Failure of Ultra-High Molecular Weight Polyethylene Yarns (UHMPWE) Under Transverse Loading Using Different Indenter Geometries

Karan Shah

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FAILURE OF ULTRA-HIGH MOLECULAR WEIGHT POLYETHYLENE YARNS (UHMPWE) UNDER TRANSVERSE LOADING USING DIFFERENT INDENTER GEOMETRIES

by

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Bachelor of Engineering
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For the Degree of Master of Science in

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2020

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DEDICATION

This thesis is dedicated to my family and relatives. Without the support from my loving parents, Uncle Purvesh, Uncle Nikhil and Uncle Dinesh, and my aunts Himal, Meena and Nayna, I wouldn’t have reached so far. I thank you all for believing in me and supporting me in my graduate studies.

—Karan
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I would like to give thanks to undergraduate student, John Cooley and high school senior student Johnathan Maxwell for their help in performing experiments. I would like to give special thanks to Drs. Ulrich Heisserer, Harm van der Werff and Mark Hazard of DSM Dyneema® for their useful discussions. Additionally, I would also like to acknowledge DSM Dyneema for providing Dyneema® SK-76 yarns, ARL for providing projectiles used in this work and University of South Carolina for providing funding for this project.
ABSTRACT

Ultra-high molecular weight polyethylene (UHMWPE) Dyneema® SK-76 fibers are widely used in personnel protection systems. Transverse ballistic impact onto these fibers results in complex multiaxial deformation modes such as axial tension, axial compression, transverse compression, and transverse shear. Previous impact studies on high performance yarns and quasi-static transverse loading of single fibers using different indenter geometries show premature failure of yarns and single fiber caused by the degradation of tensile failure strain due to the presence of such multi-axial deformation modes. However, there is a dearth of failure criterion in the literature for ballistic applications that considers the contribution of multi-axial stress or strains induced by transverse impact. This work lays the foundation towards developing a multi-axial failure criterion that can better predict ballistic performance of a material. Quasi-static transverse loading experiments are conducted on Dyneema® SK-76 single yarn at different starting angles (5°, 10°, 15°, and 25°) and using four different indenter geometries: round (radius of curvature (ROC) = 3.8 mm), intermediate (ROC = 0.2 mm), sharp (ROC = 0.02 mm) and razor blade (ROC = 0.002 mm). Experimental results show that for round and intermediate indenter, the yarn fails mainly in tension whereas for sharp indenter and razor blade the yarns fails via cutting or transverse shear and in a progressive manner. There is a significant degradation in the tensile failure strain for sharp indenter (0.73%) and razor blade (0.6-0.7%) compared to uniaxial tension (3.1%). However, the failure strain is approximately constant for all the angles considered. 3D finite element models
are developed to investigate the degree of multi-axial loading and strain concentration
developed in the yarn due to transverse loading by round projectile and sharp indenter.
Finite element results for yarn loaded by sharp indenter show that transverse shear strains
are dominant in the yarn-projectile contact zone and may significantly contribute towards
the failure of the yarn.
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<tbody>
<tr>
<td>$F_A$</td>
<td>Axial force.</td>
</tr>
<tr>
<td>$F_T$</td>
<td>Transverse force.</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Angle made by the transverse wave with the horizontal axis.</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Strain.</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stress</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Starting angle or failure angle of the yarn with respect to the horizontal axis.</td>
</tr>
<tr>
<td>$V_{50}$</td>
<td>Ballistic limit of a fabric or yarn.</td>
</tr>
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## LIST OF ABBREVIATIONS

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>ACr</td>
<td>Axial compression degradation factor</td>
</tr>
<tr>
<td>AT</td>
<td>Axial tension</td>
</tr>
<tr>
<td>FSP</td>
<td>Fragment simulating projectile</td>
</tr>
<tr>
<td>QS</td>
<td>Quasi-static</td>
</tr>
<tr>
<td>ROC</td>
<td>Radius of curvature</td>
</tr>
<tr>
<td>SCF</td>
<td>Strain concentration factor</td>
</tr>
<tr>
<td>TCr</td>
<td>Transverse compression degradation factor</td>
</tr>
<tr>
<td>TSr</td>
<td>Transverse shear degradation factor</td>
</tr>
<tr>
<td>UHMWPE</td>
<td>Ultra-high molecular weight polyethylene</td>
</tr>
<tr>
<td>UMAT</td>
<td>User defined material routine</td>
</tr>
</tbody>
</table>
CHAPTER 1
INTRODUCTION

1.1 Background and Motivation

High performance organic fibers are used for applications that require high strength, high stiffness, high toughness, damage tolerance, durability, dimensional stability and flame resistance [1]. They are mainly used in the form of woven fabrics or polymer-matrix composites [2–4] for ballistic applications like soft-body armor systems used in personnel protection. Fibers used in such applications mainly require a combination of high strength, high stiffness but low density. Examples of high performance organic fibers used in soft-body armor systems include aramid fibers like Kevlar and Twaron, Ultra-high molecular weight polyethylene (UHMWPE) fibers like Dyneema and Spectra, PBO fibers like Zylon, etc. Aramid and PBO fibers are rigid chain polymers whereas UHMWPE is a flexible chain polymer [2]. UHMWPE is a linear flexible polymer of \( C_2H_4 \) molecules that has a molecular weight of at least 3 million. Dyneema® fibers are manufactured using a patented gel-spun process developed by DSM in Netherlands. The gel spinning and drawing process results in a long chain of parallel molecules in a crystal lattice compared to a melt-crystallized block of the same molecules. This long chain of parallel molecules is oriented along the fiber axis which gives UHMWPE (Dyneema) a high tensile strength and stiffness [5].
It is cost and time intensive to perform impact experiments on full-scale fabrics and composites to investigate the halting capability of the ballistic panel. Thus, many researchers [6–8] have performed impact experiments on yarn or single layer fabrics to understand the material response under impact. The classical theory for transverse impact onto homogeneous yarns was first developed by Smith et al [9,10]. It described the transverse impact of the projectile onto the yarn in terms of the longitudinal and transverse waves induced in the material on impact. The longitudinal wave travels away from the impact site and the transverse wave travels in the direction of the projectile into the material. It is assumed that yarn failure occurs only in pure tension. The transverse wave creates an angle, $\gamma$, with the horizontal axis which remains unchanged with the transverse displacement of yarn until failure. The strain in the fiber behind the longitudinal wave and the Euler transverse wave speed, $u_{lab}$, is given by equation 1.1 and 1.2 respectively. The angle, $\gamma$, made by the transverse wave with the horizontal direction is given by equation 1.3 and 1.4

\[ V = c \sqrt{2\epsilon \sqrt{\epsilon(1+\epsilon)} - \epsilon^2} \]  

\[ u_{lab} = c(\sqrt{\epsilon(1+\epsilon)} - \epsilon) \]  

where $\epsilon$ is the strain in the fiber and $V$ is the impact velocity

\[ \gamma = \tan^{-1}\left(\frac{V}{u_{lab}}\right) \]  

\[ \gamma = \tan^{-1}\left(\frac{2V}{c}\right)^{1/3} \text{ for } u_{lab} \ll c \]
Another theory available in the literature for measuring the ballistic resistance of the material was developed by Cunniff [11]. The ballistic performance of a multi-ply fabric or panel system was measured using the ballistic limit, $V_{50}$, found from impact experiments. $V_{50}$ describes the ballistic limit of the fabric system in terms of the velocity of the projectile required for full penetration. It is calculated as $\frac{2}{\sqrt{U^*}}$ where $U^*$ is related to the specific yarn toughness and the longitudinal wave speed of the material as shown in equation 1.5.

\[
U^* = \frac{\sigma \varepsilon}{2 \rho \sqrt{E \rho}} \quad (1.5)
\]

\[
V = \frac{2}{\sqrt{U^*}} \quad (1.6)
\]

It measures the amount of energy that can be absorbed by the fiber material and the velocity at which the energy can be dissipated away from the impact location. Cunniff considered various fibers for his experiments including UHMWPE Spectra 1000. He found that the critical velocity $V_{50}$ predicted by the parameter mentioned above over-predicted the critical velocity found from experiments by 16%. This was understood to be caused by the strength degradation of UHMWPE fibers during impact.

Several authors [12–17] have reported in literature that ballistic performance of a fabrics or yarn is not solely controlled by tension properties of the material. Instead a combination of multiple factors such as multi-axial loading, stress gradients, inter-filament friction, projectile shape and velocity, boundary conditions etc. Walker and Chocron et al. [12] have understood this early failure of yarns to be caused by bounce of the yarn near the front of the projectile. The bounce can lead to the yarn having up to
twice the velocity of the projectile, thereby inducing higher stresses and causing the yarn to fail at lower velocities. Hudspeth et al.[7,13], however, reported that the premature failure of yarns is due to the degradation of the tensile failure strain of the material. Through quasi-static transverse loading experiments of high-performance single fibers like Kevlar KM2 and Dyneema® SK-76 they have reported a significant reduction in the uniaxial failure strain of fibers loaded using a fragment simulating projectile (FSP) and razor blade [18]. Further, axial tensile failure strain of these fiber also reduces with increasing fiber starting angle with respect to the horizontal axis. Additionally, through transverse impact experiments on high performance yarns like Kevlar and Dyneema® they also show that the breaking velocity of the yarn is significantly lower than that predicted using traditional Smith theory in literature [7]. This reduction in the uniaxial failure strain and breaking velocity of high-performance fibers and yarn is attributed to the presence of multi-axial stress states during transverse loading. Multi-axial stress states are induced in the form of axial tension, axial compression, transverse compression and transverse shear near the projectile-fiber contact zone and can lead to degradation of material strength eventually leading to failure. Sockalingam et. al have also shown the presence of such multi-axial stresses and stress concentration near the projectile-fiber contact region via 3D finite element models for Kevlar KM2 [17,19] and Dyneema® SK-76 fibers [20].

Axial tensile properties are main contributors to ballistic performance. Hence, then it becomes important to study how these different stress-states of transverse compression, transverse shear, and axial compression affect the tensile strength of the material. High strain rate transverse compression of Dyneema® SK-76 single fibers show
a 20% reduction in tensile strength of fibers compressed up to 77% average nominal strains[21]. Hudspeth et al. [22]developed a biaxial failure surface that showed that the tensile strength of Dyneema SK-76 fibers degrades significantly when subjected to shear stresses higher than 1GPa.

As described before, there are several studies in the literature which report the presence of such multi-axial stress states during transverse loading of ballistic armor systems. However, there is an absence of a relevant failure criteria that includes multi-axial stress states to allow for the better prediction of yarn or fabric performance. To the author’s knowledge, only Sockalingam et al. [19] has developed a failure criterion that accounts for the degradation effect of multi-axial stresses on the axial tensile failure strain of Kevlar KM2 and Dyneema® SK-76 [20] single fibers.

In this study, we aim to lay the foundation towards developing a multi-axial failure criterion, specifically for Dyneema® SK-76 yarns that can offer more accurate predictions of the ballistic performance of a material. This thesis is organized as follows: The first chapter describes the transverse loading experiments on Dyneema® SK-76 single yarns using different indenter geometries. Next, we study yarn behavior under plane quasi-static transverse compression via experiments and finite element modeling techniques to investigate its effect on yarn tensile strength. Finally, we model the transverse loading experiments for round projectile and sharp indenter using a UMAT developed by Sockalingam et al. to gain insights into the strain concentration developed in the yarn near the projectile-yarn contact zone.
CHAPTER 2
YARN FAILURE UNDER TRANSVERSE LOADING USING DIFFERENT GEOMETRIES

In this chapter, we investigate the effect of multi-axial loading on Dyneema® SK-76 yarns through transverse loading experiments. We follow the same experimental methodology developed by Hudspeth et al. [18] for single fibers and extend it to yarns. Through quasi-static transverse loading experiments, they have shown that projectile geometry and fiber starting angle have significant effect on the tensile failure strain of various high-performance fibers. Experiments were performed at quasi-static loading rates to negate the effects of wave mechanics that exists in a ballistic impact. Similarly, we consider a) Investigating the effect of the projectile geometry on yarn failure and b) Investigating the effect of starting angle of the fiber (angle between the fiber and the horizontal axis) on yarn failure. Four different indenter geometries with increasing amount of sharpness are investigated to understand its effect on stress concentration developed near the projectile-yarn contact area. The starting angle of yarn is varied to determine whether the geometry of the system affects yarn failure like the angle, $\gamma$, formed by the transverse wave in an impact environment.

Dyneema® SK-76 single fibers were obtained from DSM Dyneema®, Netherlands, in the form a spool of yarn without any twist. Yarn properties provided from the manufacturer are presented in Table 2.1.
### Table 2.1 Yarn properties provided by the manufacturer

<table>
<thead>
<tr>
<th>Linear density (dtex)</th>
<th>Mass density (g/cm$^3$)</th>
<th>No. of filaments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1760</td>
<td>0.980</td>
<td>780</td>
</tr>
</tbody>
</table>

Yarn cross-sectional area is calculated to be $0.1796 \text{ mm}^2$ using equation 2.1 given below and the properties in Table 2.1. Figure 2.1 shows the schematic that depicts the system being investigated.

\[
\text{Yarn cross-sectional area} = \frac{\text{Linear density (dtex)}}{10000 \times \rho \left(\frac{\text{dtex}}{\text{cm}^3}\right)}
\]  

(2.1)

#### 2.1 Yarn transverse loading experiments

Transverse loading experiments on yarns were conducted using an Instron 5944 single column tester with pneumatic bollard grips. Figure 2.2 shows the experimental set-up. The yarn is constrained to move in the horizontal direction using the pneumatic bollard grips. A custom designed ‘C’ fixture acts as an adaptor for the different indenters and is connected to the load cell attached to the system crosshead. As the crosshead travels upwards, the indenter loads the yarn in the transverse direction until failure.

Four different indenter geometries with increasing amount of sharpness are investigated. Table 2.2 presents the four indenter geometries with their approximate...
radius of curvatures which are illustrated in Figure 2.3. The starting angle is defined as the angle between the yarn and the horizontal axis as shown in Fig 2.1. The horizontal distance between the grips is 342.9 mm and is maintained for all experiments using different indenters. Thus, for a given starting angle the indenter height can be calculated knowing the horizontal distance and geometry of the system. That is, the yarn was positioned at a given starting angle by adjusting the height of the indenter using the system crosshead. Similarly, the angle at failure can also be calculated from the output crosshead displacement and system geometry. The displacement rate was set to 171.5 mm/min for round and intermediate indenter and 30 mm/min for sharp indenter and razor blade. All yarn specimens were end tabbed using cardstock paper for easy handling of specimens. A double-sided carbon tape (Tadpella) was applied to gripping platens and yarn ends were coated in athletic rosin powder to prevent slippage of the yarn between the grips.

Figure 2.2 Transverse loading experimental set-up
Table 2.2 Different indenter geometries investigated.

<table>
<thead>
<tr>
<th>Indenter geometry</th>
<th>Radius of curvature (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Razor blade</td>
<td>~ 0.002</td>
</tr>
<tr>
<td>Sharp</td>
<td>~ 0.02</td>
</tr>
<tr>
<td>Intermediate</td>
<td>~ 0.2</td>
</tr>
<tr>
<td>Round</td>
<td>3.8</td>
</tr>
</tbody>
</table>

Figure 2.3 a) Round projectile (ROC = 3.8 mm), b) Intermediate indenter ((ROC ~ 0.2 mm), c) Sharp indenter (ROC ~ 0.02 mm), and d) Razor blade (ROC ~0.002 mm).
Videos captured during the experiment indicate that yarn failure depends on the indenter geometry. Figures 2.4-2.7 present the transverse force vs. displacement response for different indenters at various starting angles investigated. Note that there are variations in the radius of curvature of the indenter and sharp indenter which may have contributed to the variation in transverse force vs. displacement responses for the respective indenters. Comparing the transverse force vs. displacement response for different indenters we can observe that yarn response under transverse loading is similar for round projectile and intermediate indenter and is also similar for sharp indenter and razor blade. However, the yarn fails progressively and at significantly lower loads under sharp indenter and razor blade whereas the failure is almost instantaneous under loading from round projectile and intermediate indenter. Yarn failure for round projectile and intermediate indenter is like the one observed for the quasi-static tensile failure of yarns, where the fibers snap back and form a bulbous mass at failure as was also shown by Hudspeth et al. [16]. Figure 2.8(a) shows the failure surface of yarn loaded by a 0.2 mm indenter. For yarns loaded using sharp indenter and razor blade, the failure seems to be dominated by cutting type or shear behavior. As seen from the experimental video, failure is initiated by individual fiber breaks until a maximum load point, after which multiple fiber breaks are observed until it eventually fails. Figures 2.8(b) shows the failure surface for yarn transversely loaded by razor blade. In general, a smaller ROC (sharp) of the indenter leads to a more shear type failure than a larger ROC (blunt).

A better comparison between different indenters is shown in Figure 2.9 shows the effect of starting angle on the average maximum transverse force before failure. Note that for sharp indenter and razor, this corresponds to the max load point after which we
observe a progressive degradation in forces until the yarn fails completely. For all indenter geometries, transverse force at failure increases with increasing yarn starting angle. The transverse forces are understood to be more dependent on the fiber-indenter contact zone which differs for each starting angle. As the starting angle increases, the contact area between the indenter and the yarn and increases as well leading to a higher transverse failure load. Lim et al. [23] also observed a similar trend of increasing transverse failure forces in their study on Twaron yarns with increasing radius of curvature of projectile.
Figure 2.4 Transverse force vs. displacement curves for round projectile at different starting angles: a) 10-degree b) 20-degree and c) 25-degree.
Figure 2.5. Transverse force vs. displacement curve for intermediate indenter at different starting angles: a) 10-degree and b) 25-degree.
Figure 2.6. Transverse force vs. displacement curve for sharp indenter at different starting angles: a) 10-degree and b) 25-degree.
Figure 2.7. Transverse force vs. displacement response for razor blade at different starting angles: a) 5-degree, b) 10-degree, c) 15-degree, and d) 25-degree.
Figure 2.8 Failure surface of yarn under transverse loading from a) 0.2 mm indenter and b) razor blade indenter.

Figure 2.9 Effect of starting angle on transverse force for different indenter geometries.
Axial tensile properties are key contributors to ballistic performance and thus we are more interested in studying the effect of indenter geometry on axial strength and failure strain of the yarn. Axial strength is calculated from axial forces obtained from the free body diagram of a simplified system shown in Figure 2.10 and trigonometric relations. Using the free-body diagram, the axial force is given by $F_A = F_T / (2 \sin \theta)$, where $F_A$ is the axial force and $F_T$ and $\theta$ are the transverse force and angle at failure. Figure 2.11 shows that the axial strength is approximately constant for the round indenter at different starting angles and close to the uniaxial tensile strength for yarn. This is expected and is caused by the absence of stress concentration in the fiber-projectile contact zone due to the bluntness of the indenter. For razor blade and sharp indenter, the axial strength is significantly lower than the uniaxial tensile strength and decrease slightly with increasing starting angle. This indicates a presence of stress concentration in yarn caused by the sharp geometry of the indenter. Similar trends in transverse loads and axial forces have been reported by Lim et al. [23] in their study on Twaron yarns. They also report that axial loads decrease with increasing sharpness of projectile geometry for a starting angle of 45°. The effect of breaking angle on the axial failure strain of the yarn shown in Figure 2.12 is discussed in the next section in comparison with single fiber results from the literature.

Figure 2.10 Free body diagram of transverse loading experiment.
Figure 2.11 Effect of starting angle on axial strength for different indenter geometries.
2.2 Comparison to single fiber transverse loading

Figure 2.13 presents how the axial strain varies with the failure angle or breaking angle for single fibers. Dyneema SK-76 single fiber results are obtained from the transverse loading study on high performance fibers by Hudspeth et al [18]. In their study, Hudspeth et al. investigated the effect of round projectile, an FSP with a sharp corner radius of 20 µm and razor blade on several high-performance single fibers.
Comparing Figures 2.13 and 2.12 for single fibers and yarn we observe that the axial failure strain does not vary significantly with the breaking angle for round projectile and razor blade for both single fibers and yarn. It is important to here that axial failure strain of yarn plotted in Figure 2.12 for sharp indenter and razor blades corresponds to the axial strain at the maximum load point in Figures 2.8 and 2.9, just before the onset of progressive degradation in the load carrying capacity of the yarn. However, for the FSP with a corner radius of 20 µm, there is a significant degradation of failure strain with breaking angle for single fibers unlike that for yarns where the response for the sharp indenter is close to that of a razor blade and does not vary significantly with the breaking angle. A striking difference to note here is the amount of degradation in axial failure strain between single fibers and yarns. For single fibers, there is approximately a 20-25% degradation in failure strain for breaking angles between 25-30 degrees and the degradation increases with further increase in breaking angle. However, for yarns the degradation of axial failure strain is approximately 75%, 3 times higher than that.
observed for single fibers. For the intermediate indenter and round projectile, the axial failure strain does not change significantly with breaking angle and is close to the uniaxial failure strain of yarn. As explained earlier, this may be because of the absence of stress-concentration in the fibers due to a large difference in the radius of curvature of the indenter and fiber diameter of 17 µm.

2.3 Transverse loading of yarn using FSP

For completeness, we also study yarns subjected to transverse loading using a 0.3 caliber fragment simulating projectile (FSP) made to specification MIL-DTL-46593B. Transverse loading experiments are performed at various starting angles like in section 2.1 and Figure 2.14 shows a representative transverse force vs. displacement response for FSP at a starting angle of 15°. Figure 2.15 shows the axial strength and failure strain found from experiments in comparison with other indenter geometries. Comparing Figure 2.14 to Figures 2.4 and 2.5, we can see that the transverse vs. displacement response for FSP is similar to round and intermediate indenter. The axial strength and failure strain for FSP do not change significantly with starting angle or breaking angle and is close to the uniaxial failure strain of the yarn. This suggests an absence of stress concentration in the projectile-yarn contact zone caused by the blunt geometry of the FSP. To validate this claim, we use an optical microscope to capture images of the FSP near the contact region as shown in 2.16. The radius of curvature of the FSP projectile was measured to be approximately 200 µm as shown in Figure 2.16.
Figure 2.14 Transverse force vs. displacement response at starting angle of 15 degrees.
Radius of curvature of FSP. The radius of curvature of the FSP is approximately 10 times higher than the diameter of Dyneema® SK-76 single fiber which is approximately 17.0 µm. This suggests that the radius of curvature of the FSP is not sharp enough to induce significant stress concentration in the fibers within the yarn which may explain the failure strain being approximately same as uniaxial failure strain. Thus, the FSP considered in this
work acts as blunt indenter just like a round projectile and causes no degradation in the uniaxial failure strain of the yarn.

This chapter has shown that sharp indenters cause a degradation in the uniaxial failure strain of the yarn. This is to be due to the presence of multi-axial stress states near the projectile-yarn contact region and its contribution to the stress concentration seen in the yarn. In the next chapter, we aim to study the effects of one of the multi-axial stress states: transverse compression, on the residual tensile strength of yarns.
CHAPTER 3
EFFECT OF TRANSVERSE COMPRESSION

As described in the last chapter, multi-axial stress states are induced in fibers as axial tension, axial compression, transverse compression, and transverse shear. To understand the effect of multi-axial stress state on tensile strength, we must first understand the effect individual stress-states have on the tensile strength. In this section, we investigate the effect of quasi-static transverse compression on yarn tensile strength. The effect of transverse compression on yarns can be quantified by the reduction in tensile strength of compressed yarns. Thus, we approach this problem by studying a) Quasi-static tensile strength of uncompressed yarns to establish a base-line or reference strength, b) Quasi-static compression of yarns and c) Quasi-static tensile strength of compressed yarns and compare it to the base-line or reference strength found in (a).

Dyneema® SK-76 fibers have very low surface friction coefficient and hence are prone to slipping within the grips during experiments. Different researchers have used different ways of preventing fiber slippage during experiments in the literature[7,24,25]. In our study, we use the same procedure as used by Hudspeth et al. [7] to prevent slippage of fibers or yarns within the grips. Carbon tape from Tad Pella is applied to the fixed clamp surface of pneumatic bollard grips and athletic rosin powder from HUMCO is applied to yarn ends to increase friction between the fibers and the grips. Additionally, yarn specimens are end tabbed using a high strength neoprene sheet for ease of handling.
3.2 Transverse compression experiments

Quasi-static compression tests of yarns were conducted on an INSTRON 5944 single column tester with compression platens. The experimental set-up is shown in Figure 3.1. The platform besides the platens is raised using cardboard slabs to prevent the yarn from sagging around the edges. Before starting the experiment, yarns were slightly stretched to ensure that it lays flat on the bottom platen and a preload of 2.0 N was applied to ensure contact between the yarn and the platens. The length of yarn specimen under compression was 46.0 mm and a permanent marker was used to mark this length. Each yarn specimen was compressed up to a load of 1800.0 N at a displacement rate of 0.6 mm/min. We tested 44 yarn specimens under transverse compression.

Figure 3.2 shows the nominal stress vs. strain response for 44 transverse compression experiments. Nominal stress and nominal strain are calculated as shown in equations (2) and (3). $F$ is defined as applied load or force, $A$ is yarn cross-sectional area, $u$ is displacement and $l$ is the original length. Note $l$ is equal to original thickness, ‘$t$’, for transverse compression.

![Figure 3.1 Transverse compression experimental setup](image-url)
Figure 3.2 Yarn quasi-static compression experiment results.

\[ \bar{\sigma} = \frac{F}{A} \]  
\[ \bar{\varepsilon} = \frac{\Delta l}{l} \]  

The thickness of the yarn ‘t’ is obtained by assuming an elliptical cross-section of Dyneema® SK-76 yarn with a width of 1.20 mm and a hexagonal closed packing of fibers. This yarn width is found from the study on Dyneema® composites in the literature [26]. Hence, the thickness is calculated as:

\[ t = \frac{2A}{\pi a} \]  
\[ t = \frac{2 * 0.1796}{\pi * 0.6} = 0.191 \, mm \]

‘a’ refers to the half of the major axis of an ellipse.
The cross-sectional area, ‘A’, required for calculating nominal stress is obtained by the product of width of the yarn, $w$, and length, $L$, of the yarn under transverse compression. It is assumed that the width of the yarn and hence the area does not change significantly during compression. From figure 3.2 we can see that the yarns are compressed to varying levels of nominal strain for the same amount of applied nominal stress. The level of strain varies between 62.0-95.0 % for the same amount of applied nominal stress of 32.60 MPa. This suggests that fibers within the yarn are not compressed uniformly and some fibers experience higher levels of load than others.

A similar conclusion can be drawn from Figure 3.3 which shows a microscope image of a compressed yarn after the experiment. If we observe closely, we can see small regions of compressed fibers dispersed across the length of the yarn as shown by the red markers. Sockalingam et al. [27] have also observed a similar response for Kevlar KM2 fabric where they find that the stress-strain distribution within the Kevlar fibers depends on their spatial location. They attribute this non-uniform loading to fiber spreading and fiber-fiber contact interactions during loading.

Figure 3.4 shows nominal stress vs. nominal strain response for quasi-static tension of baseline (uncompressed) yarn specimens. Quasi-static tension tests for baseline and compressed yarn specimens were conducted in accordance to the ASTM Standard D7269-07 [28] using a slack start procedure. We conducted the tension experiments on the same INSTRON Model 5944 single column tester using pneumatic bollard grips. Single yarns specimens of gage length 254 mm were pulled at a displacement rate equivalent to 50.0% of gage length, i.e., 127 mm/min. Athletic rosin powder was applied
to yarn ends before testing to increase friction between the clamping surface and fibers to prevent slipping.

Figure 3.5 shows the quasi-static tension response of compressed yarns. Comparing Figure 3.5 and Figure 3.4, there is no significant difference in the response between the compressed and baseline yarn specimens. However, from Figure 3.6 we can see that the average tensile strength of compressed yarns is lower than the average tensile strength of baseline yarns. On an average there is 4% reduction in the average tensile strength of compressed yarns.

Figure 3.3 Compressed image of yarn specimen.

Figure 3.4 Yarn quasi-static tension experiment results (Baseline).
To validate this reduction in tensile strength of compressed yarns we fit a two parameter Weibull distribution described in equation 3.4 to the tensile strength data of compressed and baseline yarns. In equation 3.4, $F(\sigma)$ represents the cumulative

Figure 3.5 Yarn tension after compression experiment results.

Figure 3.6 Average tensile strength with their standard deviations.
probability of failure, $\sigma$ is the yarn tensile strength, $m$ is the shape parameter and $\sigma_0$ is the scale parameter. Figure 3.7 shows the plot of the fitted Weibull shape and scale parameters that are also presented in Table 3.1 for both types of specimens.

$$F(\sigma) = 1 - \exp\left[\left(-\frac{\sigma}{\sigma_0}\right)^m\right] \quad (3.4)$$

Figure 3.7 Fitted Weibull plots for compressed and baseline yarns.

Table 3.1 Weibull fitted modulus and scale parameters.

<table>
<thead>
<tr>
<th>Specimen type</th>
<th>Weibull modulus ($m$)</th>
<th>Scale parameter ($\sigma_0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline yarn</td>
<td>21.47</td>
<td>3152.55</td>
</tr>
<tr>
<td>Compressed yarn</td>
<td>17.56</td>
<td>3045.71</td>
</tr>
</tbody>
</table>
Figure 3.7 depicts a left shift of the fitted line for compressed yarn specimen. The left shift signifies a shift to lower strength values for compressed yarns compared to baseline yarns. The compressed yarn specimens also show more variability in their strength as indicated by the reduced Weibull modulus, m. Based on the fitted Weibull parameters, we can create a Weibull cumulative probability of failure vs. tensile strength plot for both specimens as shown in Figure 3.8. For a 50% probability of failure, the estimated tensile strength from the plot for compressed and baseline yarn is approximately 2982.57 MPa and 3097.19 MPa respectively.

3.2 Modeling yarn transverse compression

As shown in the last section, quasi-static transverse compression of Dyneema SK-76 yarns show a mere 4% reduction in their tensile strength when compressed up to average nominal strains of 77%. In this section, we use finite element models to gain further insights into local stress-strain response within the yarn under transverse compression. We use a 2-D plane strain finite element formulation for the yarn consisting of individual fiber cross-sections to model the experiments. Cross-section images of Dyneema® composite laminates shows an approximate hexagonal close packing of fibers [6][29]. A single fiber of Dyneema® SK-76 fiber has an approximately circular cross-section with an average diameter of 17.0 µm and a yarn consists of 780 such single fibers. Another micrograph image of a ply of Dyneema® composite laminate shows that the cross-section of yarn can be approximated as an ellipse with a width or major axis as 1.20 mm [26]. The minor axis of the yarn is calculated based on the cross-sectional area of the yarn of 0.1796 mm$^2$ available from the manufacturer. Figure 3.8 shows the 2D finite element model of yarn composed of 780 single fibers with a circular cross-section
of diameter 17.0 µm arranged in a hexagonally closed pack manner for a maximum fiber volume fraction of 0.907. Each fiber cross-section is created using 84 fully integrated 2-D elements and fibers exhibit non-linear inelastic behavior using material properties presented in Table 3.2. The yarn is compressed by rigid platens at a displacement rate of 0.6 mm/min. Fiber-fiber and fiber-platen interactions are considered using a 2-D single surface contact with a friction coefficient of 0.2. Like experiments, the yarns is unconstrained in the lateral direction and is free to spread during compression. However, in fabrics the spreading of the yarn is restrained due to presence of other yarns in the warp and weft directions. Such complex boundary conditions are not considered in our model. Factors such as fiber entanglements, actual fiber packing etc. are also not accounted by the model.

Figure 3.9 shows the compression response at increasing nominal strains during loading. At 10.0 % nominal strain, fibers within the yarn have displaced themselves but show no signs of deformation. Increasing the nominal strain to 30.0% results in further spreading of the fibers and formation of gaps in the yarn packing with no signs of fiber deformation. At 45.0% nominal strain, the fibers have significantly spread themselves and show some sign of deformation due to compression as seen by the strain contours in Figure 3.9 (c). At 55.0 % nominal strains, fibers near the center of the cross-section are further compressed as the gaps between the fibers collapse. An important thing to note is that the fibers are not loaded uniformly under compression. This can be observed from Figure 3.9(c) and 3.9(d) where the fibers near the center show higher levels of strain compared to the fibers near the end. The nominal stress vs. nominal strain response of the model correlates very well to experimental results as shown in Figure 3.10.
Both the model curve and the experimental curve show little to no increase in stress with increasing applied nominal strain up to 40.0%. This is due to initial spreading and spatial rearrangement of fibers on loading as observed from Figures 3.9(a) and 3.9(b). On further loading, both the experimental and model results show a steep rise in nominal stress vs. nominal strain. The model curve shows a slight drop again in stress at a nominal strain of 45.0% compared to the experimental curve. This is thought to be caused by further spreading or sliding of fibers as they rearrange themselves. The model stress vs. strain response is stiffer than experimental response may be because of the assumption of hexagonal close packing of fibers (maximum fiber volume fraction of 0.907). The actual packing and volume of fraction of fibers within the yarn may be less than that assumed by maximum fiber volume fraction for a hexagonally closed pack.
structure. The actual microstructure induces lower stress for a given applied nominal strain in experiments compared to the model.
Figure 3.9 True compressive strain contours at a) 10%, b) 30%, c) 45% and d) 55% applied nominal strain from yarn Q-S compression model.
3.3 Comparison to single fiber transverse compression

McDaniel and Sockalingam et al. investigated the quasi-static and high strain rate transverse compression response of Dyneema SK-76 single fibers[21,30]. Their results report a non-linear stress-strain response of Dyneema SK-76 single fibers at both quasi-static and high strain rates. Table 3.3 shows the said non-linear elastic properties for Dyneema SK-76 single fiber at both quasi-static and high strain states [21]. A recent study by Golovin and Phoenix [31] also shows a reduction of about 44% and 25% in the tensile strength of Dyneema® SK-76 single fibers that are transversely compressed by hand method and machine roll method respectively. Experimental results from a study by Thomas et al. [32] show a 20% reduction in the Q-S tensile strength of Dyneema® SK-76 fibers compressed at average nominal strains of 77%. In this section, we aim to understand this difference in reduction in tensile strength observed between yarns and single fibers by modeling the transverse compression response of SK-76 single fiber and
compare it to yarns. We consider two separate models to investigate both quasi-static and high strain rate transverse compression of SK-76 single fiber.

The single fiber transverse compression response was also modeled using 2-D plane strain elements (element formulation 13) in LS-DYNA. Sockalingam et al. have shown that a minimum of 300 finite elements are required to accurately capture the transverse compression response of Dyneema® SK-76 single fiber [30]. Figure 3.11 shows the quarter symmetric fiber model created using 300 finite elements. The fiber has a radius of 8.5 µm and exhibits non-linear inelastic behavior as described by the material properties presented in Table 3.2. A 2-D single surface contact with a friction coefficient of 0.2 is used to define the contact between the fiber and rigid platens. The fiber is compressed at a displacement rate of 1 µm/s by the rigid platens.

Figure 3.12 shows the quasi-static compression response of the fiber at applied nominal strain of 55.0% and 77.0%. From Figure 3.12, we can observe that the center of the experiences high compressive strains and strains reduce away from the center. At higher applied nominal strain, a higher portion of the fiber experiences a strain concentration. Another important observation that can be made from Figure 3.12 is that the local (true) strains are much higher than the applied nominal strain. On an average, maximum local strains are approximately 1.5 times than that of the applied nominal strain. This observation can also be made for the high strain rate transverse compression response of the fiber where the local strains are 1.5-2 times higher than the applied nominal strain of 77.0% as reported in our work [33]. However, the nominal stresses developed in the fiber are twice as high for high strain-rate compression compared to quasi-static compression. McDaniel et al. [30] have shown that at high nominal strains,
like the ones observed in this study, fibrillation is a dominant mode of deformation in Dyneema® SK-76 single fibers. They have found that Dyneema® SK-76 fibers have weaker inter-fibrillar interactions compared to Kevlar fibers which possess hydrogen bonding to influence off-axis stability. Fibrillation begins at small loads and nominal strains in Dyneema® SK-76 single fibers and causes the fibril network to break into micro-fibrils leading to void formation and permanent deformation. Therefore, the reduction in fiber tensile strength observed in experiments [32] may be due to fibrillation caused by high level local strains during transverse compression as found from our model.

Comparing the simulations results for single fibers and yarns we can see that unlike single fibers, the local strains in the yarn are lower for a given applied nominal strain. For yarns, a large proportion of the fibers are deformed to between 20%-30% strain for an applied nominal strain of 55%. This can explain the lower reduction in tensile strength of yarns (4%) compared to single fibers (20%) as seen from our experiments caused by non-uniform loading and significant spreading of fibers within the yarn.

This information can prove to be very useful when modeling transverse loading experiments of yarn as described in the next section. It can be used to understand the role of transverse compressive strains in the multi-axial failure of yarns loaded by different indenter geometries.
Figure 3.11 Single fiber quarter symmetric finite element model

Figure 3.12 True compressive strain at (a) 50% and (b) 77% applied nominal strain from single fiber Q-S compression model.
CHAPTER 4

MODELING YARN TRANSVERSE LOADING EXPERIMENT

4.1 Modeling yarn transverse loading experiments

For yarn transverse loading experiments, the yarn was positioned at a given starting angle by moving the indenter to a required height that was calculated from the system geometry. Similarly, a positioning simulation is first run in LS-DYNA using an implicit FE solver to position the yarn at a given starting angle. Figure 4.1 shows the half symmetric yarn model for the positioning simulation. For simplicity and to reduce computation time we consider the yarn to be homogenous instead of being comprised of a bundle of fibers. The yarn is modeled to have an elliptical cross-section with width and thickness as 1.2 mm and 0.190 mm respectively, same as transverse compression models.

For transverse loading using round projectile, we model the yarn using 576 reduced integration elements in the cross-section and a mesh size of 50 μm up to a length of 2 mm of the yarn and an increasingly coarser mesh becoming over the remaining length of yarn.

For transverse loading using a sharp indenter, we model the yarn using 676 reduced integration elements in the cross-section and mesh size of 1.1 μm up to length of 20 μm of the yarn and an increasingly coarser mesh over the remaining length of the yarn. The round projectiles and sharp indenter are modeled as rigid bodies. An automatic surface to surface contact with soft formulation and friction coefficient of 0.2 is used to define the contact between the yarn and the projectile. As seen from Figure 4.1, the initial position of the yarn is parallel to the horizontal axis. The indenter is placed under the yarn and is
perpendicular to the yarn and the horizontal axis. During transverse loading experiments, the yarn was positioned at a given starting angle by moving the indenter upwards and into the yarn and allowing the yarn to slide within the grips. To mimic this positioning of the yarn in our finite element models, the far end of the yarn is free to move in the horizontal direction and is constrained in other directions. As the projectile moves upwards with a prescribed constant velocity, it displaces the yarn upwards and inwards along the horizontal axis, increasing the angle between the yarn and the horizontal axis as shown in Figure 4.1 (b). The desired start angle is obtained by measuring theta start from the simulation output states as shown in Figure 4.1 (b).

The simulation state corresponding to a desired starting angle is obtained and then used for transverse loading simulations with the far end fixed in all directions. The yarn is modeled as a transversely isotropic material with an inelastic transverse compressive behavior using a validated user defined material (UMAT) developed by Sockalingam et al. [34]. Table 4.1 presents the transversely isotropic elastic properties used for the yarn. As mentioned in chapter 3, Dyneema® SK-76 fibers exhibit non-linear inelastic response on transverse compression with a small elastic limit of 2% strain and an initial yield stress of 20 MPa. The yield-stress effective plastic-strain required as an input in the UMAT is also obtained from Sockalingam et al. [35].

Table 4.1 Transversely isotropic elastic properties used for the yarn.

<table>
<thead>
<tr>
<th>$\rho$ (g/cm$^3$)</th>
<th>d (µm)</th>
<th>$E_1$ (GPa)</th>
<th>$E_3$ (GPa)</th>
<th>$G_{13}$ (GPa)</th>
<th>$\nu_{31}$</th>
<th>$\nu_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.97</td>
<td>17.0</td>
<td>1.0</td>
<td>116.0</td>
<td>3.0</td>
<td>0.60</td>
<td>0.40</td>
</tr>
</tbody>
</table>
Figure 4.1 a) Yarn transverse loading positioning model b) Starting angle, $\theta_{\text{start}} = 20^\circ$.

Fig 4.2 (a) shows the round projectile transverse loading model for starting angle, $\theta_{\text{start}} = 20^\circ$. The round projectile radius of curvature, ROC = 3.8 mm is significantly greater than the fiber diameter, 17 µm. Hence, we expect there to be no axial strain concentration in the yarn including the projectile-yarn contact area. The evolution of strain concentration factor (SCF) is shown in Figure 4.2 (b). SCF is approximately equal to 1.4 at a failure angle, $\theta_{\text{fail}} = 23.9^\circ$. This is slightly higher than the expected SCF of 1 and we are currently working on our models to investigate this issue. It may be caused by numerical issues in the model due to non-physical modes. The model predicts negligible axial compression in the yarn throughout the loading history. Transverse compressive
strain in the yarn monotonically increases during loading and reaches a maximum value of 3.24% at an applied average nominal strain of 3%. Surprisingly, our model predicts that there is an increase in transverse shear strains as the yarn is transversely loaded by the round projectile. We do not expect a blunt indenter such as a round projectile to induce shear strains in the yarn and may also contribute to the higher SCF factor seen in the yarn-projectile contact zone.

![Figure 4.2](image-url)

**Figure 4.2** a) Round projectile transverse loading and b) Evolution of axial tensile failure strain, $\theta_{\text{fail}} = 23.9^\circ$.  

Max strain

Applied average axial strain
Experimental results for sharp indenter transverse loading have shown a significant degradation in axial tensile failure strain of yarn (see Figure 2.9(b)). This indicates a presence of stress-concentration in the fibers within the yarn caused by the sharp radius of curvature of 20 µm. Figure 4.3 (a) shows the evolution of the maximum axial strain in the yarn at the projectile-yarn contact zone for a failure angle of $\theta_{\text{fail}} = 10.4^\circ$. The axial tensile strain in the contact zone monotonically increases, reaching a value of 2.7% at an applied axial strain of 0.73%. Fig 4.3(a) also shows the evolution of the strain concentration factor (SCF) of 3.7 up to failure for the given failure angle. This strain concentration occurs over a contact length of approximately 0.243 mm as depicted in Figure 4.3 (b) which is approximately 10x larger than the ROC of the sharp indenter (~0.02 mm). Unlike the round projectile, transverse shear strains contribute significantly to the deformation of the yarn in the yarn-projectile contact zone as shown in Figure 4.4(a). However, transverse compressive strain contribution to yarn deformation is astonishingly negligible and maximum axial compressive strains are slightly higher than the maximum axial tensile strains. Transverse shear strains achieve a maximum value of 39% compared to 4.2% for axial compression at experimentally applied average failure strain of 0.73%. Note that LS-DYNA outputs tensorial shear strains. Thus, the shear strains predicted from the model were converted to engineering shear strain in Figure 4.4(a) to make an accurate comparison of strains between different deformation modes.

The model suggests that transverse compression and axial compression do not contribute significantly to the yarn deformation and it is only the shear strains that contribute to degradation of the tensile strength seen in experiments. It also suggests that the yarn failure must be dominated by shear as was observed from experiments. The model gives
a new insight that the SCF occurs over a longer length of the yarn compared to the indenter ROC which can potentially explain the higher degradation in strength observed in yarns compared to single fibers for the same indenter geometry.

![Graph](image)

**Figure 4.3** a) Evolution of axial tensile strain and b) contours of axial strain at θ\text{fail} = 10°.

![Graph](image)
Figure 4.4 a) Evolution of AC, TS, TC strains at $\theta_{\text{fail}} = 10^\circ$ and b) Shear strain contours.

For the sharp indenter model, we attempt at predicting failure using a failure criterion developed by Sockalingam et al. that incorporates the degradation effects of multi-axial loading on axial tensile failure strain for Kevlar KM2 [19] and Dyneema SK-
76 [20] single fibers. We apply the same failure criteria towards predicting failure of yarn which is described in equations 4.1 and 4.2 below.

\[
\frac{\varepsilon_{3,\text{max}}}{\varepsilon_{3,\text{fail}}} = 1 \tag{4.1}
\]

where

\[
\varepsilon_{3,\text{max}} = SCF \times \varepsilon_{3,\text{avg}}
\]

\[
\varepsilon_{3,\text{fail}} = \varepsilon_3(L_c, ACr, TCr, TSr)
\]

\[
\varepsilon_3 = \varepsilon_3(L_c) \times (1 - ACr) \times (1 - TCr) \times (1 - TSr) \tag{4.2}
\]

where \(\varepsilon_{3,max}\) is the maximum axial tensile strain predicted by the model. \(\varepsilon_{3,avg}\) is the average far-field strain. \(\varepsilon_{3,\text{fail}}\) is the axial tensile strain as a function of a failure strain based on the Weibull model at a gage length equal to contact length \((L_c)\). \((1 - ACr)\), \((1 - TCr)\), \((1 - TSr)\) are the reduction factors for the respective individual deformation modes of axial compression, transverse compression and transverse shear. \(ACr, TCr, TSr\) are the degradation percentages for the respective deformation modes of axial compression, transverse compression and transverse shear.

The maximum axial tensile strain \(\varepsilon_{3,max}\) is found from the FE model as function of the loading angle and applied average strain as shown in Figure 4.3(a) for the sharp indenter. The yarn is subjected to a strain concentration of 3.7 over a length, \(L_c\), of 0.243 mm as shown in Figure 4.3(b). The tensile strain to failure \(\varepsilon_3(L_c)\) is based on the Weibull weakest-link model [36] as described in equation 4.3, where the gage-length of the yarn is equal to the length \((L_c)\). For 50\% probability of failure, we can find the average failure strain for a given gage length by setting \(P = 0.5\) in equation 4.3 and solving for \(\varepsilon\) as shown in equation 4.4.
\[
P(\varepsilon, L) = 1 - \exp \left( \frac{-L}{L_0} \left( \frac{\varepsilon}{\varepsilon_0} \right)^m \right) \tag{4.3}
\]

\[
\varepsilon = \varepsilon_0 \left( \frac{-L}{L_0} \ln(0.5) \right)^{\frac{1}{m}} \tag{4.4}
\]

where \(L_0\) is the reference gage length at which the scale parameter \(\varepsilon_0 = 0.0327\) and shape parameter, \(m = 9.468\) are determined from the baseline tension data in Chapter 3. \(P(\varepsilon, L)\) is the cumulative probability of failure of gage length \(L\) at strain level \(\varepsilon\). Using Weibull scaling with a reference gage length \(L_0\) of 254 mm, the extrapolated failure strain for a gage length of 0.243 mm is 6.55%.

Figure 4.3(a) shows that the maximum tensile strain in the projectile-yarn contact region are significantly higher than the experimentally measured average far field failure strains. Knowing the average failure strain for a contact length of \(L_c = 0.243\) mm we can employ equation 4.2 to calculate the failure strain and compare it to the maximum tensile strain during the loading history to predict failure. The percentage degradation due to axial compressive strains (ACr) and transverse compressive strains (TCr) are considered negligible, given that they are relatively small compared to the shear strains (see Figure 4.4a).

Figure 4.5 shows the effect of shear strains on the residual tensile strength of SK-76 single fibers from a study by Hudspeth et al. [22]. For shear strains of the order of 40\%, the residual tensile strength of the fibers is approximately 2 GPa. This results in a degradation factor, TSr of approximately 43\%. Using this TSr, along with \(\varepsilon_3(L_c)\) and \(\varepsilon_{3,max}\) during the loading history we can predict yarn failure. However, it must be noted here that in their study Hudspeth et al. applied shear strains by longitudinally (1-3 plane, 3 being fiber direction.) twisting individual fibers to different levels of strain. In our
model, we assume that transverse shear strains (2-3 plane) as shown in Figure 4.4(b) result in the same level of degradation in tensile strength as longitudinal shear strains (1-3 plane).

Figure 4.6 shows the failure criterion plotted against applied average failure strain. Failure occurs when failure criterion equals 1 or in other words when \( \varepsilon_{3,max} = \varepsilon_{3,fail} \). This occurs at an applied average axial strain of 0.0082 as seen from Figure 4.6. There is a sudden jump in the failure criterion seen after an average axial strain of 0.008 which may be caused by non-physical modes seen during these time steps. It is possible to reduce this jump in failure criterion by running the model at smaller time steps.

![Figure 4.5 Effect of shear strain on residual tensile strength of SK-76 single fibers [22].](image)

Figure 4.5 Effect of shear strain on residual tensile strength of SK-76 single fibers [22].
Experimentally, for a failure angle $\theta_{\text{fail}} = 10^\circ$, the yarn fails at average axial strain of $0.0073 \pm 0.0007$. Thus, the model gives a reasonable prediction of failure strain, although it slightly overpredicts the failure compared to experiments. Note that the point of failure discussed here corresponds to maximum transverse load point before the onset of progressive degradation in the load carrying capacity of the yarn. Thus, the model results show that it can reasonably predict the strain at maximum transverse load point. In order to predict final failure (transverse load equal zero), progressive failure of fibers as seen in Figures 2.6 and 2.7 needs to be incorporated into the failure criterion. It has been shown by several authors in the literature [37–39] that breaking of single fibers is a dynamic event because it generates a stress-wave in the broken fiber. Ganesh and Sockalingam et al. [38,39] have reported that stress waves generated on fiber break in a glass-fiber composite induce dynamic stress concentration in the neighboring fibers that are significantly higher and have a larger zone of influence than the corresponding static stress concentration factors. Thus, in future fiber-scale yarn modeling studies are required.

Figure 4.6 Failure prediction for transverse loading of yarn using sharp indenter.
to study the dynamic effects of fiber break and its effect on the progressive failure of fibers which can then be incorporated into the failure criterion. Additionally, our aim is to run more simulations to collect more information at different failure angles, $\theta_{\text{fail}}$. 
CHAPTER 5

CONCLUSIONS AND FUTURE WORK

This work presented the experimental and preliminary finite element simulation results of transverse loading of yarns by four different indenter geometries. It lays the foundation for developing a multi-axial failure criterion that accounts for the different deformation modes observed in yarn impact scenarios and can offer better prediction of yarn performance. Transverse loading experimental results using blunt indenters (ROC = 3.8 mm and 0.2 mm) show no significant degradation in the axial failure strain compared to the uniaxial failure strain of the yarn and fail in tension. However, there is approximately 75% degradation in the axial failure strain of the yarn when loaded using sharp indenters (ROC = 0.02 mm and 0.002 mm). For blunt indenters yarns fail in tension whereas for sharp indenters failure occurs via cutting type shear behavior and is much more progressive in nature. Additionally, experimental and modeling results of quasi-static transverse compression of single yarns show that transverse compression only causes a 4% reduction in the quasi-static tensile strength of the yarn. This is understood to be caused by non-uniform loading and significant spreading of fibers within the yarn on transverse compression. Preliminary finite element simulations of transverse loading experiment using sharp indenter show that shear strains contribute significantly to yarn deformation whereas transverse compressive strain contribution is insignificant. Thus, it can be gleaned from experimental and finite element results that yarn failure is progressive in nature and is mainly controlled by transverse shear. In future, fiber-scale
yarn modeling studies are required to understand the dynamic effects of fiber break and its effect on the progressive failure of fibers which can then be incorporated into the multi-axial failure criterion. Additionally, we aim to run more simulations to collect more information at different failure angles, $\theta_{fail}$. 
REFERENCES


[34] Sockalingam S, Bremble R, Gillespie JW, Keefe M. Composites : Part A
Transverse compression behavior of Kevlar KM2 single fiber. Compos PART A

Investigation of the High Strain Rate Transverse Compression Behavior of

Mech 1951.

fiber friction on fiber damage propagation and ballistic limit of 2-D woven fabrics
doi:10.1016/j.ijimpeng.2016.06.007.

[38] Ganesh R, Sockalingam S, Haque BZ, Gillespie JW. Dynamic effects of single
fiber break in unidirectional glass fiber-reinforced composites. J Compos Mater

unidirectional glass fiber-reinforced polymer composites: Effects of matrix