Fall 2021

Evaluation of a Trigonometric Grey Model for Estimating And Predicting Pavement Condition

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EVALUATION OF A TRIGONOMETRIC GREY MODEL FOR ESTIMATING AND PREDICTING PAVEMENT CONDITION

by

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Bachelor of Science
Benedict College, 2018

Submitted in Partial Fulfillment of the Requirements

For the Degree of Master of Science in

Civil Engineering

College of Engineering and Computing

University of South Carolina

2021

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DEDICATION

To my spouse, mother, family, friends and ancestors, who were always there for me each step of the way.

ACKNOWLEDGEMENTS

I would like to thank Dr. Nathan Huynh for the time, guidance, technical input, encouragement and support he provided during this research project and my academic career. Special thanks to Dr. Robert Mullen, Dr. Gurcan Comert, and Dr. Negash Begashaw for their support, encouragement, technical inputs, comments, and suggestions. I also would like to thank Dr. Sarah Gassman and Dr. Charles Pierce for their support, technical input, comments, and suggestions.

The data used in this work is from the South Carolina Department of Transportation. The author gratefully acknowledges Chad Rawls, John Watson, Brian Fulmer, Eric Carrol, Kim Dahae, and Todd Anderson for their assistance and guidance throughout the course of the research project. The results and opinions expressed in this paper are solely those of the author, and they do not necessary reflect the view or policies of the SCDOT.

ABSTRACT

States, counties, and municipalities rely on pavement performance curves to forecast future pavement conditions in their jurisdictions. Accurate prediction is essential for budget planning and the identification of candidates for rehabilitation. This study investigates the use of a grey model (GM) to estimate and predict pavement conditions. An advantage of the GMs is that they do not require a large sample size for model estimation. This aspect is important since smaller towns and municipalities often cannot afford to collect pavement condition data frequently due to cost. There are other situations where sample size may be limited, such as using project-level pavement condition data to determine the optimal maintenance plan to prolong the life of the pavement. To this end, a novel trigonometric GM is applied to estimate and predict pavement conditions. The model's performance is compared with the performance of the first-order GM (i.e., GM(1,1)) and two S-shaped nonlinear models using pavement data from South Carolina. The estimation results indicate that the proposed GM (1,1|cos(ωt)) model outperforms the S-shaped nonlinear models and GM(1,1) model for the two considered pavement types (bituminous and bituminous over concrete) and two rehabilitation methods (mill-and-replace 2 to 4 inches + 2-inch overlay and mill-and-replace 1 to 2 inches + 4-inch overlay) in terms of Mean Absolute Percentage Error (MAPE) and Root Mean Square Error (RMSE).

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LIST OF ABBREVIATIONS

BIT .... Bituminous (or Asphalt)

BOC .... Bituminous Over Concrete

BMP .... Beginning Mile Point

EMP .... Ending Mile Point

ESAL .... Equivalent Single Axle Load

GM .... Grey Model

HPMA .... Highway Performance Management Application

IRI .... International Roughness Index

MR .... Mill and Replace

MAPE .... Mean Absolute Percentage Error

OL .... Overlay

PCI .... Present Condition Index

PDI .... Present Distress Index

PQI .... Pavement Quality Index

PSI .... Present Serviceability Index

PSL .... Predicted Service Life

PSY .... Pond Per Square Yard

PMS .... Pavement Management System

RMSE .... Root Mean Square Error

CHAPTER 1: INTRODUCTION

An integral element of a pavement preservation program for any state highway agency, county or municipality is the ability to predict future conditions of pavements. Pavement performance models are developed for this purpose. In addition to being used to identify sections of roadways that need to be rehabilitated, they are also used to estimate and rationally allocate budget at the network level (*1*), evaluate the effectiveness of various rehabilitation treatments, and perform cost and benefit analyses (*2*). For these reasons, it is essential that pavement performance models yield accurate prediction of pavement conditions.

The process of developing pavement performance models involves collecting pavement conditions data, identifying parameters that affect pavement deterioration, and determining the mathematical model that best describes the relationship between identified parameters and pavement conditions. Deterministic models have been the most widely adopted due to their simplicity (*3*, *4*). Among the deterministic models, the model having a characteristic of an S-shaped curve or sigmoid curve is the most popular. Examples of S-shaped models from previous work (*5–7*) are:

- $Pavement\ Condition\ Rating = 90 - a[exp(age^{b}) - 1] \times \log\left[\frac{ESAL}{MSN^{c}}\right]$ (1)

- $Pavement\ Condition\ Index = a + \frac{b}{1+exp(c \times T+d)}$ where $T = t + f(PCI)$ (2)

- $Pavement\ Serviceability\ Rating = O - exp\left[a - b \times c^{\ln\left(\frac{1}{age}\right)}\right]$ (3)

To determine the parameters in the above S-shaped models or other deterministic models, a sufficiently large sample size is needed. According to Montenegro (*8*), as a rule of thumb, a sample size of at least 10 times the number of parameters is needed when the ordinary least squares method is used to estimate the parameters. For example, Equation 1 (*5*) above has 3 parameters (a, b, and c), so the sample size should have at least 30 observations. Equations 2 (*6*) and 3 (*7*) have 4 parameters each, so the sample size should have at least 40 observations. This sample size requirement is problematic for smaller agencies which do not have the resources to collect pavement condition data frequently or they may have a small number of roadways with a certain pavement type. Also, there are situations where sample size may be limited, such as using project-level pavement condition data to determine the optimal maintenance plan to prolong the life of the pavement.

To overcome the problem with sample size, some researchers have begun to explore the use of grey models (GMs) which is based on grey system theory. A GM is a system model based on a differential equation where some of the model parameters are unknown (i.e., between a completely unknown black box and completely known white box). The GMs are often identified by two parameters, i and j as in GM(i,j) where i is the order of the ordinary differential equation (ODE) and j is the number of variables. In a GM, the unknown function in the ODE is not constructed. It is replaced by a surrogate model constructed from observed data. GMs have been applied widely and proven to be useful in solving uncertain problems with small samples and inadequate information. However,

their application in the area of pavement condition estimation and prediction is quite limited. Nevertheless, the general findings from these studies indicate that GMs are suitable for estimating pavement condition with small samples. The gaps in the current body of work are twofold. First, no study has applied or developed a GM beyond GM(1,2). Second, only one study has applied a GM to model pavement deterioration after receiving a micro-surfacing treatment. It remains unclear whether GMs are suitable for other pavement types and rehabilitation treatments and whether more advanced GM models should be utilized. This study seeks to fill this gap in the literature.

The objective of this thesis is to apply a novel trigonometric GM to estimate and predict pavement deterioration for two types of pavements and two types of rehabilitation methods. The pavement types considered are asphalt and asphalt over concrete. These two pavement types make up the majority of pavement types on interstates in South Carolina. The pavement type considered rehabilitation methods are mill-and-replace 2 to 4 inches + 200 PSY (pounds per square yard) overlay and mill-and-replace 1 to 2 inches + 400 PSY overlay. These two are the most frequently used treatments in the last 10 years in South Carolina by number of projects and lane-miles. According to the 2019 SCDOT Transportation Asset Management Plan (TAMP), the added service life of mill-and-replace 2 to 4 inches + 200 PSY and mill-and-replace 1 to 2 inches + 400 PSY is 15 and 20 years, respectively (*9*). The performance of the proposed trigonometric model is compared with the GM(1,1) model and S-shaped models using pavement condition data from South Carolina. Model performance is assessed using mean absolute percentage error (MAPE) and root mean square error (RMSE).

The remainder of this thesis is organized as follows. Section 2 provides a review of relevant studies. Section 3 presents the methodology and data used in this study. Section 4 presents and discusses the results. Lastly, Section 5 provides a summary of the study and concluding remarks.

CHAPTER 2: LITERATURE REVIEW

Over the last several decades, there have been numerous studies that have applied GMs. Readers are referred to the work of Liu, Lin, and Forrest (*10*) for the theoretical foundations underpinning GMs and the work of Deng (*11*, *12*) for their applications. A review of GM application in transportation can be found in the work by Bezuglov and Comert (*13*), Comert, Begashaw, and Huynh (*14*). The following review is focused on studies that have applied GMs to estimate pavement condition.

Du and Shen (*15*) developed a pavement permanent deformation (i.e., rutting) prediction model (PDPM) using GM. The purpose of their PDPM is to predict the rut depth and correct the predicted rut depth when the current rut depth is measured. They derived a multivariate GM where the function contains the previous measured rut depth and the number of loading cycles representing the traffic loading. Since there are two variables, their GM is a GM(1,2). In general, a GM(1, n) model takes $n - 1$ correlative variables as an associated series in addition to the predicted series. Their analysis showed that 95 of the 96 rutting predictions were within the ±2.5 mm tolerance level. The authors concluded that GM(1,2) is suitable for making predictions of rut depth.

Yu et al. (*16*) proposed a new Pavement Quality Index (PQI) model, which is a weighted function of four factors: 1) pavement condition index, 2) riding quality index, 3)

rut depth, and 4) skid resistance index. Due to the need to determine the appropriate weights for these four factors, the authors proposed the use of grey relational analysis. The authors reasoned that this method is suitable for situations where experiments are ambiguous or when the experimental method cannot be carried out exactly. From their analysis of the proposed PQI model for predicting service life of micro-surfacing, the authors concluded that the application of GM(1,1) and grey relational analysis for this application is reasonable to a certain degree.

Zhang et al. (*17*) applied GM(1,1) models to estimate pavement smoothness, rut, and skid resistance. They assessed their models' performances against field measured data using residuals and gray absolute correlation as metrics. They stated that the biggest residual for the above three indices is smaller than 8.09%, and the grey absolute correlation degrees all exceed 0.9, which means the accuracy of the GM(1,1) is excellent.

Tang and Xiao (*18*) applied a GM(1,1) to estimate asphalt pavement PQI and found that their model outperformed other models for the same dataset. In their study, they specifically addressed the ill-conditioned nature of the GM(1,1) model matrix, which according to the authors could limit the GM(1,1) model computational accuracy. They found that if the original PQI values are used, the GM(1,1) model matrix would be ill-conditioned. To overcome this issue, they proposed the use of "attenuations", defined as the difference in PQI between successive years. For example, for the period 2013 to 2014, the attenuation is obtained by subtracting the 2014 PQI value from the 2013 value. Since PQI values decrease over time, this approach produced a nonnegative smooth sequence required by the GM(1,1). They evaluated the matrix condition numbers using annual, quarterly, monthly, bimonthly, and daily attenuations, and found that monthly yielded the

lowest number of conditions. Monthly attenuations were obtained by dividing the annual attenuations by 12.

The above review indicates there are two gaps in the current body of work. First, no study has applied or developed a GM beyond a GM(1,1) or GM(1,2) model for estimating and predicting pavement condition. Second, the use of GM to estimate pavement deterioration for a specific rehabilitation or preservation treatment is limited to asphalt micro-surfacing. A natural progression in this line of research is to ascertain whether GMs are suitable for other pavement types and rehabilitation/preservation treatments and whether more advanced GM models should be utilized. To this end, this study applies a trigonometric GM that incorporates a cosine function into the GM(1,1) model to capture the nonlinear trends on pavement deterioration (11). It is applied to two different pavement types, asphalt and asphalt over concrete, and two different rehabilitation methods, mill-and-replace 2 to 4 inches + 2-inch overlay and mill-and-replace 1 to 2 inches + 4-inch overlay.

Table 2.1 Literature review summary table

Authors	Modeling methods	Response and significant variables	Findings
Deng (1982)	Grey system theory	Inter-disciplinarity	Theoretical foundations, methods and techniques of practical application of Grey system theory.
Deng (1988)	Grey system theory	Inter-disciplinarity	Theoretical foundations, methods and techniques of practical application of Grey system theory.
Liu, Lin, and Forrest (2010)	Grey system theory	Exploration of theoretical foundations, methods and techniques of practical application of Grey system theory in all fields of science.	The intent of this monograph was to explore the fundamental theory, methods, and techniques of practical application of grey systems theory, initiated by Professor Deng Julong in 1982. This volume presents most of the recent advances of the theory accomplished by scholars from around the world.
Bezuglov and Comert (2016)	GM(1,1), GM(1,1) with Fourier error corrections, and Grey Verhulst model with Fourier error corrections.	Traffic parameters: speed and travel time.	This study applied Grey models to predict traffic parameters. the GMs are found to be simple, adaptive, able to deal better with abrupt parameter changes, and not requiring many data points for prediction updates. Based on the sample data used, Grey models consistently demonstrated lower prediction errors compared to nonlinear time series models in term of MAPE and RMSE.
Comert, Begashaw, and Huynh (2021)	GM(1,1|cos(ωt)), GM(1,1|sin(ωt), cos(ωt)), and GM(1,1|exp(-at), sin(ωt), cos(ωt)).	Traffic parameters: speed, travel time, volume, flow, and occupancy.	The three proposed models, GM(1,1|cos(ωt)), GM(1,1|sin(ωt), cos(ωt)), and GM(1,1|exp(-at), sin(ωt), cos(ωt)), outperformed the GM(1,1) and the Grey Verhulst model in term of MAPE. It is observed that the proposed Grey models are more adaptive to location, and they do not require as many data points for training.

Table 2.2 Literature review summary table (continued)

Authors	Modeling methods	Response and significant variables	Findings
Du and Shen (2005)	GM(1,2)	Rut depth	Their analysis showed that 95 of the 96 rutting predictions were within the ±2.5 mm tolerance level. The authors found that the GM(1,2) is suitable for making predictions of rut depth
Yu, Zhang, and Xiong (2017)	GM(1,1) and grey relational analysis.	PQI	From their analysis of the proposed PQI model for predicting service life of micro-surfacing, the authors concluded that the application of GM(1,1) and grey relational analysis for this application is reasonable to a certain degree.
Zhang et al. (2019)	GM(1,1)	Pavement smoothness, rut, and skid resistance.	The authors assessed their models' performances against field measured data using residuals and grey absolute correlation as metrics. The biggest residual for the predicted indices is smaller than 8.09%, and the grey absolute correlation degrees all exceed 0.9, which means the accuracy of the GM(1,1) is excellent.
Tang and Xiao (2019)	GM(1,1)	PQI	The ill-conditioned nature of the GM(1,1) model matrix, which according to the authors could limit the GM(1,1) model computational accuracy. They found that if the original PQI values are used, the GM(1,1) model matrix would be ill-conditioned. To overcome this issue, they proposed the use of "attenuations", defined as the difference in PQI between successive years.

CHAPTER 3: METHODOLOGY

3.1 Data Description

The primary interest of this study is to model the pavement deterioration over time after rehabilitation. The pavement serviceability index (PSI), a measure of pavement rideability, is selected to represent pavement functional condition for which we want to estimate. PSI is one of two indices used by the SCDOT to quantify its pavement functional condition. The other index being the pavement distress index (PDI). PSI is related to the International Roughness Index (IRI): $PSI = 5 \times e^{-0.004 \times IRI}$. This equation yields a PSI value between 0 and 5, where 5 represents a perfectly smooth pavement surface. IRI is collected every one-tenth of a mile and is measured in inches per mile. For modeling purposes, an average PSI is used for the entire segment. Thus, each segment, regardless of its length, will have only one PSI value. For example, if a segment is 1 mile long, then the 10 IRI measurements are used to compute an average IRI per year, from which the average PSI is determined using the mentioned equation. The breakdown of PSI data used to train and evaluate the GMs is shown in Tables 3.1 to 3.4 for each pavement type and rehabilitation method. BIT (bituminous) in Table 1 refers to asphalt, and BOC refers to bituminous over concrete. Note that 200 PSY overlay is equivalent to 2-inch overlay and 400 PSY overlay is equivalent to 4-inch overlay.

The mill and replace 2-4 inches + 200 PSY treatment consists of milling the existing open-graded friction course (OGFC) and replacing Surface approximately 3-4 inches. This treatment uses 200 PSY Surface Type A mixture and 120-140 PSY OGFC. According to

the current SCDOT performance model, the added service life is 13 years for asphalt pavement and 20 years for asphalt over concrete pavement. On the other hand, the mill and replace 1-2 inches + 400 PSY consists of milling the existing OGFC and replacing Surface approximately 1-2 inches. It uses 200 PSY Surface Type A mixture, another 200 PSY Intermediate Type B mixture, and 120-140 PSY OGFC. The service life added is 18 years for asphalt pavement and 14 years for asphalt over concrete pavement.

Surface types A and B are both surface mixtures, but they are mainly different in mix design process. The asphalt binder grade for Surface Type A is PG 76-22, and PG 64-22 for Surface type B *(*19*)*. Asphalt binders are most commonly graded by their physical properties. An asphalt binder's physical properties directly describe how it will perform as a constituent in asphalt concrete pavement. The asphalt binder grading system is measured in Performance Graded (PG) used by all the 49 states in the United States. The PG grading system is based on climate (temperature in °C), so the grade notation consists of two portions: high and low pavement service temperature. High temperature increases rutting or fatigue cracking of pavement failure mode, while the cold temperature can increase thermal cracking. Thus, an average of 7 days maximum pavement temperature is used for describing the high-temperature climate. On the low-temperature side, thermal cracking can happen during one freezing night; therefore, the minimum pavement temperature describes the low-temperature climate. For both high and low-temperature grades, PG grades are graded in 6°C increments. The average 7-day maximum pavement temperature typically ranges from 46 to 82°C, and minimum pavement temperature typically ranges from -46°C to -10°C *(*20*)*. Based on the above asphalt binder description, Surface Type A

has good rutting resistance for high temperatures (PG 76-22) compared to Surface Type B (PG 64-22).

Table 3.1 Segments containing PSI data for MR 2 to 4 inches + OL 200 PSY for asphalt pavement

Route	**Number of years of data available**	**Date range**		**Length (miles)**
I-185 N	7	2011	2020	2.57
I-26 W	7	2013	2020	8.73
I-77 N	6	2013	2020	16.74
I-20 W	6	2012	2020	6.5
I-85 N	7	2011	2020	1.73
I-26 W	5	2012	2020	16.88
I-26 W	10	2012	2020	4.85
I-77 N	8	2011	2020	3.57
I-85 S	9	2011	2020	6.73
I-85 S	10	2011	2020	2.6
Total				70.90

Table 3.2 Segments containing PSI data for MR 2 to 4 inches + OL 200 PSY for asphalt over concrete pavement

Route	**Number of years of data available**	**Date range**		**Length (miles)**
I-26 E	7	2010	2020	5.1
I-26 E	5	2011	2020	8.68
I-85 N	5	2012	2020	3.99
I-26 W	6	2012	2020	5.72
I-77 S	7	2011	2020	3.99
I-526 E	6	2013	2020	2.02
I-20 W	6	2007	2020	14.16
I-95 S	7	2007	2020	15.33
I-85 S	7	2009	2020	7.92
I-385 S	7	2008	2020	4.37
Total				71.28

Table 3.3 Segments containing PSI data for MR 1 to 2 inches + OL 400 PSY for asphalt pavement

Route	**Number of years of data available**	**Date range**		**Length (miles)**
I-20 E	7	2015	2020	21.3
I-20 E	7	2015	2020	21.3
I-20 W	6	2014	2020	6.36
I-20 W	3	2014	2020	10.6
I-20 W	4	2013	2020	5.81
I-26 E	6	2001	2020	9.25
I-26 E	6	2012	2020	4.55
I-77 S	6	2013	2020	15.94
I-85 N	5	2009	2020	7.38
I-20 E	8	2012	2020	5.78
Total				108.27

Table 3.4 Segments containing PSI data for MR 1 to 2 inches + OL 400 PSY for asphalt over concrete pavement

Route	**Number of years of data available**	**Date range**		**Length (miles)**
I-20 E	3	2011	2020	14.54
I-20 E	3	2011	2020	6.1
I-185 S	3	2012	2020	2.07
I-26 E	4	2015	2020	7.15
I-20 W	5	2014	2020	11.14
I-26 E	16	2012	2020	4.44
I-26 E	6	2012	2020	17.5
I-26 E	5	2012	2020	1.02
I-26 E	9	2013	2020	12.68
I-95 S	6	2010	2020	5.35
Total				81.99

Figures 3.1 to 3.4 show the average PSI of each segment over time for each treatment method and pavement type. Note that year 0 denotes the year the pavement segment was rehabilitated. It can be seen that the average PSI trends do not exhibit a monotonic decreasing trend. Take MR 1-2 in. + OL 400 PSY asphalt over concrete, for example, the PSI decreased slightly from year 14 to 15 and increased from year 15 to 16. Possible reasons include the use of different segments across the entire state and use of different equipment/vendor to collect the IRI. Also, note that the PSI data exhibit serial correlation and are non-stationary.

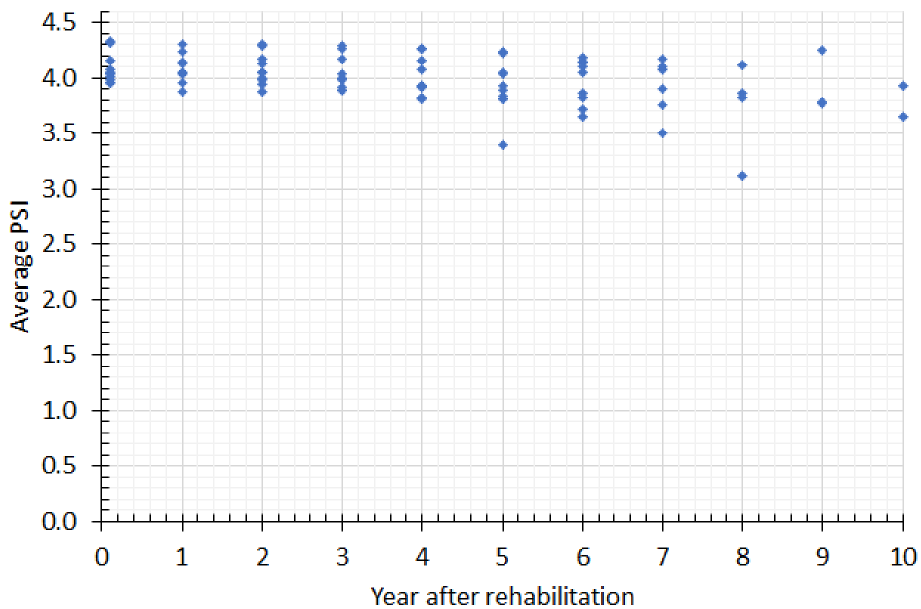

Figure 3.1 Average PSI of 10 segments for MR 2 to 4 inches + OL 200 PSY for asphalt pavement

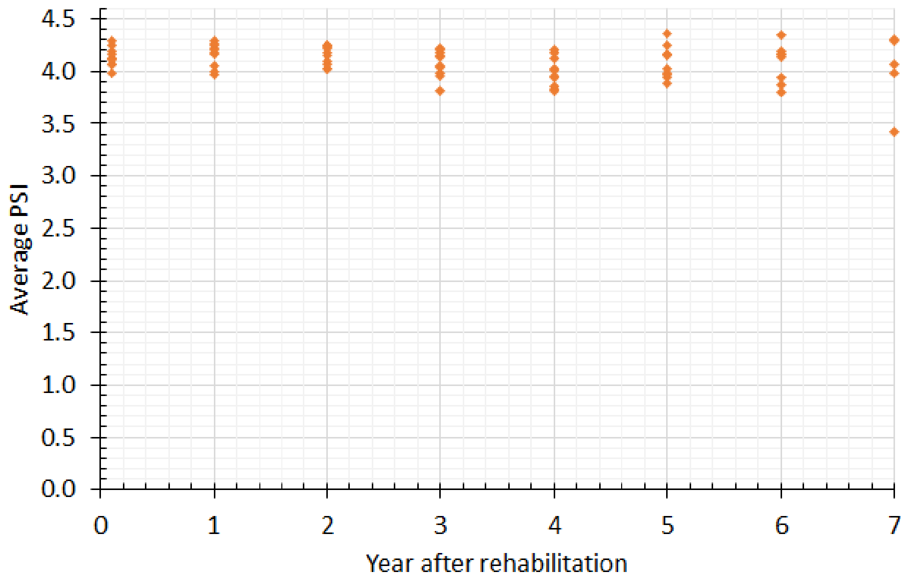

Figure 3.2 Average PSI of 10 segments for MR 2 to 4 inches + OL 200 PSY for asphalt over concrete pavement

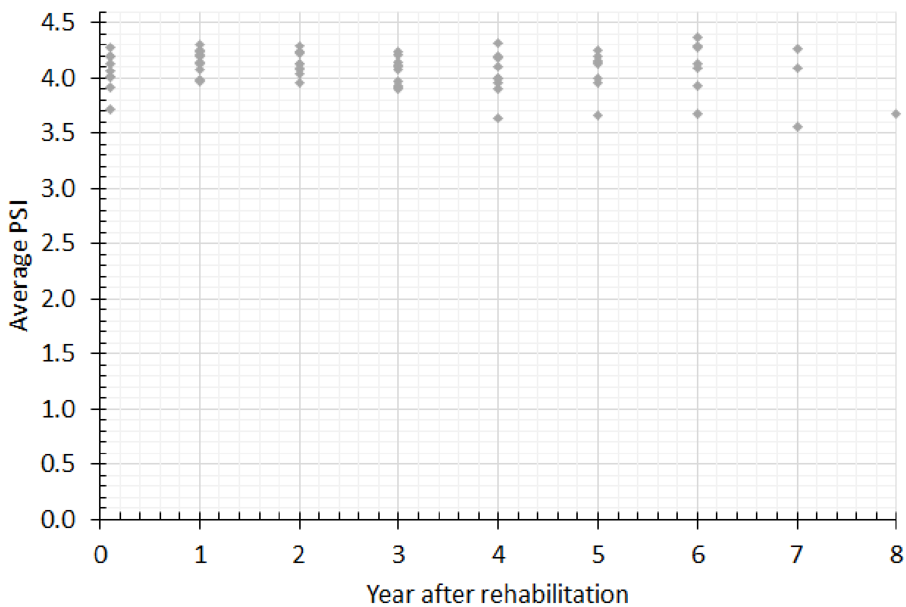

Figure 3.3 Average PSI of 10 segments for MR 1 to 2 inches + OL 400 PSY for asphalt pavement

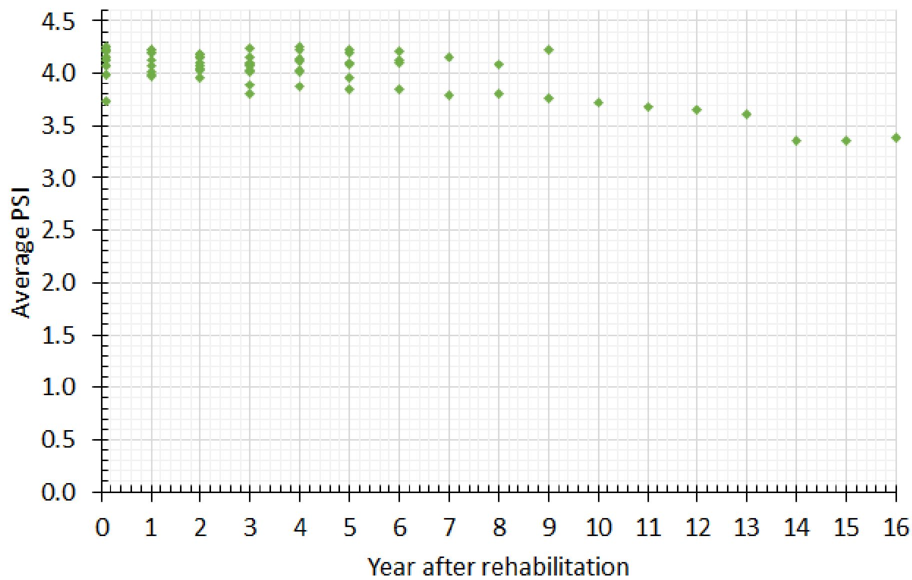

Figure 3.4 Average PSI of 10 segments for MR 1 to 2 inches + OL 400 PSY for asphalt over concrete pavement

3.2 Grey System Theory-Based Models

3.2.1 GM(1,1) Model

To model a time series, the Grey System theory (*11*) provides a family of GMs, where the most basic one is the first-order GM with one variable, often referred to as GM(1,1). The principles and estimation of GM(1,1) is briefly discussed here. Readers are referred to the work of Deng (*11*) for additional information. Suppose that $X^{(0)} = (x^{(0)}(1),x^{(0)}(2),\ldots,x^{(0)}(k))$ denotes a sequence of k nonnegative observations of a stochastic process and $X^{(1)} = (x^{(1)}(1),x^{(1)}(2),\ldots,x^{(1)}(k))$ is an accumulation sequence of $X^{(0)}$ computed as:

$$x^{(1)}(k) = \sum_{j=1}^{k} x^{(0)}(j) \qquad (4)$$

The original form of the GM(1,1) is:

$$x^{(0)}(k) + ax^{(1)}(k) = b \qquad (5)$$

Let $z^{(1)} = (z^{(1)}(2), z^{(1)}(3), \ldots, z^{(1)}(k))$ be a mean sequence of $x^{(1)}$ calculated using Equation 6.

$$z^{(1)}(k) = \left[x^{(1)}(k-1) + x^{(1)}(k)\right]/2 \tag{6}$$

The basic form of GM(1,1) where *a* and *b* are parameters to be estimated is:

$$x^{(0)}(k) + az^{(1)}(k) = b \tag{7}$$

If $\left(\hat{a}, \hat{b}\right) = (a, b)^T$ and

$$\boldsymbol{Y} = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(k) \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(k) & 1 \end{bmatrix}$$

Then, as presented in Liu and Lin (*21*), the least squares estimates of the GM(1,1) model is $\left(\hat{a}, \hat{b}\right) = (\boldsymbol{B}^T \boldsymbol{B})^{-1} \boldsymbol{B}^T \boldsymbol{Y}$. Suppose that $\hat{x}^{(0)}(k)$ and $\hat{x}^{(1)}(k)$ represent the original time response sequence and the accumulated time response sequence of the GM at time *k*, respectively, then the latter can be obtained by solving Equation 8 (whitenization equation of the GM(1,1) model). The solution to Equation 8 is shown in Equation 9.

$$\frac{dx^{(1)}}{dt} + ax^{(1)}(k) = b \tag{8}$$

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right) e^{-ak} + \frac{b}{a} \tag{9}$$

According to the definition in Equation 4, the restored values of $\hat{x}^{(0)}(k+1)$ are calculated as $\hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k)$. Essentially, we are taking derivatives at step k because we are calculating the slope for this interval. Performing this step results in

$$\hat{x}^{(0)}(k+1) = (1 - e^{a}) \left(x^{(0)}(1) - \frac{b}{a}\right) e^{-ak} \tag{10}$$

which can be used to produce forecasts for $x^{(0)}(k+1)$, $x^{(0)}(k+2)$, and so on.

3.2.3 GM(1,1|cos(ωt)) Model

The GM(1,1) model has been shown to work well when the data exhibits a steady growth or decline, but it may not perform well when the data exhibit nonlinear trends, oscillations or saturated sigmoid sequences as indicated by Mao et al. (*22*) for traffic flow data, Bezuglov and Comert for short-term traffic speed and travel time prediction (*13*), and Comert et al. (*14*) for queue length data at traffic signals. To account for nonlinearity in the data, the trigonometric version of the GM(1,1) model from Comert et al. (*14*) is adopted for this study. The basic form of the GM(1,1|cos(ωt)) is given as:

$$\frac{dx^{(1)}}{dt} + ax^{(1)}(k) = b_1 cos(\omega t) + b_2 \tag{11}$$

The estimated parameters are denoted by $(\hat{a}, \hat{b}_1, \hat{b}_2)$ for $(a, b_1, b_2)^T$ and can be obtained simultaneously using $(\boldsymbol{B^T B})^{-1}\boldsymbol{B^T X}$ where $\boldsymbol{X}$ are observations up to k and the coefficient matrix $\boldsymbol{B}$ is given by

$$\boldsymbol{B} = \begin{bmatrix} -z^{(1)}(2) & cos(\omega 2) & 1 \\ -z^{(1)}(3) & cos(\omega 3) & 1 \\ \vdots & \vdots & \\ -z^{(1)}(k) & cos(\omega k) & 1 \end{bmatrix} \tag{12}$$

Since ω is within the cosine function, it is not estimated simultaneously with $\boldsymbol{B}$ but rather via a trial-and-error approach.

The solution to the differential equation (Equation 11, whitenization equation for GM cosine model) is given by

$$x^{(1)}(k+1) = Ke^{-k} + \frac{\left(a^2 b_2 + b_2 \omega^2 + a^2 b_1 \cos(\omega k) + a b_1 \omega \sin(\omega k)\right)}{\left(a(a^2+\omega^2)\right)} \tag{13}$$

where, K is obtained from the initial condition $x^{(1)}(1) = x^{(0)}(1)$ and is given by

$$K = e^{a}\left[x^{(0)}(1) - \frac{\left(a^2 b_2 + b_2\omega^2 + a^2 b_1\cos(\omega) + ab_1\omega\sin(\omega)\right)}{\left(a(a^2+\omega^2)\right)}\right] \tag{14}$$

Following the same approach to derive Equation 10, Equation 13 is used to obtain the estimation equation:

$$x^{(0)}(k+1) = Ke^{-ak}(1-e^{a}) + \frac{1}{a^3 + a\omega^2}\Big[-ab_1\omega\left(\sin(\omega k) - \sin\big(\omega(k-1)\big)\right) + a^2 b_1\left(\cos\big(\omega(k-1)\big) - \cos(\omega k)\right)\Big] \tag{15}$$

which can be used to make forecasts $\hat{x}^{(0)}(k+1)$, $\hat{x}^{(0)}(k+2)$, and so on.

In this study, a rolling GM approach is taken. That is, the GM(1,1|*cos*(*ωt*)) model (Equation 11) is used to estimate and forecast one or more future data points: $\hat{x}^{(0)}(k+w+1)$, $\hat{x}^{(0)}(k+w+2)$, etc. using a fixed interval, *w*, of prior pavement condition data: $x^{(0)}(k+1),x^{(0)}(k+2),...,x^{(0)}(k+w)$, where $w \geq 4$. Then the process is repeated where the fixed interval is shifted to the next period and the model is used to calculate $x^{(0)}(k+2)$, $x^{(0)}(k+3)$, …, $x^{(0)}(k+n)$, where *n* denotes the desired future n^{th} year for which data need to be estimated (*23*).

The following illustrates the application of the proposed trigonometric GM(1,1|cos(*ωt*)) model. Suppose we want to predict year 11's PSI value using the past 10 years of data for the treatment MR 2-4 inches + 200 PSY overlay as shown in Table 5. To apply Equation 15, we first need to obtain values for the four parameters (a, b_1, b_2, and ω) for each year. These are found using the procedure discussed above and shown in Table 5. To predict the PSI value for year 11, Equation 15 is used with k = 1 and a, b_1, b_2, and ω equal to those values shown in Table 1 for year 10. The 1-step predicted PSI for year 11 is 3.7564. To predict the PSI value for year 12, Equation 14 is used with k = 2 and a, b_1, b_2,

and ω also equal to those values shown in Table 5 for year 10. The 2-step predicted PSI for year 12 is 3.7335. Note that to predict beyond year 11, the values for parameters a, b_1, b_2, and ω from year 10 are used.

Table 3.5 Average PSI and parameter values for MR 2 to 4 inches + OL 200 PSY for asphalt pavement

Years	Observed Avg PSI	Estimated PSI	a	b_1	b_2	ω
0	4.0802	4.0804	0.3222	0.4290	1.0483	0.2584
1	4.0806	4.0805	0.3220	0.4271	1.0456	0.2599
2	4.0778	4.0780	0.3379	0.4259	1.0440	0.2607
3	4.0399	4.0417	0.3411	0.4252	1.0432	0.2612
4	4.0145	4.0128	0.3379	0.4258	1.0440	0.2606
5	3.9194	3.9233	0.3355	0.4262	1.0446	0.2599
6	3.9643	3.9551	0.3201	0.4282	1.0486	0.2542
7	3.9413	3.9490	0.3081	0.4292	1.0517	0.2489
8	3.7303	3.7371	0.3059	0.4293	1.0522	0.2478
9	3.9331	3.9065	0.2824	0.4293	1.0582	0.2336
10	3.7874	3.8179	0.2766	0.4292	1.0596	0.2301

As noted in the literature review, the condition number of the GM model matrix needs to be small in order for it to produce accurate estimates. A condition number for a matrix and related computational task measures how sensitive the answer is to perturbations in the input data and to roundoff errors made during the solution process. The definition of the condition number depends on the choice of norm. When a matrix is said to be "ill-conditioned", it refers to the sensitivity of its inverse, i.e., of the condition number for inversion, and not of all the other condition numbers. If the condition number is not too much larger than one, the matrix is well-conditioned, which means that its inverse can be computed with good accuracy. If the condition number is very large, then the matrix is said to be ill-conditioned. Practically, such a matrix is almost singular, and the

computation of its inverse, or solution of a linear system of equations is prone to large numerical errors. If a matrix is not invertible, the condition number is taken to be infinity. When applying the GMs to estimate pavement condition using the South Carolina PSI data shown in Table 1, its matrix was found to be ill-conditioned. This is due to having values that are similar or repeating in the input data. This general problem of the grey GM(1,1) model has been observed by Limin Tang and Duyang Xiao (*18*). To overcome this issue, a Gaussian noise ~ N(0,0.0001) was added to make values slightly different from each other to allow one of the multiple solutions to be found.

3.3 S-shaped Regression Models

To evaluate the performance of the proposed GM (1,1|cos(ωt)) model, the estimated PSI are compared with those obtained by using the GM(1,1) model and the following S-shaped models in Equation 15 (*6*) and Equation 16 (*24*):

$$y_i = \frac{a}{1+exp\left(-\frac{x_i-b_1}{c}\right)} \tag{15}$$

$$y_i = O - e^{\left(a-b_1\times c^{\ln\left(\frac{1}{x_i}\right)}\right)} \tag{16}$$

where y_i is the measure to be estimated in the ith year; x_i is the corresponding pavement age; a, b_1, c, and O are parameters in the model to be estimated based on observed data. An accurate initial estimate of a, b_1, c, and O parameters is the necessary first step for obtaining the best-fit nonlinear regression model. Essentially, a in Equation 15 and O in Equation 16 represents the maximum or the asymptote values of the performance rating, which is the initial or starting point of the curve; a in Equation 16, b_1 and c in Equation 15

and 16 are related to the horizontal shift of the deterioration curve and the slope with which the deterioration curve reaches the asymptote (*24*).

The S-shaped models used in this study employ nonlinear least-squares method to estimate parameters in the regression. According to Prozzi and Madanat (*25*), the term nonlinear refers to the procedure used to estimate the parameters of a specific regression model rather than to the specification form. Thus, a general form of the nonlinear regression model can be represented as follows:

$$y_i = h(x_i, \beta) + \varepsilon_i \tag{17}$$

where y_i is a dependent or explained variable, x_i is a vector of independent or explanatory variables; β is a vector of parameters, and ε_i is the random error term, and h is a nonlinear function of β.

Suppose the ε_i in Equation 17 is normally distributed with mean zero and constant variance ρ^2. In that case, it can be deduced that the value of the parameters that minimize the sum of the squared errors will be the maximum likelihood estimators and the nonlinear least-squares estimators. The objective function (Z_{ols}) is given by

$$Z_{OLS}(\beta) = \frac{1}{2}\sum_{i=1}^{n} \varepsilon_i^2 = \frac{1}{2}\sum_{i=1}^{n}[y_i - h(x_i, \beta)]^2 \tag{18}$$

Unlike linear regression, the first-order conditions for least-squares estimation are nonlinear functions of the parameters. The values b of the parameters β obtained by minimizing Equation (18) are referred to as the least-squares estimates of β or the Maximum Likelihood estimators (MLE) estimates of β (*25*).

The following illustrates the process of solving the unknown model parameters of a nonlinear model using the least-squares method of finding the best fit. In Equation 16,

The values of the coefficients a, b_1 and c can be calculated from observed performance data using the least-squares method of finding the best fit. Equation 19 can be used to solve for the coefficient c using a simple numerical analysis method.

$$\left(\sum z_j \times t_j \times ct_j\right) - \frac{\left(\sum ct_j^2\right)\times\left(\sum z_j\right)-\left(\sum t_j\right)\times\left(\sum t_j \times ct_j\right)}{m\times\left(\sum ct_j^2\right)-\left(\sum ct\right)^2} \times \left(\sum t_j \times ct_j\right) +$$

$$\frac{-m\times\left(\sum z_j \times ct_j\right)+\left(\sum ct_j\right)\times\left(\sum z_j\right)}{m\times\left(\sum ct_j^2\right)-\left(\sum ct_j\right)^2} \times \left(\sum t_j \times ct_j^2\right) = 0 \qquad (19)$$

where,

m = number of observations

$\sum$ = sum over all observations, j = 1 to m

$t = \ln\left(\frac{1}{x}\right)$, x is the age of the pavement

$z_j = \ln\left(PSI_0 - PSI_f\right)$

$ct_j = c^{t_j}$

The determination of c becomes accurate as the number of observations increases. Once the value or the coefficient c is determined, the values for coefficients a and b_1 can be calculated using the following equations:

$$a = \frac{\left(\sum ct_j^2\right)\times\left(\sum z_j\right)-\left(\sum ct_j\right)\times\left(\sum t_j \times ct_j\right)}{m\times\left(\sum ct_j^2\right)-\left(\sum ct_j\right)^2} \qquad (20)$$

$$b_1 = \frac{-m\times\left(\sum z_t \times ct_j\right)+\left(\sum ct_j\right)\times\left(\sum z_j\right)}{m\times\left(\sum ct_j^2\right)-\left(\sum ct_j\right)^2} \qquad (21)$$

Since the sigmoidal model is designed to be a site-specific model, these coefficients need to be determined for each pavement section based on the measured pavement performance data. The initial PSI value of a newly constructed pavement in South Carolina

is 4.6. Thus, the *a* and *O* parameter in Equation 15 and 16, respectively, indicate the initial or starting point of the curve. To avoid forcing the curve to start at a value higher than the observed data, parameters *a* in Equation 15 and *O* in Equation 16 are set to be estimated in the models following the same least-squares procedure. The estimation of parameters *a* and *O* increases the accuracy of the performance curve to fit the data.

CHAPTER 4: FINDINGS

To evaluate the performance of the proposed GM (1,1|cos(ωt)) model, the estimated PSI are compared with those obtained by using the GM(1,1) model, the S-shaped model 1 (Equation 15), and the S-shaped model 2 (Equation 16). The metrics used for comparison are Mean Absolute Percentage Error (MAPE) and Root Mean Square Error:

$$MAPE = (\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\left|PSI_{i,act}-PSI_{i,est}\right|}{PSI_{i,act}}\right)) \times 100 \qquad (22)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}\left[PSI_{i,act}-PSI_{i,est}\right]^2}{N}} \qquad (23)$$

where,

N = number of observations

PSI_{act} = observed PSI

PSI_{pred} = estimated PSI

4.1 Mill and Replace 1-2 inches + OL 400 PSY for Asphalt Pavement

Figure 5 shows the estimated average PSI of the proposed GM(1,1|cos(ωt)) model compared to the GM(1,1) model, S-shaped model 1, and S-shaped model 2 for asphalt pavement up to 8 years after receiving the MR 1-2 inches + OL 400 PSY rehabilitation treatment. In this result and others shown subsequently, both GMs used an interval size,

w, of 5 years. That is, 5 observations are used to estimate/fit the PSI data. To estimate year 0's PSI, we generated 4 pseudo values by adding Gaussian noise ~ N(0,0.0001) to year 0's observed PSI and used the parameters obtained for year 0. In the rolling horizon framework, k = 4+1 = 5. It can be seen in Figure 5 that the fitted PSI values from the GM(1,1|cos(ωt)) model are closest to the observed PSI values. It has a MAPE of 0.09% compared to 0.99%, 3.22%, and 3.27% of the GM (1,1), S-shaped model 1, and S-shaped 2, respectively. The RMSE are 0.0049, 0.0465, 0.1676, and 4.91E-05 for the GM(1,1|cos(ωt)) model, GM (1,1) model, S-shaped model 1, and S-shaped model 2, respectively.

The 4-step (4-year) prediction of the GMs shows a steady and slow decline of the PSI, but the S-shaped models have an abrupt decline where the PSI reaches zero value. The GM(1,1|cos(ωt)) model 4-step prediction has a steady slow decline similar to the trend observed in the actual data for the first 8 years. In terms of service life, when the PSI value reaches 2.7, the GM (1,1|cos(ωt)) model predicted a service life of 17 years, compared to 16, 9 and 9 by the GM(1,1) model, S-shaped model 1 and S-shaped model 2, respectively.

It can be seen that the proposed GM captures the fluctuations in PSI well. In practice, a pavement's functional condition should not improve with time without some type of preservation or rehabilitation. Thus, the fluctuations seen here are due to measurement differences (equipment changes, calibration errors, operator differences, etc.). One possible approach to adopt when applying the proposed GM model is to take the minimum of the current year's PSI and next year's predicted PSI. For example, if this year's PSI value is 3.5 and the predicted PSI value is 3.6, then take minimum (3.5, 3.6) =

3.5 to be the predicted PSI, assuming we know with certainty that no maintenance has been performed on that pavement. This approach will ensure the predicted PSI will follow a monotonic decreasing function.

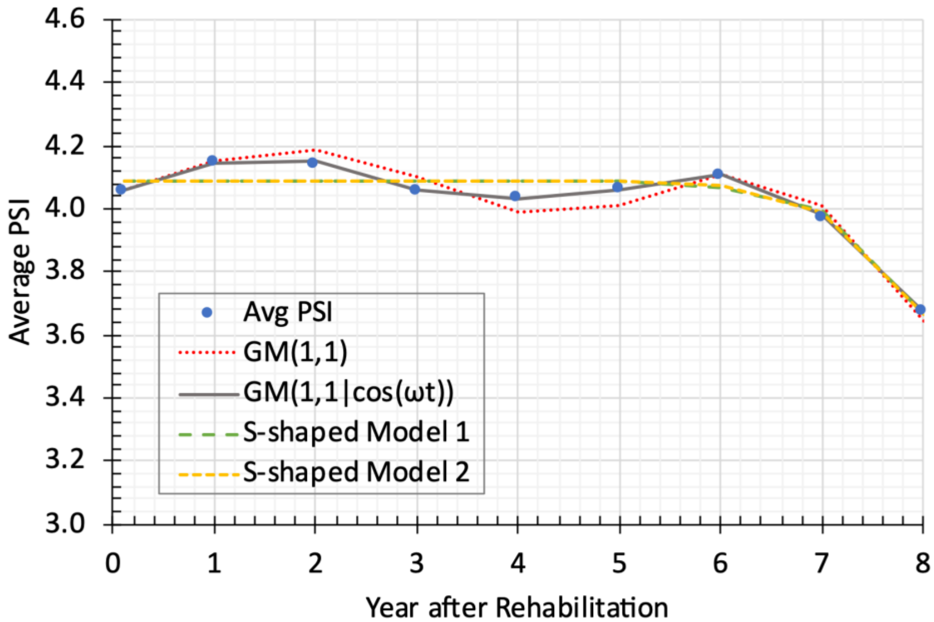

Figure 4.1 Comparison of estimated PSI for asphalt pavement and MR 1 to 2 inches + OL 400 PSY rehabilitation treatment

Table 4.1 Short-term (4-year) PSI prediction of GMs and S-shaped models for asphalt pavement and MR 1 to 2 inches + OL 400 PSY rehabilitation treatment

Year	Avg PSI (observed)	GM(1,1)	GM(1,1|cos(ωt))	S-shaped Model 1	S-shaped Model 2
0.1	4.1	4.1	4.1	4.1	4.1
1	4.1	4.1	4.1	4.1	4.1
2	4.1	4.2	4.1	4.1	4.1
3	4.1	4.1	4.1	4.1	4.1
4	4.0	4.0	4.0	4.1	4.1
5	4.1	4.0	4.1	4.1	4.1
6	4.1	4.1	4.1	4.1	4.1
7	4.0	4.0	4.0	4.0	4.0

8	3.7	3.6	3.7	3.7	3.7
9		3.4	3.6	2.6	2.7
10		3.3	3.4	1.1	0.3
11		3.2	3.5	0.3	N/A
12		3.1	3.1	0.1	N/A

Table 4.2 Models' evaluation and predicted service life of MR 1 to 2 inches + OL 400 PSY treatment for asphalt pavement

Parameters	GM(1,1)	GM(1,1|cos(ωt))	S-shaped Model 1	S-shaped Model 2
a	-0.0043	-0.0042	4.0895	15.1334
b_1	4.0124	0.0395	-9.3721	63.2865
b_2		4.0058		
c			-0.6351	1.9375
o				4.1
ω		0.2301		
w	5	5		
RMSE	0.0465	0.0049	0.1676	4.91E-05
MAPE (%)	0.99	0.09	3.28	3.27
Expected rehab year	16	17	9	9
PSI threshold	2.7	2.7	2.7	2.7

4.2 Mill and Replace 1-2 inches + OL 400 PSY for Asphalt over Concrete Pavement

Figure 6 shows the estimated average PSI of the proposed GM(1,1|cos(ωt)) model compared to the GM(1,1) model, S-shaped model 1, and S-shaped model 2 for asphalt over concrete pavement up to 16 years after receiving the MR 1-2 inches + OL 400 PSY rehabilitation treatment. It can be seen in Figure 6 that the fitted PSI values from the GM(1,1|cos(ωt)) model are closest to the observed PSI values. It has a MAPE of 0.2% compared to 1.29%, 2.66%, and 2.57% of the GM (1,1), S-shaped model 1, and S-shaped model 2, respectively. The RMSE are 0.0098, 0.068, 0.1291, and 2.44E-07 for the

GM(1,1|cos(ωt)) model, GM(1,1) model, S-shaped model 1, and S-shaped model 2, respectively. The 4-step prediction trend of the models has a steady slow decline similar to the trend observed in the actual data for the first 16 years. In terms of service life, the GM (1,1|cos(ωt)) model predicted a service life of 24 years, compared to 25, 19 and 22 by the GM(1,1) model, S-shaped model 1 and S-shaped model 2, respectively.

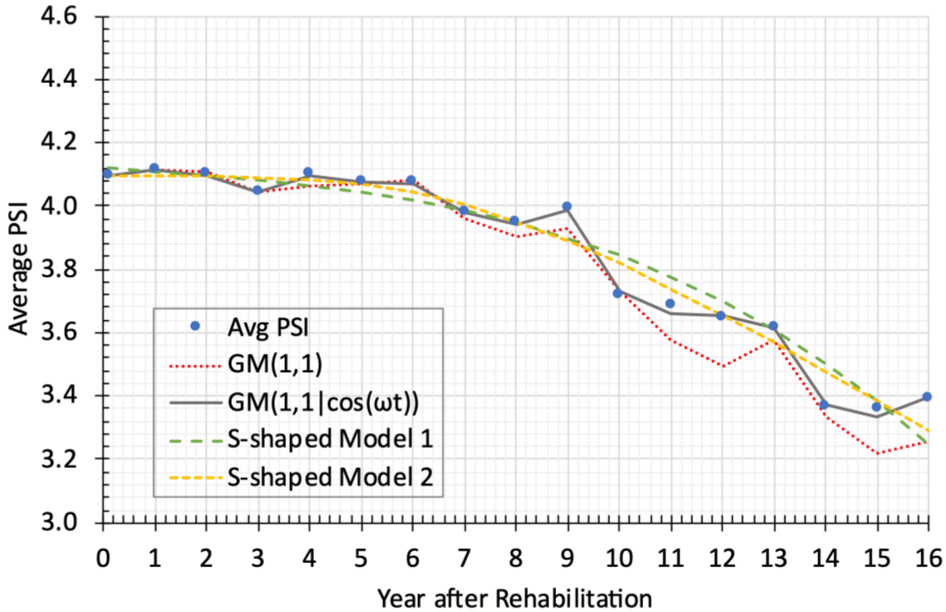

Figure 4.2 Comparison of estimated PSI for asphalt over concrete pavement and MR 1-2 inches + OL 400 PSY rehabilitation treatment

Table 4.3 Short-term (4-year) prediction of GMs and S-shaped models for asphalt over concrete pavement and MR 1-2 inches + OL 400 PSY rehabilitation treatment

Year	Avg PSI (observed)	GM(1,1)	GM(1,1|cos(ωt))	S-shaped Model 1	S-shaped Model 2
0.1	4.1	4.1	4.1	4.1	4.1
1	4.1	4.1	4.1	4.1	4.1
2	4.1	4.1	4.1	4.1	4.1
3	4.0	4.0	4.0	4.1	4.1

4	4.1	4.1	4.1	4.1	4.1
5	4.1	4.1	4.1	4.0	4.1
6	4.1	4.1	4.1	4.0	4.0
7	4.0	4.0	4.0	4.0	4.0
8	3.9	3.9	3.9	3.9	4.0
9	4.0	3.9	4.0	3.9	3.9
10	3.7	3.7	3.7	3.8	3.8
11	3.7	3.6	3.7	3.8	3.7
12	3.6	3.5	3.7	3.7	3.7
13	3.6	3.6	3.6	3.6	3.6
14	3.4	3.3	3.4	3.5	3.5
15	3.4	3.2	3.3	3.4	3.4
16	3.4	3.3	3.4	3.2	3.3
17		3.2	3.2	3.1	3.2
18		3.1	3.2	2.9	3.1
19		3.1	3.1	2.7	3.0
20		3.0	3.0	2.5	2.9

Table 4.4 Models' evaluation and predicted service life of MR 1 to 2 inches + OL 400 PSY treatment for asphalt over concrete pavement

Parameters	GM(1,1)	GM(1,1|cos(ωt))	S-shaped Model 1	S-shaped Model 2
a	0.0275	0.0272	4.16462	2.177
b_1	3.8786	-0.1303	-22.1902	21.176
b_2		3.9022		
c			-4.91054	2.194
o				4.1
ω		0.2301		
w	5	5		
RMSE	0.0680	0.0098	0.1291	2.44E-07
MAPE (%)	1.29	0.2	2.66	2.57
Expected rehab year	25	24	19	22
PSI threshold	2.7	2.7	2.7	2.7

4.3 Mill and Replace 2-4 inches + OL 200 PSY for Asphalt Pavement

Figure 7 shows the estimated average PSI of the proposed GM(1,1|cos(ωt)) model compared to the GM(1,1) model, S-shaped model 1, and S-shaped model 2 for asphalt pavement up to 10 years after receiving the MR 2-4 inches + Overlay 200 PSY rehabilitation treatment. It can be seen in Figure 7 that the fitted PSI values from the GM(1,1|cos(ωt)) model are closest to the observed PSI values. It has a MAPE of 0.21% compared to 0.64%, 3.72%, and 3.71% of the GM (1,1), S-shaped model 1, and S-shaped model 2, respectively. The RMSE are 0.0130, 0.0421, 0.1882, and 0.1879 for the GM(1,1|cos(ωt)) model, GM(1,1) model, S-shaped model 1, and S-shaped model 2, respectively. The 4-step prediction trend of the models has a steady slow decline similar to the trend observed in the actual data for the first 10 years. The predicted values of the models are similar. In terms of service life, the GM (1,1|cos(ωt)) model predicted a service life of 57 years, compared to 57, 27 and 106 by the GM(1,1) model, S-shaped model 1 and S-shaped model 2, respectively. This service life is longer than expected. It is most likely due to the proposed GM having to predict far into the future without the benefit of having updated parameters when used in a rolling horizon framework. There is also the possibility that improvement in construction practices and materials is prolonging the life of pavements.

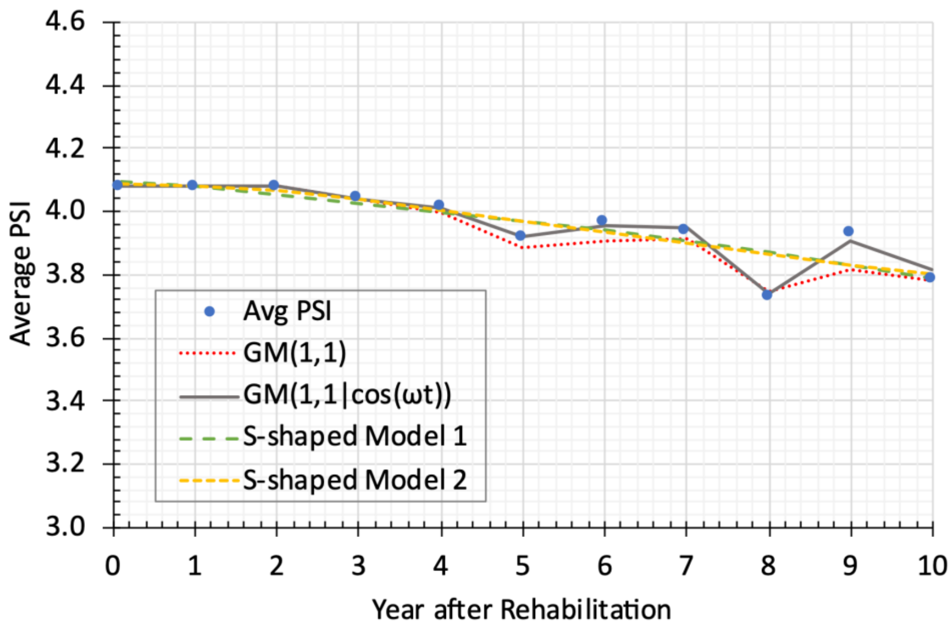

Figure 4.3 Comparison of estimated PSI for asphalt pavement and MR 2-4 inches + OL 200 PSY rehabilitation treatment

Table 4.5 Short-term (4-year) prediction of GMs and S-shaped models of MR 2 to 4 inches + OL 200 PSY treatment for asphalt pavement

Year	Avg PSI (observed)	GM(1,1)	GM(1,1|cos(ωt))	S-shaped Model 1	S-shaped Model 2
0.1	4.1	4.1	4.1	4.1	4.1
1	4.1	4.1	4.1	4.1	4.1
2	4.1	4.1	4.1	4.1	4.1
3	4.0	4.0	4.0	4.0	4.0
4	4.0	4.0	4.0	4.0	4.0
5	3.9	3.9	3.9	4.0	4.0
6	4.0	3.9	4.0	3.9	3.9
7	3.9	3.9	3.9	3.9	3.9
8	3.7	3.7	3.7	3.9	3.9
9	3.9	3.8	3.9	3.8	3.8
10	3.8	3.8	3.8	3.8	3.8
11		3.7	3.8	3.7	3.8
12		3.7	3.7	3.7	3.7
13		3.7	3.7	3.7	3.7

14		3.7	3.7	3.6	3.7

Table 4.6 Models' evaluation and predicted service life of MR 2 to 4 inches + OL 200 PSY treatment for asphalt pavement

Parameters	GM(1,1)	GM(1,1|cos(ωt))	S-shaped Model 1	S-shaped Model 2
a	0.0150	0.2766	4.3826	1.1812
b_1	4.0667	0.4292	-32.6947	6.7912
b_2		1.0596		
c			-12.2163	1.5602
o				4.1
ω		0.2301		
w	5	5		
RMSE	0.0421	0.0130	0.1882	0.1879
MAPE (%)	0.64	0.21	3.72	3.71
Expected rehab year	57	57	27	106
PSI threshold	2.7	2.7	2.7	2.7

4.4 Mill and Replace 2-4 inches + OL 200 PSY for Asphalt over Concrete Pavement

Figure 8 shows the estimated average PSI of the proposed GM(1,1|cos(ωt)) model compared to the GM(1,1) model, S-shaped model 1, and S-shaped model 2 for asphalt over concrete pavement up to 7 years after receiving the MR 2-4 inches + OL 200 PSY rehabilitation treatment. It can be seen in Figure 8 that the fitted PSI values from the GM(1,1|cos(ωt)) model are closest to the observed PSI values. It has a MAPE of 0.1% compared to 0.58%, 3.03%, and 3.03% of the GM (1,1), S-shaped model 1, and S-shaped model 2, respectively. The RMSE are 0.0053, 0.0347, 0.1518, and 0.152 for the GM(1,1|cos(ωt)) model, GM(1,1) model, S-shaped model 1, and S-shaped model 2, respectively. The 4-step prediction trend of the models has a steady slow decline similar to the trend observed in the actual data for the first 7 years. The predicted values of the models

are similar. In terms of service life, the GM (1,1|cos(ωt)) model predicted a service life of 62 years, compared to 67 and 94 and 141 by the GM(1,1) model, S-shaped model 1 and S-shaped model 2, respectively. As mentioned, this higher than expected service life is most likely due to the proposed GM having to predict far into the future without the benefit of having updated parameters.

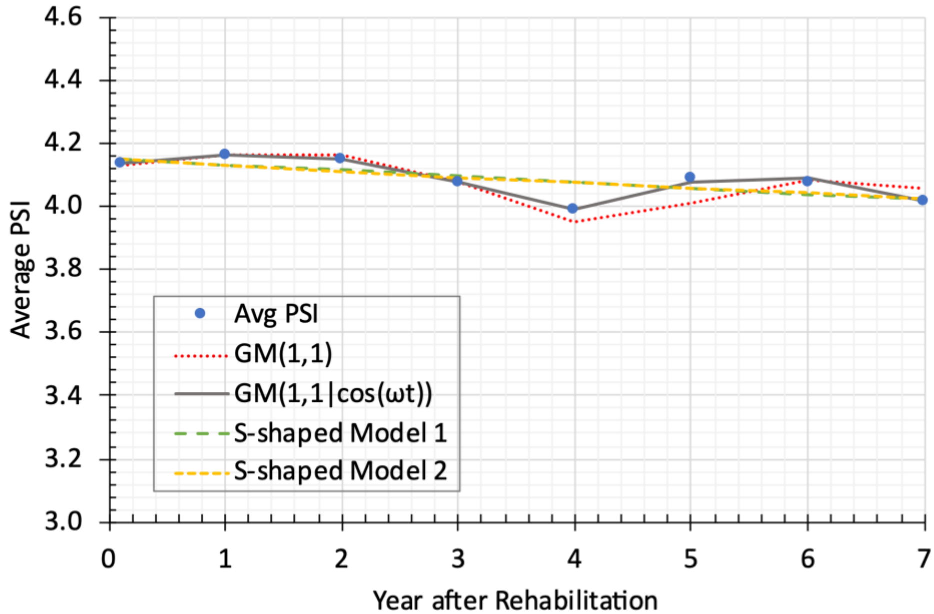

Figure 4.4 Comparison of estimated PSI for asphalt over concrete pavement and MR 2-4 inches + OL 200 PSY rehabilitation treatment

Table 4.7 Short-term (4-year) prediction of GMs and S-shaped models of MR 2 to 4 inches + OL 200 PSY treatment for asphalt over concrete pavement

Age	Avg PSI (observed)	GM(1,1)	GM(1,1|cos(ωt))	S-shaped Model 1	S-shaped Model 2
0.1	4.1	4.1	4.1	4.1	4.2
1	4.2	4.2	4.2	4.1	4.1
2	4.1	4.2	4.1	4.1	4.1
3	4.1	4.1	4.1	4.1	4.1

4	4.0	4.0	4.0	4.1	4.1
5	4.1	4.0	4.1	4.1	4.1
6	4.1	4.1	4.1	4.0	4.0
7	4.0	4.1	4.0	4.0	4.0
8		4.0	3.9	4.0	4.0
9		4.0	4.0	4.0	4.0
10		3.9	4.0	4.0	4.0
11		3.9	3.9	3.9	4.0

Table 4.8 Models' evaluation and predicted service life of MR 2 to 4 inches + OL 200 PSY treatment for asphalt over concrete pavement

Parameters	GM(1,1)	GM(1,1|cos(ωt))	S-shaped Model 1	S-shaped Model 2
a	0.0063	0.0066	48.9158	19.8228
b_1	4.1531	0.1012	482.7809	23.5870
b_2		4.1356		
c			-202.9981	1.0398
o				4.2
ω		0.2301		
w	5	5		
RMSE	0.0347	0.0053	0.1518	0.152
MAPE (%)	0.58	0.1	3.03	3.03
Expected rehab year	67	62	94	141
PSI threshold	2.7	2.7	2.7	2.7

4.5 Discussion

The results from this study indicated that the proposed GM (1,1|cos(ωt)) model only needs 5 observations to make accurate estimation. By utilizing a rolling horizon approach, the model parameters can be updated with each new observation, enabling it to better capture trends in the data. From this study, it can also be concluded that GMs can handle the serial correlation in PSI data as well as the nonstationary nature of the PSI data. When we attempted to force the proposed GM to have estimated PSIs follow a monotonic

decreasing trend by making the fixed interval, w, larger than or equal to the number of observations, it was observed that it lowered the accuracy of the model. Regarding the longer than expected service life, it is due to the use of a shorter interval ($w = 4$) for the rolling horizon and predicting with the last estimated hyperparameters. If a larger interval w is used, then it would produce a shorter service life for the two cases where it produced much higher than expected values (57 and 62 years). However, a larger w does not capture abrupt changes in the data as well. This is an area that will be explored further in future work. According to the 2019 SCDOT Transportation Asset Management Plan (TAMP), the added service life of mill-and-replace 2 to 4 inches + 200 PSY and mill-and-replace 1 to 2 inches + 400 PSY is 15 and 20 years, respectively (9). It remains to be seen whether current construction practices and materials, assuming traffic volume and weather-related impact are unchanged, have greatly extended pavement service life.

CHAPTER 5: SUMMARY AND CONCLUSIONS

The objective of this thesis is to apply a novel trigonometric GM to estimate and predict pavement deterioration for two types of pavements and two types of rehabilitation methods. The pavement types considered are asphalt and asphalt over concrete. These two pavement types make up the majority of pavement types on interstates in South Carolina. The pavement type considered rehabilitation methods are mill-and-replace 2 to 4 inches + 200 PSY (pounds per square yard) overlay and mill-and-replace 1 to 2 inches + 400 PSY overlay. These two are the most frequently used treatments in the last 10 years in South Carolina by the number of projects and lane-miles. This study proposed a novel trigonometric GM to estimate and predict pavement conditions. The model's performance was compared with the performance of the GM(1,1) and two S-shaped nonlinear models using pavement data from South Carolina, and the estimation results indicated that the proposed GM $(1,1|\cos(\omega t))$ model outperforms the GM(1,1) model and S-shaped nonlinear models in terms of MAPE for the two most common pavement types on interstates in South Carolina and the two most frequently used rehabilitation methods on interstates in South Carolina. By utilizing a rolling horizon approach with the proposed GM, the prediction results indicated that it captures the nonlinear trends in the data well with just 4 prior observations. This study makes two contributions to the field. It is the first study to apply a trigonometric GM to estimate and predict pavement condition, and it is the first to use

GM to model pavement deterioration for asphalt and asphalt over concrete pavement types after receiving either mill-and-replace 2 to 4 inches + 2 inch overlay or mill-and-replace 1 to 2 inches + 4 inch overlay.

While the GM is able to provide accurate fitting of the PSI data, it was found that the prediction may not be as accurate when it has to predict far into the future as was the case when we sought to predict how long it would take before the PSI deteriorates to a value of 2.7 (the threshold which determines when the interstate pavement should be rehabilitated). While it might be possible that a rehabilitation treatment extends a pavement's service life by 50+ years, it is speculated that the inability to update the hyperparameters with new data is a contributing factor. It remains to be seen whether current construction practices and materials, assuming traffic volume and weather-related impact are unchanged, have greatly extended pavement service life. According to the 2019 SCDOT Transportation Asset Management Plan (TAMP), the added service life of mill-and-replace 2 to 4 inches + 200 PSY and mill-and-replace 1 to 2 inches + 400 PSY are 15 and 20 years, respectively (*9*). Future work should seek to develop a guideline for how far into the future a GM can be used to predict PSI. A limitation of this thesis is that the GM's predicted PSIs do not adhere to the fact that a pavement cannot have a higher PSI in a future year unless maintenance work was performed on it. Future work should attempt to overcome this issue by smoothing (filtering) the data and combining unique features of the S-shaped nonlinear model and the GM. That is, the predicted PSI should follow a monotonic decreasing function and the rate of deterioration is driven by the GM.

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