{XT}$ in Table 4.2. The second ratio relating the toughness ratio $G_c$ and the toughness ratios of modes I and II, $G_{c1}$ and $G_{c2}$ respectively, as shown in equation 4.2.2 and referred to as the longitudinal tensile fracture toughness ratio $f_{GXT}$ in Table 4.2. Finally, the penalty stiffnesses $K_{SS1}$, $K_{SS2}$, and $K_{SS3}$ for modes I, II, and III respectively are provided in Table 4.2 and refer to the slope of the first part of the linear law for each mode.
\[
\begin{align*}
\sigma_{c1} &= n \cdot \sigma_c \\
\sigma_{c2} &= (1 - n) \cdot \sigma_c
\end{align*}
\] (4.2.1)

\[
\begin{align*}
G_{c1} &= n \cdot G_c \\
G_{c2} &= (1 - n) \cdot G_c
\end{align*}
\] (4.2.2)

Table 4.1: Material properties (stiffness)

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</tr>
<tr>
<td>RATIO ((\nu_{12}))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MINOR POISSON</td>
<td>0.32</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>RATIO ((\nu_{13}))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DENSITY ((\rho))</td>
<td>1.57e-09</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

RESULTS FROM SIMPLE DEFECT REPRESENTATION

As a first attempt to capture the influence of AFP-induced defect on the mechanical behavior of OHT specimens, the simple defect model approach was used to generate ten configurations to study. They are summarized in Table B.1 in Appendix B with a name in the left column alongside with a schematic of the defect location in the right column of the table. The aforementioned configurations correspond to those described in the previous section (section 4.1) where a combination of a gap and an overlap are placed at different positions with respect to the hole location. The analysis results obtained are presented in this section and include the stress-strain curves and the evolution of the damage and the delamination in each layer.
Table 4.2: Material properties (strengths)

<table>
<thead>
<tr>
<th></th>
<th>Matrix Tensile Strength, mode 1 (YT)</th>
<th>Matrix Compressive Strength, mode 1 (YC)</th>
<th>288.2 MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix strength, mode 2 (SL)</td>
<td>97.6 MPa</td>
<td>Tensile Fiber Strength (XT)</td>
<td>2326.2 MPa</td>
</tr>
<tr>
<td>Compressive Strength (XC)</td>
<td>1730.6 MPa</td>
<td>Matrix fracture toughness compressive, mode 1 (G1CY)</td>
<td>0.24</td>
</tr>
<tr>
<td>Matrix fracture toughness tensile, mode 1 (G1TY)</td>
<td>0.24 $M Pa.mm^{1/2}$</td>
<td>Matrix fracture toughness, mode 2 (G2CY)</td>
<td>0.739 $M Pa.mm^{1/2}$</td>
</tr>
<tr>
<td>Matrix fracture toughness, mode 3 (G3CY)</td>
<td>0.739 $M Pa.mm^{1/2}$</td>
<td>Penalty Stiffness, mode 1 (KSS1)</td>
<td>4.76e5</td>
</tr>
<tr>
<td>Penalty Stiffness, mode 2 (KSS2)</td>
<td>2.29e5</td>
<td>Penalty Stiffness, mode 3 (KSS3)</td>
<td>2.29e5</td>
</tr>
<tr>
<td>Mode I Strength, Nominal Stress, Normal-only Mode (Mode1Str)</td>
<td>80.1 MPa</td>
<td>Mode II Strength, Nominal Stress, First Direction (Mode2Str)</td>
<td>97.6 MPa</td>
</tr>
<tr>
<td>Mode III Strength, Nominal Stress Second Direction (Mode3Str)</td>
<td>97.6 MPa</td>
<td>Mode Mixity Exponent (eta BK)</td>
<td>2.07</td>
</tr>
<tr>
<td>Long. Tensile Fracture Toughness (GXT)</td>
<td>205 $M Pa.mm^{1/2}$</td>
<td>Long. Compressive Fracture Toughness (GXC)</td>
<td>61 $M Pa.mm^{1/2}$</td>
</tr>
<tr>
<td>Long. Tensile Fracture Toughness ratio (IGXT)</td>
<td>0.5</td>
<td>Long. tensile strength ratio (XT)</td>
<td>0.2</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\sigma_1 &= \frac{RF_1}{n_{\text{ply}} w t_{\text{ply}}} \\
\epsilon_1 &= 2 \cdot 100 \cdot \frac{U_1}{l}
\end{align*}
\]  

(4.2.3)

The displacement and the reaction force were requested as an output from Abaqus at a node far from the damage region of the laminate. This data is used to generate the stress-strain curves in the far-field, away from the hole introduced in the laminate. This is done adopting equation 4.2.3 where $w$, $l$, $n_{\text{ply}}$, and $t_{\text{ply}}$ are the laminates width, the laminates length, the number of plies, and the ply thickness respectively. The numerical results are presented in Figure 4.10 in black for the pristine configuration, in orange for configuration 1, in red for configuration 2, and in blue and configuration 99.
It can be observed that the global behavior of the laminate in the linear portion of the stress-strain curve, when either no or a limited amount of damage has occurred yet, does not seem affected much by the introduction of the gap-overlap. A linear interpolation was used to evaluate the slope, and therefore the longitudinal modulus $E_1$, of the four configurations presented in Figure 4.10. The results are presented in Table 4.3 alongside with the Classical Laminate Theory (CLT) prediction of the longitudinal modulus of an unnotched laminate with the same stacking sequence. The longitudinal stiffness of the pristine configuration is in good accordance with the results obtained with the CLT since, as expected, the CLT prediction for the longitudinal stiffness is slightly higher for the unnotched laminate than that of the pristine specimen as it does not account for the presence of a hole. One can also note that, as the hole moves to the left relative to the gap, and therefore the configuration goes from configuration 1 to configuration 2 and then configuration 3, the longitudinal stiffness reduces as the effective volume of the gap in the OHT laminate increases while the volume of the overlap progressively reduces. It is however, unexplained why the longitudinal stiffness of configuration 2 is not equal to that of the pristine even though the volumes of both defects are equal and should cancel out each other.

Figure 4.10: Stress-Strain curves of three main configurations
Table 4.3: Longitudinal stiffness of main configurations

<table>
<thead>
<tr>
<th></th>
<th>Pristine</th>
<th>Conf.1</th>
<th>Conf.2</th>
<th>Conf.3</th>
<th>CLT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$ (MPa)</td>
<td>53828</td>
<td>53809</td>
<td>53761</td>
<td>53742</td>
<td>54029</td>
</tr>
<tr>
<td>Variation with pristine (%)</td>
<td>NA</td>
<td>-0.04</td>
<td>-0.12</td>
<td>-0.16</td>
<td>+0.37</td>
</tr>
</tbody>
</table>

The global behavior of the four combinations is, as shown by the linear portion of the stress-strain curves, relatively similar up until the very end of the loading phase. This is not a surprising result as the failure mechanism of composite laminates in tension is generally described as a sudden-death, that is to say that little to no damage is observed before a catastrophic failure occurs. However, the stacking sequence selected based on the work of Wisnom et al. [110–112], includes ply blocks of two plies with the same orientation. Including two-layer ply blocks has been identified, by the aforementioned authors, to be a transition configuration between configurations driven by fiber failure and hence subject to abrupt failure when no ply-blocks are included in the stacking sequence, and configurations driven by delamination for which the presence of blocks of plies with the same orientation facilitate the propagation of delamination making it the main source of damage in the laminate. It is also important to note from the work of Wisnom et al. [110–112] that, despite slight differences between the material properties and ply thickness used between the present document and the mentioned articles, the order of magnitude for the failure stress in the present document is in good agreement with the experimental results presented by the authors. A $[45_2/90_2/-45_2/0_2]_{2s}$, with a hole of the same dimension as the laminates in the present numerical study, yielded at an average of 458 MPa experimentally when, as shown in Table 4.4, the pristine configuration failed at 454.54 MPa. Similar experimental study of open-hole specimens have been conducted by Gorlich [113] and by Green [114] and the final failure of the pristine laminate was established at 461 MPa and 463 MPa respectively by the authors.
To compare the three other configurations to each other, the value of the stress measured is normalized by multiplying it by the ratio between the longitudinal stiffness of the configuration of interest, reported in Table 4.3, and the longitudinal stiffness of the pristine configuration. Therefore, the values reported in Table 4.4 are the normalized values.

**Table 4.4: Failure stress/strain of main configurations**

<table>
<thead>
<tr>
<th></th>
<th>Pristine</th>
<th>Conf.1</th>
<th>Conf.2</th>
<th>Conf.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_1$ (MPa)</td>
<td>454.54</td>
<td>461.05</td>
<td>447.39</td>
<td>470.23</td>
</tr>
<tr>
<td>Variation with pristine (%)</td>
<td>NA</td>
<td>+1.43</td>
<td>-1.57</td>
<td>+3.45</td>
</tr>
<tr>
<td>$\epsilon_1$ (%)</td>
<td>0.866</td>
<td>0.87</td>
<td>0.843</td>
<td>0.895</td>
</tr>
<tr>
<td>Variation with pristine (%)</td>
<td>NA</td>
<td>+0.49</td>
<td>-2.55</td>
<td>+3.42</td>
</tr>
</tbody>
</table>

The zoom near the end of the loading phase on the behavior of the four laminates presented in Figure 4.11 allows to give a more individualized look at each configuration. In addition, to be able to compare the behavior of the various configurations, the normalized curves have been added and are represented by dashed lines. A first note must be made on the final behavior of the four laminates types. Two of them, namely the pristine and configuration 3, seem to exhibit an abrupt failure while configurations
1 and 2 present a more progressive drop in the load carrying capability. This difference must not be considered as an indication of a different mechanical behavior but simply the reflection of the limitation of a numerical model when approaching the final failure load. Indeed, within the last increments of the simulation some finite elements are excessively distorted which eventually leads to a convergence issue with CompDam. This leads to the creation, by Abaqus, of a final frame containing the information of the latest computed increment before too many elements failed to excessive distortion. Therefore, in some cases this occurs at a high load level and sometimes at a low load level in the specimen which explains the visual differences in the final behavior of the four laminates.

It can be observed that configuration 2 is the first to initiate a visible decline in stiffness, for a value of the strain around 0.825%, while the stress-strain curves of the other two configurations remain close to the pristine. In the case of configuration 2, the gap runs through the stress concentration zone generated by the hole therefore placing a weaker material susceptible to matrix cracking in an area where the stress is already higher which leads to a higher potential for damage and delamination initiation. In addition, the overlap is running through the stress concentration on the opposite side of the hole which means that a local stiffness increase is placed in an area where the stress is already higher leading to an even higher stress level precipitating damage and delamination. The combination of both defects running through the stress concentration zones on each side of the hole therefore has the potential to cause more damage and delamination in this configuration than in any of the other ones. This is in fact confirmed with Figures 4.12a and 4.12b where the percentage of elements damaged and the percentage of elements delaminated are respectively presented as a function of time. The percentages are based only on the elements in the damage region presented in Figure 4.6. For the percentage of damaged elements (in Figure 4.12a), a Python script is used to count how many brick elements,
in all the layers combined, exhibit a matrix damage higher than 1%. The results are then divided by the total number of brick elements in the damage region and multiplied by a hundred to yield a percentage. A similar process is conducted with the cohesive elements in the damage region to evaluate the percentage of elements delaminated. The symmetry of the position of the two defect types with respect to the hole in configuration 2 results in an equal volume of reduced properties for the gap and of increased properties for the overlap. Therefore resulting in a global stiffness of the laminate approaching that of the pristine configuration which explains the close proximity of the stress-strain curves up until the failure of the laminates. When it comes to the difference in failure load however, the defects still play a role as they are both, at least partially, running through the stress concentration created by the hole in the two middle zero-degree plies. The load that can no longer be carried by the damaged regions on both sides of the hole then needs to be transferred to the adjacent regions through interlaminar shear which results in a premature rupture of the laminate.

Looking at configuration 3 in Figure 4.11, we observe that the failure load and strain to failure are the highest of the four specimens studied. The gap in this configuration is fully positioned in the stress concentration area which allows, as will be shown in the following pages, for the initiation of the damage and the delamination to take place earlier in the loading process than it does in the other configurations, including the pristine. By generating more matrix cracks and delamination at a lower load level in the stress concentration area, the load redistribution can take place earlier and allow for the stress to be shifted towards regions with a higher safety factor and therefore allowing the specimen to pursue a bit longer in the loading phase. Damage and delamination then initiate in the stress concentration area on the other side of the hole, near the overlap. On this side, the presence of the overlap next to the hole, but not positioned at all in the stress concentration area, offers a region of higher
stiffness and lower failure index for the load to be redistributed from the damaged area. This, combined with the phenomenon described for the side where the gap is, explains why the specimen exhibits less damage and delamination globally than the other configurations and is able to sustain a higher stress/strain to failure.

A detailed stress-strain curve of configuration 3 with screenshots of the damaged elements at key points of the loading is presented in Figure 4.14. Four points of interest were identified and a screenshot of the damaged elements at these time points were captured to understand the initiation and propagation of damage in each configuration.

Figure 4.12: Percentage of elements damaged/delaminated
The first time point corresponds to the damage initiation at which the first element damaged is observed. This is represented with a yellow diamond in Figure 4.14. The associated screenshot indicates that the damage is initiating in the vicinity of the hole, in the ply containing the defect, and on the side of the overlap. This is coherent with the fact that the overlap, being stiffer, is subject to a higher stress than the other parts of the laminate. Therefore, the failure criterion in this region is satisfied earlier than it is in the other regions. The second time point corresponds to the loading point where a thousand elements are showing matrix damage. It is represented by an orange square and the associated screenshot allows to observe that the damage is concentrated around the hole in a seemingly balanced manner. The third point corresponds to the time at which the last frame is recorded by Abaqus, before the laminate’s failure. It is represented by a red triangle, and it can be observed that 45-degree-oriented cracks have propagated from the hole towards the edges of the laminate illustrating the upcoming final failure of the specimen. The final point corresponds to the failure point of the laminate and is represented by a black star in a circle in Figure 4.14. At
At this point, the damage has fully propagated through the laminate, two splits in the zero-degree plies (represented in green) have propagated from one end of the laminate to the other, and matrix cracks have spread in the 90-degree plies (represented in red). For a more progressive view of the propagation of the damage in configuration 3, the reader is referred to Appendix B where screenshots of the damage at other time points are presented in Figure B.5. It is important to note that the superposed stress-strain and delamination/damage curves are not synchronized. To read the percentage of damaged/delaminated elements at a given stress-strain level, the reader must look at the time stamp associated with the frame presented in the left side of the figure and then use the given time in seconds to find the number of elements damaged/delaminated. In the screenshots presented in Appendix B it can be observed that the matrix cracks in the 90-degree plies are following a 45-degree oriented line going from the hole towards the edges of the laminate. A line that actually matches with the orientation and location of matrix cracks in the adjacent 45-degree oriented plies.

Figure 4.14: Damage progression in configuration 3
Similarly to Figure 4.14 for configuration 3, the stress-strain curves of Configurations 1 and 2 with added screenshots of the state of the damage in the laminate are presented in Figures 4.15 and 4.16 respectively. Additionally, more details about the damage progression for these two configurations can also be found in Appendix B. Globally, the damage patterns and the amount of elements damaged in the three configurations are comparable. With the exception of configuration 3 which initiates the damage on the gap side of the defect, the configurations exhibit a first damage on the overlap side of the hole. The initiation is then followed by a propagation of the damage that stays limited to the vicinity of the hole until late in the loading phase. Prior to failure, cracks in the ±45 degree plies spread from the hole towards the edges of the laminate and the redistributed stress in the adjacent layers lead to multiple matrix cracks, following the ±45 degree crack line, in the adjacent 90 degree plies. The fiber pull-out failure mechanism observed by some authors (Wisnom et al. [112], Gorlich [113], and Green [114]) in specimens with thick ply blocks can be locally observed in specimens with defects on the side of the overlap in the form of matrix cracks, in green in Figures 4.15, 4.16, and 4.14, in the 0 degree oriented plies. Even though the general shape and amount of damage observed in the three configurations are similar, they still differ throughout time. Indeed, as illustrated in Figures B.6 and B.7, the load level at which the damage initiates and appears in a thousand elements respectively varies significantly with the pristine configuration. It can be observed that, as mentioned previously, configuration 3 is exhibiting damage at a lower stress level than any other while configuration 2, which fails at a lower stress/strain than the pristine configuration, is damaging earlier than the pristine and also shows a rate lower rate of elements degradation as the first thousand damaged elements only occur for a higher stress than it does for the pristine. Therefore, less stress redistribution can occur before approaching critical loads which can explain the premature failure of this configuration.
As emphasized by the reflections of the previous paragraphs on the zoom on the stress-strain curves at failure, the influence of the two defect types on the behavior of the laminates are interacting with each other making it difficult to identify which one has the major influence. This is the reason that justifies the interest given to what was referred to, in the previous pages, as the sub-configurations. They are analyzed to try to independently evaluate the influence of gaps and overlaps. Sub-configurations are limited to one defect type, gap or overlap, and the rest of the laminate remains identical to the parent configuration (1, 2, or 3). This therefore allows to shift the
focus on the influence of one single defect to try to understand the relative influence of a gap compared to an overlap for each of the main three configurations. For each of three main configurations, a cluster of graphs including the stress-strain curves, a failure load comparison with the pristine specimen, and a step by step representation of the damage initiation and propagation is presented in Figures 4.17, 4.18, and 4.19 for Configurations 1, 2, and 3 respectively.

Focusing on Configuration 1 first, it can be observed on the stress-strain curves presented in Figure 4.17a that globally the gap sub-configuration is doing better than its parent configuration and the overlap configuration is doing worse than the other two. It is important to note than no major variations within the stress-strain curves can be observed between the three specimens before a strain close to 0.81%. The differences only appear during the phase of damage propagation and until failure. This observation is also confirmed with the screenshots of the damage state in the laminates presented in Figure 4.17c. Looking at the first column on the left, it can be seen that the damage initiates at a similar simulation time and at the same physical location. The first damaged element appears to be in the area of the overlap (on the top of the 3D view). This is consistent with previous remarks made in earlier paragraphs where it was highlighted that the higher stress caused by the higher stiffness of an overlap combined with the fact that it is positioned in a stress concentration zone led to the failure index to be satisfied earlier in the loading phase for this area of the specimen. However, in the special case of sub-configuration 1 gap, no overlap is introduced in the laminate. Nonetheless, the side where the overlap should have been present is still subject to higher stress levels than the opposite side of the hole as it constitutes of a nominal tow with stiffer properties when compared to the gap present on the opposite side of the hole.
The failure load of the different variations of Configuration 1 are reported in Figure 4.17b in addition to which the variations of the failure load of the specimen with respect to the pristine configuration is also evaluated and displayed. The most significant of these variations is observed for the specimen containing only a gap. This aligns with previous comments on overlapping the hole on the gap region without placing the former in the stress concentration area. Indeed, the higher stress level in the stress concentration area in the vicinity of the hole will facilitate the creation
of local matrix cracks leading to the redistribution of the load through interlaminar shear in the adjacent layers and also in the region next to the damaged elements. This alleviates the laminate’s most sensitive areas and allows for the specimen to sustain more load before reaching failure by redistributing the load away from the stress concentration. In addition, since the bordering region is composed of a gap modeled by the material properties of pure matrix, cracks can initiate and propagate in this region too. Therefore allowing to delay reaching failure by creating matrix cracks in a non-primordial region for load carrying. In an opposing manner, for the sub-configuration 1 with an overlap placed in the stress concentration area, the combination with higher stress due to the higher stiffness facilitates the creation of cracks in a region supposed to carry load for the specimen. In Figure 4.17c, in the middle column, a screenshot of the location of the first thousand damaged elements is presented. In the case of C1-O, this screenshot was taken four frames earlier than it is for the other two configurations as the damage propagates faster due to the high stress level in the overlap. As a matter of fact, C1-O exhibits 20% more damaged elements than the parent configuration when looking at the damage status of both laminates at the same computational time. The increased rate of element damage in the laminate explains why C1-O failed before C1 and the pristine. It is also interesting to note that the amount of damage in the zero-degree oriented plies (highlighted in green in Figure 4.17c) is concentrated mainly in the overlap region and almost nonexistent in the C1-G configuration. This is an observation that adds to the explanation for the earlier failure of C1-O compared to the others as the damage is concentrating in a primary load carrying region.

Shifting the focus on Configuration 2 and following the format for the data used for the previous configuration, the stress-strain curves are presented in Figure 4.18a, the failure loads for each sub-configuration of Configuration 2 are shown in Figure 4.18b and compared to the value for the pristine model, finally the propagation of damage
is illustrated in Figure 4.18c. As a first observation, it can be seen that both C2-G and C2-O are doing either worse than or almost as good as Configuration 2 which was already doing worse than the pristine specimen. Several findings in the charts presented for Configuration 2 are comparable to what was identified for Configuration 1. First, one can note from the first column of Figure 4.18c, which represents the damage initiation for the three configurations, that elements on the side of the overlap are first to show damage and in the case of C2-G the damage still initiates on the
same side as where the overlap would have been since of the two sides it is the
stiffer one with a nominal tow compared to a gap. This is consistent with previous
observation made with Configuration 1. Secondly, no major deviations between the
specimens can be observed before approaching the laminate failure load. The only
difference in behavior, as mentioned previously, occurs during the short phase of
damage propagation.

Removing the gap from configuration 2 does not affect much the global behavior
of this specimen. The failure stress and strain levels (shown in Figure 4.18b) stay
comparable to that of Configuration 2 and the amount and location of damage
(displayed in Figure 4.18c) remain highly similar. On the other hand, removing the
overlap leads to the worst load carrying capability of all configurations tested in this
work. The presence of a gap in the stress concentration zone and in the bordering
area leads to premature failure of the laminate as matrix cracks easily form in the
stress concentration area. This leads to a stress redistribution in a weak region, a
gap, that will also crack.

Finally, turning our attention towards Configuration 3, the stress-strain curves of
the three configurations can be seen in Figure 4.19a, the failure loads are compared
with the pristine specimen in Figure 4.19b, and the damage initiation and propagation
is displayed in Figure 4.19c. A global observation of the results indicates that both
subsets C3-G and C3-O are yielding worse results than the parent Configuration 3
while still behaving better than the pristine specimen. Globally, both subsets behave
almost identically. No major differences can be observed between them up until the
very end of the loading phase and the final failure. Their respective failure loads are
almost equal and only the failure strain of the overlap configuration differs a little
by being slightly smaller. Even the damage propagation and location in C3-G and
C3-O (presented in Figure 4.19c) are similar. They, however, do differ on the location
of the damage initiation. Indeed, C3-G and C3 exhibit a first damaged element on
the top of the 3D view presented in Figure 4.19c which corresponds to the location of the gap area. On the other hand, C3-O initially damages on the opposite side of the hole in a nominal tow region. As opposed to the two configurations presented before, the overlap is not positioned in the stress concentration area, but the gap is. The weak material properties of the pure resin assigned to the gap combined with the high stress generated by the hole explain that the initiation of the damage occurs on the top part of the laminate for C3 and C3-G. In the case of C3-O, since the weakest
spot created by the gap is removed, the hole is surrounded on both sides by nominal tow material. The overlap is positioned flushed to the hole but outside of the stress concentration area. Therefore, both regions of higher stress, one due to the stress concentration and the other one due to the higher stiffness of the material, are next to each other and the damage logically occurs on this side of the laminate as it is more stressed.

Valuable information on the influence of defects can be reaped from comparing the sub-configurations containing the same defect type but placed at different location with respect to the hole. The stress-strain curves of C1-G, C2-G, and C3-G are plotted together in Figure 4.20a and their respective failure loads are reminded to the reader in Figure 4.20b. In order of the most beneficial to the most detrimental positioning of a gap defect, the positions go from overlapping with the hole but outside of the stress area, fully in the stress area and then halfway in the stress concentration area and halfway outside (C1-G - C3-G - (P) - C2-G). Both Configurations 1 and 3 with only a gap show a beneficial influence of the defect when the former is either placed fully outside of the stress concentration area or fully inside of it respectively. In the case of C3-G the formation of matrix cracks in the stress concentration area is facilitated by the presence of a weak material and the stress is redistributed in the more resilient
bordering regions effectively blunting the stress concentration by redistributing it. A similar phenomenon occurs in C1-G but at a higher load level since the material properties used in the stress concentration area is that of a nominal tow which only exhibits matrix cracks at a later loading stage. However, once cracked the stress is still redistributed in the same zone of the laminate which in this scenario is a gap. The gap material is prone to the formation of matrix cracks allowing to further spread the load redistribution and therefore delaying the final failure of the laminate. On the other hand, C2-G combines a gap in both the stress concentration area and in the zone of the laminate where the stress is normally redistributed after the apparition of matrix cracks in the vicinity of the hole. Therefore, the load can neither be redistributed in a zone with higher reserve factor nor can it be redistributed in a weaker zone capable of generating cracks to redistribute the load since the gap is already cracked. Hence the weakest behavior of the three configurations studied.

![Stress-Strain of Overlap configurations](image1)

![Failure load of Overlap configurations](image2)

(a) Stress-Strain of Overlap configurations  
(b) Failure load of Overlap configurations

Figure 4.21: Overlap configurations

A similar analysis can be conducted on the Overlap configuration C1-O, C2-O, and C3-O for which the stress-strain curves are presented in Figure 4.21a and the failure loads for the three specimens are reminded in Figure 4.21b. The order from the most beneficial to the most detrimental location to place the overlap with respect to the gap turned out to be the same as for the gap i.e. fully outside of the stress
concentration area, then fully in the stress concentration area, and finally halfway in and out of the stress concentration area (C3-O - (P) - C1-O - C2-O). The explanation behind the similar behavior is related to the fact that instead of having a weak material that generates matrix crack and therefore allows for the redistribution of local stress, overlaps are stiffer than the nominal material and consequently are subject to higher stress. The higher stress in the stiffer region combined with either the stress concentration of the hole, as in Configuration 3, or stress originating from a stress redistribution, as in Configuration 1, leads to earlier formation of matrix cracks than what would normally happen with the nominal material properties. In this way, gaps and overlaps influence the laminate in a similar manner. One must however note that the degree of variation of the final failure caused by the introduction of an overlap is always lower than that of a gap as shown in Figures 4.20b and 4.21b.

**Results from Advanced Defect Representation**

In the previous section, the results of the tensile behavior of ten specimens were presented. For nine of them at least one manufacturing defect was included in the vicinity of the hole using the simple approach presented in Chapter 3. This means that the results shown above do not take into consideration the geometrical perturbation caused by the introduction of a defect on the layers placed above it but rather only focus on the local changes on the material properties while neglecting the through-the-thickness perturbation caused by the defects. Following the advanced defect model
presented in Chapter 3, the results presented in this section for configuration 1, 2, and 3 also includes the through-the-thickness deformation of the laminates layers above the defects for the overlaps. To illustrate the deformation associated with the introduction of an overlap in the laminate, a cross-sectional view of the modified mesh in the OHT specimen is presented in Figure 4.22 where the elements highlighted in red correspond to the physical location of the overlap. It can be observed that the perturbation’s influence on the location of the nodes of the layers above the defects is not limited to the initial location of the overlap. Indeed, a transition zone can be identified between the defect - where the thickness is nearly double of the standard ply thickness - and the nominal tow away from the defect - where the thickness goes back to the standard ply thickness. This extended zone of influence of the defect, for an overlap configuration, appears to be extending no further than half of a defect width in each side of the actual physical location of the overlap. This means that the effective zone of the defect is no longer limited to the half a tow width physical location of the overlap but actually spreads for a total approaching a full tow width.

Figure 4.23: Configuration 1 with advanced defect model
Starting with configuration 1, the stress-strain curve of the advanced solution is presented in Figure 4.23a alongside the pristine configuration and the result from the simple model approach. Additionally, a comparison of the failure load level is shown in Figure 4.23b. The global behavior of the advanced solution is, in the elastic portion of the response, comparable to that of the simple approach. However, once the damage initiates in C1-Adv several drops in the stress can be observed. These drops are associated with the creation of damage in the laminate and seem to drive the laminate to fail at a lower load level than configuration 1 did as shown in Figure 4.23b. In the work conducted by Sawicki and Minguet [7], where the focus was on the compressive strength of open hole laminates, the conclusions drawn by the authors that the introduction of out of plane waviness through the introduction of a manufacturing defect is the main source of the 5 to 27% reduction in strength does not seem to transfer to open hole tension laminates presented here. Indeed, the behavior of C1-Adv, even though showing more frequent drops in stress, is still relatively similar to that of configuration 1 with the simple defect representation. The observed reduction in failure strength and strain for the laminate using the advanced modeling of the overlap confirms the observations made in the previous section with the simple model (section 4.2) that introducing an overlap in the stress concentration area will lead to premature cracking and failure of the laminate. The difference between the two approaches remains however limited.

Similar to the results presented in the previous paragraph, the advanced defect model was applied to configuration 2 and the generated stress-strain curve is presented in Figure 4.24a alongside its simple model counterpart and the pristine specimen. It can be observed that, even more than it was the case for configuration 1, the stress-strain curve is barely affected by the modifications conducted on the geometry of the laminate. The main difference appears to be that the stress-strain curve is smoother. Therefore indicating that the damage and the delamination are occurring
in a more continuous manner during the loading process and not intermittently when approaching the end of the loading phase as the simple model approach does. This result is not surprising as configuration 2 had already been identified as the most critical position for the defects to be placed in when looking at the results from the simple model. The overlap and the gap are indeed both placed in the stress concentration area and the zone which, in the pristine specimen, is used to support the stress redistribution following the generation of the first cracks in the vicinity of the hole. Adding the through-the-thickness modifications associated with the introduction of an overlap only spreads a little bit further the zone of influence of the defect through the transition zone mentioned previously. Yet, since it was observed that the transition zone does not extend any further than half a defect width on each side of the overlap, the zone effectively affected by adding an overlap remains confined to the same regions of the laminate than for the simple model as it does not lead to a direct interaction of the gap and the overlap. This observation explains why the behaviors of the two modeling approaches are so similar. Even the failure stresses, presented in Figure 4.24b, only differ by less than a percent indicating that the location, more than the representation mode, is the main driver of the influence of manufacturing defects on the quasi-static behavior of OHT laminates.
Finally, switching the focus to configuration 3, which was the specimen exhibiting the highest stress to failure using the simple model approach, the results from the advanced model are presented in Figure 4.25a for the stress-strain curves and in Figure 4.25b for the failure loads and the associated comparisons with the prediction established for the pristine specimen. A drastic change in the ultimate failure stress and strain can be observed. The specimen goes from a 3.43% increase in the failure load when compared to the pristine specimen to a 4.13% decrease for the advanced solution. As mentioned in the previous section, placing the overlap next to the stress concentration area and overlapping the hole leads to a stiffer material in the zone where the stress is redistributed after the first damage occurs on the left side of the specimen represented in the top-left corners of Figures 4.25a and 4.25b. However, with the addition of the geometrical imperfection caused by the introduction of an overlap, the variation from the pristine does not limit itself to material properties changes in the location of the defect itself but it also extends on both sides of the overlap. Indeed, the deformation of the top layers goes through a smooth transition from the defect area - where the thickness is doubled - to the nominal area - where the thickness goes back to the standard ply thickness as represented in Figure 4.22. This means, that the overlap, which originally was placed flush to the hole, but outside of
the stress concentration area, is now also introducing deformations in the region of higher stress. Therefore, what initially looked like configuration 3 is now in reality much closer to what was observed with configuration 2 with a defect spreading its influence in both the stress concentration area and in the region normally used during the stress redistribution phase which explains why the global behavior of C3-Adv is not only worse than the pristine configuration but is also comparable to what was seen with the simple model of C2.

Validation

Finally, to validate the numerical results presented above, a search in the published data for a similar laminate configuration which had been tested experimentally was conducted. In their work, O.J. Nixon et al. [115], experimentally studied the quasi-static and fatigue behavior of a pristine laminate of the $[45_2/90_2/-45_2/0_2]_s$ stacking sequence with a laminate of the dimensions presented in Figure 4.26. The specimens were manufactured by the authors’ team with the same material, IM7-8552, as the one used for the simulations shown in the present document. The average failure stress of the experimental specimens was evaluated by Nixon et al. [115] for two batches of coupons. The first batch of five specimens yielded at an average stress of 418 MPa with a coefficient of variation of 6.1%, while the second batch of three specimens yielded at an average stress of 447 MPa with a coefficient of variation of 2.58%. To validate the FE model method used to generate the data presented in the previous sections, a new FEM of the dimensions presented in Figure 4.26 was created. The failure load was evaluated numerically at 433 MPa using CompDam progressive damage analysis capability and it therefore in good agreement with the experimental results of Nixon et al. [115].
Figure 4.26: Geometry of validation specimen from [115]

(a) X-Ray CT-Scan from [115] at 80% of failure load

(b) Ply-by-ply damage in simulation

(c) Overall Damage in simulation

Figure 4.27: Comparison of CT-scan [115] and simulation

In addition to the stress at failure, the damage initiation and propagation prediction generated with the numerical simulation tools presented in this document can also be compared with the X-ray CT-scans images as shown in Figure 4.27. The X-ray CT-scan presented in Figure 4.27a were obtained by Nixon et al. by interrupting the test at 80% of the static failure load. To compare with the experimental results, screenshots of the damage state in the FEM at the first frame directly following the
specimen reaching 80% of the failure strain are presented on a ply-by-ply basis in Figure 4.27b - where the ply orientation is indicated at the bottom of each window - and as an overall view in Figure 4.27. A good concordance between the numerical results and the experimental observations can be seen both for the amount of damage present in the laminate as well as the location at which the damage initiates. Looking at ply orientations individually, one can note that the propagation of the matrix cracks is properly following the fiber orientation and that, in the 90 degree plies, the matrix cracks growth is indeed oriented at 90 degree but the initiation of new cracks is also following the 45 degree oriented trend observed in the CT scans due to the 45 degree orientation of the adjacent layer in the stacking sequence studied. Finally, a good agreement is observed between the simulation and the experimental results on the differences of damage propagation happening in the ±45 degree oriented layers where a single major crack develops in the +45 degree layers and a multitude of small cracks concentrated next to the hole are observed in the stress concentration area in the -45 degree layers. Consequently, despite the fact that the laminate used for validation is composed of half the number of layers in the specimen studied in the previous sections, the good agreement between the numerical and the experimental results observed for a specimen with the same stacking sequence but with thinner ply blocks is a good indication of the reliability of the results presented in the present document.

4.2.1 Conclusion

In this chapter two different modeling approaches were presented and applied to the case of two overlaps, stacked above each other in the two middle plies of a thirty-two plies OHT laminate under unidirectional tension loading. One of the models, namely the simple model, relied exclusively on representing the local modification to the material properties caused by the defects while the second approach, namely the
advanced defect model, also included the geometrical imperfections generated through-the-thickness of the laminate by the introduction of an overlap. Three different relative locations of the defect with respect to the hole were tested. The most critical of these configurations appeared to be the one for which both the overlap and the gap are both half overlapping the hole while the over half goes through the stress concentration area on each side of the hole. This specimen (configuration 2, presented in Figure 4.18 for the simple model and in Figure 4.24 for the advanced model) yields at a stress level 1.57% and 0.84% lower than the pristine specimen for the simple and the advanced model, respectively. Both approaches confirmed the same behavior of the laminate for which including a stiffer material, the overlap, in the stress concentration area facilitates the creation of cracks. Cracks which in turn lead to a stress redistribution away from the damaged area into what is still a stiffer material and therefore has a lower safety factor. On the other hand, specimens for which the gap is fully positioned in the stress concentration area and the overlap is fully over the hole like it is the case in configuration 3 (Figures 4.19 and 4.25 for the simple and the advanced models respectively) are yielding different results depending on the defect representation used. A non-negligible 3.45% increase of the ultimate failure stress is predicted by the simple defect model as the presence of the weaker material in the stress concentration area allows for early damage creation and therefore allows to alleviate the influence of the stress concentration by redistributing the stress away in a region with a higher reserve factor. In a completely opposite way, the advanced defect model for the same configuration exhibits a 4.13% reduction in the ultimate stress when compared to the pristine specimen. This significant change in behavior between the two approaches are due to the transition zone between the location of the physical defect and the return of the laminate to a nominal thickness unaffected by the overlap. In conclusion, both the location of the defect and the modeling approach seem to have a significant influence on the global behavior of the laminate and also seem to interact with each
other as was shown with the case of configuration 3. Globally, the influence of the introduction of defects in the middle plies of an OHT laminate on the quasi-static tensile behavior seems to be limited to a few percent variation on the ultimate failure stress. Additionally, depending on the relative location with respect to the hole, the influence can be either detrimental or beneficial. Finally, the introduction of a defect seems to drive the location of the initiation of the damage as it consistently appears first on the side of the overlap.
CHAPTER 5

MODELING THE INFLUENCE OF DEFECTS IN FATIGUE

In Chapter 4, the focus was given to the influence of the relative position of a selected type of defect, namely a combination of a gap and an overlap on opposite sides of a tow shifted by half its width from the nominal position, and a hole centrally located in a quasi-isotropic laminate loaded in longitudinal tension. Variation of the ultimate failure stress and strain ranged from a reduction of 4.13% to a augmentation of 4.11% of the ultimate failure stress. Additionally, variation in the location of the damage initiation was observed. Consequently, even if the order of magnitude of the influence of a single defect in a thirty-two plies laminate is relatively limited, it is in the logical continuity of the previous work to shift the focus onto the Tension-Tension fatigue behavior of similar laminate and defects configuration. In order to ensure consistency between the results presented in the current chapter and the previous one, the FEM generated previously are only slightly altered to include a second step for the fatigue analysis while the first one is used to introduce, quasi-statically, the maximum load. The modifications to CompDam to account for the fatigue loading are presented in section 5.1 before introducing the results in section 5.2. Finally, conclusions on the numerical analysis of the influence of defects on the fatigue behavior of the selected laminate are listed in section 5.3 alongside with comparison with already existing fatigue analysis techniques for pristine laminates.
5.1 Description of the model

Previously, for the quasi-static analysis of the OHT specimens, a consistent use of a single cohesive law within the user-subroutine CompDam was described and applied to capture both inter- and intra-laminar damage in the form of matrix cracks and delaminations respectfully. The aforementioned cohesive approach followed a conventional bi-linear law similar to the form of that presented in Figures 5.1 and 5.2 from the work of Carlos Davila [60] on a fatigue adaptation of the law. Any point outside out the domain delimited by the OET triangle corresponds to a state where the cohesive element is broken. On the OE portion, the cohesive element behaves linearly as no damage has yet occurred. Upon reaching the point E, the stress seen by the cohesive element reaches a critical value $\sigma_c$. Beyond this point, upon additional loading, the cohesive element starts exhibiting damage until it reaches the critical displacement value at point T and breaks. However, in fatigue, the element may never encounter a stress $\sigma_c$. Therefore, with the previous definition of the cohesive law, it is impossible to capture the initiation and propagation of damage. In his work, Carlos Davila [60] proposed the implementation of an additional damage variable to account for the reduction of the material properties as a function of the number of cycles applied $N$. For a point A, as shown in Figure 5.1, the maximum stress applied in the load spectrum is $\sigma_{\text{max}}$ which is smaller than $\sigma_c$. The initial slope, before applying any cycles to the model, of the portion OA is equal to $K$, the penalty stiffness. As the number of applied cycles increases, the point A travels towards point F as the fatigue damage variable $d^f$ increases and the slope of OA reduces, equating $(1 - d)K$. From Figure 5.2, it can be seen that upon reaching point F, the applied load $\sigma_{\text{max}}$ is equal to the residual strength $\sigma_{\text{res}}$ and the element breaks in fatigue without ever crossing the AEF section. In CompDam, this new definition of the cohesive law is implemented consistently for the capture of matrix cracks and the delamination of layers.
To ensure proper consistency between the fatigue analyses presented in this section and the quasi-static results presented in Chapter 4, the FEM generated previously are re-used. This ensures that the mesh, the material properties, and the geometrical characteristic of the laminates and of the defects, following the advanced defect approach presented in Chapter 3, are identical for the fatigue analyses. Only two modifications are conducted on the models. First, a second analysis step for the fatigue is added. During this step, the load is kept constant and equal to the maximum load seen in the fatigue spectrum. Secondly, since the imposed displacement in the first analysis step was initially set at a high value to ensure that, during the allocated time for the step, ultimate tensile failure would occur, the imposed displacement is
reduced to match the load severity level that needs to be simulated based on the final failure strain/displacement identified from the results of the pristine specimen presented in Figure 4.13 in Chapter 4. The pristine specimen was identified to fail at a displacement of 1.3mm in static and therefore, fatigue tests conducted at X% severity load are imposed a displacement of X% times 1.3mm.

5.2 **Open Hole Tension simulations**

The objective of the work presented in this chapter is to identify the influence of AFP-induced defects on the Tension-Tension fatigue behavior of the same composite laminate that was introduced in Chapter 4. This influence of the defect is measured by observing several indicators in the FEM simulations. First, the number of cycles to failure provides an initial and global understanding of the extent of the role played by the defect. Secondly, the type of damage, the location at which it initiates, as well as the final failure mode are all valuable information on the effect of the introduced perturbation. These information can, for instance, turn out to be useful in the design of an inspection plan. In order to be able to identify the influence of the gap/lap, it is necessary to first generate values of reference, similarly to what was done for the quasi-static approach, by focusing on a pristine specimen.

The transition from quasi-static to fatigue analyses leads to the introduction of new parameters and therefore increases even more the already significantly large design space identified in the previous chapters. To reduce the amount of configurations to study, the present work is only focusing on the Tension-Tension fatigue analyses with a load ratio $R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} = 0.1$. This selection is motivated by the fact that it is a fairly commonly used load ratio which increases the chances of finding, in the literature, data that can be used for validation of the results generated for the pristine specimen. Additionally, the energy release rate used for the definition of the cohesive law proposed by Carlos Davila [60] has been experimentally characterized for this
load ratio. Beside the load ratio, the load severity must also be considered. The load severity is defined as the ratio between the applied load (or displacement) to the quasi-static ultimate tensile failure load (or displacement). As the work conducted by Elsherbini [89] tends to indicate that the fatigue behavior of unidirectional laminates is more severely influenced by defects at high load severity than it is for lower load severity, the focus was given to 80%, 70%, and 60% load severity. Additionally, focusing on high load severity allows to reduce the computational time as specimens will fail at lower number of cycles. The user-subroutine CompDam [2] uses a cycle-jump approach governed by a series of parameters which allow to vary the number of cycles per time increment based on the damage propagation rate. Consequently, in the results presented afterwards, the elapsed simulation time is not directly proportional to the number of cycles applied. Similarly to the validation of the numerical results for static testing presented in Section 4.2, fatigue analysis of the specimen experimentally tested by Nixon et al. [116] were reproduced numerically at load severity of 80% and 70% to be compared with the experimental results presented in Figure 5.3. The numerical models yielded at 764 cycles and 5,120 cycles at load severity of 80% and 70% respectively which means that they are both in agreement with the results presented in Figure 5.3 since they fall within the observed experimental scatter.

Figure 5.3: Experimental Fatigue Results [115]
Figure 5.4: Deterministic SN Curve Pristine configuration

Shifting the focus back on the $[45_2/90_2/\text{−}45_2/0_2]_2s$ stacking sequence of interest for the present document, the results of the fatigue analyses at load severities 80%, 70%, and 60% for the pristine specimen are presented in Figure 5.4 alongside with a linear interpolation which will be referred to as the 'deterministic SN curve' in the following parts of this document. The number of cycles to failure for each load severity are also reported in Table 5.1. It is important to note that the results provided in this section are simulation of a strain-controlled fatigue test. A strain-control approach was favored to the load-control approach as, as the simulation progresses, the compliance of the laminate tends to increase due to the initiation and growth of damage and delamination. This increase in compliance would result, in the case of a load-controlled configuration, in a potential increase in the damage creation and propagation rate as the stress seen by the laminate would increase. On the opposite, with a strain-controlled approach, the increase in compliance combined with the imposed maximum strain results in a reduced stress level which can lead to a potential decrease in the damage propagation rate and therefore offers a more stable simulation capability.
### Table 5.1: Cycles to Failure for Deterministic Pristine Specimens

<table>
<thead>
<tr>
<th>Load severity (%)</th>
<th>Cycles to Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>80%</td>
<td>832</td>
</tr>
<tr>
<td>70%</td>
<td>6309</td>
</tr>
<tr>
<td>60%</td>
<td>25124</td>
</tr>
</tbody>
</table>

For each load severity tested, the stress curves as a function of time synchronized with the percentage of damaged/delaminated elements in the central region of the laminate are presented in Figures 5.5a, 5.5b, and 5.5c for load severities of 80%, 70%, and 60% respectively. The above mentioned figures follow the previously used color scheme for the stress, damage, and delamination curves. The stress curves are represented with a solid blue line, the damage curves (representing the percentage of elements exhibiting a minimum of 1% of damage in the central region of the laminate show in Figure 4.6) are a red solid line for the static loading phase and dashed during fatigue, and similarly the delamination curve is an orange solid line for the quasi-static loading phase and dashed during the fatigue analysis. Additionally, vertical yellow dashed lines were added at time points of interest in the loading phase and will be used later in this document to present the damage/delamination state of the laminate.

The first observation that can be made about the stress state presented in these figures is the suddenness of the failure. The reduction in the stress levels, and therefore of the longitudinal stiffness of the laminate, even though observable towards the end of the analyses in the 70% and 60% load severity cases, is relatively limited which is consistent with a fatigue Tension-Tension test for high-load severity/low cycles. One can also observe that the total amount of elements damaged or delaminated in the 60% load severity case is an order of magnitude smaller than it is for the other two cases which is consistent with the fact that the stress level is low compared to the stress to failure. As explained above, with a displacement-controlled specimen, the compliance increase due to the propagation of damage leads to a potential reduction
of the stress in the laminate. Therefore, this reduces the possibility for damage to initiate due to the local stress redistribution since the difference between the stress seen in the laminate and the critical static failure stress is much higher than it is for the higher load severity configurations. Consequently, the damage mainly initiates and propagates in fatigue through the properties reductions associated with the increasing number of cycles applied. The final observation that can be made on Figures 5.5a-5.5c is that, as the load severity decreases, the number of identifiable phases in the damage/delamination propagation increases as indicated by the amount of time points of interest delimited by the yellow dash lines.
As mentioned in the previous paragraphs, several phases in the initiation and propagation of damage and delamination can be identified on the curves shown in Figures 5.5a-5.5c. Screenshots of the damage and delamination states of a pristine laminate loaded at an 80% severity load at the time points A, B, and C identified in Figure 5.5a are presented in Figure 5.6. As already stated in Chapter 4 regarding the quasi-static behavior, the damage and delamination initiation occurs in the stress concentration area on both sides of the hole and spreads towards both edges of the laminate. Initially, a small amount of fiber splitting occurs in the middle zero degree
plies, quickly followed by matrix cracks in the 90 and ±45 degree plies. The fiber splitting does not propagate far from the hole but stays limited to a length roughly equal to the hole diameter. This is the starting point for the fatigue step as shown in Figures 5.6a and 5.6b for the damage and the delamination respectively.

During the first phase of the fatigue loading at a load severity of 80%, the specimen exhibits a rapid increase in the number of damaged/delaminated elements before reaching a “plateau” during which the damage/delamination propagation rate drastically reduced. Over this first phase, very little fiber splitting occurs in the zero degree plies as can be seen in the middle of the laminate in Figure 5.6c. However, a large amount of matrix cracks in the 90 and ±45 degree layers is generated while the delamination, while still propagating from the hole towards the edges of the laminates, lags behind and does not spread towards the grips as Figure 5.6d shows. However, upon reaching failure at point C, highlighted in Figure 5.5a, one can notice that the damage and delamination during the “plateau” phase was dominated by splitting in the zero degree plies spreading from the hole towards the grips and delamination propagating in-between the splits.

Following the same approach as previously used for the 80% load severity analysis and using the same formatting as indicated in Figures 5.6g and 5.6h for the damage and the delamination respectively, the evolution of the damage and the delamination during a 70% load severity fatigue analysis of a pristine specimen is presented in Figure 5.7 at the critical time points identified in Figure 5.5b. Starting by observing the similarities between the results for the two load severity (80% shown in Figure 5.6 and 70% shown in Figure 5.7), one can first notice that the location and the shape of the damage and the delamination regions are identical at point B. Likewise, the chronology of damage/delamination occurrence remains unchanged. Despite these similarities, the two simulations differ on several points. The first difference resides in the amount of damage and delamination present in the specimen at the beginning of
Figure 5.7: Damage/Delamination state - Pristine - 70% load severity

the fatigue analysis. Given the lower load levels in the lower load severity configuration, it is logical the damage state is less. Another primary difference is the total duration of the simulation, and consequently the number of cycles the laminate is subject to, which drastically increases with the reduced load severity. Since CompDam relies on a cycle jump approach proposed by Carlos Davila [60] which defines the maximum elemental damage increment per time solution increment, the number of cycles applied is not directly proportional to the simulation time. The focus is therefore given to the number of cycles and the damage growth rate. For example, Point B at 80% load
severity (Figure 5.6c) is reached faster than it is at 70% load severity (Figure 5.7c) since the damage growth rate for the lower load severity is lower. Finally, even after the load redistribution following the propagation of the damage between the fiber splits propagating from the hole towards the grips as depicted in both Figures 5.6e at 80% and 5.7e at 70%, the specimen loaded at 70% load severity does not suddenly break but enters a second slow phase of damage/delamination propagation leading to Point D (Figures 5.7g and 5.7h) and final failure of the specimen.

Moving on to the lowest load severity simulated for this document (60%), the damage and delamination states at the critical time points identified in Figure 5.5c are illustrated in Figure 5.8. The observations expressed in the previous paragraphs regarding the similarity of the damage and delamination location and shape are still valid. Despite starting from the least damaged/delaminated state of all the load severities studied, as illustrated by Figures 5.8a and 5.8b at point A for the damage and the delamination respectively, the chronology of the damage initiation remains unchanged. It is indeed possible to notice that point B, for a load severity of 80% (Figures 5.6c and 5.6d) and a load severity of 70% (Figures 5.7c and 5.7d), correspond exactly to the damage/delamination state exhibited by the 60% load case at Point C (Figures 5.8e and 5.8f). The major difference resides in the fact that the lower stress state allows for a more progressive propagation of the damage and consequently the identification of additional intermediate “plateau” such as Point B (Figures 5.8c and 5.8d). Despite the numerous resemblances in the failure patterns of the different severity scenarios, one must note the significant difference exhibited by the 60% load severity case regarding the total amount of damaged/delaminated elements upon failure of the specimen. This difference is approaching one order of magnitude as the results show in Figure 5.8 indicate that the damage is restricting itself to the main damage cracks and the secondary cracks in the 90 degree plies and off-axis plies are drastically reduced when compared to a higher load severity loading.
Figure 5.8: Damage/Delamination state - Pristine - 60% load severity scenario. Understandably, given the lower stress state throughout the laminate when compared to the other to cases, the potential for damage generation is either provided by the stress concentration introduced by the hole, which justifies the location of
the damage initiation as shown in Figure 5.8a, or by the local stress redistribution directly following nearby damage and delamination. Either way, the potential for the generation of secondary cracks is limited to a small geographical location on the specimen due to the lower stress state.

In the following paragraph, the results obtained for the defect configuration C3, presented in Chapter 4 in Figure 4.3, will be displayed using the same format that was used previously for the pristine configuration. The only exception relies in the fact that, applying a commonly used failure analysis threshold, specimens were considered to have failed in fatigue once the ratio between the residual longitudinal stiffness $E$ over the initial longitudinal stiffness at the beginning of the cyclic loading $E_0$ reached a critical value of 90%. This threshold, even though also applied to the pristine specimen, did not influence the results presented previously as the pristine specimen exhibited a sudden-death behavior for which no significant reduction in longitudinal stiffness was observed before an abrupt breakage of the laminate. However, when applied to the defect configuration, the number of cycles to failure for the C3 configuration is reduced as indicated in Table 5.2.

<table>
<thead>
<tr>
<th>Load severity (%)</th>
<th>Cycles to Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>80%</td>
<td>466</td>
</tr>
<tr>
<td>70%</td>
<td>2701</td>
</tr>
<tr>
<td>60%</td>
<td>24232</td>
</tr>
</tbody>
</table>

Combining the results for the pristine configuration documented in Table 5.1 and those of the configuration 3 shown in Table 5.2, two S-N curves have been generated and are presented in Figure 5.9. One can note that the higher the load severity is the larger the reduction in the fatigue life of the specimen is when introducing a defect. This result seems to be consistent with the observation from Elsheribi [89] on unidirectional laminates containing a centrally located gap. This observation is also
consistent with previous comments regarding the general stress state of the laminate at various load severity cases. It was indeed observed that the higher the load severity is the higher the percentage of damaged elements in the damage region ends up being at failure. An influence of defects being justified by the fact that the stress redistribution directly following the initiation of damage in the laminate leads to the redistribution of the stress in a laminate already exhibiting a generally more elevated stress state than its counterparts loaded at lower load severity. The potential for an element to reach the critical stress value to initiate damage is therefore enhanced. The local addition of a perturbation in the material properties and geometry of the laminate in the shape of a defect exacerbates this risk of creating more damage during the stress redistribution. Therefore, at high load severity, the defect acts as a catalyst as it increases the damage propagation rate and consequently reduces the number of cycles to failure when compared to a pristine specimen. In the following parts, screenshots of the damage and delamination state in the different specimens will be presented in Figures 5.11, 5.12, and 5.13 for load severities of 80%, 70%, and 60% respectively.

![SN Curves Pristine and C3](image)

Figure 5.9: Deterministic SN Curves
As it was done for the pristine configuration, the stress, damage, and delamination curves for each load severity configuration are plotted against the simulation time, which can be correlated to the number of cycles applied during the fatigue step, in Figures 5.10. Critical points have been identified based on the evolution of the percentage of elements damaged in the specimen and screenshots of the damage/delamination state in the laminate at these yellow dashed line highlighted time points are presented in the following paragraphs. At first glance, the evolution of the damage state in the laminates, regardless of the load severity level applied, are smoother than they were for the pristine configuration. Particularly, the 60% load severity case does not display clearly delimited “plateau”. The presence of the defect running the entire length of the laminate provides the environment necessary for the damage to more continuously develop throughout the cycles.

The evolution of the damage and the delamination state in the C3 laminate at the critical points highlighted in Figure 5.10a are presented in Figure 5.11. Focusing initially on the damage state at the beginning of the fatigue analysis shown in Figure 5.11a, at point A, it can be observed that, when compared to the starting point for the pristine configuration shown in Figure 5.6a, the distribution of the damage, even though highly similar, still presents small differences. The repartitions of matrix cracks on both sides of the hole in Configuration 3 are not symmetric like they were for the pristine configuration. Slightly more elements are damaged on the top part of Figure 5.6a which corresponds to the side of the gap. This is consistent with the fact that the resin rich region is a weaker material which leads to an earlier onset of damage and therefore more damage occurs on this side leading to the propagation all the way to the free edge of the laminate even before cyclic loading. On the other hand, the delamination (Figure 5.11b) does not show any significant differences with the pristine configuration. When shifting the focus on point B of the damage/delamination state (Figures 5.11c and 5.11d) the damage state in Configuration 3 is
Figure 5.10: Stress curves - Conf.3 specimen

showing a distribution of damage mainly in the 90 and ±45 degree layers and minimal splitting in the zero degree plies, similar to, though less severe, than the pristine configuration. Contrarily, the delamination state in the defect configuration (Figure 5.11d) is influenced by the defect when compared to the pristine (Figure 5.6d). A slight imbalance between the gap and overlap side can be observed. On the top part of Figure 5.11d, at the location of the gap, it can be observed a wider region of delamination between the inner zero degree plies, containing the defect, and the adjacent -45 degree plies represented by the region of purple color. This is consistent with the previous remarks regarding the influence of gaps on the initiation of damage which inherently leads to the redistribution of stress through interlaminar shear and eventually delamination. The imbalance perpetuates all the way to the
laminate failure as shown in Figure 5.11f. Globally, however, the amount of damage and delamination at failure in the configuration containing a defect is the same as it was for the pristine specimen. The difference resides in the location, as highlighted in the previous comments, and in the propagation rate as can be seen in the curves shown in Figures 5.5a and 5.10a for the pristine and C3 respectively. The initial rapid increase of the amount of damage is relatively similar between the two specimens with C3 being slightly less damaged but the “plateau”-phase for C3 shows a propagation rate of the damage and delamination sufficiently higher than for the pristine that it leads to a similar final state of damage/delamination between the two configurations.

Moving on to the 70% load severity case, the damage and delamination states in the laminate at the critical time point identified in Figure 5.10b are shown in Figure 5.12. When observing the damaging state of the laminate at Point A (Figure 5.12a) and comparing it to that of the pristine equivalent at the same time point (Figure 5.7a),
at the end of the quasi-static loading up to the maximum load in the cycling loading but before the beginning of the cyclic loading itself, no significant variation in the damage and delamination state can be observed apart from a slightly reduced amount of splitting in the zero degree plies on the side of the overlap. If the focus is shifted towards the second point of interest - point B - in Figures 5.12c and 5.12d for the damage and the delamination respectively, the distribution of the delamination is, just like it was at 80% load severity, still imbalanced and more heavily leaning towards the -45/0 interface surrounding the gap. Finally, upon reaching failure, the total amount and the distribution of damage in both specimens are similar while the delamination tends to be a bit more concentrated around the location of the defect.

Figure 5.12: Damage/Delamination state - Conf.3 - 70% load severity
As for the other two load severity configurations, the damage/delamination state in the laminate at the critical points identified in Figure 5.10c are shown in Figure 5.13 for the 60% load severity case. As it was mentioned in Chapter 4 on the analysis of the influence of the same defect configuration of the quasi-static behavior of the laminate, the main variations of behavior are observed upon reaching the quasi-static failure of the laminate. It is therefore, not surprising to observe the damage state of the defect configuration at the beginning of the fatigue step (Figure 5.13a) is quasi-identical to that of the pristine configuration at the same time point (Figure 5.8a).
The first observation that can be made when comparing the pristine configuration and the defect configuration at load severity of 60% is that it is not possible to identify clearly delimited phases in the propagation of the damage/delamination as attested by the smoother aspect of the damage and delamination curves shown in Figure 5.10c. Globally, the distribution of both the damage and the delamination for this load configuration is consistent with the results observed for higher load severities. In particular, the delamination surrounding the gap is still present.

5.3 Conclusion

In this chapter, the focus was given to the deterministic analysis of the fatigue behavior of the same composite laminate presented in Chapter 4 when subjected to Tension-Tension fatigue loading at a stress ratio $R = 0.1$. Studies were conducted at a load severity ranging from 80% to 60% of the ultimate static failure strain. From the numerical simulations, the number of cycles to failure were quantified and used to generate a S-N curve for the pristine specimen and screenshots at critical time points during the loading scenario were presented to highlight the various phases of the evolution of the damage and delamination states throughout the laminate. It was identified that for highest load severity levels, the amount of damage in the laminate at failure was higher than for the lower load severity cases because of the local stress redistribution occurring at failure of elements and resulting in a larger amount of matrix cracks in the 90 and $\pm45$ layers due to the already higher failure indices. For a second time, the results for a selected defect configuration, namely Configuration-3 introduced in Chapter 4 in Figure 4.3, were also presented for the same loading scenario that was imposed to the pristine specimen. The first conclusion was that the higher the load severity is, the more significant the reduction in the number of cycles to failure of the specimen containing a defect is when compared to the results of the pristine configuration. On the other hand, upon reaching a 60% load severity
the influence of the defect on the number of cycles to failure was less significant. The
global behavior of the two specimens, pristine or containing a defect, were found to be
highly similar in the way the damage initiates, where the first damage is observed and
how it spreads throughout the laminate. Slight differences were however identified
and more particularly so for the high load severity cases. A small imbalance in the
distribution of the delamination at the beginning of the cyclic loading was observed.
A higher amount of delaminated elements in the cohesive layers surrounding the inner
zero degree plies, which contain the defects, was observed on the side of the gap and is
believed to be due to the higher propensity of the resin rich region to generate matrix
cracks. This asymmetric distribution of the delamination during the early phases of
the cyclic loading were more and more difficult to identify as the load severity reduced
but was still observable at the failure in the form of delamination surrounding the
gap.

The influence of the presence of a defect in a composite laminate loaded in fatigue
seems to progressively diminish as the load severity does the same. To confirm this
observation, it is the authors belief that experimental testing should be conducted.
Interrupting the cyclic loading at various amounts of cycles applied to conduct non
destructive inspections to quantify the amount and location of damage/delamination
with, for instance, CT scans could provide valuable insight on the driving phenomenon
behind the differences with the pristine specimen. Additionally, focusing on other
loading cases such as compression could allow to identify other influences of the defects
such as the tendency to create local buckling in the layers adjacent to the defect due
to the induced out-of-plane waviness as identified by Sawicki and Minguet [7].
Chapter 6

Stochastic representation of material properties

Characterizing the influence of manufacturing defects on the quasi-static (chapter 4) and fatigue (chapter 5) behavior of composite laminates is heavily reliant on the accuracy and reliability of the results of the simulation of pristine specimens (not containing defects) as it sets a baseline for the failure load to be expected. Furthermore, damage related information such as the load level at which the damage initiates and how it propagates both in the first damaged layer and through the thickness of the laminate may be influenced substantially due to stochastic variation in properties. Indeed, the accuracy of the results generated for the pristine laminates regarding the damage initiation load level, the damage propagation path and the failure load levels significantly affect the ability to detect and quantify small disturbances provoked by the introduction of manufacturing defects. The progressive damage modeling solution employed in the previous chapters via CompDam, as many FEM based solutions do, is facing a mesh dependent issue on the damage initiation load level. Load levels at initiation in a pristine specimen cannot be compared to experimental results as they are driven by the free edge effects and therefore are typically mesh dependent. An example of this situation is illustrated in Figure 6.1 with the work of Brian Justusson and Imran Hyder on a unidirectional $[0]_4$ laminate. The results of a mesh convergence study are shown in Figure 6.1a and indicate that the threshold of 0.15mm for the mesh size is sufficient to reach the mesh convergence. However, the load levels for the
damage initiation in the same unidirectional laminate \([0\]_4\) are presented for different mesh sizes in Figure 6.1b. As can be seen, even after mesh convergence has been attained for an element size inferior or equal to 0.15mm, the load level for the damage initiation keeps on decreasing with the mesh size. This behavior is consistent with the local increase of the stress concentration near the free-edges with the reduction in size of the finite elements. The spurious stress increase at the free-edge of the laminate leads to spurious activation of the failure index used in Progressive Failure Analysis (PFA).

In previous attempts to tackle the issue of free-edge stresses causing the damage initiation [117], OHT specimens have been studied. The geometrical imperfection introduced in the form of a hole in the specimen was intended to introduce a stress concentration area in the expectation that damage would occur next to the hole first and not in the free-edge area. In other words, the hole mitigates the influence of the free-edge on the damage initiation load level. In addition, to counter the creation of spurious matrix cracks during the loading sequence, the evaluation of the state variable for the matrix damage was enabled, as explained in Chapter 4, only for a limited number of material strips (“matrix strips”) spatially spaced by a fixed number of rows of finite elements for which the matrix damage was disabled. Empirical results for matrix strips spaced every four rows of elements have shown to yield good results.
Once the issue of spurious damage initiation tackled, a second issue with regard to the ultimate failure load yielded by the FE model arises. Indeed, in the previous chapters that covered the quasi-static (chapter 4) and the fatigue (chapter 5) behavior of composite laminates containing a defect, results were presented using material properties obtained following ASTM standards and therefore the output of the analysis was deterministic: i.e., for a given specimen configuration, the output was limited to a single value of the ultimate failure strength. This provides an estimation of the average behavior of a specimen as it is based on average material property values but it however, fails to capture the scatter typically observed with experimental testing of the same coupons. The output, whichever the number of runs conducted on the model, is limited to one data point for the ultimate strength. To account for the variability of the material properties of composite materials, a common practice for the certification process is to evaluate the A- and B-basis values. A-values, used for single path load carrying components, are values that 99% of a population is supposed to equate or exceed with a 95% confidence. B-values, used for multiple load path carrying components, are values that 90% of a population is supposed to equate or exceed with a 95% confidence. However, the deterministic PFA presented in both Chapters 4 and 5 does not allow to generate a list of ultimate failure strengths for a given population. As a consequence, an industrial application for a validation of A- and B-basis values cannot be envisioned.

If the intention is to transition from a first order estimation of the influence of defects on the mechanical behavior of an industrial part towards the support of the certification process through the modeling of damage initiation and propagation, then it is necessary to be able to capture the propagation of uncertainty from the material properties level all the way to the ultimate strength or the number of cycles to failure for a given load configuration. In another words, the proposed numerical model must be able to take into account the non-homogeneous distribution of the
material properties exhibited by physical coupons and propagate the influence of the variations of the material properties from the microscopic scale towards the macroscopic behavior of the laminate under load. The work presented in this section is an attempt to develop such a methodology. The objective is to incorporate the ability to attribute an element-based material properties, which follows a distribution selected by the user, to each element in a FEM.

In the current configuration of the tool presented thereafter, the only supported type of distribution is a normal distribution with a user-given mean and standard deviation. The objective of the presented tool is to identify if, given an initial distribution for the material properties, the failure load or the number of cycles to failure can be represented by a stochastic distribution in which parameters can be related to those of the distribution used as an input for the model. In addition, the tool is tested to assert if the introduction of local perturbations in the material properties can contribute, similarly to what the introduction of the hole in specimens did in Chapters 4 and 5, to reducing the influence of the free-edges on the damage initiation load level in pristine specimens. Finally, an assessment of the potential application for industrial use is done to evaluate if the proposed solution would allow to reduce the need of lengthy and expensive experimental campaigns.

6.1 Implementation of a Stochastic Distribution

CompDam, the Abaqus VUMAT subroutine presented in Chapters 4 and 5, generates a single array of material properties per material specified by the user in the Abaqus input file. Which means that, as it is conventionally the case of FE models, only an average of each of the components of the material properties can be provided by the user. Usually two main materials are defined for PFA models: a cohesive element for cohesive interfaces between layers and a conventional material definition in the damage region to capture intraply damage. In the event that the damage region does not cover
the full length of the specimen, as it is the case in the models shown in Chapters 4 and 5 to reduce the computational cost of the progressive failure analysis, a “far-regions” material properties definition is also defined. The material properties definition in the “far-regions” is equivalent to the material property definition used for the damage region with the exception that all the features offered by CompDam through state variables are disabled. In other words, the far-region’s material properties are similar to the material properties used for the damage region with the exception that the damage will not be evaluated in these regions. Globally, this is a relatively conventional setup using average values of the material properties obtained through an extensive material characterization process. Experimentally, however, the results come in the form of a range of values, a distribution, due to the non-homogeneity of the material properties. Some fibers may exhibit a higher longitudinal modulus than others, some matrix to fiber bonds may be locally weaker than at other locations within the laminate.

In the previous section (section 6.1), issues regarding the initiation of damage have been identified and a solution based on a stochastic representation of the material properties has been proposed. In this section, a way to implement the solution in the already existing Abaqus subroutine CompDam is presented. The implementation of a statistical representation of the material properties in a FE is divided in several steps. First, one must determine the properties of primary interest. Since the tests presented in the previous sections on static (Chapter 4) and fatigue (Chapter 5) were tension tests, the longitudinal modulus is the variable selected to be modified. Secondly, since extensive material characterization campaigns for IM7-8552 have been conducted [118], the mean and standard error of the longitudinal modulus are recovered from NIAR’s experimental characterization in that reference. Finally, since the main difficulty resides in the implementation of the statistical law with the FE subroutine and not in the type of law itself, a normal distribution of the longitudinal
modulus is implemented. The influence of such a distribution on a AS4/PEEK system has been studied in the work of Naderi and Khonsari [119], and a good reproducibility of the experimental damage was observed with simulations with various distributions. The following paragraphs explain the implementation process followed as well as the normality tests conducted to verify the quality of the data generated.

6.1.1 Generate a normal distribution

Assuming that the material property of interest is best represented by a normal distribution based on the available experimental data, a potential way of implementing a number generator following a normal distribution is to use the basic form of the Box-Muller transform. The Box-Muller transform is a method to generate a pair of random numbers, $Z_1$ and $Z_2$, following independent normalized pseudo-normal standard distributions $\mathcal{N}(0, 1)$ in $[0, 1]$ from a pair, $U_1$ and $U_2$, of uniformly distributed random numbers $\mathcal{U}(0, 1)$ in $[0, 1]$. The generation method for the two normally distributed variables $Z_1$ and $Z_2$ is presented in equation 6.1.1. We must however, first generate a set of independent and uniformly distributed variables $U_1$ and $U_2$ and one of the simplest way to do so is to use a Linear Congruential Generator (LCG).

$$\begin{align*}
Z_1 &= \sqrt{-2 \cdot ln(U_1)} \cdot \cos(2\pi U_2) \\
Z_2 &= \sqrt{-2 \cdot ln(U_1)} \cdot \sin(2\pi U_2)
\end{align*}$$  

(6.1.1)

LCG’s are one of the oldest ways to generate a pseudo-random number. They also present the interest of being fast and to require a small amount of memory. Since the intended application is for a progressive failure FE model which is generally meshed densely, the periodicity of the random number generator is of significant importance. The periodicity indeed defines the length of consecutive outputs of the recurrence relation presented in equation 6.1.2 before the output repeats itself with the same pattern. It is therefore necessary to have the longest periodicity possible. Thankfully,
if the parameters for the recurrence law presented thereafter are selected in a proper manner, the periodicity of an LCG can be as long as needed. Given an initial seed $X_0$, a LCG will generate the next random number in a uniform distribution by following the recurrence relation 6.1.2 where “$m$” is the modulo, “$a$” is the multiplier, and “$c$” is the increment. A simplified version of LCG for which the value of the increment is set to zero, and the multiplier is taken as a positive integer smaller than “$m$” is called a Multiplicative Congruential Generator (MCG) and is used for this work.

$$X_{n+1} = (a \cdot X_n + c) \pmod{m} \quad (6.1.2)$$

In addition, to ensure a periodicity of $m - 1$ for the random number generator, one can select a prime modulo “$m$” and a multiplier “$a$” that is primitive root of “$m$”. A number “$a$” is said to be a primitive root modulo “$m$” if every number “$b$” coprime to “$m$” is congruent to a power of “$a$” modulo “$m$”. That is to say that for every integer “$b$” coprime to “$m$”, there is some integer ‘$k$’ for which $a^k \equiv b \pmod{m}$. In this configuration, the initial state of the recurrence “$X_0$” can be freely selected in the range $[0, m - 1]$. The generator implemented and presented in this section is, in order to avoid repetitions in the FE model, be a MCG with a prime modulo. More specifically, “$m$” will be a Mersenne prime: $m = 2^{31} - 1$ and the multiplier “$a$” is taken equal to $a = 7^5 = 16807$ as it is a popular choice that has already been widely studied and implemented.

An implementation in Python of the approach described above is available in the appendix A.1. Following the MCG recurrence presented in equation 6.1.2, two independent, normalized, and uniformly distributed variables $U_1$ and $U_2$ are created and are then used as input for the Box-Muller transform summarized with equation 6.1.1 to generate the normalized pseudo-normal distributions $Z_1$ and $Z_2$. Finally, to obtain a variable $Z$ following a normal distribution with a mean $\mu$ and a standard deviation $\sigma \mathcal{N}(\mu, \sigma^2)$, $Z_1$ is used as presented in equation 6.1.3.
To ensure that the proposed Python tool implementation presented in appendix A.1 is capable of generating a variable following the desired statistical law, an example of 5,000 samples as been generated. As an input for the Python tool to generate the distribution, an initial seed $X_0 = 123456789$, a requested standard deviation $\sigma = 1$, and a requested mean value $\mu = 0$ were provided. Graphical representations as the histogram (Figure 6.2a) and the Q-Q (Quantile-Quantile) plot (Figure 6.2b) presented in Figure 6.2 are available to quickly and visually assess how close the population is to a normal distribution. QQ-plots are a statistical, graphic method to represent a distribution to verify the quality of the distribution generated. The Q-Q plot compares the quantiles of two distributions. Either two theoretical distributions against each other or, as is the case here, the quantiles of the population of interest on the y-axis against the quantiles of the theoretical normal distribution on the x-axis are compared. As a result, the closer the points are from the bisector line y=x in Figure 6.2b, the closer the population studied is to the theoretical expectation. As for the histogram in Figure 6.2a, a typical bell-curve shape is expected for a normal distribution. In Figure 6.2, a best fit of the data by a normal distribution is presented in a red dashed line. The parameters of the best fit are also evaluated as $\mu = -0.009$ and $\sigma = 0.996$ and included in the title of the figure. Both values are really close to the mean (0.0) and standard deviation (1.0) requested during the generation process. Regarding the Q-Q plot in Figure 6.2b, the points seems to fall on the y=x curve. The visual inspection confirms the normal distribution hypothesis, however, as a safety measure, more mathematically rigorous tests than a simple visual validation of the normality of a distribution exist and can be used to verify the quality of the data created.
(a) Histogram: \( \mu = -0.009, \sigma = 0.996 \)

(b) QQ Plot

Figure 6.2: Normality test on python generated sample

Several tests are commonly brought into play to check the normality of a sample. The most common ones are the Shapiro-Wilk test [120], the D’Agostino-Pearson test [121], and the Anderson-Darling test [122]. The null hypothesis, that we will call \( H_0 \), of these tests is that the provided population is normally distributed. Two indicators are available to verify if the null hypothesis is valid or not: the \( p \)-value and the test statistic \( W \). The closer to one \( W \) is and the higher the \( p \)-value is compared to the significance level \( \alpha \), the better. The test statistic \( W \) is a single numerical value, specific to the test conducted, that represents the sample being tested. The closer to one it is, the higher the chances of the null hypothesis to be true are. It is, however, very important to note that if the provided population is too small the test is rarely going to reject the null hypothesis. Similarly, if the population is too large, 5000 values being the cut-off, any small differences from the normal distribution will lead
to the rejection of $H_0$. Hence the choice of 5000 values in the sample data used to verify the quality of the generator. Technically, these tests, through the value of the test statistic $W$ and the $p$-value, are only indicating if a significant variation from the theoretical normal distribution is detected.

The normality tests mentioned previously are implemented in the Python library SciPy, and were theretofore used. It is important to note that SciPy’s documentation for the Shapiro-Wilk test warns the user of a 5% risk of false negative result regardless of the dimension of the dataset. No significant departure from normality was reported by the Shapiro-Wilk test (Test statistic $W$: 0.999, $p$-value: 0.267), nor by the D'Agostino-Pearson test (Test statistic $W$: 1.068, $p$-value: 0.586). The Anderson-Darling relies on a set of pre-tabulated values called the significance level ($sl$) for a set of given critical values ($cv$). This pre-tabulated data depends on the statistical law that the user wants to test their data against and, in the case of a normal distribution, the tabulated data are as shown in the first two columns of Table 6.1. SciPy’s Anderson-Darling test for the sample presented before provided the results summarized in Table 6.1 with a test statistic $W = 0.644$. The null hypothesis $H_0$ is rejected at the critical value when the test statistic is larger than the significance level, which in the current table corresponds to the values of the pre-tabulated data for a normal distributions. Given the visual inspection and the results of the normality test, we can conclude that the population generated is unlikely to not follow a normal distribution. For brevity’s sake, any further mention of the population generated will say that it follows a normal distribution.

We now know that we can easily generate a population which follows a standard normal distribution ($X \sim \mathcal{N}(0, 1)$) and therefore we can adapt the generated population to material properties following a normal distribution with a mean $\mu$ and a standard deviation $\sigma$ ($X \sim \mathcal{N}(\mu, \sigma^2)$). To incorporate the distribution presented in the paragraphs above into a FEM, a new state variable is created for the material
Table 6.1: Results of the Anderson-Darling test for normality

<table>
<thead>
<tr>
<th>Critical Value (cv)</th>
<th>Significance Level (sl)</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>15%</td>
<td>0.576</td>
<td>Fail $H_0$</td>
</tr>
<tr>
<td>10%</td>
<td>0.655</td>
<td>Pass $H_0$</td>
</tr>
<tr>
<td>5%</td>
<td>0.786</td>
<td>Pass $H_0$</td>
</tr>
<tr>
<td>2.5%</td>
<td>0.917</td>
<td>Pass $H_0$</td>
</tr>
<tr>
<td>1%</td>
<td>1.091</td>
<td>Pass $H_0$</td>
</tr>
</tbody>
</table>

applied in the damage region of CompDam. Additionally, the first value of the seed used by the normal number generator is hard-coded in the Fortran code of the subroutine. Therefore, for each element of the model, at the beginning of the analysis, at $t = 0$, the state variable is initialized to a value from the normally distributed population and the seed is updated so as to be used for the next element. A flowchart representation of the initialization process used for the state variable $SV_{Distri}$ is presented in Figure 6.3.

Figure 6.3: Flowchart of state variable initialization
6.2 Application to OHT Static

In the previous section, a mathematical solution was presented to generate a continuous stochastic variable following a normal distribution by initiating it with a seed value chosen by the user. This solution was incorporated within CompDam, the ABAQUS user-subroutine used to generate the quasi-static and fatigue results discussed in chapters 4 and 5 respectively, in the form of an additional state variable. Therefore, it is possible to create an element-based integration of local material properties variation in a FE model. The process presented in the previous section is used at the initialization of the FE analysis to generate, element by element, a value following a normal distribution with a null mean $\mu$ and a unit standard deviation $\sigma$ as well as an updated value of the seed to be used to generate the next value in the normal distribution. Once attributed a value, the state variable can be used as a coefficient of correction to apply to any of the material properties of interest before they are used by ABAQUS to evaluate the local stiffness matrix of an individual element. It is important to note that the selection, by the user, of the initial value of the seed used by the normal distribution generator offers the ability to track the specimen and its associated distribution.

![Normal distribution in OHT specimen](image)

Figure 6.4: Normal distribution in OHT specimen
The objective of this chapter being more of a show of capability to understand the possibility to implement a propagation of uncertainty in a commercial FE software, only one state variable was generated and consequently only one of the material properties can be represented by a stochastic variable to capture the local variations of stiffness. It is however easy to extend the proposed approach to a multitude of material properties through the use of several state variables following independent distributions. However, since the configurations studied consist of OH specimens in tension, the longitudinal modulus $E_1$ is the main driver of the load carrying capability. The longitudinal modulus $E_1$ is, therefore, the selected variable to be modified in the models presented below. As an example, the spacial distribution of the stochastic coefficient of correction to apply to the longitudinal stiffness in an OHT model is presented in Figure 6.4. The value of the stochastic coefficient of correction is applied to the longitudinal stiffness of each element following Equation 6.2.1. Just as the example presented in Figure 6.2a, it was requested from the normal distribution generator to create a population following a normal distribution with a mean of zero and a standard deviation of one. Since the number of finite elements in the OH model is too large to be checked with the most common normality tests and that the 5,000 data sample tested in section 6.1.1 yielded satisfactory results, a simple visual inspection of the quality of the distribution is conducted. It can be observed that the image, in Figure 6.4, appears to be predominantly green which is a visual confirmation that the mean value of the generated population is approaching the requested null value. No direct confirmation of the standard deviation can be obtained from the figure but we can however notice that the minimum and maximum values are close to the $\pm 3\sigma$ from the mean value of zero that delimits 99.7% of the population of a normal distribution.
A direct application of the stochastic representation of material properties has been conducted on a pristine specimen with the same geometrical characteristics as the laminate studied in Chapters 4 and 5. It is therefore a 300mm long by 36mm wide, 32-ply conventional laminate with a $[45_2/90_2/-45_2/0_2/45_2/90_2/-45_2/0_2]_s$ stacking sequence, and a hole of 6.0mm diameter in the middle as presented in Figure 4.1. The only modification to the model was, before the first increment of the simulation (for an elapsed time $\Delta t = 0$), evaluating the new state variable for the correction coefficient to follow a normal distribution. The longitudinal stiffness of each element is then modified before its used in the evaluation of the stiffness matrix following equation 6.2.1 where $E_1$ is the longitudinal modulus, $CV$ is the coefficient of variation of the material property defined by $CV = \mu/\sigma$, and $X$ is the statistical variable following a standard normal distribution. Effectively, the initial standard deviation generated for the state variable is stretched by a factor $\sigma$ and shifted by a factor $\mu$ both condensed in the coefficient of variation $CV$ extracted from the experimental
data of Clarskon et al. [123]. The results, in the form of stress-strain curves, obtained with ten different values of the initial seed for the normal distribution generator are presented in Figure 6.5 where a zoom on the final failure of the ten specimens is depicted.

\[ E_1 = \sigma \cdot (1 + CV \cdot X) \]  

(6.2.1)

Before diving into the details of the variations observed near the end of the loading process for the ten specimens with a different distribution of the material properties, it is interesting to take a closer look at the longitudinal stiffness of the laminate during the elastic loading phase. The average value of the longitudinal stiffness of the twenty specimens (53.87 GPa) only differs from the reference pristine by 0.08% and the standard error is limited to 7.68 MPa. Therefore, we can safely conclude that the inclusion of the element-based distribution does not modify the linear behavior of the laminate.

However, going back to the stress-strain curves presented in Figure 6.5, both the ultimate failure load and the ultimate failure strain are subject to changes comparable to those observed with the introduction of defects in Chapter 4. The limits, for the configurations analyzed, are bounded by Seed 0 for the lower bound of the ultimate failure strain of 0.838% and Seed 7 for the upper bound with a strain of 0.901%. Similarly, Seed 0 also defines the lower limit for the ultimate failure strength with 448.72 MPa and Seed 7 defines the upper limit as 475.14 MPa. It must be mentioned here again that the values listed above correspond to the strain and stress level in the far-region at the peak of the stress-strain curve and not at the numerical final failure. Additionally, the number of different distributions was limited to twenty due to the duration of the analysis. The variations of the ultimate failure stress and strain are represented in Figures 6.6a and 6.6b respectively where they are compared to the value of the pristine specimen and the standard error of the generated population.
is represented by the black error bar. On average the failure load of the stochastic specimen is 459.94 MPa which is 1.19% higher than the pristine specimen and the standard error is equal to 3.1 MPa. The average failure strain is 0.869% which corresponds to a 0.37% increase compared to the pristine specimen. The standard error for the failure strain is 0.007%. In the end, one can conclude that the introduction of the normal distribution of the longitudinal modulus seems to affect the stress-strain response in a significant manner similar to the specimens containing defects in Chapter 4.

Some specimens exhibit several local peaks in the stress-strain curves, indicating several phases in the development of damage/delamination in the layers. The most predominant example shown in Figure 6.6 is Seed 7 with three local maxima at respective strain levels of approximately 0.865%, 0.88%, and 0.9%. On the other hand, some specimens show a less progressive reduction in the longitudinal stiffness of the laminate without major drops in the load which is an indicator of a more spontaneous propagation of the damage/delamination than for the specimens mentioned previously. A typical representation of this behavior is portrayed by Seed 3 for which the stress-strain curve only trends down past the 0.85% strain point. One important observation that can be made from both of these damage propagation modes in the stochastic specimens is that specimens in the first category, i.e. specimens exhibiting a more continuous propagation of damage, demonstrate a more damage tolerant behavior as
they fail at a higher ultimate failure stress/strain than their counterparts in the second category. This goes in the same direction as the observations made in Chapter 4 on the influence of defects regarding the importance for the laminate to be able to, from as early as possible in the loading phase, generate matrix cracks that allow to redistribute local stress concentrations on a larger surface.

In addition to affecting the rate of propagation of damage/delamination, the variations in the spatial distribution of material properties also affect the amount of elements exhibiting damage/delamination. Screen-shots of the state of the damage, at the last frame of the analysis, in the top half plies of the laminate in Seed 3 and Seed 7 are presented in Figures 6.7a and 6.8a respectively. The damage is presented via planar views of the damaged elements (in red) for one layer of each two-plies blocks.
in the top half of the stacking sequence. The ply index and the fiber orientation angle are specified in the bottom right corner of each cut view. Each window presents an X-Y view of the laminate with the x-axis in the horizontal direction and the y-axis in the transverse direction. The loading is applied in the horizontal direction.

Figure 6.8: Failure states in Seed 7

Figure 6.9: Breaking modes in OHT specimens [112]
First, note that globally the shape of the damaged area in each ply is consistent between configurations. In 45 degree oriented plies, the main cracks can be observed following a 45 degree oriented line initiating from the side of the hole and spreading towards the edges of the laminate. A screen-shot, for each configuration, of the damage in the laminates’ outermost 45 degree plies is presented in Figure 6.10. From this figure, one can identify what can be called a primary damage in the form of a 45 degree oriented line spreading from the side of the hole towards the edges of the laminate. Additionally, note that the initiation point is not at the center of the stress concentration area but more towards the front and back sides of the hole. This primary damage appears at the same location in all the specimens. It is one of the driving phenomenon of failure and can be seen in physical specimens breaking either by pull-out or delamination as shown in Figure 6.9 from the work of Wisnom et al. [112]. It is identifiable by the presence of 45 degree oriented broken edges. Then, secondary matrix cracks, still following a 45 degree orientation, can be observed. This secondary type of damage is where the variation of the material properties seems to have the most influence. One can, for instance, focus again on the Seed 3 and Seed 7 cases and observe that the primary damage is shared by both configurations but the matrix cracks density for the secondary damage is higher in Seed 7 than it is for Seed 3. Similarly, for the 90 degree oriented plies, matrix cracks also initiate from the stress concentration area in the vicinity of the hole before spreading transversely. A screen-shot of the final stage of the damage in the outermost 90 degree plies in each of the stochastic specimens is presented in Figure 6.11. It is worth mentioning that, since the adjacent layers are ± 45 degree layers, subsequent matrix crack initiations follow a global ± 45 degree line following the primary damage in the ± 45 degree plies and appear to be bounded, in width, by the secondary matrix cracks in the adjacent layers.
Secondly, the plot of stress-strain curves and the damage/delamination curves confirms that the specimen Seed 7 is globally more damaged/delaminated than Seed 3 by the end of the loading phase with around 10% of elements damaged compared to 8% for Seed 3. This is supported by comparing the off-axis plies between Figures 6.7a and 6.8a. In the case of Seed 7, a significantly higher amount of matrix cracks in ply 29 (90 degree) can be observed. Additionally, a slight increase in the number of damaged elements can also be seen in the ± 45 degree plies (plies 31 and 23, and plies 27 and 19 respectively). Interestingly too, splitting appeared in the middle, 0 degree oriented, four-ply block. This reminds us that the selected configuration was identified by Wisnom et al.[110, 111] to be a transition configuration between specimens for which the failure was dominated by fiber failure and specimens whose failure was governed by delamination when the plies orientations were dispersed or
in blocks respectively. Introducing local perturbations in the form of the normal
distribution seems to help tip the failure mode to one option or the other therefore
confirming that a two-ply blocks configuration can either fail by delamination or fiber
splitting.

In the work of Wisnom et al. [110], specimens failing by delamination did so after
exhibiting the largest amount of delamination at the interface between the zero-degree
plies in the middle of the laminate and the surrounding minus forty-five-degree plies.
This delamination was associated with the first major drop in load observed on the
stress-strain curves. Similarly to the damage state discussed in the above paragraphs,
the delamination in each cohesive layer in the top half of the laminate are presented
in Figure 6.7b and 6.8b for Seed 3 and Seed 7 respectively and a particular focus can
be given to the layers corresponding to the interface of minus forty-five-degree layers
and zero-degree layers (bottom row). One can note that, all layers combined, Seed 7
shows more delamination than Seed 3 with 6% of elements delaminated versus 4.5%
respectively. Globally, the interface between the four-ply block in the middle of the

Figure 6.11: Damage in outer-most 90 degree ply
lamine and the minus forty-five layers appears to be exhibiting a similar amount of delamination and an akin pattern between the two specimens. More significant variations can however be observed on the first 45/90 interface/cohesive layer in the secondary delamination. A more in-depth comparison of the delaminations in this same interface, for all the stochastic configurations, is presented in Figure 6.12. Just as it was for the case for the damage presented previously, one can identify a primary delamination pattern at the same location as the primary damage. This is a localization that is consistent with the fact that the creation of the first damaged elements in the primary pattern lead to a load redistribution through interlaminar shear into the adjacent layers which eventually leads to local delaminations right above and below the damaged elements. The reader’s attention is directed towards **Seed 9** in particular. Since the delamination presented in Figure 6.12 corresponds to the interface between the outer 45 degree plies and the adjacent 90 degree plies, it is interesting to note that the localization of delamination is in good concordance with the damage presented in Figure 6.11 for the 90 degree plies.

![Figure 6.12: Delamination in outer-most 45 degree ply](image-url)

Figure 6.12: Delamination in outer-most 45 degree ply
6.3 Application to OHT Fatigue

In Chapter 5, the results of the fatigue analyses for the pristine specimen alongside with two defect configurations, using a deterministic approach, were presented. However, in the continuity of the approach presented in this chapter, a stochastic implementation of the material properties was also used to understand the influence of local changes in the material properties on the number of cycles to failure as well as the failure modes exhibited by the specimens. The objective of the work presented in this section is to compare multiple approaches to generate S-N curves to capture the behavior in fatigue of the OHT specimens with defects. These methods are: a stochastic representation of the material properties as described previously in this chapter, a parallel curve approach based on the statistical properties in static, and the Load Enhancement Factor presented by Whitehead et al. [124]. Each of these methods will be discussed in more detail in this section.
6.3.1 **Stochastic Approach**

In the previous section, a local modification of material properties through a user-subroutine was implemented and demonstrated on the static tension test of OHT specimens. In the continuity of this work, the same concept is applied to the fatigue analyses presented in Chapter 5. The objective of applying this concept of stochastic material properties, only to the pristine specimen, is to define an interval of confidence for the number of cycles to failure at different load severities. In an attempt to develop a consistent approach, the seed values used to generate the normally distributed material properties in section 6.2 are re-used for the fatigue analyses and will be referred to with the same denominations. The work presented in this chapter is an attempt to confirm the Strength-Life Equal Rank Assumption (SLERA) first introduced by H.T. Hahn and R.Y. Kim [125] which states that the rank of a specimen in fatigue life is equal to its rank in static strength. This therefore allows, knowing the distribution followed by the static strength population generated with the stochastic approach in subsection 6.2, to determine the B-basis value of the fatigue life and an interval of cycles to failure at various load severities for the pristine specimen using the 10th percentile and 90th percentile of the statistical law identified for the static tests.

As explained above, the prediction of the interval of confidence for the number of cycles to failure relies on the identification of the statistical law followed by the static strength population. Commonly, this type of data is well represented with a two-parameter Weibull law for composite materials as explained in the work of Lee et al. [126]. This was therefore the selected law for the characterization of IM7/8552 used for this work. The FEM being displacement controlled, the determination of the two-parameter Weibull law was conducted on the ultimate failure strains reported in Table 6.2 where the “Seed Index” column corresponds to the index given to the
different simulations of the pristine specimen conducted with a different “seed” for the normal number generator introduced in Section 6.1.1. The specimen referred to as “P” in Table 6.2 is the reference point generated with the conventional deterministic approach.

<table>
<thead>
<tr>
<th>Seed Index</th>
<th>Failure Strain (%)</th>
<th>Seed Index</th>
<th>Failure Strain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.83552</td>
<td>11</td>
<td>0.96051</td>
</tr>
<tr>
<td>1</td>
<td>0.882166</td>
<td>12</td>
<td>0.962868</td>
</tr>
<tr>
<td>2</td>
<td>0.850059</td>
<td>13</td>
<td>0.964355</td>
</tr>
<tr>
<td>3</td>
<td>0.848385</td>
<td>14</td>
<td>0.955901</td>
</tr>
<tr>
<td>4</td>
<td>0.840475</td>
<td>15</td>
<td>0.874643</td>
</tr>
<tr>
<td>5</td>
<td>0.882401</td>
<td>16</td>
<td>0.855083</td>
</tr>
<tr>
<td>6</td>
<td>0.884277</td>
<td>17</td>
<td>0.981275</td>
</tr>
<tr>
<td>7</td>
<td>0.901065</td>
<td>18</td>
<td>0.930656</td>
</tr>
<tr>
<td>8</td>
<td>0.882401</td>
<td>19</td>
<td>1.0033</td>
</tr>
<tr>
<td>9</td>
<td>0.881696</td>
<td>20</td>
<td>0.935918</td>
</tr>
<tr>
<td>10</td>
<td>0.927591</td>
<td>P</td>
<td>0.865889</td>
</tr>
</tbody>
</table>

When identifying the parameters of a Weibull distribution, four commonly used methods can be employed and were compared by Pobočíková et al. [127] for different sample sizes. Given that, in the current study, the number of specimen falls into the “small” sample size identified by the authors, either the Weighted Least Square Fit (WLSF) method or the Least Square Fit (LSF) method are suitable. On consideration of the fact that every data point regarding the static failure strain was generated with the same FEM and following the same post-processing procedure, it does not make sense to attribute weights to the population. Therefore, the LSF method was preferred to the WLSF approach. This offers the opportunity for a graphical representation of the parameters’ identification process and the use of a common step-by-step procedure for the LSF. The failure strains were first sorted in an ascending order. Then, each data point was assigned a probability $p$ equal to $(i - 0.5)/j$ where $i$ is the rank of the failure strain in the sorted list and $j$ is the number of samples in the population. Then the plot of $ln(-ln(1 - p))$ vs $ln(data)$, where data is the strain to failure, was
generated. This is shown in Figure 6.14. The parameters can then be graphically determined by adding a linear interpolation curve to the scatter plot generated. The slope of the trendline is equal to the shape parameter $\kappa$ and the intersection with the x-axis is equal to $-\kappa \cdot ln(\lambda)$ where $\lambda$ is the scale parameter. The above described approach allowed to identify that a shape parameter $\kappa$ equal to 21.517 and a scale parameter $\lambda$ equal to 1.0033, corresponding to the two-parameter Weibull distribution estimated by the LSF.

![Figure 6.14: Weibull parameters identification graph](image)

Despite the previous assurances that the LSF method provides the most accurate estimation of the shape and scale parameters for small sample sizes, the Maximum Likelihood Method (MLM) was also applied to the same sample batch. The MLM and LSF estimations are represented in Figure 6.15 with a dashed red and a dashed black line respectively and alongside an histogram representation of the static failure strain data. As it can be observed, the estimation of the scale parameter $\lambda$ with the LSF approach does not allow for a good match with the strains to failure reported and therefore, the better estimation for the two-parameters Weibull law is provided by the MLM. In the following parts of this document, the following value of the scale and shape parameters will be used for the Weibull representation of the static strains to failure: $\kappa = 19.883$ and $\lambda = 0.932$. 

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The identification of the parameters for the Weibull law defining the static strain to failure allows to make use of the SLERA stating that the rank of a specimen in a population tested for its static strength is equal to its rank in fatigue. In other words, the statically strongest laminate is predicted to have the longest fatigue life. This allows to generate a Probabilistic S-N (PSN) curve using the data generated in Chapter 5 as a mean value to create a base SN curve and the statistical data presented in this section to evaluate an interval for the number of cycles to failure for a given load severity. The interval is bounded at the bottom by the B-basis value, or 10th percentile of the static failure strain, and at the top by the 90th percentile of the same statistical law. All of this is illustrated in Figure 6.16 where the data for the pristine specimen loaded in static and fatigue are summarized on a semi-log plot representing the fatigue strain normalized by the geometrical mean of the failure strain of the static stochastic specimens versus the number of cycles to failure. For different load severities, namely 80%, 70%, and 60%, data points were generated using
the stochastic approach mentioned in the previous sections. Similarly, the stochastic data for the static failure strain is reported at the abscissa of 1 cycle. Using the number of cycles to failure predicted by the deterministic approach, a mean S-N curve, displayed in a solid blue line in Figure 6.16, was generated. The equation of the trendline associated with the mean SN curve, as well as the correlation ratio R are also displayed on the graph. Finally, the probability density functions of the identified Weibull distributions for the static strain to failure as well as the distributions for the number of cycles to failure at different load levels are presented with black curves directly above the data they are associated with. Alongside the Weibull probability density functions, the 10th and 90th percentile are indicated with a dashed line in red for the 90th percentile and in green for the 10th. As mentioned previously, the 10th percentile, or B-basis, and the 90th percentile of the static strain to failure were used to define an interval of confidence for the number of cycles to failure by shifting the mean SN curve to pass through the static 90th percentile and 10th percentile. This therefore generates the two parallel curves for the parallel curves approach.

The main advantage of the proposed solution is that it does not technically require to run stochastic models in fatigue since the interval for the number of cycles to failure relies only on deterministic data for the mean SN curve and on the statistical distribution of the static tests for the two parallel curves. To compare the parallel curve approach with the stochastic approach introduced in this chapter, stochastic fatigue analyses were conducted at load severity level ranging from 80% to 60%. The objective was to ensure that the number of cycles to failure obtained with these simulations would either fall into the aforementioned interval or yield at a number of cycles to failure higher than the upper limit of the interval and therefore indicate that the method is conservative. A good agreement between the stochastic fatigue analyses results at a load severity of 80%, represented by the blue circles in Figure 6.16,
and the interval of confidence is observed. With the exceptions of Seeds 1, 2, and 8 respectively failing at 2042, 2082, and 2345 cycles and therefore approaching or slightly exceeding the upper bound of the interval, all the specimens failed within the predicted interval.

To generate the results presented above, the fatigue analyses done with Abaqus were conducted up until the processor faced errors and terminated the analyses due to excessive distortion of the cohesive elements. In some rare cases, such as Seeds 1, 2, 8, and 9 loaded with an 80% load severity, this lead to an unrealistic deformation of the cohesive elements. In order to ground the numerical simulation in reality it is necessary to relate the numerical failure to a physical criterion used to consider an experimental fatigue test as finished. It is common practice, while conducting fatigue experimental tests, to use an indicator, such as $E/E_0$ where $E$ is the residual stiffness and $E_0$ is the initial stiffness of the laminate, as a threshold to interrupt the
cyclic loading. In practice, the value of the threshold used to consider that a part has failed depends on the structural part being tested. One can however consider a 15% degradation in the laminate stiffness as a good order of magnitude. The evaluation of the number of cycles to failure were conducted once again using threshold values of 10%, 15%, and 20% on the longitudinal stiffness degradation as a stopping factor. A summary of the configurations for which using the reduction in residual stiffness lead to a variation in the estimated number of cycles to failure is presented in Table 6.3 where $SR$ is the stiffness reduction factor equal to $1 - E/E_0$.

Table 6.3: Cycles to Failure at 80% load severity

<table>
<thead>
<tr>
<th>Seed Index</th>
<th>$N_f$ (ref.)</th>
<th>$N_f$ (SR = 10%)</th>
<th>$N_f$ (SR = 15%)</th>
<th>$N_f$ (SR = 20%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2042</td>
<td>453</td>
<td>2042</td>
<td>2042</td>
</tr>
<tr>
<td>2</td>
<td>2082</td>
<td>457</td>
<td>2042</td>
<td>2042</td>
</tr>
<tr>
<td>8</td>
<td>2345</td>
<td>414</td>
<td>2345</td>
<td>2345</td>
</tr>
<tr>
<td>9</td>
<td>927</td>
<td>359</td>
<td>927</td>
<td>927</td>
</tr>
</tbody>
</table>

As indicated in Table 6.3, no variations of the number of cycles to failure was observed for values of SR superior to 10%. However, at SR = 10% reductions in the number of cycles to failure for the configurations associated with Seed 1, 2, 8, and 9 were identified. To illustrate the phenomenon, an isometric view of the damage state in Seed 8 at the time frame at which the ratio $E/E_0$ reaches 90% is represented in Figure 6.17 alongside with synchronized stress versus time and damage/delamination versus time curves. It can be noted that the yellow vertical line, corresponding to the simulation time at which the residual stiffness reaches the defined threshold, matches with the end of the drastic increase in damage and delamination propagation rates illustrated by the abrupt increase shown by the dashed red and orange curves respectively.

Adjusting the number of cycles to failure for the configurations listed above allows to re-evaluate the 2-parameter Weibull for the 80% load severity case. The shape and scale parameters of the adjusted 80% population are $\kappa = 2.9984$ and $\lambda = 553.413$ cycles, respectively. The adjusted Probabilistic SN curve is shown in Figure 6.18.
More specifically, the SN curve itself, shown in blue in the figure, is not modified since it relies only on the stochastic results in static and the deterministic results in fatigue but the Weibull’s probability density function represented by a solid black line is adjusted to the new parameters values. As can be seen on the Probabilistic SN curve, the estimated cycles to failure obtained for specimen containing a stochastic representation of the material properties are all falling within the interval of confidence predicted by the parallel curve approach presented previously.

In order to compare several probabilistic specimens together and with the deterministic results introduced in Chapter 5, screenshots of the damage state in probabilistic specimens Seed 1 and Seed 6 were taken at the same time stamps as those presented in Figure 5.6 for the deterministic pristine specimen. The choice of the two seeds is based on the fact that Seed 1, as mentioned previously, reached a reduction of the longitudinal stiffness superior to ten percent of the initial stiffness. Seed 6, on the other hand, was selected for its behavior characteristic of the average specimen loaded at 80% load severity. Looking first at the screenshots for Seed 1 in Figures 6.19a, 6.19c, and 6.19e at time points A, B and C respectively, one can note the long “plateau”
Figure 6.18: Adjusted Probabilistic SN curve pristine specimen

from time mark 1.57s up until the abrupt drop of the stress curve. During this phase, no additional damage or delamination is created in the laminate since it has already failed and reached a ratio of $E/E_0 = 0.9$ as explained previously in the section on the adjustment of the SN curve for specimens loaded at 80% load severity. Making abstraction of the aforementioned time period in the analysis of Seed 1, it is possible to observe the high similitude of the damage/delamination kinematics shared with Seed 6 illustrated with Figures 6.19b, 6.19d, and 6.19f at the same time points A, B, and C respectively. Two main phases of the damage/delamination propagation can be identified once entering the fatigue loading step at time point A. A first fast progression of the damage/delamination propagation from point A to a time stamp approaching 0.5s, followed by a slower propagation phase up until point B, and finally an abrupt increase in the damage/delamination rate up until failure of the laminate at point C. Despite locally being assigned different material properties with the normal
distribution generator, Seed 1 and Seed 6 are still a representation of the same laminate for which the same normal law is used to represent the material properties. It is therefore not surprising to see that, globally, the damage/delamination state is similar between the two specimens but still display local differences such as the location of matrix cracks in the 90 degree plies as shown in Figures 6.19c and 6.19d for Seed 1 and Seed 6 at point B respectively. In fact, the chronology of the damage initiation and propagation is the same between the two stochastic models. An initial splitting in the inner zero degree plies next to the hole, quickly followed by matrix cracks in the 90 degree and ±45 degree plies in the stress concentration area are created during the quasi-static loading. When entering the fatigue loading phase, the first significant increase in the damage propagation rate corresponds to the development of the matrix cracks in the off-axis plies spreading towards the free-edges of the laminate while the matrix cracks in the 90 degree plies concentrate around the newly formed cracks in the ±45 plies. The zero degree plies splitting extend only slightly towards the grips. During the ‘plateau’ phase, the matrix cracks are propagating towards the grip area. The final phase, which corresponds to the jump in the propagation rate, is associated with a fast propagation of the splitting all the way to the grip area while matrix cracks in the 90 and ±45 plies starts to fill the gap created in between the two splits on each sides of the hole.

A comparison of the results of the stochastic representation with the results presented in Chapter 5 for the same specimen modeled using a deterministic approach are in order. The deterministic results for a pristine specimen loaded with a load severity of 80% are presented in Figure 5.6. Globally, the deterministic and the stochastic approaches are yielding the same results. The predicted chronology of events as well as the location of the damage/delamination are highly similar. The only noticeable difference resides in the fact that the spread of the damage in the 90 and ±45 degree plies in the deterministic specimen seems to be more uniform than it is
in the stochastic specimens. One can indeed observe, in Seed 1 (Figure 6.19c) and Seed 6 (Figure 6.19d), for the example of time point B, the presence of vastly distributed off-axis cracks running from the middle of the specimen all the way to the free-edges. This was not the case in the deterministic approach. It is perfectly consistent with the fact that, introducing local weak points through the normal distribution of material properties creates the opportunity for cracks initiations and therefore contributes to the perception of a more randomly distributed propagation of the matrix cracks during the “plateau” phase identified in the previous paragraph.
Regarding the case of a load severity of 70%, given the increased duration of each analysis and the fact that the kinetic to internal energy ratio remained well below the commonly used threshold of 6% for specimens loaded at a load severity of 80%, the *initial number of cycles per increment* and the *cycles per increment modification* parameters of CompDam where both increased to allow for a more timely analysis of the ten stochastic specimens. Therefore, the time points used previously for the deterministic approach and shown in Figure 5.7 cannot directly be associated with the same time points in the stochastic approach. Consequently, three critical time points have been selected and used to present the state of damage in Seed 3 under a load severity of 70% in Figure 6.20. For this load severity, the same observations that were made previously for the comparison between the stochastic and deterministic results at a load severity of 80% are still valid. The global chronology of the damage initiation and propagation is identical to that of the deterministic approach but the geographical distribution of the damage throughout the laminate is not as uniform as it was in the deterministic approach. Similarly to the results presented previously, the propagation rate using the stochastic approach is showing more clearly delimited phases whereas the damage propagates without major variation in rate when using the deterministic approach.

Finally, completing the interest given to the pristine configuration, the damage evolution for a load severity of 60% using the stochastic approach is presented in Figure 6.21 for Seed 1. The duration of the analysis for the deterministic and the stochastic approach for a load severity of 60% being similar, no changes were made.
Figure 6.21: Damage in Seed 1 at 60% load severity

to the previously mentioned initial cycles per increment and cycles per increment modification parameters between the two approaches and therefore the same time points can be used to extract the damage state presented in Figures 5.8 and 6.21 for the deterministic and the stochastic approach respectively. For this particular case of the load severity, the conclusions are the same as they were in Chapter 5 on the influence of a defect on the behavior of a pristine specimen constrained with the same load severity. The local variation of the material properties, whether they are introduced by a defect or a stochastic representation of the material properties, upon reaching a low enough global stress level in the laminate have no influence on the number of cycles to failure or the location of the damage throughout the loading scenario. Indeed, the damage can only initiate in the vicinity of the hole or next to already existing damage due to the stress concentration associated with either the hole or the localized stress redistribution directly following the formation of matrix cracks. This explains both the relatively low amount of elements exhibiting damage in the 60% load severity configuration and the rather sparse distribution of matrix cracks in the 90 and ±45 degree layers when comparing to the results for the 80% and 70% load severity cases. This indicates one limit of the current stochastic approach.
To conclude on the stochastic approach proposed in this section, it was observed that the prediction on the number cycles to failure for every load severity tested fell in the interval of number of cycles to failure predicted by the parallel curve approach. However, the computational cost associated with this method is significantly higher than only conducting a stochastic analysis of the static behavior to generate the parallel curves solution. The stochastic technique only presents the advantage to generate a map of the potential zones of damage initiation and propagation which could help design the inspection protocol when studying a component higher in the building block.

6.3.2 Load Enhancement Factor (LEF) Approach

The Load Enhancement Factor (LEF) is a commonly used certification method used to reduce the duration of the fatigue testing conducted for the certification process and relies on the technique presented by R.S. Whitehead et al. [124]. A graphical representation of the principal behind the the LEF approach is presented in Figure 6.22. The curve represents a conventional mean residual strength of a composite versus the number of fatigue lifetimes while the dash line corresponds to the the B-basis curve. The objective of this technique consists in slightly augmenting, by the LEF factor F, the maximum applied stress in the loading scenario so that if during experimental testing a single test specimen does not fail for N applied cycles with a maximum load of $F \cdot P$ then 90% of the specimens cyclic loaded with a load $P$ will fail at a number of cycles higher than N. Which correspond to say that $P$ is the B-basis value for N cycles. This is illustrated in Figure 6.22 as follows. Assuming that the maximum applied stress is $P_F$ and the mean residual strength of a laminate at one lifetimes cycles is $P_T$ then one can demonstrate B-basis reliability one of two ways: either successfully test a specimen at $P_T$ for one lifetime or successfully test a specimen to $N_F$ cycles at a maximum applied stress $P_F$. 

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Based on the work presented by Whitehead et al. [124] it is possible to express the LEF factor \( F = P_T / P_F \) as a function of the scale and shape parameters \( \kappa_R \) and \( \lambda_R \) of the Weibull distribution fitting the residual strength at a certain fatigue lifetime. This expression of the LEF is given in equation 6.3.1. The coefficients necessary to evaluate the load enhancement factor are described in Table 6.4.

\[
\begin{align*}
F &= \frac{\mu \cdot \Gamma(\frac{\kappa_R + 1}{\kappa_R})}{\left[ \frac{-\ln(p)}{\chi^2(2n)/2n} \right]^{\pi_R}} \\
p &= \exp[\ln(l) \cdot N^\kappa_L] \\
\mu &= \frac{\left[ \Gamma(\frac{\kappa_L + 1}{\kappa_L}) \right]^{\kappa_L}}{\Gamma(\frac{\kappa_L + 1}{\kappa_L})}
\end{align*}
\tag{6.3.1}
\]

Going through the needed coefficients one by one, the scatter measure in fatigue for the material IM7/8552 was evaluated based on the experimental results available in the work conducted by Mukhopadhyay et al. [84] and Nixon-Pearson et al. [115] by Dr. Kassapoglou. The evaluation yielded a value of \( \kappa_L = 1.516972 \). The scatter of the static test was identified using the stochastic approach presented previously in Section 6.2 and yielded a value of \( \kappa_R = 19.883 \). The reliability was set to 95%
Table 6.4: Parameters of the Load Enhancement Factor F

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>Coefficient equal to 1 when $N = N_F$</td>
</tr>
<tr>
<td>$\kappa_R$</td>
<td>Shape parameter of the static tests</td>
</tr>
<tr>
<td>$\kappa_L$</td>
<td>Shape parameter of the fatigue tests</td>
</tr>
<tr>
<td>$l$</td>
<td>Required reliability at $\gamma$ level of confidence</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Level of confidence (Equal to 0.9 for B-basis)</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>The Gamma function</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of samples tested at the Enhanced Load (n=1)</td>
</tr>
<tr>
<td>$\chi^2_{\gamma}(2n)$</td>
<td>Chi-square percentile at $\gamma$ with 2n D.O.F.</td>
</tr>
</tbody>
</table>

and the level of confidence to 90% to correspond to the B-basis value. Finally, the number of sample n was set to one. All of this combined allowed to evaluate the Load Enhancement Factor (LEF) at one lifetime: $F = 1.288$. The new prediction of the B-Basis value using the LEF approach is represented by a black dashed line in Figure 6.23. It can be observed that the highest the load severity is the more conservative the LEF approach is compared to the parallel curve solution proposed. Extrapolating from the results presented in Figure 6.23, the two approaches are presenting a similar evaluation of the B-basis value when reaching a load severity of 40%.

Comparing the stochastic solution presented in Section 6.3 and the LEF approach presented here, it appears that the data necessary to generate the two approaches is relatively similar. While the stochastic approach relies on the characterization of the static behavior through a series of numerical analyses to generate a reliable population, the LEF approach relies on both the statistical representation of the static behavior to evaluate $\mu$, and the statistical representation of the fatigue behavior to obtain the scatter coefficient $\kappa_L$. Given the much lower financial and time cost in conducting static tests to characterize the Weibull law of the static population, the parallel curve approach appears more suited for application for which the static
sequence has already been defined. On the other hand, the LEF approach, despite the need for experimental fatigue data and the more conservative results generated for the estimation of the B-basis value, presents the advantage of necessitating coefficients which are only material dependent ($\kappa_R$ and $\kappa_L$).

6.3.3 Stochastic approach applied to a defect configuration

In Chapter 4, a study of the influence of three defect configurations on the static behavior of a thirty-two plies laminate was conducted and then used as a foundation for the fatigue analysis of the same defect configuration presented in Chapter 5. This work allowed to establish that the defect appeared to have a predominant influence on the number of cycles to failure at high load level severity. To test the applicability of the parallel curve approach to a defect configuration, a series of stochastic static analyses of the defect configuration C3 were conducted to establish a large enough
population to evaluate the 2-parameter Weibull law. As it was established with the pristine specimen that a sample size of twenty stochastic analyses was sufficient to properly represent the static behavior, the same amount of stochastic C3 models were analyzed generating the population presented in Table 6.5. Based on this population, and using the same MLM approach as the one used for the pristine specimen in Section 6.2 to evaluate the parameters of a fitting Weibull law, an evaluation of the shape and the scale parameters of a 2-parameter Weibull law for the static failure strain of the configuration C3 are established and presented in Figure 6.24. The evaluated shape parameter $\kappa$ is equal to 17.288 and the scale parameter $\lambda$ is evaluated at 0.993.

Table 6.5: Failure Strains of Stochastic Models (C3)

<table>
<thead>
<tr>
<th>Seed Index</th>
<th>Failure Strain (%)</th>
<th>Seed Index</th>
<th>Failure Strain (%)</th>
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<tbody>
<tr>
<td>1</td>
<td>1.06161</td>
<td>12</td>
<td>0.9987</td>
</tr>
<tr>
<td>2</td>
<td>0.912497</td>
<td>13</td>
<td>0.93292</td>
</tr>
<tr>
<td>3</td>
<td>0.993267</td>
<td>14</td>
<td>0.894673</td>
</tr>
<tr>
<td>4</td>
<td>1.024007</td>
<td>15</td>
<td>0.992307</td>
</tr>
<tr>
<td>5</td>
<td>0.96254</td>
<td>16</td>
<td>0.91342</td>
</tr>
<tr>
<td>6</td>
<td>0.93936</td>
<td>17</td>
<td>0.96408</td>
</tr>
<tr>
<td>7</td>
<td>0.91514</td>
<td>18</td>
<td>0.908233</td>
</tr>
<tr>
<td>8</td>
<td>1.0577</td>
<td>19</td>
<td>0.930087</td>
</tr>
<tr>
<td>9</td>
<td>1.07968</td>
<td>20</td>
<td>0.930993</td>
</tr>
<tr>
<td>10</td>
<td>0.925884</td>
<td>C3</td>
<td>0.843169</td>
</tr>
<tr>
<td>11</td>
<td>0.975793</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Despite using the Advanced model approach to represent the defect in the stochastic configuration C3, the order of magnitude of the strain to failure illustrated by the fitting Weibull distribution in Figure 6.24 is more in concordance with the results obtained with the Simple model approach presented in Chapter 3 which indicated an increased strain to failure when compared to pristine specimen. Consequently, since the deterministic fatigue results for the defect configuration were already in the interval of confidence defined by the parallel curve solution of the pristine specimen, the interval of confidence generated for the defect configuration using the newly
evaluated Weibull law parameters almost entirely overlaps the interval of confidence for the pristine configuration. This is shown in Figure 6.25 in blue dashed lines for the C3 configuration. It is therefore impossible to conclude what influence the defect has on the fatigue behavior using only the stochastic approach as it is. One can only note the slightly higher slope of the estimated SN curve for the defect configuration. The greater slope of the SN curve for the defect specimen is in fact driven by the predicted reduction in fatigue life for the highest value of the load severity but does not reflect an influence of the defect on the low values of the load severity.
6.4 Conclusion

In this chapter a solution was proposed to represent the local variability of the material properties at the origin of the scatter observed during experimental testing. This was presented in the form of a normal distribution representation of the longitudinal stiffness applied at the element level through an Abaqus user-subroutine. This stochastic approach was first applied to the quasi-static OH tension test in section 6.2 which allowed to confirm observations made by Wisnom et al. [110] who identified that the laminate selected for the present document is a transition stacking sequence between specimens whose failure is driven by delamination because of thicker ply blocks of the same orientation and laminates whose failure is driven by fiber failure when opting for thinner ply blocks of the same orientation but with still the same total amount of plies. The stochastic static results were then leveraged to implement a parallel curve solution to generate an interval of confidence on the number of cycles to failure for an OH specimen loaded in fatigue Tension-Tension using only deterministic results at various load severity levels in fatigue and a two-parameter Weibull law representation of the stochastic static data. The parallel curve concept was then set side by side with a common certification practice called the Load Enhancement Factor (LEF) and shown to have the potential to be less conservative. The stochastic approach was also applied to the fatigue loading of OH specimens to ensure the good agreement of the predicted number of cycles to failure with the parallel curves solution. Despite falling in the proper interval predicted by the parallel curve solution, the stochastic approach, when applied to simple coupons, does not provide enough additional information on the failure modes to justify the much higher time cost associated with the data generation. This is however a good note on the stochastic approach as it confirms that the local perturbations associated with the material properties do affect the fatigue life of the laminate but the global aspect of the distribution of the damage throughout the different simulations of an otherwise identical laminate remains identical.
Chapter 7

Conclusion

The work presented in this document initially stems from a simple observation made during the manufacturing process using an AFP machine. Just like any other manufacturing technique, AFP is subject to its own specific type of defects occurring during the production phase. Even though understanding the sources of the creation of defects would allow to look for mitigation techniques to reduce their frequency, it is not always suitable for the manufacturing plant to incorporate the inspection tools and protocols needed for the reduction of the number of defects generated during the layup process. The work presented in the previous chapters is an attempt to develop numerical tools and procedures to provide an in-depth understanding of the influence of the introduction of AFP-induced defects in composite laminates on their geometry, local material properties distribution, quasi-static tensile behavior, and Tension-Tension fatigue response. However, the proposed tools are not limited to the aforementioned list and can be applied to a variety of load cases.

First, the present work offers the possibility to numerically represent the geometry and the influence on the material distribution in a composite laminate for the most common types of defects, as shown in Chapter 3. Two different approaches are detailed in Chapter 3: the simple and the advanced representations. The former offers a representation of a defect that only captures the influence of defects on the local distribution of resin and fibers by using gaps and overlaps as being blocks for other types of defects. This is accomplished by decomposing each defect type into sub-regions for which the material properties assigned are either that of a gap or an overlap.
Even capable of providing a fast first order understanding of the influence of a defect on the quasi-static tensile behavior of a laminate in most cases, as shown in Chapter 4, this approach lacks a crucial representation of the geometrical imperfection caused in the layers above a defect. The advanced solution, based on a compaction simulation and presented in Chapter 3.1.2, allows to capture both the material properties and geometric perturbations engendered by a defect.

The second step conducted for this work on the numerical characterization of AFP-induced defects is presented in Chapter 4 and consists in the study of three main defect configurations in OH coupons loaded in tension. The selection of an OHT configuration was justified by two considerations. First, it allows for the introduction of a stress concentration in the vicinity of the hole which allows to alleviate the issue of spurious damage initiation at the free-edges of the FE model. Secondly, this provides an additional parameter for the study of the influence of defects in the form of the relative position of the selected defect type, a half gap-overlap, with respect to the hole and its stress concentration. Globally, it is observed that the introduction of a single, tow-wide, defect in a thirty-two layers laminate has a relatively limited influence of the quasi-static mechanical response. It is also observed that, depending on the location of the defect with respect to the stress concentration, detrimental or beneficial influence of the defect on the ultimate failure strain of the laminate in the order of magnitude of 3-4% can be expected. The local resin-rich region, or gap, introduced in the laminate appeared to be the source of initiation and propagation of a larger amount of matrix cracks when compared to a pristine specimen which allowed, in the configurations where the gap is running through the stress concentration, for the redistribution of the stress. The local introduction of an overlap, and therefore a stiffer material than the in pristine specimen, led to a consistent initiation of the damage, in all the coupons simulated, on the side where the overlap was positioned. The quasi-static analysis
of the influence of AFP-induced defects confirmed previous observations made in the literature that defects have a limited influence on the tensile behavior of composite laminates but it also allowed to select the most critical configuration to study in Tension-Tension fatigue in the next chapter.

The third step of the present work consists in the analysis of the influence of AFP-induced defects on the fatigue behavior of composite laminates and is presented in Chapter 5. The focus is initially given to the pristine configuration as to establish a reference SN curve. This reference is then used to quantify, at various load severity levels, the influence of a defect on the number of cycles to failure of the laminate. The attention is given, for two reasons, to the highest load severities: \( \varepsilon_{\text{max}} / \varepsilon_{\text{UTS}} = 80\%, 70\%, \text{ and } 60\% \). The first reason being that previous studies, like that work of Elsherbini [89], indicated that the influence of defects on the number of cycles to failure was more significant at higher load severity. This is also beneficial as numerical simulations at higher load severity are faster to compute. A comparison of the fatigue behavior of the pristine specimen with that of Configuration 3, first introduced in Chapter 4, is conducted, and Elsherbini’s conclusion about the influence of defects at high load severity levels on unidirectional laminates is extended to quasi-isotropic laminates. A reduction approximating 50\% of the number of cycles to failure for the two highest load severities was indeed unveiled by the numerical simulation of configuration 3. Upon reaching a load severity of 60\%, the number of cycles to failure of the defect configuration reached the same order of magnitude as the pristine specimen. However, a difference in the distribution of the delaminations was observed for every load severity in the form of a delamination in the cohesive layers surrounding the gap region and therefore at the interface between the inner zero degree ply block and the adjacent -45 degree plies.
Finally, the well-known scatter observed in experimental testing results of composite laminates has led to the implementation of conservative certification techniques tools such as the A- and B-basis allowables to account for the variability in failure stress in quasi-static and in the number of cycles to failure when subject to fatigue loading. In Chapter 6, an attempt to numerically model the local variations of material properties in finite elements, using a statistical number generator to assign a normal distribution to the longitudinal stiffness of OH specimens was proposed as a solution to quantify the influence local changes can have on both the quasi-static tension failure and the number of cycles to failure of specimens under Tension-Tension fatigue loading at a stress ratio R = 0.1. Given the nature of the modification, which is limited to small local changes of a single variable (i.e. the longitudinal stiffness), the global behavior of the laminates including the proposed stochastic approach was similar to the the deterministic approach in the sense that the global distribution, amount, and chronology of the damage/delamination initiation and propagation were not significantly modified. However, the proposed parallel curve solution presented in detail in Chapter 6 allows for an estimation of an interval of number of cycles to failure for pristine specimens based only on the deterministic prediction of the SN curve and a stochastic analysis of the static failure strain. This approach appears to be less conservative than the Load Enhancement Factor technique also presented in the aforementioned chapter and could provide a valuable tool for estimations of the fatigue life of a laminate. It is however important to note that the application of this parallel curve approach to the configuration containing a manufacturing defect was not capable of providing conclusive predictions on the influence of the defect since the intervals of confidence for the pristine configuration and the interval of confidence for the defect configuration are overlapping.
In conclusion, it is observed that the studied configurations containing a single AFP-induced defect in a thirty-two layer laminate are only exhibiting a small reduction or increase of the static tensile failure load when compared to the pristine specimen and that the location of the defect with respect to the hole is an important factor driving towards either a beneficial or a detrimental influence of the defect. Globally, the observations made in this document regarding the influence of defects on the quasi-static tensile behavior of composite laminates is in good concordance with previously published data as presented in Chapter 2. When shifting the focus towards fatigue, it is observed that the potential influence of manufacturing defects is concentrated at high load severity loading scenarios and that, as the load severity reduces, the fatigue life of the specimen returns to that of the pristine configuration. In general, the numerical representation techniques developed in this document to include defects in finite element simulations has allowed for an improved understanding of the mechanical phenomena that the introduction of defects cause in fatigue and quasi-static loading conditions. It is however the authors’ belief that, despite the amount of information already gathered on the influence of defects through the present document and previous studies published in the literature, a change of focus towards complex loading scenarios in both static and fatigue for defect configuration closer to the manufacturing reality, in the sense that it is not limited to single defect in the middle plies, would allow for a better global understanding of the influence of defects and therefore the elaboration of criteria applicable during the manufacturing process to dispose of a decision process regarding the need for a manual repair of identified defects.


[63] Cheryl A Rose, Carlos G Dávila, and Frank A Leone. NASA/TM-2013-218024-


[94] Dassault Systèmes Simulia Corp. Abaqus/CAE 2018.


Appendix A

Stochastic approach and associated code

A.1 Random Number Generator

"""Pseudorandom Number Generator
"""

#########################################################################

#########################################################################

# import libraries

# Basic libraries
import math
import sys
import random

# 3rd party imports

#########################################################################

#########################################################################

def real_uniformDistribution(seed):
    """Generate pseudorandom uniformly distributed numbers using a
    Multiplicative Congruential Generator (MCG) with m prime (and c=0)

    Parameters
    --------

    seed: integer

    Returns
    -------

    A list of pseudorandom uniformly distributed numbers

    Example
    -------

    >>> real_uniformDistribution(12345)
    [0.21873, 0.54689, 0.79307, ...]
    """

    seed = int(input("Enter a seed: "))
    m = 1000000
    c = 0
    a = 73573
    x = seed
    y = 0
    n_samples = 100
    x = (a * x + c) % m
    for _ in range(n_samples):
        y = (a * x + c) % m
        yield y / m
seed (int)

Initial value for the Multiplicative Congruential Generator

Returns
-------

seed (real)

seed for next call to real_uniformDistribution

Real

Uniformly distributed number

m = 2147483647  # Modulus (2^31-1) -- prime
a = 16807  # Multiplier a <= sqrt(m) (Lehmer RNG --> MINSTD)
q = m//a  # Quotient of m divided by a
r = m - m//a * a  # Remainder of m divided by a

seed = a*(seed%q) - r*(seed//q)

if seed <= 0:
    seed = seed + m

return seed, float(seed)/float(m)

def real_standardNormalDistribution(seed):
    """Generate pseudorandom standard normal numbers using a Box-Muller transformation"

    Parameters
    ----------

    seed (int)
Initial value the generation of two independant uniformly distributed numbers

Returns
-------
seed (real)
    seed for calls to real_uniformDistribution
Real
    Standard normal distribution number

s, u1 = real_uniformDistribution(seed)
s, u2 = real_uniformDistribution(s)

    return s, float(math.sqrt(-2.0*math.log(u1))*math.cos(2*math.pi*u2))

def real_sigmaMuNormalDistribution(seed, sigma, mu):
    """Generate pseudorandom normal numbers using a Box-Muller transform
    with a mean mu and a standard variation sigma
    Parameters
    ----------
    seed (int)
        Initial value the generation of two independant uniformly distributed numbers
    sigma (double)
        Standard variation
    mu (double)
        Mean
    Returns
    -------
    s, u1 = real_uniformDistribution(seed)
s, u2 = real_uniformDistribution(s)
Seed (real)
    seed for calls to real_uniformDistribution

Real
    Standard normal distribution number

""

s, u1 = real_uniformDistribution(seed)
s, u2 = real_uniformDistribution(s)
x = float(math.sqrt(-2.0*math.log(u1))*math.cos(2*math.pi*u2))

return s, float(mu) + x*float(sigma)

if __name__ == '__main__':

    f = open("normal.txt", "a")

    if len(sys.argv) >= 4: # arguments include seed, standard variation and mean
        seed = int(sys.argv[-3])
    elif len(sys.argv) == 2: # arguments contain seed but no mean or standard variation
        seed = int(sys.argv[-1])
    elif len(sys.argv) == 1: #argument limited to the name of the pyhton file to call
        seed = random.random()
    else:
        raise ValueError('Please specify no argument, one argument (seed), or three arguments (seed, standard variation, and mean)!')
if len(sys.argv) != 4:
    a, b = real_standardNormalDistribution(seed)
    f.write(str(b) + '\n')
    print('Seed: {}, New seed: {}, Output: {}'.format(seed, a, b))

for i in range(3000):
    seed = a
    a, b = real_standardNormalDistribution(seed)
    f.write(str(b) + '\n')
    print('Seed: {}, New seed: {}, Output: {}'.format(seed, a, b))
else:
    a, b = real_sigmaMuNormalDistribution(seed, sys.argv[-2],
                                          sys.argv[-1])
    f.write(str(b) + '\n')
    print('Seed: {}, New seed: {}, Output: {}'.format(seed, a, b))

for i in range(5000):
    seed = a
    a, b = real_sigmaMuNormalDistribution(seed, sys.argv[-2],
                                          sys.argv[-1])
    f.write(str(b) + '\n')
    print('Seed: {}, New seed: {}, Output: {}'.format(seed, a, b))

f.close()
Appendix B

OHT Configurations

Figure B.1: Three main OHT configurations staggered

Figure B.1: Three main OHT configurations staggered
<table>
<thead>
<tr>
<th>Name of the Configuration</th>
<th>Defect location</th>
</tr>
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<tbody>
<tr>
<td>Pristine</td>
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<tr>
<td>Configuration 1</td>
<td></td>
</tr>
<tr>
<td>Configuration 1 Gap</td>
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</tr>
<tr>
<td>Configuration 1 Overlap</td>
<td></td>
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<tr>
<td>Configuration 2</td>
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<tr>
<td>Configuration 2 Gap</td>
<td></td>
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<tr>
<td>Configuration 2 Overlap</td>
<td></td>
</tr>
<tr>
<td>Configuration 3</td>
<td></td>
</tr>
<tr>
<td>Configuration 3 Gap</td>
<td></td>
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<tr>
<td>Configuration 3 Overlap</td>
<td></td>
</tr>
</tbody>
</table>
Figure B.2: Damage propagation in pristine
Figure B.3: Damage propagation in configuration 1
Figure B.4: Damage propagation in configuration 2
Figure B.5: Damage propagation in configuration 3
Figure B.6: Damage Initiation Load Level

Figure B.7: Load Level comparison at 1000 damaged elements