Summer 2021

Large-N_C Constraints for One- and Two-Nucleon Currents in Effective Field Theory

Thomas Richardson

Large-N_c Constraints for One- and Two-Nucleon Currents in Effective Field Theory

by

Thomas Richardson

Bachelor of Science
Furman University 2015

Submitted in Partial Fulfillment of the Requirements

for the Degree of Doctor of Philosophy in

Physics

College of Arts and Sciences

University of South Carolina

2021

Accepted by:

Matthias Schindler, Major Professor

Steffen Strauch, Committee Member

Fred Myhrer, Committee Member

Roxanne Springer, Committee Member

Tracey L. Weldon, Interim Vice Provost and Dean of the Graduate School

Dedication

To the three women from whom I have learned more than any class, textbook, or paper can teach me: my wife, our daughter, and my sister.

Acknowledgments

There are many people to whom I am greatly indebted for their investment in my life throughout graduate school. First, I must thank Matthias Schindler for his constant guidance and patience with me as I have cut my teeth on the research contained in this dissertation. Likewise, Roxanne Springer has taught me much about working and writing to the best of my ability. Matthias and Roxanne have also shown me the great value of laughter, often at their own expense, in a collaborative environment as well as the value kindness. I am also thankful for many useful conversations (and laughter) with Xincheng Lin, Son Ngyuen, Saori Pastore, and Jared Vanasse.

Although this work would not have come to fruition without my nuclear theory colleagues, I also owe a debt of gratitude to several astrophysicists. Steve Rodney fostered my love for teaching interesting content in equally interesting ways. His drive to work well and to help those around him succeed is truly an inspiration. Kyle O'Connor and Justin Roberts-Pierel have also been stalwarts of the Jones basement with me. Much like a quarterback needs a running back with a funny mustache or like Aragorn needs Legolas and Gimli to fight back the dark forces of Mordor in Minas Tirith, I have needed their camaraderie to endure the endless sets of problems from Jackson's *Classical Electrodynamics* and to figure out why my Python codes are not working. Sam Beals is not an astrophysicist, but he may also be the most helpful person I have ever met. None of us would succeed without him.

I am also grateful for the endless support of my parents, my sister, my in-laws, and my niece and nephew. My parents gave me a bottomless source of support from my birth through the present. Similarly, my sister and all of my in-laws have provided

much needed encouragement, even if they still do not know exactly what I do all day, and inspiration in ways of which they are completely unaware. My niece and nephew have also pushed me in ways that I hope they will understand one day. Few people are as funny as they are, and they used their humor many times to lift my spirits. While he is not family, I would be remiss if I did not thank John Kinard, my high school physics teacher. He was the first to introduce me to many interesting problems in physics, and he lit a fire under me when I needed it most as he has done for so many other students.

Finally, I am most indebted to my wife, Kelsey. She has been patient when I found it impossible to step away from my work, and she has encouraged me when I doubted myself. Moreover, she has taught me how to be more selfless and empathetic. Then there is our daughter who will be joining us soon. We have not seen her face-to-face yet, but she has changed me already in ways too deep to verbalize. I am proud to be her father, and I hope she will be just as proud to be my daughter.

Abstract

There is a long-standing goal of understanding nuclear physics in terms of quarks and gluons, the constituent particles of nucleons. However, the underlying theory is strongly coupled at the scales relevant for nuclear physics; therefore, this goal can only be achieved through nonperturbative calculations. Additionally, nuclear targets are employed in a variety of experiments ranging from electroweak processes to Beyond the Standard Model physics. Thus, it is important to have a strong theoretical foundation for nuclear physics in the presence of external fields in order to interpret experimental results. Effective field theory (EFT) for nuclear physics constitutes a systematic and model-independent approach to performing the necessary calculations to achieve this understanding in a manner that is consistent with the symmetries of quantum chromodynamics (QCD), the theory of the strong interactions. Yet, an EFT contains *a priori* undetermined coefficients that must be obtained either from data or from nonperturbative calculations. In the absence of these determinations, it is imperative to constrain the coefficients through other means. Here, the large-N_c limit of QCD, where N_c is the number of colors, is combined with nuclear EFTs to derive such constraints for one and two nucleons in external fields. A general set of constraints for the nuclear currents is derived, and these constraints are applied to the particular cases of electroweak currents, neutrinoless double β decay in the context of the light Majorana exchange mechanism, and the direct detection of dark matter.

Table of Contents

Dedication iii

Acknowledgments iv

Abstract vi

List of Figures ix

Chapter 1 Introduction 1

Chapter 2 Effective Field Theory 5

2.1 Pionless Effective Field Theory 8

2.2 Chiral Perturbation Theory and Chiral Effective Field Theory 14

Chapter 3 Large-N_c Quantum Chromodynamics 26

3.1 Planar Expansion 26

3.2 Mesons 29

3.3 Baryons 31

3.4 Two-Nucleon Interactions 38

Chapter 4 One- and Two-Nucleon Currents 42

4.1 Vector Currents 44

4.2 Axial Vector Currents 62

4.3 Axial Charge 66

4.4 Scalar Currents 72

4.5 Two-Nucleon Spurion Operators 75

Chapter 5 Applications 85

5.1 Electroweak Currents 85

5.2 Lepton Number Violation and Isospin Violation 93

5.3 Dark Matter Direct Detection 99

Chapter 6 Conclusion 111

Bibliography 116

Appendix A Fierz Transformations 138

List of Figures

Figure 2.1 Integrating out pion exchange. Figure from Ref. [1]. 8

Figure 2.2 Sum of two-nucleon bubble diagrams in EFT$_{\not\pi}$. Figure from Ref. [2]. 11

Figure 3.1 Double-line notation of (a) quark and (b) gluon propagators. Figure from Ref. [3]. 27

Figure 3.2 Double-line notation of (a) quark-gluon vertex, (b) three-gluon vertex, and (c) four-gluon vertex. Figure from Ref. [3]. 28

Figure 3.3 Single quark loop that produces a sum over N_c colors. 29

Figure 3.4 $O(N_c)$ quark loop with three planar gluons and the corresponding double-line graph. Figure from Ref. [3]. 29

Figure 3.5 Baryon-meson scattering via quark and gluon exchange. Figure from Ref. [4]. 33

Figure 3.6 Baryon-baryon scattering in large-N_c QCD. Figure from Ref. [3]. . 34

Figure 3.7 Insertion of axial current in a baryon. Figure from Ref. [5]. 34

Figure 3.8 Tree-level pion-nucleon diagrams 34

Figure 3.9 (a) Tree-level one meson exchange, (b) two-meson exchange box diagram, and (c) cross-box diagram. Figure from Ref. [6]. 41

Figure 5.1 The ratio $|{}^{\not\pi}L_2/{}^{\not\pi}L_1|$ as a function of the renormalization scale μ. The solid line corresponds to solutions of the renormalization group equations combined with the values of Eq. (5.20). The dashed line corresponds to Eq. (5.23), while the dark (light) gray band corresponds to 10% (30%) corrections. 90

Figure 5.2 The irreducible two-point function. The solid lines are nucleons, the crossed circle is the interpolating deuteron field, and the black square is the two-nucleon C_2 vertex. 106

Figure 5.3 The irreducible four point function. The double lines are dark matter, the small black dots are vertices from the dark matter-single-nucleon Lagrangian, and the large black dot is a vertex from the dark matter-two-nucleon Lagrangian. 106

Chapter 1

Introduction

Quantum chromodynamics (QCD) is the SU(3) gauge theory of the strong interactions describing the interactions between quarks, which are spin-1/2 fermions, and gluons, which are the spin-1 gauge bosons [7–9]. There are three light quark flavors: up (u), down (d), and strange (s), and there are three heavy quark flavors: charm (c), bottom (b), and top (t). Individual quarks and gluons have not been observed in nature, rather QCD exhibits color confinement such that the only observable particles are color neutral [10]. These particles are broadly referred to as hadrons. Specifically, hadrons are either mesons, i.e., bosonic bound states, or baryons, i.e., fermionic bound states. In the simple quark model, the observed mesons are bound states of a quark and an antiquark, but there are possible exotic states. Conventional baryons, on the other hand, contain three quarks in the simple quark model. In particular, nucleons, which are the focus of this thesis, consist of u and d quarks. The modern understanding of this picture is that nucleons contain u and d valence quarks as well as virtual "sea" quarks and gluons. Because nucleons possess this QCD structure, there is a long-standing goal of describing nucleons directly in terms of the underlying quark and gluon dynamics.

Additionally, it has been shown that the theory is asymptotically free, i.e. the quarks and gluons are weakly coupled, at high energies [10, 11]. Conversely, QCD is strongly coupled, and therefore nonperturbative, at the scales relevant for nuclear physics, which frustrates the attempts to describe nuclei in terms of the underlying gauge theory. One approach to achieving this description is lattice QCD, QCD formu-

lated on a discrete space-time lattice and pioneered in Ref. [12]. Despite the fact that lattice QCD provides a nonperturbative description of baryons through numerical calculations, it is computationally expensive. As resources continue to improve and more techniques are developed, it is expected that uncertainties in these calculations will abate and that more properties of hadronic systems will become accessible. In the meantime, though, it is necessary to explore other options that connect nuclear physics to the underlying theory.

One method for studying baryons in terms of quarks and gluons is working in the large-N_c limit of QCD, where N_c is the number of quark colors [13, 14]. In large-N_c QCD, the gauge group is taken to be SU(N_c) with N_c tending towards infinity, and QCD acquires an expansion in powers of $1/N_c$. There are many interesting qualitative results regarding mesons in this limit [4, 13–22], and the description of baryons in large-N_c QCD was initiated in Ref. [4]. In order to perform these studies, it is often assumed that confinement persists in the large-N_c limit; although, this particular aspect is still an active area of research. For mesons, no modification for the quark content is necessary because a quark-antiquark pair is still color neutral. However, baryons must contain N_c quarks in order to be color neutral; thus, a large-N_c description of baryons is considerable more difficult than the description of mesons. Regardless, as we will see, the large-N_c limit produces interesting constraints for baryons that are in agreement with phenomenology.

Effective field theories (EFTs) written directly in terms of nucleon fields, and meson fields when relevant, are also invaluable tools that can encapsulate the symmetries of QCD [23–29]. The power of EFT is summarized by a "theorem" due to Weinberg that the most general S-matrix consistent with unitarity, analyticity, etc. can be obtained by writing down the most general Lagrangian consistent with the symmetries of a system and calculating matrix elements [30]. In particular, theories such as pionless EFT ($\mathrm{EFT}_{\not\pi}$) for few-nucleon systems [31–36], chiral perturbation theory (χPT)

for a single nucleon [26, 27, 37], and chiral EFT (ChEFT) for two or more nucleons [28, 29] constitute a variety of EFTs that are consistent with the symmetries of QCD. The complicated dynamics of QCD are also subsumed in undetermined low energy coefficients (LECs) in these theories that can be fixed by lattice QCD calculations or by fitting to experiment. Once these coefficients are determined, an EFT becomes predictive. Although these EFTs do not describe QCD directly, they form a powerful approach to relating nuclear physics to the underlying strong interactions.

In the absence of data or lattice calculations, it is necessary to obtain theoretical constraints for the LECs from other means. The large-N_c expansion can be combined with the EFT paradigm in order to provide such constraints on the relative sizes of LECs in the EFTs [38–42]. This combined framework has been employed to study the properties of single nucleon matrix elements as well as several features of purely two- and three-nucleon systems (see, e.g., [6, 38, 39, 43–45]).

The combined large-N_c and EFT expansion, however, may also be used to provide novel constraints for few-nucleon systems in external fields. We will obtain these new constraints for one- and two-nucleon currents coupled to external fields throughout this dissertation. The external fields considered here will be Standard Model fields such as electroweak probes, which are important for obtaining a clear theoretical picture of the Standard Model interactions in nuclei and some beyond-the-Standard-Model (BSM) field such as a dark matter field for which it is pertinent to obtain theoretical constraints for these systems in the absence of data. One outcome of this work is that it sheds light on the QCD constraints for nuclei interacting with external probes. We also desire that it will help to prioritize future lattice QCD calculations while providing an additional hierarchy for terms at each order in the EFT expansion that are employed in many-body calculations. These constraints will not be rigorous predictions, but rather the constraints should be interpreted as general trends among the various LECs.

The structure of this dissertation is as follows: the relevant background of nuclear EFTs is contained in Ch. 2. In Ch. 3, large-N_c QCD is explored, paying close attention to the constraints that have been derived for one- and two-nucleon systems. The remaining chapters of this dissertation contain the central new results. Novel constraints for currents in $\mathrm{EFT}_{\not\pi}$ and isospin breaking currents in ChEFT are obtained in Ch. 4 through the dual large-N_c and EFT expansion. Targeted applications of these constraints are contained in Ch. 5. Specifically, the large-N_c constraints from Ch. 4 are used to analyze electroweak currents, lepton number violation, isospin breaking, and the direct detection of dark matter. A summary of the results can be found in Ch. 6. Finally, details for the Fierz transformations frequently used throughout this work are presented in the Appendix.

Chapter 2

Effective Field Theory

It is often the case in physics that a full theory for a process exists for a wide domain of energies. However, these theories can also become intractable for many systems. Effective theories, which are pervasive in the physics catalog, have become invaluable tools in simplifying the necessary calculations for those initially intractable problems. A textbook example is the multipole expansion of the electrostatic potential in classical electrodynamics. For a point outside of the charge distribution generating the potential, the expansion is in powers of r'/r where r' characterizes the charge distribution and r is the observation point. When $r \gg r'$, the potential is insensitive to the finite size and structure of the charge distribution; therefore, the expansion may be truncated in order to achieve an accurate approximation of the potential at r.

This example illustrates a key feature of effective field theory (EFT): the separation of scales. The following discussion is strongly influenced by Refs. [1, 2, 46–54]. In general, an EFT consists of an expansion in disparate momentum or energy scales. The small scale, call it p, typically describes the characteristic momentum of the constituents of the system of interest. The high scale, denoted by Λ, describes the breakdown scale of the EFT. Thus, observables calculated in an EFT occur in an expansion in powers of p/Λ.

In addition to the scales relevant for a given system, one must identify the appropriate degrees of freedom. For example, the Fermi theory of the weak interaction consists only of four fermions that interact via a local contact term. Now, it is well known that a weak process such as beta decay is mediated by $W^{\pm}$ bosons; however,

if the energy scale of interest is well below the W mass, then these bosons can be "integrated out" by performing the path integral over gauge bosons. That is to say that the only degrees of freedom are the four fermions. This implies that low energy physics is generally insensitive to the details of the high energy physics above the EFT cutoff [55] much like in the case of the multipole expansion in electrostatics when the details of the charge distribution are not important.

With the scales set and the degrees of freedom determined, all of the possible interactions between the degrees of freedom need to be derived. All allowed interactions can be constructed through symmetry principles. First, the operators must satisfy either Lorentz or Galilean invariance. The operators can also be constructed to have any desired transformations under parity (P), time-reversal (T), and charge conjugation (C) as long as they are invariant under the combined CPT. Gauge symmetries can also be included systematically. For example, if electromagnetic interactions are to be included, the theory must be U(1) gauge invariant. This procedure, in general, leads to an infinite tower of operators.

Before actually doing meaningful physics with these operators, their relative importance must be discerned through power counting. In some cases, it is possible to determine the importance of an operator from its mass dimension. The operator's mass dimension can be translated into powers of p/Λ; therefore, a higher dimension operator will be suppressed compared to low dimension operators. However, this picture of power counting is overly simplistic, and this topic will be revisted in the following sections. Regardless, the infinite number of operators in the EFT is distilled to a finite number of operators at each order in the power counting.

Once the basic ingredients have been pinned down, it is possible to perform calculations. The effective Lagrange density is written as

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \sum_n \left(\frac{p}{\Lambda}\right)^n C_{\mathcal{O}_n} \mathcal{O}_n \,, \tag{2.1}$$

where $\mathcal{L}_0$ is the free part of the effective Lagrange density, $\mathcal{O}_n$ is an operator at order n, and $C_{\mathcal{O}}$ is a Wilson or low energy coefficient (LEC) corresponding to $\mathcal{O}_n$. The LECs can not be fixed by symmetry principles; thus, they must be determined either from data or by matching to the full theory. Typically, the LECs are assumed to be of natural size, i.e. on the order of unity, once any dimensional factors have been removed. The sum in Eq. (2.1) is infinite, so in order to perform calculations it is necessary to determine the desired precision. Then the Lagrange density is truncated, and the standard machinery of quantum field theory is applied to calculate objects such as correlation functions and scattering amplitudes. Divergences can appear when evaluating loop diagrams, and these will require renormalization. However, there is a finite number of terms in the Lagrange density at each order in the power counting such that the EFT is renormalizable order by order.

In summary, there are several advantages in this EFT paradigm. First, even if a full theory is known, it is often much easier to work with an EFT through a judicious choice for the degrees of freedom and the expansion parameter. Second, one can choose the level of precision at which to work due to the power expansion in p/Λ. Therefore, EFTs can be improved systematically, and theoretical uncertainties can be estimated in a fairly simple way. Last, a full theory may not be known, or the full theory might be nonperturbative at the scales of interest as is the case with QCD at the scales relevant for nuclear physics, so EFT provides a standardized way to parameterize all of the necessary interactions. In this case, the LECs are treated as free parameters, but their impact can be estimated through power counting. Additionally, the LECs themselves contain all of the information about physics above the breakdown scale Λ.

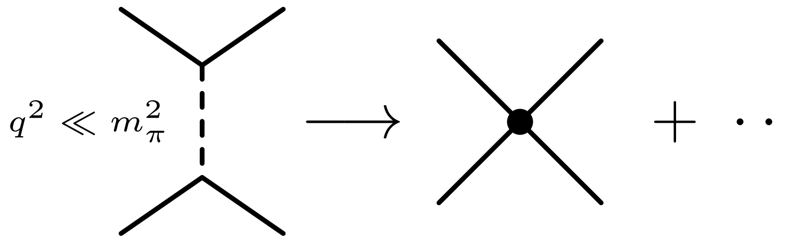

Figure 2.1: Integrating out pion exchange. Figure from Ref. [1].

2.1 Pionless Effective Field Theory

Phenomenological potentials [56–58], especially meson exchange potentials [59, 60], have dominated the nuclear theory landscape for a few decades. When the meson exchange potential consists of one-pion-exchange, then the range is set by the pion mass, m_π. Therefore, for momenta well below the pion mass, i.e. $Q \ll \Lambda_{\not{\pi}} = m_\pi$, the pion can be integrated out and nuclear systems are reduced to a set of short-range contact interactions as shown schematically in Fig. 2.1. The resulting effective theory is pionless EFT ($\text{EFT}_{\not{\pi}}$), which constitutes a systematic framework capable of describing few-nucleon interactions including external fields in a renormalizable (in the modern EFT sense) field theory. For reviews and notes, see Refs. [1, 2, 33, 50, 51, 61–63].

Before moving into $\text{EFT}_{\not{\pi}}$, it is necessary to review a few basic facts from nonrelativistic quantum mechanics for the two-nucleon system. Nucleons are spin-1/2 and isospin-1/2, so they can combine in a state of total spin $S = 0$ or 1 and total isospin $I = 0$ or 1. The particular combination that they occupy will depend on what the orbital angular momentum of the system is due to the fact that the overall wave function must be antisymmetric under the exchange of the two nucleons. At low energies, S wave interactions, which have zero orbital momentum, should dominate. The spatial part of the S wave is symmetric under the interchange of particles; therefore, the possible configurations are a spin singlet-isospin triplet (${}^1S_0$) or a spin triplet-isospin

singlet ($^3S_1$), where the partial waves are written in spectroscopic notation $^{2S+1}L_J$ with total spin S, orbital angular momentum L, and total angular momentum J.

In S-wave scattering, the scattering amplitude in the center-of-mass frame is related to the phase shift $\delta^{(s)}$, where s denotes the partial wave, by

$$\mathcal{A}^{(s)} = \frac{4\pi}{m_N} \frac{1}{p \cot \delta^{(s)} - ip} \,, \tag{2.2}$$

where $p = |\vec{p}|$ is the magnitude of the three-momentum of one of the incoming nucleons, and s denotes either the $^3S_1$ or $^1S_0$ channel. The quantity $p \cot \delta^{(s)}$ has the expansion [64]

$$p \cot \delta^{(s)} = -\frac{1}{a^{(s)}} + \frac{1}{2} r_0^{(s)} p^2 + \cdots \,, \tag{2.3}$$

where $a^{(s)}$ is the scattering length, $r_0^{(s)}$ is the effective range, and the ellipsis denotes terms of higher order in p. This is the Effective Range Expansion (ERE). Keeping only the first term in the scattering amplitude yields

$$\mathcal{A}^{(s)} = -\frac{4\pi}{m_N} \frac{1}{\frac{1}{a^{(s)}} + ip} \,. \tag{2.4}$$

In the complex momentum plane, the amplitude has a pole at $p = \frac{i}{a^{(s)}}$. The scattering lengths in the $^1S_0$ and $^3S_1$ channels are [65]

$$a^{(^1S_0)} = -23.71 \text{ fm} \,, \tag{2.5}$$

$$a^{(^3S_1)} = 5.42 \text{ fm} \,. \tag{2.6}$$

Therefore, the pole in the $^3S_1$ channel is on the positive imaginary axis, which corresponds to a bound state, i.e. the deuteron. On the other hand, the pole in $^1S_0$ channel is on the negative imaginary axis, which corresponds to a nearly bound virtual state. The EFT should reproduce these features, but it will also go beyond the ERE to include effects due to external fields for example.

In EFT$_{\not\pi}$, nucleons are nonrelativistic, and the free part of the effective Lagrangian is

$$\mathcal{L}_0 = N^\dagger \left(i\partial_0 + \frac{1}{2m_N} \nabla^2 \right) N \,, \tag{2.7}$$

where N is a spin-isospin doublet and m_N is the nucleon mass. The classical equations of motion produce the Schrödinger equation for the field N. Interactions can be added to the Lagrangian that are invariant under rotations and isospin transformations. In the two-nucleon sector, there are four possible operators with zero derivatives,

$$\mathcal{L}^0_{NN} = \tilde{C}_1 \left(N^\dagger N\right)^2 + \tilde{C}_2 \left(N^\dagger \sigma^i N\right)^2 + \tilde{C}_3 \left(N^\dagger \tau^a N\right)^2 + \tilde{C}_4 \left(N^\dagger \sigma^i \tau^a N\right)^2 . \tag{2.8}$$

However, these operators are not linearly independent. Fierz transformations can be used to eliminate the terms proportional to $\tilde{C}_3$ and $\tilde{C}_4$ such that the Lagrangian can be written as [28, 29, 31, 32]

$$\mathcal{L}^0_{NN} = -\frac{1}{2} C_S \left(N^\dagger N\right)^2 - \frac{1}{2} C_T \left(N^\dagger \sigma^i N\right)^2 . \tag{2.9}$$

Alternatively, the two-nucleon Lagrangian may be formulated in terms of nucleon bilinears written in partial waves. The Lagrangian in this basis is [63]

$$\mathcal{L}^0_{NN} = -C_0^{(^1S_0)} \left(N^T \bar{P}^a N\right)^\dagger \left(N^T \bar{P}^a N\right) - C_0^{(^3S_1)} \left(N^T P^i N\right)^\dagger \left(N^T P^i N\right) , \tag{2.10}$$

where

$$\bar{P}^a = \frac{1}{\sqrt{8}} \sigma^2 \tau^2 \tau^a , \tag{2.11}$$

$$P^i = \frac{1}{\sqrt{8}} \sigma^2 \sigma^i \tau^2 , \tag{2.12}$$

are the projectors onto the $^1S_0$ and $^3S_1$ states, respectively. These projectors are defined such that they are normalized to

$$\mathrm{Tr}\left(P^i P^j\right) = \frac{1}{2} \delta^{ij} . \tag{2.13}$$

The LECs in the two bases are related according to

$$C_0^{(^3S_1)} = C_S + C_T , \tag{2.14}$$

$$C_0^{(^1S_0)} = C_S - 3C_T . \tag{2.15}$$

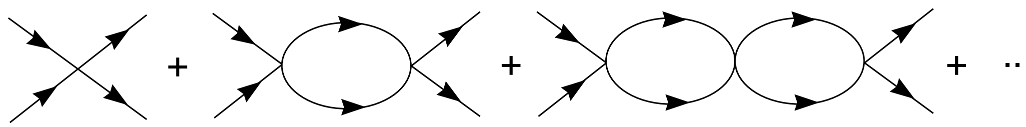

Figure 2.2: Sum of two-nucleon bubble diagrams in $\text{EFT}_{\not\pi}$. Figure from Ref. [2].

In a theory where the only scale apart from momentum is the cutoff $\Lambda_{\not\pi}$, one might expect that power counting is done through dimensional analysis. In this case, the LECs are $O(1/m_N \Lambda_{\not\pi})$. The factor of the nucleon mass appears in the denominator due to the fact that the kinetic energy is included in the free part of the Lagrangian and thus included in the nucleon propagator, so the loop expansion will carry factors of m_N in the numerator. However, there is another low energy scale set by the inverse scattering lengths, that is

$$\frac{1}{a^{(s)}} \ll \Lambda_{\not\pi} \,. \tag{2.16}$$

This feature will result in a fine-tuning that necessitates a careful analysis of the power counting in $\text{EFT}_{\not\pi}$.

Now, it is possible to begin computing Feynman diagrams with the aim of calculating the scattering amplitude. Since there are only nonrelativistic nucleons, the amplitude will be a sum of bubble diagrams shown in Fig. 2.2. Thus, the amplitude has the expansion

$$i\mathcal{A}^{(s)} = -iC_0^{(s)} + (-iC_0^{(s)}) I (-iC_0^{(s)}) + \cdots \,, \tag{2.17}$$

where the loop integral using dimensional regularization is

$$\begin{aligned} I &= \left(\frac{\mu}{2}\right)^n \int \frac{d^n q}{(2\pi)^n} \frac{i}{E - q_0 - \frac{q^2}{2m_N} + i\epsilon} \frac{i}{q_0 - \frac{q^2}{2m_N} + i\epsilon} \\ &= -i m_N \left(\frac{\mu}{2}\right)^n \int \frac{d^{n-1} q}{(2\pi)^{n-1}} \frac{1}{q^2 - m_N E - i\epsilon} \\ &= -\frac{i m_N}{(4\pi)^{(n-1)/2}} \left(\frac{\mu}{2}\right)^n (-m_N E - i\epsilon)^{(n-3)/2} \,\Gamma\left(\frac{3-n}{2}\right) . \end{aligned} \tag{2.18}$$

In dimensional regularization, divergences appear as poles in the gamma function. Using the minimal subtraction scheme amounts to subtracting a pole at $n = 4$; however, this integral does not have a pole at $n = 4$ and is finite.

Now, summing Eq. (2.17) to all orders results in

$$\mathcal{A}^{(s)} = -\frac{4\pi}{m_N} \frac{1}{\frac{4\pi}{m_N C_0^{(s)}} + ip} , \tag{2.19}$$

where the relation $-ip = \sqrt{-mE - i\epsilon}$ has been used. Matching Eq. (2.19) to Eq. (2.4) implies [31]

$$C_0^{(s)} = \frac{4\pi a^{(s)}}{m_N} . \tag{2.20}$$

If $\frac{1}{a^{(s)}} \sim \Lambda$, then this would be acceptable, and the sum in Eq. (2.17) could then be done in perturbation theory without resumming all of the terms, or the resummed amplitude can equivalently be expanded in powers of $a^{(s)}p$. However, the fact that the scattering lengths are large leads to unnaturally large values for the LECs [31, 32]. A method to resolve this issue is a new subtraction scheme, Power Divergence Subtraction (PDS) [32]. This scheme consists of subtracting poles in $n = 4$ dimensions as well as poles that might occur in $n = 3$ dimensions from Eq. (2.18), which correspond to power law divergences in four dimensions. These poles are in fact a generic feature of nonrelativistic theories.

Employing PDS in the two-nucleon system amounts to adding a counterterm to the one loop diagram such that the one loop diagram plus the counterterm is

$$\left[(-iC_{0,R}^{(s)})^2 I + \delta C_0^{(s)}\right]_{n=4} = i\frac{m_N}{4\pi} C_{0,R}^{(s)2} \left(ip + \mu\right) , \tag{2.21}$$

where the subscript R indicates that this is the renormalized LEC. In this case, the bubble sum for the amplitude yields

$$\mathcal{A}^{(s)} = \frac{4\pi}{m_N} \frac{1}{\frac{4\pi}{m_N C_{0,R}^{(s)}} + \mu + ip} , \tag{2.22}$$

and matching to the ERE implies

$$C_{0,R}^{(s)}(\mu) = \frac{4\pi}{m_N} \frac{1}{\frac{1}{a^{(s)}} - \mu} . \tag{2.23}$$

Thus, an explicit dependence on the subtraction point has been introduced in the LEC. This relation could also be derived from the requirement that the amplitude be invariant under the renormalization group and using the value of the scattering lengths as boundary conditions. The new power counting is now obtained with μ and $\frac{1}{a^{(s)}} \sim Q$, where Q is the low momentum scale, while keeping $\frac{1}{a^{(s)}} - \mu \sim Q$. With this power counting, all of the terms in the bubble sum are $O(1/Q)$. Therefore, each term in the sum is equally important, which justifies the resummation of the series.

Two-nucleon contact terms with derivatives can also be added to the Lagrangian as well as relativistic corrections. All of these terms can be treated perturbatively, and the two-derivative terms can be fit to reproduce to the effective range. Additionally, any operators that connect two S-wave states will also exhibit the $1/m_N Q$ enhancement from the dimensional analysis power counting similar to the LO interactions [51].

It is also possible to fit the ${}^3S_1$ LECs in order to reproduce the location of the deuteron pole [63],

$$\gamma_t = \sqrt{m_N B}, \tag{2.24}$$

where $\gamma_t \approx 45.7$ MeV is the binding momentum and $B \approx 2.22$ MeV is the deuteron binding energy. The ERE about this value has the form

$$p \cot \delta^{({}^3S_1)} = -\gamma_t + \frac{1}{2}\rho_d \left(p^2 + \gamma^2\right) + \cdots\,, \tag{2.25}$$

where ρ_d is the effective range. In this case, the LECs themselves are expanded in powers of Q and then matched to the ERE parameters. Another alternative is the Z-parameterization [66]. In this approach, the LECs are fit to reproduce the residue of the deuteron pole at NLO. The LECs have an expansion in Q similar to fitting to the ERE about the deuteron binding momentum; however, this approach demonstrates faster convergence. While the correct residue of the deuteron pole can be approached perturbatively in the ERE schemes, the number of diagrams at higher orders in the

EFT proliferates. It is also possible to bypass the need for two-nucleon contact terms through the use of auxiliary fields for the deuteron and spin-singlet virtual state [67]. In this scheme, the effective range is typically summed to all orders, and the dibaryon fields are convenient for three-body processes such as neutron-deuteron scattering [34, 68–70].

Finally, external fields such as magnetic fields can be included in EFT$_{\not\pi}$ in a simple manner [63, 71]. Gauge fields can be included through the minimal coupling prescription. Any general external field can be specified along with the desired transformation properties of the nucleon operators under parity and time-reversal, and all of the operators consistent with the symmetries can be systematically derived. Single-nucleon currents will obey a power counting based on dimensional analysis while two-nucleon currents will be counted like the two-nucleon contact terms. Specifically, operators that connect two-nucleon ${}^3S_1$ or ${}^1S_0$ states will have the same $1/m_N Q$ enhancement present in the two-nucleon scattering LECs while operators connecting other partial waves will generally be suppressed [51]. These currents constitute the main focus of this work.

2.2 Chiral Perturbation Theory and Chiral Effective Field Theory

2.2.1 Chiral Symmetry in QCD

Another useful EFT for systems containing a single nucleon is chiral perturbation theory (χPT), and the generalization to few-nucleon systems is referred to as chiral EFT (ChEFT) [1, 26, 27, 37, 48, 49, 52, 72–81]. What follows recapitulates the main results from this body of literature and is particularly influenced by Ref. [48]. The low-momentum scales for these EFTs are set by the momenta of the external particles and the pion mass m_π. The breakdown scale for the single nucleon sector of χPT is roughly $\Lambda_\chi = 4\pi F$ where F is the pion decay constant in the chiral limit; although, the breakdown scale could be closer to mass of the ρ meson. Since the Δ-nucleon

mass splitting is around 300 MeV, it may also be the case that χPT without the Δ resonance has an even lower cutoff. In ChEFT, there is evidence that the breakdown scale is closer to 500 MeV or even lower. These two EFTs are low-energy theories of QCD that exploit the approximate chiral symmetry of the light quarks. In the chiral limit, i.e. the limit of massless light quarks, the QCD Lagrangian is

$$\mathcal{L}_{\text{QCD},0} = \sum_{f=u,d} \bar{q}_f i \not{D} q_f - \frac{1}{2} \text{Tr}(G^{\mu\nu} G_{\mu\nu}) \,, \tag{2.26}$$

where q_f is a quark field with isospin index $f = u, d$ where u (d) is the up (down) quark. The gauge covariant derivative is

$$D_\mu q = \left(\partial_\mu + i g A_\mu^i \lambda^i\right) q \,, \tag{2.27}$$

where λ^i is the SU(3) generator in color space, g is the quark-gluon coupling, and A_μ^i is the gluon field. The gluon field strength tensor is $G^{\mu\nu}$ and the trace of the gluon kinetic term is over the color indices. It is possible to rewrite the Lagrangian in terms of left- and right-handed quark fields as

$$\mathcal{L} = \left(\bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R\right) - \frac{1}{2} \text{Tr}(G^{\mu\nu} G_{\mu\nu}) \,, \tag{2.28}$$

where

$$\begin{aligned} q_R &= P_R q \,, \\ q_L &= P_L q \,, \end{aligned} \tag{2.29}$$

and the subscripts denote the chirality of the quark field. The right- and left-handed projection operators are defined as

$$P_{R/L} = \frac{1}{2} \left(\mathbb{1} \pm \gamma^5\right) \,. \tag{2.30}$$

At this stage, it is also sufficient to consider the quark fields as isospin doublets, i.e., $q = (u\ d)^T$. This Lagrangian is classically invariant under $\text{U}(2)_L \times \text{U}(2)_R$, which would suggest that there are 8 Noether currents. In principle, strange quarks could

be included and the symmetry group would then be $\mathrm{U}(3)_L \times \mathrm{U}(3)_R$; however, it is sufficient to consider only the two-flavor case for systems containing only nucleons because they only contain valence up and down quarks, so effects due to strange quarks should be suppressed. Additionally, the size of the strange quark mass relative to the masses of the up and down quark masses suggests that chiral symmetry is stronger in the two-flavor case. However, the $\mathrm{U}(1)_A$ symmetry is anomalously broken; therefore the symmetry group is $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_V$.

Based on chiral symmetry alone, it might be expected that the hadrons come in parity doublets and fill out representations of the chiral symmetry group. However, it appears that the hadrons possess an SU(2) symmetry rather than $\mathrm{SU}(2) \times \mathrm{SU}(2)$ because there are no observed parity partners. Additionally, the smallness of the pion mass compared to the vector mesons insinuates that the pions could be the pseudo-Goldstone bosons of a spontaneously broken symmetry. Observations do indeed support the conclusion that the $\mathrm{SU}(2)_A$ subgroup is spontaneously broken, which implies that there are three pseudoscalar pseudo-Goldstone bosons, one for each generator of the broken symmetry.

If the light quarks really were massless then the pions would be massless Goldstone bosons, but the quark masses break chiral symmetry explicitly leading to the small masses of the pions. Including the mass term in Eq. (2.28) yields

$$\mathcal{L}_{\mathrm{QCD}} = \left(\bar{q}_L i\not{D} q_L + \bar{q}_R i\not{D} q_R\right) - \bar{q}_R M q_L - \bar{q}_L M^\dagger q_R - \frac{1}{2}\,\mathrm{Tr}(G^{\mu\nu}G_{\mu\nu})\,, \tag{2.31}$$

where $M = \mathrm{diag}(m_u, m_d)$ is the quark mass matrix. If the quark mass matrix transformed under the chiral symmetry group as

$$M \mapsto RML^\dagger\,, \tag{2.32}$$

then the Lagrangian would still be invariant under the chiral symmetry group.

External fields can also be incorporated in the Lagrangian in a way that is consistent with chiral symmetry like the mass term. The Lagrangian with external fields

is

$$\mathcal{L} = \mathcal{L}_{\mathrm{QCD},0} + \bar{q}\gamma_\mu \left(v^\mu + \frac{1}{3} v^\mu_{(s)} + \gamma^5 a^\mu \right) q - \bar{q} \left(s - i\gamma^5 p \right) q \,. \tag{2.33}$$

The external isovector fields v^μ and a^μ can be written as

$$\begin{aligned} v^\mu &= \sum_{a=1}^{3} \tau^a v^\mu_a \,, \\ a^\mu &= \sum_{a=1}^{3} \tau^a a^\mu_a \,, \end{aligned} \tag{2.34}$$

and thus couple to the isovector quark currents. The isoscalar vector current couples to $v^\mu_{(s)}$, but the isoscalar axial vector is omitted because of the axial anomaly. Analgously the scalar and pseudoscalar fields can be written as

$$\begin{aligned} s &= \sum_{a=0}^{3} \tau^a s_a \,, \\ p &= \sum_{a=0}^{3} \tau^a p_a \,, \end{aligned} \tag{2.35}$$

where $a = 0$ corresponds to isospin identity. This reduces to the Lagrangian in Eq. (2.31) with v^μ, a^μ, $v^\mu_{(s)}$, and $p = 0$ and $s = \mathcal{M}$. The Lagrangian can also be arranged into the form

$$\begin{aligned} \mathcal{L} = \mathcal{L}_{\mathrm{QCD},0} &+ \bar{q}_L \gamma^\mu \left(l_\mu + \frac{1}{3} v^{(s)}_\mu \right) q_L + \bar{q}_R \gamma^\mu \left(r_\mu + \frac{1}{3} v^{(s)}_\mu \right) q_R \\ &- \bar{q}_R \gamma^\mu \left(s + ip \right) q_L - \bar{q}_L \gamma^\mu \left(s - ip \right) q_R \,, \end{aligned} \tag{2.36}$$

where

$$r_\mu = v_\mu + a_\mu \,, \tag{2.37}$$

$$l_\mu = v_\mu - a_\mu \,. \tag{2.38}$$

Under local chiral transformations, this Lagrangian will be invariant if

$$r_\mu \mapsto R r_\mu R^\dagger + iR\partial_\mu R^\dagger \,, \tag{2.39}$$

$$l_\mu \mapsto L l_\mu L^\dagger + iL\partial_\mu L^\dagger \,, \tag{2.40}$$

$$v^{(s)}_\mu \mapsto v^{(s)}_\mu - \partial_\mu \Theta \,, \tag{2.41}$$

$$s + ip \mapsto R\,(s + ip)\,L^\dagger\,, \tag{2.42}$$

$$s - ip \mapsto L\,(s - ip)\,R^\dagger\,. \tag{2.43}$$

This procedure is suitable for external fields such as electroweak fields or even BSM fields. For example, interactions with an external electromagnetic field A_μ are obtained by setting $r_\mu = l_\mu = -\frac{e}{2}\tau^3 A_\mu$ and $v_\mu^{(s)} = -\frac{e}{2}A_\mu$. However, a modification similar to the procedure followed for the mass matrix is necessary if one wishes to have dynamical photons, leptons, and BSM fields. These modifications will be required in this work and will be reviewed in a following subsection.

2.2.2 Meson Chiral Perturbation Theory

In this section, the chiral meson Lagrangian is constructed following Refs. [23, 26, 27]. All of the features described above can be incorporated in an effective Lagrangian written in terms of hadronic degrees of freedom and as an expansion in p, the momenta of external particles. Since the expansion parameters for the effective Lagrangian are p and m_π, the power counting for all of the terms in the Lagrangian will be determined by the number of derivatives and powers of the pion mass associated with each term. In the meson sector, the pions are implemented through a nonlinear realization U of the chiral symmetry group [24, 25, 72], where

$$U = \exp\left(\frac{i}{F_0}\phi_a\tau^a\right)\,, \tag{2.44}$$

and ϕ_a, $a = \{1, 2, 3\}$, are the Cartesian pion fields. This matrix exponential transforms under chiral transformations as

$$U \mapsto RUL^\dagger\,, \tag{2.45}$$

and is $O(1)$ in the χPT power counting. In the chiral limit, the lowest order chirally invariant pionic Lagrangian is [23, 26, 27]

$$\mathcal{L}_\pi^{(2)} = \frac{F_0^2}{4}\,\mathrm{Tr}\Big(\partial_\mu U \partial^\mu U^\dagger\Big)\,, \tag{2.46}$$

where the subscript π indicates that this is the pion component of the chiral Lagrangian, the superscript indicates that this is the Lagrangian at $O(p^2)$ due to the factors of p that enter through the derivatives, and F is treated as an LEC that can be extracted from semileptonic pion decay. The exponential U can be expanded in the number of pion fields, which results in the kinetic term for a single pion and interactions for even numbers of pions. Explicit symmetry breaking due to the quark masses is included through constructing terms that are invariant under chiral transformations that contain powers of U and $\mathcal{M}$.

At first order in $\mathcal{M}$, the Lagrangian becomes [23, 26, 27]

$$\mathcal{L}_{\pi}^{(2)} = \frac{F_0^2}{4}\,\mathrm{Tr}\Big(\partial_\mu U \partial^\mu U^\dagger\Big) + \frac{1}{2}F_0^2 B_0\,\mathrm{Tr}\Big(MU^\dagger + UM^\dagger\Big)\,. \tag{2.47}$$

In the isospin limit, i.e $m = m_u = m_d$, the pion masses are related to the LEC B_0 through

$$m_\pi^2 = 2B_0 m\,. \tag{2.48}$$

The value for B_0 can be determined from the value of the chiral quark condensate.

The external vector fields can also be included through the definition of the covariant derivative acting on U,

$$D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu\,. \tag{2.49}$$

Given the transformation properties of the external fields in Eqs. (2.39)-(2.43), this object transforms in the same way as U. Also, external fields count as a power of momentum because they are included in the covariant derivative. The traceless field strength tensors, which are $O(p^2)$, for r_μ and l_μ are defined according to

$$\begin{aligned} F_{\mu\nu}^R &= \partial_\mu r_\nu - \partial_\nu r_\mu - i\left[r_\mu, r_\nu\right]\,, \\ F_{\mu\nu}^L &= \partial_\mu l_\nu - \partial_\nu l_\mu - i\left[l_\mu, l_\nu\right]\,. \end{aligned} \tag{2.50}$$

For the scalar and pseudoscalar fields, the mass matrix in the effective Lagrangian is generalized to the form

$$\chi = 2B_0\left(s + ip\right)\,, \tag{2.51}$$

where $s = \mathcal{M}$ again when the external fields vanish. The most general Lagrangian with these ingredients at lowest order is [23, 26, 27]

$$\mathcal{L}_{\pi}^{(2)} = \frac{F_0^2}{4}\,\mathrm{Tr}\Big(D_\mu U D^\mu U^\dagger\Big) + \frac{F_0^2}{4}\,\mathrm{Tr}\Big(\chi U^\dagger + U\chi^\dagger\Big)\,, \tag{2.52}$$

and the subscript indicates that the Lagrangian is $O(p^2)$ at this order. Higher order components of the Lagrangian have been derived, and when doing so redundant terms can be eliminated through integration by parts and the lowest order equations of motion [26, 82–84].

2.2.3 Baryon Chiral Perturbation Theory

Two-flavor χPT can also be extended to include the interactions of nucleons, pions, and external fields [37]. The proton and neutron form an isospin doublet

$$\psi = (p \quad n)^T\ , \tag{2.53}$$

where the proton (neutron) field p (n) is a four-component Dirac spinor. This field transforms under the chiral symmetry group as

$$\psi \mapsto K(L,R,U)\psi\,, \tag{2.54}$$

where the SU(2) valued matrix K is defined as

$$K\left(L,R,U\right) = \sqrt{RUL^\dagger}^{-1} R\sqrt{U}\,. \tag{2.55}$$

When $L = R$, K reduces to a $\mathrm{SU}(2)_V$ matrix, which is just an isospin transformation on the nucleon field. The unitary square root of U is required, i.e. $u^2 = U$. This matrix transforms under the chiral symmetry group as

$$u \mapsto RuK^\dagger = KuL^\dagger\,, \tag{2.56}$$

The construction of a Lagrangian invariant under local $\mathrm{SU}(2)_L{\times}\mathrm{SU}(2)_R{\times}\mathrm{U}(1)_V$ transformations including the external fields in Eq. (2.33) necessitates a covariant

derivative acting on nucleon fields. This derivative is defined as

$$D_\mu \psi = \left(\partial_\mu + \Gamma_\mu - i v_\mu^{(s)}\right) \psi \,, \tag{2.57}$$

where the so-called chiral connection is

$$\Gamma_\mu = \frac{1}{2}\left[u^\dagger \left(\partial_\mu - i r_\mu\right) + u \left(\partial_\mu - i l_\mu\right) u^\dagger\right] \,, \tag{2.58}$$

which transforms as

$$D_\mu \psi \mapsto e^{-i\Theta} K D_\mu \psi \,. \tag{2.59}$$

Additionally, there is the chiral vielbein

$$u_\mu = i\left[u^\dagger \left(\partial_\mu - i r_\mu\right) - u \left(\partial_\mu - i l_\mu\right) u^\dagger\right] \,, \tag{2.60}$$

which transforms as

$$u_\mu \mapsto K u_\mu K^\dagger \,. \tag{2.61}$$

Using all of these ingredients allows for the construction of the most general Lagrangian that is chirally invariant. The Lagrangian will contain bilinears of the form $\bar{\psi}\mathcal{O}\psi$ where $\mathcal{O} \mapsto K\mathcal{O}K^\dagger$. The leading order component of the Lagrangian is thus [37]

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\psi}\left(i \not{D} - m + \frac{g_A}{2}\gamma^\mu \gamma^5 u_\mu\right)\psi \,, \tag{2.62}$$

where m is the nucleon mass in the chiral limit, and g_A is the axial coupling in the chiral limit. In order to construct higher order terms that may also include terms proportional to the quark masses and external scalar and pseudoscalar fields it is convenient to work with

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u \,. \tag{2.63}$$

In this form, the parity of the resulting bilinears can be read off from the sign of $\chi_\pm$. There are also the field strength tensors in Eq. (2.50) in addition to

$$v_{\mu\nu}^{(s)} = \partial_\mu v_\nu^{(s)} - \partial_\nu v_\mu^{(s)} \,, \tag{2.64}$$

$$f_{\mu\nu}^{\pm} = u f_{L\mu\nu}^{\pm} u^{\dagger} \pm u^{\dagger} f_{R\mu\nu}^{\pm} u \,. \tag{2.65}$$

While this formulation of baryon χPT is covariant, it suffers from the fact that derivatives acting on ψ will bring down powers of the nucleon mass which is on the order of Λ_χ [37]. There are renormalization schemes that are Lorentz invariant; however, a popular approach to these issues is heavy baryon χPT (HBχPT) [73, 74]. The idea is to separate the large and small components of the Dirac spinor ψ and integrate out the small components.

First, split the momentum of the nucleon into the form

$$p_\mu = m v_\mu + k_\mu \,, \tag{2.66}$$

where k_μ is a soft residual momentum and

$$\begin{aligned} v^2 &= 1 \\ v_0 &\geq 1 \,. \end{aligned} \tag{2.67}$$

In the nucleon rest frame $v_\mu = (1, \vec{0})$. These conditions also imply

$$v^\mu k_\mu = -\frac{k_\mu k^\mu}{2m} \,, \tag{2.68}$$

and in the nucleon rest frame this becomes

$$k_0 = E_{\mathrm{NR}} = E - m \,. \tag{2.69}$$

Now, define the projection operators

$$P_\pm = \frac{1}{2} \left(\mathbb{1} \pm \not{v} \right) \,. \tag{2.70}$$

The field ψ can now be decomposed in terms of velocity dependent fields as

$$\psi = e^{-imv\cdot x} \left[N_v + H_v \right] \,, \tag{2.71}$$

where

$$\begin{aligned} N_v &= e^{imv\cdot x} P_+ \psi \,, \\ H_v &= e^{imv\cdot x} P_- \psi \,. \end{aligned} \tag{2.72}$$

Inserting this decompostion into the Lagrangian in Eq. (2.62) and integrating out the field H_v results in

$$\mathcal{L}_{\pi N} = \bar{N}_v \left(iv \cdot D + g_A S_v \cdot u\right) N_v + O(1/m)\,, \tag{2.73}$$

which is valid up to $O(1/m)$ corrections, and the spin matrix is defined as

$$S_v^\mu = \frac{i}{2}\gamma^5 \sigma^{\mu\nu} v_\nu\,. \tag{2.74}$$

Additional operators can be constructed that contribute at higher orders, and then invariance under infinitesimal Lorentz transformations [85, 86] or equivalently reparameterization invariance [87] can be used to constrain the LECs.

2.2.4 Chiral Effective Field Theory

Chiral EFT is the effective theory for systems with two or more nucleons incorporating the constraints of chiral symmetry. In the so-called Weinberg Power Counting (WPC) scheme, the nuclear potential is defined as the sum of irreducible diagrams that do not have intermediate nucleon propagators and include two-nucleon contact terms along with the exchange of one or more pions [28, 29]. Again, the two-nucleon system requires a resummation of the diagrams, so the two-nucleon scattering amplitude is then obtained through a Lippman-Schwinger equation. Specifically, the leading order terms that are resummed in WPC are the diagrams that contain vertices proportional to C_S and C_T as well as one-pion-exchange. While this prescription is phenomenologically successful, it requires the promotion of subleading operators in WPC to leading order in order to achieve renormalization [31]. An alternative power counting was suggested in Refs. [32, 88], generally referred to as KSW power counting, in which interactions with pions are relegated to subleading order and treated perturbatively. However, the KSW prescription with perturbative pions was later shown not to converge in several partial waves [89]. Subsequently, reformulations of

KSW approach have been presented but without widespread adoption (see Ref. [1] and references therein).

2.2.5 Spurion Fields

While the chiral Lagrangian with external fields is sufficient to generate all Feynman diagrams including virtual particles, there will in general be divergences that must be renormalized by local counterterms. The use of spurion operators similar to the inclusion of the quark mass matrix in the chiral Lagrangian is a convenient and popular method to construct the counterterms [90–98]. These operators represent the effects of virtual particles with momenta above the breakdown scale of the EFT.

Consider, for example, the QCD Lagrangian for two quark flavors with minimal coupling to an electromagnetic field. The Lagrangian in terms of left- and right-handed quark fields is

$$\mathcal{L} = i\bar{q}_L \not{\partial} q_L + i\bar{q}_R \not{\partial} q_R - \bar{q}_L M^\dagger q_R - \bar{q}_R M q_L + ieA_\mu \left[\bar{q}_L \gamma^\mu Q_q q_L + \bar{q}_R \gamma^\mu Q_q q_R\right], \quad (2.75)$$

where $Q_q = \mathrm{diag}(\frac{2}{3}, -\frac{1}{3})$ is the quark charge matrix. The terms proportional to the quark charge break chiral symmetry explicitly. However, the pattern of symmetry breaking can be mapped onto the effective Lagrangian by splitting the charge matrix into Q_L and Q_R such that the electromagnetic component of the Lagrangian can be written as

$$\mathcal{L}_{EM} = ieA_\mu \left[\bar{q}_L \gamma^\mu Q_L q_L + \bar{q}_R \gamma^\mu Q_R q_R\right], \quad (2.76)$$

and requiring the charge matrices to transform under the chiral symmetry group as

$$Q_R \mapsto R Q_R R^\dagger, \quad (2.77)$$

$$Q_L \mapsto L Q_L L^\dagger. \quad (2.78)$$

Now, it is possible to construct all terms in the effective Lagrangian using the spurion fields Q_L and Q_R such that the operators are invariant under the chiral

symmetry group. This procedure works for any generic spurion field, and at lowest order there will be two insertions of the spurions. After constructing all possible operators, the specific form of the spurions can be chosen. Continuing the example of electromagnetic interactions, it is conventional to continue using the quark charge matrix in the meson sector. However, it is also common to use the nucleon charge matrix $Q_N = \frac{1}{2}(\mathbb{1} + \tau^3)$, and the difference compared to using the quark charge matrix amounts to a shift by an unobservable constant [98].

When constructing the spurion operators in the nucleon sector, it is convenient to use the combinations

$$Q_\pm = \frac{1}{2}\left[u^\dagger Q_R u \pm u Q_L u^\dagger\right] , \tag{2.79}$$

which transform under the chiral symmetry group as

$$Q_\pm \to K Q_\pm K^\dagger \,. \tag{2.80}$$

Lastly, it is also useful to separate the spurions into isoscalar and isovector components,

$$Q_\pm = \frac{1}{2}\,\mathrm{Tr}(Q_\pm)\mathbb{1} + \tilde{Q}_\pm \,. \tag{2.81}$$

where

$$\tilde{Q}_\pm = \frac{1}{2}\,\mathrm{Tr}(Q_\pm \tau^a)\tau^a \,. \tag{2.82}$$

Thus, all of the spurion operators can be derived in terms of $\mathrm{Tr}(Q_\pm)$ and $\tilde{Q}_\pm$.

Chapter 3

Large-N_c Quantum Chromodynamics

3.1 Planar Expansion

As mentioned in Ch. 1, QCD is the SU(3) gauge theory of the strong interaction. At the scales relevant for hadronic physics, QCD is nonperturbative. However, by considering the limit $N_c \to \infty$, where N_c is the number of colors in QCD, we can deduce some of the important features of hadrons in an expansion in powers of $1/N_c$ [4, 13, 14]. The discussion in this chapter highly reflects Refs. [3, 4, 99, 100].

The QCD Lagrangian is

$$\mathcal{L} = \sum_f \bar{q}_f \left(i\gamma^\mu D_\mu - m_f\right) q_f - \frac{1}{2}\,\mathrm{Tr}(G^{\mu\nu} G_{\mu\nu}), \tag{3.1}$$

where the sum over f is over quark flavors and the trace is a sum over color indices. The gauge covariant derivative is

$$D_\mu q = \left(\partial_\mu + igA^i_\mu \lambda^i\right) q, \tag{3.2}$$

where A^a_μ is the gluon field, λ^i is an SU(N_c) generator, and g is the quark-gluon coupling. The generators are normalized such that

$$\mathrm{Tr}\left(\lambda^i \lambda^j\right) = \frac{1}{2}\delta^{ij}\ . \tag{3.3}$$

The gluon field strength tensor is

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig\left[A_\mu, A_\nu\right], \tag{3.4}$$

where $A_\mu = A^a_\mu \lambda^a$. It is also possible to write the field strength tensor as an object with a single adjoint index rather than two indices in the fundamental representation

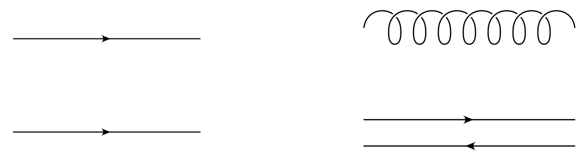

Figure 3.1: Double-line notation of (a) quark and (b) gluon propagators. Figure from Ref. [3].

of SU(N_c) through the defintion $G_{\mu\nu} = \lambda_a G^a_{\mu\nu}$. This second definition has the explicit form

$$G^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu - g f_{ijk} A^j_\mu A^k_\nu \, , \tag{3.5}$$

where f_{ijk} are the SU(N_c) structure constants.

At the one-loop level, the β-function is

$$\mu \frac{dg}{d\mu} = -\frac{g^3}{(4\pi)^2} \left[\frac{11}{3} N_c - \frac{2}{3} N_f \right] \, , \tag{3.6}$$

which is clearly not well defined as $N_c \to \infty$. However, if $g^2 N_c = \hat{g}$ is held fixed in the limit $N_c \to \infty$, then the beta function for $\hat{g}$,

$$\mu \frac{d\hat{g}}{d\mu} = -\frac{\hat{g}^3}{(4\pi)^2} \left[\frac{11}{3} - \frac{2N_f}{3N_c} \right] \, , \tag{3.7}$$

is well defined [13]. Therefore, in the forthcoming discussion of Feynman diagrams for large-N_c QCD, a factor of $1/\sqrt{N_c}$ will be associated with every quark-gluon vertex and three-gluon vertex, and a factor of $1/N_c$ will be associated with every four-gluon vertex. This scaling of the coupling, along with factors of N_c that appear due to color traces, will furnish a perturbative expansion in powers of $1/N_c$.

In order to study the consequences of the large-N_c limit for hadrons, it is beneficial to understand how to determine the N_c scaling of any given Feynman diagram (see Refs. [3, 4, 13, 14] for greater detail). It is convenient to adopt the 't Hooft double line notation to perform this analysis; this amounts to analyzing the flow of color in the diagrams. First, quark propagators are given in the standard form

$$\langle 0 | \, T \left\{ q^a(x) \bar{q}_b(y) \right\} | 0 \rangle = \delta^a_b S_F(x-y) \, , \tag{3.8}$$

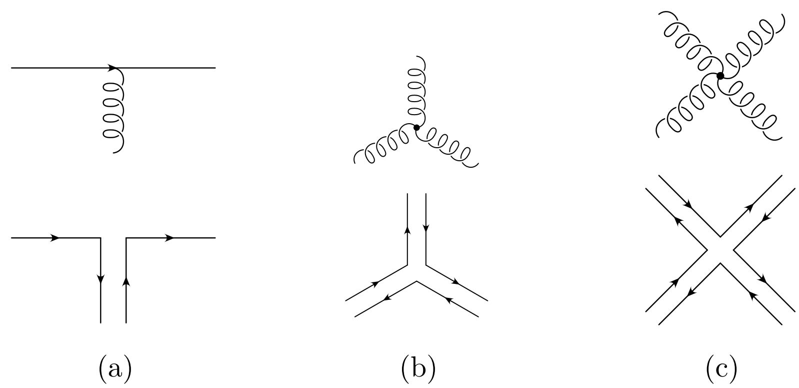

Figure 3.2: Double-line notation of (a) quark-gluon vertex, (b) three-gluon vertex, and (c) four-gluon vertex. Figure from Ref. [3].

where $S_F(x-y)$ is the fermion propagator. The typical form of the gluon propagator is

$$\langle 0| T\left\{A^i_\mu(x)A^j_\nu(y)\right\}|0\rangle = \delta^{ij}D_{\mu\nu}(x-y)\ , \tag{3.9}$$

where $D_{\mu\nu}$ is the propagator for a single gauge field. Instead of expressing the gluon propagator in terms of the adjoint indices i and j, it may be alternatively expressed in terms of the color indices as

$$\langle 0| T\left\{A^i_\mu(x)\lambda^{ai}_b A^j_\nu(y)\lambda^{cj}_d\right\}|0\rangle = \frac{1}{2}D_{\mu\nu}(x-y)\left(\delta^a_d\delta^c_b - \frac{1}{N_c}\delta^a_b\delta^c_d\right) \tag{3.10}$$

after use of the identity

$$\lambda^{ai}_b\lambda^{ci}_d = \frac{1}{2}\left(\delta_{ad}\delta_{cb} - \frac{1}{N_c}\delta_{ab}\delta_{cd}\right)\ . \tag{3.11}$$

The second term in this identity appears due to the fact that the SU(N_c) generators are traceless. However, this term vanishes as $N_c \to \infty$, so it may be neglected such that the gluon matrices are in the adjoint representation of U(N_c). This approximation is valid at leading order in the N_c expansion, but corrections are $1/N_c^2$ suppressed and may be ignored. Gluons may now be represented as a double line with a single color index on each line, one in the fundamental representation and one in the antifundamental represenation as shown in Fig. 3.1. The QCD vertices can now be translated into this double line notation and are shown in Fig. 3.2.

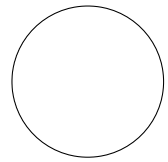

Figure 3.3: Single quark loop that produces a sum over N_c colors.

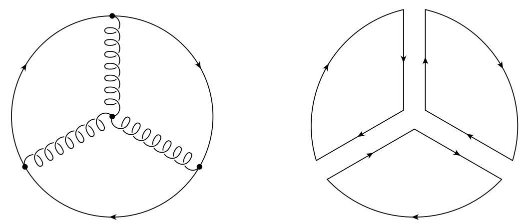

Figure 3.4: $O(N_c)$ quark loop with three planar gluons and the corresponding double-line graph. Figure from Ref. [3].

Now, this formalism can be used to analyze vacuum QCD diagrams. In particular, the leading diagrams contain only planar gluons and are $O(N_c^2)$. At $O(N_c)$, there is a diagram that consists only of a single quark loop. Any number of planar gluons can be added to the quark loop such that the quark loop forms the edge of the diagram without changing the large-N_c scaling. For example, the single quark loop in Fig. 3.3 contains a single color trace and is thus $O(N_c)$. In Fig. 3.4, the double line notation makes explicit the fact that there are three color traces, and accounting for the factors of N_c from vertices leads to an overall scaling of N_c. Additional quark loops may be included, but they require quark-gluon vertices; therefore, each additional quark loop costs a factor of $1/N_c$. Diagrams that contain nonplanar gluons are also suppressed by $1/N_c^2$.

3.2 Mesons

There are many interesting qualitative features of mesons that can be derived in the large-N_c limit [4, 13–22]. Let J be a current that creates a meson. The two-point

correlation function for this current can be written as [4]

$$\langle J(k)J(-k)\rangle = \sum_n \frac{|a_n|^2}{k^2 - m_n^2}, \tag{3.12}$$

where $a_n = \langle 0|\, J\,|n\rangle$ and the states $|n\rangle$ are one-meson states. Since the two-point function should be a sum of planar diagrams in the large-N_c limit, Eq. (3.12) is $O(N_c)$. For this statement to hold true for all values of k^2, it is necessary that the meson masses m_n be independent of N_c. In addition, it is known that the large-momentum behavior of the two-point function on the left-hand side of Eq. (3.12) depends logarithmically on k^2. However, if there is a finite number of meson states, then the sume on the right-hand side will behave as $1/k^2$. Therefore, there must be an infinite number of meson states so that both sides agree. As a result, the amplitude to create a meson from the vacuum, or equivalently the meson decay constant, a_n is $O(\sqrt{N_c})$. Additionally, the single-particle poles are all on the real axis; therefore, the mesons are stable.

Similar diagrammatic analyses of the three- and four-point functions reveal that the three-meson vertex is $O(1/\sqrt{N_c})$ and the four-meson vertex is $O(1/N_c)$, and this analysis can be extended to an arbitary number of mesons. This indicates that mesons are weakly interacting for large but finite N_c while they are free for $N_c = \infty$. As a result, meson-meson scattering at leading order in the large-N_c expansion consists of tree-level diagrams with meson exchanges which are $O(1/N_c)$.

These features are also incorporated in the meson sector of χPT [40–42] since chiral symmetry is still broken in the large-N_c limit [101]. The LO pion Lagrangian from Eq. (2.52) is

$$\mathcal{L} = \frac{F^2}{4}\,\mathrm{Tr}\Big(D_\mu U D^\mu U^\dagger\Big) + \frac{F^2}{4} B_0\,\mathrm{Tr}\Big(M^\dagger U + M U^\dagger\Big)\,.$$

Now, a single flavor trace in χPT corresponds to a single quark loop in large-N_c QCD and therefore scales as N_c. Mapping the scaling of the traces onto the LECs implies that $F \sim O(\sqrt{N_c})$ while B_0, which is determined by the pion mass, is $O(1)$.

Additionally, by expanding Eq. (2.44) in the number of pion fields, it can be seen that each additional pion leads to a $1/\sqrt{N_c}$ suppression due to the factors of $1/F$ that will appear. These features are exactly what was just determined through the large-N_c analysis.

It has also been shown that the $\mathrm{U}(1)_A$ anomaly does not persist as $N_c \to \infty$. Therefore, when strange quarks are included the chiral symmetry group breaks down from $\mathrm{U}(3)_R \times \mathrm{U}(3)_L$ to $\mathrm{U}(3)_V$. As a consequence the η' is the ninth Goldstone boson, and should be included in $N_f = 3$ χPT [41, 42]. For more information on this version of the incorporation of the η' in large-N_c χPT, see Refs. [102–107].

3.3 Baryons

3.3.1 Baryon Masses

Despite the success of the diagrammatic approach in the meson sector of large-N_c QCD, the case is not so simple for baryons. The analysis of this section follows Refs. [3, 4, 100]. Since baryons are fermions, they must obey the Pauli exclusion principle; therefore, a ground state baryon must consist of N_c quarks completely antisymmetric in their color indices and totally symmetric in the combination of their spin and flavor indices.

Now, if one tries to perform an analysis of the diagrams in perturbation theory for an N_c-quark propagating state, it will quickly be seen that the diagrams grow with increasing N_c. The first correction to the free propagation of quarks consists of single gluon exchange, and we determine the large-N_c scaling of the diagram through combinatorics. There are $\frac{1}{2}N_c\,(N_c - 1)$ ways to choose a pair of quarks in the baryon. When we include the factors of $1/\sqrt{N_c}$ from the quark-gluon vertices, the contribution is $O(N_c)$. Going a step further, a connected n-quark interaction will yield an overall contribution that is $O(N_c)$. This process can be conducted for a diagram with m disconnected pieces, and these diagrams will contribute at $O(N_c^m)$.

At first, this situation seems hopeless. However, it was pointed out in Ref. [4] that the amplitude for a baryon to propagate from time $t' = 0$ to $t' = t$ is $e^{-im_B t}$, where m_B is the baryon mass. Expanding this exponential yields

$$e^{-im_B t} = 1 - im_B t - \frac{1}{2} m_B^2 t^2 + \cdots \,, \tag{3.13}$$

Thus, the growth of the interactions can be understood as the baryon mass being $O(N_c)$ [3, 4, 100].

3.3.2 Scattering Processes

In baryon-meson scattering, the meson carries a single quark that can be exchanged with a quark from the baryon. Recall that baryons are composed of N_c quarks that are antisymmetric in the color indices. Therefore if the two quarks are of the same color then there is only one possible exchange that can occur without gluon exchange and this contribution is thus $O(N_c^0)$. If, however, the quark from the meson is exchanged with a quark from the baryon of a different color, then there must be a gluon exchange so that the baryon will be a color singlet. In this case, there are $N_c - 1$ choices for the quark exchange and the quark-gluon vertices contribute a factor of $1/N_c$ resulting in an overall contribution on the order of $O(N_c^0)$. Since the baryon mass is $O(N_c)$, then this interaction does little to affect its motion while the interaction is on the same order as the meson mass. Thus, the meson scatters off of the baryon [4].

For baryon-baryon scattering (see Fig. 3.6), there are N_c ways to choose a quark from the first baryon. The chosen quark can be exchanged with a quark of the same color from the second baryon without any gluon exchange. There is only one such quark, so this interaction is $O(N_c)$. On the other hand, the quark from the first baryon can be exchanged with one of $N_c - 1$ quarks from the second baryon, but this must be accompanied by gluon exchange. Again, the combinatorics along with the scaling of the quark-gluon coupling imply that the interaction is $O(N_c)$. By analyzing two-

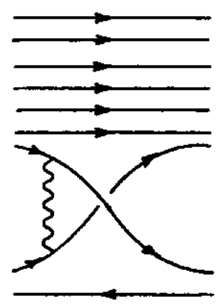

Figure 3.5: Baryon-meson scattering via quark and gluon exchange. Figure from Ref. [4].

nucleon scattering through additional quark-gluon diagrams, it is possible to show that a connected diagram involving r quarks will be on the order of $N_c^{1-r}N_c^r = N_c$.

As discussed in Ref. [4], it seems problematic that this interaction is $O(N_c)$. Specifically, how can there be a smooth limit for the scattering amplitude as $N_c \to \infty$? To resolve this issue, Ref. [4] takes the baryon velocities to be fixed with respect to N_c. The kinetic energy $\frac{1}{2}m_B v^2$ is then $O(N_c)$ because of the factor of the baryon mass. Therefore, the kinetic energy is of the same order as the interaction. If instead the momentum transfer is held fixed with respect to N_c, then the kinetic energy will be $O(1/N_c)$. In this case, nuclear matter forms a crystal and does not scatter [39]. However, this kinematic regime will be the case of interest in this work since it justifies the use of a nonrelativistic EFT like EFT$_{\not{\pi}}$. Additionally, Ref. [39] advocates that the large-N_c expansion for nuclear forces is useful because it can shed light on certain qualitative features of these forces. The following section examines the role of spin and isospin towards that end.

3.3.3 Consistency Conditions

First, consider the baryon-meson interaction. The size of the coupling can be determined by inserting a current J that creates a meson on one of the baryon's quark lines as shown in Fig. 3.7. The current can be inserted on any of the N_c quarks in the baryon, and accounting for the amplitude to create a meson from the vacuum

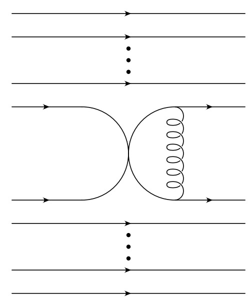

Figure 3.6: Baryon-baryon scattering in large-N_c QCD. Figure from Ref. [3].

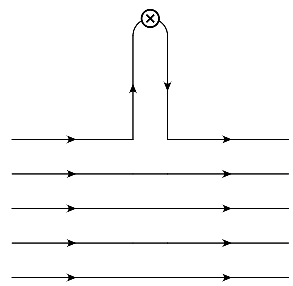

Figure 3.7: Insertion of axial current in a baryon. Figure from Ref. [5].

through the LSZ reduction formula, the overall amplitude and thus the meson-baryon coupling is $O(\sqrt{N_c})$.

Recall that the meson-baryon scattering amplitude is $O(1)$. At tree-level in terms of nucleon and pion fields, there are three possible contributions to the scattering amplitude, two of which are shown in Fig. 3.8. The third contribution is a seagull diagram which is usually argued to be suppressed in the large-N_c limit; however, this detail is not necessary for the following. Since there are two $O(\sqrt{N_c})$ baryon-pion couplings, one might expect that the overall result is $O(N_c)$. This is clearly at odds with the fact that the scattering amplitude should be fixed with respect to N_c.

Figure 3.8: Tree-level pion-nucleon diagrams

The solution to this problem is the existence of so-called consistency conditions [108–111] (see Ref. [5] for a review). Essentially, the intermediate baryon should include a sum over all baryon states which are degenerate with the incoming and outgoing nucleons. It will now be shown that this tower of intermediate baryon states leads to the necessary cancellations in order to produce an amplitude independent of N_c.

The baryon-pion interaction can be parameterized as

$$\frac{\partial_\mu \phi^a}{F_\pi} \langle B'| \bar{q}\gamma^\mu\gamma^5\tau^a |B\rangle \ . \tag{3.14}$$

In the limit of interest the baryon is static, so the matrix elements of the time component of the axial current vanish. The matrix elements of the spatial components of the axial current may be written as

$$\langle B'| \bar{q}\gamma^\mu\gamma^5\tau^a q\,|B\rangle = gN_c \langle B'| X^{ia} |B\rangle \ . \tag{3.15}$$

The factor of gN_c has been removed so that the baryon-pion coupling is $gN_c/F_\pi \sim O(\sqrt{N_c})$ with g fixed and the matrix elements of X^{ia} are at most fixed with respect to N_c. Now, the scattering amplitude takes the form

$$i\frac{g^2N_c^2}{E_\pi F_\pi^2}\left[X^{ia}, X^{jb}\right] \ . \tag{3.16}$$

Therefore, the amplitude will be $O(1)$ if

$$\left[X^{ia}, X^{jb}\right] \lesssim O(1/N_c)\,. \tag{3.17}$$

The cancellations that occur to produce this scaling are the aforementioned consistency conditions.

In general, the operator X^{ia} has a $1/N_c$ expansion of the form

$$X^{ia} = \sum_{n=0} N_c^{-n} X_n^{ia} \ , \tag{3.18}$$

where the matrix elements of the X_n^{ia} are $O(1)$. Thus, for Eq. (3.17) to be true it is necessary that

$$\left[X_0^{ia}, X_0^{jb}\right] = 0\,, \tag{3.19}$$

which is the first consistency condition.

Now, if the process is nucleon-pion scattering and there is only an intermediate nucleon which has $I = J = \frac{1}{2}$, then the operator X^{ia} is just proportional to $\sigma^i \tau^a$ and the matrix elements do not commute. If, however, the Δ resonance with $I = J = \frac{3}{2}$ is included as an intermediate state, then the necessary cancellation can occur. As a consequence, the consistency condition leads to the constraint [110]

$$\frac{g_{\pi N \Delta}}{g_{\pi NN}} = \frac{3}{2} + O(1/N_c^2)\,. \tag{3.20}$$

This calculation can be performed for Δ-pion scattering in order to constrain the $I = J = \frac{5}{2}$ coupling, and this can be carried out recursively. This demonstrates the necessity of an infinite tower of baryon states with $I = J = \frac{1}{2}, \frac{3}{2}, \cdots$ to ensure the required cancellations for the consistency condition in Eq. (3.19). This paradigm may also be used to systematically calculate $1/N_c$ corrections to observables.

Since the operator X^{ia} is a spin-1 isospin-1 operator, there are commutation relations in addition to Eq. (3.19) which are satisfied:

$$\begin{aligned} \left[J^i, X_0^{jb}\right] &= i\epsilon^{ijk} X_0^{kb}, \\ \left[I^a, X_0^{jb}\right] &= i\epsilon^{abc} X_0^{jc}. \end{aligned} \tag{3.21}$$

In addition, there are the familiar commutation relations

$$\begin{aligned} \left[J^i, J^j\right] &= i\epsilon^{ijk} J^k, \\ \left[I^a, I^b\right] &= i\epsilon^{abc} I^c, \\ \left[J^i, I^a\right] &= 0. \end{aligned} \tag{3.22}$$

Now, consider the Lie algebra for SU(4). There are the $\mathrm{SU}(2)_J \times \mathrm{SU}(2)_I$ generators as well as as a combined spin-flavor $G^{ia} = J^i \otimes I^a$. The commutation relations involving

G^{ia} are

$$\left[J^i, G^{jb}\right] = i\epsilon^{ijk}G^{kb},$$
$$\left[I^a, G^{jb}\right] = i\epsilon^{abc}G^{jc}, \tag{3.23}$$
$$\left[G^{ia}, G^{jb}\right] = \frac{i}{4}\left(\epsilon^{ijk}J^k\delta^{ab} + \delta^{ij}\epsilon^{abc}I^c\right).$$

The matrix elements of the G^{ia} are generally $O(N_c)$, so G^{ia}/N_c will be fixed in the large-N_c limit [111, 112]. Therefore, the algebra in terms of the operator X^{ia} can be obtained from the $SU(4)$ algebra through

$$X^{ia} = \lim_{N_c\to\infty}\frac{G^{ia}}{N_c}. \tag{3.24}$$

This limiting process is what is known as a contraction. Thus, the operators J^i, I^a, and X_0^{ia} are said to generate a contracted $SU(4)$ symmetry under which the large-N_c baryons transform [108–110]. One can also see that the result generalizes to $SU(6)$ for the case of 3 quark flavors.

3.3.4 Spin-Flavor Symmetry and the $1/N_c$ Expansion

At this point, it is natural to wonder if the usual SU(4) generators may be used to classify the N_c dependence of baryon matrix elements. After all, the contracted SU(4) symmetry is only exact as $N_c \to \infty$. Indeed, this is possible since the only difference in using X^{ia} compared to G^{ia}/N_c amounts to a reorganization of subleading corrections.

The general strategy is to classify the baryon matrix elements of various operators according to the spin-flavor representations. In order to do so, the operator basis is taken to be

$$\hat{J}^i = q^\dagger J^i q$$
$$\hat{I}^a = q^\dagger I^a q \tag{3.25}$$
$$\hat{G}^{ia} = q^\dagger G^{ia} q,$$

which is also the basis of the nonrelativistic quark model. There is also the operator $q^\dagger q$; however, this is the quark number operator and can be reduced to $N_c \mathbb{1}$. A generic m-body QCD operator can be expanded in this basis as [112]

$$\mathcal{O}^{(m)}_{QCD} = N_c^m \sum_{n,s,t} c_n \left(\frac{\hat{J}^i}{N_c}\right)^s \left(\frac{\hat{I}^a}{N_c}\right)^t \left(\frac{\hat{G}^{ia}}{N_c}\right)^{n-s-t} , \tag{3.26}$$

where the coefficient of each term in the expansion c_n is independent of N_c and can in principle be determined from the QCD dynamics. Many of the operators that occur in the expansion are redundant and can be removed through various reduction rules. For two quark flavors, these rules amount to the elimination of any operators in which spin or isospin indices are contracted with a Kronecker delta or with the Levi-Civita symbol except for the operator J^2 [112].

The axial vector current, for example, has the expansion [112]

$$\bar{q}\gamma^i\gamma^5\tau^a q = N_c \left[\frac{G^{ia}}{N_c} + \frac{J^i I^a}{N_c^2} + \cdots\right] . \tag{3.27}$$

In this form, it is clear that the LO contribution is from $G^{ia} \sim O(N_c)$, there are corrections on the order of $1/N_c^2$ for baryons with fixed spin and isospin, i.e. the physical baryon states. This strategy has also been implemented to study other static properties such as the baryon masses and magnetic moments [5, 112–116]. Additionally, when working with 3 quark flavors, the effects of flavor symmetry breaking can be incorporated systematically [112, 116–121].

3.4 Two-Nucleon Interactions

As previously mentioned, the two-nucleon interaction is expected to be $O(N_c)$, which can be described by a relativistic Hartree Hamiltonian [4]. While the exact Hamiltonian cannot be constructed, the general structure and its consequences may be deduced in terms of the spin-flavor expansion. In the following, the operator $\hat{J}^i$ will

be replaced with $\hat{S}^i$. The Hamiltonian has the expansion [111, 112, 115, 122]

$$H = N_c \sum_{n,s,t} v_{stn} \left(\frac{\hat{S}^i}{N_c}\right)^s \left(\frac{\hat{I}^a}{N_c}\right)^t \left(\frac{\hat{G}^{ia}}{N_c}\right)^{n-s-t} , \tag{3.28}$$

where the coefficients v_{stn} are at most $O(1)$ and are momentum dependent.

The two-nucleon matrix elements of operators appearing in the Hamiltonian factorize in the large-N_c limit [38],

$$\langle N_\alpha N_\beta | \mathcal{O}_1 \mathcal{O}_2 | N_\gamma N_\delta \rangle \to \langle N_\alpha | \mathcal{O}_1 | N_\gamma \rangle \langle N_\beta | \mathcal{O}_2 | N_\delta \rangle + \text{ crossed} , \tag{3.29}$$

where α, β, γ and δ denote the spin, isospin, and momenta of the fields. For physical baryons, those with $S = I \sim O(1)$, the scaling of the single-nucleon matrix elements of the operators appearing in the expansion are determined from

$$\langle N' | \frac{\mathcal{O}^{(n)}_{I,S}}{N_c^n} | N \rangle \lesssim \frac{1}{N_c^{|I-S|}} , \tag{3.30}$$

where $\mathcal{O}^{(n)}$ is an n-quark operator with spin and isospin S and I respectively [38, 39]. For the SU(4) generators, this produces the expected scalings

$$\begin{aligned} \langle N' | \frac{\mathbb{1}}{N_c} | N \rangle \sim \langle N' | \frac{G^{ia}}{N_c} | N \rangle \lesssim 1 , \\ \langle N' | \frac{S^i}{N_c} | N \rangle \sim \langle N' | \frac{I^a}{N_c} | N \rangle \lesssim \frac{1}{N_c} . \end{aligned} \tag{3.31}$$

These large-N_c spin-flavor properties make it possible to constrain the two-nucleon potential

$$V(\mathbf{p}_+, \mathbf{p}_-) = \langle N_\alpha N_\beta | H | N_\gamma N_\delta \rangle , \tag{3.32}$$

where the Greek subscripts denote the spin, isospin, and momentum dependence of the nucleons, and the momenta are given in terms of the incoming (outgoing) relative momentum $\mathbf{p}$ ($\mathbf{p}'$)

$$\mathbf{p}_\pm = \frac{1}{2} (\mathbf{p}' \pm \mathbf{p}) . \tag{3.33}$$

For elastic scattering, the central potential has the form [39]

$$V_{\text{central}} = V_0^0 + V_\sigma^0 \sigma_1 \cdot \sigma_2 + \left(V_0^1 + V_\sigma^1 \sigma_1 \cdot \sigma_2\right) \tau_1 \cdot \tau_2. \tag{3.34}$$

As a result, the scalings of each component of the central potential are

$$\begin{aligned} V_0^0 &\sim V_\sigma^1 \sim N_c \ , \\ V_\sigma^0 &\sim V_0^1 \sim \frac{1}{N_c} \ . \end{aligned} \tag{3.35}$$

These scalings provide an explanation of the approximate Wigner $\mathrm{SU}(4)_W$ symmetry of light nuclei [38, 39]. The nucleon states $p\uparrow$, $p\downarrow$, $n\uparrow$, and $n\downarrow$ transform under the fundamental representation of $\mathrm{SU}(4)_W$. In even partial waves, V_0^0 and V_σ^1 respect the $\mathrm{SU}(4)_W$ symmetry while the terms that break the symmetry, V_σ^0 and V_0^1, are relatively $1/N_c^2$ suppressed. This analysis was also performed for the spin-orbit, tensor, and quadrupole contributions to the potential Ref. [39]. Additionally, the results were compared to the Nijgmegen potential, and the sizes of couplings in the Nijmegen potential fall roughly within the large-N_c predictions.

When working to higher orders in the momenta, there are additional N_c suppressions that can arise due to relativistic corrections. For example, in t-channel meson exchange, $\mathbf{p}_-$ is considered to be $O(1)$ while $\mathbf{p}_+$ only occurs in relativistic corrections as $\mathbf{p}_+/m_N$ and is therefore suppressed. In the u-channel, the roles of the momenta are reversed which leads to identical results.

These results are also consistent with meson exchange as demonstrated in Ref. [6]. Given that the meson-baryon coupling is $O(\sqrt{N_c})$, one might assume that the tree level diagram in Fig. 3.9 is $O(N_c)$ while the box- and cross-diagram are both $O(N_c^2)$. If more meson exchanges are included, then this problem will be exacerbated. However, it has already been pointed out that nucleons and the Δ resonance are degenerate in the large-N_c limit. In pion-baryon scattering, it was necessary to include all possible intermediate baryons in order to achieve a scattering amplitude with the correct N_c scaling. In an analogous way, Ref. [6] showed that the inclusion of the Δ in the two-meson exchange diagrams leads to cancellations that result in a potential that is $O(N_c)$. Additionally, Ref. [123] found that three- and higher-meson exchange contributions do not necessarily yield the correct large-N_c scalings; however, this issue

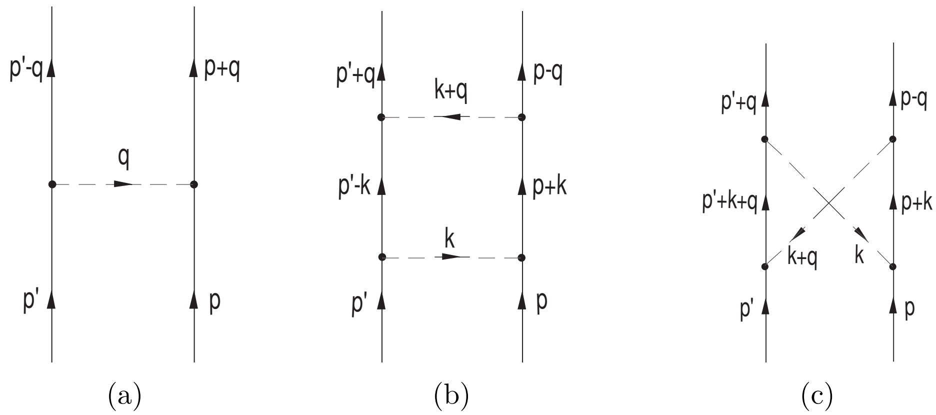

Figure 3.9: (a) Tree-level one meson exchange, (b) two-meson exchange box diagram, and (c) cross-box diagram. Figure from Ref. [6].

is resolved through the observation that energy independent potentials adhere to the large-N_c scaling while energy dependent potentials can yield different N_c scalings [124].

In addition to the analysis of the parity conserving potential, the spin-flavor algebra has been used to derive constraints for the three-nucleon potential [43] and parity- and time-reversal-invariance-violating potentials [125, 126]. In the context of EFT$_{\not\pi}$, these techniques have been applied to the parity violation in the two-nucleon sector [44, 127] and time-reversal-invariance violation in the three-nucleon sector [128]. Also, constraints for two-derivative two-nucleon contact operators were derived in Ref. [45] along with a detailed discussion of the tension between large-N_c constraints and the renormalization group in the PDS scheme.

Chapter 4

One- and Two-Nucleon Currents

In this chapter, the one- and two-nucleon scalar, vector, and axial currents through one derivative are derived, and the large-N_c scalings of the corresponding LECs are deduced from the derivative and spin-flavor structure of each operator. It is unnecessary to study the pseudoscalar currents at the order to which we will work as they are related to the axial current [129, 130]. These currents are important for a variety of processes including electroweak properties of nuclei, lepton-nucleus scattering, and BSM applications such as neutrinoless double beta decay and the direct detection of dark matter. The applications of these currents in those contexts will be demonstrated in the subsequent chapter. For the one-nucleon currents there are isoscalar and isovector operators, while the two-nucleon currents also include isotensor operators. The isotensor operators derived here are also symmetric and traceless. They are symmetric because the irreducible representations of SU(2) are completely symmetric, any antisymmetric isotensor can always be reduced to isovector operators. Also, they are traceless in order to remove the $I = 0$ component from each operator.

The operators, that is the products of the external field and the currents, in the Lagrangian are constrained by rotational symmetry to be scalars. Also, the Lagrangian should be invariant under infinitesimal boosts or Galilean transformations and Hermitian. The constraints provided by boosts introduce factors of the nucleon mass in the denominator of certain operators. These factors will be made explicit in the large-N_c scaling of the LECs. Also, when including gauge fields such as electro-

magnetic fields, the operators should also be gauge invariant; however, this case is reserved for the application to electromagnetic fields in Ch. 5.

In the following, the currents are divided into the categories vector, axial vector, axial charge, and scalar. This nomenclature refers strictly to the parity of each current, unless otherwise noted, and whether or not it has a free spatial index to couple to the external field. In some cases there is only one option for the time-reversal property of a current; in other cases the time-reversal properties of the currents can fall into one of two groups. In principle, the currents with the "wrong" time-reversal can be obtained from the nonrelativistic reduction of Lorentz covariant operators such that the nonrelativistic operators receive contributions from several relativistic operators; therefore, it is not always possible to pin down the origin of the nonrelativistic operator. It is for this reason and for the sake of completeness that the currents with non-standard time-reversal properties are included in each section.

After deriving the parity invariant Lagrangian, the same currents can then be coupled to other external fields in order to construct symmetry violating interactions. For example, one could couple the vector currents to an external axial field to obtain parity violating terms. Then, as long as the external field is not a QCD object, the large-N_c scalings remain intact for the symmetry violating LECs.

Finally, each component of the Lagrangian below is denoted as $\mathcal{L}_{\phi}^{(s,v,t;T)}$, where s, v, and t indicate the component with isoscalar, isovector, or isotensor currents. The index ϕ is the type of field that the currents are coupled to, and T indicates the time-reversal of the currents. In the isovector and isotensor cases, the time-reversal is indicated by the free isospin indices. The external fields are allowed to carry isospin indices so that the Lagrangian will be an isoscalar; however, these indices are implicit throughout this chapter so that these currents can be used in more general isospin breaking applications. There will also be the subscript N or NN depending on the number of nucleons involved in the interactions, which should become clear. The

LECs in the overcomplete Lagrangian are denoted as $\tilde{C}^{(s,v,t)}_{\phi,i}$, and in the minimal basis the tildes are removed.

4.1 Vector Currents

Consider a one- or two-nucleon system in an external vector field V^i with the Cartesian index $i = 1, 2, 3$. There are two cases of the time-reversal of the external field to consider. First, the vector field can be odd under both parity and time-reversal like the electromagnetic vector potential, $\mathbf{A}$. Second, the vector field can be even under time-reversal like the electric field strength $\mathbf{E}$; however, this analogy should not be taken too literally as the actual currents that couple to $\mathbf{E}$ are constrained by gauge invariance. Therefore, we will derive currents for all possible time-reversal properties.

4.1.1 One-Nucleon Currents

First, consider the currents that are odd under time-reversal. There are two possible single-nucleon isoscalar vector currents,

$$\mathcal{L}^{(s,-)}_{VN} = V^i \left[\frac{i}{2m_N} \not{T} g^{(s)}_{1,V} \left(N^\dagger \overleftrightarrow{\nabla}^i N \right) + \not{T} g^{(s)}_{2,V} \epsilon^{ijk} \nabla^j \left(N^\dagger \sigma^k N \right) \right] , \tag{4.1}$$

If the external vector field were the electromagnetic potential A^i, then the first operator would result from the minimal coupling prescription while the second operator contributes to the anomalous magnetic moment. However, as will be discussed in the following chapter, if the nucleon charge matrix is included caution is needed to determine the correct large-N_c scaling since the nucleon charge can be treated either as $O(1)$ or $O(N_c)$. Here, the former scaling of the nucleon charge is chosen because it the closest to the real world, but the scalings listed in this section are for a more general external vector field. Thus, the LECs scale as

$$\frac{1}{2m_N} \not{T} g^{(s)}_{1,V} , \ \not{T} g^{(s)}_{2,V} \ \sim \ O(1) . \tag{4.2}$$

Although $\not{g}_{1,V}^{(s)}$ appears to be LO-in-N_c, the explicit factor of the nucleon mass in the denominator of the operator indicates that contributions from both operators will be of the same order in N_c. In addition, there are two isovector currents that transform under time-reversal as $J_{VN}^{ia} \xrightarrow{T} (-1)^a J_{VN}^{ia}$ such that they are odd for $a = 3$,

$$\mathcal{L}_{VN}^{(s,a)} = V^i \left[\frac{i}{2m_N} \not{g}_{1,V}^{(v)} \left(N^\dagger \overleftrightarrow{\nabla}^i \tau^a N \right) + \not{g}_{2,V}^{(v)} \epsilon^{ijk} \nabla^j \left(N^\dagger \sigma^k \tau^a N \right) \right] , \tag{4.3}$$

where the LECs scale as

$$\frac{1}{m_N} \not{g}_{1,V}^{(v)} \sim O(1/N_c), \tag{4.4}$$
$$\not{g}_{2,V}^{(v)} \sim O(N_c)\,. \tag{4.5}$$

Additional constraints arise when a specific form for the vector field is chosen such that the Lagrangian will be invariant under Galilean transformations or infinitesimal Lorentz transformations.

The isoscalar operators with the opposite time-reversal are obtained by interchanging the derivative structure in Eq. (4.1)

$$\mathcal{L}_{VN}^{(s,+)} = V^i \left[\not{g}_{3,V}^{(s)} \epsilon^{ijk} \nabla^j \left(N^\dagger N \right) + \frac{i}{2m_N} \not{g}_{4,V}^{(s)} \epsilon^{ijk} \left(N^\dagger \overleftrightarrow{\nabla}^j \sigma^k N \right) \right] , \tag{4.6}$$

where the LECs scale as

$$\not{g}_{3,V}^{(s)} \sim O(N_c) \tag{4.7}$$
$$\frac{1}{2m_N} \not{g}_{4,V}^{(s)} \sim O(1/N_c)\,. \tag{4.8}$$

The isovector operators are also obtained by interchanging the derivative structure in Eq. (4.3)

$$\mathcal{L}_{VN}^{(s,a+1)} = V^i \left[\not{g}_{5,V}^{(v)} \epsilon^{ijk} \nabla^j \left(N^\dagger \tau^a N \right) + \frac{i}{2m_N} \not{g}_{6,V}^{(v)} \left(N^\dagger \overleftrightarrow{\nabla}^i \sigma^k \tau^a N \right) \right] , \tag{4.9}$$

where the LECs scale as

$$\not{g}_{5,V}^{(v)} \sim O(1)\,, \tag{4.10}$$

$$\not{g}_{6,V}^{(v)} \sim O(1)\,. \tag{4.11}$$

4.1.2 Two-Nucleon Currents

In this section, the two-nucleon contact vector currents are derived along with the large-N_c constraints. Analogous structures can be found in Refs. [130–133]. However, these references are specifically concerned with the context of electromagnetic currents. Regardless, it is necessary to derive an overcomplete basis for the Lagrangian in order to discern which LECs are LO in the large-N_c expansion and which are subleading without hiding any scalings through Fierz transformations. A direct comparison with Refs. [131, 132] is contained in Sec. 5.1. For a comparison of the pion exchange contributions from time-ordered perturbation theory and the method of unitary transformations to the vector current, see Ref. [130].

There are twelve possible two-nucleon isoscalar vector currents. The Lagrangian with a nonminimal set of operators can be written as

$$\begin{aligned}
\mathcal{L}_{VNN}^{(s,-)} = V^i \Bigg[& \frac{i}{2m_N}\tilde{C}_{V,1}^{(s)} \left(N^\dagger \overleftrightarrow{\nabla}^i N\right)\left(N^\dagger N\right) + \frac{i}{2m_N}\tilde{C}_{V,2}^{(s)} \left(N^\dagger \overleftrightarrow{\nabla}^i \tau^a N\right)\left(N^\dagger \tau^a N\right) \\
& + \frac{i}{2m_N}\tilde{C}_{V,3}^{(s)} \left(N^\dagger \overleftrightarrow{\nabla}^i \sigma^j N\right)\left(N^\dagger \sigma^j N\right) + \frac{i}{2m_N}\tilde{C}_{V,4}^{(s)} \left(N^\dagger \overleftrightarrow{\nabla}^i \sigma^j \tau^a N\right)\left(N^\dagger \sigma^j \tau^a N\right) \\
& + \frac{i}{2m_N}\tilde{C}_{V,5}^{(s)} \left(N^\dagger \overleftrightarrow{\nabla}^j \sigma^i N\right)\left(N^\dagger \sigma^j N\right) + \frac{i}{2m_N}\tilde{C}_{V,6}^{(s)} \left(N^\dagger \overleftrightarrow{\nabla}^j \sigma^i \tau^a N\right)\left(N^\dagger \sigma^j \tau^a N\right) \\
& + \frac{i}{2m_N}\tilde{C}_{V,7}^{(s)} \left(N^\dagger \overleftrightarrow{\nabla}^j \sigma^j N\right)\left(N^\dagger \sigma^i N\right) + \frac{i}{2m_N}\tilde{C}_{V,8}^{(s)} \left(N^\dagger \overleftrightarrow{\nabla}^j \sigma^j \tau^a N\right)\left(N^\dagger \sigma^i \tau^a N\right) \\
& + \tilde{C}_{V,9}^{(s)} \epsilon^{ijk} \nabla^j \left(N^\dagger \sigma^k N\right)\left(N^\dagger N\right) + \tilde{C}_{V,10}^{(s)} \epsilon^{ijk} \nabla^j \left(N^\dagger \sigma^k \tau^a N\right)\left(N^\dagger \tau^a N\right) \\
& + \tilde{C}_{V,11}^{(s)} \epsilon^{ijk} \nabla^j \left(N^\dagger N\right)\left(N^\dagger \sigma^k N\right) + \tilde{C}_{V,12}^{(s)} \epsilon^{ijk} \nabla^j \left(N^\dagger \tau^a N\right)\left(N^\dagger \sigma^k \tau^a N\right) \Bigg] \,. \qquad (4.12)
\end{aligned}$$

Including explicit factor of the nucleon mass, there are eight LECs which are $O(1)$,

$$\begin{matrix} \frac{1}{2m_N}\tilde{C}_{V,1}^{(s)},\ \frac{1}{2m_N}\tilde{C}_{V,4}^{(s)},\ \frac{1}{2m_N}\tilde{C}_{V,6}^{(s)},\ \frac{1}{2m_N}\tilde{C}_{V,8}^{(s)}, \\ \tilde{C}_{V,9}^{(s)},\ \tilde{C}_{V,10}^{(s)},\ \tilde{C}_{V,11}^{(s)},\ \tilde{C}_{V,12}^{(s)} \end{matrix} \sim O(1)\,, \qquad (4.13)$$

while there are four that are $1/N_c^2$ suppressed,

$$\frac{1}{2m_N}\tilde{C}_{V,2}^{(s)},\ \frac{1}{2m_N}\tilde{C}_{V,3}^{(s)},\ \frac{1}{2m_N}\tilde{C}_{V,5}^{(s)},\ \frac{1}{2m_N}\tilde{C}_{V,7}^{(s)} \ \sim\ O(1/N_c^2)\,. \qquad (4.14)$$

Denoting the current corresponding to the LEC $\tilde{C}^{(s)}_{V,n}$, including the nucleon mass when appropriate, by $\tilde{J}^i_{V,n}$, Fierz transformations lead to the system of equations:

$$0 = 5\tilde{J}^i_{V,1} + \tilde{J}^i_{V,2} + \tilde{J}^i_{V,3} + \tilde{J}^i_{V,4}\,, \tag{4.15}$$

$$0 = 3\tilde{J}^i_{V,1} + 3\tilde{J}^i_{V,2} + 3\tilde{J}^i_{V,3} - \tilde{J}^i_{V,4}\,, \tag{4.16}$$

$$0 = 9\tilde{J}^i_{V,1} - 3\tilde{J}^i_{V,2} - 3\tilde{J}^i_{V,3} + 5\tilde{J}^i_{V,4} \tag{4.17}$$

$$0 = \tilde{J}^i_{V,1} + \tilde{J}^i_{V,2} - \tilde{J}^i_{V,3} - \tilde{J}^i_{V,4} + 5\tilde{J}^i_{V,5} + \tilde{J}^i_{V,6} + \tilde{J}^i_{V,7} + \tilde{J}^i_{V,8} + \tilde{J}^i_{V,9} + \tilde{J}^i_{V,10} - \tilde{J}^i_{V,11} - \tilde{J}^i_{V,12}\,, \tag{4.18}$$

$$0 = 3\tilde{J}^i_{V,1} - \tilde{J}^i_{V,2} + 3\tilde{J}^i_{V,3} - \tilde{J}^i_{V,4} + 3\tilde{J}^i_{V,5} + 3\tilde{J}^i_{V,6} + 3\tilde{J}^i_{V,7} - \tilde{J}^i_{V,8} + 3\tilde{J}^i_{V,9} - \tilde{J}^i_{V,10} - 3\tilde{J}^i_{V,11} + \tilde{J}^i_{V,12}\,, \tag{4.19}$$

$$0 = \tilde{J}^i_{V,1} + \tilde{J}^i_{V,2} - \tilde{J}^i_{V,3} - \tilde{J}^i_{V,4} + \tilde{J}^i_{V,5} + \tilde{J}^i_{V,6} + 5\tilde{J}^i_{V,7} + \tilde{J}^i_{V,8} - \tilde{J}^i_{V,9} - \tilde{J}^i_{V,10} + \tilde{J}^i_{V,11} + \tilde{J}^i_{V,12}\,, \tag{4.20}$$

$$0 = 3\tilde{J}^i_{V,1} - \tilde{J}^i_{V,2} + 3\tilde{J}^i_{V,3} - \tilde{J}^i_{V,4} + 3\tilde{J}^i_{V,5} - \tilde{J}^i_{V,6} + 3\tilde{J}^i_{V,7} + 3\tilde{J}^i_{V,8} - 3\tilde{J}^i_{V,9} + \tilde{J}^i_{V,10} + 3\tilde{J}^i_{V,11} - \tilde{J}^i_{V,12}\,, \tag{4.21}$$

$$0 = \tilde{J}^i_{V,5} + \tilde{J}^i_{V,6} - \tilde{J}^i_{V,7} - \tilde{J}^i_{V,8} + 5\tilde{J}^i_{V,9} + \tilde{J}^i_{V,10} + \tilde{J}^i_{V,11} + \tilde{J}^i_{V,12} \tag{4.22}$$

$$0 = 3\tilde{J}^i_{V,5} - \tilde{J}^i_{V,6} - 3\tilde{J}^i_{V,7} + \tilde{J}^i_{V,8} + 3\tilde{J}^i_{V,9} + 3\tilde{J}^i_{V,10} + 3\tilde{J}^i_{V,11} - \tilde{J}^i_{V,12} \tag{4.23}$$

$$0 = \tilde{J}^i_{V,5} + \tilde{J}^i_{V,6} - \tilde{J}^i_{V,7} - \tilde{J}^i_{V,8} - \tilde{J}^i_{V,9} - \tilde{J}^i_{V,10} - 5\tilde{J}^i_{V,11} - \tilde{J}^i_{V,12} \tag{4.24}$$

$$0 = 3\tilde{J}^i_{V,5} - \tilde{J}^i_{V,6} - 3\tilde{J}^i_{V,7} + \tilde{J}^i_{V,8} - 3\tilde{J}^i_{V,9} + \tilde{J}^i_{V,10} - 3\tilde{J}^i_{V,11} - 3\tilde{J}^i_{V,12}\,. \tag{4.25}$$

This system reduces the set of operators to five independent choices. One choice is to retain $\tilde{J}^i_{V,1}$, $\tilde{J}^i_{V,8}$, $\tilde{J}^i_{V,10}$, $\tilde{J}^i_{V,11}$, $\tilde{J}^i_{V,12}$ such that the Lagrangian may be written as

$$\begin{aligned}\mathcal{L}^{(s,-)}_{VNN} = V^i \Bigg[&\frac{i}{2m_N} C^{(s)}_{V,1} \left(N^\dagger \overleftrightarrow{\nabla}^i N\right)\left(N^\dagger N\right) + \frac{i}{2m_N} C^{(s)}_{V,2} \left(N^\dagger \overleftrightarrow{\nabla}^j \sigma^j \tau^a N\right)\left(N^\dagger \sigma^i \tau^a N\right) \\ &+ C^{(s)}_{V,3} \epsilon^{ijk} \nabla^j \left(N^\dagger \sigma^k \tau^a N\right)\left(N^\dagger \tau^a N\right) + C^{(s)}_{V,4} \epsilon^{ijk} \nabla^j \left(N^\dagger N\right)\left(N^\dagger \sigma^k N\right) \\ &+ C^{(s)}_{V,5} \epsilon^{ijk} \nabla^j \left(N^\dagger \tau^a N\right)\left(N^\dagger \sigma^k \tau^a N\right)\Bigg]\,,\end{aligned} \tag{4.26}$$

where the LECs are related to those from the overcomplete basis according to

$$\frac{1}{2m_N}C^{(s)}_{V,1} = \frac{1}{2m_N}\tilde{C}^{(s)}_{V,1} - \frac{1}{2m_N}\tilde{C}^{(s)}_{V,2} - \frac{1}{2m_N}\tilde{C}^{(s)}_{V,3} - \frac{3}{2m_N}\tilde{C}^{(s)}_{V,4} - \frac{1}{m_N}\tilde{C}^{(s)}_{V,6} - \frac{1}{m_N}\tilde{C}^{(s)}_{V,7}\,, \tag{4.27}$$

$$\frac{1}{2m_N}C^{(s)}_{V,2} = \frac{1}{2m_N}\tilde{C}^{(s)}_{V,5} - \frac{3}{2m_N}\tilde{C}^{(s)}_{V,6} - \frac{3}{2m_N}\tilde{C}^{(s)}_{V,7} + \frac{1}{2m_N}\tilde{C}^{(s)}_{V,8}\,, \tag{4.28}$$

$$C^{(s)}_{V,3} = -\frac{1}{6m_N}\tilde{C}^{(s)}_{V,5} + \frac{1}{2m_N}\tilde{C}^{(s)}_{V,6} - \frac{1}{3}\tilde{C}^{(s)}_{V,9} + \tilde{C}^{(s)}_{V,10}\,, \tag{4.29}$$

$$C^{(s)}_{V,4} = \frac{1}{2m_N}\tilde{C}^{(s)}_{V,5} - \frac{3}{2m_N}\tilde{C}^{(s)}_{V,7} - \tilde{C}^{(s)}_{V,9} + \tilde{C}^{(s)}_{V,11}\,, \tag{4.30}$$

$$C^{(s)}_{V,5} = \frac{1}{3m_N}\tilde{C}^{(s)}_{V,5} - \frac{1}{2m_N}\tilde{C}^{(s)}_{V,6} - \frac{1}{2m_N}\tilde{C}^{(s)}_{V,7} - \frac{1}{3}\tilde{C}^{(s)}_{V,9} + \tilde{C}^{(s)}_{V,12}\,, \tag{4.31}$$

and the LECs in the minimal basis are all $O(1)$ with respect to N_c.

Transforming to the partial wave basis results in

$$\begin{aligned}
\mathcal{L}^{(s,-)}_{VNN} = V^i \Bigg\{ & \left[-\frac{1}{2m_N}C^{(s)}_{V,1} + \frac{3}{2m_N}C^{(s)}_{V,2}\right] i\nabla^i \left(N^T P^j N\right)^\dagger \left(N^T P^j N\right) \\
&+ \left[-\frac{1}{2m_N}C^{(s)}_{V,1} + \frac{1}{2m_N}C^{(s)}_{V,2}\right] i\nabla^i \left(N^T \bar{P}^a N\right)^\dagger \left(N^T \bar{P}^a N\right) \\
&+ \left[-\frac{3}{2m_N}C^{(s)}_{V,2} + 3C^{(s)}_{V,3} - C^{(s)}_{V,4} + 3C^{(s)}_{V,5}\right] i\nabla^j \left(N^T P^i N\right)^\dagger \left(N^T P^j N\right) \\
&+ \left[-\frac{3}{2m_N}C^{(s)}_{V,2} - 3C^{(s)}_{V,3} + C^{(s)}_{V,4} + 3C^{(s)}_{V,5}\right] i\nabla^j \left(N^T P^j N\right)^\dagger \left(N^T P^i N\right) \\
&+ \left[\frac{1}{2m_N}C^{(s)}_{V,1} - 3C^{(s)}_{V,3} - C^{(s)}_{V,4} + 3C^{(s)}_{V,5}\right] \epsilon^{ijk} \left(N^T \overleftrightarrow{\nabla}^j P^0 N\right)^\dagger \left(N^T P^k N\right) \\
&+ \left[\frac{1}{2m_N}C^{(s)}_{V,1} + C^{(s)}_{V,3} - C^{(s)}_{V,4} - C^{(s)}_{V,5}\right] \epsilon^{ijk} \left(N^T \overleftrightarrow{\nabla}^j P^{ka} N\right)^\dagger \left(N^T \bar{P}^a N\right) + \text{H.c} \Bigg\}\,,
\end{aligned} \tag{4.32}$$

where we have used the projectors [45, 89]

$$P^0 = \frac{1}{\sqrt{8}}\sigma^2\tau^2\,, \tag{4.33}$$

$$P^{ia} = \frac{1}{\sqrt{8}}\sigma^2\sigma^i\tau^2\tau^a\,. \tag{4.34}$$

Again, each term is of the same order in N_c. However, the relative signs of the large-N_c basis LECs could lead to cancellations such that certain partial wave LECs will

be suppressed. A derivative acting from the outside on a bilinear will be proportional to a factor of the total momentum of the state, which does not change the orbital component of the partial wave, so the first four terms connect two S-wave states. On the other hand, the last two terms connect P-wave to a S-wave states and are therefore expected to be suppressed in the EFT$_{\not\pi}$ power counting [51].

The currents that are even under time-reversal are obtained by interchanging the type of derivative occurring in each operator. Therefore, the overcomplete Lagrangian is

$$\begin{aligned}
\mathcal{L}_{VNN}^{(s,+)} = V^i \Big[& \tilde{C}_{V,13}^{(s)} \nabla^i \left(N^\dagger N\right)\left(N^\dagger N\right) + \tilde{C}_{V,14}^{(s)} \nabla^i \left(N^\dagger \tau^a N\right)\left(N^\dagger \tau^a N\right) \\
& + \tilde{C}_{V,15}^{(s)} \nabla^i \left(N^\dagger \sigma^j N\right)\left(N^\dagger \sigma^j N\right) + \tilde{C}_{V,16}^{(s)} \nabla^i \left(N^\dagger \sigma^j \tau^a N\right)\left(N^\dagger \sigma^j \tau^a N\right) \\
& + \tilde{C}_{V,17}^{(s)} \nabla^j \left(N^\dagger \sigma^i N\right)\left(N^\dagger \sigma^j N\right) + \tilde{C}_{V,18}^{(s)} \nabla^j \left(N^\dagger \sigma^i \tau^a N\right)\left(N^\dagger \sigma^j \tau^a N\right) \\
& + \tilde{C}_{V,19}^{(s)} \nabla^j \left(N^\dagger \sigma^j N\right)\left(N^\dagger \sigma^i N\right) + \tilde{C}_{V,20}^{(s)} \nabla^j \left(N^\dagger \sigma^j \tau^a N\right)\left(N^\dagger \sigma^i \tau^a N\right) \\
& + \frac{i}{2m_N} \tilde{C}_{V,21}^{(s)} \epsilon^{ijk} \left(N^\dagger \overleftrightarrow{\nabla}^j \sigma^k N\right)\left(N^\dagger N\right) \\
& + \frac{i}{2m_N} \tilde{C}_{V,22}^{(s)} \epsilon^{ijk} \left(N^\dagger \overleftrightarrow{\nabla}^j \sigma^k \tau^a N\right)\left(N^\dagger \tau^a N\right) \\
& + \frac{i}{2m_N} \tilde{C}_{V,23}^{(s)} \epsilon^{ijk} \left(N^\dagger \overleftrightarrow{\nabla}^j N\right)\left(N^\dagger \sigma^k N\right) \\
& + \frac{i}{2m_N} \tilde{C}_{V,24}^{(s)} \epsilon^{ijk} \left(N^\dagger \overleftrightarrow{\nabla}^j \tau^a N\right)\left(N^\dagger \sigma^k \tau^a N\right) \Big] \,.
\end{aligned} \tag{4.35}$$

There are four LECs at $O(N_c)$

$$\tilde{C}_{V,13}^{(s)},\, \tilde{C}_{V,16}^{(s)},\, \tilde{C}_{V,18}^{(s)},\, \tilde{C}_{V,20}^{(s)} \sim O(N_c)\,, \tag{4.36}$$

while the remaining LECs are $O(1/N_c)$. Fierz transformations produce the homogeneous system

$$0 = 5\tilde{J}_{V,13}^i + \tilde{J}_{V,14}^i + \tilde{J}_{V,15}^i + \tilde{J}_{V,16}^i\,, \tag{4.37}$$

$$0 = 3\tilde{J}_{V,13}^i + 3\tilde{J}_{V,14}^i + 3\tilde{J}_{V,15}^i - \tilde{J}_{V,16}^i\,, \tag{4.38}$$

$$0 = 9\tilde{J}_{V,13}^i - 3\tilde{J}_{V,14}^i - 3\tilde{J}_{V,15}^i + 5\tilde{J}_{V,16}^i\,, \tag{4.39}$$

$$
\begin{aligned}
0 &= \tilde{J}^i_{V,13} + \tilde{J}^i_{V,14} - \tilde{J}^i_{V,15} - \tilde{J}^i_{V,16} + 5\tilde{J}^i_{V,17} + \tilde{J}^i_{V,18} + \tilde{J}^i_{V,19} + \tilde{J}^i_{V,20} \\
&\quad - \tilde{J}^i_{V,21} - \tilde{J}^i_{V,22} + \tilde{J}^i_{V,23} + \tilde{J}^i_{V,24}\,, \qquad (4.40) \\
0 &= 3\tilde{J}^i_{V,13} - \tilde{J}^i_{V,14} - 3\tilde{J}^i_{V,15} + \tilde{J}^i_{V,16} + 3\tilde{J}^i_{V,17} + 3\tilde{J}^i_{V,18} + 3\tilde{J}^i_{V,19} - \tilde{J}^i_{V,20} \\
&\quad - 3\tilde{J}^i_{V,21} + \tilde{J}^i_{V,22} + 3\tilde{J}^i_{V,23} - \tilde{J}^i_{V,24}\,, \qquad (4.41) \\
0 &= \tilde{J}^i_{V,13} + \tilde{J}^i_{V,14} - \tilde{J}^i_{V,15} - \tilde{J}^i_{V,16} + \tilde{J}^i_{V,17} + \tilde{J}^i_{V,18} + 5\tilde{J}^i_{V,19} + \tilde{J}^i_{V,20} \\
&\quad + \tilde{J}^i_{V,21} + \tilde{J}^i_{V,22} - \tilde{J}^i_{V,23} - \tilde{J}^i_{V,24}\,, \qquad (4.42) \\
0 &= 3\tilde{J}^i_{V,13} - \tilde{J}^i_{V,14} - 3\tilde{J}^i_{V,15} + \tilde{J}^i_{V,16} + 3\tilde{J}^i_{V,17} - \tilde{J}^i_{V,18} + 3\tilde{J}^i_{V,19} + 3\tilde{J}^i_{V,20} \\
&\quad + 3\tilde{J}^i_{V,21} - \tilde{J}^i_{V,22} - 3\tilde{J}^i_{V,23} + \tilde{J}^i_{V,24}\,, \qquad (4.43) \\
0 &= -\tilde{J}^i_{V,17} - \tilde{J}^i_{V,18} + \tilde{J}^i_{V,19} + \tilde{J}^i_{V,20} + 5\tilde{J}^i_{V,21} + \tilde{J}^i_{V,22} + \tilde{J}^i_{V,23} + \tilde{J}^i_{V,24}\,, \qquad (4.44) \\
0 &= -3\tilde{J}^i_{V,17} + \tilde{J}^i_{V,18} + 3\tilde{J}^i_{V,19} - \tilde{J}^i_{V,20} + 3\tilde{J}^i_{V,21} + 3\tilde{J}^i_{V,22} + 3\tilde{J}^i_{V,23} - \tilde{J}^i_{V,24}\,, \qquad (4.45) \\
0 &= \tilde{J}^i_{V,17} + \tilde{J}^i_{V,18} - \tilde{J}^i_{V,19} - \tilde{J}^i_{V,20} + \tilde{J}^i_{V,21} + \tilde{J}^i_{V,22} + 5\tilde{J}^i_{V,23} + \tilde{J}^i_{V,24}\,, \qquad (4.46) \\
0 &= 3\tilde{J}^i_{V,17} - \tilde{J}^i_{V,18} - 3\tilde{J}^i_{V,19} + \tilde{J}^i_{V,20} + 3\tilde{J}^i_{V,21} - \tilde{J}^i_{V,22} + 3\tilde{J}^i_{V,23} + 3\tilde{J}^i_{V,24}\,, \qquad (4.47)
\end{aligned}
$$

which reduces the number of independent operators to six. Here, the choice is made to write the minimal Lagrangian as

$$
\begin{aligned}
\mathcal{L}^{(s,+)}_{VNN} = V^i \Big[& C^{(s)}_{V,6} \nabla^i \left(N^\dagger N\right)\left(N^\dagger N\right) + C^{(s)}_{V,7} \nabla^i \left(N^\dagger \tau^a N\right)\left(N^\dagger \tau^a N\right) \\
& + \tilde{C}^{(s)}_{V,8} \nabla^j \left(N^\dagger \sigma^i N\right)\left(N^\dagger \sigma^j N\right) + C^{(s)}_{V,9} \nabla^j \left(N^\dagger \sigma^i \tau^a N\right)\left(N^\dagger \sigma^j \tau^a N\right) \\
& + C^{(s)}_{V,10} \nabla^j \left(N^\dagger \sigma^j \tau^a N\right)\left(N^\dagger \sigma^i \tau^a N\right) \\
& + \frac{i}{2m_N} C^{(s)}_{V,11} \epsilon^{ijk} \left(N^\dagger \overleftrightarrow{\nabla}^j \sigma^k \tau^a N\right)\left(N^\dagger \tau^a N\right)\Big]\,, \qquad (4.48)
\end{aligned}
$$

where the LECs are related to those from the overcomplete Lagrangian according to

$$
\begin{aligned}
C^{(s)}_{V,6} &= \tilde{C}^{(s)}_{V,13} - 2\tilde{C}^{(s)}_{V,15} - 3\tilde{C}^{(s)}_{V,16} - 2\tilde{C}^{(s)}_{V,19} + \frac{1}{2m_N}\tilde{C}^{(s)}_{V,21} - \frac{3}{2m_N}\tilde{C}^{(s)}_{V,24}\,, \qquad (4.49) \\
C^{(s)}_{V,7} &= \tilde{C}^{(s)}_{V,14} - \tilde{C}^{(s)}_{V,15} - \frac{2}{3}\tilde{C}^{(s)}_{V,19} + \frac{1}{6m_N}\tilde{C}^{(s)}_{V,21} - \frac{1}{2m_N}\tilde{C}^{(s)}_{V,24}\,, \qquad (4.50) \\
C^{(s)}_{V,8} &= \tilde{C}^{(s)}_{V,17} - \tilde{C}^{(s)}_{V,19} + \frac{1}{2m_N}\tilde{C}^{(s)}_{V,21} - \frac{3}{2m_N}\tilde{C}^{(s)}_{V,24}\,, \qquad (4.51) \\
C^{(s)}_{V,9} &= -\frac{1}{3}\tilde{C}^{(s)}_{V,19} + \frac{1}{6m_N}\tilde{C}^{(s)}_{V,21} - \frac{1}{6m_N}\tilde{C}^{(s)}_{V,23}\,, \qquad (4.52)
\end{aligned}
$$

$$C^{(s)}_{V,10} = -\frac{1}{3}\tilde{C}^{(s)}_{V,19} + \frac{1}{6m_N}\tilde{C}^{(s)}_{V,23} - \frac{1}{2m_N}\tilde{C}^{(s)}_{V,24}\,, \tag{4.53}$$

$$C^{(s)}_{V,11} = -\frac{1}{6m_N}\tilde{C}^{(s)}_{V,21} - \frac{1}{6m_N}\tilde{C}^{(s)}_{V,23} + \frac{1}{2m_N}\tilde{C}^{(s)}_{V,24}\,. \tag{4.54}$$

In the minimal Lagrangian, there are three $O(N_c)$ LECs,

$$C^{(s)}_{V,6},\, C^{(s)}_{V,9},\, C^{(s)}_{V,10} \sim O(N_c)\,, \tag{4.55}$$

and three $O(1/N_c)$ LECs,

$$C^{(s)}_{V,7},\, C^{(s)}_{V,8},\, C^{(s)}_{V,11} \sim O(1/N_c)\,. \tag{4.56}$$

Transforming to the partial wave basis yields

$$\begin{aligned}
\mathcal{L}^{(s,+)}_{VNN} = V^i \Big\{ & \left[C^{(s)}_{V,6} + C^{(s)}_{V,7} - C^{(s)}_{V,8} - C^{(s)}_{V,9} - C^{(s)}_{V,10}\right] \nabla^i \left(N^T \bar{P}^a N\right)^\dagger \left(N^T \bar{P}^a N\right) \\
& + \left[C^{(s)}_{V,6} - 3C^{(s)}_{V,7} + C^{(s)}_{V,8} - 3C^{(s)}_{V,9} - 3C^{(s)}_{V,10}\right] \nabla^i \left(N^T P^j N\right)^\dagger \left(N^T P^j N\right) \\
& + \left[-C^{(s)}_{V,8} + 3C^{(s)}_{V,9} + 3C^{(s)}_{V,10} + \frac{3}{2m_N}C^{(s)}_{V,11}\right] \nabla^j \left(N^T P^i N\right)^\dagger \left(N^T P^j N\right) \\
& + \left[-C^{(s)}_{V,8} + 3C^{(s)}_{V,9} + 3C^{(s)}_{V,10} - \frac{3}{2m_N}C^{(s)}_{V,11}\right] \nabla^j \left(N^T P^j N\right)^\dagger \left(N^T P^i N\right) \\
& + \left[C^{(s)}_{V,8} - 3C^{(s)}_{V,9} + 3C^{(s)}_{V,10} + \frac{3}{2m_N}C^{(s)}_{V,11}\right] i\epsilon^{ijk} \left(N^T \overleftrightarrow{\nabla}^j P^0 N\right)^\dagger \left(N^T P^k N\right) \\
& + \left[-C^{(s)}_{V,8} - C^{(s)}_{V,9} + C^{(s)}_{V,10} - \frac{1}{2m_N}C^{(s)}_{V,11}\right] i\epsilon^{ijk} \left(N^T \overleftrightarrow{\nabla}^j P^{ka} N\right)^\dagger \left(N^T P^a N\right) \\
& + \mathrm{H.c} \Big\}\,.
\end{aligned} \tag{4.57}$$

Again, each operator received $O(N_c)$ contributions, but it is possible that the relative signs of the LECs could lead to suppressions or enhancements in this basis. Additionally, the first four terms connect two S-wave states, so they are expected to be enhanced in the EFT$_{\not\pi}$ power counting.

Now, consider the two-nucleon isovector current operators that transform under time-reversal as $J^{ia}_V \xrightarrow{T} (-1)^a\, J^{ia}_V$ with no sum over the isospin index a so that the current will be odd under T for $a = 3$. There are 18 possible operators such that the overcomplete Lagrangian is

$$\mathcal{L}^{(v,a)}_{VNN} = V^i \left[\tilde{C}^{(v)}_{V,1}\epsilon^{ijk}\nabla^j \left(N^\dagger\sigma^k N\right)\left(N^\dagger\tau^a N\right) + \tilde{C}^{(v)}_{V,2}\epsilon^{ijk}\nabla^j \left(N^\dagger\sigma^k\tau^a N\right)\left(N^\dagger N\right)\right.$$

$$
\begin{aligned}
&+\tilde{C}^{(v)}_{V,3}\epsilon^{ijk}\nabla^j\left(N^\dagger N\right)\left(N^\dagger\sigma^k\tau^a N\right)+\tilde{C}^{(v)}_{V,4}\epsilon^{ijk}\nabla^j\left(N^\dagger\tau^a N\right)\left(N^\dagger\sigma^k N\right)\\
&+\frac{i}{2m_N}\tilde{C}^{(v)}_{V,5}\left(N^\dagger\overleftrightarrow{\nabla}^i N\right)\left(N^\dagger\tau^a N\right)+\frac{i}{2m_N}\tilde{C}^{(v)}_{V,6}\left(N^\dagger\overleftrightarrow{\nabla}^i\tau^a N\right)\left(N^\dagger N\right)\\
&+\frac{i}{2m_N}\tilde{C}^{(v)}_{V,7}\left(N^\dagger\overleftrightarrow{\nabla}^i\sigma^j N\right)\left(N^\dagger\sigma^j\tau^a N\right)+\frac{i}{2m_N}\tilde{C}^{(v)}_{V,8}\left(N^\dagger\overleftrightarrow{\nabla}^i\sigma^j\tau^a N\right)\left(N^\dagger\sigma^j N\right)\\
&+\frac{i}{2m_N}\tilde{C}^{(v)}_{V,9}\left(N^\dagger\overleftrightarrow{\nabla}^j\sigma^i N\right)\left(N^\dagger\sigma^j\tau^a N\right)+\frac{i}{2m_N}\tilde{C}^{(v)}_{V,10}\left(N^\dagger\overleftrightarrow{\nabla}^j\sigma^i\tau^a N\right)\left(N^\dagger\sigma^j N\right)\\
&+\frac{i}{2m_N}\tilde{C}^{(v)}_{V,11}\left(N^\dagger\overleftrightarrow{\nabla}^j\sigma^j N\right)\left(N^\dagger\sigma^i\tau^a N\right)+\frac{i}{2m_N}\tilde{C}^{(v)}_{V,12}\left(N^\dagger\overleftrightarrow{\nabla}^j\sigma^j\tau^a N\right)\left(N^\dagger\sigma^i N\right)\\
&+\tilde{C}^{(v)}_{V,13}\epsilon^{abc}\nabla^i\left(N^\dagger\tau^b N\right)\left(N^\dagger\tau^c N\right)+\tilde{C}^{(v)}_{V,14}\epsilon^{abc}\nabla^i\left(N^\dagger\sigma^j\tau^b N\right)\left(N^\dagger\sigma^j\tau^c N\right)\\
&+\tilde{C}^{(v)}_{V,15}\epsilon^{abc}\nabla^j\left(N^\dagger\sigma^i\tau^b N\right)\left(N^\dagger\sigma^j\tau^c N\right)+\tilde{C}^{(v)}_{V,16}\epsilon^{abc}\nabla^j\left(N^\dagger\sigma^j\tau^b N\right)\left(N^\dagger\sigma^i\tau^c N\right)\\
&+\frac{i}{2m_N}\tilde{C}^{(v)}_{V,17}\epsilon^{ijk}\epsilon^{abc}\left(N^\dagger\overleftrightarrow{\nabla}^j\tau^b N\right)\left(N^\dagger\sigma^k\tau^c N\right)\\
&+\frac{i}{2m_N}\tilde{C}^{(v)}_{V,18}\epsilon^{ijk}\epsilon^{abc}\left(N^\dagger\overleftrightarrow{\nabla}^j\sigma^k\tau^b N\right)\left(N^\dagger\tau^c N\right)\Big]\,. \qquad (4.58)
\end{aligned}
$$

There are five LECs at LO-in-N_c,

$$
\tilde{C}^{(v)}_{V,2},\ \tilde{C}^{(v)}_{V,3},\ \tilde{C}^{(v)}_{V,14},\ \tilde{C}^{(v)}_{V,15},\ \tilde{C}^{(v)}_{V,16}\sim O\left(N_c\right)\,, \qquad (4.59)
$$

while the remaining LECs are all relatively $1/N_c^2$ suppressed:

$$
\begin{matrix}
\tilde{C}^{(v)}_{V,1},\ \tilde{C}^{(v)}_{V,4},\ \frac{1}{m_N}\tilde{C}^{(v)}_{V,5},\ \frac{1}{m_N}\tilde{C}^{(v)}_{V,6},\ \frac{1}{m_N}\tilde{C}^{(v)}_{V,7},\ \frac{1}{m_N}\tilde{C}^{(v)}_{V,8},\ \frac{1}{m_N}\tilde{C}^{(v)}_{V,9},\\
\frac{1}{m_N}\tilde{C}^{(v)}_{V,10},\ \frac{1}{m_N}\tilde{C}^{(v)}_{V,11},\ \frac{1}{m_N}\tilde{C}^{(v)}_{V,12},\ \tilde{C}^{(v)}_{V,13},\ \frac{1}{m_N}\tilde{C}^{(v)}_{V,17},\ \frac{1}{m_N}\tilde{C}^{(v)}_{V,18}
\end{matrix}
\sim O(1/N_c)\,. \qquad (4.60)
$$

Fierz transformations lead to the system of equations:

$$
0=3\tilde{J}^{ia}_{V,1}+\tilde{J}^{ia}_{V,2}+3\tilde{J}^{ia}_{V,3}+\tilde{J}^{ia}_{V,4}-\tilde{J}^{ia}_{V,17}-\tilde{J}^{ia}_{V,18}\,, \qquad (4.61)
$$

$$
0=2\tilde{J}^{ia}_{V,1}-2\tilde{J}^{ia}_{V,3}+\tilde{J}^{ia}_{V,9}+\tilde{J}^{ia}_{V,10}-\tilde{J}^{ia}_{V,11}-\tilde{J}^{ia}_{V,12}+\tilde{J}^{ia}_{V,15}-\tilde{J}^{ia}_{V,16}\,, \qquad (4.62)
$$

$$
0=\tilde{J}^{ia}_{V,1}+3\tilde{J}^{ia}_{V,2}+\tilde{J}^{ia}_{V,3}+3\tilde{J}^{ia}_{V,4}+\tilde{J}^{ia}_{V,17}+\tilde{J}^{ia}_{V,18}\,, \qquad (4.63)
$$

$$
0=2\tilde{J}^{ia}_{V,2}-2\tilde{J}^{ia}_{V,4}+\tilde{J}^{ia}_{V,10}-\tilde{J}^{ia}_{V,11}-\tilde{J}^{ia}_{V,12}-\tilde{J}^{ia}_{V,15}+\tilde{J}^{ia}_{V,16}\,, \qquad (4.64)
$$

$$
0=5\tilde{J}^{ia}_{V,5}+\tilde{J}^{ia}_{V,6}+\tilde{J}^{ia}_{V,7}+\tilde{J}^{ia}_{V,8}+\tilde{J}^{ia}_{V,13}+\tilde{J}^{ia}_{V,14}\,, \qquad (4.65)
$$

$$
0=\tilde{J}^{ia}_{V,5}+5\tilde{J}^{ia}_{V,6}+\tilde{J}^{ia}_{V,7}+\tilde{J}^{ia}_{V,8}-\tilde{J}^{ia}_{V,13}-\tilde{J}^{ia}_{V,14}\,, \qquad (4.66)
$$

$$0 = 3\tilde{J}^{ia}_{V,5} + 3\tilde{J}^{ia}_{V,6} + 3\tilde{J}^{ia}_{V,7} - \tilde{J}^{ia}_{V,8} + 3\tilde{J}^{ia}_{V,13} - \tilde{J}^{ia}_{V,14}\,, \tag{4.67}$$

$$0 = 3\tilde{J}^{ia}_{V,5} + 3\tilde{J}^{ia}_{V,6} - \tilde{J}^{ia}_{V,7} + 3\tilde{J}^{ia}_{V,8} - 3\tilde{J}^{ia}_{V,13} + \tilde{J}^{ia}_{V,14}\,, \tag{4.68}$$

$$\begin{aligned} 0 = {} & \tilde{J}^{ia}_{V,1} + \tilde{J}^{ia}_{V,2} - \tilde{J}^{ia}_{V,3} - \tilde{J}^{ia}_{V,4} + \tilde{J}^{ia}_{V,5} + \tilde{J}^{ia}_{V,6} - \tilde{J}^{ia}_{V,7} - \tilde{J}^{ia}_{V,8} + 3\tilde{J}^{ia}_{V,9} \\ & + 3\tilde{J}^{ia}_{V,10} + \tilde{J}^{ia}_{V,11} + \tilde{J}^{ia}_{V,12}\,, \end{aligned} \tag{4.69}$$

$$0 = 2\tilde{J}^{ia}_{V,9} - 2\tilde{J}^{ia}_{V,10} + \tilde{J}^{ia}_{V,13} - \tilde{J}^{ia}_{V,14} + \tilde{J}^{ia}_{V,15} + \tilde{J}^{ia}_{V,16} + \tilde{J}^{ia}_{V,17} - \tilde{J}^{ia}_{V,18}\,, \tag{4.70}$$

$$\begin{aligned} 0 = {} & \tilde{J}^{ia}_{V,1} + \tilde{J}^{ia}_{V,2} - \tilde{J}^{ia}_{V,3} - \tilde{J}^{ia}_{V,4} - \tilde{J}^{ia}_{V,5} - \tilde{J}^{ia}_{V,6} + \tilde{J}^{ia}_{V,7} + \tilde{J}^{ia}_{V,8} - \tilde{J}^{ia}_{V,9} \\ & - \tilde{J}^{ia}_{V,10} - 3\tilde{J}^{ia}_{V,11} - 3\tilde{J}^{ia}_{V,12}\,, \end{aligned} \tag{4.71}$$

$$0 = 2\tilde{J}^{ia}_{V,11} - 2\tilde{J}^{ia}_{V,12} + \tilde{J}^{ia}_{V,13} - \tilde{J}^{ia}_{V,14} + \tilde{J}^{ia}_{V,15} + \tilde{J}^{ia}_{V,16} - \tilde{J}^{ia}_{V,17} + \tilde{J}^{ia}_{V,18}\,, \tag{4.72}$$

$$0 = \tilde{J}^{ia}_{V,5} - \tilde{J}^{ia}_{V,6} + \tilde{J}^{ia}_{V,7} - \tilde{J}^{ia}_{V,8} + 2\tilde{J}^{ia}_{V,13}\,, \tag{4.73}$$

$$0 = 3\tilde{J}^{ia}_{V,5} - 3\tilde{J}^{ia}_{V,6} - \tilde{J}^{ia}_{V,7} + \tilde{J}^{ia}_{V,8} + 2\tilde{J}^{ia}_{V,14}\,, \tag{4.74}$$

$$0 = \tilde{J}^{ia}_{V,9} - \tilde{J}^{ia}_{V,10} + \tilde{J}^{ia}_{V,11} - \tilde{J}^{ia}_{V,12} + \tilde{J}^{ia}_{V,15} + \tilde{J}^{ia}_{V,16}\,, \tag{4.75}$$

$$0 = \tilde{J}^{ia}_{V,1} - \tilde{J}^{ia}_{V,2} - \tilde{J}^{ia}_{V,3} + \tilde{J}^{ia}_{V,4} + \tilde{J}^{ia}_{V,15} - \tilde{J}^{ia}_{V,16}\,, \tag{4.76}$$

$$0 = \tilde{J}^{ia}_{V,1} - \tilde{J}^{ia}_{V,2} + \tilde{J}^{ia}_{V,3} - \tilde{J}^{ia}_{V,4} - \tilde{J}^{ia}_{V,17} - \tilde{J}^{ia}_{V,18}\,, \tag{4.77}$$

$$0 = \tilde{J}^{ia}_{V,9} - \tilde{J}^{ia}_{V,10} - \tilde{J}^{ia}_{V,11} + \tilde{J}^{ia}_{V,12} + \tilde{J}^{ia}_{V,17} - \tilde{J}^{ia}_{V,18}\,. \tag{4.78}$$

The solution to this system requires that both $\tilde{J}^{ia}_{V,9}$ and $\tilde{J}^{ia}_{V,16}$ vanish while $\tilde{J}^{ia}_{V,13} = \tilde{J}^{ia}_{V,14}$. The remaining operators can be expressed in terms of the leading operators $\tilde{J}^{ia}_{V,3}$, $\tilde{J}^{ia}_{V,14}$, and $\tilde{J}^{ia}_{V,15}$ and the subleading operators $\tilde{J}^{ia}_{V,12}$ and $\tilde{J}^{ia}_{V,17}$. Therefore, the Lagrangian may be written as

$$\begin{aligned} \mathcal{L}^{(v,a)}_{VNN} = V^i \Big[& C^{(v)}_{V,1} \epsilon^{ijk} \nabla^j \left(N^\dagger N\right) \left(N^\dagger \sigma^k \tau^a N\right) + C^{(v)}_{V,2} \epsilon^{abc} \nabla^i \left(N^\dagger \sigma^j \tau^b N\right) \left(N^\dagger \sigma^j \tau^c N\right) \\ & + C^{(v)}_{V,3} \epsilon^{abc} \nabla^j \left(N^\dagger \sigma^i \tau^b N\right) \left(N^\dagger \sigma^j \tau^c N\right) + \frac{i}{2m_N} C^{(v)}_{V,4} \left(N^\dagger \overleftrightarrow{\nabla}^j \sigma^j \tau^a N\right) \left(N^\dagger \sigma^i N\right) \\ & + \frac{i}{2m_N} C^{(v)}_{V,5} \epsilon^{ijk} \epsilon^{abc} \left(N^\dagger \overleftrightarrow{\nabla}^j \tau^b N\right) \left(N^\dagger \sigma^k \tau^c N\right) \Big]\,, \end{aligned} \tag{4.79}$$

where these LECs are related to those from the overcomplete basis according to

$$C^{(v)}_{V,1} = \tilde{C}^{(v)}_{V,1} - \tilde{C}^{(v)}_{V,2} + \tilde{C}^{(v)}_{V,3} + \frac{1}{2m_N} \tilde{C}^{(v)}_{V,5} + \frac{1}{2m_N} \tilde{C}^{(v)}_{V,6} - \frac{3}{2m_N} \tilde{C}^{(v)}_{V,7}$$

$$- \frac{3}{2m_N}\tilde{C}^{(v)}_{V,8} - \frac{1}{m_N}\tilde{C}^{(v)}_{V,10} - \frac{1}{m_N}\tilde{C}^{(v)}_{V,11} + \frac{2}{m_N}\tilde{C}^{(v)}_{V,18}\,, \tag{4.80}$$

$$C^{(v)}_{V,2} = -\frac{1}{4m_N}\tilde{C}^{(v)}_{V,5} + \frac{1}{4m_N}\tilde{C}^{(v)}_{V,6} - \frac{1}{4m_N}\tilde{C}^{(v)}_{V,7} + \frac{1}{4m_N}\tilde{C}^{(v)}_{V,8} + \tilde{C}^{(v)}_{V,13} + \tilde{C}^{(v)}_{V,14}\,, \tag{4.81}$$

$$\begin{aligned} C^{(v)}_{V,3} = &-\tilde{C}^{(v)}_{V,1} + \frac{1}{2}\tilde{C}^{(v)}_{V,2} + \frac{1}{2}\tilde{C}^{(v)}_{V,4} - \frac{1}{4m_N}\tilde{C}^{(v)}_{V,5} - \frac{1}{4m_N}\tilde{C}^{(v)}_{V,6} + \frac{3}{4m_N}\tilde{C}^{(v)}_{V,7} \\ &+ \frac{3}{4m_N}\tilde{C}^{(v)}_{V,8} + \frac{3}{4m_N}\tilde{C}^{(v)}_{V,10} + \frac{1}{4m_N}\tilde{C}^{(v)}_{V,11} + \tilde{C}^{(v)}_{V,15} - \frac{1}{m_N}C^{(v)}_{V,18}\,, \end{aligned} \tag{4.82}$$

$$\frac{1}{2m_N}C^{(v)}_{V,4} = \tilde{C}^{(v)}_{V,1} + \tilde{C}^{(v)}_{V,4} - \frac{1}{2m_N}\tilde{C}^{(v)}_{V,10} + \frac{1}{2m_N}\tilde{C}^{(v)}_{V,12} + \frac{1}{m_N}C^{(v)}_{V,18}\,, \tag{4.83}$$

$$\begin{aligned} \frac{1}{2m_N}C^{(v)}_{V,5} = &-\frac{1}{4m_N}\tilde{C}^{(v)}_{V,5} - \frac{1}{4m_N}\tilde{C}^{(v)}_{V,6} + \frac{3}{4m_N}\tilde{C}^{(v)}_{V,7} + \frac{3}{4m_N}\tilde{C}^{(v)}_{V,8} + \frac{1}{2m_N}\tilde{C}^{(v)}_{V,17} \\ &- \frac{1}{2m_N}\tilde{C}^{(v)}_{V,18}\,, \end{aligned} \tag{4.84}$$

and the large-N_c scalings are

$$C^{(v)}_{V,1},\ C^{(v)}_{V,2},\ C^{(v)}_{V,3} \sim O(N_c)\,, \tag{4.85}$$

$$\frac{1}{2m_N}C^{(v)}_{V,4},\ \frac{1}{2m_N}C^{(v)}_{V,5} \sim O(1/N_c)\,. \tag{4.86}$$

In comparison to the isoscalar couplings, the three leading isovector couplings are dominant. Specifically, the isoscalar couplings are $1/N_c$ suppressed relative to the leading isovector couplings. Transforming to the partial wave basis results in

$$\begin{aligned} \mathcal{L}^{(v,a)}_{VNN} = V^i \Bigg\{ &\left[C^{(v)}_{V,1} + 2C^{(v)}_{V,3} - \frac{1}{2m_N}C^{(v)}_{V,4} + \frac{1}{m_N}C^{(v)}_{V,5}\right] \epsilon^{ijk}\nabla^j \left(N^T\bar{P}^a N\right)^\dagger \left(N^T P^k N\right) \\ &+ \left[C^{(v)}_{V,1} + 2C^{(v)}_{V,3} + \frac{1}{2m_N}C^{(v)}_{V,4} - \frac{1}{m_N}C^{(v)}_{V,5}\right] \epsilon^{ijk}\nabla^j \left(N^T P^k N\right)^\dagger \left(N^T\bar{P}^a N\right) \\ &+ \left[C^{(v)}_{V,1} - \frac{1}{2m_N}C^{(v)}_{V,4}\right] i\epsilon^{ijk}\epsilon^{abc} \left(N^T \overleftrightarrow{\nabla}^j P^{kb} N\right)^\dagger \left(N^T\bar{P}^c N\right) \\ &+ \left[C^{(v)}_{V,1} + 2C^{(v)}_{V,3} + \frac{1}{2m_N}C^{(v)}_{V,4} + \frac{1}{m_N}C^{(v)}_{V,5}\right] i \left(N^T \overleftrightarrow{\nabla}^j P^{ia} N\right)^\dagger \left(N^T P^j N\right) \\ &+ \left[-C^{(v)}_{V,1} + 2C^{(v)}_{V,3} + \frac{1}{2m_N}C^{(v)}_{V,4} - \frac{1}{m_N}C^{(v)}_{V,5}\right] i \left(N^T \overleftrightarrow{\nabla}^j P^{ja} N\right)^\dagger \left(N^T P^i N\right) \\ &+ \left[-6C^{(v)}_{V,2} - 2C^{(v)}_{V,3} + \frac{1}{2m_N}C^{(v)}_{V,4}\right] i \left(N^T \overleftrightarrow{\nabla}^i P^0 N\right)^\dagger \left(N^T\bar{P}^a N\right) \\ &+ \left[-2C^{(v)}_{V,2} - 2C^{(v)}_{V,3} - \frac{1}{2m_N}C^{(v)}_{V,4}\right] i \left(N^T \overleftrightarrow{\nabla}^i P^{ja} N\right)^\dagger \left(N^T P^j N\right) \end{aligned}$$

$$+\frac{1}{2m_N} C^{(v)}_{V,4} \epsilon^{abc} \nabla^i \left(N^T \bar{P}^b N\right)^\dagger \left(N^T \bar{P}^c N\right) + \text{H.c.}\Big\} \,. \tag{4.87}$$

All but the last term contain $O(N_c)$ contributions, so the last term is $O(1/N_c)$. Additionally, the first, second, and last terms connect two S-wave states. Therefore, these three terms are expected to be enhanced in the EFT$_{\not\pi}$ power counting. All together, there are two leading currents in both the EFT$_{\not\pi}$ power counting and in the large-N_c expansion in the partial wave basis, i.e., the first and second terms.

The currents that transform under time-reversal as $J^{ia}_V \xrightarrow{T} (-1)^{a+1} J^{ia}_V$ can be obtained by interchanging the type of derivative occuring in each operator. Thus, the overcomplete Lagrangian is

$$\begin{aligned}
\mathcal{L}^{(v,\,a+1)}_{VNN} = V^i \Bigg[& \frac{i}{2m_N} \tilde{C}^{(v)}_{V,19} \epsilon^{ijk} \left(N^\dagger \overleftrightarrow{\nabla}^j \sigma^k N\right) \left(N^\dagger \tau^a N\right) \\
& + \frac{i}{2m_N} \tilde{C}^{(v)}_{V,20} \epsilon^{ijk} \left(N^\dagger \overleftrightarrow{\nabla}^j \sigma^k \tau^a N\right) \left(N^\dagger N\right) \\
& + \frac{i}{2m_N} \tilde{C}^{(v)}_{V,21} \epsilon^{ijk} \left(N^\dagger \overleftrightarrow{\nabla}^j N\right) \left(N^\dagger \sigma^k \tau^a N\right) \\
& + \frac{i}{2m_N} \tilde{C}^{(v)}_{V,22} \epsilon^{ijk} \left(N^\dagger \overleftrightarrow{\nabla}^j \tau^a N\right) \left(N^\dagger \sigma^k N\right) \\
& + \tilde{C}^{(v)}_{V,23} \nabla^i \left(N^\dagger N\right) \left(N^\dagger \tau^a N\right) + \tilde{C}^{(v)}_{V,24} \nabla^i \left(N^\dagger \tau^a N\right) \left(N^\dagger N\right) \\
& + \tilde{C}^{(v)}_{V,25} \nabla^i \left(N^\dagger \sigma^j N\right) \left(N^\dagger \sigma^j \tau^a N\right) + \tilde{C}^{(v)}_{V,26} \nabla^i \left(N^\dagger \sigma^j \tau^a N\right) \left(N^\dagger \sigma^j N\right) \\
& + \tilde{C}^{(v)}_{V,27} \nabla^j \left(N^\dagger \sigma^i N\right) \left(N^\dagger \sigma^j \tau^a N\right) + \tilde{C}^{(v)}_{V,28} \nabla^j \left(N^\dagger \sigma^i \tau^a N\right) \left(N^\dagger \sigma^j N\right) \\
& + \tilde{C}^{(v)}_{V,29} \nabla^j \left(N^\dagger \sigma^j N\right) \left(N^\dagger \sigma^i \tau^a N\right) + \tilde{C}^{(v)}_{V,30} \nabla^j \left(N^\dagger \sigma^j \tau^a N\right) \left(N^\dagger \sigma^i N\right) \\
& + \frac{i}{2m_N} \tilde{C}^{(v)}_{V,31} \epsilon^{abc} \left(N^\dagger \overleftrightarrow{\nabla}^i \tau^b N\right) \left(N^\dagger \tau^c N\right) \\
& + \frac{i}{2m_N} \tilde{C}^{(v)}_{V,32} \epsilon^{abc} \left(N^\dagger \overleftrightarrow{\nabla}^i \sigma^j \tau^b N\right) \left(N^\dagger \sigma^j \tau^c N\right) \\
& + \frac{i}{2m_N} \tilde{C}^{(v)}_{V,33} \epsilon^{abc} \left(N^\dagger \overleftrightarrow{\nabla}^j \sigma^i \tau^b N\right) \left(N^\dagger \sigma^j \tau^c N\right) \\
& + \frac{i}{2m_N} \tilde{C}^{(v)}_{V,34} \epsilon^{abc} \left(N^\dagger \overleftrightarrow{\nabla}^j \sigma^j \tau^b N\right) \left(N^\dagger \sigma^i \tau^c N\right) \\
& + \tilde{C}^{(v)}_{V,35} \epsilon^{ijk} \epsilon^{abc} \nabla^j \left(N^\dagger \tau^b N\right) \left(N^\dagger \sigma^k \tau^c N\right)
\end{aligned}$$

$$+\tilde{C}^{(v)}_{V,36}\epsilon^{ijk}\epsilon^{abc}\nabla^j\left(N^\dagger\sigma^k\tau^b N\right)\left(N^\dagger\tau^c N\right)\Big] \,. \tag{4.88}$$

Most of the LECs are $O(1)$ with the exception of three LECs at $O(1/N_c^2)$,

$$\frac{1}{2m_N}\tilde{C}^{(v)}_{V,19},\ \frac{1}{2m_N}\tilde{C}^{(v)}_{V,22},\ \frac{1}{2m_N}\tilde{C}^{(v)}_{V,31} \sim O(1/N_c^2)\,. \tag{4.89}$$

Therefore, relative to the isovector couplings of the opposite time-reversal, these couplings are at least $1/N_c$ suppressed. Fierz transformations lead to the homogeneous system

$$0 = 3\tilde{J}^i_{V,19} + 3\tilde{J}^i_{V,20} + \tilde{J}^i_{V,21} + \tilde{J}^i_{V,22} - \tilde{J}^i_{V,27} - \tilde{J}^i_{V,28} + \tilde{J}^i_{V,29} + \tilde{J}^i_{V,30}\,, \tag{4.90}$$

$$0 = 2\tilde{J}^i_{V,19} - 2\tilde{J}^i_{V,20} + \tilde{J}^i_{V,33} - \tilde{J}^i_{V,34} + \tilde{J}^i_{V,35} + \tilde{J}^i_{V,36}\,, \tag{4.91}$$

$$0 = \tilde{J}^i_{V,19} + \tilde{J}^i_{V,20} + 3\tilde{J}^i_{V,21} + 3\tilde{J}^i_{V,22} + \tilde{J}^i_{V,27} + \tilde{J}^i_{V,28} - \tilde{J}^i_{V,29} - \tilde{J}^i_{V,30}\,, \tag{4.92}$$

$$0 = 2\tilde{J}^i_{V,21} - 2\tilde{J}^i_{V,22} - \tilde{J}^i_{V,33} + \tilde{J}^i_{V,34} + \tilde{J}^i_{V,35} + \tilde{J}^i_{V,36}\,, \tag{4.93}$$

$$0 = 3\tilde{J}^i_{V,23} + 3\tilde{J}^i_{V,24} + \tilde{J}^i_{V,25} + \tilde{J}^i_{V,26}\,, \tag{4.94}$$

$$0 = 2\tilde{J}^i_{V,23} - 2\tilde{J}^i_{V,24} - \tilde{J}^i_{V,31} - \tilde{J}^i_{V,32}\,, \tag{4.95}$$

$$0 = 2\tilde{J}^i_{V,25} - 2\tilde{J}^i_{V,26} - 3\tilde{J}^i_{V,31} + \tilde{J}^i_{V,32}\,, \tag{4.96}$$

$$\begin{aligned}0 = {} & \tilde{J}^i_{V,23} + \tilde{J}^i_{V,24} - \tilde{J}^i_{V,25} - \tilde{J}^i_{V,26} + 3\tilde{J}^i_{V,27} + \tilde{J}^i_{V,28} + 3\tilde{J}^i_{V,29} + \tilde{J}^i_{V,30} - \tilde{J}^i_{V,31} + \tilde{J}^i_{V,32} \\ & - \tilde{J}^i_{V,33} - \tilde{J}^i_{V,34}\,,\end{aligned} \tag{4.97}$$

$$0 = \tilde{J}^i_{V,19} + \tilde{J}^i_{V,20} - \tilde{J}^i_{V,21} - \tilde{J}^i_{V,22} - 2\tilde{J}^i_{V,27} + 2\tilde{J}^i_{V,29} - \tilde{J}^i_{V,35} + \tilde{J}^i_{V,36}\,, \tag{4.98}$$

$$\begin{aligned}0 = {} & \tilde{J}^i_{V,23} + \tilde{J}^i_{V,24} - \tilde{J}^i_{V,25} - \tilde{J}^i_{V,26} + \tilde{J}^i_{V,27} + 3\tilde{J}^i_{V,28} + 3\tilde{J}^i_{V,29} + 3\tilde{J}^i_{V,30} + \tilde{J}^i_{V,31} - \tilde{J}^i_{V,32} \\ & + \tilde{J}^i_{V,33} + \tilde{J}^i_{V,34}\,,\end{aligned} \tag{4.99}$$

$$0 = \tilde{J}^i_{V,19} + \tilde{J}^i_{V,20} - \tilde{J}^i_{V,21} - \tilde{J}^i_{V,22} - 2\tilde{J}^i_{V,28} + 2\tilde{J}^i_{V,30} + \tilde{J}^i_{V,35} - \tilde{J}^i_{V,36}\,, \tag{4.100}$$

$$0 = \tilde{J}^i_{V,23} - \tilde{J}^i_{V,24} + \tilde{J}^i_{V,25} - \tilde{J}^i_{V,26} - 2\tilde{J}^i_{V,31}\,, \tag{4.101}$$

$$0 = 3\tilde{J}^i_{V,23} - 3\tilde{J}^i_{V,24} - \tilde{J}^i_{V,25} + \tilde{J}^i_{V,26} - 2\tilde{J}^i_{V,32}\,, \tag{4.102}$$

$$0 = \tilde{J}^i_{V,23} - \tilde{J}^i_{V,24} - \tilde{J}^i_{V,25} + \tilde{J}^i_{V,26} + \tilde{J}^i_{V,27} - \tilde{J}^i_{V,28} + \tilde{J}^i_{V,29} - \tilde{J}^i_{V,30} - \tilde{J}^i_{V,33} - \tilde{J}^i_{V,34}\,, \tag{4.103}$$

$$0 = \tilde{J}^i_{V,19} - \tilde{J}^i_{V,20} - \tilde{J}^i_{V,21} + \tilde{J}^i_{V,22} + \tilde{J}^i_{V,33} - \tilde{J}^i_{V,34}\,, \tag{4.104}$$

$$0 = \tilde{J}^i_{V,19} - \tilde{J}^i_{V,20} + \tilde{J}^i_{V,21} - \tilde{J}^i_{V,22} + \tilde{J}^i_{V,35} + \tilde{J}^i_{V,36}\,, \tag{4.105}$$

$$0 = \tilde{J}^i_{V,27} - \tilde{J}^i_{V,28} - \tilde{J}^i_{V,29} + \tilde{J}^i_{V,30} + \tilde{J}^i_{V,35} - \tilde{J}^i_{V,36}\,. \tag{4.106}$$

The solution to this system requires $\tilde{J}^i_{V,29} = 0$. Also, the number of independent operators is reduced to six. Here, the minimal set is chosen such that the Lagrangian may be written as

$$\begin{aligned}
\mathcal{L}^{(v,\,a+1)}_{VNN} = V^i \Big[& C^{(v)}_{V,6} \nabla^i \left(N^\dagger N\right)\left(N^\dagger \tau^a N\right) + C^{(v)}_{V,7} \nabla^i \left(N^\dagger \tau^a N\right)\left(N^\dagger N\right) \\
& + C^{(v)}_{V,8} \nabla^i \left(N^\dagger \sigma^j N\right)\left(N^\dagger \sigma^j \tau^a N\right) + \frac{i}{2m_N} C^{(v)}_{V,9} \epsilon^{abc} \left(N^\dagger \overleftrightarrow{\nabla}^j \sigma^j \tau^b N\right)\left(N^\dagger \sigma^i \tau^c N\right) \\
& + C^{(v)}_{V,10} \epsilon^{ijk} \epsilon^{abc} \nabla^j \left(N^\dagger \tau^b N\right)\left(N^\dagger \sigma^k \tau^c N\right) \\
& + C^{(v)}_{V,11} \epsilon^{ijk} \epsilon^{abc} \nabla^j \left(N^\dagger \sigma^k \tau^b N\right)\left(N^\dagger \tau^c N\right) \Big]\,,
\end{aligned} \tag{4.107}$$

where the LECs are related to the overcomplete basis according to

$$\begin{aligned}
C^{(v)}_{V,6} = & -\frac{1}{m_N}\tilde{C}^{(v)}_{V,20} + \frac{1}{m_N}\tilde{C}^{(v)}_{V,22} + \tilde{C}^{(v)}_{V,23} - 3\tilde{C}^{(v)}_{V,26} - 2\tilde{C}^{(v)}_{V,27} - \tilde{C}^{(v)}_{V,28} + \tilde{C}^{(v)}_{V,30} \\
& + \frac{1}{m_N}\tilde{C}^{(v)}_{V,31} - \frac{2}{m_N}\tilde{C}^{(v)}_{V,33}\,,
\end{aligned} \tag{4.108}$$

$$\begin{aligned}
C^{(v)}_{V,7} = & \frac{1}{4m_N}\tilde{C}^{(v)}_{V,19} - \frac{5}{4m_N}\tilde{C}^{(v)}_{V,20} - \frac{1}{4m_N}\tilde{C}^{(v)}_{V,21} + \frac{5}{4m_N}\tilde{C}^{(v)}_{V,22} + \tilde{C}^{(v)}_{V,24} - 3\tilde{C}^{(v)}_{V,26} \\
& - 2\tilde{C}^{(v)}_{V,27} - \tilde{C}^{(v)}_{V,28} + \tilde{C}^{(v)}_{V,30} + \frac{1}{2m_N}\tilde{C}^{(v)}_{V,31} - \frac{3}{2m_N}\tilde{C}^{(v)}_{V,32} - \frac{3}{m_N}\tilde{C}^{(v)}_{V,33}\,,
\end{aligned} \tag{4.109}$$

$$\begin{aligned}
C^{(v)}_{V,8} = & \frac{1}{4m_N}\tilde{C}^{(v)}_{V,19} - \frac{3}{4m_N}\tilde{C}^{(v)}_{V,20} - \frac{1}{4m_N}\tilde{C}^{(v)}_{V,21} + \frac{3}{4m_N}\tilde{C}^{(v)}_{V,22} + \tilde{C}^{(v)}_{V,25} - \tilde{C}^{(v)}_{V,26} - \tilde{C}^{(v)}_{V,27} \\
& + \tilde{C}^{(v)}_{V,30} + \frac{1}{2m_N}\tilde{C}^{(v)}_{V,31} - \frac{1}{2m_N}\tilde{C}^{(v)}_{V,32} - \frac{2}{m_N}\tilde{C}^{(v)}_{V,33}\,,
\end{aligned} \tag{4.110}$$

$$\begin{aligned}
C^{(v)}_{V,9} = & \frac{1}{4m_N}\tilde{C}^{(v)}_{V,19} - \frac{1}{4m_N}\tilde{C}^{(v)}_{V,20} - \frac{1}{4m_N}\tilde{C}^{(v)}_{V,21} + \frac{1}{4m_N}\tilde{C}^{(v)}_{V,22} - \frac{1}{2m_N}\tilde{C}^{(v)}_{V,33} \\
& + \frac{1}{2m_N}\tilde{C}^{(v)}_{V,34}\,,
\end{aligned} \tag{4.111}$$

$$\begin{aligned}
C^{(v)}_{V,10} = & -\frac{1}{2m_N}\tilde{C}^{(v)}_{V,20} - \frac{1}{4m_N}\tilde{C}^{(v)}_{V,21} + \frac{3}{4m_N}\tilde{C}^{(v)}_{V,22} - \frac{3}{2}\tilde{C}^{(v)}_{V,27} + \frac{1}{2}\tilde{C}^{(v)}_{V,28} + \tilde{C}^{(v)}_{V,30} \\
& - \frac{3}{2m_N}\tilde{C}^{(v)}_{V,33} + \tilde{C}^{(v)}_{V,35}\,,
\end{aligned} \tag{4.112}$$

$$\begin{aligned}
C^{(v)}_{V,11} = & -\frac{1}{4m_N}\tilde{C}^{(v)}_{V,19} + \frac{3}{4m_N}\tilde{C}^{(v)}_{V,20} - \frac{1}{2m_N}\tilde{C}^{(v)}_{V,22} + \frac{3}{2}\tilde{C}^{(v)}_{V,27} - \frac{1}{2}\tilde{C}^{(v)}_{V,28} - \tilde{C}^{(v)}_{V,30} \\
& + \frac{3}{2m_N}\tilde{C}^{(v)}_{V,33} + \tilde{C}^{(v)}_{V,36}\,.
\end{aligned} \tag{4.113}$$

All six of these LECs are $O(1)$ in the large-N_c expansion. In terms of partial waves, the Lagrangian may be written as

$$\begin{aligned}
\mathcal{L}_{VNN}^{(v,\,a+1)} = V^i \Bigg\{ & \left[-C_{V,6}^{(v)} - C_{V,7}^{(v)} + 3C_{V,8}^{(v)}\right] i\epsilon^{abc}\nabla^i \left(N^T \bar{P}^b N\right)^\dagger \left(N^T \bar{P}^c N\right) \\
& + \left[-C_{V,6}^{(v)} + C_{V,7}^{(v)} - 3C_{V,8}^{(v)} - \frac{1}{m_N} C_{V,9}^{(v)}\right] \left(N^T \overleftrightarrow{\nabla}^i P^0 N\right)^\dagger \left(N^T \bar{P}^a N\right) \\
& + \left[-C_{V,6}^{(v)} + C_{V,7}^{(v)} + C_{V,8}^{(v)} - \frac{1}{m_N} C_{V,9}^{(v)}\right] \left(N^T \overleftrightarrow{\nabla}^i P^{ja} N\right)^\dagger \left(N^T \bar{P}^j N\right) \\
& + \left[\frac{1}{m_N} C_{V,9}^{(v)} + 2C_{V,10}^{(v)} - 2C_{V,11}^{(v)}\right] i\epsilon^{ijk}\nabla^j \left(N^T \bar{P}^a N\right) \left(N^T P^k N\right) \\
& + \left[\frac{1}{m_N} C_{V,9}^{(v)} - 2C_{V,10}^{(v)} + 2C_{V,11}^{(v)}\right] i\epsilon^{ijk}\nabla^j \left(N^T \bar{P}^k N\right) \left(N^T P^a N\right) \\
& + \left[-2C_{V,10}^{(v)} - 2C_{V,11}^{(v)}\right] \left(N^T \overleftrightarrow{\nabla}^j P^{ia} N\right)^\dagger \left(N^T P^j N\right) \\
& + \left[2C_{V,10}^{(v)} + 2C_{V,11}^{(v)}\right] \left(N^T \overleftrightarrow{\nabla}^j P^{ja} N\right)^\dagger \left(N^T P^i N\right) + \text{H.c.} \Bigg\} \, .
\end{aligned} \tag{4.114}$$

Thus, each partial wave current is $O(1)$ in the large-N_c expansion with the same caveat that the relative signs of the LECs might affect the overall sizes of each contribution. Regardless, the four terms with a derivative acting on a bilinear from the outside connect two S-wave states and will therefore be enhanced.

Lastly, consider the isotensor currents. The operators will be of the form J_V^{iab}, and they will be symmetric under the interchange of a and b. Additionally, we begin by considering the currents that transform under time-reversal as $J_V^{iab} \xrightarrow{T} (-1)^{a+b+1} J_v^{iab}$ so that the currents will be odd for $a = b = 3$. There are six possible operators:

$$\begin{aligned}
\mathcal{L}_{VNN}^{(t,\,a+b+1)} = V^i \Bigg\{ & \frac{1}{2} \Bigg[\frac{i}{2m_N} \tilde{C}_{V,1}^{(t)} \left(N^\dagger \overleftrightarrow{\nabla}^i \tau^a N\right) \left(N^\dagger \tau^b N\right) \\
& + \frac{i}{2m_N} \tilde{C}_{V,2}^{(t)} \left(N^\dagger \overleftrightarrow{\nabla}^i \sigma^j \tau^a N\right) \left(N^\dagger \sigma^j \tau^b N\right) \\
& + \frac{i}{2m_N} \tilde{C}_{V,3}^{(t)} \left(N^\dagger \overleftrightarrow{\nabla}^j \sigma^i \tau^a N\right) \left(N^\dagger \sigma^j \tau^b N\right) \\
& + \frac{i}{2m_N} \tilde{C}_{V,4}^{(t)} \left(N^\dagger \overleftrightarrow{\nabla}^j \sigma^j \tau^a N\right) \left(N^\dagger \sigma^i \tau^b N\right) \\
& + \tilde{C}_{V,5}^{(t)} \epsilon^{ijk}\nabla^j \left(N^\dagger \sigma^k \tau^a N\right) \left(N^\dagger \tau^b N\right) + \tilde{C}_{V,6}^{(t)} \epsilon^{ijk}\nabla^j \left(N^\dagger \tau^a N\right) \left(N^\dagger \sigma^k \tau^b N\right)
\end{aligned}$$

$$
\begin{aligned}
&+ a \leftrightarrow b\Big] \\
&- \frac{1}{3}\delta^{ab}\left[\frac{i}{2m_N}\tilde{C}^{(t)}_{V,1}\left(N^\dagger \overleftrightarrow{\nabla}^i \tau^c N\right)\left(N^\dagger \tau^c N\right)\right. \\
&+\frac{i}{2m_N}\tilde{C}^{(t)}_{V,2}\left(N^\dagger \overleftrightarrow{\nabla}^i \sigma^j \tau^c N\right)\left(N^\dagger \sigma^j \tau^c N\right) \\
&\frac{i}{2m_N}\tilde{C}^{(t)}_{V,3}\left(N^\dagger \overleftrightarrow{\nabla}^j \sigma^i \tau^c N\right)\left(N^\dagger \sigma^j \tau^c N\right) \\
&+\frac{i}{2m_N}\tilde{C}^{(t)}_{V,4}\left(N^\dagger \overleftrightarrow{\nabla}^j \sigma^j \tau^c N\right)\left(N^\dagger \sigma^i \tau^c N\right) \\
&\left.+\tilde{C}^{(t)}_{V,5}\epsilon^{ijk}\nabla^j\left(N^\dagger \sigma^k \tau^c N\right)\left(N^\dagger \tau^c N\right) + \tilde{C}^{(t)}_{V,6}\epsilon^{ijk}\nabla^j\left(N^\dagger \tau^c N\right)\left(N^\dagger \sigma^k \tau^c N\right)\right]\Big\}\ ,
\end{aligned}
\tag{4.115}
$$

where there are five LECs that are $O(1)$,

$$
\frac{1}{2m_N}\tilde{C}^{(t)}_{V,2},\ \frac{1}{2m_N}\tilde{C}^{(t)}_{V,3},\ \frac{1}{2m_N}\tilde{C}^{(t)}_{V,4},\ \tilde{C}^{(t)}_{V,5},\ \tilde{C}^{(t)}_{V,6} \sim O(1)\,, \tag{4.116}
$$

and one LEC that is $1/N_c^2$ suppressed

$$
\frac{1}{2m_N}\tilde{C}^{(t)}_{V,1} \sim O(1/N_c^2)\,. \tag{4.117}
$$

Fierz transformations lead to the system of equations

$$
0 = 2\tilde{J}^{iab}_{V,1} + \tilde{J}^{iab}_{V,2}\,, \tag{4.118}
$$

$$
0 = \tilde{J}^{iab}_{V,3} + \tilde{J}^{iab}_{V,4}\,, \tag{4.119}
$$

$$
0 = \tilde{J}^{iab}_{V,3} - \tilde{J}^{iab}_{V,4} + \tilde{J}^{iab}_{V,5} - \tilde{J}^{iab}_{V,6}\,, \tag{4.120}
$$

$$
0 = \tilde{J}^{iab}_{V,5} + \tilde{J}^{iab}_{V,6}\,, \tag{4.121}
$$

$$
0 = \tilde{J}^{iab}_{V,3} - \tilde{J}^{iab}_{V,4} - \tilde{J}^{iab}_{V,5} + \tilde{J}^{iab}_{V,6}\,. \tag{4.122}
$$

The first, second, and third equations yield

$$
\tilde{J}^{iab}_{V,1} = -\frac{1}{2}\tilde{J}^{iab}_{V,2}\,, \tag{4.123}
$$

$$
\tilde{J}^{iab}_{V,3} = -\tilde{J}^{iab}_{V,4}\,, \tag{4.124}
$$

$$
\tilde{J}^{iab}_{V,5} = -\tilde{J}^{iab}_{V,6}\,, \tag{4.125}
$$

so that the remaining two equations both become

$$0 = \tilde{J}^{iab}_{V,3} + \tilde{J}^{iab}_{V,5}\,. \tag{4.126}$$

Therefore, there are only two independent operators such that Lagrangian can be written as

$$\begin{aligned}
\mathcal{L}^{(t,\,a+b+1)}_{VNN} = V^i \Bigg\{ \frac{1}{2} \Bigg[& \frac{i}{2m_N} C^{(t)}_{V,1} \left(N^\dagger \overset{\leftrightarrow}{\nabla}{}^i \sigma^j \tau^a N \right) \left(N^\dagger \sigma^j \tau^b N \right) \\
& + C^{(t)}_{V,2} \epsilon^{ijk} \nabla^j \left(N^\dagger \sigma^k \tau^a N \right) \left(N^\dagger \tau^b N \right) + a \leftrightarrow b \Bigg] \\
& - \frac{1}{3} \delta^{ab} \Bigg[\frac{i}{2m_N} C^{(t)}_{V,1} \left(N^\dagger \overset{\leftrightarrow}{\nabla}{}^i \sigma^j \tau^c N \right) \left(N^\dagger \sigma^j \tau^c N \right) \\
& + C^{(t)}_{V,2} \epsilon^{ijk} \nabla^j \left(N^\dagger \sigma^k \tau^c N \right) \left(N^\dagger \tau^c N \right) \Bigg] \Bigg\}\,,
\end{aligned} \tag{4.127}$$

where both LECs are $O(1)$ and they are related to the original couplings according to

$$\frac{1}{2m_N} C^{(t)}_{V,1} = -\frac{1}{4m_N} \tilde{C}^{(t)}_{V,1} + \frac{1}{2m_N} \tilde{C}^{(t)}_{V,2}\,, \tag{4.128}$$

$$C^{(t)}_{V,2} = -\frac{1}{2m_N} \tilde{C}^{(t)}_{V,3} + \frac{1}{2m_N} \tilde{C}^{(t)}_{V,4} + \tilde{C}^{(t)}_{V,5} - \tilde{C}^{(t)}_{V,6}\,. \tag{4.129}$$

The Lagrangian in terms of partial waves may be written as

$$\begin{aligned}
\mathcal{L}^{(t,\,a+b+1)}_{VNN} = V^i \Bigg\{ \frac{1}{2} \Bigg[& -\frac{3i}{m_N} C^{(t)}_{V,1} \nabla^i \left(N^T \bar{P}^a N \right)^\dagger \left(N^T \bar{P}^b N \right) \\
& - 2 C^{(t)}_{V,2} \epsilon^{ijk} \left(N^T \overset{\leftrightarrow}{\nabla}{}^j P^{ka} N \right) \left(N^T \bar{P}^b N \right) + a \leftrightarrow b \Bigg] \\
& - \frac{1}{3} \delta^{ab} \Bigg[-\frac{3i}{m_N} C^{(t)}_{V,1} \nabla^i \left(N^T \bar{P}^c N \right)^\dagger \left(N^T \bar{P}^c N \right) \\
& - 2 C^{(t)}_{V,2} \epsilon^{ijk} \left(N^T \overset{\leftrightarrow}{\nabla}{}^j P^{kc} N \right) \left(N^T \bar{P}^c N \right) \Bigg] + \text{H.c} \Bigg\}\,.
\end{aligned} \tag{4.130}$$

Again, both terms are $O(1)$ in the large-N_c expansion, but the first term will be enhanced in the EFT$_{\not\pi}$ power counting.

The analogous currents with the opposite time-reversal transformation property lead to the overcomplete Lagrangian

$$\mathcal{L}^{(t,\,a+b)}_{VNN} = V^i \Bigg\{ \frac{1}{2} \Big[\tilde{C}^{(t)}_{V,7} \nabla^i \left(N^\dagger \tau^a N \right) \left(N^\dagger \tau^b N \right) + \tilde{C}^{(t)}_{V,8} \nabla^i \left(N^\dagger \sigma^j \tau^a N \right) \left(N^\dagger \sigma^j \tau^b N \right)$$

$$
\begin{aligned}
&+\tilde{C}^{(t)}_{V,9}\nabla^j\left(N^\dagger\sigma^i\tau^a N\right)\left(N^\dagger\sigma^j\tau^b N\right)+\tilde{C}^{(t)}_{V,10}\nabla^j\left(N^\dagger\sigma^j\tau^a N\right)\left(N^\dagger\sigma^i\tau^b N\right)\\
&+\frac{i}{2m_N}\tilde{C}^{(t)}_{V,11}\epsilon^{ijk}\left(N^\dagger\overleftrightarrow{\nabla}^j\sigma^k\tau^a N\right)\left(N^\dagger\tau^b N\right)\\
&+\frac{i}{2m_N}\tilde{C}^{(t)}_{V,12}\epsilon^{ijk}\left(N^\dagger\overleftrightarrow{\nabla}^j\tau^a N\right)\left(N^\dagger\sigma^k\tau^b N\right)+a\leftrightarrow b\Big]\\
&-\frac{1}{3}\delta^{ab}\Big[\tilde{C}^{(t)}_{V,7}\nabla^i\left(N^\dagger\tau^c N\right)\left(N^\dagger\tau^c N\right)+\tilde{C}^{(t)}_{V,8}\nabla^i\left(N^\dagger\sigma^j\tau^c N\right)\left(N^\dagger\sigma^j\tau^c N\right)\\
&+\tilde{C}^{(t)}_{V,9}\nabla^j\left(N^\dagger\sigma^i\tau^c N\right)\left(N^\dagger\sigma^j\tau^c N\right)+\tilde{C}^{(t)}_{V,10}\nabla^j\left(N^\dagger\sigma^j\tau^c N\right)\left(N^\dagger\sigma^i\tau^c N\right)\\
&+\frac{i}{2m_N}\tilde{C}^{(t)}_{V,11}\epsilon^{ijk}\left(N^\dagger\overleftrightarrow{\nabla}^j\sigma^k\tau^c N\right)\left(N^\dagger\tau^c N\right)\\
&+\frac{i}{2m_N}\tilde{C}^{(t)}_{V,12}\epsilon^{ijk}\left(N^\dagger\overleftrightarrow{\nabla}^j\tau^c N\right)\left(N^\dagger\sigma^k\tau^c N\right)\Big]\Big\}\,, \qquad (4.131)
\end{aligned}
$$

and the LECs scale as

$$\tilde{C}^{(t)}_{V,8},\ \tilde{C}^{(t)}_{V,9},\ \tilde{C}^{(t)}_{V,10}\sim O(N_c)\,, \qquad (4.132)$$

$$\tilde{C}^{(t)}_{V,7},\ \frac{1}{2m_N}\tilde{C}^{(t)}_{V,11},\ \frac{1}{2m_N}\tilde{C}^{(t)}_{V,12}\sim O(1/N_c)\,. \qquad (4.133)$$

Fierz transformations yield the system

$$0=3\tilde{J}^{i\,ab}_{V,7}+\tilde{J}^{i\,ab}_{V,8}\,, \qquad (4.134)$$

$$0=\tilde{J}^{i\,ab}_{V,7}-\tilde{J}^{i\,ab}_{V,8}+3\tilde{J}^{i\,ab}_{V,9}+\tilde{J}^{i\,ab}_{V,10}-\tilde{J}^{i\,ab}_{V,11}+\tilde{J}^{i\,ab}_{V,12}\,, \qquad (4.135)$$

$$0=\tilde{J}^{i\,ab}_{V,7}-\tilde{J}^{i\,ab}_{V,8}+\tilde{J}^{i\,ab}_{V,9}+3\tilde{J}^{i\,ab}_{V,10}+\tilde{J}^{i\,ab}_{V,11}-\tilde{J}^{i\,ab}_{V,12}\,, \qquad (4.136)$$

$$0=-\tilde{J}^{i\,ab}_{V,9}+\tilde{J}^{i\,ab}_{V,10}+3\tilde{J}^{i\,ab}_{V,11}+\tilde{J}^{i\,ab}_{V,12}\,, \qquad (4.137)$$

$$0=\tilde{J}^{i\,ab}_{V,9}-\tilde{J}^{i\,ab}_{V,10}+\tilde{J}^{i\,ab}_{V,11}+3\tilde{J}^{i\,ab}_{V,12}\,, \qquad (4.138)$$

which reduces the number of independent operators to two. Here, the choice is made to retain $\tilde{J}^{i\,ab}_{V,8}$ and $\tilde{J}^{i\,ab}_{V,10}$ such that the Lagrangian may be written as

$$
\begin{aligned}
\mathcal{L}^{(t,\,a+b)}_{VNN}=V^i\Big\{\frac{1}{2}&\Big[C^{(t)}_{V,3}\nabla^i\left(N^\dagger\sigma^j\tau^a N\right)\left(N^\dagger\sigma^j\tau^b N\right)+C^{(t)}_{V,4}\nabla^j\left(N^\dagger\sigma^j\tau^a N\right)\left(N^\dagger\sigma^i\tau^b N\right)\\
&+a\leftrightarrow b\Big]-\frac{1}{3}\delta^{ab}\Big[C^{(t)}_{V,3}\nabla^i\left(N^\dagger\sigma^j\tau^c N\right)\left(N^\dagger\sigma^j\tau^c N\right)\\
&+C^{(t)}_{V,4}\nabla^j\left(N^\dagger\sigma^j\tau^c N\right)\left(N^\dagger\sigma^i\tau^c N\right)\Big]\Big\}\,, \qquad (4.139)
\end{aligned}
$$

where the LECs in the minimal basis are related to those from the overcomplete basis according to

$$C_{V,3}^{(t)} = -\frac{1}{3}\tilde{C}_{V,7}^{(t)} + \tilde{C}_{V,8}^{(t)} + \frac{2}{3}\tilde{C}_{V,9}^{(t)} + \frac{1}{6m_N}\tilde{C}_{V,11}^{(t)} - \frac{1}{6m_N}\tilde{C}_{V,12}^{(t)}\,, \tag{4.140}$$

$$C_{V,4}^{(t)} = -\tilde{C}_{V,9}^{(t)} + \tilde{C}_{V,10}^{(t)} - \frac{1}{2m_N}\tilde{C}_{V,11}^{(t)} + \frac{1}{2m_N}\tilde{C}_{V,12}^{(t)}\,. \tag{4.141}$$

Also, both LECs in the minimal basis are $O(N_c)$. In the partial wave basis, the Lagrangian takes the form

$$\begin{aligned}\mathcal{L}_{VNN}^{(t,\,a+b)} = V^i \Bigg\{ \frac{1}{2} \bigg[& \left(6C_{V,3}^{(t)} + 2C_{V,4}^{(t)}\right) \nabla^i \left(N^T \bar{P}^a N\right)^\dagger \left(N^T \bar{P}^b N\right) \\ & -2iC_{V,4}^{(t)} \epsilon^{ijk} \left(N^T \overleftrightarrow{\nabla}^j P^{ka} N\right) \left(N^T \bar{P}^b N\right) + a \leftrightarrow b \bigg] \\ & - \frac{1}{3}\delta^{ab} \bigg[\left(6C_{V,3}^{(t)} + 2C_{V,4}^{(t)}\right) \nabla^i \left(N^T \bar{P}^c N\right)^\dagger \left(N^T \bar{P}^c N\right) \\ & -2iC_{V,4}^{(t)} \epsilon^{ijk} \left(N^T \overleftrightarrow{\nabla}^j P^{kc} N\right) \left(N^T \bar{P}^c N\right) \bigg] + \text{H.c.} \Bigg\}\,. \end{aligned} \tag{4.142}$$

While both LECs are $O(N_c)$, the term proportional only to $C_{V,4}^{(t)}$ connects an S-wave state to a P-wave state while the term proportional to the sum of the LECs connects two S-wave states and will therefore be enhanced.

4.2 Axial Vector Currents

Consider an external axial vector field A^i, which is even under parity. In order to derive parity conserving interactions, the operators must contain an even number of derivatives. Here, we only consider terms with no derivatives; therefore, the free vector index on the currents must come from Pauli spin matrices. Additionally, the only way to change the time-reversal of a current is to interchange the various spin and isospin structures. Again, terms that violate parity in the Lagrangian can be constructed by stitching together these currents with other external fields.

4.2.1 One-Nucleon Currents

At LO, there are two independent currents, an isoscalar and an isovector,

$$\mathcal{L}_{A^i N} = A^i \left[\not{T} g_s^{(A)} \left(N^\dagger \sigma^i N \right) + \not{T} g_v^{(A)} \left(N^\dagger \sigma^i \tau^a N \right) \right] \tag{4.143}$$

The LECs scale as

$$\not{T} g_s^{(A)} \sim O(1)\,, \tag{4.144}$$

$$\not{T} g_v^{(A)} \sim O(N_c)\,. \tag{4.145}$$

The scaling of $\not{T} g_v^{(A)}$ is a familiar result from large-N_c studies of the baryon axial current matrix elements [110–112]. For the single-nucleon currents, it is not possible to construct currents with different time-reversal properties without including derivatives.

4.2.2 Two-Nucleon Currents

First, consider the isoscalar currents. There are two possible operators,

$$\mathcal{L}_{A^i NN}^{(s,-)} = A^i \left[\tilde{C}_{A^i,1}^{(s)} \left(N^\dagger N \right) \left(N^\dagger \sigma^i N \right) + \tilde{C}_{A^i,2}^{(s)} \left(N^\dagger \tau^a N \right) \left(N^\dagger \tau^a \sigma^i N \right) \right] \tag{4.146}$$

The spin-isospin structure of these operators, which has already been analyzed in the previous section, implies the scalings

$$\tilde{C}_{A^i,1}^{(s)},\ \tilde{C}_{A^i,2}^{(s)} \quad \sim \quad O(1) \tag{4.147}$$

$$\tag{4.148}$$

However, these currents are not linearly independent as can be shown through Fierz transformations. Precisely, the terms are related to each other through

$$\tilde{J}_{A^i,2}^{i} \quad = \quad -3 \tilde{J}_{A^i,1}^{i}\,. \tag{4.149}$$

Therefore, the Lagrangian can be reduced to

$$\mathcal{L}_{A^i NN}^{(s,-)} \quad = \quad C_{A^i}^{(s)} A^i \left(N^\dagger N \right) \left(N^\dagger \sigma^i N \right)\,, \tag{4.150}$$

where

$$C^{(s)}_{A^i} = \tilde{C}^{(s)}_{A^i,1} - 3\tilde{C}^{(s)}_{A^i,2} \sim O(1)\,. \tag{4.151}$$

Again, it is not possible to construct currents that transform differently under time-reversal because the possible combinations of spin and isospin have been exhausted. Making a change to the partial wave basis the Lagrangian can be written as

$$\mathcal{L}^{(s,-)}_{A^i NN} = 2iC^{(s)}_{A^i}\epsilon^{ijk}A^i\left(N^T P^j N\right)^\dagger\left(N^T P^k N\right)\,. \tag{4.152}$$

This form of the Lagrangian will be used in the next chapter in order to draw comparisons to previous analyses of the deuteron magnetic moment and neutrino-deuteron scattering in EFT$_{\not\pi}$.

Now, consider the two-nucleon isovector axial currents. There are three possible currents that couple to the external field and transform under time-reversal as $J^{ia}_{A^i,NN} \xrightarrow{T} (-1)^a\, J^{ia}_{A^i,NN}$,

$$\begin{aligned}\mathcal{L}^{(v,a)}_{A^i NN} = A^i\Big[&\tilde{C}^{(v)}_{A^i,1}\epsilon^{ijk}\epsilon^{abc}\left(N^\dagger\sigma^j\tau^b N\right)\left(N^\dagger\sigma^k\tau^c N\right) + \tilde{C}^{(v)}_{A^i,2}\left(N^\dagger\sigma^i\tau^a N\right)\left(N^\dagger N\right)\\ &+\tilde{C}^{(v)}_{A^i,3}\left(N^\dagger\sigma^i N\right)\left(N^\dagger\tau^a N\right)\Big]\,.\end{aligned} \tag{4.153}$$

The large-N_c counting rules imply the scalings

$$\tilde{C}^{(v)}_{A^i,1},\ \tilde{C}^{(v)}_{A^i,2} \sim \mathcal{O}(N_c) \tag{4.154}$$

$$\tilde{C}^{(v)}_{A^i,3} \sim \mathcal{O}(N_c^{-1}) \tag{4.155}$$

However, there is only one linearly independent operator. Fierz transformations lead to the following system of equations:

$$0 = \tilde{J}^{ia}_{A^i,1} + 2\tilde{J}^{ia}_{A^i,2} + 6\tilde{J}^{ia}_{A^i,3}\,, \tag{4.156}$$

$$0 = \tilde{J}^{ia}_{A^i,1} - 6\tilde{J}^{ia}_{A^i,2} - 2\tilde{J}^{ia}_{A^i,3}\,, \tag{4.157}$$

Therefore, both $\tilde{J}^{ia}_{A^i,2}$ and $\tilde{J}^{ia}_{A^i,3}$ can be eliminated in favor of $\tilde{J}^{ia}_{A^i,1}$. Thus, the Lagrangian with the minimal set of operators is

$$\mathcal{L}^{(v,\,a)}_{A^iNN} = C^{(v)}_{A^i}\epsilon^{ijk}\epsilon^{abc}A^i\left(N^\dagger\sigma^j\tau^b N\right)\left(N^\dagger\sigma^k\tau^c N\right)\,, \tag{4.158}$$

where

$$C^{(v)}_{A^i,1} = \tilde{C}^{(v)}_{A^i,1} + \frac{1}{4}\left(\tilde{C}^{(v)}_{A^i,2} - \tilde{C}^{(v)}_{A^i,3}\right) \sim O(N_c)\,. \tag{4.159}$$

Again, a change to the partial wave basis yields

$$\mathcal{L}^{(v,\,a)}_{A^iNN} = 8\tilde{C}^{(v)}_{A^i,1}A^i\left[\left(N^T\bar{P}^a N\right)^\dagger\left(N^T P^i N\right) + \text{h.c.}\right]\,. \tag{4.160}$$

Assuming that these LECs are natural apart from their large-N_c scaling, an immediate consequence of these scalings is that the isoscalar coupling is suppressed relative to the isovector coupling, i.e.

$$\frac{C^{(s)}_{A^i}}{C^{(v)}_{A^i,1}} \sim O(1/N_c)\,. \tag{4.161}$$

There are two possible currents that transform under time-reversal as $J^{ia}_{A^i} \xrightarrow{T} (-1)^{a+1}\,J^{ia}_{A^i}$,

$$\mathcal{L}^{(v,\,a+1)}_{A^iNN} = A^i\left[\tilde{C}^{(v)}_{A^i,4}\epsilon^{ijk}\left(N^\dagger\sigma^j\tau^a N\right)\left(N^\dagger\sigma^k N\right) + \tilde{C}^{(v)}_{A^i,5}\epsilon^{abc}\left(N^\dagger\sigma^i\tau^a N\right)\left(N^\dagger\tau^b N\right)\right]\,, \tag{4.162}$$

where both LECs are $O(N_c)$. However, these operators are related through Fierz transformations such that the Lagrangian can be written as

$$\mathcal{L}^{(v,\,a+1)}_{A^iNN} = A^iC^{(v)}_{A^i,2}\epsilon^{ijk}\left(N^\dagger\sigma^j\tau^a N\right)\left(N^\dagger\sigma^k N\right)\,, \tag{4.163}$$

where the LECs are related according to

$$C^{(v)}_{A^i,2} = \tilde{C}^{(v)}_{A^i,4} - \tilde{C}^{(v)}_{A^i,5}\,. \tag{4.164}$$

Transforming to the partial wave basis yields

$$\mathcal{L}_{A^i,NN}^{(v,\,a+1)} = 4A^i C_{A^i,2}^{(v)} \left[i\left(N^T P^i N\right)^\dagger \left(N^T \bar{P}^a N\right) + \text{H.c}\right] . \tag{4.165}$$

Lastly, consider the isotensor current. There is a single axial vector isotensor operator such that the Lagrangian can be written

$$\begin{aligned}\mathcal{L}_{ANN}^{(t)} = A^i C_A^{(t)} \Big\{ & \frac{1}{2}\left[\left(N^\dagger \sigma^i \tau^a N\right)\left(N^\dagger \tau^b N\right) + \left(N^\dagger \sigma^i \tau^b N\right)\left(N^\dagger \tau^a N\right)\right] \\ & -\frac{1}{3}\delta^{ab}\left(N^\dagger \sigma^i \tau^c N\right)\left(N^\dagger \tau^c N\right)\Big\} ,\end{aligned} \tag{4.166}$$

where the last term is present in order to form a traceless tensor and to isolate the $I = 1$ representation. However, Fierz transformations reveal that this current actually vanishes.

4.3 Axial Charge

Here, we consider the axial charge, which is derived by coupling either one or two nucleons to the time component of an axial 4-vector, i.e. A^0. The axial charge and the pseudoscalar current can be distinguished by their properties under time-reversal, i.e., the axial charge is even under time-reversal while the pseudoscalar current is odd. Thus, only the currents that are even under time-reversal are considered in this section. For the isovector currents, this is taken to mean that the currents are even under time-reversal when the isospin index $a = 3$. Similarly, the isotensor currents will be even under time-reversal when both isospin indices $a = b = 3$.

4.3.1 One-Nucleon Charge

For a single nucleon, there is one possible isoscalar operator,

$$\mathcal{L}_{A^0 N}^{(s,\,+)} = \frac{i}{2m_N} \not{g}_{A^0}^{(s)} A^0 \left(N^\dagger \overleftrightarrow{\nabla}^i \sigma^i N\right) , \tag{4.167}$$

where

$$\frac{1}{2m_N} \not{g}_{A^0}^{(s)} \sim O(1/N_c) . \tag{4.168}$$

Analogously, there is one possible isovector operator,

$$\mathcal{L}_{A^0 N}^{(v,\,a+1)} = \frac{i}{2m_N} \not{g}_{A^0}^{(v)} A^0 \left(N^\dagger \overset{\leftrightarrow}{\nabla}{}^i \sigma^i \tau^a N \right) , \tag{4.169}$$

where

$$\frac{1}{2m_N} \not{g}_{A^0}^{(v)} \sim O(1) \, . \tag{4.170}$$

4.3.2 Two-Nucleon Charge

There are six possible isoscalar axial charge operators. The Lagrangian with an overcomplete set is

$$\begin{aligned}
\mathcal{L}_{A^0 NN}^{(s,\,+)} = A^0 \Bigg[& \frac{i}{2m_N} \tilde{C}_{A^0,\,1}^{(s)} \left(N^\dagger \overset{\leftrightarrow}{\nabla}{}^i \sigma^i N \right) \left(N^\dagger N \right) + \frac{i}{2m_N} \tilde{C}_{A^0,\,2}^{(s)} \left(N^\dagger \overset{\leftrightarrow}{\nabla}{}^i N \right) \left(N^\dagger \sigma^i N \right) \\
& + \tilde{C}_{A^0,\,3}^{(s)} \epsilon^{ijk} \nabla^i \left(N^\dagger \sigma^j N \right) \left(N^\dagger \sigma^k N \right) + \frac{i}{2m_N} \tilde{C}_{A^0,\,4}^{(s)} \left(N^\dagger \overset{\leftrightarrow}{\nabla}{}^i \sigma^i \tau^a N \right) \left(N^\dagger \tau^a N \right) \\
& + \frac{i}{2m_N} \tilde{C}_{A^0,\,5}^{(s)} \left(N^\dagger \overset{\leftrightarrow}{\nabla}{}^i \tau^a N \right) \left(N^\dagger \sigma^i \tau^a N \right) \\
& + \tilde{C}_{A^0,\,6}^{(s)} \epsilon^{ijk} \nabla^i \left(N^\dagger \sigma^j \tau^a N \right) \left(N^\dagger \sigma^k \tau^a N \right) \Bigg] ,
\end{aligned} \tag{4.171}$$

where there is one LEC at leading order,

$$\tilde{C}_{A^0,\,6}^{(s)} \sim O(N_c) \, , \tag{4.172}$$

and the remaining LECs are $1/N_c^2$ suppressed,

$$\frac{1}{m_N} \tilde{C}_{A^0,\,1}^{(s)}, \; \frac{1}{m_N} \tilde{C}_{A^0,\,2}^{(s)}, \; \tilde{C}_{A^0,\,3}^{(s)}, \; \frac{1}{m_N} \tilde{C}_{A^0,\,4}^{(s)}, \; \frac{1}{m_N} \tilde{C}_{A^0,\,5}^{(s)} \sim O(1/N_c) \, . \tag{4.173}$$

Fierz relations yield the system of equations

$$0 = 5\tilde{J}_{A^0,\,1} + \tilde{J}_{A^0,\,2} - \tilde{J}_{A^0,\,3} + \tilde{J}_{A^0,\,4} + \tilde{J}_{A^0,\,5} - \tilde{J}_{A^0,\,6} \, , \tag{4.174}$$

$$0 = \tilde{J}_{A^0,\,1} + 5\tilde{J}_{A^0,\,2} + \tilde{J}_{A^0,\,3} + \tilde{J}_{A^0,\,4} + \tilde{J}_{A^0,\,5} + \tilde{J}_{A^0,\,6} \, , \tag{4.175}$$

$$0 = 3\tilde{J}_{A^0,\,1} + 3\tilde{J}_{A^0,\,2} - 3\tilde{J}_{A^0,\,3} + 3\tilde{J}_{A^0,\,4} - \tilde{J}_{A^0,\,5} + \tilde{J}_{A^0,\,6} \, , \tag{4.176}$$

$$0 = 3\tilde{J}_{A^0,1} + 3\tilde{J}_{A^0,2} + 3\tilde{J}_{A^0,3} - \tilde{J}_{A^0,4} + 3\tilde{J}_{A^0,5} - \tilde{J}_{A^0,6}\,, \tag{4.177}$$

$$0 = \tilde{J}_{A^0,1} - \tilde{J}_{A^0,2} - 2\tilde{J}_{A^0,3} + \tilde{J}_{A^0,4} - \tilde{J}_{A^0,5}\,, \tag{4.178}$$

$$0 = 3\tilde{J}_{A^0,1} - 3\tilde{J}_{A^0,2} - \tilde{J}_{A^0,4} + \tilde{J}_{A^0,5} - 2\tilde{J}_{A^0,6}\,. \tag{4.179}$$

The solution to this system implies that there are three independent operators. Here, the choice is made to retain $\tilde{J}_{A^0,4}$, $\tilde{J}_{A^0,5}$, and $\tilde{J}_{A^0,6}$ so that the the Lagrangian can be written as

$$\begin{aligned}\mathcal{L}^{(s,+)}_{A^0NN} = A^0 \Big[& C^{(s)}_{A^0,1}\epsilon^{ijk}\nabla^i \left(N^\dagger\sigma^j\tau^a N\right)\left(N^\dagger\sigma^k\tau^a N\right) \\ & + \frac{i}{2m_N}C^{(s)}_{A^0,2}\left(N^\dagger \overleftrightarrow{\nabla}^i \sigma^i\tau^a N\right)\left(N^\dagger\tau^a N\right) \\ & + \frac{i}{2m_N}C^{(s)}_{A^0,3}\left(N^\dagger \overleftrightarrow{\nabla}^i \tau^a N\right)\left(N^\dagger\sigma^i\tau^a N\right)\Big]\,,\end{aligned} \tag{4.180}$$

where the new LECs are related to the original couplings according to

$$C^{(s)}_{A^0,1} = \frac{1}{6m_N}\tilde{C}^{(s)}_{A^0,1} - \frac{1}{6m_N}\tilde{C}^{(s)}_{A^0,2} + \tilde{C}^{(s)}_{A^0,3} + \tilde{C}^{(s)}_{A^0,6}\,, \tag{4.181}$$

$$\frac{1}{2m_N}C^{(s)}_{A^0,2} = -\frac{1}{6m_N}\tilde{C}^{(s)}_{A^0,2} + \frac{2}{3}\tilde{C}^{(s)}_{A^0,3} + \frac{1}{2m_N}\tilde{C}^{(s)}_{A^0,4}\,, \tag{4.182}$$

$$\frac{1}{2m_N}C^{(s)}_{A^0,3} = -\frac{1}{6m_N}\tilde{C}^{(s)}_{A^0,2} - \frac{2}{3}\tilde{C}^{(s)}_{A^0,3} + \frac{1}{2m_N}\tilde{C}^{(s)}_{A^0,5}\,, \tag{4.183}$$

and the LECs scale as

$$C^{(s)}_{A^0,1} \sim O(N_c)\,, \tag{4.184}$$

$$\frac{1}{2m_N}C^{(s)}_{A^0,2},\ \frac{1}{2m_N}C^{(s)}_{A^0,3} \sim O(1/N_c)\,. \tag{4.185}$$

The partial wave basis Lagrangian is

$$\begin{aligned}\mathcal{L}^{(s,+)}_{A^0NN} = A^0 \Bigg\{ & \left[-C^{(s)}_{A^0,1} + \frac{3}{2m_N}C^{(s)}_{A^0,2} - \frac{3}{2m_N}C^{(s)}_{A^0,3}\right] i\left(N^T \overleftrightarrow{\nabla}^i P^0 N\right)^\dagger \left(N^T P^i N\right) \\ & + \left[-C^{(s)}_{A^0,1} - \frac{1}{2m_N}C^{(s)}_{A^0,2} + \frac{1}{2m_N}C^{(s)}_{A^0,3}\right] i\left(N^T \overleftrightarrow{\nabla}^i P^{ia} N\right)^\dagger \left(N^T \bar{P}^a N\right) \\ & + \left[\frac{3}{2m_N}C^{(s)}_{A^0,2} + \frac{3}{2m_N}C^{(s)}_{A^0,3}\right]\epsilon^{ijk}\nabla^i\left(N^T P^j N\right)^\dagger\left(N^T P^k N\right) + \text{H.c.}\Bigg\}\,. \end{aligned} \tag{4.186}$$

Only the first and second terms contain $O(N_c)$ contributions. However, only the third term receives the power counting enhancement.

There are nine possible isovector charge operators that one can write down. The Lagrangian takes the form

$$\begin{aligned}
\mathcal{L}^{(v,a+1)}_{A^0NN} = A^0 \Bigg[& \frac{i}{2m_N}\tilde{C}^{(v)}_{A^0,1}\left(N^\dagger \overleftrightarrow{\nabla}^i \sigma^i\tau^a N\right)\left(N^\dagger N\right) \\
& + \frac{i}{2m_N}\tilde{C}^{(v)}_{A^0,2}\left(N^\dagger \overleftrightarrow{\nabla}^i \sigma^i N\right)\left(N^\dagger \tau^a N\right) \\
& + \frac{i}{2m_N}\tilde{C}^{(v)}_{A^0,3}\left(N^\dagger \overleftrightarrow{\nabla}^i \tau^a N\right)\left(N^\dagger \sigma^i N\right) + \frac{i}{2m_N}\tilde{C}^{(v)}_{A^0,4}\left(N^\dagger \overleftrightarrow{\nabla}^i N\right)\left(N^\dagger \sigma^i\tau^a N\right) \\
& + \tilde{C}^{(v)}_{A^0,5}\epsilon^{ijk}\nabla^i\left(N^\dagger \sigma^j\tau^a N\right)\left(N^\dagger \sigma^k N\right) + \tilde{C}^{(v)}_{A^0,6}\epsilon^{ijk}\nabla^i\left(N^\dagger \sigma^j N\right)\left(N^\dagger \sigma^k\tau^a N\right) \\
& + \frac{i}{2m_N}\tilde{C}^{(v)}_{A^0,7}\epsilon^{ijk}\epsilon^{abc}\left(N^\dagger \overleftrightarrow{\nabla}^i \sigma^j\tau^b N\right)\left(N^\dagger \sigma^k\tau^c N\right) \\
& + \tilde{C}^{(v)}_{A^0,8}\epsilon^{abc}\nabla^i\left(N^\dagger \sigma^i\tau^b N\right)\left(N^\dagger \tau^c N\right) + \tilde{C}^{(v)}_{A^0,9}\epsilon^{abc}\nabla^i\left(N^\dagger \tau^b N\right)\left(N^\dagger \sigma^i\tau^c N\right)\Bigg] \,,
\end{aligned} \tag{4.187}$$

where there are seven LECs that scale as

$$\begin{matrix} \frac{1}{2m_N}\tilde{C}^{(v)}_{A^0,1},\ \frac{1}{2m_N}\tilde{C}^{(v)}_{A^0,4},\ \tilde{C}^{(v)}_{A^0,5}, \\ \tilde{C}^{(v)}_{A^0,6},\ \frac{1}{2m_N}\tilde{C}^{(v)}_{A^0,7},\ \tilde{C}^{(v)}_{A^0,8},\ \tilde{C}^{(v)}_{A^0,9} \end{matrix} \sim O(1)\,, \tag{4.188}$$

and the remaining LECs scale as

$$\frac{1}{2m_N}\tilde{C}^{(v)}_{A^0,2},\ \frac{1}{2m_N}\tilde{C}^{(v)}_{A^0,4} \sim O(1/N_c^2)\,. \tag{4.189}$$

Again, Fierz transformations lead to the system

$$0 = 3\tilde{J}^a_{A^0,1} + 3\tilde{J}^a_{A^0,2} + \tilde{J}^a_{A^0,3} + \tilde{J}^a_{A^0,4} - \tilde{J}^a_{A^0,5} - \tilde{J}^a_{A^0,6}\,, \tag{4.190}$$

$$0 = 2\tilde{J}^a_{A^0,1} - 2\tilde{J}^a_{A^0,2} - \tilde{J}^a_{A^0,7} - \tilde{J}^a_{A^0,8} - \tilde{J}^a_{A^0,9}\,, \tag{4.191}$$

$$0 = \tilde{J}^a_{A^0,1} + \tilde{J}^a_{A^0,2} + 3\tilde{J}^a_{A^0,3} + 3\tilde{J}^a_{A^0,4} + \tilde{J}^a_{A^0,5} + \tilde{J}^a_{A^0,6}\,, \tag{4.192}$$

$$0 = 2\tilde{J}^a_{A^0,3} - 2\tilde{J}^a_{A^0,4} + \tilde{J}^a_{A^0,7} - \tilde{J}^a_{A^0,8} - \tilde{J}^a_{A^0,9}\,, \tag{4.193}$$

$$0 = \tilde{J}^a_{A^0,1} + \tilde{J}^a_{A^0,2} - \tilde{J}^a_{A^0,3} - \tilde{J}^a_{A^0,4} - \tilde{J}^a_{A^0,5} - \tilde{J}^a_{A^0,6}\,, \tag{4.194}$$

$$0 = \tilde{J}^a_{A^0,5} - \tilde{J}^a_{A^0,6} + \tilde{J}^a_{A^0,8} - \tilde{J}^a_{A^0,9}\,, \tag{4.195}$$

$$0 = \tilde{J}^a_{A^0,1} - \tilde{J}^a_{A^0,2} - \tilde{J}^a_{A^0,3} + \tilde{J}^a_{A^0,4} - \tilde{J}^a_{A^0,7}\,, \tag{4.196}$$

$$0 = \tilde{J}^a_{A^0,1} - \tilde{J}^a_{A^0,2} + \tilde{J}^a_{A^0,3} - \tilde{J}^a_{A^0,4} - \tilde{J}^a_{A^0,8} - \tilde{J}^a_{A^0,9}\,, \tag{4.197}$$

$$0 = \tilde{J}^a_{A^0,5} - \tilde{J}^a_{A^0,6} + \tilde{J}^a_{A^0,8} - \tilde{J}^a_{A^0,9}\,. \tag{4.198}$$

This system leads to four independent operators such that the Lagrangian can be written as

$$\begin{aligned}\mathcal{L}^{(v,\,a+1)}_{A^0NN} = A^0 \Big[& C^{(v)}_{A^0,1}\epsilon^{ijk}\nabla^i\left(N^\dagger\sigma^j N\right)\left(N^\dagger\sigma^k\tau^a N\right) \\ & + \frac{i}{2m_N}C^{(v)}_{A^0,2}\epsilon^{ijk}\epsilon^{abc}\left(N^\dagger\overleftrightarrow{\nabla}^i\sigma^j\tau^b N\right)\left(N^\dagger\sigma^k\tau^c N\right) \\ & + C^{(v)}_{A^0,3}\epsilon^{abc}\nabla^i\left(N^\dagger\sigma^i\tau^b N\right)\left(N^\dagger\tau^c N\right) + C^{(v)}_{A^0,4}\epsilon^{abc}\nabla^i\left(N^\dagger\tau^b N\right)\left(N^\dagger\sigma^i\tau^c N\right)\Big]\,,\end{aligned} \tag{4.199}$$

where these LECs are related to those from the overcomplete Lagrangian according to

$$\begin{aligned} C^{(v)}_{A^0,1} &= \frac{1}{4m_N}\tilde{C}^{(v)}_{A^0,1} + \frac{1}{4m_N}\tilde{C}^{(v)}_{A^0,2} - \frac{1}{4m_N}\tilde{C}^{(v)}_{A^0,3} \\ &\quad - \frac{1}{4m_N}\tilde{C}^{(v)}_{A^0,4} + \tilde{C}^{(v)}_{A^0,5} + \tilde{C}^{(v)}_{A^0,6}\,, \end{aligned} \tag{4.200}$$

$$\begin{aligned} \frac{1}{2m_N}C^{(v)}_{A^0,2} &= \frac{1}{8m_N}\tilde{C}^{(v)}_{A^0,1} - \frac{1}{8m_N}\tilde{C}^{(v)}_{A^0,2} - \frac{1}{8m_N}\tilde{C}^{(v)}_{A^0,3} \\ &\quad + \frac{1}{8m_N}\tilde{C}^{(v)}_{A^0,4} + \tilde{C}^{(v)}_{A^0,7}\,, \end{aligned} \tag{4.201}$$

$$C^{(v)}_{A^0,3} = -\tilde{C}^{(v)}_{A^0,5} + \tilde{C}^{(v)}_{A^0,8}\,, \tag{4.202}$$

$$\begin{aligned} C^{(v)}_{A^0,1} &= \frac{1}{4m_N}\tilde{C}^{(v)}_{A^0,1} - \frac{1}{4m_N}\tilde{C}^{(v)}_{A^0,2} + \frac{1}{4m_N}\tilde{C}^{(v)}_{A^0,3} \\ &\quad - \frac{1}{4m_N}\tilde{C}^{(v)}_{A^0,4} + \tilde{C}^{(v)}_{A^0,5} + \tilde{C}^{(v)}_{A^0,9}\,, \end{aligned} \tag{4.203}$$

and all of the LECs are $O(1)$. This choice of basis is nearly identical to that of Ref. [134] with the exception of the term proportional to $C^{(v)}_{A^0,2}$. In Ref. [134] (see Appendix C in that work), a combination of $\tilde{J}^a_{A^0,1}$, $\tilde{J}^a_{A^0,2}$, $\tilde{J}^a_{A^0,3}$, and $\tilde{J}^a_{A^0,4}$ is retained. In that case, the large-N_c scaling is set by $\tilde{J}^a_{A^0,1}$ and $\tilde{J}^a_{A^0,4}$ while $\tilde{J}^a_{A^0,2}$ and $\tilde{J}^a_{A^0,3}$

provide subleading corrections. A similar comparison holds for the axial charge in Refs. [129, 130]. Transforming to the partial wave basis yields

$$\begin{aligned}\mathcal{L}_{A^0NN}^{(v,\,a+1)} = A^0 \Big\{ &\Big[C_{A^0,\,1}^{(v)} - \frac{2}{m_N} C_{A^0,\,2}^{(v)} - 2C_{A^0,\,3}^{(v)} + 2C_{A^0,\,4}^{(v)}\Big] i\nabla^i \left(N^T P^i N\right)^\dagger \left(N^T \bar{P}^a N\right) \\ &+ \Big[-C_{A^0,\,1}^{(v)} - \frac{2}{m_N} C_{A^0,\,2}^{(v)} + 2C_{A^0,\,3}^{(v)} - 2C_{A^0,\,4}^{(v)}\Big] i\nabla^i \left(N^T \bar{P}^a N\right)^\dagger \left(N^T P^i N\right) \\ &- C_{A^0,\,1}^{(v)} \epsilon^{abc} \left(N^T \overleftrightarrow{\nabla}^i P^{ib} N\right)^\dagger \left(N^T \bar{P}^c N\right) \\ &+ \Big[-2C_{A^0,\,3}^{(v)} - 2C_{A^0,\,4}^{(v)}\Big] \epsilon^{ijk} \left(N^T \overleftrightarrow{\nabla}^i P^{ja} N\right)^\dagger \left(N^T P^k N\right) + \text{H.c.} \Big\} \, . \end{aligned} \tag{4.204}$$

Each term is still $O(1)$ with the possibility still that suppressions or enhacements can arise depending on the relative signs of the LECs. Additionally, only the first two terms receive the EFT$_{\not\pi}$ power counting enhancement because they connect two S-wave states.

Recall that the isotensor axial charge is defined here such that it will be odd under time-reversal when the isospin indices $a = b = 3$. There are three possible operators such that the overcomplete Lagrangian is

$$\begin{aligned}\mathcal{L}_{A^0NN}^{(t)} = A^0 \Big\{ &\frac{1}{2} \Big[\tilde{C}_{A^0,\,1}^{(t)} \epsilon^{ijk} \nabla^i \left(N^\dagger \sigma^j \tau^a N\right) \left(N^\dagger \sigma^k \tau^b N\right) \\ &+ \frac{i}{2m_N} \tilde{C}_{A^0,\,2}^{(t)} \left(N^\dagger \overleftrightarrow{\nabla}^i \sigma^i \tau^a N\right) \left(N^\dagger \tau^b N\right) + \frac{i}{2m_N} \tilde{C}_{A^0,\,3}^{(t)} \left(N^\dagger \overleftrightarrow{\nabla}^i \tau^a N\right) \left(N^\dagger \sigma^i \tau^b N\right) \\ &+ a \leftrightarrow b\Big] - \frac{1}{3} \delta^{ab} \Big[\tilde{C}_{A^0,\,1}^{(t)} \epsilon^{ijk} \nabla^i \left(N^\dagger \sigma^j \tau^c N\right) \left(N^\dagger \sigma^k \tau^c N\right) \\ &+ \frac{i}{2m_N} \tilde{C}_{A^0,\,2}^{(t)} \left(N^\dagger \overleftrightarrow{\nabla}^i \sigma^i \tau^c N\right) \left(N^\dagger \tau^c N\right) \\ &+ \frac{i}{2m_N} \tilde{C}_{A^0,\,3}^{(t)} \left(N^\dagger \overleftrightarrow{\nabla}^i \tau^c N\right) \left(N^\dagger \sigma^i \tau^c N\right) \Big]\Big\} \, , \end{aligned} \tag{4.205}$$

where the LECs have the large-N_c scalings

$$\tilde{C}_{A^0,\,1}^{(t)} \sim O(N_c) \, , \tag{4.206}$$

$$\frac{1}{2m_N} \tilde{C}_{A^0,\,2}^{(t)}, \; \frac{1}{2m_N} \tilde{C}_{A^0,\,3}^{(t)} \sim O(1/N_c) \, . \tag{4.207}$$

Fierz transformations yield the system

$$0 = \tilde{J}_{A^0,1}^{(t)} - \tilde{J}_{A^0,2}^{(t)} + \tilde{J}_{A^0,3}^{(t)} \, , \tag{4.208}$$

$$0 = \tilde{J}^{(t)}_{A^0,1} - 3\tilde{J}^{(t)}_{A^0,2} - \tilde{J}^{(t)}_{A^0,3}\,, \tag{4.209}$$

$$0 = \tilde{J}^{(t)}_{A^0,1} + \tilde{J}^{(t)}_{A^0,2} + 3\tilde{J}^{(t)}_{A^0,3}\,, \tag{4.210}$$

which reduces the number of independent operators to one. The Lagrangian may then be written as

$$\begin{aligned}\mathcal{L}^{(t)}_{A^0NN} = A^0 C^{(t)}_{A^0}\Big\{&\frac{1}{2}\epsilon^{ijk}\left[\nabla^i\left(N^\dagger\sigma^j\tau^a N\right)\left(N^\dagger\sigma^k\tau^b N\right) + a\leftrightarrow b\right]\\ &-\frac{1}{3}\delta^{ab}\epsilon^{ijk}\nabla^i\left(N^\dagger\sigma^j\tau^c N\right)\left(N^\dagger\sigma^k\tau^c N\right)\Big\}\,,\end{aligned} \tag{4.211}$$

where $C^{(t)}_{A^0} \sim O(N_c)$ and is related to the original LECs according to

$$C^{(t)}_{A^0} = \tilde{C}^{(t)}_{A^0,1} + \frac{1}{4m_N}\tilde{C}^{(t)}_{A^0,\,2} - \frac{1}{4m_N}\tilde{C}^{(t)}_{A^0,\,3}\,. \tag{4.212}$$

This Lagrangian in the partial wave basis is

$$\begin{aligned}\mathcal{L}^{(t)}_{A^0NN} = 4C^{(t)}_{A^0}A^0\Bigg\{&\frac{1}{2}\left[i\left(N^T\overset{\leftrightarrow}{\nabla}{}^i P^{ia}N\right)^\dagger\left(N^T\bar{P}^b N\right) + a\leftrightarrow b\right]\\ &-\frac{i}{3}\delta^{ab}\left(N^T\overset{\leftrightarrow}{\nabla}{}^i P^{ic}N\right)^\dagger\left(N^T\bar{P}^c N\right) + \text{H.c}\Bigg\}\,.\end{aligned} \tag{4.213}$$

The current here only connects an S-wave state to a P-wave state; therefore, it will not be enhanced in the EFT$_{\not\pi}$ power counting.

4.4 Scalar Currents

Finally, consider an external scalar field S. These currents are even under parity, thus they must have an even number of derivatives. Again, we only consider zero derivative currents. In addition, the scalar currents have the same structure as the vector charge at leading order.

4.4.1 One-Nucleon Currents

There is a single one-nucleon isoscalar operator that leads to the Lagrangian

$$\mathcal{L}^{(s,+)}_{SN} = {}^{\not\pi}g^{(s)}_S S\left(N^\dagger N\right), \tag{4.214}$$

where the scaling of the LEC is

$$\not{g}_S \sim O(N_c). \tag{4.215}$$

Analogously, there is a single isovector operator

$$\mathcal{L}_{SN}^{(v,\,a+1)} = \not{g}_S^{(v)} S \left(N^\dagger \tau^a N\right) , \tag{4.216}$$

where

$$\not{g}_S^{(v)} \sim O(1) . \tag{4.217}$$

Direct comparison of Eqs. (4.215) and (4.217) indicate that the isovector coupling is $1/N_c$ suppressed relative to the isoscalar coupling.

4.4.2 Two-Nucleon Currents

There are four possible isoscalar two-nucleon currents. Specifically, they have the exact same structure as the two-nucleon scattering operators

$$\begin{aligned}\mathcal{L}_{SNN}^{(s,+)} = S \Big[& \tilde{C}_{S,1}^{(s)} \left(N^\dagger N\right)\left(N^\dagger N\right) + \tilde{C}_{S,2}^{(s)} \left(N^\dagger \sigma^i N\right)\left(N^\dagger \sigma^i N\right) \\ & + \tilde{C}_{S,3}^{(s)} \left(N^\dagger \tau^a N\right)\left(N^\dagger \tau^a N\right) + \tilde{C}_{S,4}^{(s)} \left(N^\dagger \sigma^i \tau^a N\right)\left(N^\dagger \sigma^i \tau^a N\right)\Big] .\end{aligned} \tag{4.218}$$

The LECs scale as

$$\tilde{C}_{S,1}^{(s)},\ \tilde{C}_{S,4}^{(s)} \sim O\left(N_c\right) \tag{4.219}$$

$$\tilde{C}_{S,2}^{(s)},\ \tilde{C}_{S,3}^{(s)} \sim O\left(N_c^{-1}\right) . \tag{4.220}$$

As usual, two of these terms can be eliminated through Fierz transformations. In particular, the currents are related through

$$\tilde{J}_{S,4} = -3\tilde{J}_{S,1}^{(s)} , \tag{4.221}$$

$$\tilde{J}_{S,3} = -2\tilde{J}_{S,1}^{(s)} - \tilde{J}_{S,2}^{(s)} . \tag{4.222}$$

Therefore, the Lagrangian can be written as

$$\mathcal{L}_{SNN}^{(s,+)} = S \left[C_{S,1}^{(s)} \left(N^\dagger N\right)\left(N^\dagger N\right) + C_{S,2}^{(s)} \left(N^\dagger \sigma^i N\right)\left(N^\dagger \sigma^i N\right)\right] , \tag{4.223}$$

where

$$C^{(s)}_{S,1} = \tilde{C}^{(s)}_{S,1} - 2\tilde{C}^{(s)}_{S,3} - 3\tilde{C}^{(s)}_{S,4} \sim O(N_c)\,, \tag{4.224}$$

$$C^{(s)}_{S,2} = \tilde{C}^{(s)}_{S,2} - \tilde{C}^{(s)}_{S,3} \sim O(1/N_c)\,. \tag{4.225}$$

In the partial wave basis, the Lagrangian becomes

$$\begin{aligned}\mathcal{L}^{(s,+)}_{SNN} = S\Big[&2\left(C^{(s)}_{S,1} + C^{(s)}_{S,2}\right)\left(N^T P^i N\right)^\dagger\left(N^T P^i N\right)\\ &+2\left(C^{(s)}_{S,1} - 3C^{(s)}_{S,2}\right)\left(N^T \bar{P}^a N\right)^\dagger\left(N^T \bar{P}^a N\right)\Big]\,.\end{aligned} \tag{4.226}$$

There are two possible isovector currents that transform under time-reversal $\tilde{J}^a_S \xrightarrow{T} (-1)^{a+1}\,\tilde{J}^a_S$ with no sum over the isospin index. At this order, there are no currents that transform otherwise. The corresponding overcomplete Lagrangian is

$$\mathcal{L}^{(v,\,a+1)}_{SNN} = S\left[\tilde{C}^{(v)}_{S,1}\left(N^\dagger\tau^a N\right)\left(N^\dagger N\right) + \tilde{C}^{(v)}_{S,2}\left(N^\dagger\sigma^i\tau^a N\right)\left(N^\dagger\sigma^i N\right)\right]\,, \tag{4.227}$$

and the LECs scale as

$$\tilde{C}^{(v)}_{S,1},\ \tilde{C}^{(v)}_{S,2} \sim O(1)\,. \tag{4.228}$$

Fierz transformations lead to the relation

$$\tilde{J}^a_{S,2} = -5\tilde{J}^a_{S,1}\,. \tag{4.229}$$

Thus, the Lagrangian becomes

$$\mathcal{L}^{(v,\,a+1)}_{SNN} = C^{(v)}_{S,1} S\left(N^\dagger\tau^a N\right)\left(N^\dagger N\right)\,, \tag{4.230}$$

where

$$C^{(v)}_{S,1} = \tilde{C}^{(v)}_{S,1} - 5\tilde{C}^{(v)}_{S,2} \sim O(1)\,. \tag{4.231}$$

Transforming to the partial wave basis results in

$$\mathcal{L}^{(v,\,a+1)}_{SNN} = -2iC^{(v)}_{S,1}\epsilon^{abc} S\left(N^T \bar{P}^b N\right)^\dagger\left(N^T \bar{P}^c N\right)\,. \tag{4.232}$$

Finally, there are two possible symmetric, traceless isotensors that can be written down. The Lagrangian is

$$\mathcal{L}_{SNN}^{(t,\,a+b)} = S\left\{\tilde{C}_{S,\,1}^{(t)}\left[\left(N^\dagger\sigma^i\tau^a N\right)\left(N^\dagger\sigma^i\tau^b N\right) - \frac{1}{3}\left(N^\dagger\sigma^i\tau^c N\right)\left(N^\dagger\sigma^i\tau^c N\right)\right]\right.$$
$$\left. +\tilde{C}_{S,\,1}^{(t)}\left[\left(N^\dagger\tau^a N\right)\left(N^\dagger\tau^b N\right) - \frac{1}{3}\left(N^\dagger\tau^c N\right)\left(N^\dagger\tau^c N\right)\right]\right\}, \tag{4.233}$$

where

$$\tilde{C}_{S,\,1}^{(t)} \sim O(N_c)\,, \tag{4.234}$$

$$\tilde{C}_{S,\,2}^{(t)} \sim O(1/N_c)\,. \tag{4.235}$$

However, these two operators are related through a Fierz transformation such that

$$\tilde{J}_{S,\,2}^{ab} = -\frac{1}{3}\tilde{J}_{S,\,1}^{ab}\,. \tag{4.236}$$

Therefore, the Lagrangian is

$$\mathcal{L}_{SNN}^{(t,\,a+b)} = SC_S^{(t)}\left[\left(N^\dagger\sigma^i\tau^a N\right)\left(N^\dagger\sigma^i\tau^b N\right) - \frac{1}{3}\left(N^\dagger\sigma^i\tau^c N\right)\left(N^\dagger\sigma^i\tau^c N\right)\right], \tag{4.237}$$

where

$$C_S^{(t)} = \tilde{C}_{S,\,1}^{(t)} - \frac{1}{3}\tilde{C}_{S,\,2}^{(t)} \sim O(N_c)\,. \tag{4.238}$$

The transormation to the partial wave basis produces the Lagrangian

$$\mathcal{L}_{SNN}^{(t,\,a+b)} = 12C_S^{(t)}S\left\{\frac{1}{2}\left[\left(N^T\bar{P}^a N\right)^\dagger\left(N^T\bar{P}^b N\right) + \left(N^T\bar{P}^b N\right)^\dagger\left(N^T\bar{P}^a N\right)\right]\right.$$
$$\left. -\frac{1}{3}\delta^{ab}\left(N^T\bar{P}^c N\right)^\dagger\left(N^T\bar{P}^c N\right)\right\}, \tag{4.239}$$

which introduces an overall factor of 12.

4.5 Two-Nucleon Spurion Operators

In this section, the most general set of operators with two insertions of spurion fields as described in Sec. 2.2.5 is derived. The large-N_c counting rules have a straightforward extension to these operators with additional suppressions arising from the inclusion

of pion fields. While the primary focus in this work are the two-nucleon currents in EFT$_{\not\pi}$, pions are necessary for the particular application to lepton number violation and isospin breaking in Sec. 5.2. Additionally, there will be operator redundancies that can be eliminated through a general set of Fierz transformations while also retaining the manifestly dominant operators in the large-N_c expansion.

As previously discussed, the spurion operators parameterize the exchange of virtual particles, e.g. photons, above the breakdown scale of χPT or ChEFT [90–98]. Also, the terms that do not contain pions, the non-vanishing terms in the expansion of u at $O(\phi^0)$, can appear in EFT$_{\not\pi}$ even though the discussion in this section is more general. In either EFT, the operators in the Lagrangian appear with explicit factors of e^2 or G_F^2 depending on the specific spurion under consideration, but these factors may be omitted in the determination of the large-N_c scalings as they do not change the relative sizes.

The most general set of operators with two spurion insertions is

$$B_1 = \mathrm{Tr}(Q_+)^2 \left(N^\dagger \Gamma N\right)^2 , \tag{4.240}$$

$$B_2 = \mathrm{Tr}(Q_+) \left(N^\dagger \Gamma N\right) \left(N^\dagger \tilde{Q}_+ \Gamma N\right) , \tag{4.241}$$

$$B_3 = \left(N^\dagger \tilde{Q}_+ \Gamma N\right)^2 , \tag{4.242}$$

$$B_4 = \mathrm{Tr}(Q_-)^2 \left(N^\dagger \Gamma N\right)^2 , \tag{4.243}$$

$$B_5 = \mathrm{Tr}(Q_-) \left(N^\dagger \Gamma N\right) \left(N^\dagger \tilde{Q}_- \Gamma N\right) , \tag{4.244}$$

$$B_6 = \left(N^\dagger \tilde{Q}_- \Gamma N\right)^2 , \tag{4.245}$$

$$B_7 = \mathrm{Tr}(Q_+)\,\mathrm{Tr}(Q_-) \left(N^\dagger \Gamma N\right) \left(N^\dagger \Gamma N\right) , \tag{4.246}$$

$$B_8 = \mathrm{Tr}(Q_-) \left(N^\dagger \tilde{Q}_+ \Gamma N\right) \left(N^\dagger \Gamma N\right) , \tag{4.247}$$

$$B_9 = \mathrm{Tr}(Q_+) \left(N^\dagger \tilde{Q}_- \Gamma N\right) \left(N^\dagger \Gamma N\right) , \tag{4.248}$$

$$B_{10} = \left(N^\dagger \tilde{Q}_+ \Gamma N\right) \left(N^\dagger \tilde{Q}_- \Gamma N\right) , \tag{4.249}$$

$$B_{11} = \mathrm{Tr}\left(\tilde{Q}_+^2 + \tilde{Q}_-^2\right) \left(N^\dagger \Gamma N\right)^2 = \frac{1}{2}\,\mathrm{Tr}\left(\tilde{Q}_R^2 + \tilde{Q}_L^2\right) \left(N^\dagger \Gamma N\right)^2 , \tag{4.250}$$

$$B_{12} = \mathrm{Tr}\left(\tilde{Q}_+^2 - \tilde{Q}_-^2\right) \left(N^\dagger \Gamma N\right)^2 = \mathrm{Tr}\left(U \tilde{Q}_L U^\dagger \tilde{Q}_R\right) \left(N^\dagger \Gamma N\right)^2 , \tag{4.251}$$

$$B_{13} = \mathrm{Tr}\Big(\tilde{Q}_+\tilde{Q}_-\Big)\Big(N^\dagger\Gamma N\Big)^2 = \mathrm{Tr}\Big(\tilde{Q}_R^2 - \tilde{Q}_L^2\Big)\Big(N^\dagger\Gamma N\Big)^2 , \tag{4.252}$$

where Γ can be $\mathbb{1}$, σ^i, τ^a, or $\sigma^i\tau^a$. These four possibilities are redundant. Each operator also appears with an LEC $\bar{\mathcal{C}}_{i,j}$, where the first index i refers to the B_i from which each operator originates while the second index j refers to $\Gamma = \mathbb{1},\ \sigma^i,\ \tau^a,\ \sigma^i\tau^a$ for $j = 1, 2, 3, 4$, respectively.

The operators that contain only traces of the spurions have bilinears of the form $(N^\dagger\Gamma N)^2$. Thus, the only possibility for $1/N_c$ suppressions to enter is through the appearance of pion fields. Fierz transformations eliminate the operators with $\Gamma = \tau^a$ and $\sigma^i\tau^a$. Therefore, the subset of relevant operators becomes

$$\mathcal{O}_{1,1} = \mathrm{Tr}(Q_+)^2\Big(N^\dagger N\Big)^2 , \tag{4.253}$$

$$\mathcal{O}_{1,2} = \mathrm{Tr}(Q_+)^2\Big(N^\dagger\sigma^i N\Big)^2 , \tag{4.254}$$

$$\mathcal{O}_{4,1} = \mathrm{Tr}(Q_-)^2\Big(N^\dagger N\Big)^2 , \tag{4.255}$$

$$\mathcal{O}_{4,2} = \mathrm{Tr}(Q_-)^2\Big(N^\dagger\sigma^i N\Big)^2 , \tag{4.256}$$

$$\mathcal{O}_{7,1} = \mathrm{Tr}(Q_+)\,\mathrm{Tr}(Q_-)\Big(N^\dagger N\Big)\Big(N^\dagger N\Big) , \tag{4.257}$$

$$\mathcal{O}_{7,2} = \mathrm{Tr}(Q_+)\,\mathrm{Tr}(Q_-)\Big(N^\dagger\sigma^i N\Big)\Big(N^\dagger\sigma^i N\Big) , \tag{4.258}$$

$$\mathcal{O}_{11,1} = \mathrm{Tr}\Big(\tilde{Q}_+^2 + \tilde{Q}_-^2\Big)\Big(N^\dagger N\Big)^2 = \frac{1}{2}\,\mathrm{Tr}\Big(\tilde{Q}_R^2 + \tilde{Q}_L^2\Big)\Big(N^\dagger N\Big)^2 , \tag{4.259}$$

$$\mathcal{O}_{11,2} = \mathrm{Tr}\Big(\tilde{Q}_+^2 + \tilde{Q}_-^2\Big)\Big(N^\dagger\sigma^i N\Big)^2 = \frac{1}{2}\,\mathrm{Tr}\Big(\tilde{Q}_R^2 + \tilde{Q}_L^2\Big)\Big(N^\dagger\sigma^i N\Big)^2 , \tag{4.260}$$

$$\mathcal{O}_{12,1} = \mathrm{Tr}\Big(\tilde{Q}_+^2 - \tilde{Q}_-^2\Big)\Big(N^\dagger N\Big)^2 = \mathrm{Tr}\Big(U\tilde{Q}_L U^\dagger\tilde{Q}_R\Big)\Big(N^\dagger N\Big)^2 , \tag{4.261}$$

$$\mathcal{O}_{12,2} = \mathrm{Tr}\Big(\tilde{Q}_+^2 - \tilde{Q}_-^2\Big)\Big(N^\dagger\sigma^i N\Big)^2 = \mathrm{Tr}\Big(U\tilde{Q}_L U^\dagger\tilde{Q}_R\Big)\Big(N^\dagger\sigma^i N\Big)^2 , \tag{4.262}$$

$$\mathcal{O}_{13,1} = \mathrm{Tr}\Big(\tilde{Q}_+\tilde{Q}_-\Big)\Big(N^\dagger N\Big)^2 = \mathrm{Tr}\Big(\tilde{Q}_R^2 - \tilde{Q}_L^2\Big)\Big(N^\dagger N\Big)^2 , \tag{4.263}$$

$$\mathcal{O}_{13,2} = \mathrm{Tr}\Big(\tilde{Q}_+\tilde{Q}_-\Big)\Big(N^\dagger\sigma^i N\Big)^2 = \mathrm{Tr}\Big(\tilde{Q}_R^2 - \tilde{Q}_L^2\Big)\Big(N^\dagger\sigma^i N\Big)^2 , \tag{4.264}$$

where the operator $\mathcal{O}_{i,j}$ has the same indices as the corresponding LECs $\bar{\mathcal{C}}_{i,j}$. Once a specific form of the spurion is chosen, there are additional redundancies that can be absorbed in a redefinition of the corresponding LECs attending each operator in the Lagrangian.

For the operators containing either one or two insertions of the traceless part of the spurion fields inside the nucleon bilinears, the spurions can be expanded through Eq. (2.82). However, the insertion of $\Gamma = \tau^a$ or $\sigma^i\tau^a$ leads to terms that do not match onto the Hartree Hamiltonian, rather they contain products of Pauli matrices that must be reduced using

$$\tau^a\tau^b = \delta^{ab}\mathbb{1} + i\epsilon^{abc}\tau^c\,. \tag{4.265}$$

The largest possible N_c scaling of the resulting operators can then be determined from the spin-isospin structure. All of the possible suppressions arising from the presence of pion fields are contained in coefficients defined by $c_{a,\pm} = \frac{1}{2}\operatorname{Tr}\left(\tilde{Q}_\pm\tau^a\right)$. Therefore, for B_3, B_6, and B_{10}, the matrices $\Gamma = 1$ and σ^i, respectively, lead to

$$\left(N^\dagger\tilde{Q}_\pm N\right)\left(N^\dagger\tilde{Q}_\pm N\right) = c_{a,\pm}c_{b,\pm}\left(N^\dagger\tau^a N\right)\left(N^\dagger\tau^b N\right)\,, \tag{4.266}$$

$$\left(N^\dagger\tilde{Q}_\pm\sigma^i N\right)\left(N^\dagger\tilde{Q}_\pm\sigma^i N\right) = c_{a,\pm}c_{b,\pm}\left(N^\dagger\tau^a\sigma^i N\right)\left(N^\dagger\tau^b\sigma^i N\right)\,. \tag{4.267}$$

For $\Gamma = \tau^c$,

$$\begin{aligned}\left(N^\dagger\tilde{Q}_\pm\tau^c N\right)\left(N^\dagger\tilde{Q}_\pm\tau^c N\right) &= c_{a,\pm}c_{a,\pm}\left(N^\dagger N\right)^2 - c_{a,\pm}c_{a,\pm}\left(N^\dagger\tau^b N\right)^2\\ &\quad + c_{a,\pm}c_{b,\pm}\left(N^\dagger\tau^a N\right)\left(N^\dagger\tau^b N\right)\,,\end{aligned} \tag{4.268}$$

and the operator $\left(N^\dagger\tau^b N\right)^2$ can be removed through Fierz transformations to obtain

$$\begin{aligned}\left(N^\dagger\tilde{Q}_\pm\tau^c N\right)\left(N^\dagger\tilde{Q}_\pm\tau^c N\right) &= 3c_{a,\pm}c_{a,\pm}\left(N^\dagger N\right)^2 + c_{a,\pm}c_{a,\pm}\left(N^\dagger\sigma^i N\right)^2\\ &\quad + c_{a,\pm}c_{b,\pm}\left(N^\dagger\tau^a N\right)\left(N^\dagger\tau^b N\right)\,.\end{aligned} \tag{4.269}$$

The first two terms in Eq. (4.269) have the same bilinear structure as the operators $\mathcal{O}_{1,1}$ and $\mathcal{O}_{1,2}$, respectively. Their contributions can be absorbed into a redefinition of the LECs of these operators. The third term in Eq. (4.269) is Eq. (4.266). For $\Gamma = \sigma^i\tau^c$, the expansion of the spurions leads to

$$\begin{aligned}\left(N^\dagger\tilde{Q}_\pm\sigma^i\tau^c N\right)\left(N^\dagger\tilde{Q}_\pm\sigma^i\tau^c N\right) &= 3c_{a,\pm}c_{a,\pm}\left(N^\dagger N\right)^2 + c_{a,\pm}c_{a,\pm}\left(N^\dagger\sigma^i N\right)^2\\ &\quad + c_{a,\pm}c_{b,\pm}\left(N^\dagger\sigma^i\tau^a N\right)\left(N^\dagger\sigma^i\tau^b N\right)\,.\end{aligned} \tag{4.270}$$

The first line is the same as the first line of Eq. (4.269) while the single term in the second line is Eq. (4.267). Therefore, $\Gamma = \tau^c$ and $\sigma^i\tau^c$ in B_3, B_6, and B_{10} do not yield independent operators.

Additional relationships exist among some of the operators corresponding to $\Gamma = \mathbb{1}$ and $\Gamma = \sigma^i$. For B_3, B_6, and B_{10}, $\Gamma = \mathbb{1}$ can be eliminated by applying Fierz transformations to Eq. (4.266) along with the decomposition in Eq. (2.82). Using

$$\mathrm{Tr}\left(\tilde{Q}_\pm^2\right) = 2c_{a,\pm}c_{a,\pm}\,, \tag{4.271}$$

the Fierz transformation for Eq. (4.266) leads to

$$\begin{aligned} -3\left[\left(N^\dagger\tilde{Q}_\pm N\right)\left(N^\dagger\tilde{Q}_\pm N\right) - \frac{1}{6}\,\mathrm{Tr}\left(\tilde{Q}_\pm\tilde{Q}_\pm\right)\left(N^\dagger\tau^a N\right)^2\right] \\ = \left(N^\dagger\tilde{Q}_\pm N\right)\left(N^\dagger\sigma^i\tilde{Q}_\pm N\right) - \frac{1}{6}\,\mathrm{Tr}\left(\tilde{Q}_\pm\tilde{Q}_\pm\right)\left(N^\dagger\sigma^i\tau^a N\right)^2\,, \end{aligned} \tag{4.272}$$

which can be arranged, with the help of additional Fierz transformations, to be

$$\begin{aligned} \left(N^\dagger\tilde{Q}_\pm N\right)\left(N^\dagger\tilde{Q}_\pm N\right) &= -\frac{1}{3}\left(N^\dagger\tilde{Q}_\pm N\right)\left(N^\dagger\sigma^i\tilde{Q}_\pm N\right) - \frac{1}{2}\,\mathrm{Tr}\left(\tilde{Q}_\pm\tilde{Q}_\pm\right)\left(N^\dagger N\right)^2 \\ &\quad -\frac{1}{6}\,\mathrm{Tr}\left(\tilde{Q}_\pm\tilde{Q}_\pm\right)\left(N^\dagger\sigma^i N\right)^2\,. \end{aligned} \tag{4.273}$$

Therefore, the choice of $\Gamma = \mathbb{1}$ can be eliminated from B_3, B_6, and B_{10} in favor of combinations of $\Gamma = \sigma^i$ and operators from B_{11}, B_{12}, and B_{13}.

Following the same procedure for operators from B_2, B_5, B_8, and B_9 results in

$$\mathrm{Tr}(Q_\pm)\left(N^\dagger\tilde{Q}_\pm N\right)\left(N^\dagger N\right) = c_{a,\pm}\,\mathrm{Tr}(Q_\pm)\left(N^\dagger\tau^a N\right)\left(N^\dagger N\right)\,, \tag{4.274}$$

$$\mathrm{Tr}(Q_\pm)\left(N^\dagger\tilde{Q}_\pm\sigma^i N\right)\left(N^\dagger\sigma^i N\right) = c_{a,\pm}\,\mathrm{Tr}(Q_\pm)\left(N^\dagger\sigma^i\tau^a N\right)\left(N^\dagger\sigma^i N\right)\,, \tag{4.275}$$

for $\Gamma = \mathbb{1}$ and σ^i, respectively. When $\Gamma = \tau^b$, application of Eq. (4.265) produces

$$\mathrm{Tr}(Q_\pm)\left(N^\dagger\tilde{Q}_\pm\tau^b N\right)\left(N^\dagger\tau^b N\right) = c_{a,\pm}\,\mathrm{Tr}(Q_\pm)\left(N^\dagger\tau^a N\right)\left(N^\dagger N\right)\,, \tag{4.276}$$

which is the same as Eq. (4.274). Similarly, $\Gamma = \sigma^i\tau^b$ leads to

$$\mathrm{Tr}(Q_\pm)\left(N^\dagger\tilde{Q}_\pm\sigma^i\tau^b N\right)\left(N^\dagger\sigma^i\tau^b N\right) = c_{a,\pm}\,\mathrm{Tr}(Q_\pm)\left(N^\dagger\sigma^i\tau^a N\right)\left(N^\dagger\sigma^i N\right)\,, \tag{4.277}$$

which is the same as Eq. (4.275). This shows that again the operators with $\Gamma = \tau^a$ and $\Gamma = \sigma^i\tau^a$ neither lead to independent operators nor obscure the dominant large-N_c contributions. As before, additional relationships exist between the $\Gamma = \mathbb{1}$ and $\Gamma = \sigma^i$ operators. Eliminating the remaining redundancy through Fierz transformations leads to

$$-3\left(N^\dagger N\right)\left(N^\dagger \tilde{Q}_\pm N\right) = \left(N^\dagger \sigma^i N\right)\left(N^\dagger \sigma^i \tilde{Q}_\pm N\right) . \tag{4.278}$$

Therefore, either $\Gamma = \mathbb{1}$ or $\Gamma = \sigma^i$ may be retained, and both choices give the same large-N_c counting.

This process eliminates the operators that possess a subleading spin-flavor structure as well. Any additional factors of N_c that might be present will arise from pion fields in the expansion of u; however, these factors will not change the spin-flavor structure of the nucleon bilinears, and only lead to additional $1/\sqrt{N_c}$ suppressions arising from factors of $1/F$.

4.5.1 Electromagnetic Spurions

Now, consider the explicit choice of electromagnetic spurions. It is convenient to write the Lagrangian in terms of the nucleon charge matrix,

$$Q = \frac{1}{2}\left(\mathbb{1} + \tau^3\right) . \tag{4.279}$$

The difference between using the nucleon charge matrix and using the quark charge matrix amounts to a shift by an unobservable constant [97]. Here, the nucleon charge matrix is independent of N_c, which implies that the up and down quark charges are N_c-dependent.

Using Eq. (4.279) as the spurion field in Eq. (4.240) yields operators with a clear spin-flavor structure. Setting $Q_R = Q_L = Q$ gives

$$\tilde{Q}_\pm = \frac{1}{4}\left[u^\dagger \tau^3 u \pm u \tau^3 u^\dagger\right] . \tag{4.280}$$

The corresponding traces of operators are [98]

$$\begin{aligned}
\mathrm{Tr}(Q_+) &= 1, && (4.281)\\
\mathrm{Tr}(Q_-) &= 0, && (4.282)\\
\mathrm{Tr}\left(\tilde{Q}_+^2 + \tilde{Q}_-^2\right) &= \mathrm{Tr}\left(\tilde{Q}^2\right) = 1/2, && (4.283)\\
\mathrm{Tr}\left(\tilde{Q}_+^2 - \tilde{Q}_-^2\right) &= \mathrm{Tr}\left(U\tilde{Q}U^\dagger\tilde{Q}\right), && (4.284)\\
\mathrm{Tr}\left(\tilde{Q}_+\tilde{Q}_-\right) &= 0. && (4.285)
\end{aligned}$$

Since

$$\mathrm{Tr}\left(\tilde{Q}_+^2 + \tilde{Q}_-^2\right)\left(N^\dagger \mathcal{O} N\right)^2 \sim \mathrm{Tr}(Q_+)^2\left(N^\dagger \mathcal{O} N\right)^2 , \tag{4.286}$$

the operators from B_{11} can be absorbed into those from B_1. Therefore, the only independent operators are those from B_1, B_2, B_3, B_6, B_9, B_{10}, and B_{12}. The operators from B_{10} vanish at least through $O(\phi^2)$ when u is expanded and can be neglected at this order. After using the results of the Fierz transformations above, the minimal set of operators is

$$\begin{aligned}
\mathcal{O}_{1,1} &= \left(N^\dagger N\right)^2 , && (4.287)\\
\mathcal{O}_{1,2} &= \left(N^\dagger \sigma^i N\right)^2 , && (4.288)\\
\mathcal{O}_{2} &= \left(N^\dagger N\right)\left(N^\dagger \tilde{Q}_+ N\right) , && (4.289)\\
\mathcal{O}_{3} &= \left(N^\dagger \sigma^i \tilde{Q}_+ N\right)\left(N^\dagger \sigma^i \tilde{Q}_+ N\right) , && (4.290)\\
\mathcal{O}_{6} &= \left(N^\dagger \sigma^i \tilde{Q}_- N\right)\left(N^\dagger \sigma^i \tilde{Q}_- N\right) , && (4.291)\\
\mathcal{O}_{9} &= \left(N^\dagger \tilde{Q}_- N\right)\left(N^\dagger N\right) , && (4.292)\\
\mathcal{O}_{12,1} &= \mathrm{Tr}\left(U\tilde{Q}U^\dagger\tilde{Q}\right)\left(N^\dagger N\right)^2 , && (4.293)\\
\mathcal{O}_{12,2} &= \mathrm{Tr}\left(U\tilde{Q}U^\dagger\tilde{Q}\right)\left(N^\dagger \sigma^i N\right)^2 , && (4.294)
\end{aligned}$$

where the first subscript i in $\mathcal{O}_{i,j}$ indicates the B_i from which each operator originates, and the second index j, where necessary, refers to a specific operator within the B_i, $j = 1, 2, 3, 4$ for $\Gamma = \mathbb{1}$, σ^i, τ^a, $\sigma^i\tau^a$, respectively.

Finally, as discussed in Sec. 3.2, each additional pion field introduces a factor of $1/F \sim 1/\sqrt{N_c}$. Expanding each operator to second order in the pion fields to determine the maximum N_c scaling of the corresponding LECs yields

$$\mathcal{O}_{1,1} = \left(N^\dagger N\right)^2 + \cdots\,, \tag{4.295}$$

$$\mathcal{O}_{1,2} = \left(N^\dagger \sigma^i N\right)^2 + \cdots\,, \tag{4.296}$$

$$\mathcal{O}_{2} = \frac{1}{2}\left(1 - \frac{1}{2F^2}\phi_a\phi_a\right)\left(N^\dagger N\right)\left(N^\dagger \tau^3 N\right) + \frac{1}{4F^2}\phi_3\phi_a\left(N^\dagger N\right)\left(N^\dagger \tau^a N\right) + \cdots\,, \tag{4.297}$$

$$\mathcal{O}_{3} = \frac{1}{4}\left(1 - \frac{1}{F^2}\phi_a\phi_a\right)\left(N^\dagger \sigma^i\tau^3 N\right)^2 + \frac{1}{4F^2}\phi_3\phi_a\left(N^\dagger \sigma^i\tau^3 N\right)\left(N^\dagger \sigma^i\tau^a N\right) + \cdots\,, \tag{4.298}$$

$$\mathcal{O}_{6} = \frac{1}{4F^2}\epsilon^{3ab}\epsilon^{3cd}\phi_a\phi_c\left(N^\dagger \sigma^i\tau^b N\right)\left(N^\dagger \sigma^i\tau^d N\right) + \cdots\,, \tag{4.299}$$

$$\mathcal{O}_{9} = -\frac{1}{2F}\epsilon^{3ab}\phi_a\left(N^\dagger \tau^b N\right)\left(N^\dagger N\right) + \cdots\,, \tag{4.300}$$

$$\mathcal{O}_{12,1} = \left[\frac{1}{2} - \frac{4}{F^2}\left(\phi_a\phi_a - \phi_3\phi_3\right)\right]\left(N^\dagger N\right)^2 + \cdots\,, \tag{4.301}$$

$$\mathcal{O}_{12,2} = \left[\frac{1}{2} - \frac{4}{F^2}\left(\phi_a\phi_a - \phi_3\phi_3\right)\right]\left(N^\dagger \sigma^i N\right)^2 + \cdots \tag{4.302}$$

where the ellipses indicate additional pion fields. The scaling of the LECs $\bar{\mathcal{C}}_{i,j}$ multiplying $\mathcal{O}_{i,j}$ in the Lagrange density is given by

$$\bar{\mathcal{C}}_{1,1} \sim N_c\,, \tag{4.303}$$

$$\bar{\mathcal{C}}_{1,2} \sim N_c^{-1}\,, \tag{4.304}$$

$$\bar{\mathcal{C}}_{2} \sim 1\,, \tag{4.305}$$

$$\bar{\mathcal{C}}_{3} \sim N_c\,, \tag{4.306}$$

$$\bar{\mathcal{C}}_{6} \sim 1\,, \tag{4.307}$$

$$\bar{\mathcal{C}}_{9} \sim N_c^{-1/2}\,, \tag{4.308}$$

$$\bar{\mathcal{C}}_{12,1} \sim N_c\,, \tag{4.309}$$

$$\bar{\mathcal{C}}_{12,2} \sim N_c^{-1}\,. \tag{4.310}$$

The operators $\mathcal{O}_{1,1}$ and $\mathcal{O}_{12,1}$ differ only at the multi-pion level. Therefore, differences between the two will be $1/N_c$ suppressed. The same holds for the operators $\mathcal{O}_{1,2}$ and $\mathcal{O}_{12,2}$. The operator $\mathcal{O}_6$ provides a concrete example of an earlier point: the generic spin-flavor structure of the operator, before expanding u in the number of pion fields, indicates that it could be $O(N_c)$, but the first nonzero term has two pion fields and is thus suppressed by an additional factor of $1/N_c$.

The Lagrangian at LO and next-to-leading order (NLO) in the large-N_c expansion is

$$\mathcal{L}_{\text{LO-in-}N_c} = e^2 \left\{ \left[\bar{\mathcal{C}}_{1,1} + \bar{\mathcal{C}}_{12,1} \operatorname{Tr}\left(U\tilde{Q}U^\dagger \tilde{Q} \right) \right] \left(N^\dagger N \right)^2 + \bar{\mathcal{C}}_3 \left(N^\dagger \sigma^i \tilde{Q}_+ N \right)^2 \right\} , \quad (4.311)$$

$$\mathcal{L}_{\text{NLO-in-}N_c} = e^2 \left\{ \bar{\mathcal{C}}_2 \left(N^\dagger N \right) \left(N^\dagger \tilde{Q}_+ N \right) + \bar{\mathcal{C}}_6 \left(N^\dagger \sigma^i \tilde{Q}_- N \right)^2 \right\} . \quad (4.312)$$

4.5.2 Weak Spurions

For weak interactions, $Q_L = \tau^+$ while $Q_R = 0$, which gives

$$Q_\pm = \tilde{Q}_\pm = \pm \frac{1}{2} u Q_L u^\dagger = \pm \frac{1}{2} u \tau^+ u^\dagger . \quad (4.313)$$

As a result, all traces in Eqs. (4.240) vanish and therefore operators from B_1, B_2, B_4, B_5, B_7, B_8, B_9, B_{11}, B_{12}, and B_{13} do not contribute. Since $\tilde{Q}_+ = -\tilde{Q}_-$, the only nonvanishing term is

$$\left(N^\dagger u \tau^+ u^\dagger \Gamma N \right)^2 , \quad (4.314)$$

and the structures B_3, B_6, and B_{10} become identical. As pointed out in Ref. [135], the two operators corresponding to $\Gamma = \mathbb{1}$ and σ^i in this term are related through a Fierz identity and are not independent at $O(\phi^0)$. The authors of Ref. [135] choose to retain $\Gamma = \mathbb{1}$; that is, the operator $\left(N^\dagger \tau^+ N \right)^2$. According to Eq. (3.31), this operator does not appear at LO in the large-N_c expansion. However, eliminating the operator $\left(N^\dagger \sigma^i \tau^+ N \right)^2$ through the Fierz transformation

$$\left(N^\dagger \sigma^i \tau^+ N \right)^2 = -3 \left(N^\dagger \tau^+ N \right)^2 \quad (4.315)$$

introduces a hidden LO-in-N_c contribution in the term proportional to $\left(N^\dagger\tau^+N\right)^2$. As a result, after removing the overall factor of N_c from the Hartree Hamiltonian, the LEC for this operator will be $O(N_c)$. In Sec. (5.2), this LEC will be explicitly introduced as g_ν^{NN}.

Chapter 5

Applications

5.1 Electroweak Currents

In this section, the large-N_c constraints from Ch. 4 are applied to two-nucleon electroweak currents. These electromagnetic operators must be $\mathrm{U}(1)_{\mathrm{EM}}$ gauge invariant; therefore, the currents can either be minimally coupled to the electromagnetic field A_μ, or they can be coupled to the electric and magnetic field strengths, E^i and B^i, respectively. The weak currents couple the two-nucleon system to an external weak current where the gauge bosons have been integrated out; thus, these operators are four-fermion contact operators. The two-nucleon currents in this section have been studied in a variety of processes involving the deuteron [63, 71, 136, 137].

5.1.1 Minimal Coupling for Two-Derivative Contact Interactions

In the single nucleon sector, U(1) gauge invariant interactions can be obtained by promoting time derivatives and gradients to be the covariant derivatives

$$\mathcal{D}_0 N = \partial_0 N + ieA_0 Q N\,, \tag{5.1}$$

$$\mathcal{D}^i N = \nabla^i N - ieA^i Q N\,, \tag{5.2}$$

where Q is the charge matrix $Q = \frac{1}{2}\left(\mathbb{1} + \tau^3\right)$. As mentioned in Sec. 4.1.1, the nucleon charge is taken to be independent of N_c in this work. Thus, if one were to split the resulting operators into isoscalar and isovector pieces it is important to not count their respective N_c scalings differently, but rather to count both components as being $O(1)$. For a nucleon bilinear $N^\dagger \Gamma N$ where $\Gamma = \{\mathbb{1},\ \sigma^i,\ \tau^a,\ \sigma^i\tau^a\}$, the independent

combinations of covariant derivatives are

$$\begin{aligned} \mathcal{D}^i \left(N^\dagger \Gamma N\right) &= \nabla^i \left(N^\dagger \Gamma N\right) + ieA^i N^\dagger \left[Q, \Gamma\right] N \,, \\ N^\dagger \overleftrightarrow{\mathcal{D}}^i \Gamma N &= N^\dagger \overleftrightarrow{\nabla}^i \Gamma N - ieA^i N^\dagger \left\{Q, \Gamma\right\} N \,. \end{aligned} \tag{5.3}$$

This procedure can also be applied to the two-derivative two-nucleon contact terms in Ref. [45]. A similar approach was used in Ref. [131]; however, a different operator basis is retained in this work. Because the following currents are obtained by gauging two-derivative contact terms, the couplings are determined by two-nucleon scattering observables. When the derivative replacement is made, there will be two-nucleon-one-photon and two-nucleon-two-photon interactions, which are required by gauge invariance, but the latter are not considered explicitly here. Although, these terms can contribute to processes such as Compton scattering.

From Eq. (5.3), it can be seen if either Γ_1 or Γ_2, where the subscript refers to a specific nucleon bilinear, do not contain isospin matrices other than the identity, then these operators will not couple to the electromagnetic field. Therefore, it is not necessary to consider the terms proportional to $C_{1\cdot 1}$, $C_{\sigma\cdot\sigma}$, and $C'_{\sigma\cdot\sigma}$ in the notation of Ref. [45]. The remaining currents at LO-in-N_c are

$$J^i_{G\cdot G} = 2eC_{G\cdot G}\epsilon^{3ab}\nabla^i(N^\dagger\sigma^j\tau^a N)(N^\dagger\sigma^j\tau^b N)\,, \tag{5.4}$$

$$J'^i_{G\cdot G} = 2eC'_{G\cdot G}\epsilon^{3ab}\nabla^j(N^\dagger\sigma^j\tau^a N)(N^\dagger\sigma^i\tau^b N)\,, \tag{5.5}$$

while at next-to-next-to-leading order in the large-N_c scaling (N^{2}LO-in-N_c) they are

$$\overleftrightarrow{J}^i_{1\cdot 1} = 4ie\overleftrightarrow{C}_{1\cdot 1}(N^\dagger\overleftrightarrow{\nabla}^i N)(N^\dagger Q N)\,, \tag{5.6}$$

$$J^i_{\tau\cdot\tau} = 2eC_{\tau\cdot\tau}\epsilon^{3ab}\nabla^i(N^\dagger\tau^a N)(N^\dagger\tau^b N)\,, \tag{5.7}$$

$$\overleftrightarrow{J}^i_{G\cdot G} = 2ie\overleftrightarrow{C}_{G\cdot G}\left[(N^\dagger\sigma^j\tau^a\overleftrightarrow{\nabla}^i N)(N^\dagger\sigma^j\tau^a N) + (N^\dagger\sigma^j\tau^3\overleftrightarrow{\nabla}^i N)(N^\dagger\sigma^j N)\right]\,, \tag{5.8}$$

$$\overleftrightarrow{J}^i_{1\cdot\sigma} = e\overleftrightarrow{C}_{1\cdot\sigma}\epsilon^{ijk}\left[\nabla^k(N^\dagger\sigma^j N)(N^\dagger Q N) + \nabla^k(N^\dagger N)(N^\dagger\sigma^j Q N)\right]\,, \tag{5.9}$$

$$\overleftrightarrow{J}'^i_{G\cdot G} = 2ie\overleftrightarrow{C}'_{G\cdot G}\left[(N^\dagger\overleftrightarrow{\nabla}^j\sigma^j\tau^a N)(N^\dagger\sigma^i\tau^a N) + (N^\dagger\overleftrightarrow{\nabla}^j\sigma^j\tau^3 N)(N^\dagger\sigma^i N)\right]\,, \tag{5.10}$$

$$\overleftrightarrow{J}^{i}_{G\cdot\tau} = \frac{e}{2}\overleftrightarrow{C}_{G\cdot\tau}\epsilon^{ijk}\Bigg\{ \left[\nabla^k\left(N^\dagger\sigma^j\tau^a N\right)\left(N^\dagger\tau^a N\right) + \nabla^k\left(N^\dagger\tau^a N\right)\left(N^\dagger\sigma^j\tau^a N\right)\right.$$
$$\left. + \nabla^k\left(N^\dagger\sigma^j\tau^3 N\right)\left(N^\dagger N\right) + \nabla^k\left(N^\dagger\tau^3 N\right)\left(N^\dagger\sigma^j N\right)\right] \tag{5.11}$$
$$+ i\epsilon^{3ab}\left[\left(N^\dagger\sigma^j\tau^b N\right)\left(N^\dagger\overleftrightarrow{\nabla}^k\tau^a N\right) + \left(N^\dagger\tau^b N\right)\left(N^\dagger\overleftrightarrow{\nabla}^k\sigma^j\tau^a N\right)\right]\Bigg\}.$$

Making use of Fierz transformations for the N^{2}LO-in-N_c terms reduces the set of operators to

$$J^i_{\text{LO-in-}N_c} = 2eC_{G\cdot G}\epsilon^{3ab}\nabla^i\left(N^\dagger\sigma^j\tau^a N\right)\left(N^\dagger\sigma^j\tau^b N\right)$$
$$+ 2eC'_{G\cdot G}\epsilon^{3ab}\nabla^j(N^\dagger\sigma^j\tau^a N)(N^\dagger\sigma^i\tau^b N)\,, \tag{5.12}$$
$$J^i_{\text{N}^2\text{LO-in-}N_c} = 2e\left(C_{\tau\cdot\tau} - \frac{1}{4}\overleftrightarrow{C}_{1\cdot 1} - \frac{3}{4}\overleftrightarrow{C}_{G\cdot G} + \frac{1}{4}\overleftrightarrow{C'}_{G\cdot G}\right)\epsilon^{3ab}\nabla^i(N^\dagger\tau^a N)(N^\dagger\tau^b N)$$
$$+ e\left(\overleftrightarrow{C}_{1\cdot\sigma} + \overleftrightarrow{C}_{G\cdot\tau}\right)\epsilon^{ijk}\left[\nabla^k(N^\dagger\sigma^j N)(N^\dagger QN) + \nabla^k(N^\dagger N)(N^\dagger\sigma^j QN)\right]. \tag{5.13}$$

Performing the Fierz transformations also introduces subleading corrections in $J^i_{\text{LO-in-}N_c}$; however, these are not shown explicitly. This is also the same result one would obtain by reducing the overcomplete set of operators to a minimal set and then gauging these operators. Therefore, the order of gauging derivatives and performing Fierz transformation is irrelevant in this case.

As previously mentioned, this process is analogous to Refs. [131, 132] with the difference being the choice of which operators to retain. The operators here can be mapped to the operators of Ref. [132], which shows that the second and third terms in Eq. (2.20) of Ref. [132] are LO-in-N_c, while the first and fourth terms are N^{2}LO-in-N_c. In addition, the currents scalings are in agreement with those in Ref. [138] in which the currents are derived in terms of Fermi invariants.

5.1.2 Magnetic Couplings

In addition to nucleons minimally coupled to the photon A^i, the nucleons can couple directly to the magnetic field $\mathbf{B} = \nabla\times\mathbf{A}$. The Lagrangian with consists of Eqs. (4.152)

and (4.160) only with a magnetic field in place of the general axial field,

$$\mathcal{L} = eB_i \left[{}^{\not\pi}L_1 \left(N^T P_i N \right)^\dagger \left(N^T \bar{P}_3 N \right) - i\epsilon^{ijk}\, {}^{\not\pi}L_2 \left(N^T P_j N \right)^\dagger \left(N^T P_k N \right) \right] + \text{h.c.}\,. \quad (5.14)$$

The Lagrangian has also been expressed as

$$\mathcal{L} = eB_i \left\{ C'_{15} \left(N^\dagger \sigma^i N \right) \left(N^\dagger N \right) + C'_{16} \left[\left(N^\dagger \sigma^i \tau^3 N \right) \left(N^\dagger N \right) - \left(N^\dagger \sigma^i N \right) \left(N^\dagger \tau^3 N \right) \right] \right\} , \quad (5.15)$$

in Ref. [139]. As shown in Sec. 4.2.2, the operators in this Lagrangian are related through Fierz transformations to operators that are manifestly leading order in the large-N_c expansion. In particular, the Lagrangian consists of Eqs. (4.150) and (4.158) again with a magnetic field in place of the general axial field,

$$\mathcal{L} = eB^i \left[C_s^{(M)} \left(N^\dagger \sigma^i N \right) \left(N^\dagger N \right) + C_v^{(M)} \epsilon^{ijk} \epsilon^{3ab} \left(N^\dagger \sigma^j \tau^a N \right) \left(N^\dagger \sigma^k \tau^b N \right) \right] , \quad (5.16)$$

where

$$C_s^{(M)} \sim \mathcal{O}(N_c^0)\,, \quad (5.17)$$

$$C_v^{(M)} \sim \mathcal{O}(N_c)\,. \quad (5.18)$$

Again, these LECs are related to those in the partial wave basis through

$${}^{\not\pi}L_1 = 8C_v^{(M)}, \quad {}^{\not\pi}L_2 = -C_s^{(M)}\,. \quad (5.19)$$

${}^{\not\pi}L_1$ can be obtained from a fit to the experimental cross section for the radiative capture $np \to d\gamma$, while ${}^{\not\pi}L_2$ can be determined from the deuteron magnetic moment. The fit values of these LECs at $\mu = m_\pi$ are [63]

$${}^{\not\pi}L_1(m_\pi) = 7.24\,\text{fm}^4\,, \quad {}^{\not\pi}L_2(m_\pi) = -0.149\,\text{fm}^4\,. \quad (5.20)$$

Looking at these values, it is clear that there is a size disparity, a fact that is noted explicitly in Ref. [140]. As mentioned in Ch. 2, naturalness is a working assumption in EFTs, i.e. LECs are $O(1)$ after the proper dimensional factors have been removed.

Phrased another way, LECs occuring at the same order in the power counting should roughly be the same size. However, the ratio of the two fitted LECs at $\mu = m_\pi$ is

$$\left|\frac{{}^{\not\pi}L_2}{{}^{\not\pi}L_1}\right|_{\text{exp}} \approx 0.021\,, \tag{5.21}$$

which indicates that the relative sizes of the LECs in the partial wave are in conflict with the naturalness assumption.

While the large-N_c analysis demonstrates the suppression of the isoscalar coupling relative to the isovector coupling by a factor of N_c, setting $N_c = 3$ as it is in the physical world does not account for the entire size of the suppression. Fierz transformations to the partial wave basis as shown in Sec. 4.2 do introduce additional factors of 1/8 from the normalization of the partial wave projection operators. However, the values of the large-N_c basis LECs can be determined using the fit values of ${}^{\not\pi}L_1$ and ${}^{\not\pi}L_2$,

$$C_s^{(M)} = 0.149\,\text{fm}^4, \quad C_v^{(M)} = 0.905\,\text{fm}^4\,. \tag{5.22}$$

In this case, $C_v^{(M)} \approx 6C_s^{(M)}$ or $C_v^{(M)} \approx 2N_c C_s^{(M)}$ at the physical value $N_c = 3$, that is to say that $\left|C_v^{(M)}\right| \sim N_c \left|C_s^{(M)}\right|$ apart from a residual factor of 2, which can be accommodated in the naturalness assumption. Therefore, if it is assumed that $\left|C_v^{(M)}\right| \sim N_c \left|C_s^{(M)}\right|$ from the outset, then the ratio of the partial wave couplings including the 1/8 suppression is

$$\left|\frac{{}^{\not\pi}L_2}{{}^{\not\pi}L_1}\right|_{N_c} \approx \frac{1}{8N_c} \approx 0.042\,. \tag{5.23}$$

Since the large-N_c constraints are really upper bounds on the LECs, it appears that the large-N_c constraint is consistent with the values of the coefficients obtained in Ref. [63]. This result also furnishes an explicit example of the basis dependence of the naturalness assumption. While the partial wave basis is amenable to the PDS scheme, the LECs in this case are not natural. Therefore, care must be taken when quantifying naturalness in attempts to estimate the values of LECs through e.g. Bayesian analysis.

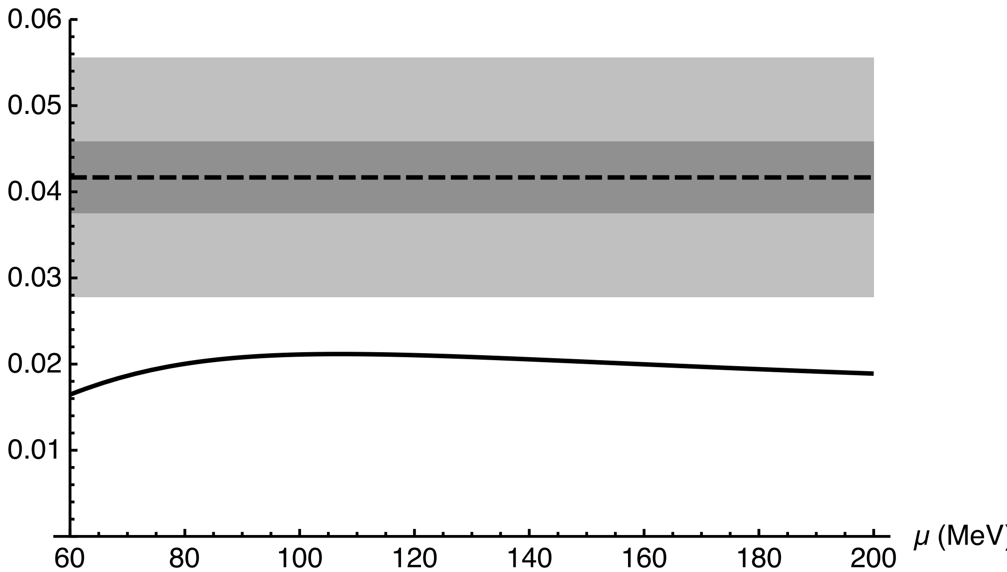

Figure 5.1: The ratio $|{}^{\not\pi}L_2/{}^{\not\pi}L_1|$ as a function of the renormalization scale μ. The solid line corresponds to solutions of the renormalization group equations combined with the values of Eq. (5.20). The dashed line corresponds to Eq. (5.23), while the dark (light) gray band corresponds to 10% (30%) corrections.

A major factor in this suppression is the factor of 8 that enters through Fierz transformations. As discussed in Ch. 4, it possible that these somewhat large factors do not occur for a different choice of the LO-in-N_c operators to retain when eliminating redundancies. Therefore, this suppression should not be treated as a rigorous prediction, but rather it should be seen as a general result tied to the spin-isospin structure of the two-nucleon operators as they emerge from large-N_c QCD.

Additionally, it has been recognized that large-N_c constraints are only valid for certain choices of the subtraction point [38, 45]. The LECs ${}^{\not\pi}L_1$ and ${}^{\not\pi}L_2$ were both fit at $\mu = m_\pi$, which appears to be within the acceptable range for which large-N_c constraints are valid. The running of the ratio $\left|{}^{\not\pi}L_2/{}^{\not\pi}L_1\right|$ is shown in Fig. 5.1. Here, the ratio does not vary much in the range of μ considered; therefore, the large-N_c constraints should not be sensitive to cutoff in this case.

5.1.3 Axial two-nucleon contact terms

This analysis can be generalized to two-nucleon operators coupled to an arbitrary external axial field, which is relevant for processes such as neutrino-deuteron scattering. The relevant forms of the Lagrangian are in Eqs. (4.150), (4.152), (4.158), and (4.158) and are repeated here for convenience. There is the isoscalar Lagrangian

$$\mathcal{L}^{(s)}_{ANN} = C^{(s)}_A A^i \left(N^\dagger N\right)\left(N^\dagger \sigma^i N\right) \tag{5.24}$$

$$= 2iC^{(s)}_A \epsilon^{ijk} A^i \left(N^T P_j N\right)^\dagger \left(N^T P_k N\right) . \tag{5.25}$$

and the isovector Lagrangian

$$\mathcal{L}^{(v)}_{ANN} = C^{(v)}_A \epsilon^{ijk}\epsilon^{abc} A^i \left(N^\dagger \sigma^j \tau^b N\right)\left(N^\dagger \sigma^k \tau^c N\right) \tag{5.26}$$

$$= 8C^{(v)}_{A,1}\left[\left(N^T \bar{P}_a N\right)^\dagger \left(N^T P_i N\right) + \text{h.c.}\right] \tag{5.27}$$

These LECs are related to those in Ref. [136] according to

$$L_{2,A} = -C^{(s)}_A\,, \tag{5.28}$$

$$L_{1,A} = 8C^{(v)}_{A,1}\,. \tag{5.29}$$

The same prediction for the ratio of LECs as for the magnetic LECs holds,

$$\left|\frac{L_{2,A}}{L_{1,A}}\right|_{N_c} = \left|\frac{C^{(A)}_s}{8C^{(A)}_v}\right| \sim \frac{1}{8N_c}, \tag{5.30}$$

where for the last relation it has been assumed $\left|C^{(v)}_A\right| \sim N_c\left|C^{(s)}_A\right|$.

The values of these LECs are estimated to be [136]

$$|L_{1,A}| \sim |L_{2,A}| \sim \frac{4\pi}{M}\frac{1}{\mu^2} \approx 5\,\text{fm}^3, \tag{5.31}$$

at $\mu = m_\pi$, and in this approach are expected to be of the same size. In principle, the value of $L_{1,A}$ can be determined from proton-proton fusion, neutrino-deuteron reactions, and tritium β-decay [136, 137, 141–145]. On the other hand, the impact of $L_{2,A}$ in these processes is negligible. The NPLQCD collaboration recently performed

a lattice QCD calculation of the proton-proton fusion process at $m_\pi^{\rm lat} = 806\,\text{MeV}$ and used the result to extract the LEC $L_{1,A}$ with μ at the physical pion mass to be [146]

$$L_{1,A}^{\rm NPLQCD}(\mu = m_\pi) = 3.9(0.2)(1.0)(0.4)(0.9)\,\text{fm}^3, \tag{5.32}$$

where the values in parentheses denote the statistical, systematic fitting and analysis, and systematic mass extrapolation uncertainties, as well as an estimate of higher-order corrections in the $\text{EFT}_{\not\pi}$ power counting. Within uncertainties, this value is in agreement with the extractions based on neutrino-deuteron reactions and tritium β-decay.

Once again, while $L_{1,A}$ and $L_{2,A}$ occur at the same order in the $\text{EFT}_{\not\pi}$ power counting and are thus expected to be roughly the same size, the results concerning the magnetic LECs suggest that it could be rather the LECs in the large-N_c basis instead that might natural. Using, for example, the NPLQCD value in the large-N_c relationship of Eq. (5.30) with $N_c = 3$ would result in

$$L_{2,A} \sim 0.1625\,\text{fm}^3\,, \tag{5.33}$$

which would provide even stronger justification for neglecting this contribution in the processes that have been considered.

In Ref. [147], the two-nucleon magnetic and axial LECs for $\text{EFT}_{\not\pi}$ in a finite volume are matched to lattice data. Each LEC is extracted in at least two ways. For example, $L_{2,A}$ is fit to the matrix element of the axial current between deuteron states as well as between triton states. While the calculations in Ref. [147] are performed using a Gaussian regulator and can not be compared directly to the fit in PDS, it is worthwhile comparing the relative sizes of LECs from the finite volume calculation to the large-N_c predictions. Note, however, that the fits are for regions of allowed values consistent with the lattice data.

The allowed regions for ${}^{\not\pi}L_1$ and ${}^{\not\pi}L_2$ in Ref. [147] appear to be consistent with the $1/8N_c$ suppression up to a relative factor of about 2. Additionally, $L_{2,A}$ does indeed

appear to suppressed relative to $L_{1,A}$ for certain regions of the parameter space. It is noted that there is a mild tension in the values extracted for $L_{2,A}$ from $np \to d$ matrix elements and ^{3}H matrix elements. Reference [147] suggests that this tension indicates "the potential need for higher-order terms in the EFT description."

5.2 Lepton Number Violation and Isospin Violation

Several experimental efforts are centered around the search for lepton number violation (LNV). In particular, LNV in the mode of neutrinoless double beta decay would unequivocally demonstrate that neutrinos are Majorana particles [148] while also shedding light on the neutrino mass hierarchy [149, 150] and the matter-antimatter asymmetry in the universe [151].

The inverse of the $0\nu\beta\beta$ half-life can be expressed as (see Refs. [152–154] for reviews)

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}|M_{0\nu}|^2 m_{\beta\beta}^2\,, \tag{5.34}$$

where $m_{\beta\beta}$ is the effective Majorana neutrino mass, $G_{0\nu}$ is a phase space factor, and $M_{0\nu}$ is the corresponding nuclear matrix element (NME). Clearly, a thorough understanding of the NME is required in order to interpret experimental results. In order to obtain this understanding, it is necessary to have a strong grasp on the multi-nucleon operators that are used in many-body calculations.

It has been repeatedly emphasized that the EFTs of interest here constitute model-independent approaches to nuclear physics. An initial step towards the application of EFT to $0\nu\beta\beta$ was taken in Ref. [155] in the context of chiral effective field theory; although, it was also assumed on the grounds of Weinberg power counting that a possible two-nucleon contact operator would be suppressed. However, there are multiple examples of contact terms being required at LO for renormalization. An isospin symmetric contact term is required to renormalize the two-nucleon amplitude when the LO in ChEFT one-pion-exchange is dressed by C_0 vertices [31]. Similarly,

an isospin breaking contact term is required for renormalization when Coulomb exchange is included in the two-proton system in EFT$_{\not\pi}$ [156]. Recently, it has been demonstrated that the light-Majorana exchange mechanism for $0\nu\beta\beta$ is no different [135, 157–161]. The contact term in the LO Lagrangian is [160, 161]

$$\mathcal{L}^{NN}_{|\Delta L=2|} = \left(2\sqrt{2}G_F V_{ud}\right)^2 m_{\beta\beta}\bar{e}_L C \bar{e}_L^T \frac{g_\nu^{NN}}{4}\left[\left(\bar{N}u\tilde{Q}_L^w u^\dagger N\right)^2 - \frac{1}{6}\operatorname{Tr}\left(\tilde{Q}_L^w\tilde{Q}_L^w\right)\left(\bar{N}\tau^a N\right)^2\right] + \text{H.c.}\,, \tag{5.35}$$

where e_L is the left-handed electron, the charge conjugation matrix is $C = i\gamma^2\gamma^0$, G_F is the Fermi constant, V_{ud} is an element of the Cabibbo-Kobayashi-Maskawa matrix, and

$$\tilde{Q}_L^w = \tau^+ = \frac{1}{2}\left(\tau^1 + i\tau^2\right)\;. \tag{5.36}$$

The matrix u is

$$u = \exp\left(\frac{i}{2F}\phi_a\tau^a\right), \tag{5.37}$$

where the ϕ_a $(a = 1, 2, 3)$ are the pion fields in Cartesian coordinates, the τ^a are Pauli matrices in isospin space, and F is the pion decay constant in the chiral limit. The renormalization group (RG) requirement to include a contact term at LO means that an additional unknown LEC, g_ν^{NN}, must be determined in order to analyze and interpret current and future measurements of $0\nu\beta\beta$ decay.

As previously mentioned, this isospin violating counterterm is analogous to other required contact terms, but it is connected by chiral symmetry to charge independence breaking (CIB) in the two-nucleon force [159–161]. When the electromagnetic mass splitting of the pion is included in OPE dressed by C_0 vertices, which leads to CIB in the two-nucleon force, there is a logarithmic divergence that requires the promotion of an isotensor contact term to LO for renormalization. The CIB counterterm Lagrangian has been presented in Refs. [162–164], and in Ref. [161] it is written as

$$\mathcal{L}^{NN}_{CIB} = \frac{e^2}{4}\left\{\mathcal{C}_1\left[\left(\bar{N}u^\dagger\tilde{Q}_R u N\right)^2 + \left(\bar{N}u\tilde{Q}_L u^\dagger N\right)^2 - \frac{1}{6}\operatorname{Tr}\left(\tilde{Q}_L^2 + \tilde{Q}_R^2\right)\left(\bar{N}\tau^a N\right)^2\right]\right.$$

$$+\mathcal{C}_2\left[2\left(\bar{N}u^\dagger\tilde{Q}_R uN\right)\left(\bar{N}u\tilde{Q}_L u^\dagger N\right)-\frac{1}{3}\,\mathrm{Tr}\!\left(U\tilde{Q}_L U^\dagger\tilde{Q}_R\right)\left(\bar{N}\tau^a N\right)^2\right]\Bigg\},\tag{5.38}$$

where $U = u^2$ and here

$$\tilde{Q}_L = \tilde{Q}_R = \frac{1}{2}\tau^3.\tag{5.39}$$

Specifically, chiral symmetry dictates $g_\nu^{NN} = \mathcal{C}_1$ [160, 161]. It is also worth noting that the Argonne v_{18} potential [56], the CD-Bonn potential [57], and several potentials derived from ChEFT [81, 165–167] include short-range CIB and CSB operators in order to reproduce scattering data.

Presently, it is not possible to disentangle effects from $\mathcal{C}_1$ and $\mathcal{C}_2$. In particular, only $\mathcal{C}_1 + \mathcal{C}_2$ is constrained by available data, it can be determined from two-nucleon scattering data, while $\mathcal{C}_1 - \mathcal{C}_2$ is sensistive to two-nucleon-multi-pion interactions and is thus inaccessible. In order to estimate the impact of g_ν^{NN} in NMEs, Refs. [160, 161] assumed that $g_\nu^{NN} = \frac{1}{2}(\mathcal{C}_1 + \mathcal{C}_2)$, which amounts to the assumption that $\mathcal{C}_1 \approx \mathcal{C}_2$, i.e. the CIB LECs are about the same size and sign.

Lattice QCD calculations of double-β decay matrix elements are beginning to appear [168, 169] as well as $0\nu\beta\beta$ matrix elements in the meson sector [169, 170] (see also Refs. [171, 172] and Ref. [173] for a general review). However, there are currently no first-principles determinations of g_ν^{NN}. Therefore, it is necessary to obtain theoretical constraints from other means. Recently, Refs. [174, 175] calculated $\mathcal{C}_1 + \mathcal{C}_2$ and $\mathcal{C}_1$ using a method analogous to the Cottingham formula [176, 177]. These results support the assumption $g_\nu^{NN} = \frac{1}{2}(\mathcal{C}_1 + \mathcal{C}_2)$. In this section, the spurion analysis of Sec. 4.5 is applied as an independent check of the consistency of this assumption.

5.2.1 Large-N_c hierarchy of charge dependent nuclear forces

In the derivation of Eqs. (4.287), a large-N_c hierarchy of the classes of charge dependence in nuclear forces [178, 179] arises. The classes of the forces are defined as:

(I) isospin invariant and charge symmetric: $\mathbb{1}_1\mathbb{1}_2$, $\vec{\tau}_1 \cdot \vec{\tau}_2$,

(II) CIB but not charge-symmetry-breaking (CSB), which have the isotensor form: $\tau_1^3\tau_2^3 - \frac{1}{3}\vec{\tau}_1 \cdot \vec{\tau}_2$,

(III) CSB (and thus CIB) terms that are symmetric in spin and isospin indices: $\tau_1^3 + \tau_2^3$,

(IV) CSB with isospin mixing (these vanish on *nn* and *pp* systems, but not *np*, and only occur in $L \neq 0$ partial waves): $\tau_1^3 - \tau_2^3$, $(\vec{\tau}_1 \times \vec{\tau}_2)^3$,

where the subscripts above denote nucleon bilinears one and two. Previously, the relative sizes of the classes have been estimated through phenomenological considerations and meson exchange models [164, 179, 180]. The hierarchy based on their observations is that Class (I) > Class (II) > Class (III) > Class (IV).

The operators derived in Sec. 4.5.1 fall into the categories

$$\text{(I)} \qquad \mathcal{O}_{1,1},\ \mathcal{O}_{1,2}\ , \tag{5.40}$$

$$\text{(II)} \qquad \mathcal{O}_{3}\ , \tag{5.41}$$

$$\text{(III)} \qquad \mathcal{O}_{2}\,. \tag{5.42}$$

The large-N_c analysis of these operators indicates that Class (I) and (II) interactions appear at the same order in the large-N_c expansion. However, it should be noted that these operators occur at $O(e^2)$, i.e. the operators in Class (I) here are $O(e^2)$ corrections to the strong component of isospin symmetric interactions. Class (III) terms are suppressed by $1/N_c$ relative to Classes (I) and (II). Class (IV) operators, which are not shown explicitly, scale at most as $O(N_c^0)$, but as noted above they only occur in $L \neq 0$ partial waves. Therefore, these operators have derivatives and will be suppressed in the ChEFT power counting. In addition, the operators in Class (IV) in Ref. [179] are related through Fierz transformations, and are thus not independent in the two-nucleon Lagrangian. However, it has also been discussed that the impact

Fierz transformations on few-body calculations with local regulators is ambiguous [181–183]. Regardless, the large-N_c hierarchy of these forces is in line with previous estimates [164, 179, 180].

5.2.2 Large-N_c consistency of lepton number violating and charge-independence-breaking contact terms

Here, the consistency of the approximation $g_\nu^{NN} = \frac{1}{2}(\mathcal{C}_1 + \mathcal{C}_2)$ is demonstrated from the large-N_c point of view. If these LECs were not of the same order in N_c, then the incosistency could cast doubt on the approximation. The Lagrangian in Eq. (4.311) can be rearranged as

$$\begin{aligned}\mathcal{L}_{\text{LO-in-}N_c} =& e^2 \left\{ \frac{1}{2}\left[2\bar{\mathcal{C}}_{1,1} + \bar{\mathcal{C}}_{12,1} - \bar{\mathcal{C}}_3\right] \mathrm{Tr}\left(\tilde{Q}_+^2\right)\left(N^\dagger N\right)^2 \right. \\ & \left. + \bar{\mathcal{C}}_3 \left[\left(N^\dagger \sigma^i \tilde{Q}_+ N\right)^2 - \frac{1}{6}\mathrm{Tr}\left(\tilde{Q}_+^2\right)\left(N^\dagger \sigma^i \tau^a N\right)^2\right]\right\} ,\end{aligned} \tag{5.43}$$

where the second term proportional to $\bar{\mathcal{C}}_3$ is now a symmetric traceless isotensor. The included trace term appears at the same order in the large-N_c expansion. This rearrangement also modifies the NLO-in-N_c Lagrangian in Eq.(4.312) so that it becomes

$$\begin{aligned}\mathcal{L}_{\text{NLO-in-}N_c} =& e^2 \left\{ \frac{1}{2}\left[2\bar{\mathcal{C}}_{1,1} - \bar{\mathcal{C}}_{12,1} - \bar{\mathcal{C}}_6\right] \mathrm{Tr}\left(\tilde{Q}_-^2\right)\left(N^\dagger N\right)^2 + \bar{\mathcal{C}}_2\left(N^\dagger N\right)\left(N^\dagger \tilde{Q}_+ N\right) \right. \\ & \left. + \bar{\mathcal{C}}_6 \left[\left(N^\dagger \sigma^i \tilde{Q}_- N\right)^2 - \frac{1}{6}\mathrm{Tr}\left(\tilde{Q}_-^2\right)\left(N^\dagger \sigma^i \tau^a N\right)^2\right]\right\} .\end{aligned} \tag{5.44}$$

Again, the third term is a symmetric traceless isotensor. Both of the isotensor terms can be rewritten through Fierz transformations,

$$\mathcal{L}^{\Delta I=2}_{\text{LO-in-}N_c} = -3e^2 \bar{\mathcal{C}}_3 \left[\left(N^\dagger \tilde{Q}_+ N\right)^2 - \frac{1}{6}\mathrm{Tr}\left(\tilde{Q}_+^2\right)\left(N^\dagger \tau^a N\right)^2\right] , \tag{5.45}$$

$$\mathcal{L}^{\Delta I=2}_{\text{NLO-in-}N_c} = -3e^2 \bar{\mathcal{C}}_6 \left[\left(N^\dagger \tilde{Q}_- N\right)^2 - \frac{1}{6}\mathrm{Tr}\left(\tilde{Q}_-^2\right)\left(N^\dagger \tau^a N\right)^2\right] . \tag{5.46}$$

The Lagrangian of Eq. (5.38) can be written as

$$\mathcal{L}^{NN}_{CIB} = \frac{e^2}{2}\left\{(\mathcal{C}_1 + \mathcal{C}_2)\left[\left(N^\dagger \tilde{Q}_+ N\right)^2 - \frac{1}{6}\mathrm{Tr}\left(\tilde{Q}_+^2\right)\left(N^\dagger \tau^a N\right)^2\right]\right.$$

$$+ (\mathcal{C}_1 - \mathcal{C}_2) \left[\left(N^\dagger \tilde{Q}_- N \right)^2 - \frac{1}{6} \mathrm{Tr}\left(\tilde{Q}_-^2 \right) \left(N^\dagger \tau^a N \right)^2 \right] \Bigg\} \, . \tag{5.47}$$

Direct comparison with Eqs. (5.45) and (5.46) shows that

$$\frac{1}{2} (\mathcal{C}_1 + \mathcal{C}_2) = -3\bar{\mathcal{C}}_3 \, , \tag{5.48}$$

$$\frac{1}{2} (\mathcal{C}_1 - \mathcal{C}_2) = -3\bar{\mathcal{C}}_6 \, , \tag{5.49}$$

which demonstrates that $\mathcal{C}_1 + \mathcal{C}_2 \sim N_c$ and $\mathcal{C}_1 - \mathcal{C}_2 \sim O(1)$, i.e. the difference of the two LECs is $1/N_c$ suppressed relative to the sum. This implies that $\mathcal{C}_1$ and $\mathcal{C}_2$ are of the same size and sign. This conclusion is easier to see if we invert Eqs. (5.48) and (5.49),

$$\mathcal{C}_1 = -3\bar{\mathcal{C}}_3 - 3\bar{\mathcal{C}}_6 = -3\bar{\mathcal{C}}_3 \left[1 + O(1/N_c) \right] , \tag{5.50}$$

$$\mathcal{C}_2 = -3\bar{\mathcal{C}}_3 + 3\bar{\mathcal{C}}_6 = -3\bar{\mathcal{C}}_3 \left[1 + O(1/N_c) \right] . \tag{5.51}$$

Recall that g_ν^{NN} is $O(N_c)$ as shown in Sec. 4.5.2. Thus, $\mathcal{C}_1 = \mathcal{C}_2$ up to 30% corrections, which supports the assumption of Ref. [161] that $g_\nu^{NN} \approx \frac{1}{2}(\mathcal{C}_1 + \mathcal{C}_2)$.

As previously mentioned, Refs. [174, 175] present a method analogous to the Cottingham formula [176, 177] to determine $\mathcal{C}_1$ or equivalently g_ν^{NN}. This paragraph recapitulates the results of these references. First, dimensionless couplings are defined as

$$\tilde{\mathcal{C}}_i = \left(\frac{m_N}{4\pi} C \right)^2 \mathcal{C}_i \, , \tag{5.52}$$

where $C = C_0$ in EFT$_{\not\pi}$ and $C = C_0 + \frac{g_A^2}{4\pi F^2}$ in ChEFT and $i = 1, 2$. The CIB LECs, which depend on the renormalization scheme, are then determined in minimal subtraction at $\mu = m_\pi$,

$$\tilde{\mathcal{C}}_1(\mu = m_\pi) = 1.3(6) \, , \tag{5.53}$$

$$\left(\tilde{\mathcal{C}}_1 + \tilde{\mathcal{C}}_2 \right) (\mu = m_\pi) = 2.9(1.2) \, . \tag{5.54}$$

Now, if only the central values of the LECs are considered, then $\tilde{\mathcal{C}}_1\,(\mu = m_\pi) = 1.3$ and $\tilde{\mathcal{C}}_2 = 1.6$. Then the ratio of the LECs is

$$\frac{\tilde{\mathcal{C}}_1}{\tilde{\mathcal{C}}_2} \approx 0.81\,, \tag{5.55}$$

which is within 30% corrections of 1 and in line with the large-N_c prediction. However, it should be noted that these LECs have different RG runnings [161], so there is the potential that this relationship is violated for different choices of the subtraction point μ. Although, it is possible that RG running of this ratio is mild.

5.3 Dark Matter Direct Detection

The detection of dark matter constitutes a major experimental effort in searches for Beyond the Standard Model physics. Direct detection experiments, which measure the recoil of nuclei ranging from fluorine to xenon, are able to provide some of the most sensitive constraints on dark matter candidates (see, e.g., Refs. [184–194]). Therefore, it is imperative to obtain a clear theoretical picture of the interactions involved in the nuclear matrix elements and the impact on cross sections. While the most stringent constraints on the dark matter-nucleon cross section are from these underground direct detection experiments, cosmological constraints obtained from a variety of sources are maturing rapidly [195–200]. The cosmological constraints aim to obtain constraints on the dark matter-proton cross section directly without employing larger nuclei. Recently, Ref. [201] emphasized that extra care is needed when comparing the limits obtained from different sources based on assumptions regarding the scaling of a cross section with the mass number A. There it was also suggested that detectors employing light nuclei can be of some utility. Additionally, the use of helium isotopes as direct detection targets has been proposed in several other works [202–206].

Several EFT studies in this vein have been conducted in chiral EFT [207–215], a nonrelativistic EFT for the nucleus as a whole [216], and a nonrelativistic EFT for single nucleons [217, 218]. For elastic dark-matter scattering off of light nuclei, the momentum transfer has an upper bound of a few MeV for light nuclei; therefore, EFT$_{\not\pi}$ should be ideal for these systems. An advantage of this approach is that it has a well understood power counting in few-nucleon systems and it allows for calculations involving the deuteron to be performed analytically [71]. Additionally, two-nucleon currents in chiral EFT studies are generally suppressed in the EFT power counting. However, it has been pointed out throughout this text that two-nucleon currents that connect two S-wave states typically occur at next-to-leading order (NLO) with an infrared enhancement [51] and can therefore have a sizeable impact.

The basis of operators in EFT$_{\not\pi}$ is similar to the basis found in Ref. [217], and the operators are distinguished by being either WIMP spin independent (SI) or spin dependent (SD). In EFT$_{\not\pi}$, the LO contributions consist of single-nucleon zero-derivative currents coupled to the external dark matter current. The WIMP current consists of bilinears of the form $\chi^\dagger\Gamma\chi$, where $\Gamma = 1$ for a spin-0 particle and $\Gamma = \mathbb{1}$ or σ^i for a spin-1/2 particle. Therefore, the single-nucleon Lagrangian is

$$\begin{aligned}\mathcal{L}_{\chi N}^{(PT)} = &C_{1,\chi N}^{(PT)}\left(N^\dagger N\right)\left(\chi^\dagger\chi\right) + C_{2,\chi N}^{(PT)}\left(N^\dagger\sigma^i\tau^3 N\right)\left(\chi^\dagger\sigma^i\chi\right)\\ &+ C_{3,\chi N}^{(PT)}\left(N^\dagger\sigma^i N\right)\left(\chi^\dagger\sigma^i\chi\right) + C_{4,\chi N}^{(PT)}\left(N^\dagger\tau^3 N\right)\left(\chi^\dagger\chi\right). \end{aligned} \tag{5.56}$$

The superscript on the left hand side of Eq. (5.56) indicates that this portion of the Lagrangian is invariant under parity and time-reversal separately. This Lagrangian corresponds to those from Eqs. (4.143), (4.214), and (4.216) with $S = \chi^\dagger\chi$ and

$A^i = \chi^\dagger \sigma^i \chi$, and with the LECs replaced according to

$$\begin{aligned}
\not{T} g_S^{(s)} &\to C_{1,\chi N}^{(PT)}\,, \\
\not{T} g_{A^i}^{(v)} &\to C_{2,\chi N}^{(PT)}\,, \\
\not{T} g_{A^i}^{(s)} &\to C_{3,\chi N}^{(PT)}\,, \\
\not{T} g_S^{(v)} &\to C_{4,\chi N}^{(PT)}\,.
\end{aligned} \tag{5.57}$$

Recall the large-N_c scalings of the LECs

$$C_{1,\chi N}^{(PT)},\ C_{2,\chi N}^{(PT)} \sim O(N_c)\,, \tag{5.58}$$

$$C_{3,\chi N}^{(PT)},\ C_{4,\chi N}^{(PT)} \sim O(1)\,, \tag{5.59}$$

At NLO in the single-nucleon sector, there are $\not{P}T$ and $\not{P}\not{T}$ operators containing one derivative. These operators will consist of a product of a nucleon axial current with a dark matter vector current and other similar combinations. As discussed at the end of Sec. 2.2.3, Galilean invariance or invariance under infinitesimal Lorentz transformations restricts the operators to appear in certain linear combinations, some of which will contain inverse factors of m_N or m_χ [86, 218]. The $\not{P}T$ component of the Lagrangian is

$$\begin{aligned}
\mathcal{L}_{\chi N}^{(\not{P}T)} = {}& C_{5,\chi N}^{(\not{P}T)} \epsilon^{ijk} \nabla^j \left(N^\dagger \sigma^k \tau^3 N\right)\left(\chi^\dagger \sigma^i \chi\right) + C_{6,\chi N}^{(\not{P}T)} \epsilon^{ijk} \nabla^j \left(N^\dagger \sigma^k N\right)\left(\chi^\dagger \sigma^i \chi\right) \\
&+ i C_{7,\chi N}^{(\not{P}T)} \left[\frac{1}{2m_N}\left(N^\dagger \overleftrightarrow{\nabla}^i \sigma^i \tau^3 N\right)\left(\chi^\dagger \chi\right) + \frac{1}{2m_\chi}\left(N^\dagger \sigma^i \tau^3 N\right)\left(\chi^\dagger \overleftrightarrow{\nabla}^i \chi\right)\right] \\
&+ i C_{8,\chi N}^{(\not{P}T)} \left[\frac{1}{2m_N}\left(N^\dagger \overleftrightarrow{\nabla}^i N\right)\left(\chi^\dagger \sigma^i \chi\right) + \frac{1}{2m_\chi}\left(N^\dagger N\right)\left(\chi^\dagger \overleftrightarrow{\nabla}^i \sigma^i \chi\right)\right] \\
&+ i C_{9,\chi N}^{(\not{P}T)} \left[\frac{1}{2m_N}\left(N^\dagger \overleftrightarrow{\nabla}^i \sigma^i N\right)\left(\chi^\dagger \chi\right) + \frac{1}{2m_\chi}\left(N^\dagger \sigma^i N\right)\left(\chi^\dagger \overleftrightarrow{\nabla}^i \chi\right)\right] \\
&+ i C_{10,\chi N}^{(\not{P}T)} \left[\frac{1}{2m_N}\left(N^\dagger \overleftrightarrow{\nabla}^i \tau^3 N\right)\left(\chi^\dagger \sigma^i \chi\right) + \frac{1}{2m_\chi}\left(N^\dagger \tau^3 N\right)\left(\chi^\dagger \overleftrightarrow{\nabla}^i \sigma^i \chi\right)\right]\,,
\end{aligned} \tag{5.60}$$

where the LECs scale as

$$C_{5,\chi N}^{(\not{P}T)} \sim O(N_c)\,, \tag{5.61}$$

$$C_{8,\chi N}^{(\not{P}T)},\ C_{7,\chi N}^{(\not{P}T)} \sim O(1) + O\left(\frac{N_c}{m_\chi}\right)\,, \tag{5.62}$$

$$C_{6,\chi N}^{(\not{P}T)} \sim O(1)\,, \tag{5.63}$$

$$C_{9,\chi N}^{(\not{P}T)},\; C_{10,\chi N}^{(\not{P}T)} \sim O\left(\frac{1}{N_c}\right) + O\left(\frac{1}{m_\chi}\right)\,. \tag{5.64}$$

The $\not{P}\not{T}$ component of the Lagrangian is

$$\begin{aligned}
\mathcal{L}_{\chi N}^{\not{P}\not{T}} = {} & C_{11,\chi N}^{(\not{P}\not{T})}\nabla^i\left(N^\dagger\sigma^i\tau^3 N\right)\left(\chi^\dagger\chi\right) + C_{12,\chi N}^{(\not{P}\not{T})}\nabla^i\left(N^\dagger N\right)\left(\chi^\dagger\sigma^i\chi\right) \\
& + C_{13,\chi N}^{(\not{P}\not{T})}\nabla^i\left(N^\dagger\tau^3 N\right)\left(\chi^\dagger\sigma^i\chi\right) + C_{14,\chi N}^{(\not{P}\not{T})}\nabla^i\left(N^\dagger\sigma^i N\right)\left(\chi^\dagger\chi\right) \\
& + i\epsilon^{ijk}C_{15,\chi N}^{(\not{P}\not{T})}\left[\frac{1}{2m_N}\left(N^\dagger\overleftrightarrow{\nabla}^j\sigma^k\tau^3 N\right)\left(\chi^\dagger\sigma^i\chi\right) + \frac{1}{2m_\chi}\left(N^\dagger\sigma^k\tau^3 N\right)\left(\chi^\dagger\overleftrightarrow{\nabla}^j\sigma^i\chi\right)\right] \\
& + i\epsilon^{ijk}C_{16,\chi N}^{(\not{P}\not{T})}\left[\frac{1}{2m_N}\left(N^\dagger\overleftrightarrow{\nabla}^j\sigma^k N\right)\left(\chi^\dagger\sigma^i\chi\right) + \frac{1}{2m_\chi}\left(N^\dagger\sigma^k N\right)\left(\chi^\dagger\overleftrightarrow{\nabla}^j\sigma^i\chi\right)\right]\,,
\end{aligned} \tag{5.65}$$

where the LECs scale as

$$C_{11,\chi N}^{(\not{P}\not{T})},\; C_{12,\chi N}^{(\not{P}\not{T})} \sim O(N_c)\,, \tag{5.66}$$

$$C_{15,\chi N}^{(\not{P}\not{T})} \sim O(1) + O\left(\frac{N_c}{m_\chi}\right)\,, \tag{5.67}$$

$$C_{13,\chi N}^{(\not{P}\not{T})},\; C_{14,\chi N}^{(\not{P}\not{T})} \sim O(1)\,, \tag{5.68}$$

$$C_{16,\chi N}^{(\not{P}\not{T})} \sim O(1/N_c) + O\left(\frac{1}{m_\chi}\right)\,. \tag{5.69}$$

In principle, two-nucleon currents can also contribute at NLO. The leading two-nucleon currents will have no derivatives and can be found in Secs. 4.2 and 4.4,

$$\begin{aligned}
\mathcal{L}_{\chi NN} = {} & C_{1,\chi NN}^{(s)}\left(\chi^\dagger\chi\right)\left(N^\dagger N\right)\left(N^\dagger N\right) + C_{2,\chi NN}^{(s)}\left(\chi^\dagger\chi\right)\left(N^\dagger\sigma^i N\right)\left(N^\dagger\sigma^i N\right) \\
& + C_{3,\chi NN}^{(s)}\left(\chi^\dagger\sigma^i\chi\right)\left(N^\dagger\sigma^i N\right)\left(N^\dagger N\right) + C_{1,\chi NN}^{(v)}\left(\chi^\dagger\chi\right)\left(N^\dagger\tau^3 N\right)\left(N^\dagger N\right) \\
& + \epsilon^{ijk}\epsilon^{3ab}C_{2,\chi NN}^{(v)}\left(\chi^\dagger\sigma^i\chi\right)\left(N^\dagger\sigma^j\tau^a N\right)\left(N^\dagger\sigma^k\tau^b N\right) \\
& + C_{\chi NN}^{(t)}\left(\chi^\dagger\chi\right)\left[\left(N^\dagger\sigma^i\tau^3 N\right)\left(N^\dagger\sigma^i\tau^3 N\right) - \frac{1}{3}\left(N^\dagger\sigma^i\tau^a N\right)\left(N^\dagger\sigma^i\tau^a N\right)\right] \\
& + C_{3,\chi NN}^{(v)}\epsilon^{ijk}\left(\chi^\dagger\sigma^i\chi\right)\left(N^\dagger\sigma^j\tau^3 N\right)\left(N^\dagger\sigma^k N\right)\,.
\end{aligned} \tag{5.70}$$

All of the operators except for the last term are invariant under both parity and time-reversal; the last term violates time-reversal-invariance but respects parity. In

the partial wave basis this is

$$
\begin{aligned}
\mathcal{L}_{\chi NN} = & \, 2\left(C^{(s)}_{1,\chi NN} + C^{(s)}_{2,\chi NN}\right)\left(\chi^\dagger\chi\right)\left[\left(N^T P^i N\right)^\dagger \left(N^T P^i N\right)\right] \\
& + 2\left(C^{(s)}_{1,\chi NN} - 3C^{(s)}_{2,\chi NN}\right)\left(\chi^\dagger\chi\right)\left[\left(N^T \bar{P}^a N\right)^\dagger \left(N^T \bar{P}^a N\right)\right] \\
& + 12 C^{(t)}_{1,\chi NN}\left(\chi^\dagger\chi\right)\left[\left(N^T \bar{P}^3 N\right)^\dagger \left(N^T \bar{P}^3 N\right) - \frac{1}{3}\left(N^T \bar{P}^a N\right)^\dagger \left(N^T \bar{P}^a N\right)\right] \\
& + 8 C^{(v)}_{1,\chi NN}\left(\chi^\dagger\sigma^i\chi\right)\left[\left(N^T \bar{P}^3 N\right)^\dagger \left(N^T P^i N\right) + \text{H.c.}\right] \\
& - 2i\epsilon^{ijk} C^{(s)}_{3,\chi NN}\left(\chi^\dagger\sigma^i\chi\right)\left(N^T P^j N\right)^\dagger \left(N^T P^k N\right) \\
& - 2i\epsilon^{3ab} C^{(v)}_{1,\chi NN}\left(\chi^\dagger\chi\right)\left(N^T \bar{P}^i N\right)^\dagger \left(N^T \bar{P}^b N\right) \\
& + 4\left(\chi^\dagger\sigma^i\chi\right) C^{(v)}_{3,\chi NN}\left[i\left(N^T P^i N\right)^\dagger \left(N^T \bar{P}^a N\right) + \text{H.c}\right].
\end{aligned}
\tag{5.71}
$$

The first two terms differ only at $O(1/N_c^2)$; therefore, the SI isoscalar dark matter-two-nucleon interaction has the same strength at LO in the large-N_c expansion for two nucleons combined in both the $^1S_0$ and $^3S_1$ states.

Before presenting the results for nucleons and the deuteron, it is useful to briefly review the formalism for the dark matter-nucleus cross section. The differential cross section for dark matter scattering off of a nucleus initially at rest is related to the scattering amplitude $\mathcal{M}$ through

$$
\frac{d\sigma}{dE_R} = \frac{1}{32\pi m_\chi^2 m_T v_\chi^2}|\mathcal{M}|^2 \,, \tag{5.72}
$$

where E_R is the recoil energy, m_T is the mass of the target nucleus, and v_χ is the dark matter velocity in the lab frame. Since we take the nucleus to be at rest initially, the recoil energy can be expressed as $E_R = q^2/2m_T$, where q is the momentum transfer. Alternatively, the differential cross section can be expressed in terms of SI and SD components as

$$
\frac{d\sigma}{dE_R} = \frac{m_T}{2m_r^2 v_\chi^2}\left[\sigma_0^{\text{SI}} F_{\text{SI}}^2\left(E_R\right) + \sigma_0^{\text{SD}} F_{\text{SD}}^2\left(E_R\right)\right] \,, \tag{5.73}
$$

where m_r is the dark matter-nucleus reduced mass, $\sigma_0^{\text{SI}\,(\text{SD})}$ is the total SI (SD) cross section at zero momentum transfer, and $F^2_{\text{SI}\,(\text{SD})}\left(E_R\right)$ is the SI (SD) nuclear form

factor. Equating the left hand sides of Eqs. (5.72) and (5.73) leads to

$$|\mathcal{M}|^2 = 16\pi \left(m_T + m_\chi\right)^2 \left[\sigma_0^{\rm SI} F_{\rm SI}^2\left(E_R\right) + \sigma_0^{\rm SD} F_{\rm SD}^2\left(E_R\right)\right] . \tag{5.74}$$

Therefore, the calculation of the amplitude facilitates the calculation of either the differential cross section or the form factors. In the following, we calculate the unpolarized cross sections; thus, the amplitude in Eqs. (5.72) and (5.74) should also include the average over incoming spins and the sum over outgoing spins.

Consider an incoming (outgoing) nucleon with spin ν (μ), isospin b (a), and momentum $\mathbf{p} = 0$ ($\mathbf{p}' = \mathbf{q}$) and an incoming (outgoing) dark matter particle with spin r (s) and momentum $\mathbf{k}$ ($\mathbf{k}'$). At leading order in the EFT$_{\not\pi}$ expansion, the amplitude only receives contributions from Eq. (5.56). The unpolarized amplitude can be expressed as

$$\frac{1}{4}\sum_{\rm spins} |\mathcal{M}|^2 = 16 m_N^2 m_\chi^2 \left[\left(C_{1,\chi N}^{(PT)} \pm C_{4,\chi N}^{(PT)}\right)^2 + 3\left(C_{2,\chi N}^{(PT)} \pm C_{3,\chi N}^{(PT)}\right)^2\right] , \tag{5.75}$$

where the upper signs corresponds to proton-dark matter scattering and lower signs correspond to neutron-dark matter scattering. Additionally, the first term contains the SI interactions while the second term contains the SD interactions. From Eq. (5.74), we find

$$\left(C_{1,\chi N}^{(PT)} \pm C_{4,\chi N}^{(PT)}\right)^2 = \frac{\pi}{m_{\chi N}^2}\sigma_{0,\,p/n}^{\rm SI} F_{{\rm SI},\,p/n}^2(q^2)\,, \tag{5.76}$$

$$\left(C_{2,\chi N}^{(PT)} \pm C_{3,\chi N}^{(PT)}\right)^2 = \frac{\pi}{3m_{\chi N}^2}\sigma_{0,\,p/n}^{\rm SD} F_{{\rm SD},\,p/n}^2(q^2)\,, \tag{5.77}$$

where $m_{\chi N}$ is the dark matter-nucleon reduced mass. As $q^2 \to 0$, the form factors approach one. Additionally, the higher order terms introduce explicit momentum dependence and will vanish in this limit. Therefore, at $q^2 = 0$ we have

$$\left(C_{1,\chi N}^{(PT)} \pm C_{4,\chi N}^{(PT)}\right)^2 = \frac{\pi}{m_{\chi N}^2}\sigma_{0,\,p/n}^{\rm SI}\,, \tag{5.78}$$

$$\left(C_{2,\chi N}^{(PT)} \pm C_{3,\chi N}^{(PT)}\right)^2 = \frac{\pi}{3m_{\chi N}^2}\sigma_{0,\,p/n}^{\rm SD}\,, \tag{5.79}$$

which is exact. Therefore, at LO-in-N_c the ratio of the cross sections is

$$\frac{\sigma_{0,N}^{\mathrm{SI}}}{\sigma_{0,N}^{\mathrm{SD}}} = \frac{1}{3}\left[\frac{C_{1,\chi N}^{(PT)}\left(1 \pm \frac{C_{4,\chi N}^{(PT)}}{C_{1,\chi N}}\right)}{C_{2,\chi N}^{(PT)}\left(1 \pm \frac{C_{3,\chi N}^{(PT)}}{C_{2,\chi N}^{(PT)}}\right)}\right]^2 \approx \frac{1}{3}\left(\frac{C_{1,\chi N}^{(PT)}}{C_{2,\chi N}^{(PT)}}\right)^2 \sim \frac{1}{3}\,, \tag{5.80}$$

for all proton and neutron combinations if both $C_{1,\chi N}^{(PT)}$ and $C_{2,\chi N}^{(PT)}$ are considered to be of natural size apart from their N_c scalings. We expect this ratio to receive roughly 30% corrections at NLO in the large-N_c expansion.

This result stands in contrast with the relative sizes of the cross section bounds from direct detection experiments (see for example Refs. [187–189]). However, drawing a clear comparison between this result and the experimental analyses is not straightforward due to the different assumptions involved in the analyses. For example, many analyses place constraints on the SD cross section by considering either proton only interactions or neutron only interactions, but the proton and neutron couplings at LO in the large-N_c expansion are equal and opposite again up to 30% corrections.

On one hand, despite the assumptions in SD analyses, the assumption in direct detection analyses that the SI interaction is isoscalar is not inconsistent with the LO-in-N_c cross section. Thus, a LO-in-N_c upper bound of $\left|C_{1,\chi N}^{(PT)}\right|_{\text{LO-in-}N_c}$ can be determined from the zero momentum transfer cross section in, e.g., Ref. [189]. However, on the other hand, there are "xenonphobic" scenarios for isospin violating dark matter, where the canonical ratio of the couplings is $|C_{i,\chi n}/C_{i,\chi p}| \approx 0.7$ [219–222]. In this case, the ratio of the couplings in the isospin basis is within the large-N_c bounds, but these scenarios require more stringent analyses of the experimental data. For instance, Ref. [219] examined the relief that isospin violating dark matter can provide for the tension between several direct detection experiments.

Before calculating the WIMP-deuteron cross section, we review the technology developed in Ref. [71] for calculating amplitudes involving the deuteron. Elements of

Figure 5.2: The irreducible two-point function. The solid lines are nucleons, the crossed circle is the interpolating deuteron field, and the black square is the two-nucleon C_2 vertex.

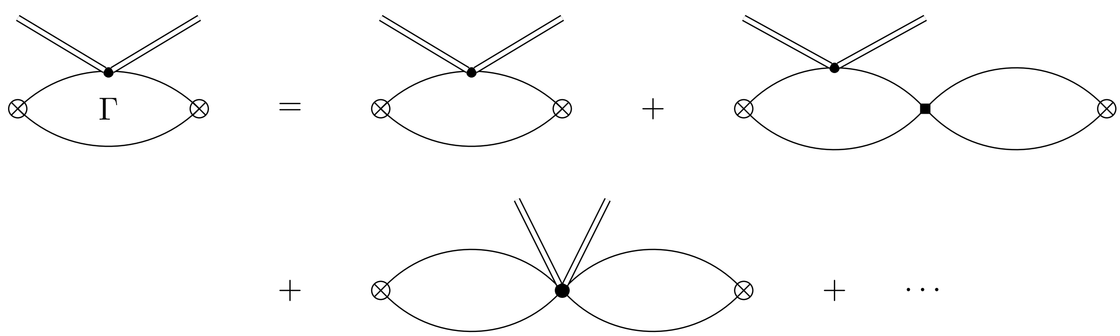

Figure 5.3: The irreducible four point function. The double lines are dark matter, the small black dots are vertices from the dark matter-single-nucleon Lagrangian, and the large black dot is a vertex from the dark matter-two-nucleon Lagrangian.

the S-matrix and thus the scattering amplitude are related to correlation functions through

$$\langle p', m; k', r | \, S \, | p, n; k, s\rangle = i \left[\frac{\Gamma(q^2, \bar{E}, \bar{E}')}{d\Sigma(\bar{E})/dE}\right]_{\bar{E}, \bar{E}' \to -B} , \tag{5.81}$$

where Γ is the irreducible four-point correlation function, Σ is the irreducible two-point function, p (p') is the momentum of the incoming (outgoing) deuteron, m (n) is the polarization of the incoming (outgoing) deuteron, k (k') is the momentum of the incoming (outgoing) dark matter, q is the momentum transfer,and s (r) is the incoming (outgoing) projection of spin along the z-axis for spin-1/2 dark matter. The relevant Feynman diagrams can be found in Figs. 5.2 and 5.3. The two-nucleon center-of-mass energy $\bar{E}$ is given by

$$\bar{E} = E - \frac{p^2}{4m_N} , \tag{5.82}$$

where the energy of each nucleon is $E/2$. An analogous relationship holds for the energy of the outgoing deuteron E'.

The functions Σ and Γ are expanded in powers of Q,

$$\Sigma\left(\bar{E}\right) = \sum_{n=1} \Sigma_{(n)}\left(\bar{E}\right) , \tag{5.83}$$

$$\Gamma\left(q^2, \bar{E}, \bar{E}'\right) = \sum_{n=-1} \Gamma_{(n)}\left(q^2, \bar{E}, \bar{E}'\right) , \tag{5.84}$$

where n denotes the order in Q of each term. Since $d/d\bar{E} \sim Q^{-2}$, the derivative of the two-point function begins at $d\Sigma(\bar{E})/d\bar{E} \sim Q^{-1}$. In PDS, the functions for Σ_1 and Σ_2 are

$$\Sigma_{(1)}(\bar{E}) = -\frac{im_N}{4\pi}\left(\mu - \sqrt{-m_N\bar{E} - i\epsilon}\right) , \tag{5.85}$$

$$\Sigma_{(2)}(\bar{E}) = -iC_2 m_N \bar{E}\Sigma^2_{(1)}(\bar{E}) . \tag{5.86}$$

To NLO, Eq. (5.81) will be

$$\langle p', m; k', r| \, S \, |p, n; k, s\rangle = i\left[\frac{\Gamma_{(-1)}}{d\Sigma_{(1)}/d\bar{E}} + \frac{\Gamma_{(0)}\left(d\Sigma_1/d\bar{E}\right) - \Gamma_{(-1)}\left(d\Sigma_{(2)}/d\bar{E}\right)}{\left(d\Sigma_{(1)}/d\bar{E}\right)^2}\right] . \tag{5.87}$$

Thus, the LO contributions come from the isoscalar operators in Eq. (5.56) dressed by C_0 vertices. At NLO, there are contributions from two-derivative two-nucleon operators with an insertion of the single-nucleon currents in Eq. (5.56), from the dark matter-two-nucleon contact terms in Eq. (5.70) or equivalently (5.71), and from the symmetry violating terms in Eqs. (5.60) and (5.65); although, the last contributions are not considered in this work. For the two-nucleon currents, we are only interested in the terms that connect $^3S_1$ states.

Now, we must calculate the contributions from the irreducible four-point diagrams. At LO, we find

$$\Gamma_{(-1)}(q^2, E, E') \;=\; -\frac{m_N^2}{\pi q}\left[i\delta^{rs}\delta^{mn}C_{1,\chi N} + \sigma^i_{rs}\epsilon^{imn}C_{3,\chi N}\right]\tan^{-1}\left(\frac{q}{4\gamma}\right) . \tag{5.88}$$

The one-body contributions at NLO are

$$\Gamma_{(0),1b} = -\frac{2m_N^3}{(4\pi)^2}C_2\left(iC_{1,\chi N}\delta^{mn}\delta^{rs} + C_{3,\chi N}\epsilon^{imn}\sigma^i_{rs}\right)$$

$$\times(\mu-\gamma)\left[\mu-\gamma-\frac{4\gamma^2}{q}\tan^{-1}\left(\frac{q}{4\gamma}\right)\right]. \tag{5.89}$$

Additionally, there are contributions from two-body currents, which yield

$$\Gamma_{(0),2b}=-\left(\frac{m_N}{4\pi}\right)^2(\mu-\gamma)^2\left[-2i\left(C^{(s)}_{1,\chi NN}+C^{(s)}_{2,\chi NN}\right)\delta^{mn}\delta^{rs}-2\epsilon^{imn}C^{(s)}_{3,\chi NN}\sigma^i_{rs}\right]. \tag{5.90}$$

Thus, at NLO the connected S-matrix elements, and therefore the amplitude, is given by

$$\begin{aligned}i\mathcal{M}=4m_d m_\chi&\left\{\frac{8\gamma}{q}\tan^{-1}\left(\frac{q}{4\gamma}\right)\left[i\delta^{rs}\delta^{mn}C^{(PT)}_{1,\chi N}+\sigma^i_{rs}\epsilon^{imn}C^{(PT)}_{3,\chi N}\right]\right.\\&+\frac{m\gamma}{\pi}C_2\left[i\delta^{rs}\delta^{mn}C^{(PT)}_{1,\chi N}+\sigma^i_{rs}\epsilon^{imn}C^{(PT)}_{3,\chi N}\right](\mu-\gamma)^2\left[1-\frac{4\gamma}{q}\tan^{-1}\left(\frac{q}{4\gamma}\right)\right]\\&\left.-\frac{\gamma}{\pi}(\mu-\gamma)^2\left[i\left(C^{(s)}_{1,\chi NN}+C^{(s)}_{2,\chi NN}\right)\delta^{mn}\delta^{rs}+\epsilon^{imn}C^{(s)}_{3,\chi NN}\sigma^i_{rs}\right]\right\}.\end{aligned} \tag{5.91}$$

The μ dependence of the dark matter-two-nucleon couplings is determined by requiring the amplitude to be renormalization group invariant. Because the first two terms in the amplitude are already invariant, the third term should separately be invariant, so

$$\left(C^{(s)}_{1,\chi NN}+C^{(s)}_{2,\chi NN}\right),\ C^{(s)}_{3,\chi NN}\propto(\mu-\gamma)^{-2}. \tag{5.92}$$

Therefore, the familiar infrared enhancement occurs when $\mu\sim Q$, and although it is likely that there is another scale in the problem on the order of the mass of a mediator, this scaling is assumed to justify the inclusion of the two-body current at NLO in this work. It has been discussed that large-N_c constraints for two-nucleon contact terms conflict with the renormalization group for $\mu<m_\pi$ [45]. However, in analogy to two-nucleon magnetic and axial couplings, the RG running of the ratio of the SI and SD couplings should be fairly mild because they individually have the same running. Again, it is possible that these constraints could be obscured through the inclusion of another momentum scale.

Averaging over initial spins and summing over final spins in the squared amplitude yields

$$\frac{1}{6}\sum_{\text{spins}}|\mathcal{M}|^2 = 16m_d^2m_\chi^2\left\{\left[2C_{1,\chi N}^{(PT)}\left(\frac{4\gamma}{q}\tan^{-1}\left(\frac{q}{4\gamma}\right)+\frac{m_N\gamma}{2\pi}C_2\left(\mu-\gamma\right)^2\right.\right.\right.$$
$$\left.-\frac{2m_N\gamma^2}{\pi q}C_2\left(\mu-\gamma\right)^2\tan^{-1}\left(\frac{q}{4\gamma}\right)\right)-\frac{\gamma}{\pi}\left(\mu-\gamma\right)^2\left(C_{1,\chi NN}^{(s)}+C_{2,\chi NN}^{(s)}\right)\bigg]^2$$
$$+2\left[2C_{3,\chi N}^{(PT)}\left(\frac{4\gamma}{q}\tan^{-1}\left(\frac{q}{4\gamma}\right)+\frac{m_N\gamma}{2\pi}C_2\left(\mu-\gamma\right)^2\right.\right.$$
$$\left.\left.\left.-\frac{2m_N\gamma^2}{\pi q}C_2\left(\mu-\gamma\right)^2\tan^{-1}\left(\frac{q}{4\gamma}\right)\right)-\frac{\gamma}{\pi}\left(\mu-\gamma\right)^2C_{3,\chi NN}^{(s)}\right]^2\right\}. \tag{5.93}$$

Therefore, the cross sections in the limit $q^2\to 0$ are

$$\sigma_{0,d}^{\text{SI}} = \frac{m_{\chi d}^2}{\pi}\left[2C_{1,\chi N}^{(PT)}-\frac{\gamma}{\pi}\left(\mu-\gamma\right)^2\left(C_{1,\chi NN}^{(s)}+C_{2,\chi NN}^{(s)}\right)\right]^2, \tag{5.94}$$
$$\sigma_{0,d}^{\text{SD}} = \frac{2m_{\chi d}^2}{\pi}\left[2C_{3,\chi N}^{(PT)}-\frac{\gamma}{\pi}\left(\mu-\gamma\right)^2C_{3,\chi NN}^{(s)}\right]^2, \tag{5.95}$$

where $m_{\chi d}$ the dark matter-deuteron reduced mass while the form factors at LO are given by

$$F_{\text{SI}}\left(q\right) = F_{\text{SD}}\left(q\right) = \frac{4\gamma}{q}\tan^{-1}\left(\frac{q}{4\gamma}\right). \tag{5.96}$$

In the SI cross section, the two-body contributions are of the same order in the large-N_c expansion as the one-body contribution, i.e., they are both $O(N_c)$. The one-body and two-body contributions to the SD cross section are also of the same order in N_c, but they are both $O(1)$. Therefore, if we only retain the LO terms in both the EFT$_{\not\pi}$ power counting and the large-N_c expansion, then the ratio of the cross sections at LO-in-N_c is

$$\frac{\sigma_{0,d}^{\text{SD}}}{\sigma_{0,d}^{\text{SI}}} = \frac{2C_{3,\chi N}^{(PT)2}}{C_{1,\chi N}^{(PT)2}} \sim \frac{2}{N_c^2}, \tag{5.97}$$

and this is expected to receive $O(1/N_c^2)$ corrections. Additionally, the ratio of the SI deuteron-WIMP cross section to the corresponding nucleon-WIMP cross section at leading order in both the combined EFT and large-N_c expansion is

$$\frac{\sigma_{0,d}^{\text{SI}}}{\sigma_{0,N}^{\text{SI}}} = \frac{4m_{\chi d}^2}{m_{\chi N}^2} = \frac{4m_d^2}{m_N^2}\left(\frac{m_N+m_\chi}{m_d+m_\chi}\right)^2. \tag{5.98}$$

Thus, the deuteron-WIMP SI cross section is fixed with respect to N_c, but it is roughly an order of magnitude greater than the nucleon-WIMP SI cross section due to the deuteron mass. The ratio of the SD cross sections is

$$\frac{\sigma_{0,d}^{\mathrm{SD}}}{\sigma_{0,N}^{\mathrm{SD}}} = \frac{8m_{\chi d}^2}{3m_{\chi N}^2} \frac{1}{\left(1 \pm \frac{C_{2,\chi N}^{(PT)}}{C_{3,\chi N}^{(PT)}}\right)^2} \,. \tag{5.99}$$

Therefore, the SD cross section for the deuteron is $O(1/N_c^2)$ suppressed relative to the corresponding nucleon cross section. However, at the physical value $N_c = 3$, the other factors yield roughly $O(N_c^3)$. Thus, this suppression should not be over interpreted, but rather it should be seen as a general trend due to the spin and isospin structure of the interactions.

Chapter 6

Conclusion

In this work, a general set of two-nucleon contact currents relevant for EFT$_{\not\pi}$ and ChEFT has been derived. The undetermined LECs that attend each current have been constrained using the emergent SU(4) spin-flavor symmetry in the large-N_c limit of QCD. Targeted applications of these constraints to electroweak currents, lepton number violation, isospin breaking, and dark matter direct detection have also been explored.

In the electroweak application, a partial explanation of the suppression of the two-nucleon magnetic isoscalar current relative to the isovector current has been provided. Specifically, the isoscalar couplings is $1/N_c$ suppressed relative to the isovector coupling in the large-N_c basis. These couplings can be determined from the deuteron magnetic moment and radiative neutron capture, respectively [63]. Ref. [63] found that the values of the corresponding couplings in the partial wave basis differ nearly by a factor of 50, and the isoscalar coupling is "significantly smaller than naively expected" [140]. Thus, it appears that these LECs can not be simultaneously natural, i.e., of the same size because they occur at the same order in the power counting. However, the $1/N_c$ suppressions alone is not sufficient to explain this size disparity. Although, using Fierz transformations to change to the partial wave basis introduces an additional factor of 8 such that the partial wave isoscalar coupling is $1/8N_c$ suppressed relative to the isovector coupling. At the physical value $N_c = 3$, this ratio is 1/24; therefore, there is still a residual factor of roughly 2 between the large-N_c constraint and the experimental ratio that is not accounted for, but this factor can

likely be accomodated in the naturalness assumption. This result suggests that naturalness assumptions also might be basis dependent. While the analysis does not provide a definitive prescription, it does imply that extra caution should be taken when quantifying naturalness in, e.g., Bayesian priors.

Additionally, the renormalization group evolution of the ratio of these couplings, shown in Fig. 5.1, is fairly stable; therefore, this constraint is possibly valid over a wide range of subtraction points in the PDS scheme. If the running exceeded the large-N_c estimated, then we would conclude that there is an inconsistency between the large-N_c constraints and renormalization group constraints as was the case for the two-derivative two-nucleon couplings in Ref. [45].

Similar constraints were obtained for a more general external axial vector field. These couplings are relevant for processes such as neutrino-deuteron scattering, proton-proton fusion, and tritium beta decay [136, 137, 141–145]. Using the value of the isovector coupling $L_{1,A}$ from a recent lattice calculation [146], we estimated the size of the isoscalar coupling to be $L_{2,A} \sim 0.1625$ fm^3, which provides some justification for neglecting the isoscalar term in existing calculations [136, 137]. Also, these constraints are not inconsistent with recent results that match finite volume EFT$_{\not\pi}$ calculations to lattice data [147]. However, this comparison is only qualitative as the lattice data only restricts the EFT$_{\not\pi}$ LECs to a region of allowed values.

In regards to lepton number violation and charge independence breaking, the light Majorana exchange mechanism for neutrinoless double beta decay requires the inclusion of a contact term proportional to g_ν^{NN} at LO in both EFT$_{\not\pi}$ and ChEFT for renormalization [160, 161]. The requirement to include this term at LO can potentially have a strong impact on the calculations of the nuclear matrix elements relevant for neutrinoless double beta decay, but the coefficient is not determined by data and there is not yet a lattice calculation. This contact term is, however, related to charge-independence-breaking in the two-nucleon force via chiral symmetry [159–

161], but there are two independent CIB terms proportional to $\mathcal{C}_1$ and $\mathcal{C}_2$, and chiral symmetry dictates that $g_\nu^{NN} = \mathcal{C}_1$. The sum of the LECs can be determined by two-nucleon scattering data while the difference is sensitive to two-nucleon-multi-pion interactions and it is currently inaccessible. Therefore, it is not possible yet to disentangle the $\mathcal{C}_1$ and $\mathcal{C}_2$. In order to estimate the impact of this contact term on the matrix elements of various nuclei, Ref. [161] made the assumption $g_\nu^{NN} = \frac{1}{2}(\mathcal{C}_1 + \mathcal{C}_2)$, i.e., $\mathcal{C}_1$ and $\mathcal{C}_2$ are of the same size and sign. Here, this assumption has been justified from the large-N_c perspective. Recently, the relative sizes of the LECs predicted by the large-N_c expansion have been corroborated by Refs. [174, 175].

As a byproduct of this study, a large-N_c hierarchy of charge symmetry breaking of the nuclear force emerges. In particular, charge symmetric forces and CIB forces are $O(N_c)$ while CSB forces are at most $O(1)$. It is important to note that the charge symmetric forces considered here are $O(e^2)$ corrections to the strong component of the nuclear force. Regardless, the hierarchy in this thesis is in agreement with phenomenological estimates [164, 179, 180].

Lastly, the large-N_c constraints for one- and two-nucleon currents have been used to analyze dark matter-nucleon and dark matter-deuteron elastic scattering. In the absence of conclusive data, these constraints can hopefully provide some theoretical input for experimental analyses. Specifically, the constraints on the couplings of the currents to dark matter can be translated into constraints on the dark matter spin independent and spin dependent cross sections at zero momentum transfer. At LO in the large-N_c expansion, the dark matter-single-nucleon coupling is isoscalar while isovector contributions are expected to be $1/N_c$ suppressed. Thus, if one only considers the LO contributions, then the assumption of several analyses that the dark matter-nucleon coupling is isoscalar is a reasonable approximation up to 30% corrections. However, when the subleading isovector terms are incorporated, the ratios of the neutron and proton couplings fall within xenonphobic scenarios in which po-

tential signals for xenon-based detectors are severely suppressed. These xenonphobic scenarios have also been shown to relieve the tension between different detection experiments [219]. Therefore, it is imperative that additional experiments be done, potentially with lighter nuclei [201], and the large-N_c constraints contained in this work can be used to constrain the relative sizes of the parameter space. An interesting extension of this study would be to examine the impact of the large-N_c constraints for ^{4}He in light of recent proposals for its use as direct detection target.

While this analysis is quite general, and it is carried out at the level of the Lagrangian without reference to any particular nucleus, there are open questions regarding large-N_c techniques that should be addressed. First, it is well known and pointed out in Ch. 3 that the Δ resonance is degenerate with the nucleons as $N_c \to \infty$ and plays an important role in baryon-pion scattering as well as meson-exchange models of the two-nucleon interaction. The inclusion of the Δ in $^1S_0$ NN scattering in EFT$_{\not\pi}$ began in Ref. [223], but large-N_c studies of the NN interaction without the Δ do not contradict available data [38, 39, 45]. Including the Δ in large-N_c χPT and ChEFT is also a delicate issue because some quantities depend on the ratio δ/m_π, where δ is the nucleon-Δ mass splitting. Therefore, these quantities are sensitive to the order in which the chiral and large-N_c limits are taken. Despite these issues, the scalings of the LECs in Ch. 4 do not change; however, including the Δ could impact the scaling of the observables calculated in the EFT.

Second, the binding energy per nucleon in nuclear matter is predicted to be on the order of $m_N \sim O(N_c)$ in Skyrme models, but the observed binding energies are typically on the order of a few MeV. This calls into question the applicability of the large-N_c expansion in nuclear matter. Again, however, the analysis in this dissertation is performed in the general context of the spin-flavor structure of the Lagrangian; therefore, we do not expect this question to impact these results. Also, similar large-N_c analyses have been shown to be consistent with NN scattering data

[38, 39, 45] as well as Wigner SU(4) symmetry and beta decays of some medium-mass nuclei [38].

Third, there is currently no systematic framework for determining the large-N_c scaling of couplings in the dibaryon formulation of EFT$_{\not\pi}$ without matching to the theory without dibaryons as done in Refs. [44, 128]. Because of the dibaryon's utility, especially in three-body processes, the development of such a framework could streamline the study of large-N_c techniques for three- and higher-body systems. While this is not a high priority question, a solution would be beneficial.

In principle, the couplings in this work can be determined through lattice QCD calculations. Hopefully, the large-N_c constraints presented here can provide some guidance and priorities to lattice extractions of LECs. Also, these currents can be included in many-body calculations for heavier nuclei; therefore, the combined large-N_c and EFT expansion can produce a large-N_c hierarchy of currents at each order in the EFT power counting such that the computational input can be reduced up to a given order. Finally, further studies of the combinded expansion for additional BSM processes will be useful in order to provide theoretical constraints for novel BSM searches.

Bibliography

[1] H.-W. Hammer, S. König, and U. van Kolck, Reviews of Modern Physics **92**, 025004 (2020).

[2] S. R. Beane, P. F. Bedaque, W. C. Haxton, D. R. Phillips, and M. J. Savage, in *At The Frontier of Particle Physics: Handbook of QCD*, Vol. 1, edited by M. Shifman (World Scientific, 2001) pp. 133–269.

[3] A. V. Manohar, arXiv:hep-ph/9802419 (1998), arXiv: hep-ph/9802419.

[4] E. Witten, Nucl.Phys. **B160**, 57 (1979).

[5] E. Jenkins, Annual Review of Nuclear and Particle Science **48**, 81 (1998), arXiv: hep-ph/9803349.

[6] M. K. Banerjee, T. D. Cohen, and B. A. Gelman, Phys.Rev. **C65**, 034011 (2002).

[7] H. Fritzsch, M. Gell-Mann, and H. Leutwyler, Physics Letters B **47**, 365 (1973).

[8] D. J. Gross and F. Wilczek, Physical Review D **8**, 3633 (1973).

[9] S. Weinberg, Physical Review Letters **31**, 494 (1973).

[10] D. J. Gross and F. Wilczek, Physical Review Letters **30**, 1343 (1973).

[11] H. D. Politzer, Physical Review Letters **30**, 1346 (1973).

[12] K. G. Wilson, Physical Review D **10**, 2445 (1974).

[13] G. 't Hooft, Nucl.Phys. **B72**, 461 (1974).

[14] G. 't Hooft, Nuclear Physics B **75**, 461 (1974).

[15] G. Veneziano, Nuclear Physics B **117**, 519 (1976).

[16] C. G. Callan, N. Coote, and D. J. Gross, Physical Review D **13**, 1649 (1976).

[17] M. B. Einhorn, Physical Review D **14**, 3451 (1976).

[18] M. B. Einhorn, S. Nussinov, and E. Rabinovici, Physical Review D **15**, 2282 (1977).

[19] J. Cardy, Physics Letters B **61**, 293 (1976).

[20] R. Brower, J. Ellis, M. Schmidt, and J. Weis, Nuclear Physics B **128**, 131 (1977).

[21] R. Brower, J. Ellis, M. Schmidt, and J. Weis, Nuclear Physics B **128**, 175 (1977).

[22] S.-S. Shei and H.-S. Tsao, Nuclear Physics B **141**, 445 (1978).

[23] S. Weinberg, Physical Review **166**, 1568 (1968).

[24] S. Coleman, J. Wess, and B. Zumino, Physical Review **177**, 2239 (1969).

[25] C. G. Callan, S. Coleman, J. Wess, and B. Zumino, Physical Review **177**, 2247 (1969).

[26] J. Gasser and H. Leutwyler, Annals Phys. **158**, 142 (1984).

[27] J. Gasser and H. Leutwyler, Nucl.Phys. **B250**, 465 (1985).

[28] S. Weinberg, Phys.Lett. **B251**, 288 (1990).

[29] S. Weinberg, Nucl.Phys. **B363**, 3 (1991).

[30] S. Weinberg, *The Quantum Theory of Fields: Modern applications*, Vol. 2 (Cambridge Univ. Press, Cambridge, 2013).

[31] D. B. Kaplan, M. J. Savage, and M. B. Wise, Nucl.Phys. **B478**, 629 (1996).

[32] D. B. Kaplan, M. J. Savage, and M. B. Wise, Phys.Lett. **B424**, 390 (1998).

[33] U. van Kolck, Nucl. Phys. **A645**, 273 (1999), _eprint: nucl-th/9808007.

[34] P. F. Bedaque and U. van Kolck, Physics Letters B **428**, 221 (1998), arXiv: nucl-th/9710073.

[35] P. F. Bedaque, H.-W. Hammer, and U. van Kolck, Nuclear Physics A **676**, 357 (2000), arXiv: nucl-th/9906032.

[36] J. Vanasse, Physical Review C **88**, 044001 (2013), arXiv: 1305.0283.

[37] J. Gasser, M. Sainio, and A. Svarc, Nucl.Phys. **B307**, 779 (1988).

[38] D. B. Kaplan and M. J. Savage, Phys.Lett. **B365**, 244 (1996).

[39] D. B. Kaplan and A. V. Manohar, Phys.Rev. **C56**, 76 (1997).

[40] H. Leutwyler, Physics Letters B **374**, 163 (1996).

[41] R. Kaiser and H. Leutwyler, The European Physical Journal C **17**, 623 (2000).

[42] R. Kaiser and H. Leutwyler, arXiv:hep-ph/9806336 (1998), arXiv: hep-ph/9806336.

[43] D. R. Phillips and C. Schat, Physical Review C **88**, 034002 (2013), arXiv: 1307.6274.

[44] M. R. Schindler, R. P. Springer, and J. Vanasse, Phys.Rev. **C93**, 025502 (2016).

[45] M. R. Schindler, H. Singh, and R. P. Springer, Phys.Rev. **C98**, 044001 (2018).

[46] D. B. Kaplan, arXiv:nucl-th/0510023 (2005), arXiv: nucl-th/0510023.

[47] A. V. Manohar, arXiv:1804.05863 [hep-ph] (2018), arXiv: 1804.05863.

[48] S. Scherer and M. R. Schindler, *A Primer for Chiral Perturbation Theory*, Lect.Notes Phys., Vol. 830 (Springer Berlin Heidelberg, 2012) publication Title: Lect.Notes Phys.

[49] V. Bernard, N. Kaiser, and U.-G. Meißner, Int.J.Mod.Phys. **E4**, 193 (1995), _eprint: hep-ph/9501384.

[50] U. van Kolck, Prog.Part.Nucl.Phys. **43**, 337 (1999), _eprint: nucl-th/9902015.

[51] P. F. Bedaque and U. van Kolck, Ann.Rev.Nucl.Part.Sci. **52**, 339 (2002).

[52] E. Epelbaum, H.-W. Hammer, and U.-G. Meißner, Rev.Mod.Phys. **81**, 1773 (2009), _eprint: 0811.1338.

[53] E. Epelbaum and U.-G. Meißner, Annual Review of Nuclear and Particle Science **62**, 159 (2012).

[54] H. Georgi, Ann.Rev.Nucl.Part.Sci. **43**, 209 (1993).

[55] T. Appelquist and J. Carazzone, Physical Review D **11**, 2856 (1975).

[56] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Physical Review C **51**, 38 (1995).

[57] R. Machleidt, Physical Review C **63**, 024001 (2001).

[58] V. G. J. Stoks, R. A. M. Klomp, C. P. F. Terheggen, and J. J. de Swart, Physical Review C **49**, 2950 (1994).

[59] A. Jackson, D. Riska, and B. Verwest, Nuclear Physics A **249**, 397 (1975).

[60] M. Lacombe, B. Loiseau, J. M. Richard, R. V. Mau, P. Pires, and R. de Tourreil, Physical Review D **12**, 1495 (1975).

[61] L. Platter, Few Body Syst. **46**, 139 (2009).

[62] D. Phillips, Czechoslovak Journal of Physics **52**, B49 (2002), arXiv: nucl-th/0203040.

[63] J.-W. Chen, G. Rupak, and M. J. Savage, Nucl.Phys. **A653**, 386 (1999).

[64] H. A. Bethe and C. Longmire, Phys. Rev. **77**, 647 (1950).

[65] J. J. de Swart, C. P. F. Terheggen, and V. G. J. Stoks, arXiv:nucl-th/9509032 (1995), arXiv: nucl-th/9509032.

[66] D. R. Phillips, G. Rupak, and M. J. Savage, Phys. Lett. **B473**, 209 (2000), _eprint: nucl-th/9908054.

[67] D. B. Kaplan, Nucl.Phys. **B494**, 471 (1997).

[68] P. F. Bedaque, H.-W. Hammer, and U. van Kolck, arXiv:nucl-th/9802057 (1998), 10.1103/PhysRevC.58.641, arXiv: nucl-th/9802057.

[69] P. F. Bedaque, H. Hammer, and U. van Kolck, Phys.Rev. **C58**, 641 (1998), _eprint: nucl-th/9802057.

[70] F. Gabbiani, P. F. Bedaque, and H. W. Griesshammer, Nuclear Physics A **675**, 601 (2000), arXiv: nucl-th/9911034.

[71] D. B. Kaplan, M. J. Savage, and M. B. Wise, Phys.Rev. **C59**, 617 (1999).

[72] S. Weinberg, Physica **A96**, 327 (1979).

[73] E. Jenkins and A. V. Manohar, Physics Letters B **255**, 558 (1991).

[74] E. E. Jenkins and A. V. Manohar, in *Workshop on Effective Field Theories of the Standard Model*, edited by U.-G. Meissner (1991) pp. 113–137.

[75] S. Scherer, in *Adv.Nucl.Phys.*, Vol. 27 (2003) p. 277, _eprint: hep-ph/0210398.

[76] J. Bijnens, Prog.Part.Nucl.Phys. **58**, 521 (2007), _eprint: hep-ph/0604043.

[77] V. Bernard and U.-G. Meissner, Ann.Rev.Nucl.Part.Sci. **57**, 33 (2007), _eprint: hep-ph/0611231.

[78] V. Bernard, Prog.Part.Nucl.Phys. **60**, 82 (2008), _eprint: 0706.0312.

[79] M. Birse and J. McGovern, in *In: Close, F. (ed.) et al.: Electromagnetic interactions and hadronic structure* (2007) pp. 229–270.

[80] E. Epelbaum, W. Gloeckle, and U.-G. Meissner, Nucl. Phys. **A637**, 107 (1998), _eprint: nucl-th/9801064.

[81] R. Machleidt and D. Entem, Phys.Rept. **503**, 1 (2011), _eprint: 1105.2919.

[82] H. W. Fearing and S. Scherer, Physical Review D **53**, 315 (1996).

[83] J. Bijnens, G. Colangelo, and G. Ecker, Journal of High Energy Physics **1999**, 020 (1999).

[84] J. Bijnens, G. Colangelo, and G. Ecker, Annals of Physics **280**, 100 (2000).

[85] J. Heinonen, R. J. Hill, and M. P. Solon, Physical Review D **86**, 094020 (2012).

[86] R. J. Hill, G. Lee, G. Paz, and M. P. Solon, Physical Review D **87**, 053017 (2013).

[87] M. Luke and A. V. Manohar, Physics Letters B **286**, 348 (1992).

[88] D. B. Kaplan, M. J. Savage, and M. B. Wise, Nucl.Phys. **B534**, 329 (1998).

[89] S. Fleming, T. Mehen, and I. W. Stewart, Nuclear Physics A **677**, 313 (2000), arXiv: nucl-th/9911001.

[90] R. Urech, Nuclear Physics B **433**, 234 (1995), arXiv: hep-ph/9405341.

[91] M. Knecht and R. Urech, Nuclear Physics B **519**, 329 (1998), arXiv: hep-ph/9709348.

[92] H. Neufeld and H. Rupertsberger, Zeitschrift für Physik C Particles and Fields **68**, 91 (1995).

[93] H. Neufeld and H. Rupertsberger, Zeitschrift für Physik C: Particles and Fields **71**, 131 (1996), arXiv: hep-ph/9506448.

[94] G. Ecker, J. Gasser, A. Pich, and E. De Rafael, Nuclear Physics B **321**, 311 (1989).

[95] M. Knecht, H. Neufeld, H. Rupertsberger, and P. Talavera, The European Physical Journal C **12**, 469 (2000), arXiv: hep-ph/9909284.

[96] U.-G. Meißner, G. Müller, and S. Steininger, Physics Letters B **406**, 154 (1997), arXiv: hep-ph/9704377.

[97] U.-G. Meißner and S. Steininger, Physics Letters B **419**, 403 (1998).

[98] G. Müller and U.-G. Meißner, Nuclear Physics B **556**, 265 (1999), arXiv: hep-ph/9903375.

[99] S. Coleman, *Aspects of Symmetry: Selected Erice Lectures* (Cambridge University Press, Cambridge, U.K., 1985).

[100] R. F. Lebed, Mesons and light nuclei. Proceedings, 11th Indian-Summer School on Intermediate-Energy Physics, Prague, Czech Republic, September 7-11, 1998 **49**, 1273 (1999), _eprint: nucl-th/9810080.

[101] S. Coleman and E. Witten, Physical Review Letters **45**, 100 (1980).

[102] B. Moussallam, Physical Review D **51**, 4939 (1995).

[103] H. Leutwyler, Nuclear Physics B - Proceedings Supplements **64**, 223 (1998).

[104] X.-K. Guo, Z.-H. Guo, J. A. Oller, and J. J. Sanz-Cillero, Journal of High Energy Physics **2015**, 175 (2015).

[105] P. Herrera-Siklódy, Physics Letters B **442**, 359 (1998).

[106] P. Herrera-Siklódy, J. Latorre, P. Pascual, and J. Taron, Nuclear Physics B **497**, 345 (1997).

[107] B. Borasoy, The European Physical Journal C **34**, 317 (2004).

[108] J. L. Gervais and B. Sakita, Physical Review D **30**, 1795 (1984).

[109] J. L. Gervais and B. Sakita, Physical Review Letters **52**, 87 (1984).

[110] R. F. Dashen and A. V. Manohar, Phys.Lett. **B315**, 425 (1993).

[111] R. F. Dashen, E. E. Jenkins, and A. V. Manohar, Physical Review D **49**, 4713 (1994).

[112] R. F. Dashen, E. E. Jenkins, and A. V. Manohar, Phys.Rev. **D51**, 3697 (1995).

[113] E. E. Jenkins and A. V. Manohar, Phys.Lett. **B335**, 452 (1994).

[114] M. A. Luty, J. March-Russell, and M. White, Physical Review D **51**, 2332 (1995).

[115] M. A. Luty and J. March-Russell, Nucl.Phys. **B426**, 71 (1994).

[116] E. Jenkins and R. F. Lebed, Physical Review D **52**, 282 (1995), arXiv: hep-ph/9502227.

[117] J. Dai, R. F. Dashen, E. E. Jenkins, and A. V. Manohar, Phys.Rev. **D53**, 273 (1996).

[118] E. E. Jenkins, Physical Review D **77**, 034012 (2008).

[119] E. Jenkins, Nuclear Physics B - Proceedings Supplements **94**, 246 (2001).

[120] E. Jenkins, Physical Review D **54**, 4515 (1996).

[121] E. E. Jenkins, Phys. Rev. **D85**, 065007 (2012), _eprint: 1111.2055.

[122] C. Carone, H. Georgi, and S. Osofsky, Phys.Lett. **B322**, 227 (1994).

[123] A. V. Belitsky and T. D. Cohen, Phys. Rev. **C65**, 064008 (2002), _eprint: hep-ph/0202153.

[124] T. D. Cohen, Phys. Rev. **C66**, 064003 (2002), _eprint: nucl-th/0209072.

[125] D. R. Phillips, D. Samart, and C. Schat, Phys. Rev. Lett. **114**, 062301 (2015), _eprint: 1410.1157.

[126] D. Samart, C. Schat, M. R. Schindler, and D. R. Phillips, Phys.Rev. **C94**, 024001 (2016).

[127] S. T. Nguyen, M. R. Schindler, R. P. Springer, and J. Vanasse, Physical Review C **103**, 054004 (2021).

[128] J. Vanasse and A. David, arXiv:1910.03133 [nucl-th] (2019), arXiv: 1910.03133.

[129] H. Krebs, E. Epelbaum, and U.-G. Meißner, Annals Phys. **378**, 317 (2017).

[130] H. Krebs, The European Physical Journal A **56**, 234 (2020).

[131] S. Pastore, R. Schiavilla, and J. L. Goity, Phys.Rev. **C78**, 064002 (2008).

[132] M. Piarulli, L. Girlanda, L. E. Marcucci, S. Pastore, R. Schiavilla, and M. Viviani, Phys. Rev. **C87**, 014006 (2013), _eprint: 1212.1105.

[133] S. Kolling, E. Epelbaum, H. Krebs, and U.-G. Meißner, Phys.Rev. **C84**, 054008 (2011).

[134] A. Baroni, L. Girlanda, S. Pastore, R. Schiavilla, and M. Viviani, Phys.Rev. **C93**, 015501 (2016).

[135] V. Cirigliano, W. Dekens, J. de Vries, M. L. Graesser, and E. Mereghetti, Journal of High Energy Physics **2017**, 82 (2017).

[136] M. Butler and J.-W. Chen, Nucl.Phys. **A675**, 575 (2000).

[137] M. Butler, J.-W. Chen, and X. Kong, Phys.Rev. **C63**, 035501 (2001).

[138] D. O. Riska, Nucl. Phys. **A710**, 55 (2002), _eprint: nucl-th/0204016.

[139] S. Pastore, L. Girlanda, R. Schiavilla, M. Viviani, and R. B. Wiringa, Phys.Rev. **C80**, 034004 (2009).

[140] J.-W. Chen, G. Rupak, and M. J. Savage, Phys.Lett. **B464**, 1 (1999), _eprint: nucl-th/9905002.

[141] M. Butler, J.-W. Chen, and P. Vogel, Phys.Lett. **B549**, 26 (2002).

[142] J.-W. Chen, K. M. Heeger, and R. G. H. Robertson, Phys. Rev. **C67**, 025801 (2003), _eprint: nucl-th/0210073.

[143] S. Ando, J. W. Shin, C. H. Hyun, S. W. Hong, and K. Kubodera, Phys. Lett. **B668**, 187 (2008), _eprint: 0801.4330.

[144] H. De-Leon, L. Platter, and D. Gazit, Phys. Rev. **C100**, 055502 (2019), _eprint: 1611.10004.

[145] B. Acharya and S. Bacca, Phys. Rev. **C101**, 015505 (2020), _eprint: 1911.12659.

[146] M. J. Savage, P. E. Shanahan, B. C. Tiburzi, M. L. Wagman, F. Winter, S. R. Beane, E. Chang, Z. Davoudi, W. Detmold, and K. Orginos, Phys.Rev.Lett. **119**, 062002 (2017).

[147] W. Detmold and P. Shanahan, Physical Review D **103**, 074503 (2021).

[148] J. Schechter and J. W. F. Valle, Physical Review D **25**, 2951 (1982).

[149] J. Vergados, H. Ejiri, and F. Šimkovic, Int. J. Mod. Phys. E **25**, 1630007 (2016), _eprint: 1612.02924.

[150] H. Päs and W. Rodejohann, New J. Phys. **17**, 115010 (2015), _eprint: 1507.00170.

[151] S. Davidson, E. Nardi, and Y. Nir, Phys. Rept. **466**, 105 (2008), _eprint: 0802.2962.

[152] J. Engel and J. Menéndez, Reports on Progress in Physics **80**, 046301 (2017).

[153] C. Drischler, W. Haxton, K. McElvain, E. Mereghetti, A. Nicholson, P. Vranas, and A. Walker-Loud, arXiv:1910.07961 [hep-ex, physics:hep-lat, physics:hep-ph, physics:nucl-ex, physics:nucl-th] (2020), arXiv: 1910.07961.

[154] F. T. Avignone, S. R. Elliott, and J. Engel, Rev. Mod. Phys. **80**, 481 (2008), publisher: American Physical Society.

[155] G. Prézeau, M. Ramsey-Musolf, and P. Vogel, Physical Review D **68**, 034016 (2003).

[156] X. Kong and F. Ravndal, Nuclear Physics A **665**, 137 (2000), arXiv: hep-ph/9903523.

[157] V. Cirigliano, W. Dekens, J. de Vries, M. L. Graesser, and E. Mereghetti, Journal of High Energy Physics **2018**, 97 (2018).

[158] V. Cirigliano, W. Dekens, M. Graesser, and E. Mereghetti, Phys. Lett. B **769**, 460 (2017), _eprint: 1701.01443.

[159] V. Cirigliano, W. Dekens, E. Mereghetti, and A. Walker-Loud, Physical Review C **97**, 065501 (2018).

[160] V. Cirigliano, W. Dekens, J. De Vries, M. L. Graesser, E. Mereghetti, S. Pastore, and U. Van Kolck, Phys.Rev.Lett. **120**, 202001 (2018).

[161] V. Cirigliano, W. Dekens, J. de Vries, M. L. Graesser, E. Mereghetti, S. Pastore, M. Piarulli, U. van Kolck, and R. B. Wiringa, Physical Review C **100**, 055504 (2019), arXiv: 1907.11254.

[162] E. Epelbaum and U.-G. Meißner, Physics Letters B **461**, 287 (1999), arXiv: nucl-th/9902042.

[163] M. Walzl, U.-G. Meißner, and E. Epelbaum, Nuclear Physics A **693**, 663 (2001), arXiv: nucl-th/0010019.

[164] U. Van Kolck, *Soft Physics: Applications of Effective Chiral Lagrangians to Nuclear Physics and Quark Models*, Ph.D. thesis, Texas U. (1993).

[165] M. Piarulli, L. Girlanda, R. Schiavilla, A. Kievsky, A. Lovato, L. E. Marcucci, S. C. Pieper, M. Viviani, and R. B. Wiringa, Physical Review C **94**, 054007 (2016).

[166] E. Epelbaum, A. M. Gasparyan, H. Krebs, and C. Schat, The European Physical Journal A **51**, 26 (2015), arXiv: 1411.3612.

[167] P. Reinert, H. Krebs, and E. Epelbaum, Eur. Phys. J. A **54**, 86 (2018), _eprint: 1711.08821.

[168] B. C. Tiburzi, M. L. Wagman, F. Winter, E. Chang, Z. Davoudi, W. Detmold, K. Orginos, M. J. Savage, P. E. Shanahan, and NPLQCD Collaboration, Physical Review D **96**, 054505 (2017).

[169] V. Cirigliano, W. Detmold, A. Nicholson, and P. Shanahan, Progress in Particle and Nuclear Physics **112**, 103771 (2020), arXiv: 2003.08493.

[170] W. Detmold and D. J. Murphy, arXiv:2004.07404 [hep-lat, physics:hep-ph, physics:nucl-th] (2020), arXiv: 2004.07404.

[171] Z. Davoudi and S. V. Kadam, Phys. Rev. Lett. **126**, 152003 (2021), _eprint: 2012.02083.

[172] Z. Davoudi and S. V. Kadam, Phys. Rev. D **102**, 114521 (2020), _eprint: 2007.15542.

[173] Z. Davoudi, W. Detmold, K. Orginos, A. Parreño, M. J. Savage, P. Shanahan, and M. L. Wagman, Phys. Rept. **900**, 1 (2021), _eprint: 2008.11160.

[174] V. Cirigliano, W. Dekens, J. de Vries, M. Hoferichter, and E. Mereghetti, Journal of High Energy Physics **2021**, 289 (2021).

[175] V. Cirigliano, W. Dekens, J. de Vries, M. Hoferichter, and E. Mereghetti, Physical Review Letters **126**, 172002 (2021).

[176] W. Cottingham, Annals Phys. **25**, 424 (1963).

[177] H. Harari, Phys. Rev. Lett. **17**, 1303 (1966).

[178] E. M. Henley and G. A. Miller, in *Mesons in Nuclei*, Vol. 1 (North-Holland, Amsterdam, 1979), edited by M. Rho and D. Wilkinson, pp. 405–434.

[179] G. A. Miller, A. K. Opper, and E. J. Stephenson, Annual Review of Nuclear and Particle Science **56**, 253 (2006), arXiv: nucl-ex/0602021.

[180] G. A. Miller, Chin. J. Phys. **32**, 1075 (1994), arXiv: nucl-th/9406023.

[181] J. E. Lynn, I. Tews, J. Carlson, S. Gandolfi, A. Gezerlis, K. E. Schmidt, and A. Schwenk, Phys. Rev. Lett. **116**, 062501 (2016), _eprint: 1509.03470.

[182] J. E. Lynn, I. Tews, J. Carlson, S. Gandolfi, A. Gezerlis, K. E. Schmidt, and A. Schwenk, Phys. Rev. **C96**, 054007 (2017), _eprint: 1706.07668.

[183] D. Lonardoni, S. Gandolfi, J. E. Lynn, C. Petrie, J. Carlson, K. E. Schmidt, and A. Schwenk, Phys. Rev. **C97**, 044318 (2018), _eprint: 1802.08932.

[184] R. Catena and P. Gondolo, Journal of Cosmology and Astroparticle Physics **2014**, 045 (2014).

[185] R. Catena and P. Gondolo, Journal of Cosmology and Astroparticle Physics **2015**, 022 (2015).

[186] K. Schneck, B. Cabrera, D. G. Cerdeno, V. Mandic, H. E. Rogers, R. Agnese, A. J. Anderson, M. Asai, D. Balakishiyeva, D. Barker, R. B. Thakur, D. A. Bauer, J. Billard, A. Borgland, D. Brandt, P. L. Brink, R. Bunker, D. O. Caldwell, R. Calkins, H. Chagani, Y. Chen, J. Cooley, B. Cornell, C. H. Crewdson, P. Cushman, M. Daal, P. C. F. Di Stefano, T. Doughty, L. Esteban, S. Fallows, E. Figueroa-Feliciano, G. L. Godfrey, S. R. Golwala, J. Hall, H. R. Harris, T. Hofer, D. Holmgren, L. Hsu, M. E. Huber, D. M. Jardin, A. Jastram, O. Kamaev, B. Kara, M. H. Kelsey, A. Kennedy, A. Leder, B. Loer, E. L. Asamar, P. Lukens, R. Mahapatra, K. A. McCarthy, N. Mirabolfathi, R. A. Moffatt, J. D. M. Mendoza, S. M. Oser, K. Page, W. A. Page, R. Partridge, M. Pepin, A. Phipps, K. Prasad, M. Pyle, H. Qiu, W. Rau, P. Redl, A. Reisetter, Y. Ricci, A. Roberts, T. Saab, B. Sadoulet, J. Sander, R. W. Schnee, S. Scorza, B. Serfass, B. Shank, D. Speller, D. Toback, S. Upadhyayula, A. N. Villano, B. Welliver,

J. S. Wilson, D. H. Wright, X. Yang, S. Yellin, J. J. Yen, B. A. Young, and J. Zhang, Physical Review D **91**, 092004 (2015), arXiv: 1503.03379.

[187] E. Aprile, J. Aalbers, F. Agostini, M. Alfonsi, L. Althueser, F. Amaro, M. Anthony, V. Antochi, F. Arneodo, L. Baudis, B. Bauermeister, M. Benabderrahmane, T. Berger, P. Breur, A. Brown, A. Brown, E. Brown, S. Bruenner, G. Bruno, R. Budnik, C. Capelli, J. Cardoso, D. Cichon, D. Coderre, A. Colijn, J. Conrad, J. Cussonneau, M. Decowski, P. de Perio, P. Di Gangi, A. Di Giovanni, S. Diglio, A. Elykov, G. Eurin, J. Fei, A. Ferella, A. Fieguth, W. Fulgione, A. Gallo Rosso, M. Galloway, F. Gao, M. Garbini, L. Grandi, Z. Greene, C. Hasterok, E. Hogenbirk, J. Howlett, M. Iacovacci, R. Itay, F. Joerg, S. Kazama, A. Kish, G. Koltman, A. Kopec, H. Landsman, R. Lang, L. Levinson, Q. Lin, S. Lindemann, M. Lindner, F. Lombardi, J. Lopes, E. López Fune, C. Macolino, J. Mahlstedt, A. Manfredini, F. Marignetti, T. Marrodán Undagoitia, J. Masbou, D. Masson, S. Mastroianni, M. Messina, K. Micheneau, K. Miller, A. Molinario, K. Morå, Y. Mosbacher, M. Murra, J. Naganoma, K. Ni, U. Oberlack, K. Odgers, B. Pelssers, F. Piastra, J. Pienaar, V. Pizzella, G. Plante, R. Podviianiuk, N. Priel, H. Qiu, D. Ramírez García, S. Reichard, B. Riedel, A. Rizzo, A. Rocchetti, N. Rupp, J. dos Santos, G. Sartorelli, N. Šarčević, M. Scheibelhut, S. Schindler, J. Schreiner, D. Schulte, M. Schumann, L. Scotto Lavina, M. Selvi, P. Shagin, E. Shockley, M. Silva, H. Simgen, C. Therreau, D. Thers, F. Toschi, G. Trinchero, C. Tunnell, N. Upole, M. Vargas, O. Wack, H. Wang, Z. Wang, Y. Wei, C. Weinheimer, D. Wenz, C. Wittweg, J. Wulf, Z. Xu, J. Ye, Y. Zhang, T. Zhu, J. Zopounidis, and XENON Collaboration 4, Physical Review Letters **122**, 141301 (2019).

[188] E. Aprile, J. Aalbers, F. Agostini, M. Alfonsi, L. Althueser, F. Amaro, M. Anthony, F. Arneodo, L. Baudis, B. Bauermeister, M. Benabderrahmane,

T. Berger, P. Breur, A. Brown, A. Brown, E. Brown, S. Bruenner, G. Bruno, R. Budnik, C. Capelli, J. Cardoso, D. Cichon, D. Coderre, A. Colijn, J. Conrad, J. Cussonneau, M. Decowski, P. de Perio, P. Di Gangi, A. Di Giovanni, S. Diglio, A. Elykov, G. Eurin, J. Fei, A. Ferella, A. Fieguth, W. Fulgione, A. Gallo Rosso, M. Galloway, F. Gao, M. Garbini, C. Geis, L. Grandi, Z. Greene, H. Qiu, C. Hasterok, E. Hogenbirk, J. Howlett, R. Itay, F. Joerg, B. Kaminsky, S. Kazama, A. Kish, G. Koltman, H. Landsman, R. Lang, L. Levinson, Q. Lin, S. Lindemann, M. Lindner, F. Lombardi, J. Lopes, J. Mahlstedt, A. Manfredini, T. Marrodán Undagoitia, J. Masbou, D. Masson, M. Messina, K. Micheneau, K. Miller, A. Molinario, K. Morå, M. Murra, J. Naganoma, K. Ni, U. Oberlack, B. Pelssers, F. Piastra, J. Pienaar, V. Pizzella, G. Plante, R. Podviianiuk, N. Priel, D. Ramírez García, L. Rauch, S. Reichard, C. Reuter, B. Riedel, A. Rizzo, A. Rocchetti, N. Rupp, J. dos Santos, G. Sartorelli, M. Scheibelhut, S. Schindler, J. Schreiner, D. Schulte, M. Schumann, L. Scotto Lavina, M. Selvi, P. Shagin, E. Shockley, M. Silva, H. Simgen, D. Thers, F. Toschi, G. Trinchero, C. Tunnell, N. Upole, M. Vargas, O. Wack, H. Wang, Z. Wang, Y. Wei, C. Weinheimer, C. Wittweg, J. Wulf, J. Ye, Y. Zhang, T. Zhu, and XENON Collaboration 7, Physical Review Letters **121**, 111302 (2018).

[189] E. Aprile, J. Aalbers, F. Agostini, M. Alfonsi, L. Althueser, F. Amaro, V. Antochi, E. Angelino, F. Arneodo, D. Barge, L. Baudis, B. Bauermeister, L. Bellagamba, M. Benabderrahmane, T. Berger, P. Breur, A. Brown, E. Brown, S. Bruenner, G. Bruno, R. Budnik, C. Capelli, J. Cardoso, D. Cichon, D. Coderre, A. Colijn, J. Conrad, J. Cussonneau, M. Decowski, P. de Perio, A. Depoian, P. Di Gangi, A. Di Giovanni, S. Diglio, A. Elykov, G. Eurin, J. Fei, A. Ferella, A. Fieguth, W. Fulgione, P. Gaemers, A. Gallo Rosso, M. Galloway, F. Gao, M. Garbini, L. Grandi, Z. Greene, C. Hasterok, C. Hils, E. Hogenbirk,

J. Howlett, M. Iacovacci, R. Itay, F. Joerg, S. Kazama, A. Kish, M. Kobayashi, G. Koltman, A. Kopec, H. Landsman, R. Lang, L. Levinson, Q. Lin, S. Lindemann, M. Lindner, F. Lombardi, J. Lopes, E. López Fune, C. Macolino, J. Mahlstedt, A. Manfredini, F. Marignetti, T. Marrodán Undagoitia, J. Masbou, S. Mastroianni, M. Messina, K. Micheneau, K. Miller, A. Molinario, K. Morå, Y. Mosbacher, M. Murra, J. Naganoma, K. Ni, U. Oberlack, K. Odgers, J. Palacio, B. Pelssers, R. Peres, J. Pienaar, V. Pizzella, G. Plante, R. Podviianiuk, J. Qin, H. Qiu, D. Ramírez García, S. Reichard, B. Riedel, A. Rocchetti, N. Rupp, J. dos Santos, G. Sartorelli, N. Šarčević, M. Scheibelhut, S. Schindler, J. Schreiner, D. Schulte, M. Schumann, L. Scotto Lavina, M. Selvi, P. Shagin, E. Shockley, M. Silva, H. Simgen, C. Therreau, D. Thers, F. Toschi, G. Trinchero, C. Tunnell, N. Upole, M. Vargas, G. Volta, O. Wack, H. Wang, Y. Wei, C. Weinheimer, D. Wenz, C. Wittweg, J. Wulf, J. Ye, Y. Zhang, T. Zhu, J. Zopounidis, and XENON Collaboration, Physical Review Letters **123**, 251801 (2019).

[190] J. Xia, A. Abdukerim, X. Chen, Y. Chen, X. Cui, D. Fang, C. Fu, K. Giboni, F. Giuliani, L. Gu, X. Guo, Z. Guo, K. Han, C. He, S. He, D. Huang, X. Huang, Z. Huang, P. Ji, X. Ji, Y. Ju, S. Li, H. Lin, H. Liu, J. Liu, Y. Ma, Y. Mao, K. Ni, J. Ning, X. Ren, F. Shi, A. Tan, A. Wang, C. Wang, H. Wang, M. Wang, Q. Wang, S. Wang, X. Wang, X. Wang, Z. Wang, M. Wu, S. Wu, M. Xiao, P. Xie, B. Yan, J. Yang, Y. Yang, C. Yu, J. Yuan, J. Yue, D. Zhang, H. Zhang, T. Zhang, L. Zhao, Q. Zheng, J. Zhou, N. Zhou, X. Zhou, and W. C. Haxton, arXiv:1807.01936 [hep-ex, physics:hep-ph] (2019), 10.1016/j.physletb.2019.02.043, arXiv: 1807.01936.

[191] P. Adhikari, R. Ajaj, C. E. Bina, W. Bonivento, M. G. Boulay, M. Cadeddu, B. Cai, M. Cárdenas-Montes, S. Cavuoti, Y. Chen, B. T. Cleveland, J. M.

Corning, S. Daugherty, P. DelGobbo, P. Di Stefano, L. Doria, M. Dunford, A. Erlandson, S. S. Farahani, N. Fatemighomi, G. Fiorillo, D. Gallacher, E. A. Garcés, P. G. Abia, S. Garg, P. Giampa, D. Goeldi, P. Gorel, K. Graham, A. Grobov, A. L. Hallin, M. Hamstra, T. Hugues, A. Ilyasov, A. Joy, B. Jigmeddorj, C. J. Jillings, O. Kamaev, G. Kaur, A. Kemp, I. Kochanek, M. Kuźniak, M. Lai, S. Langrock, B. Lehnert, N. Levashko, X. Li, O. Litvinov, J. Lock, G. Longo, I. Machulin, A. B. McDonald, T. McElroy, J. B. McLaughlin, C. Mielnichuk, J. Monroe, G. Oliviéro, S. Pal, S. J. M. Peeters, V. Pesudo, M.-C. Piro, T. R. Pollmann, E. T. Rand, C. Rethmeier, F. Retière, L. Roszkowski, E. S. García, T. Sánchez-Pastor, R. Santorelli, D. Sinclair, P. Skensved, B. Smith, N. J. T. Smith, T. Sonley, R. Stainforth, M. Stringer, B. Sur, E. Vázquez-Jáuregui, S. Viel, A. C. Vincent, J. Walding, M. Waqar, S. Westerdale, and A. Zuñiga-Reyes, arXiv:2005.14667 [astro-ph, physics:hep-ex, physics:hep-ph] (2020), 10.1103/PhysRevD.102.082001, arXiv: 2005.14667.

[192] Y. Wang, Z. Zeng, Q. Yue, L. T. Yang, K. J. Kang, Y. J. Li, M. Agartioglu, H. P. An, J. P. Chang, J. H. Chen, Y. H. Chen, J. P. Cheng, C. Y. Chiang, W. H. Dai, Z. Deng, C. H. Fang, X. P. Geng, H. Gong, Q. J. Guo, X. Y. Guo, H. J. He, L. He, S. M. He, J. W. Hu, T. C. Huang, H. X. Huang, H. T. Jia, L. P. Jia, X. Jiang, H. B. Li, J. M. Li, J. Li, M. X. Li, R. M. J. Li, X. Li, Y. L. Li, B. Liao, F. K. Lin, S. T. Lin, S. K. Liu, Y. D. Liu, Y. Liu, Y. Y. Liu, Z. Z. Liu, H. Ma, Y. C. Mao, Q. Y. Nie, J. H. Ning, H. Pan, N. C. Qi, J. Ren, X. C. Ruan, C. S. Shang, V. Sharma, Z. She, L. Singh, M. K. Singh, T. X. Sun, C. J. Tang, W. Y. Tang, Y. Tian, G. F. Wang, L. Wang, Q. Wang, Y. C. Wang, Y. X. Wang, Z. Wang, H. T. Wong, S. Y. Wu, Y. C. Wu, H. Y. Xing, Y. Xu, T. Xue, Y. L. Yan, N. Yi, C. X. Yu, H. J. Yu, J. F. Yue, M. Zeng, B. T. Zhang, L. Zhang, F. S. Zhang, Z. Y. Zhang, K. K. Zhao, M. G. Zhao,

J. F. Zhou, Z. Y. Zhou, and J. J. Zhu, Science China Physics, Mechanics & Astronomy **64**, 281011 (2021), arXiv: 2007.15555.

[193] D. Akerib, S. Alsum, H. Araújo, X. Bai, J. Balajthy, A. Baxter, E. Bernard, A. Bernstein, T. Biesiadzinski, E. Boulton, B. Boxer, P. Brás, S. Burdin, D. Byram, M. Carmona-Benitez, C. Chan, J. Cutter, L. de Viveiros, E. Druszkiewicz, A. Fan, S. Fiorucci, R. Gaitskell, C. Ghag, M. Gilchriese, C. Gwilliam, C. Hall, S. Haselschwardt, S. Hertel, D. Hogan, M. Horn, D. Huang, C. Ignarra, R. Jacobsen, O. Jahangir, W. Ji, K. Kamdin, K. Kazkaz, D. Khaitan, E. Korolkova, S. Kravitz, V. Kudryavtsev, N. Larsen, E. Leason, B. Lenardo, K. Lesko, J. Liao, J. Lin, A. Lindote, M. Lopes, A. Manalaysay, R. Mannino, N. Marangou, D. McKinsey, D.-M. Mei, M. Moongweluwan, J. Morad, A. Murphy, A. Naylor, C. Nehrkorn, H. Nelson, F. Neves, A. Nilima, K. Oliver-Mallory, K. Palladino, E. Pease, Q. Riffard, G. Rischbieter, C. Rhyne, P. Rossiter, S. Shaw, T. Shutt, C. Silva, M. Solmaz, V. Solovov, P. Sorensen, T. Sumner, M. Szydagis, D. Taylor, R. Taylor, W. Taylor, B. Tennyson, P. Terman, D. Tiedt, W. To, L. Tvrznikova, U. Utku, S. Uvarov, A. Vacheret, V. Velan, R. Webb, J. White, T. Whitis, M. Witherell, F. Wolfs, D. Woodward, J. Xu, C. Zhang, and LUX Collaboration, Physical Review D **103**, 122005 (2021).

[194] D. Akerib, S. Alsum, H. Araújo, X. Bai, A. Bailey, J. Balajthy, P. Beltrame, E. Bernard, A. Bernstein, T. Biesiadzinski, E. Boulton, P. Brás, D. Byram, S. Cahn, M. Carmona-Benitez, C. Chan, A. Chiller, C. Chiller, A. Currie, J. Cutter, T. Davison, A. Dobi, J. Dobson, E. Druszkiewicz, B. Edwards, C. Faham, S. Fallon, S. Fiorucci, R. Gaitskell, V. Gehman, C. Ghag, M. Gilchriese, C. Hall, M. Hanhardt, S. Haselschwardt, S. Hertel, D. Hogan, M. Horn, D. Huang, C. Ignarra, R. Jacobsen, W. Ji, K. Kamdin, K. Kazkaz,

D. Khaitan, R. Knoche, N. Larsen, C. Lee, B. Lenardo, K. Lesko, A. Lindote, M. Lopes, A. Manalaysay, R. Mannino, M. Marzioni, D. McKinsey, D.-M. Mei, J. Mock, M. Moongweluwan, J. Morad, A. Murphy, C. Nehrkorn, H. Nelson, F. Neves, K. O'Sullivan, K. Oliver-Mallory, K. Palladino, E. Pease, L. Reichhart, C. Rhyne, S. Shaw, T. Shutt, C. Silva, M. Solmaz, V. Solovov, P. Sorensen, S. Stephenson, T. Sumner, M. Szydagis, D. Taylor, W. Taylor, B. Tennyson, P. Terman, D. Tiedt, W. To, M. Tripathi, L. Tvrznikova, S. Uvarov, V. Velan, J. Verbus, R. Webb, J. White, T. Whitis, M. Witherell, F. Wolfs, J. Xu, K. Yazdani, S. Young, C. Zhang, and LUX Collaboration, Physical Review Letters **118**, 251302 (2017).

[195] V. Gluscevic and K. K. Boddy, Physical Review Letters **121**, 081301 (2018).

[196] K. K. Boddy and V. Gluscevic, Physical Review D **98**, 083510 (2018), arXiv: 1801.08609.

[197] K. Maamari, V. Gluscevic, K. K. Boddy, E. O. Nadler, and R. H. Wechsler, The Astrophysical Journal Letters **907**, L46 (2021).

[198] R. H. Cyburt, B. D. Fields, V. Pavlidou, and B. D. Wandelt, Physical Review D **65**, 123503 (2002), arXiv: astro-ph/0203240.

[199] C. Dvorkin, K. Blum, and M. Kamionkowski, Physical Review D **89**, 023519 (2014), arXiv: 1311.2937.

[200] W. L. Xu, C. Dvorkin, and A. Chael, Physical Review D **97**, 103530 (2018), arXiv: 1802.06788.

[201] M. C. Digman, C. V. Cappiello, J. F. Beacom, C. M. Hirata, and A. H. G. Peter, Physical Review D **100**, 063013 (2019), arXiv: 1907.10618.

[202] W. Guo and D. N. McKinsey, Physical Review D **87**, 115001 (2013).

[203] T. M. Ito and G. M. Seidel, Physical Review C **88**, 025805 (2013).

[204] G. Gerbier, I. Giomataris, P. Magnier, A. Dastgheibi, M. Gros, D. Jourde, E. Bougamont, X. F. Navick, T. Papaevangelou, J. Galan, J. Derre, I. Savvidis, and G. Tsiledakis, arXiv:1401.7902 [astro-ph, physics:hep-ex, physics:physics] (2014), arXiv: 1401.7902.

[205] S. Profumo, Physical Review D **93**, 055036 (2016).

[206] S. Hertel, A. Biekert, J. Lin, V. Velan, and D. McKinsey, Physical Review D **100**, 092007 (2019).

[207] V. Cirigliano, M. L. Graesser, and G. Ovanesyan, Journal of High Energy Physics **2012**, 25 (2012).

[208] G. Prézeau, A. Kurylov, M. Kamionkowski, and P. Vogel, Physical Review Letters **91**, 231301 (2003).

[209] C. Körber, A. Nogga, and J. de Vries, Physical Review C **96**, 035805 (2017).

[210] F. Bishara, J. Brod, B. Grinstein, and J. Zupan, Journal of Cosmology and Astroparticle Physics **2017**, 009 (2017).

[211] M. Hoferichter, P. Klos, and A. Schwenk, Physics Letters B **746**, 410 (2015).

[212] D. Gazda, R. Catena, and C. Forssén, Physical Review D **95**, 103011 (2017).

[213] M. Hoferichter, P. Klos, J. Menéndez, and A. Schwenk, Physical Review D **99**, 055031 (2019).

[214] P. Klos, J. Menéndez, D. Gazit, and A. Schwenk, Physical Review D **88**, 083516 (2013).

[215] J. Menéndez, D. Gazit, and A. Schwenk, Physical Review D **86**, 103511 (2012).

[216] J. Fan, M. Reece, and L.-T. Wang, Journal of Cosmology and Astroparticle Physics **2010**, 042 (2010).

[217] A. L. Fitzpatrick, W. Haxton, E. Katz, N. Lubbers, and Y. Xu, Journal of Cosmology and Astroparticle Physics **2013**, 004 (2013).

[218] R. J. Hill and M. P. Solon, Physical Review D **91**, 043505 (2015).

[219] V. Cirigliano, M. L. Graesser, G. Ovanesyan, and I. M. Shoemaker, Physics Letters B **739**, 293 (2014), arXiv: 1311.5886.

[220] J. L. Feng, J. Kumar, D. Marfatia, and D. Sanford, Physics Letters B **703**, 124 (2011).

[221] J. L. Feng, J. Kumar, and D. Sanford, Physical Review D **88**, 015021 (2013).

[222] C. E. Yaguna, Journal of Cosmology and Astroparticle Physics **2019**, 041 (2019).

[223] M. J. Savage, Phys. Rev. **C55**, 2185 (1997), _eprint: nucl-th/9611022.

Appendix A

Fierz Transformations

Throughout this work, Fierz transformations are frequently employed. There are two sets of transformations: a set of transformations that eliminates redundancies of operators in the large-N_c basis and a set of transformations that carries these operators into the partial wave basis. The first set of transformations is

$$\delta_{ab}\sigma^i_{cd} = \frac{1}{2}\left[\sigma^i_{ad}\delta_{cb} + \delta_{ad}\sigma^i_{cb} + i\epsilon^{ijk}\sigma^j_{ad}\sigma^k_{cb}\right] , \tag{A.1}$$

$$\sigma^i_{ab}\delta_{cd} = \frac{1}{2}\left[\sigma^i_{ad}\delta_{cb} + \delta_{ad}\sigma^i_{cb} - i\epsilon^{ijk}\sigma^j_{ad}\sigma^k_{cb}\right] , \tag{A.2}$$

$$\sigma^j_{ab}\sigma^k_{cd} = \frac{1}{2}\left[\sigma^j_{ad}\sigma^k_{cb} + \sigma^k_{ad}\sigma^j_{cb} + \delta^{jk}\delta_{ad}\delta_{cb} - \delta^{jk}\sigma^l_{ad}\sigma^l_{cb} + i\epsilon^{jkl}\sigma^l_{ad}\delta_{cb} - i\epsilon^{jkl}\delta_{ad}\sigma^l_{cb}\right] , \tag{A.3}$$

$$\delta_{ab}\delta_{cd} = \frac{1}{2}\left[\delta_{ad}\delta_{cb} + \sigma^j_{ad}\sigma^j_{cb}\right] . \tag{A.4}$$

The Fierz transformation in order to change to the partial wave basis are

$$\delta_{ab}\delta_{cd} = -\frac{1}{2}\sigma^2_{ac}\sigma^2_{bd} + \frac{1}{2}\left(\sigma^i\sigma^2\right)_{ac}\left(\sigma^2\sigma^i\right)_{bd} , \tag{A.5}$$

$$\sigma^i_{ab}\delta_{ad} = \frac{1}{2}\sigma^2_{ac}\left(\sigma^2\sigma^i\right)_{bd} - \frac{1}{2}\left(\sigma^i\sigma^2\right)_{ac}\sigma^2_{bd} - \frac{i}{2}\epsilon^{ijk}\left(\sigma^j\sigma^2\right)_{ac}\left(\sigma^2\sigma^k\right)_{bd} \tag{A.6}$$

$$\delta_{ab}\sigma^i_{cd} = -\frac{1}{2}\sigma^2_{ac}\left(\sigma^2\sigma^i\right)_{bd} + \frac{1}{2}\left(\sigma^i\sigma^2\right)_{ac}\sigma^2_{bd} - \frac{i}{2}\epsilon^{ijk}\left(\sigma^j\sigma^2\right)_{ac}\left(\sigma^2\sigma^k\right)_{bd} , \tag{A.7}$$

$$\begin{aligned}\sigma^i_{ab}\sigma^j_{cd} &= \frac{1}{2}\delta^{ij}\sigma^2_{ac}\sigma^2_{bd} + \frac{1}{2}\left(\delta^{ij}\delta^{kl} - \delta^{ik}\delta^{jl} - \delta^{il}\delta^{jk}\right)\left(\sigma^k\sigma^2\right)_{ac}\left(\sigma^2\sigma^l\right)_{bd} \\ &\quad + \frac{i}{2}\epsilon^{ijk}\left[\sigma^2_{ac}\left(\sigma^2\sigma^k\right)_{bd} + \left(\sigma^k\sigma^2\right)_{ac}\sigma^2_{bd}\right] .\end{aligned} \tag{A.8}$$

These relations also hold for isospin Pauli matrices. After using these transformations for the spin-isospin structure of a two-nucleon operator, an overall sign change needs to be included to account for the anticommutators of the nucleon fields.