Math Workshop and Its Impact on Higher-level Thinking Skills

Elizabeth Ross

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Math Workshop and Its Impact on Higher-level Thinking Skills

by

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Abstract

This paper addresses the problem of students not using problem solving and critical thinking skills in high school mathematics classes. It describes an action research project that looked at the impact of Math Workshop on higher-level thinking skills, number of attempts to demonstrate proficiency, and students’ feelings/attitudes about mathematics in a high school Algebra II setting. In a small high school, 20 students enrolled in an Algebra II course were selected to complete a pre-test, post-test and tasks framed by Math Workshop. A triangulated mixed-methods design was used to compare quantitative and qualitative data to answer the research questions. Descriptive statistics were used to analyze the quantitative data, while coding was used to organize and analyze the qualitative data. The two sets of data were interpreted together to find the results of this study. Math Workshop increased the use of problem solving and critical thinking skills, reduced the number of attempts to demonstrate proficiency, and had a positive impact on students’ feelings/attitudes about mathematics. These results supported research on constructivism, Math Workshop, and the use of 21st century skills. Implications for this study are the introduction of the Math Workshop framework within the school, and eventually the District where this study took place. A larger study needs to be conducted with different populations and settings in order to confirm the results.

Keywords: Math Workshop, Proficiency, Mathematics, Constructivism, 21st Century Skills, High School, Problem Solving, Critical Thinking
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Chapter 1: Research Overview

Public education in the United States has taken many approaches to teaching students over the past 100 years. Two ideologies that have been used in opposition to each other are the scholar academic and learner centered approaches (Schiro, 2013). Schiro describes that the scholar academic approach assumes that experts in each academic field should decide what content should be taught and tested based on what has been useful in preparing students in the past. Teachers need to have a strong understanding of the content they teach in order to prepare students for a future in the field (Joseph, Bravmann, Windschitl, Mikel, & Green, 2000). Dewey (1938) referred to this as the traditional approach with teachers passing on their knowledge to students and enforcing behavioral rules. Dewey also pointed out students are passive learners in this setting and are expected to acquire knowledge that other people have without judging and reflecting on their learning. The learner centered ideology focuses on the needs of individual students, allowing them to develop their own knowledge through organic experiences (Schiro, 2013). This approach nurtures risk-taking, keeps learners actively engaged, focuses on the process of learning instead of the product, and allows students to move at their own pace (Joseph et al., 2000). This progressive approach uses quality experiences to build on prior knowledge along with time for reflection and deeper thought (Dewey, 1938).

According to the website for District A (pseudonym), it is one of the first districts in Maine to fully transition to a proficiency-based, learner centered school system. The
vision describes learner centered education as putting students at the center of every
decision and focusing on effective learning for each child’s personal path. The district
joined a state cohort and created a set of Measurement Topics for each subject area that
students were expected to meet. District A then adjusted these Measurement Topics to fit
the needs of their students and created a math flow to order these topics into a logical
sequence.

Proficiency in District A is defined on their website as having specific knowledge
to meet a purpose. Proficiency-based education assumes that schooling will be focused
on the learners first. District A is committed to becoming a learner centered model of
education. This system has been in place and growing for nine years, and student
evidence is graded on a 1-4 scale. This scale gives each assessment a score of 1
(beginning), 2 (developing), 3 (proficient), or a 4 (advanced). These levels are
determined by where the required thinking skills fall on Marzano’s Taxonomy (See
Appendix A) and how each piece relates to those skills. As the level of thinking
increases, the scores on assessments do too. Students are expected to meet specific targets
in a logical progression in order to graduate. District A believes that students need to be
taught and assessed in different ways as society changes. Applied learning, a tenant of
District A, allows for different ways to demonstrate newly acquired knowledge, and Math
Workshop is a framework to apply information while learning, as well as a way to
demonstrate proficiency.

As a math teacher of 11 years, this researcher has noticed that to get the minimum
requirement score of 3, the thinking skills required are often recalling and executing on
the Marzano Taxonomy Chart (Marzano & Kendall, 2007). The level 4 work is not
required, and many students are not pushing themselves to do the extra work, leading to minimal use of thinking skills that are higher on the taxonomy, such as problem solving and critical thinking. Math lessons are still taught in the traditional format as described by Dewey (1938): the teacher stands in the front of the room, shows students an example, and then lets the students work on more examples independently. This requires little thinking on the part of the student, but now they are required to keep working on the material until they meet the standards. The teachers in District A are noticing that multiple retakes are often needed before students can be successful in demonstrating that they have truly learned the material. This has led to classes having multiple groups of students working on different topics simultaneously, leaving less time for the teacher to work with each student’s individual needs.

The state of Maine uses the SAT to determine proficiency of third year high school students, and as a whole the percentage of students meeting the state requirements are dropping (Maine Department of Education, 2018). On average 42.66% of students taking the assessment met the state level of proficiency from 2007-2010 (Maine Department of Education, n.d.). That percent rose to 47.50% from 2011-2012 (Maine Department of Education, n.d.), but has since fallen to only 37.97% on average from 2015-2018 (Maine Department of Education, 2018).

As a school, Monarch High School’s (pseudonym) percentage of students meeting proficiency in mathematics is worse than the state average. Although the school’s average exceeded the state average from 2007-2010 with 44.55% of students demonstrating proficiency, that number dropped to 43.15% when the state average rose from 2011-2012 (Maine Department of Education, n.d.). Monarch’s success continued to
fall from 2015-2018 with their percentage of students meeting proficiency dropping to only 30.49% (Maine Department of Education, 2018). District A began implementing proficiency-based education in its schools in 2011. In 2016, the SAT changed their format of questions to more realistic and application types of problems (College Board, 2019a). “The SAT Math Test covers a range of math practices, with an emphasis on problem solving” (College Board, 2019b) showing that students need to increase their use of this while learning the content to be successful on this test.

In response to school percentages dropping even faster than the state’s, District A investigated Math Workshop, a different way of teaching mathematics (Bresser & Holtzman, 2018). This framework entails students doing more assignments requiring manipulatives, having more discussion about thinking, and discovering the math themselves instead of always being told the rules (Bresser & Holtzman, 2018). Math Workshop is about building understanding through learner ownership (Hoffer, 2012). As teachers guide students through the process, perseverance and independence are promoted by allowing students to challenge themselves and choose their own paths to solutions (Hoffer, 2012). Activities and problems are thought-provoking and allow for multiple approaches, promoting the use of higher-level thinking skills (Bresser & Holtzman, 2018).

**Problem of Practice**

In pursuing a learner centered, proficiency-based system of education, District A desires to cultivate hope in its students. To cultivate hope, students need to experience success and be prepared for the world after high school. They need to be comfortable with problem-solving and analyzing mathematical situations in the real world (Joseph et
Based on Dewey’s (1938) description of traditional teaching of mathematics, it seems reasonable to conclude that it does not allow for students to use the higher-level thinking skills for this success. Critical thinking is needed to make and support decisions in the real world and needs to be taught in schools (Wilcox et al., 2017). Giving constant directions and step-by-step processes limits student independence and fails to encourage the growth the district wants. The problem of practice is that students are not using thinking beyond the proficient level. The purpose of the study is to examine a framework of teaching mathematics that encourages problem solving and critical thinking, improves communication skills, and reduces the need for students to assess multiple times before demonstrating proficiency.

**Summary of Background Literature**

John Dewey (1938) had concerns about passive learning with drills and tests limiting a student’s ability to judge and reflect. Decades later the same concerns arise as high school graduates are unable to problem-solve and think critically upon entering the real world (Spence, 2001). Students learn naturally when they are interested in content and build their own connections with the knowledge, but it is difficult for teachers to meet the individual needs of each student (Schiro, 2013). Instead of varied experiences suggested by Dewey, students still memorize information to prepare for tests that do not examine the ability to problem-solve or analyze situations (Spence, 2001). These tests produce anxiety that affects the actual learning that is taking place (Ashcraft, 2002).

Around the world, countries are developing lists of 21st century skills that they think are important for students to have when they leave high school to be successful in the workplace (Wilcox, Liu, Thall & Howley, 2017). Although there are multiple
frameworks outlining these skills, international scholars agree on the need for communication, collaboration, critical-thinking, and problem-solving (ISTE, 2019; P21, 2016; Voogt & Roblin, 2012). Even though there is international agreement that these skills need to be taught within the curriculum, they are still not being actively applied within many mathematics classrooms (Voogt & Roblin, 2012). Authentic applications that scaffold deeper thinking within the classroom will better prepare students for the real world than the move toward high stakes testing and school accountability will (Wilcox et al., 2017).

The United States has high expectations in mathematics achievement, but standardized test scores show those expectations are not being met (Hoffer, 2012). “Students arrive at the end of high school without adequate academic skills” (Bailey, Joeng, & Cho, 2010, p. 256). Bailey et al. continue to explain that more than half of all students who attend community college are required to enroll in remedial courses. Students are generally not successful in these courses and need to take them more than once to pass (Dabkowska & Sosnovski, 2016). Remedial recommendations are based on placement tests, standardized test scores, and/or academic records, and the requirements to be college-ready vary by school and state (Bailey et al., 2010). Mathematics is an abstract concept, and students need a way to make the content real and concrete (Hartshorn & Boren, 1990). Hartshorn & Boren also state that newer school standards are leading toward open-ended questions and problem-solving applications that traditional approaches to education will not fully support. Hoffer shares the need to explicitly teach students to think with tools, hands-on tasks, time to struggle, realistic experiences, and time for reflection. Math Workshop is a teaching framework that assists
students in building numeracy, connecting new activities with prior knowledge, and
deep understanding of content instead of simple memorization (Legnard & Austin,
2012). The activities within Math Workshop involve multiple approaches allowing
students to use problem solving and critical thinking to find their own answers (Hoffer,
2012; Heuser, 2002; Bresser & Holtzman, 2018).

**Theoretical Framework**

Math Workshop is a framework for teaching mathematics with a deeper understanding. Although there are many names for the parts of the workshop, all versions have an opener, mini lesson, work period and time for reflection (Hoffer, 2012; Heuser, 2002; Bresser & Holtzman, 2018). The order of these pieces can change depending on the goals of the lesson, and it may take multiple class periods to complete the full workshop (Hoffer, 2012). Each piece of the workshop has a role in making learning meaningful to students based on educational theory. Constructivism is a key component in this framework as well as making the classroom learner centered to provide students with the best options in extending their knowledge (Heuser, 2002).

A key tenet of constructivism is that students create their own knowledge from their experiences with the content (Ertmer & Newby, 2013). Jean Piaget and Bärbel Inhelder (1969) share that as children develop, they are able to see deeper relationships and make connections with concrete and abstract content. Lev Vygotsky (1978) explains that children use language and tools to solve problems and build understanding, create meaning within social situations, and understand relationships with the help of others. Building connections with prior knowledge is a key component of constructivism, but students develop at differing rates and have different life experiences creating a need for
different approaches and supports within each task (Hoffer, 2012). The learner centered ideology involves considering each student individually and putting their individual needs first within the classroom (Schiro, 2013; Montessori, 2017). In this ideology, teachers meet students where they are and allow for differing approaches for creating their own meaning (Schiro, 2013; Smith, Walters, & Leinwand, 2015).

Using an opener gets the students situated into the day’s work, allowing them to build connections to their prior knowledge (Ertmer & Newby, 2013). A problem of the day that practices old skills or introduces the day’s topic is a common opener (Hoffer, 2012). Presenting an interesting problem to engage student thinking and demonstrate what they will be able to do at the completion of the workshop is another way to begin the class (Heuser, 2002). The constructivist theory of learning states that people build knowledge by connecting what they already know with a new experience, and both methods activate that knowledge (Eggen & Kauchak, 2007; Montessori, 2017). If students are not yet able to complete the initial problem, they are able to discuss what they already know about the topic and what they may need to know to solve it (Hoffer, 2012). They also can make predictions about the answer that they will check later in the workshop, providing them practice with their own metacognition and perseverance skills (Eggen & Kauchak).

The goal of the mini lesson is to provide students with essential information or skills, model an approach to the upcoming activity, or provide practice with previous content (Hoffer, 2012). Students may be provided with manipulatives to make an abstract topic more concrete (Kolb & Kolb, 2009). The mini lesson could be an introduction to manipulatives with an explanation of how to use the tools (Bresser &
Another use of the mini lesson is to demonstrate a problem and talk students through the teacher’s thinking (Hoffer, 2012). This process helps students see how connections can be made, breaks larger problems into smaller pieces, and/or encourages appropriate strategies that can be used when trying problems independently or in small groups (Ertmer & Newby, 2013). A third use of the mini lesson is having it follow the work period (Heuser, 2002). Students are allowed to grapple with a problem or activity for a while, supporting a cognitive learning style by allowing children to work together to test their ideas and build meaning on their own (Eggen & Kauchak, 2007; Warshauer, 2014). When the mini lesson is used this way, students can explain their thinking and the teacher can help to demonstrate examples or guide them through any misunderstandings that they may have (Hoffer, 2012; Smith et al., 2015). It also gives students the chance to internalize what they worked on as a group, and solidify their personal learning (Eggen & Kauchak, 2007).

The work period can come before or after the mini lesson, but its purpose is to give students time to work with engaging and challenging tasks (Hoffer, 2012). Allowing students to decide their own approach to meaningful problems, test possible solutions without consequences, and explore content instead of being told the information promotes a deeper understanding of the material in a learner centered classroom (Joseph et al., 2000). During this time, students work with new tools, explore ideas, and have choice in their approach (Bresser & Holtzman, 2018). Students work on fewer problems than in a more traditional classroom, but these problems promote deeper thinking, put the math into context, and develop cognition instead of memorization (Heuser, 2002). The tasks can vary from learning games that revisit older content (Bresser & Holtzman,
ill-defined problems that allow multiple approaches and answers (Heuser, 2002), and/or connect mathematics content to realistic situations that are applicable outside of school (Legnard & Austin, 2012). During this time, students may work independently, in small groups, or have direct instruction from the teacher (Hoffer, 2012; Slavin, 1980). All these approaches allow students to actively create their own understanding with relevant experiences (Ertmer & Newby, 2013). Problems and activities are goal oriented instead of answer based, meaningful to students, encourage active exploration and persistence, build on previous knowledge, and provide opportunities to apply what they have learned in new situations further demonstrating a constructivist approach (Ertmer & Newby, 2013).

Another aspect of the work period is that not all students are always doing the same thing (Legnard & Austin, 2012). The problems may be tiered to meet students with differing prior knowledge, questions may be scaffolded to accommodate different guidance needs, and choices may be provided on how to practice and demonstrate their learning (Hoffer, 2012; Tomlinson, 2001). Dewey (1938) shares that experiences should arouse curiosity and prepare students to deal with future experiences. They should build on each other allowing students to build their knowledge in pieces based on what they see and do (Eggen & Kauchak, 2007; Mighton, 2003). Since problems are not well defined, students can choose to approach the solution in multiple ways and relate what they are doing to other experiences they have had outside of the classroom, further deepening their understanding (Cai, Moyer, & Gowchowski, 1999). Teachers assist students with their discoveries instead of telling them what to do, design tasks that will link student interests with the mathematical content, and allow students to move on at their own pace.
while encouraging a productive struggle making workshops learner centered (Schiro, 2013).

The reflection piece is arguably the most important part of Math Workshop. Students and teachers can uncover errors in thinking through discussion (Hoffer, 2012). Journal entries and sharing methods allow students to consolidate all the pieces of the workshop and solidify their understanding (Heuser, 2002). Reflection also reinforces the learning that occurred by giving more exposure to the content as well as other approaches to the same problem (Bresser & Holtzman, 2018). Reflecting in small groups provides students with the opportunity to strengthen their communication skills by explaining why they took the approach they did, and to see connections other students made, all strengthening their own understanding (Dewey, 1938; Slavin, 1980). Analyzing their own or other students’ work during the reflection period can build critical thinking skills companies desire in their future employees (Wilcox, et al., 2017). Experiential learning is a spiral that includes reflection in order to make decisions in different situations in the future by judging success and relationships with other problems (Kolb & Kolb, 2009; University of Chicago, n.d.). Finally, teachers can use the reflections, whether they be in journals or group discussions, to plan future lessons based on where the students are at the end of the workshop (Heuser, 2002).

Assessment is a key part of education as it is a way to see what students know and communicate that with others. Formative assessment shows where a student is in the moment and helps the teacher prepare the next lesson, problem, or task (Hoffer, 2012; Mighton, 2003). Interviews are an example of formative assessment that can show the teacher what the student is thinking, and if there are any misconceptions about the
content (Heuser, 2002). Summative assessment is given at the completion of a unit, and in a proficiency-based system, must be completed in order to move to the next topic (Marzano, Norford, Finn, & Finn, 2017). These assessments do not have to be formal tests, but instead can be another problem to solve, a project, an interview or even a portfolio of the work done during the workshop (Heuser, 2002). The learner centered ideology is reflected in this part of Math Workshop by allowing students the ability to choose how they demonstrate their knowledge when they are ready for assessment (Schiro, 2013).

**Action Research**

The purpose of this action research project was to determine if using the Math Workshop framework addressed the problem of practice: encouraging problem solving and critical thinking and reducing the need for students to assess multiple times to demonstrate proficiency. The framework of opening activities to activate knowledge, mini lessons to demonstrate examples, addresses missing skills, or provide instruction, activity time to work on challenging problems, and periods of reflection to solidify knowledge before moving on are linked to the constructivist theory of learning (Hoffer, 2012; Heuser, 2002; Bresser & Holtzman, 2018). Experiencing the content in real-life contexts allows students to create their own understandings (Eggen & Kauchak, 2007). Group tasks and reflections allow students to build from each other as well as to practice communication of their ideas (Ertmer & Newby, 2013). If students are using these methods to gain a deeper and more meaningful understanding of the content and have choices in how and when they are assessed, it seems to follow that they would be prepared to demonstrate proficiency.
These thoughts and data collection methods led to the research questions for this study:

1. What is the effect of using Math Workshop on increasing problem solving and critical thinking skills as demonstrated by successful completion of level 4 work?
2. To what degree does using Math Workshop impact the number of attempts to demonstrate learner proficiency?
3. What is the effect of Math Workshop on students’ general attitudes/feelings about math?

**Rationale**

The new SAT tests are designed to test understanding and application of mathematical content at higher taxonomy levels than the old version (College Board, 2019a). To score better on these assessments, students must practice these skills in the classroom (Hoffer, 2012). The first question addressed the problem of practice by monitoring the use of problem solving, critical thinking and communication skills. Tasks and assessments were designed to require the use of these skills. If students could complete them during the workshop, it showed that they used the skills. If students needed scaffolding to complete the tasks, the researcher was able to see which skills needed support (Eggen & Kauchak, 2007). When lessons, tasks, or reflections required student explanations, students practiced communication, both written and verbal. Journals also showed the development of these skills over time. The second question addressed the problem of practice by recording the number of attempts that students needed to demonstrate proficiency.
**Positionality**

I believe that all students can learn if given the right resources, appropriate guidance, and enough time. This is also the belief of District A, as shown by the fact that they strive to be a learner centered system. Math Workshop provides these opportunities by giving students the chance to construct their own understandings of the material in a safe, encouraging space (Hoffer, 2012). This unit was the first experience that both the students and myself had experience using Math Workshop.

In response to students needing multiple attempts to demonstrate proficiency, lack of student communication about mathematics, and students not attempting the level 4 work (therefore not using problem solving and critical thinking skills), I decided to study my own mathematical teaching practice. The results of this study could affect the future of my classroom as well as other classrooms in the district, making me an insider in this research study (Herr & Anderson, 2015). Sharing the same race as all the students in this study as well as an upbringing in the same state made me an insider, because I share many life experiences with the students (Herr & Anderson, 2015). Knowing the students and sharing similar experiences allowed me to create tasks that were more engaging (Schiro, 2013). As the research progressed, I reflected on any changes made to the lessons to ensure that they were still following the workshop framework and meeting the needs of the students.

Mertler (2014) describes a participant-observer continuum outlining the varying levels of researcher participation in their studies, ranging from an observer to a full participant. As an insider in this study, I fell closer to the full participant end of the continuum (Mertler, 2014). A full participant would be part of the group and
participating in the tasks with the students while a participant as observer is able to interact with the participants, ask questions, and be in the room without taking part in the tasks (Mertler, 2014). I fell somewhere in between those two as I was providing instruction and guidance instead of doing the tasks, but I was also part of discussions and daily interactions with the students (Mertler, 2014).

Collaboration is inevitable when using the Math Workshop framework as students are working together, making choices on their activities and assessments, and building and sharing their knowledge (Legnard & Austin, 2012; Slavin, 1980). Each class in the study had a peer tutor in the room capable of guiding students with their learning. These peers needed training in the methods of Math Workshop in order to ensure the authenticity of the study. Collaboration between the participants came in designing tasks that interest them, meet their needs and assess learning (Schiro, 2013). I still chose the content to be covered and which activities or problems were offered, but these were chosen to best help the learners progress (Hoffer, 2012).

Design

A mixed-methods research design combines qualitative and quantitative data to best represent the study (Mertler, 2014). Qualitative data is useful for gaining insight while quantitative data can look at how many times something happens (Leech & Onwuegbuzie, 2007). This action research project used a triangulation mixed-methods design: I collected qualitative and quantitative data simultaneously and analyzed them together to interpret the results (Mertler, 2014). To address the first research question, quantitative data was collected on the completion of level 4 assignments. This was supported by qualitative surveys about problem solving and critical thinking. The second
research question was addressed with the scores received on the pre- and post-assessments as well as the number of times assessments were attempted in order to demonstrate proficiency. Qualitative data was collected from observing the participants in the study and reading their journals to answer question three. This was supported by observations on the methods used to demonstrate knowledge and behaviors seen when doing so. This design was appropriate for this action research as it combined parts of qualitative and quantitative studies to get a more complete picture and make stronger connections with the data (Leech & Onwuegbuzie, 2007).

The study took place in my two Algebra II classes at Monarch High School. To be in this class, students had completed both Algebra I and Geometry. It is a graduation requirement for District A, so all students take this course at some point in their high school careers. At the time of this study, students completed four of the nine units in the course, making the workshop model the mode for teaching the fifth unit, Probability, which was taught in the spring semester of 2020.

The participants consisted of 27 students (21 females and six males). Their ages ranged from 15-18, and they were a mix of sophomores (7), juniors (18), and seniors (2). One of the students had an individual education plan (IEP) due to low working memory, one student had a 504 plan due to medical needs, and two others had 504 plans due to attention issues and anxiety. The mean unweighted grade point average (GPA) of the class was 3.482 with a standard deviation of 0.209. The mean grade in Algebra I was a 3.485 and the mean grade for Geometry was a 3.607. The mean math PSAT score for the spring of 2019 was 463.2, while the two seniors had a mean math SAT score of 375. The median number of absences this year was one. Six of the students attended the vocational
school in the afternoons, causing two of them to leave math 15 minutes early each class. The majority of students (26) were White, while one was mixed race of White and African American. The researcher taught 19 of the students Geometry while the other eight students had a different teacher.

**Data Collection and Analysis**

There were five main sources of data collection for this study: pre- and post-tests, surveys, student artifacts, student journals, and field notes. Surveys provided insight into student experiences, feelings, thinking skills, and how often they attempted level 4 work (Mills, 2007). They gave students a way to share their opinions and perceptions three times during the study (Dana & Yendol-Hoppey, 2014). The surveys were done through Google Forms as it allowed for multiple choice, short answer, scales and checklists; it was easily shared through Gmail, which all students had; and data could be automatically sent to a spreadsheet for easier analysis (Gehringer, 2010). Student artifacts were any pieces of work completed during the study (Dana & Yendol-Hoppey, 2014). These were the activities designed for different problem solving and critical thinking skills, formative assessments, and homework assignments. These artifacts showed the work that students accomplished, the process they used to find solutions, any misconceptions held by learners, and classroom activity that may not have otherwise been seen (Dana & Yendol-Hoppey, 2014). Journals were completed at the end of each workshop period. These allowed students to explain their thinking, reflect on the day’s activities, and share the skills that were used that day (Mills, 2007). Finally, I collected field notes in each class. These were from direct observation while teaching as well as from video recordings of the lessons after. Field notes described in detail what was seen during the course of the
study such as student quotes, strategies used, or diagrams of the classroom (Dana & Yendol-Hoppey, 2014).

These data sources provided both qualitative and quantitative data. The qualitative information was from open-ended survey questions, journal entries and field notes. These were organized and coded by common words or phrases and then described based on the characteristics found during coding (Leech & Onwuegbuzie, 2007). A constant comparative method allowed all the qualitative data to be analyzed together, streamlining the coding process (Mertler, 2014). As the data came together and patterns were found, the story of the study was told (Dana & Yendol-Hoppey, 2014). Quantitative data were analyzed with descriptive statistics such as the mean and standard deviation (Sullivan, 2017). These numerical outcomes were organized in tables and graphs and provided a visual of the information (Mertler, 2014).

**Significance and Limitations**

This study builds on the constructivist theory of learning. The Math Workshop framework uses this theory of building one’s own knowledge as a way for students to learn mathematics (Hoffer, 2012). This study examined if Math Workshop encouraged the use of problem solving, critical thinking, and communication skills while also reducing the need to assess multiple times to demonstrate proficiency. It also addressed the concerns of the district in how students are not using their higher-level thinking skills regularly to exceed the proficiency standards. The intended audience is high school math teachers, curriculum leaders, and administrators. The interested parties will be able to gain insight on this approach at the high school level. They will be able to see if thinking
changes over time with a different framework of teaching as well as how these changes impact the number of times students try before reaching proficiency.

Every study has limitations, and this study was no different. I had a responsibility to teach students, so if they were not learning, that had to be addressed during the study. If students were unable to work on more abstract concepts, the material needed to be scaffolded to help them reach that goal (Eggen & Kauchak, 2007). Another limitation was that there was only so much class time. The workshop design often required more time being used on problems than the traditional methods of teaching (Hoffer, 2012), but I only had 60-minute class periods. Several students left math early each day to go to the technical school. This limited their exposure and made many journal entries homework assignments. Finally, a small sample in a small school provided a limited window to the effect Math Workshop would have on a larger population (Sullivan, 2017). Depending on the results of this study, a larger study will need to follow.

**Dissertation in Practice Organization**

Chapter 1: Research Overview has identified a Problem of Practice (PoP), the purpose of the study, and the research and observations that lead to the decision to complete this study. Chapter 2: Literature Review goes more in depth into the current research involving Math Workshop and higher-level thinking skills. It contains a detailed literature review on both topics in relation to mathematics specifically. Chapter 3: Methodology outlines the study, research objectives, questions, site, and participants. It also describes the design, data collection methods and explains how the data was analyzed. Chapter 4: Data Analysis presents the data and describes the findings. It shows reflections on the material and all the information that was collected in relation to
the research. Finally, Chapter 5: Results and Recommendations summarizes the findings of the study while relating them to current literature. It also makes suggestions for future studies that are needed or next steps in the classroom to improve the impact of Math Workshop on higher-level thinking skills.

**Glossary of Terms**

*Scholar Academic:* A curricular ideology that teaches students to think and act like scholars in each discipline and where academic experts know what is best to teach (Schiro, 2013).

*Learner Centered:* A curricular ideology that focuses on student needs and interests and allows them to develop at a natural pace (Schiro, 2013).

*Measurement Topics:* The name that District A gives to their standards.

*Level 4 Work:* Work that goes above the required proficiency level and uses higher-level thinking skills to apply the knowledge.

*Proficiency-Based Learning:* A learning system where students progress as they demonstrate understanding of topics instead of completing grade levels (Marzano et al., 2017).

*Hands-on Tasks:* Tasks that require students to move or manipulate items to solve problems (Eggen & Kauchak, 2007).

*Problem Solving Skills:* The skills used to overcome obstacles when the solution is not obvious (Eggen & Kauchak, 2007; Marzano & Kendall, 2007).

*Critical Thinking:* The ability to analyze and evaluate difficult or complex problems with no automatic solution (Wilcox et al., 2017).

*Communication:* The ability to convey thoughts and ideas to others (Wilcox et al., 2017).
**Scaffolding:** Providing visual cues, steps or feedback to bridge the gap between what students can do alone and where they want to go (Eggen & Kauchak, 2007).

**Metacognition:** The awareness of one’s own thinking (Eggen & Kauchak, 2007).

**Ill-defined problems:** Problems that can be solved multiple ways and have multiple answers (Eggen & Kauchak, 2007).

**Well-defined problems:** Problems with only one correct answer and one way to reach it (Eggen & Kauchak, 2007).

**Manipulatives:** “Objects that can be touched or moved to introduce or reinforce a mathematical concept” (Hartshorn & Boren, 1990).

**21st Century Skills:** Skills needed for success in the workplace after high school (Wilcox et al., 2017).

**Collaboration:** Working together on a task or problem (Hoffer, 2012).

**Remedial (Developmental) Education:** College courses that build skills to prepare students for college level work (Bailey et al., 2010).

**Numeracy:** The ability to understand and use numbers (Bresser & Holtzman, 2018).

**Perseverance:** The ability to stick with a problem or try something new before giving up (Boaler, 2016).

**Cognition:** The mental process of acquiring knowledge (Eggen & Kauchak, 2007).

**Tiered Problems:** Parallel tasks at different levels of difficulty (Tomlinson, 2001).

**Productive Struggle:** A method of teaching perseverance by allowing students to be challenged to find solutions (Warshauer, 2014).

**Formative Assessment:** Assessment that gives the teacher and learner information that helps in creating future lessons (Eggen & Kauchak, 2007).
**Summative Assessment:** The process of assessing after instruction is complete to see if learning has occurred and often to assign grades (Eggen & Kauchak, 2007).

**Individual Education Plan:** An individual plan for a student with special needs to provide support and required accommodations (Eggen & Kauchak, 2007).

**Marzano’s Taxonomy:** A continuum of thinking skills used to determine proficiency (Marzano & Pickering, 1997).

**Math Workshop:** A teaching framework that allows students to create their own mathematics understanding through relevant experiences, collaboration, and reflection (Hoffer, 2012).

**Traditional Teaching:** A method of teaching where the educator holds the knowledge and passes it on to students through lecture and steps (Dewey, 1938).

**SAT:** The standardized assessment, made by College Board, that the state of Maine uses to assess proficiency in third year high school students.

**Constructivism:** A theory of learning where students create their own knowledge through relevant experiences (Ertmer & Newby, 2013).

**Experience:** An engaging interaction with knowledge that encourages growth (Dewey, 1938).

**Zone of Proximal Development:** The place between where students are and where they could be potentially. The level of rigor that will promote the most growth (Vygotsky, 1978).
Chapter 2: Literature Review

The desire to improve how educators teach students is not a new idea, and District A is no different. In this world, there is an international call for a new set of skills to be successful after high school (Voogt & Roblin, 2012; Wilcox, Liu, Thall, & Howley, 2017). Since switching to a proficiency-based system, District A has noticed that students are not using these higher-level thinking skills as they are not part of the required content. Students also need multiple attempts to demonstrate proficiency, even at the lower cognitive levels. The problem of practice that this action research addressed is that students needed to improve their problem solving, critical thinking, and communication skills as well as reduce the need to reassess when demonstrating proficiency.

Math Workshop is a framework for teaching mathematics that has been successful at increasing problem solving and critical thinking in lower grades (Hoffer, 2012; Bresser & Holtzman, 2018; Heuser, 2002). The structure, consisting of opening activities, mini lessons, worktime, and a period of reflection, is linked to the constructivist theory of learning by allowing students to build their own knowledge through real-life activities (Eggen & Kauchak, 2007; Hoffer, 2012). The purpose of this study was to determine if using the Math Workshop framework encouraged problem solving and critical thinking and reduced the need for students to assess multiple times to demonstrate proficiency.

Mathematics is not an easy subject to learn without context of why it matters (Hoffer, 2012). The current approach to teaching mathematics involves memorizing an
algorithm that often does not make sense to the student (Heuser, 2002). Today’s world requires people to understand what they are doing and memorizing an algorithm does not connect the mathematical ideas to reality (Heuser, 2002). In District A students are required to meet proficiency on a progression of Measurement Topics, and these are often recalling and executing, the lower levels on the Marzano Taxonomy Chart (Marzano & Kendall, 2007). Students have the choice to go above and beyond to earn a 4 (advanced) where they use skills higher on the taxonomy, but many are not pushing themselves to do this. This leads to learners having less practice with problem solving and critical thinking and in turn being less comfortable with these skills.

The research questions that this study looked to answer were as follows:

1. What is the effect of using Math Workshop on increasing problem solving and critical thinking skills as demonstrated by successful completion of level 4 work?

2. To what degree does using Math Workshop impact the number of attempts to demonstrate learner proficiency?

3. What is the effect of Math Workshop on students’ general attitudes/feelings about math?

This chapter identifies the importance of literature review and describes the strategies used to conduct one. Conceptual perspectives of traditional math teaching, constructivism, and the learner centered ideology are discussed to explain the problem of practice, causes, and possible solutions. Finally, the Math Workshop framework is described and connected to the theories and ideas previously explained.
Methodology and Purpose

A literature review is how a researcher looks into what is already known about their topic of interest (Mertler, 2014). A complex literature review gives the author a starting point for his or her research and supports the decisions made while developing and executing the study (Machi & McEvoy, 2016). It allows for deeper understanding of the problem of practice and its implications in the overarching world of education (Mills, 2007). It also provides historical perspectives on teaching and presents scholarly support for the potential solution (Machi & McEvoy, 2016). In doing a literature review, the researcher is able to see what other researchers have discovered, multiple perspectives on the problem of practice, and various approaches to solving the problem (Mills, 2007).

The sources for this literature review were found in multiple ways. Searches of ERIC, Google Scholar, and the University of South Carolina’s library of databases revealed studies done using Math Workshop at the elementary and middle levels, grades K-8, but no studies could be found at the high school stage, reinforcing the need for it to be examined more thoroughly. The same databases provided journal articles on theories of education, alternate approaches to teaching math, and proficiency-based education. Texts on educational methods, curriculum, theories, and Math Workshop were recommended by peers as well as found through Google Scholar internet searches. Texts giving guidance for middle school, grades 6-8, application of Math Workshop were available, but still nothing at the high school level.

To understand the problem of practice, the history of teaching mathematics needed to be examined. Texts and journal articles regarding psychology and ideologies behind teaching help to show where different approaches in the classroom came from as
well as how these approaches changed over time. As the world changes, the skills to be successful in it also change, no matter what a student’s plan is after high school (Voogt & Roblin, 2012; Wilcox et al., 2017). In looking at international expectations of high school graduates at universities and within the workforce, gaps in desired skills were discovered, leading to the definition of the problem of practice.

In today’s society, there are certain expectations, such as standardized test scores, that schools are required to meet (College Board, 2019c). As those expectations change, so do the issues seen in the classroom. Once the problem of practice was defined, the next step was to identify potential causes and solutions. Journal articles and books about the historical perspectives of education brought light to where the problem of practice originated and how far spread it has become. Studies on Math Workshop show its potential to improve problem solving and critical thinking skills for elementary and middle school but also the need for further research at the high school level. These same sources show the links between Math Workshop, constructivism, and the learner centered ideology, making this framework an approach to consider when addressing the problem of practice for this research.

Review of Literature

District A is looking for a way to teach mathematics that promotes higher-level thinking skills and independence in its students. The district website claims they want students to have hope for the future and be prepared for success when they graduate. “Workshops foster and nurture students’ quests for wonder and exploration in a safe risk-taking environment” (Legnard & Austin, 2012, p. 228) and are therefore worth examining. This review investigated the traditional approach of stand-and-deliver to
mathematics education and how it helped to cause this problem of practice (Dewey, 1938; Schiro, 2013; Spence, 2001; Ertmer & Newby, 2013). In a world of ever-changing technology, the skills that colleges and jobs require of students are changing (Voogt & Roblin, 2012; Wilcox et al., 2017). Research on the development of constructivism and its connection to the learner centered ideology and experiential learning showed what well known educational theorists believe about learning (Dewey, 1938; Eggen & Kauchak, 2007; Ertmer & Newby, 2013; Cai, Moyer, & Grochowski, 1999; Kolb & Kolb, 2009; Montessori, 2017; Schiro, 2013). The framework of Math Workshop has been successful with the development of problem solving and critical thinking skills and is described and linked to those historical perspectives and the theory of constructivism (Hoffer, 2012; Bresser & Holtzman, 2018; Heuser, 2002; Legnard & Austin, 2012).

Finally, alternative perspectives and comparative studies are discussed in support of the choices made in this action research (Cai et al., 1999; Dabkowska & Sosnovski, 2016; Eggen & Kauchak, 2007; Ertmer & Newby, 2013; Kolb & Kolb, 2009; Legnard & Austin, 2012; Schiro, 2013).

**Conceptual Perspectives**

The world has changed with the development of technology, but many aspects of education have not. John Dewey (1938) describes traditional education as focused on scheduling, testing and rules of order. Adult standards are imposed on children when teachers pass on their knowledge, and students are expected to be docile and passive in their reception of it (Dewey, 1938). Dewey also shares the standard practice of skill and drill may enforce fact memorization, but not the development of higher-level thinking skills. This behavioral approach focuses on facts, but not the mental processes or
thinking skills behind the learning of those facts (Ertmer & Newby, 2013). Teaching involves providing a desired response and having students practice how to give that response on command, keeping students at the recall and recognition levels of the taxonomy chart (Ertmer & Newby, 2013; Marzano & Kendall, 2007). The content in this traditional approach is chosen by experts in each field, usually scholars with years of experience studying that content, and the goal of schools is to prepare students for further education upon graduation (Schiro, 2013). In focusing on furthering the academic fields, children who have other goals can be overlooked. Spence (2001) states that it is hard to overcome the public vision of education, and that these same methods are still seen in classrooms today.

Facts are more easily available today than they have been in the past due to growing access to technology, causing the international call for understanding, problem solving, and critical thinking to be taught in schools (Voogt & Roblin, 2012). The ability to solve problems, think critically, and communicate with colleagues are the skills that are needed for success in the workplace (Wilcox et al., 2017). These skills need to be part of the curriculum by being taught within the traditional content, but in order to do this successfully teaching methods need to change (Voogt & Roblin, 2012).

Ertmer and Newby (2013) explain that the theory of constructivism asserts that students create their own knowledge from their personal interactions with the environment. Students need real-life experiences to understand the abstract content, and the more examples they have, the deeper their understanding becomes (Ertmer & Newby, 2013). When experiences are meaningful to students, they can link what is happening with prior knowledge and use that link to transfer their learning to other environments.
(Ertmer & Newby, 2013). Ertmer and Newby support learners actively exploring ideas and solutions to build their problem solving and critical thinking skills. Teachers still need to plan realistic, meaningful tasks and provide educational objectives, but with the constructivist approach they do not tell the students every detail; they allow students to discover the rules on their own and to explain the why and how of their thinking instead of just giving an expected answer (Eggen & Keuchak, 2007).

Jean Piaget was one of the first psychologists to share the constructivist approach to education. He believed that people want the world to make sense and will explain their experiences based on what they already know or adjust their personal knowledge so that it continues to do so (Piaget & Inhelder, 1969). Individuals create their own ideas as they experience the world and then test those ideas in relation to other’s (Piaget & Inhelder, 1969). Eggen and Kauchak (2007) explain that Piaget believed that once students have basic knowledge, they can work with abstract concepts, but they need concrete experiences to make those connections. The use of manipulatives and hands-on activities relevant to students are ways to make these ideas concrete and engaging (Piaget & Inhelder, 1969; Hartshorn & Boren, 1990).

Another leader in the world of constructivism was Lev Vygotsky. Like Piaget, he believed that students needed to be active participants in their learning and to build their own knowledge (Vygotsky, 1978). Unlike Piaget, he felt that people needed to work together to learn, and that social interaction and language were key in the development of building knowledge before it was able to be internalized (Vygotsky, 1978). Vygotsky discussed the Zone of Proximal Development (ZPD), the level where students can be successful with help. The ability to create new ideas and knowledge depends on what is
already known, so teachers designing the experiences need to provide scaffolding for all students to be successful and grow; they need to know their students to be aware of where each one needs to start (Vygotsky, 1978).

There are many words that describe the way that students can build their knowledge, such as task, activity, or problem. Heuser (2003) shares that problems are realistic and develop cognitive strategies by increasing in difficulty and have multiple solutions allowing for different approaches. Activities build on each other, help to address misconceptions, and connect prior and new knowledge to deepen understanding (Cai, Moyer, & Grochowski, 1999). Although there are slight differences in each term, they are all ways to experience the world and build knowledge. John Dewey (1938) describes experience as a teacher that allows more contact between individuals, arouses curiosity, accounts for the abilities of all students, and stimulates deeper thinking in learners. Internships, games, and even lectures are experiences that focus on the process of learning, provide opportunities to analyze situations, and meet all learning styles as they range from concrete to abstract and allow conceptualization and active experimentation. (Kolb & Kolb, 2009).

The theory of constructivism can be seen in the learner centered ideology of curriculum and supported by John Dewey and Maria Montessori (Schiro, 2013). Montessori (2017) thinks education should develop naturally and that schools should allow free movement for the best growth in children. Instead of disciplining students into submission, teachers should allow them to experiment and get their rewards from their own hard work (Montessori, 2017). Dewey’s (2017) learner centered school focuses on students as they are now instead of what they might become in the future by connecting
what they know from home to classroom experiences allowing them to build connections and see the meaning of their activities. Schiro (2013) describes a learner centered classroom as one that is based on student interest and meeting students where they are based on their current developmental needs. Hands-on activities and manipulatives are used to solve real-world problems and allow students to discover their own facts (Hartshorn & Boren, 1990). Students must be active in building their knowledge, but different stages of development have different needs, so the teacher needs to be prepared to meet those varying levels (Schiro, 2013).

An approach that demonstrates a teacher’s belief in his or her students, provides context for abstract mathematics, non-routine problems to solve in groups, and time for reflection can be found in the Math Workshop (Heuser, 2002). “Math Workshop offers differentiated instruction of rich, rigorous mathematics that is attainable by all learners” (Legnard & Austin, 2012, p. 228).

Wendy Hoffer (2012) provides her version of Math Workshop in grades 4-8. She believes that educators need to teach students how to struggle with hard problems in a safe space to cultivate their understanding of mathematics. This struggle will help to develop perseverance and ownership over their own success (Hoffer, 2012). Her text provides guidelines for implementing Math Workshop, guides for each piece of the process, and how to use reflection to reinforce learning. In Hoffer’s workshop model, sessions begin with a purposeful opening activity that activates student knowledge and prepares them to work with the upcoming content. A mini lesson provides essential information, models thinking skills, and/or introduces manipulatives that students will use during their work time (Hoffer, 2012). The work period offers time for students to
work with tools while attempting to solve challenging and engaging problems in their own way activating their problem solving and critical thinking skills (Hoffer, 2012). Finally, Hoffer shares that the reflection period can uncover errors in thinking, provide a chance to share ways of thinking and potential solutions, and give students an opportunity to build the connections with their prior knowledge. Believing in students, celebrating their success, and providing a place of intellectual safety helps to develop success in mathematics learning (Hoffer, 2012).

Rusty Bresser and Caren Holtzman (2018) focus on Math Workshop in grades 3-5. Their text outlines the pieces of Math Workshop, the purposes of each piece, and how to tie them together for the best results. Routines warm students up to the content by providing an introduction or allowing them to make predictions (Bresser & Holtzman, 2018). Bresser and Holtzman describe focus lessons as a way to teach thinking skills and necessary mathematical skills that students will need in their worktime. This worktime uses rich, engaging tasks, or learning stations, to get students thinking and applying the abstract mathematical content while building connections with what they already know by making it more concrete (Bresser & Holtzman, 2018). Group work allows for practice collaborating on possible solutions and explaining and defending their decisions during reflection creates deeper understanding of the material (Vygotsky, 1978). The details in this text help to plan the lessons used in this study. Mistakes are part of the learning process and slowing down to find the misconceptions students have makes a big difference in the final outcomes (Bresser & Holtzman, 2018).

Daniel Heuser (2002) explains the community already accepts Reading and Writing Workshop, so modeling a method to teach math after that is efficient. Heuser
defends a constructive approach to connect the abstract ideas of mathematics to reality as well as the power of reflection in retention. Math Workshop focuses on fewer problems than the traditional method of teaching, but those problems go more in depth, focus on the process over the answer, and allow students the chance to invent their own solutions leading to a deeper understanding of the mathematics (Heuser, 2002). In a classroom where the teacher questions the students and their thinking instead of just answering questions, students are more likely to have the ability to apply what they are learning (Heuser, 2002). Beginning with a hook to activate knowledge and a mini lesson that explains the main idea of their upcoming activity allows students to be prepared for the activity period (Heuser, 2002). Heuser’s activity period gives students a real-world problem that can be solved with critical thinking and the use of manipulatives or a game that practices the use of previously addressed content. A reflection period provides students with an opportunity to connect their new experiences with their previous ones as well as a chance to consolidate all this information into a meaningful representation for themselves (Heuser, 2002).

Danielle Legnard and Susan Austin (2012) describe their approach to teaching mathematics that follows the design of Math Workshop. They point out how it makes for differentiated instruction by providing choice to the students while also keeping the work rich and rigorous. Legnard and Austin’s approach entails offering a menu of activities for students to work on following the mini lesson that introduces the skills and purpose of the workshop. Some activities on the menu are required while others are optional, allowing for students with different skills levels, prior experiences, and interests to all be successful (Legnard & Austin, 2012). These choices lead to the development of self-
direction and understanding instead of obedience and memorization (Legnard & Austin, 2012).

**Theoretical Framework**

Psychology is a large part of how people learn, and there are three major focuses in education, behaviorism, cognitivism, and constructivism (Ertmer & Newby, 2013). Behaviorism looks at learning as a change in behavior, reinforces giving the correct response to a given stimulus, and focuses on the *recall* and *recognition* levels of taxonomy (Ertmer & Newby, 2013). While it does encourage the development of basic mathematical skills, the focus on the bottom of the taxonomy chart limits the growth of problem-solving and critical thinking. Cognitivism on the other hand promotes these deeper mental processes by focusing on how students learn instead of just what they learn (Ertmer & Newby, 2013). Instead of viewing learning as a change in behavior, cognitivism sees learning as a change in knowledge and the ability to transfer that knowledge to new situations (Ertmer & Newby, 2013). Finally, constructivism views learning as how students build their own knowledge from experiences and real-life examples (Ertmer & Newby, 2013). Piaget’s version of constructivism has students working with relevant problems to build their own understanding and uses group work to test the ideas that they have individually created (Piaget & Inhelder, 1969). Vygotsky’s (1978) social constructivism sees students working together to build a group understanding and then uses reflection to solidify personal understanding. Math Workshop connects the views of Piaget and Vygotsky by allowing both individual and group work to create learning through various learning experiences (Eggen & Kauchak, 2007).
When developing curriculum, the ideologies of the developer and teachers influence the final outcome (Schiro, 2013). The scholar academic ideology expects students to learn the accumulated knowledge of the culture from their teachers and this knowledge is chosen by the academic experts in the field (Schiro, 2013). There are universal truths shown in the classics, and some feel those should be taught to each new generation (Joseph, Bravmann, Windschitl, Mikel, & Green, 2000). Another ideology, social efficiency, focuses on training students to function in society by mastering skills (Schiro, 2013). These skills come from the current needs of the public and often include both academic and technical approaches (Joseph et al., 2000). Finally, the learner centered ideology focuses on the needs of the students instead of outside people (Schiro, 2013). This approach lets students work at their own pace, encourages risk taking and problem solving, as well as links formal knowledge with experiences allowing students to create their own understandings (Joseph et al., 2000). Math Workshop definitely matches the learner centered ideology but includes pieces of the others such as required content topics and real-world skills (Hoffer, 2012).

John Mighton (2003) explains that children see the world differently from adults. Students thrive when a teacher shows they truly think all students can learn math and provides the guidance necessary for success (Mighton, 2003). He believes all students can learn math with the right level of guidance, and experiences should excite students. Skills need to be taught, but as students progress, they can do more exploring independently (Mighton, 2003). Math Workshop differs from traditional math education in that students look at relevant problems that have multiple approaches, have options in their learning, and use reflection to solidify and clarify their learning (Hoffer, 2012).
Teachers can teach to content standards while also stimulating thinking and perseverance (Dewey, 1938). Students should think about the reasonableness of their answers and practice making their thinking visible to others (Bresser & Holtzman, 2018). Teachers can cover their content while explicitly teaching these thinking skills, self-direction, and responsibility (Legnard & Austin, 2012).

In modern-day public education, programs are being developed to bring more experiences to the mathematics classroom, but they are falling short of the connections that Dewey claimed are important (1938). Everyday Mathematics is a program that uses real-life problems to introduce concepts as well as spirals the curriculum to maximize exposure for students (University of Chicago, n.d.). However, it is lacking the reflection piece that is the foundation of Dewey’s theory. The initial experiences may be enriching, but without reflecting on how they apply to the real world, much of the intended knowledge can be lost (Hoffer, 2012). Math Workshop has several key parts that make it successful. An opening activity to engage students, a mini lesson to teach a skill or address potential problems, work time that allows students to explore the math, and a reflection period to solidify and clarify what they have learned that day (Hoffer, 2012; Heuser, 2002; Bresser & Holtzman, 2018). The activities are hands-on and allow for a productive struggle, a method of teaching perseverance by allowing students to be challenged to find solutions developing deeper understanding for students (Warshauer, 2014).

Beginning with an opening activity that sets the purpose activates prior knowledge and builds curiosity (Hoffer, 2012). This activity could be a quick review, an open-ended question, or a way to model strategies to get students prepared to learn the
day’s mathematics (Bresser & Holtzman, 2018). Jean Piaget explains that people want the world to make sense and they do this by connecting their prior experiences to what they are currently experiencing (Eggen & Kauchak, 2007). When an opening activity introduces the day’s topic, it activates those prior experiences and prepares the brain to make links with what is coming next (Hoffer, 2012). If a teacher thinks the students do not have the prior experience with a topic that would make them successful, they can use the opening activity to give them background with the content (Hoffer, 2012). Using this activity prepares students for what is coming during class, allowing their brains a chance to adjust and be ready to learn (Eggen & Kauchak, 2007).

Following the opener, a mini lesson introduces a skill, addresses potential problems that students may encounter, builds on previous content, provides essential information needed for independent work, and/or models thinking and solution strategies for students (Hoffer, 2012). These lessons are often done with the whole group but meet the needs of all the learners in the room (Bresser & Holtzman, 2018). When a mini lesson involves the teacher solving a difficult problem while sharing his or her thinking out loud, it encourages problem solving skills and perseverance in students (Heuser, 2002). For some students, a brief review can prepare them for the upcoming activity while that same review can give other students the background knowledge they need to be successful with new content (Tomlinson, 2001). A teacher should provide tools and manipulatives, scaffold content based on current needs of the students, and demonstrate the thinking, not just the answer is important; the mini lesson is the perfect time to introduce these tools (Hoffer, 2012). When used after the activity period, a teacher can use this time to address any misconceptions seen, meeting the needs of the students in the
moment after a period of productive struggle (Bresser & Holtzman, 2018; Schiro, 2013; Warshauer, 2014).

The activity or work period allows students to explore tools, build excitement, see the big picture, and test possible solutions to the problems presented (Heuser, 2002). The tasks in this section of the workshop are chosen ahead of time and should build numeracy like a reading workshop builds literacy (Leonard & Austin, 2012). Workshop leads to differentiation by allowing students choice on the rich, thought provoking tasks that are provided for them (Bresser & Holtzman, 2018). Instead of using drills to memorize a process, Math Workshop encourages students to invent their own solutions based on their personal understanding; as their skills improve, teachers can guide students to more efficient methods (Heuser, 2002). Workshops develop deeper understanding by using non-routine tasks to get students thinking (Heuser, 2002; Bresser & Holtzman, 2018). Students should be encouraged to struggle while teachers help them find ways to help themselves (Hoffer, 2012). Questions that are hard now, may not be hard in the future if conceptual understanding is built through small successes along the way (Mighton, 2003).

Cooperative learning greatly enhances achievement, allows students to learn from each other and builds deeper understanding by encouraging learners to describe their thinking (Eggen & Kauchak, 2007). Collaboration promotes understanding, engagement, and cooperation while students work to solve complex math problems (Hoffer, 2012). “Students benefit from thinking and talking about what they need to do to solve a problem” (Bresser & Holtzman, 2018, p. 80). They are able to analyze errors, brainstorm issues, approaches, and solutions when having intentional conversations about
mathematics (Hoffer, 2012). Although students are individuals and may take different
paths to solve a problem, sharing their ideas with others promotes problem solving and
guides students to the ultimate goal, understanding (Dewey, 1938). When students
“collaborate to find deeper meaning behind the mathematics” (228) they learn more and
retain it better in small groups (Legnard & Austin, 2012). Math workshop provides
success across all ability levels by allowing students to work on tasks in their zone of
proximal development (ZPD) at the same time as other students (Heuser, 2002). The
ZPD is the space between the level of development and potential development of a
student’s knowledge (Vygotsky, 1978). Every student needs different things at different
times during their education, and Math Workshop provides the chance for the teacher to
meet all these needs (Heuser, 2002). Students need to be challenged at their own level,
and small group work allows children in similar places to benefit from each other’s
knowledge (Hoffer, 2012; Schiro, 2013). Working together to solve complex tasks at
their own level provides shared ownership of both the process and the solution (Bresser
& Holtzman, 2018).

Experiences should have meaning to the students, connect to past and future ones,
and arouse curiosity through engaging activities (Dewey, 1938). These activities allow
students to connect abstract content to the real world making it relevant and meaningful
(Legnard & Austin, 2012). In what Dewey (1938) refers to as progressive education,
experiences are the teachers, while the adults in the classroom are there to guide students
through them. All parties contribute to each other’s learning while the instructors
promote continued growth and use what they see to direct and encourage next steps
(Dewey, 1938). Building connections between the activities being completed, the
learning that is happening, and its application to the real world is the best way to gain knowledge (Dewey, 1938). Required content should be taught through problems that have multiple ways to solve them, different ways to start, allow students to use new ideas, and allow them to build their own understanding of the material (Hoffer, 2012). When rote skills are necessary, they can be supplemented with conceptual exercises to promote understanding development, possibly leading to larger learning gains in the future (Mighton, 2003). As skills are developed, experiences that allow students to construct their own meaning and apply it to real life will help increase these gains (Ertmer & Newby, 2013).

Finally, reflection time allows students to organize and make connections with all they have seen, gives them a chance to develop their vocabulary while defending their methods, and practice what they have learned in different situations (Heuser, 2002). They are able to connect abstract ideas to real-life context by thinking about the purpose and meaning behind the mathematics, while also linking their words to the equations and symbols they have been using (Bresser & Holtzman, 2018). “Students learn most when they spend math work time thinking, talking, and making meaning of mathematics for themselves” (Hoffer, 2012, p. 116), so lessons should have time for reflection. This time allows students to process their thinking and meet writing expectations in the mathematics classroom (Hoffer, 2012). It provides students a chance to use their language skills while converting their experiences into knowledge that stays with them (Eggen & Kauchak, 2007). When reflection is done as a class discussion, students are able to see alternate approaches (Bresser & Holtzman, 2018), communicate mathematically with their peers (Heuser, 2002), and hear explanations in language that
they understand (Hoffer, 2012). When reflection is done through writing, students can explain their thinking independently (Bresser & Holtzman, 2018). Both methods allow for comprehension to be solidified and for students to look at how their thinking has changed (Hoffer, 2012). The think time given before and during reflections promotes self-control by limiting impulsiveness in answers and encouraging all students to participate (Dewey, 1938). Finally, reflection time can be used as a formative assessment for the teacher by pointing out proficiency or errors in their work and explanations (Hoffer, 2012).

Teachers must assess the learning of their students at various times during a course. These assessments can be precursors to the learning, sources of formative information during the lesson, or a summative review of what has been learned. Math workshop provides places for each of these assessment types within its structure. Answers to questions or descriptions of thinking during the opener or mini lesson show what students already know or are missing in relation to the days’ content (Heuser, 2002). Formative assessment can be gathered in many ways during a workshop and then used to plan future lessons. During worktime, interviews with students allow for one on one explanations of work (Bresser & Holtzman, 2018). Wrong answers to problems show underlying misconceptions and interviews give students a private chance to explain their thinking (Heuser, 2002). Reflection time also shows what students have retained from the day and where they still need support in the future (Hoffer, 2012). Summative assessments can vary when using Math Workshop. Graduated assignments start with the basics of a concept and progress in difficulty; the level of completion can show how much a student has mastered and what is needed before moving on (Mighton, 2003). If a
formal test is given, complex problems using similar skills from the workshops
demonstrate if a student has developed an independent understanding of the math
concepts (Heuser, 2002).

**Related Research**

Mark Ashcraft (2002) conducted a study to examine the effects math anxiety has
on student performance. Participants in this study took surveys to rate their own anxiety,
completed mathematical and verbal tasks for comparison, completed both timed and
untimed tests, as well as completed tasks while being asked to keep other things in mind
(Ashcraft, 2002). Ashcraft found that timed tests produced anxiety and as the difficulty
of tasks increased, levels of anxiety increased as well. When anxiety increases, the
capability of working memory decreases leading to more difficulty with the same tasks
(Ashcraft, 2002).

Studies have also been done to test the effectiveness of different approaches to
mathematics education, and these various approaches all have pieces that can be found in
Math Workshop. One such study was done by Robert Slavin (1980) on cooperative
learning. The purpose of this study was to look at the research and the general theory of
cooperative learning. Instead of doing one study, Slavin looked at various studies and
compiled the results. The studies Slavin worked with ranged from second grade to high
school and lasted from four to twelve weeks. Even in 1980, Slavin found that cooperative
learning changes the structure of the classroom by having students prepare and support
each other. Slavin found that students were on task more often than a traditional
classroom and had higher levels of self-esteem. Research still needs to be done on
specific elements, but this study created a starting point for future studies (Slavin, 1980).
In 1999 Jinfa Cai, John Mayer, and Nancy Grochowski did a study involving the algebraic mean. The purpose of this study was to explore if using a different approach with hands-on activities helped students to better understand the mathematical concept of the mean (Cai et al., 1999). The sample consisted of 42 sixth grade students who were ethnically and culturally diverse (Cai et al., 1999). In the study, a teacher introduced various problems and strategies and gave students time to work before comparing their approaches and solutions (Cai et al., 1999). These problems used a constructivist approach to apply the concept of mean to real-world examples and tested the effect of the strategy with pre- and post-tests (Cai et al., 1999). Cai et al. found that the approach increased knowledge of finding the mean, but students still struggled with transfer of the content. Students were better able to explain their strategies on the post-test, but many students simply repeated the teacher’s strategy instead of creating their own (Cai et al., 1999).

Toni Smith, Kirk Walters, and Steve Leinwand (2015) completed a study on the effect of student-centered instruction on problem solving at the high school level. Seven teachers were selected for this case study and many had their primary degree in mathematics (Smith et al., 2015). These teachers have positive environments and used student-centered practices at varying levels during the study (Smith et al., 2015). Smith et al. found that this approach promoted problem solving and fluency and supported various pathways to finding solutions. The students liked the explorations and the connections to the real world (Smith et al., 2015).

Ewa Dabkowska and Bianca Sosnovski (2016) completed a study on using a remedial math workshop at a community college. This study was to evaluate the
effectiveness of the workshop model on college students’ success and retention in remedial college mathematics courses (Dabkowska & Sosnovski, 2016). The participants were 69 college students who had failed a remedial mathematics course at least one time (Dabkowska & Sosnovski, 2016). In this study, students worked as groups to solve problems and had conversations that spiraled content beginning with the most misunderstood content giving more exposure by the end of the workshop (Dabkowska & Sosnovski, 2016). Dabkowska and Sosnovski found that 96% of participants passed the requirements through this workshop, and that as passing rates increased so did comprehension and confidence, showing that traditional remedial teaching needs to change.

Activities instead of tests will reduce anxiety (Ashcraft, 2002). Cooperation leads to deeper understanding and more time on task (Slavin, 1980). Student-centered instruction increases buy in (Smith et al., 2015). Overall, the Math Workshop framework combines these successful studies into one method of teaching math.

Summary

Math “Workshops foster and nurture students’ quests for wonder and exploration in a safe risk-taking environment” (Legnard & Austin, 2012, p. 228). Developed to match the already accepted reading and writing workshops, students are trained on expectations and work on activities that elicit growth on specific math content and numeracy (Hoffer, 2012). Students must be attentive and excited in order to learn (Mighton, 2003). Game-like activities meet a social need while building understanding of mathematical content (Heuser, 2002). Providing choice usually results in students working at their specific ability level, being attentive to the work at hand, and being
persistent in finding a solution (Heuser, 2002). Students should be encouraged to struggle while using skills and strategies that have been taught to build their conceptual understanding (Hoffer, 2012; Warshauer, 2014). If students are behind, they can catch up by learning one step at a time to master the pre-conceptual knowledge before moving into more abstract ideas (Mighton, 2003). As students shift from a focus on and need for fluency, a behavioral approach, they can move to cognition and construction (Ertmer & Newby, 2013).

All three approaches are important depending on the needs of each student, (Ertmer & Newby, 2013) but the ultimate goal of Math Workshop is to get students to build their own understanding through deeper exploration and connections (Bresser & Holtzman, 2018). Once fluency has been internalized, students can become more flexible and look for patterns to make predictions (Bresser & Holtzman, 2018). As students find the usefulness in numbers and mathematical symbols, they are better able to make sense of and justify their approaches to the presented problems (Bresser & Holtzman, 2018).

Traditional education sees the future as the next unit or grading period, while the progressive approach prepares students for their next experience, course, or career (Dewey, 1938). By connecting the activities to prior knowledge as well as making them useful in the future, there is more buy in from students and a stronger development of their conceptual understanding (Hoffer, 2012). As students work through complex problems to build their own knowledge, teachers adapt to their changing needs and keep the learning relevant (Dewey, 1938). Worktime allows teachers to confer with students, honor their thinking, fill in gaps of required knowledge, and guide students to uncover their own errors all while feeling safe to make mistakes and grow (Hoffer, 2012).
Students learn by doing and reflecting on what they have done (Dewey, 1938). The Math Workshop provides students with experiences that will not only teach the mathematics, but how and when to use higher-level thinking skills. In reflecting on what they have done and connecting it to the Measurement Topics and the real world, I believe that this method will teach students how to think at a deeper level, and allow them to do so more independently. By combining the ideas of Dewey, Piaget, and Vygotsky, and the model of Math Workshop, I hoped to address the concerns that have grown in the minds of teachers in District A.
John Dewey (1938) describes the difference between a traditional stand-and-deliver classroom and a newer, experience-based classroom. He explains that students learn best when applying the concepts and building their own understandings of the information (Dewey, 1938). District A has a mission to be learner centered and do what is best to help the students learn. The problem of practice for this study was that students are not using higher-level thinking beyond the proficient level. The purpose of the study was to examine if the Math Workshop framework of teaching mathematics encouraged problem solving and critical thinking, reduced the need for students to assess multiple times before demonstrating proficiency, and affected students’ feelings toward mathematics.

As standardized test scores decline (Maine Department of Education, 2015) the traditional method of teaching mathematics is not allowing for the development of the necessary skills to pass. Constantly directing students limits their problem-solving skills instead of developing them (Dewey, 1938). Math Workshop is an instructional framework that can be applied in any math classroom, and District A has begun introducing it to some teachers. The idea behind this framework is to guide students to discover knowledge with hands-on tasks, a constructivist approach (Legnard & Austin, 2012). While teaching lessons and guiding discussion, teachers can encourage the use of the higher-level thinking skills that are desired in their students. This method
effectively combines the constructivist ideas of John Dewey, Jean Piaget, and Lev Vygotsky and the learner centered approach of District A.

Math Workshop, the intervention used in this study, is a framework that allows students to construct their own knowledge while working on challenging and relevant tasks (Hoffer, 2012). Instead of the traditional stand-and-deliver method of teaching mathematics, this framework provides an opener, mini lesson, worktime, and a reflection period for each topic covered (Hoffer, 2012; Heuser, 2002; Bresser & Holtzman, 2018). The opener activates prior knowledge preparing students to make connections with the new content (Ertmer & Newby, 2013). The mini lesson gives guidance, an example, or directions that students need in order to do the tasks during the worktime (Heuser, 2002). The mini lesson can also be used following the worktime to clarify misunderstandings after allowing students to try and create knowledge on their own (Hoffer, 2012). The worktime allows students to play games, use manipulatives, and work on relevant problems to develop a deeper understanding of the content (Bresser & Holtzman, 2018). Finally, the reflection period allows students to consolidate their learning, ask any lingering questions, and explain their thinking to others (Hoffer, 2012).

This study attempted to answer the following research questions:

1. What is the effect of using Math Workshop on increasing problem solving and critical thinking skills as demonstrated by successful completion of level 4 work?

2. To what degree does using Math Workshop impact the number of attempts to demonstrate learner proficiency?

3. What is the effect of Math Workshop on students’ general attitudes/feelings about math?
This chapter provides an outline of the research design that was used in this study. It discusses why this approach was chosen, my role and positionality in the process, the participants involved in the study, and how they were chosen. The data collection methods are described along with the overall procedure for implementing the research and gathering data. It also provides an overview of how the data was sorted and analyzed during the study and after its completion.

**Research Design**

This action research project used a triangulation mixed methods intervention study of two Algebra II classes. Qualitative and quantitative data were collected simultaneously and analyzed together for a thorough understanding of the results (Leech & Onwuegubuzie, 2007). The one-group pre-test/post-test design (Mertler, 2014) began with a pre-test on the content covered in the probability unit. This allowed me to see where each participant was starting in relation to the topic, created a base for analyzing the data, and allowed for a learner centered design with each task assigned (Schiro, 2013). Following the pre-test, I observed students as they participated in Math Workshop activities to learn probability. The students completed the unit by taking a summative assessment, similar to the pre-test, allowing me to see what growth, if any, occurred.

A case study allows a researcher to focus on a bounded system, in this case Algebra II courses taught by the researcher (Merriam & Tisdell, 2016). A case study that uses quantitative and qualitative methods to collect data was the best choice for this project because it allowed results to be seen from the intervention without interfering with or limiting the teaching process (Mertler, 2014; Merriam & Tisdell, 2016). All students received the same instruction taking away the ability to run an experiment, but
data was still gathered from the case study. Activities and assessments provided data, but they only gave one piece of the picture. Surveys, journals, and classroom observations reflected feelings from both myself and the participants about the experience as well as what an outsider would have seen the students doing while working on mathematics. The purpose of the data was to discover if there was any impact on the use of problem solving and critical thinking skills, the number of attempts to show proficiency, and students’ feelings/attitudes about math when introducing Math Workshop to the classroom.

**Context and Setting**

The high school where this research project took place is located in central Maine. The school district has three high schools, three middle schools, and four elementary schools. Monarch High School houses 190 students in grades 9-12 with a 91% graduation rate. A student body that is 97% Caucasian is not very diverse racially. There is an approximately equal percentage of genders and approximately 25% of students receive free or reduced lunch. The population comes from families with lakefront properties as well as students who are homeless during the year. The district has been a proficiency-based, learner centered system for ten years allowing for students to progress through the Measurement Topics at their own rate. The results of this study will affect other math classrooms in the district; professional development will be given to assist other teachers in implementing this method in their classrooms.

The students in this action research project were selected based upon enrollment in my Algebra II classes. Although a convenience sample, it represented 16% of the school, and 73.8% of students currently taking Algebra II. The senior class made up 8.5% of the sample that also included 40.8% of the junior class and 14.3% of the
sophomore class. At Monarch High School grades for courses are calculated by averaging the proficiency level, 1-4, of all the Measurement Topics for each course. The overall grade point average (GPA) is calculated by averaging each course grade with weights for length of the course and honors designations. The average GPA of the students in the two classes was a 3.461 with a standard deviation of 0.24. The average Algebra I grade was a 3.477 with a standard deviation of 0.223. This suggests that students do attempt level 4 work, but either they do not do it often or are not successful with it more than half of the time.

Within the classes, students were various types of learners that were required to meet the same Measurement Topics to complete the course and graduate high school. They have moved through the district’s math flow at different rates based on their personal abilities and learning needs. Some had recently reviewed probability, while others had not worked with the content in several years. This time gap may have led to students forgetting pieces of the content, but the hope was that the study would show that Math Workshop could overcome that attrition. Parents and students had to sign a release allowing them to participate in the study (see Appendix B).

**Intervention**

Traditionally, mathematics is taught with the teacher sharing information and processes while students passively accept the knowledge (Dewey, 1938). Instead, Dewey (1938) suggests a progressive approach to education using experiences to learn material. Following constructivism ideals, students explore deep problems based on real-life examples, make personal choices on how to approach those problems, and increase the ability to transfer their knowledge in new contexts (Ertmer & Newby, 2013). Math
Workshop combines these ideas by engaging students, teaching the basic skills needed, and allowing work time to explore solutions to complex problems in a safe space (Bresser & Holtzman, 2018). This approach encourages higher-level thinking skills such as problem-solving and critical thinking (Hoffler, 2012). Those same skills are required to attempt the level 4 work, so the expectation was that Math Workshop would encourage students to attempt these more often, leading to a potential solution to the problem of practice.

In the District A model of proficiency-based education, the required level of proficiency most often calls for recalling and/or executing, limiting students’ use of higher-level skills. Math Workshop provides real-life application that allows students to build their own knowledge about the content (Hoffler, 2012). It allows time to teach skills with direct instruction, but instead of students listening and then doing, they are able to decide when they need more information and when to try the next step (Heuser, 2003). Each section of the probability unit was introduced with an activity, explored by the students, and followed up with by the teacher to ensure that learning occurred.

Class periods began with an opening problem to situate the learners to the day’s tasks. These reviewed concepts to activate prior knowledge, practiced a skill that was needed for success in the lesson, or posed a new idea to get students thinking about the upcoming topic (Hoffler, 2012; Eggen & Kauchak, 2007). Following the opener, a mini lesson showed the class a sample problem, covered any potential issues that arose, introduced a new skill, or demonstrated ways to think through confusing parts (Hoffler, 2012). Next, students were given time to work to solve a complex problem related to the
topic. They were able to choose if they wanted to work with their peers or independently each class.

These problems had multiple approaches, connected patterns to relationships, and were relevant for the students attempting them (Bresser & Holtzman, 2018; Schiro, 2013). Some class periods presented the problem before the mini lesson to give students time to create their own understanding of the material (Eggen & Kauckak, 2007). Finally, a reflection period ended each workshop. During this time, students shared their thoughts on the problems, verbalized or wrote out their strategies, assessed their progress, or set goals for the next day (Hoffler, 2012).

Variables

The variables in this study were the attempting and completion of level 4 work, the number of attempts needed to demonstrate proficiency, and student feelings/attitudes about math and the specific activities. The pre- and post-tests had level 4 questions on them, allowing me to see if students were willing to try those. Daily openers and activities during the work period had level 4 options as well, and the student artifacts showed when these were attempted and successful. I tracked the number of attempts it took students to demonstrate proficiency on this unit. The first attempt was counted when students tried the problems on the test. After being corrected, students tried any questions that were still wrong a second time. If any questions were still incorrect, students were given a guiding question to point them in the right direction. This included the post-test as well as any alternative methods of assessing. Attitudes, thoughts, and observations were collected qualitatively through field notes, surveys, and journal entries throughout the study. Math Workshop was the method used to teach the probability unit
and the activities used allowed for the data to be collected as the students completed their work.

**Teacher-Researcher**

Instructing the students, assessing their work, and responding when necessary placed me almost as a full participant on Mertler’s (2014) continuum. As a white female who grew up in the same state, I was an insider and shared many demographic characteristics with the students. There are times when I was part of the lesson or activity, and there were times when I was only watching (Herr & Anderson, 2014). Being the teacher also made me an outsider because I was ultimately in control of what and how the students learned (Herr & Anderson, 2014). On the other hand, I am a known entity in a small school, so I already had the trust of the participants in my commitment to their education. In this role, I took notes during discussions and group work and adjusted final survey questions to find any information that was missing. Unlike being a “full participant” (Mertler, 2014, p. 94) or “participant as observer” (Mertler, 2014, pg. 94), I was not completing the assignments with the students. I collected data about the students but was not an actual part of the group. The children knew that I was their teacher and not able to contribute to the data as a participant, but only through observation. In a small school, I am one of two educators who teach Algebra II, and the opportunities for collaboration with colleagues are rare. I was the only person in charge of what and how these students would learn this unit.

**Participants**

To address the research questions through the eyes of all students, I needed the sample to be as representative of the school as possible (Sullivan, 2017). In this case 27
students, 14.2% of the school, a majority of students were represented. The sample was 26% sophomores, 67% juniors, and 7% seniors. The sophomores were ahead of most of their grade level peers who were still in Geometry. The juniors were where they were expected to be along the District Math Flow, and the seniors were behind their peers who were in classes the follow Algebra II. This gave me access to students with various achievement levels, matching the general make-up of Monarch High School.

In a small, rural school, of 190 students, a convenience sample, one that is easily available, was the only option (Sullivan, 2017). Although it was a convenience sample, racially, it was the same as the school population with 3% Black students and 97% White. The ratio of males to females was slightly lower in this sample than the general population, but the percentage of students with IEPs and 504s matched the school as a whole. The only exclusion criteria were from any students being unwilling or unable to participate. Even nonparticipants were taught the same way; their information was just not used in the data collection process. There was no consequence for a student choosing not to participate.

The College Board (2019c) uses 530 as the math benchmark on the SAT. The mean PSAT (sophomores and juniors) and SAT scores (seniors) were 463.2 and 375 respectively in the spring of 2019 for the students in my sample. The discrepancy in their scores and the benchmark imply that my students needed more help with the problem solving and critical thinking skills required by the SAT -- the focus of question one in this research (College Board, 2019a). Math Workshop provides students with the chance to use these skills in a risk-free environment to complete the assigned tasks (Hoffer, 2012). It costs money to retake the SAT, so it is beneficial for students to meet the benchmark
on their first attempt (College Board, 2019c). The second research question examined
the number of attempts needed to determine a student’s proficiency in District A in hopes
that Math Workshop would increase the frequency of initially correct answers which
would transfer to high-stakes tests such as the SAT.

Sources of Data Collection

The study began with a pre-test (see Appendix B). This pre-test was designed for
this research study to see what students already knew and at what level as well as what
they were willing to attempt before beginning the unit (Mills, 2007). I looked at the
Measurement Topics for this unit as well as the previous post-test that had been used to
create pre-test questions at the required taxonomy levels. I also created questions that
required higher-level thinking such as problem solving and critical thinking to monitor
who was willing to attempt the level 4 questions.

Surveys (see Appendices F, G, & H) were given at three points during the study:
before students began using Math Workshop, halfway through, and upon demonstrating
proficiency. The surveys were developed to allow students to anonymously share their
feelings and experiences with math (Dana & Yendol-Hoppey, 2014). They had questions
that allowed for qualitative and quantitative responses. The questions gathered data on
students’ knowledge of thinking skills and their use in math, willingness to attempt level
4 work, and feelings/attitudes about mathematics to help answer the research questions.

Observations of students were made while they completed the tasks. Video
recordings of the classes were made each day to obtain field notes for all groups as the
teacher was working directly with one group. The observation forms (see Appendix D)
allowed for field notes of observations such as interventions needed, questions asked,
when students chose to work in groups, and the outcomes of specific tasks during the study (Dana & Yendel-Hoppey, 2014).

Students did some class work in a journal to keep the work together for the analysis phase of the study. At the conclusion of each class, students responded to questions, reflecting on their attitudes about the activities, relevance to real life, and which thinking skills were used during the day (see Appendix E). The journals were collected and the data from each was synthesized upon the completion of the study. Other artifacts were collected and graded to see if students chose to try the level 4 examples and if they were successful with the level 3 examples.

Finally, the unit ended with a summative post-test (see Appendix I). This showed the students’ understanding of the desired content after the workshops were complete. It was similar to the pre-test in order to show any content growth that occurred during the study. It examined whether students were more willing to attempt the optional level 4 work, and if students could demonstrate proficiency on their first attempt.

**Research Procedure**

This study occurred over the course of 14 class periods and covered four Measurement Topics (see Appendix J) on probability. During each class period, I took field notes. If there was a group that was working independently, I took the field notes following the class by watching the video recording. Each class period was 60 minutes, (unless we had a snow delay when they were shortened to 45 minutes). During some class periods, students worked with new content while other classes continued to work on activities from previous lessons. The lesson outlines can be found in Appendix K. Every
class period ended with 5-10 minutes of time for students to answer questions in their journals.

The study began with students taking an anonymous survey (see Appendix F) through Google Forms addressing their feelings about math and the thinking skills that they felt they typically used. Google Forms organized the data into spreadsheets and created tables and charts of any quantitative data. Students then took a pre-test (see Appendix C) of the content being taught at various thinking levels. This showed students’ achievement levels and allowed me to see if any growth occurred by the end of the unit. At the end of the first class, students did their first journal entry (see Appendix E). This entry focused on how students felt doing the pre-assessment as well as got them thinking about which thinking skills they used to do it.

The next set of classes (2-12) began with a problem of the day. This problem either reviewed previous content or gave students an interesting problem to get them in the mindset for the day’s activities. Class 2 introduced probability through a poker hand activity, connecting the content to a game that some of the students play, making it learner centered (Schiro, 2013). Following the activity, the mini lesson reviewed the rules that students should have learned previously. The work period let students practice these rules and get help if needed to be ready for the following class. Students ended the class with journal reflections on their feelings and use of thinking skills for the day. Class 3 began with an opener involving compound probability. This moved into a lesson about dependence and independence as well as the rules for finding compound probabilities. The work period allowed students to practice solving problems with
compound probabilities while working in groups or alone. This option gave students ownership over their own learning (Schiro, 2013).

Classes 4-8 were on expected value. The opener for Class 4 was similar to the work period for the day. Students were given a choice of four payouts based on grades. They had to choose an option and explain why. Following their decision, they worked with peers to defend their choice. This approach allowed students to find and defend their own answers, which deepened their connection to the material (Eggen & Kauchak, 2007). Following the opener, the mini lesson shared what expected value is, how to find it, and why it is useful. The work period for the day had students look at four games, make a choice as to which game to play, and use math to defend their choice. The homework was to choose an option from four more games. Class 5 began with students sharing their choices and how they made those decisions. When everyone shared, the activity period was to play the games with various stipulations put on the students. Following the games, students reflected on their results in their journals before sharing with the class. We discussed why their predictions may not have matched their actual results and how probability works. Classes 6-8 looked at expected value using various types of insurance. At the end of Class 8, students took the midterm survey to see if their feelings had changed on level 4s and mathematics in general (see Appendix G).

Classes 9-11 focused on conditional probability. Class 9 examined data recorded in tables, Class 10 worked with tree diagrams, and Class 11 worked with Venn diagrams. Each day students started with a set of data and made predictions during the opener. The mini lesson shared the method of data recording, how to organize data using that method, and how to use the data to make decisions. Students had the work period to do those same
things on data that is relevant to their lives. The reflections those days compared their predictions at the beginning of class to what they found at the end. They also looked to see if their predictions were more accurate as they were given more options to try (Eggen & Kauchak, 2007).

Class 12 was a review class. The opener, mini lesson, and worktime were combined to work on a practice assessment, but the class still ended with a journal entry. Students were able to ask about parts of the unit that they wanted more experience or help with before the assessment. This period also allowed students a chance to practice level 4 work if they had not tried it yet but were interested.

During Classes 13 and 14, students took the summative assessment (see Appendix I), wrote a final journal entry (see Appendix E), and took a final survey about the overall unit (see Appendix H). This was spread over two class periods as some students needed more time to complete the assessment. They were given one section at a time to ensure that no one was given an unfair advantage and was able to study between the two class periods. The journal entry was more complex than previous ones to gather any follow up that was needed as well as an overall reflection on the unit and Math Workshop in general. The survey was online and anonymous to see if feelings/attitudes changed over time.

Identity Protection

Work was done in the journals or on handouts that were collected upon completion. Names were on covers of the journals and on cover sheets for worksheets and assessments. This allowed me to grade them as needed for school, but the covers were then removed from the items before I analyzed them for the study. Removing the
covers allowed each student to stay protected and removes the potential for bias from knowing the students. Online surveys were anonymous to allow students comfort in sharing honest responses. Finally, when taking field notes, only overall observations were recorded without including student names. These measures helped to ensure that the students felt safe in the data analysis process. Finally, pseudonyms were used for the school and district to protect student identities. In a small school that is part of a well-known district within the state, this is necessary to ensure that no one can deduce who was being studied (Mertler, 2014).

All notebooks, field notes, and recordings were locked up when I was not using them. This ensured that no one had access to the information, that identities remained private, and that no one had unobserved access to their work during the unit. The videos were watched and field notes taken within 48 hours of the class and then deleted; the memory cards were reformatted, ensuring anonymity if they were reused in the future. As stated in the consent/assent letter (see Appendix B), all raw data will be destroyed within one year of completing the study.

Data Analysis

After the data was collected and the study was completed, the information needed to be organized and analyzed (Mills, 2007). Data needed to be sorted and put into understandable formats before each piece could be compared and evaluated in relation to the research questions (Merriam & Tisdell, 2016; Mertler, 2014; Mills, 2007).

Quantitative data, such as number of level 4 attempts and number of attempts needed to demonstrate proficiency, were recorded on spreadsheets and then explained with descriptive statistics such as means, standard deviations, and medians (Sullivan,
Differences between the pre- and post-tests were recorded, and statistics were calculated on the average of the group (Sullivan, 2017). A repeated measures t-test was used to examine if the differences in the two assessments were statistically significant (Mertler, 2014). The Likert-scale questions from the journals were analyzed as visual tables and using the median (Mertler, 2014).

Qualitative data, such as feelings about math and this intervention, were organized with coding. An inductive constant comparative analysis was used to group data according to patterns or repeated phrases (Mertler, 2014; Leech and Onwuegbuzie, 2007; Mills, 2007). Leech and Onwuegbuzie share how this process is done: the researcher first reads the data, chunking similar information, giving it a code, and then grouping similar codes into themes to interpret the vast amount of data. The codes allowed me to find specific information more easily in the vast amounts of data by reducing the information to a manageable form (Merriam & Tisdell, 2016; Mills 2007). Coding also facilitated comparing the different forms of data as the patterns cut across multiple collection types (Merriam & Tisdell, 2016).

After naming categories to organize all the important data, they were connected to the research questions and examined for any contraindications (Mertler, 2014). For example, if students reported the same feelings and use of thinking skills, this showed an overall theme within the classes. If there were no common elements, the other responses may have given a better idea of the attitudes and understandings that the students had.

Once the data was simplified, relationships, similarities, and differences were interpreted to answer the three research questions (Mertler, 2014). Analysis was extended by asking questions about and challenging the data, and findings were displayed.
In various tables and graphs (Mertler, 2014; Mills, 2007). In organizing the data, I considered breaking information into demographics of the sample (Merriam & Tisdell, 2016). This proved difficult as much of the data was collected or analyzed anonymously, so the decision was made to look at the group as a whole.

In addressing question one (What is the effect of using Math Workshop on increasing problem solving and critical thinking skills as demonstrated by successful completion of level 4 work?) both quantitative and qualitative data were analyzed. The pre-and post-tests showed how many students attempted the level 4 questions as well as how many were successful. The surveys gave quantitative data on which thinking skills were used and how that changed over the course of this study. Finally, the journal entries and field notes provided qualitative data on the level 4 work and the use of thinking skills. These were self-reported by students as well as noted by me as I taught the classes and watched the videos.

When looking at question two (To what degree does using Math Workshop impact the number of attempts to demonstrate learner proficiency?) quantitative data was collected on how many attempts each student needed to demonstrate proficiency. If students needed multiple attempts or to try different ways to show their learning, this was recorded qualitatively in the field notes. Student work was also used to examine how successful students were when practicing during the unit.

Question three (What is the effect of Math Workshop on students’ attitudes/feelings about math?) was answered with qualitative data. The surveys shared explicit feelings and reasons for those feelings about math in general, journal entries looked more specifically about feelings in relation to the unit, activities, and intervention,
and field notes were a way for me to record any observations that were made while implementing the research.

Once the data was coded, statistical values were calculated, and information was put together in a meaningful way, I saw all the information together and made recommendations for next steps. Fuller discussion of these outcomes will be provided in Chapter 4 (see Table 3.1).

Summary

This mixed-methods study observed two Algebra II classes and their progression through one 14 class period unit of mathematics instruction using the Math Workshop framework. This unit covered four Measurement Topics on probability. Data was collected through pre- and post-tests, observation field notes, journal entries, surveys, and student artifacts. The objective of this study was to examine the extent Math Workshop impacted willingness to attempt optional level 4 work, if it affected the number of attempts needed to demonstrate proficiency, and if this framework affected students’ feelings/attitudes about mathematics. Participants were selected by placement in Algebra II with me, a graduation requirement for District A. As the teacher, I was in the classroom providing instruction, as well as collecting data for this study. Qualitative and quantitative data were collected and analyzed together to triangulate an interpretation of the findings (Leech & Onwuegbuzie, 2007). Each question had a main source of data, with other sources to back up and support the main findings.

Students learn by doing and reflecting on what they have done (Dewey, 1938). The Math Workshop framework provides students with experiences that will not only teach the mathematics, but how and when to use higher-level thinking skills and give
Table 3.1: Categories of Qualitative Data

<table>
<thead>
<tr>
<th>Code Words</th>
<th>Comparison</th>
<th>Perception of Others</th>
<th>Level of Rigor</th>
<th>Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Other Classes</td>
<td>GPA Disappointment Parents Honor Chords Honor Roll Scared Worried Win Lose Look Not Finished Miss Ross</td>
<td>Hard Easy Confusing Pace Information Makes Sense Understand Complex Difficult Guess Can’t Too Much</td>
<td>Fun Tedious Care Repetitive On Task Off Task Engaged Discussion Math Question Excitement</td>
</tr>
</tbody>
</table>

students a chance to build their own knowledge (Hoffer, 2012). In reflecting on what they have done and connecting it to the Measurement Topics and the real world, I believe that this method taught students how to think at a deeper level, and in time, allowed them to do so more independently (Eggen & Kauchak, 2007). By combining the ideas of Dewey, constructivism, and the learner centered ideology into the Math Workshop framework, I hoped to address the concerns that have grown in the minds of teachers in District A.
Chapter 4: Data Analysis

Critical thinking skills are needed for success in today’s world (Wilcox et al., 2017). These skills are needed to analyze situations and solve problems in many professions (Joseph et al., 2000). Since District A has switched to a proficiency-based model, teachers have noticed a drop in the use of these skills. Math targets usually expect students to work with lower level thinking skills such as recognition and execution to demonstrate proficiency. The problem of practice for this study was that students are not using higher-level thinking skills to get beyond the proficient level. The purpose of this study was to investigate whether using the Math Workshop framework helped to encourage problem solving, critical thinking, and communication skills and reduce the number of attempts needed to demonstrate proficiency.

The data collected in this action research study helped to answer the following research questions:

1. What is the effect of using Math Workshop on increasing problem solving and critical thinking skills as demonstrated by successful completion of level 4 work?
2. To what degree does using Math Workshop impact the number of attempts to demonstrate learner proficiency?
3. What is the effect of Math Workshop on students’ general attitudes/feelings about math?

The Math Workshop model is grounded in the constructivist theory of learning to help students build their own understandings of the mathematical content.
(Hoffer, 2012). Math Workshop is a framework that can be used to present and practice mathematical concepts. The traditional model of math instruction has students take in information passively and then regurgitate that information (Dewey, 1938). Constructivism is a theory of education based on the idea that students actively build their own understandings through experiences (Eggen & Kauchak, 2007). This theory is foundational to the Math Workshop framework in that students are able to experience the content while creating their own knowledge (Hoffer, 2012).

In order to foster learning, there are four pieces to the Math Workshop framework. Each lesson begins with an opening activity. This activity can review a previous concept, introduce a new idea, or hook student interest for the upcoming topic (Heuser, 2002). Following the opener is typically a lesson that shows students how to understand what is being asked or how to use formulas (Legnard & Austin, 2012). While this lesson can mirror the traditional approach, when combined with the other aspects of Math Workshop it helps to build understanding (Dewey, 1938). Each workshop contains a work period. This can come before the lesson to allow students to come to their own solutions, or it can come after the lesson to give students a chance to practice what they have learned (Hoffer, 2012). Finally, each workshop ends with a reflection period. This time gives students a chance to connect the things that they have done, process what questions they still have, or to think about how the content can apply to their personal lives (Heuser, 2002). This piece is essential in the constructivist approach as it allows students the time to process the knowledge they built while working (Eggen & Kauchak, 2007).
This study examined how teaching probability to two Algebra II classes using the Math Workshop framework affected students’ learning and feelings. Each class started with a problem of the day as the opener. Some days moved into a lesson followed by a work period where students practiced using the content. Other days started with a work period where students were expected to build their own knowledge followed by a lesson to solidify this learning and address any misconceptions that were seen. Finally, each class ended with a reflection time where students could think about what skills they used and how they felt about the day’s lesson.

Data was collected over the course of 14 class periods. A pre-test was given at the beginning of the unit to see what base knowledge students had. The last two classes had students take a post-test to show how much they had learned during the unit. During each class, students were asked to reflect in their journals on their feelings, thinking skills, and relevance of the lesson. During the class students completed tasks that were collected and used to plan for intervention if needed. Three anonymous surveys were sent out during the study asking students about feelings toward mathematics and the workshop model. Finally, each class was recorded, and videos were reviewed after completion of the study for evidence of thinking skills, engagement, and communication during the workshop tasks.

The sample began with 27 students, but only 20 were able to complete the final assessment and survey within the time frame of this study. All students were enrolled in my two Algebra 2 classes in January 2020 and were taught using the Math Workshop framework, even if they did not complete the study due to absence or taking the assessment after the study period closed. Those seven students were able to demonstrate
proficiency, but data was no longer being collected at that time. Using the College Board benchmark of 530, the means of both the SAT (375) and PSAT (463.2) scores were well below the expectation. The final sample was made up of 13 females, six males, and one student who was transitioning from female to male during this study. While Monarch High School is 97% white, the final sample ended up being 100% white. The original group of students had three students with 504 plans and one with an IEP, but only one student with a 504 remained in the final study. The other three students needed more time and intervention after the last-minute snow days pushing the final assessment to after a vacation, and data was no longer being collected when they finished the final assessment.

This chapter identifies how the data was collected, sorted, and processed during this action research study. The quantitative data is presented with descriptive statistics that describe the results of each tool and is connected to the research questions. An overview of the qualitative data is also discussed through themes found during analysis. Following the presentation of the data, an overview of the combined results is discussed in relation to each research question. Supplemental findings are shared followed by a summary of what was found during this study.

**Data Presentation**

In reviewing the data, each tool showed various results toward answering the questions. Student journals collected both quantitative and qualitative data involving students’ feelings about each activity while also allowing them to reflect on their work. A journal entry was done during each class except Days 7 and 12, and they were used to answer research questions one and three. Day 7 was omitted as the activity of that day
took two class periods, and Day 12 was a review day that was cut short due to a snow-provoked early release, shortening classes at the last minute. Anonymous responses to the three surveys allowed students to share their true feelings without fear of repercussions. The surveys also collected both numerical and categorical data and related to all three questions.

The video recordings were able to be viewed after the fact, allowing me a chance to reflect on the lesson and see things that I may have missed in the moment. This data was all qualitative in nature and could be used to address parts of each research question. The student work showed levels of understandings and patterns of thinking while working with the material being presented. Finally, the pre- and post-tests are able to show where the students began and how they grew throughout this unit. It also showed how many attempts students needed to demonstrate proficiency. This data was quantitative in nature and supported the data for research questions one and two.

Throughout the study the tools were used to inform instruction in future lessons. Upon completion of the learning unit, each tool was revisited, and the data was gathered and sorted. Data from each collection method was separated into quantitative and qualitative groups before being analyzed.

Descriptive statistics were found for the quantitative data. The mean and standard deviations were the most common statistics used, as they show the average responses for large groups of data (Mertler, 2014). When analyzing the Likert scale questions, the median was used because the responses were so close together that one value could greatly affect the mean (Mertler, 2014). Hypothesis testing was done on the pre- and post-test to see if the outcomes were statistically significant (Mertler, 2014). Once the
data was reduced into comparable numbers, it was considered in relation to each research question and compared to interpret the overall data. It was organized into tables, graphs, and narratives to clearly present the data.

Qualitative data was organized by coding. As each collection tool was read, codes were assigned to patterns and repeated phrases that were found within the data (Mills, 2007). These codes were grouped into themes that were then used to facilitate understanding and analyze the information in relation to the research questions (Mertler, 2014). While analyzing the field notes, I periodically paused to reflect on my personal biases from knowing the students (Merriam & Tisdell, 2016). I made sure what I was coding and grouping was actual data and not just what I knew because of my time as their teacher. Once the data was processed, it was arranged in a way to best present the information to others through narrative.

Finally, the two types of data were interpreted together. Triangulation was used to consider both quantitative and qualitative data with equal weights to see if they indicated similar findings (Mertler, 2014). The results were interpreted at the same time to answer the questions and guide next steps for sharing results and future research.

**Quantitative Analysis**

Numerical data can focus on a specific variable of the study and can be compared using mathematical support (Mertler, 2014). The data from tools used at various times throughout the study can be compared to find possible correlations between the intervention and the outcomes (Mills, 2007). The student journals, surveys, work examples, and the pre- and post-test all collected numerical data to answer the research
questions. That data was then analyzed by finding means and medians where appropriate and those descriptive statistics were interpreted together (Mertler, 2014).

**Journals.** For each journal entry, students were asked which thinking skills they used that day. The number of students that chose each skill was recorded daily, and the mean and standard deviation of the 10 daily journal entries were tracked. Daily entries as well as final averages were compared to show which skills were the most and least common according to students. The final journal entry covered the entire unit, so a difference of means hypothesis test was done on the top and bottom two reported thinking skills to see if the daily means were different from the final entries.

Likert scales were completed at the end of each journal entry concerning student interest and relevance of the lesson to their lives. The median of each entry was found instead of the mean because there is a smaller range of data gathered (Mertler, 2014). Once the median for each day was calculated, a median of those responses was found. The daily medians were compared to the final journal entry to see if there were differences between the individual days and the overall feelings about the unit.

**Surveys.** The only quantitative data on the surveys was in regards to thinking skills. On the first survey students were asked about which skills they had used previously in math. The middle and final survey asked about which skills they felt they had used in this unit specifically. The surveys produced percentages of students who responded about the use of each thinking skill. The highest and lowest three skills from each survey were determined, and a 2-proportion Z-test was completed to see if the final survey had more responses than the initial survey for the top three higher-level skills (Sullivan, 2017).
**Pre- and Post-tests.** The pre- and post-tests were analyzed in two ways. The first piece that was examined was the level 4 questions for each section. The percent of students who attempted the higher-level questions as well as the percent who got them correct were collected on both assessments. These percentages were then compared for improvement using a 2-proportion Z-test. The second method of analysis involved the number of attempts students needed to demonstrate proficiency on each of the three sections of the post-test. Each level 3 question was looked at individually at first, and the mean number of attempts for each section was calculated.

**Quantitative Data Presentation**

**Thinking Skills.** District A uses Marzano’s taxonomy (see Appendix A) to determine the level of thinking required for proficiency and beyond for each Measurement Topic. In our mathematics curriculum, most topics require students to use lower-level skills such as *recognition* and *recall*, while the real world expects students to be prepared to use *critical thinking* and *problem solving* (Voogt & Roblin, 2012). The district encourages, but does not require, the use of these skills when students work on the level 4 content. Journals and surveys, as well as the scores on the pre- and post-tests, show which thinking skills students felt they were using as well as which level 4 content was attempted and/or completed.

Lower-level thinking skills such as *recognition*, *recall*, and *execution* always had the highest number of responses in the journals, with the largest maximum being 18 students using *executing* on Day 3 in a compound probability lesson. Higher-level skills, such as *analysis* and *problem solving* also had consistently high responses throughout the
### Table 4.1: Daily Thinking Skill Use

<table>
<thead>
<tr>
<th>Thinking Skill</th>
<th>Day (number of students)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>Final Mean</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>Recognition</td>
<td>15</td>
<td>9</td>
<td>4</td>
<td>7</td>
<td>10</td>
<td>6</td>
<td>7</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>7.20</td>
</tr>
<tr>
<td>Recall</td>
<td>16</td>
<td>7</td>
<td>10</td>
<td>7</td>
<td>6</td>
<td>7</td>
<td>9</td>
<td>10</td>
<td>6</td>
<td>11</td>
<td>8.40</td>
</tr>
<tr>
<td>Executing</td>
<td>7</td>
<td>7</td>
<td>18</td>
<td>14</td>
<td>16</td>
<td>15</td>
<td>16</td>
<td>12</td>
<td>12</td>
<td>13</td>
<td>13.30</td>
</tr>
<tr>
<td>Integrating</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Symbolizing</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>7</td>
<td>8</td>
<td>5</td>
<td>3.70</td>
</tr>
<tr>
<td>Matching</td>
<td>1</td>
<td>9</td>
<td>3</td>
<td>0</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>3.70</td>
</tr>
<tr>
<td>Classifying</td>
<td>3</td>
<td>4</td>
<td>8</td>
<td>3</td>
<td>7</td>
<td>4</td>
<td>9</td>
<td>9</td>
<td>6</td>
<td>11</td>
<td>6.40</td>
</tr>
<tr>
<td>Analyzing Errors</td>
<td>9</td>
<td>1</td>
<td>6</td>
<td>8</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>8</td>
<td>2</td>
<td>8</td>
<td>5.80</td>
</tr>
<tr>
<td>Generalizing</td>
<td>3</td>
<td>0</td>
<td>10</td>
<td>5</td>
<td>9</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>4.60</td>
</tr>
<tr>
<td>Specifying</td>
<td>2</td>
<td>1</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>7</td>
<td>3.20</td>
</tr>
<tr>
<td>Decision Making</td>
<td>13</td>
<td>6</td>
<td>11</td>
<td>13</td>
<td>11</td>
<td>8</td>
<td>9</td>
<td>7</td>
<td>4</td>
<td>10</td>
<td>9.20</td>
</tr>
<tr>
<td>Problem Solving</td>
<td>13</td>
<td>6</td>
<td>16</td>
<td>12</td>
<td>14</td>
<td>15</td>
<td>14</td>
<td>13</td>
<td>14</td>
<td>12</td>
<td>12.90</td>
</tr>
<tr>
<td>Experimenting</td>
<td>7</td>
<td>7</td>
<td>11</td>
<td>2</td>
<td>10</td>
<td>4</td>
<td>9</td>
<td>6</td>
<td>2</td>
<td>7</td>
<td>6.50</td>
</tr>
<tr>
<td>Investigating</td>
<td>6</td>
<td>6</td>
<td>8</td>
<td>1</td>
<td>10</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>4.60</td>
</tr>
</tbody>
</table>
study. *Problem solving* only showed one day with fewer than 10 students indicating that they had used the skill. This data is shown in Table 4.1 with the final survey data shaded.

According to the journals, *executing* had the highest overall mean of 13.30 with a standard deviation of 3.80. The second highest was *problem solving* with a mean of 12.90 and a standard deviation of 2.73. When t-tests were done to compare the daily means to the final journal results, no statistical differences were found as the p-values, 0.81 and 0.32 respectively, were both higher than alpha, 0.05 (Sullivan, 2017). The lowest used skills were *integrating* and *specifying* with means of only 2.50 and 3.20 respectively of students saying they used these skills during the unit. Hypothesis testing showed a difference in the mean and final journal entry for both these as the p-values, 0.03 and 0 respectively, were smaller than alpha. This demonstrates that students indicated their use in the overall unit more than they did in the daily journal entries.

The surveys showed similar results. The first survey showed 92.6% of students believing that they had use *recognizing* prior to this unit and 88.9% of students shared they had used *recall* and *problem solving*. The middle survey showed problem solving as the number one used skill with 92.6% of students responding they had used it during this unit. *Executing* and *decision making* tied for the next highest skill with 74.1% of students reporting their use. The final survey had 100% of students report using *problem solving* during this unit, while 90% shared the use of *execution*. The results of each survey are compared in Figure 4.1.

Hypothesis testing was done on *decision making*, *problem solving*, and *analyzing errors* as they were the higher-level skills with the highest indication of use on the surveys. The proportion of students who claimed to use these skills before the unit was
compared to the proportion of students on the final survey to see if there was statistical evidence of increased use. The p-value for problem solving was 0.06, the p-value for decision making was 0.59, and the p-value for experimenting was 0.19. None of three tests showed statistical evidence of increased use as all the p-values are less than alpha.

Figure 4.1: Thinking Skill Use Before and During This Study

Finally, the pre- and post-tests showed the percent of students who were able to successfully complete the level 4 questions. There were six questions that involved the use of higher-level thinking skills and more students were able to complete them successfully on the post-test than on the pre-test. The hypothesis test for each question yielded a p-value of zero, showing that each level 4 question has statistical evidence of improvement between the pre- and post-tests. Figure 4.2 compares the percent of correct responses on the two assessments for each level 4 question.
Overall, the quantitative data shows an increase in the successful use of higher-level thinking skills throughout the course of this unit. While some tests were not significantly significant, the data as a whole supports the idea that problem solving and critical thinking skills were used more.

Figure 4.2: Pre-test and Post-test Level 4 Percent Correct

**Number of Attempts.** Students at Monarch High School are required to take assessments until they are able to demonstrate proficiency. Teachers in District A feel that students needed too many attempts to take tests, an indication that they are memorizing the format instead of learning the material. If students are successful with their work as they progress through the unit, they should be better prepared for the final assessment (Hoffer, 2012). Upon completion of the first attempt, I corrected the assessment and returned it to the student with correct problems marked. They were able to try the incorrect questions again without any feedback. When they turned the test in a second time, each question that was still wrong was given a guiding question to direct
thinking. The questions were about pieces of the problems (for example, “How many hearts are in a deck of cards?”) rather than the mathematical content.

All 20 students were able to demonstrate proficiency within three attempts. The first section of the post-test showed that 58.33% of students were able to earn a 3 on their first attempts, 37.67% on their second, and only 4.00% needed the guiding question to show proficiency (see Figure 4.3). The second section of the post-test showed 76.00% demonstrating proficiency on their first attempt, 19.50% on the second, and 4.50% needed the guiding question (see Figure 4.4). Finally, 68%, 28%, and 4% were able to succeed on their first, second, and third attempts, respectively, on the final section of the post-test (see Figure 4.5). With approximately 4% of students needing a guiding question, the vast majority of students were able to demonstrate their knowledge without assistance. The questions did not provide guidance on the actual solution, but only led students to a small error, so while the third attempt may have skewed the completion data slightly, it should have had a minimal effect.

**Feelings.** My students often complain that they do not like math and that it is hard for them. Math Workshop provides students a chance to see the real-life application of what they are learning as well as complete tasks that are more interesting than a traditional lecture approach (Hoffer, 2012). The student journals were able to capture numerical data regarding how students felt about the math they were doing: whether the lesson was interesting, if they understood the material, and the lesson’s relevance to real life.

When asked if they found each task interesting, students’ median responses ranged from three to five. For both relevance to real life and understanding the material,
Figure 4.3: Section 1 – Mean Percent of Attempts to Demonstrate Proficiency

Figure 4.4: Section 2 – Mean Percent of Attempts to Demonstrate Proficiency

Figure 4.5: Section 3 – Mean Percent of Attempts to Demonstrate Proficiency
students gave median responses of threes and fours (see Figure 4.6). The median of each daily question overall was also calculated and compared to the final journal entry that asked about their feelings of the unit as a whole (see Figure 4.7). Interest and understanding the material had overall medians of four, both matching the final journal entry. Relevance to real life had an overall median response of three, but the final journal entry gave a median of a four. In general, the feelings addressed with the journal entries remained consistent with a spike in interest on Day 5 when they played games of chance in class.

**Summary.** Throughout the quantitative analysis, the data provided information regarding each research question. Thinking skills appear to be used more in this unit than before and students were able to demonstrate proficiency in three or fewer tries. In regard to feelings these do not appear to show large changes in the numerical data. Feelings are not always able to be captured in numbers, so qualitative data was also collected in hopes of supporting these numerical summaries.

**Qualitative Data Presentation**

Categorical data is able to tell a story better than numerical data. Allowing students to use their own words to describe what they are feeling gives meaning to the numbers found in the numerical contexts (Merriam & Tisdell, 2016). The video recordings were able to substantiate the answers that students gave in their journals and surveys (Mertler, 2014). The student work also showed the processes that were used to solve problems as well as misunderstandings that may have occurred while working. These same misconceptions and thinking patterns could not be seen when only looking at actual answers or percent correct (Mills, 2007).
Qualitative data was coded as it was read to bring some organization to the large amounts of information. Codes were assigned to patterns, repeated phrases, and other information that could help answer the research questions during analysis. When the coding was complete, units were grouped into similar categories that helped in understanding what was found (Mertler, 2014). Those categories were then evaluated for their relationship to each research question before being interpreted (Mills, 2007). All of
the data that contributed to addressing the research questions fit into one of four major themes: comparisons, perception of others, level of rigor, and interest (see Table 3.1).

**Comparison.** The theme of comparison was seen when students were asked how they liked math in general, specific tasks or lessons, and whether they preferred the more traditional way of teaching or the Math Workshop framework. Some questions led students to compare styles, while others (for example, “Do you like math? Why or why not?”) led students to make comparisons on their own.

When asked whether they liked math, students shared their feelings in relation to other subject areas. One student shared, “It is a more logic-based class, so I like it better than English.” Another said, “Well, I guess. I like it better than my other classes at least.” They also compared different subjects within math with comments like, “I like Algebra better than Geometry because it is less visual.” One student’s comment showed that feelings can depend on circumstance and not the subject of mathematics alone: “It depends on the teacher. Some do a really good job explaining math, but others make it confusing.”

Students also compared the daily lessons in the journal entries. “Today was much better than yesterday,” showed how one student felt after doing expected value on the third day. Even though they were working on the same concept, the student liked the lesson better when he already knew the formula to solve the problem. Another student shared, “I didn’t like what we did today. This type of probability is more confusing than the insurance stuff,” in relation to the tree diagrams. Instead of having a specific complaint about the conditional probability, her negative feelings were in relation to another task. Since the student identities were hidden when the journals were analyzed,
there was no way other than this comparison to see how she felt about this lesson, as I
could not see how she felt about the first one.

Finally, students were asked to specifically compare the more traditional approach
to math instruction to the Math Workshop framework we used in this unit. Among those
with a preference, feelings were split evenly between each method with seven students
each, and the other six students said they liked both equally. “I learned better because we
worked in groups and did hands-on activities,” was one response in support of Math
Workshop. A proponent of the traditional method shared, “I like it better when you just
tell us what to do, and then let us do packets on our own to practice.” One student
shared, “I like both ways. The activities were good, but sometimes it’s nice to just do the
work.” These three viewpoints compared pieces of Math Workshop to the more passive
style of teaching math, with both positive and negative feelings.

**Perception of Others.** Overall, the students in this sample were very concerned
with what others thought of them. This concern affected many students desire to attempt
the level 4 work as well as how they felt when sharing answers and trying new things
during class.

The surveys asked whether students were willing to try the level 4 questions and
why. Half of the responses indicated that they would because they wanted better grades
and a higher grade point average (GPA). One student even responded, “Yes, I would
because I don’t want to be a disappointment to my parents.” While the GPA is used for
junior and senior privileges at Monarch High School, it is also used to determine class
rank and honors at graduation. Students shared that they wanted a good GPA “so that I
can have chords at graduation,” showing that they want others to see how well they did overall.

The video recordings also demonstrated that students cared what their peers thought about them and how that changed over the course of the unit. On Day 2, students seemed reluctant to share their answers. I asked one why, and she responded, “I’m scared to be wrong in front of everyone.” On Day 4, students were willing to share which payment method they chose on the opener, but only a few were willing to defend their choice. Two of the students who tried to present a defense started with, “I don’t know if this is right, but I chose this because….” They both wanted people to know they were not sure, so if they were wrong, it was not a big deal. On Day 5 when we played games of chance, students were willing to share their winnings out loud: “Of course I want to know how much money everyone made. How else will I know if I won?”

Students started asking each other for help on Day 6. This demonstrated that they were less worried about others knowing they did not understand. Days 7 and 8 showed that students were more willing to share out during the problem of the day. The same student who was scared on Day 2 even said, “Come on guys, just share out, no one cares if it’s wrong!” She had grown into the idea that the process was just as important as the correct answer and was less nervous about being wrong in front of everyone.

Lastly, the students cared what I thought about their progress. During the insurance packet work, one student hollered across the room, “Hey, Miss Ross, look how fast I am going!” This student is often behind on his work and was very happy to be able to show me that he was doing well that day. On Day 10, the students were working with tree diagram packets that got progressively more difficult as they worked through them.
Students were directed to get as far as they could in 20 minutes. After 15 minutes, three separate students made comments about not being able to finish in time. One commented, “Miss Ross will think I can’t do it if I don’t finish.” Caring about the study was more important to one girl than her learning. She actually asked me, “What will happen if I forget everything that we learned? Will it mess everything up?” Once I reminded her that they study was to see how Math Workshop worked and that we would relearn the material another way if needed, she calmed down and began her post-test.

**Level of Rigor.** The theme of rigor, perceived and actual, was seen across all collection tools. Whether students related it to work completion or how they felt about math, their perception of difficulty played major roles. Degree of understanding, the pace of instruction, and the amount of information all played a role in how difficult the content seemed. Each survey asked students if they liked math and why. The surveys also asked if students usually tried level 4 questions and if they attempted them for this unit. Students were asked whether they thought math was easy as well. The third question directly asked about difficulty level; interestingly, the other two questions yielded many answers that included difficulty as the reason for their answer. Approximately 77% of the responses to these three questions included reasoning based on the perceived difficulty level.

The level of difficulty was key to students liking or not liking math. In general, students who liked math indicated that this was because they found it easy. For example, one student said, “I like math because it’s pretty easy for me to understand.” Another shared they only like math “when it makes sense.” Students also pointed out the thinking skills they enjoy about math. One shared. “Steps and formulas make math easy,”
showing they were comfortable working with recalling and executing. “I don’t like math because numbers confuse me and make my head hurt,” shows this logic in reverse as well, with students who think math is hard being less likely to enjoy it. Further demonstrating how rigor affected feelings about math, reasoning for another student was “I think math is okay when I understand what is going on. If I don’t understand what is going on, then I will not like it.”

When asked how they felt about each days’ lesson, the responses in the journals mirrored the surveys with positive comments. Students shared, the math was “easy to understand” on multiple days. Most of the negative comments about the lesson were explained by comments indicating the lesson was “hard,” “complex,” or “confusing.” One student said “I didn’t like the lesson at first, but practice made it better,” demonstrating that once they understood the material, they liked it better.

The level 4 tasks in class and on the pre- and post-assessments use higher-level thinking skills, making the tasks more difficult in nature. Students were asked if they usually tried the level 4 questions (Day 1 Survey), if they planned to go for the level 4 questions (Day 8 survey), and if they tried the level 4 questions on this test (Final Survey). In general, students who wanted to attempt the higher-level tasks felt that math was easy. More people said yes than no with multiple reasons having to do with difficulty level. One student claimed that they only did the “easy ones,” while another student said they would try the level 4 “because I am understanding the concept.” The problems were “too difficult,” was a response from another student, and yet another claimed they were not going to attempt the level 4 because, “I already have a hard time getting the 3s.”
Some of the daily tasks required the use of higher-level thinking. The video recordings showed how students approached these tasks as well as comments that were made while working on them. On Day 1 students worked with poker hands. Many of them were new to cards or the game of poker so prior knowledge was not available for them to make connections, causing this task to be more difficult than anticipated (Eggen & Kauchak, 2007). Instead of using actual decks of cards to investigate the answers, students quickly asked for help. “Can you just tell us the answer?” was a common question. I also overheard one student say, “This is too hard. I am just going to guess because I can’t do it.” Day 3 (compound probability) yielded similar comments. More students were willing to try and work out the answers than on Day 1, but two still asked “Can’t you just tell us the rule?” Day 10, when we looked at tree diagrams, brought more comments about the difficulty of the tasks. One student who is usually excited about math said, “I hate this. The trees make no sense.” Even though the packet was progressive and students were directed to get as far as they could, I overheard three comments of students who were worried about not finishing because it was hard and they needed more time. Two students also shared that there was “too much info for one unit. I am never going to remember it all.”

Although questions were asked in different ways, students fell back on their perception of the rigor of the unit or specific tasks when answering many of the journal and survey questions. They also reflected how they felt about a task with comments they made or pieces of the assignments that were attempted at various thinking levels.

**Interest.** The last theme that I discovered while analyzing the qualitative data was that of interest. Students shared what interested them in the surveys, and their interests
seemed to change as thinking levels changed. Feelings about the math in general, the Math Workshop framework, and individual classes also changed as their interest levels in the content and activities changed.

*Recall and execution*, two of the lowest three thinking skills, involve remembering and using a formula. In general, students liked class better when these were the thinking skills most in use. Student E shared “I like class when there are steps to follow. Formulas make it better.” The least favorite lessons were the “confusing ones” and “the ones that made me think too much.” When it came to attempting the level 4 work, a few students were not interested at all. One girl even said, “I am not going to try them, because I don’t care.”

The surveys asked the students which classes were their favorite, their least favorite, and why. The journals asked students how they felt and why following each class. These questions led to responses that showed various levels of interest linked with the activities. Overall, every student enjoyed when they got to play games on Day 5. The students found playing games and working with money fun. One stated, “I never thought I’d be gambling in school with my friends. We had so much fun while learning math!” Another student shared that their least favorite days were doing the insurance packets. They found them to be “no fun. The formulas are tedious and repetitive. They took too long to do.” Although this is the opposite of the excitement for Day 5, it demonstrates that when students were not having as much fun, they did not like lessons as much.

Interest levels of the students was often noted when reviewing the video recordings. When students were interested, there were fewer redirections needed during
the lesson, less talking about outside topics while working, and more math centered
discussion. When the material was realistic, students started discussions about the topics
and how they related to them. The largest discussion provokers were the drug test
problem and the insurance information. Students were very concerned that drug tests
were not perfect and how the probability of the test’s outcome can affect people’s jobs.
One student who is usually off task looked at the board and asked, “Why on Earth would
a company use a test that had that many wrong results?” As I got ready to answer a quiet
student in the back of the room remarked, “They are probably cheap and then retest
people who are positive, since most of the wrong ones could be found with a better test.”

Students also had lots of questions about the different types of insurances and the
math behind those decisions. While calculating car insurance one student asked about the
honor roll discount. Another responded, “Hasn’t your mom ever yelled at you for getting
bad grades and it costing her money on car insurance?” When the topic had to do with
their lives, they were able to make connections outside of the classroom and were more
interested in the mathematics. One of the several junior firefighters in this groups asked
how someone figured out the percentages for fire insurance, and when I did not have the
answer, we began a discussion about what could cause fires and how someone might
decide a house is at risk. Before using the workshop framework to design activities that
interested my students, I had not seen this level of involvement and questioning of the
material.

Any activity that involved money instantly claimed the students’ interest. Even
when the games were abstract and unlikely to be seen outside of a classroom, students
wanted to know how to make more money. When their expected values did not make
sense based on insurance costs or game payouts, students kept digging instead of giving up. One student questioned, “Why is my insurance cost negative? I doubt they are going to pay me money.” This statement led a group of four other students to look at her work and see if they could find the error. Before this lesson, students would tell me they could not figure out a problem and wait for me to show them what went wrong. After this lesson, they were more independent with their thinking and worked together before bringing me into the conversation.

The way that students responded on the surveys and journals as well as how they acted in class showed varying levels of interest. Some students like the structure of a formula, while others find it boring. The biggest effect interest played on students in class was the more excited about the topic, the more engaged they were with the material, the activities, and the discussions. Their willingness to defend their math and work together to solve problems also increased with their level of interest.

**Summary**

The qualitative data showed much more detail than the numerical data did. Reasoning for answers such as, “I like math, because it is easy” help to explain the numerical data. The video recordings showed students engaged in tasks and having fun. This showed a level of interest that was not captured in the journals or surveys. Overall, the outcomes seemed positive in relation to the use of Math Workshop.

**General Findings and Results**

The problem of practice was concerned with the use of higher-level thinking skills in mathematics. When preparing for the study three research questions evolved involving the use of those skills, the number of attempts to reach proficiency, and how students felt
about math in regard to the Math Workshop framework that was introduced. Overall positive results were seen in relation to this study.

In the beginning stages of this research study students asked a lot of questions and were timid to give their answers in fear of being wrong. As the study progressed students were more willing to try and find an answer on their own or with peers before asking for my help. They also got more comfortable with sharing their answers and comparing what they got with other students. They also chose to work together more as the unit progressed. Student feelings were more positive on days that were interesting, even when the level of rigor was higher. As students became more comfortable with making mistakes, they also became more willing to try the more difficult problems, even when they were the level 4 questions on the post-test. Although not a universal feeling, students in general were more interested in the tasks during this unit and less afraid to attempt the harder problems more than once.

In a normal unit, I often have to hunt students down in order to get them to complete their homework assignments. During this unit, most of the students stayed on pace because they wanted to be able to participate in the next activity. They wanted to play the games that were discussed on their homework, wanted to see if their answers were correct, and they discussed their results without prodding. The willingness to participate generally increased as students became more comfortable with the process and more engaged with the content.

Generally speaking, the feelings of the students were positive throughout this unit. There were some days and activities that elicited more negativity that others, but positive comments were always seen more than negative ones. They found activities fun and
interesting, did their work when it was due instead of after it was late, and engaged in discussions around the answers they found. More students attempted the level four problems, and all students in the final sample completed their assessment in three or fewer attempts, even after a vacation. Their positivity increased as the unit progressed, and the final survey and journal entry showed that even the students who didn’t like Math Workshop better overall found pieces of it that they enjoyed.

**Analysis of Data**

Quantitative and qualitative data were initially analyzed separately. After this, the results were examined together in order to address the research questions. While some questions relied more on one type of data than the other, all three questions needed both sets in order to be fully answered.

**Thinking Skills**

Question one asked, “What is the effect of using Math Workshop on increasing problem solving and critical thinking skills as demonstrated by successful completion of level 4 work?” Overall this study shows positive results in relation to using higher-level thinking skills in relation to level 4 work.

The journals showed that lower-level skills were used more than higher-level ones daily, but as the unit progressed critical thinking and problem solving were recognized more by the students. The second highest reported skill was *problem solving* with a mean of 12.90 students reporting its use each day. *Decision making* was the highest overall critical thinking skill reported, but the other skills under analysis were addressed most days.
Student work showed an increase in willingness to try or get to the more complex problems. Through the course of the unit, fewer comments were heard about being scared to be wrong. Students put more effort into their work before asking questions, and on the assignments there were fewer questions left blank as the study progressed. Although percentage of correct answers were not always high on the classwork and homework attempts, the fact that students were willing to try shows a higher level of comfort with thinking at that level.

When looking at the quantitative data for the pre- and post-tests, each section had at least one level 4 question. The questions in section one and section two asked students to make and defend decisions, requiring the use of specifying. Error analysis was required in both section two and three. Both of these skills require critical thinking in order for students to be successful. While the first section had the same percent of students attempting the level 4 questions, there were significant increases in the other two sections. The section two level 4 questions were left blank by 10% of students on the pre-test, but every student tried them on the post-test. Conditional probability, the third section, had 74% of students leave the level 4 question blank on the pre-test. This dropped to 21% on the post-test showing students were more comfortable with the material and trying the more complex problems after using the Math Workshop framework.

The daily tasks required the use of higher-level thinking skills, and in turn more students attempted the optional, more complex problems willingly on the post-test. The students’ answers on work during the unit were not always correct, but as they grew more comfortable with being wrong, their willingness to try grew, and the post-test showed a
higher percentage of students that attempted the level four work also got the correct answers. No one got all of the level 4 questions right on any section of the pre-test, but on the post-test 46% of students answered both level 4 questions correctly on section one, 51% answered the level 4 questions correctly on section two, and 75% answered the level 4 question correctly on section three. This shows that students were able to understand the processes introduced, apply them in unique situations, and find errors in other work successfully.

**Number of Attempts**

Students in District A are required to take assessments until they are able to demonstrate proficiency of the Measurement Topics. Teachers in this district are worried about the number of times that students needed to attempt the assessments since students will not have multiple attempts to find correct answers on standardized tests like the SAT. This led me to look at the second research question: to what degree does using Math Workshop impact the number of attempts to demonstrate learner proficiency?

In previous years, the probability unit has been one that required the most retakes by students. On this post-test every student was able to demonstrate proficiency within three attempts. More than half were able to finish the assessment with a level 3 on the first attempt, with only 4% needing the guiding question before the third try. Students also had fewer questions while they worked on the assessment than they did during the rest of the unit. When a large snow storm led to two days out of school and pushed the test to be taken after a week off, the success on the assessment supported the idea that this approach had a positive effect on the number of attempts as compared to previous years.
Feelings about Mathematics

Math is known for student dislike and struggle (Ashcraft, 2002; Boaler, 2016; Dabkowska & Sosnovski, 2016). In applying the Math Workshop framework and adding activities that were relevant and interesting to my students I asked questions to address the third research question: what is the effect of Math Workshop on students’ attitudes/feelings about math? I looked at feelings regarding liking math, the relevance of activities to the learning target and real life, as well as whether students thought it was easy for them to learn. The surveys also asked how things changed for the students over the course of this unit.

The level of rigor affected how students felt about math. When asked on the first survey if they liked math 19% did not because they saw it as hard and not useful. By the end of the unit, only 15% indicated they did not like math, and the only reason for the dislike was math was difficult. This demonstrates that even when working with more complex tasks, the amount of practice and relevance of the activities decreased overall dislike of mathematics.

Students were interested in many of the tasks assigned during this study. As their interest increased, so did the level of engagement, time on task, and discussions about math. When activities involved money or were relevant to student lives, students reported having more fun and a deeper understanding of the material. When tasks were tedious, students needed more redirection and talked more about things outside of the classroom. Students who claimed to not like math at the onset of this study had more positive things to say about the activities at its conclusion.
Students also appeared to care less what others thought about them by the end of the unit. They were more willing to work together to solve difficult problems at the end of the unit. Students who were scared to share their answers at first were encouraging others at the end. While still wanting higher GPAs to impress their parents, students were more likely to ask a peer for help and look for errors than they were on Day 2. More students shared answers to the opening problems as well as their reasoning, even when they knew it was wrong.

Finally, when asked specifically about comparing this method, Math Workshop, with my more traditional method of teaching the results were inconclusive. Students liked the group work, visuals, and reflections, but only 40% said they learned better with Math Workshop and 30% said they felt that they learned worse. The remaining 30% did not see a difference between the two methods. When asked which method they preferred, 45% of students said they like the more traditional approach, 30% said they liked the workshop, 20% said they liked both approaches equally, and the final 5% said they did not see a difference. It appears that while students had positive feedback during the unit, overall they are more comfortable with a more traditional approach.

Summary

This action research study examined the effects that the Math Workshop framework had on two high school Algebra II courses. Probability was taught using this framework and the data was collected and analyzed to answer three research questions. Question one asked what the effect of using this framework was on increasing problem solving and critical thinking skills. During this unit the use of both sets of skills increased as did the successful completion of level four work. The second question asked to what
degree Math Workshop impacted the number of attempts to demonstrate proficiency. Students needed fewer attempts to complete the test and no alternate versions were needed within this unit. Finally, the last question dealt with the effects of Math Workshop on students’ general attitudes and feelings about math. While students did not always like the workshop framework better, they were more engaged in the content more on task, and usually had positive things to say about the math. The only negative effects were based on being confused and the difficulty of the material.

Trying out this framework during one unit has provided a lot of data on its success with critical thinking, feelings about math, and success on assessments. Most of the research done on this framework has been with younger students, and the results of this study were somewhat different than expected based on that information. More inquiry needs to be done in the future to further the implications of using Math Workshop at the high school level.
Chapter 5: Results and Recommendations

The 21st century requires the use of problem solving and critical thinking skills after high school (P21, 2016). For graduates to be able to use these skills, they need to practice them in high school. District A switched to a proficiency-based model of instruction in 2011. Since making this change, students are required to demonstrate proficiency and have the option to exceed it by using higher-level thinking skills. The Measurement Topics in mathematics usually require lower-level skills such as recall and execution, and the teachers of the district have noticed a drop in the use of the optional, higher-level skills. The problem of practice of this study is that students are not using the critical thinking and problem solving skills that they need after high school. The purpose of this study is to apply the Math Workshop framework and determine if it helps to encourage the use of these skills.

The intervention used in this study, Math Workshop, is designed to inspire the use of higher-level thinking skills by introducing complex, interesting tasks to students and allowing them to build their own connections to the content. Each lesson during this study began with an opening problem designed to review prior material, spark interest in the upcoming content, or to get students thinking about a situation. These openers were followed by either a mini lesson to teach new content or an activity where students tried to solve problems and defend their choices. If the lesson was first, it was followed by an activity period where students worked with the math content they had learned. If the activity was first, it was followed by a lesson to answer questions, check for
understanding, and address any noticed misconceptions. Finally, each lesson ended with a reflection period when students were given the opportunity to think about the task, share questions and feelings, and sometimes compare their personal results to the correct ones to analyze what they did differently.

The data gathered during this unit was used to answer the following research questions:

1. What is the effect of using Math Workshop on increasing problem solving and critical thinking skills as demonstrated by successful completion of level 4 work?
2. To what degree does using Math Workshop impact the number of attempts to demonstrate learner proficiency?
3. What is the effect of Math Workshop on students’ general attitudes/feelings about math?

Overall, the application of the Math Workshop framework showed positive results. In the first few lessons, students asked for a lot of guidance, but the more practice they had with the Math Workshop framework, the more comfortable they became with using higher-level thinking skills independently. While scared to be wrong in front of their peers at the beginning, students were more willing to take risks and discuss their thinking later on, especially when the tasks were relevant to their lives. The relevance of tasks also led to students reporting deeper levels of understanding of the material throughout the unit. As interest in the tasks increased, so did the level of engagement, time on task, and depth of the discussions. Although hypothesis testing did not show a statistically significant increase in the daily use of problem solving and critical thinking skills, it did
show improvement on the successful use of these skills on the post-test, even after two snow days and a weeklong vacation.

This chapter discusses the findings of this action research study. It shares how the results relate to the current literature on Math Workshop and how it has transferred to high school students. Overall conclusions about the use of Math Workshop are given, as well as unexpected results unrelated to the research questions. It presents recommendations for implementation in my classroom, Monarch High School, and District A as whole. I also reflect on the action research process, make suggestions on how it could be improved, and point out limitations of this specific study. Finally, recommendations for future research on this topic are provided.

**Results**

John Dewey (1938) believed that the standard practice of skill and drill teaching does not develop the use of higher-level thinking skills. On the other hand, 21st century students need to be proficient with problem solving and critical thinking to be successful after high school (P21, 2016). Constructivism encourages the uses of these skills by allowing students to create their own knowledge with rich tasks and group collaboration (Ertmer & Newby, 2013). Math Workshop is a framework that uses rigorous tasks to get students thinking at deeper levels with relevant experiences that encourage higher-level thinking skills, while preparing students for the 21st century and future education.

**Constructivism**

A key tenet of constructivist learning is that students build their own knowledge through interaction with their environment and experiences (Ertmer & Newby, 2013). These activities should be relevant to students’ lives, provide opportunities to be hands-
on with their learning, and stimulate deeper thinking thorough connections with prior knowledge, productive struggle, and collaboration (Schiro, 2013; Legnard & Austin, 2012; Piaget & Inhelder, 1969; Dewey, 1938; Ertmer & Newby, 2013; Joseph et al., 2000). This study used tasks that reflected these aspects of constructivism and allowed students to learn the material presented.

Students are able to make connections with content when they find it relevant to their own lives (Ertmer & Newby, 2013; Vygotsky, 1978). Having the connection to the problem encourages engagement with the mathematics, and in turn builds deeper understanding of the material (Dewey, 1938; Legnard & Austin, 2012). When students were talking about car and hazard insurance they shared personal connections to the idea. They were engaged in the work, stayed on task, and were more willing to defend their answers on these activities. As they progressed through the packet, conversations began about how certain percentages were calculated. The interest in the topic led to students asking for more information because the abstract content became concrete and applicable. Even when working with abstract games that may never be seen outside of a classroom, students were invested because they involved money. Wanting to earn money and beat their peers fostered engagement with the math ahead of time so they knew which games to try on Day 5.

Hands-on activities and the use of manipulatives are methods that make abstract content more concrete and applicable to students (Piaget & Inhelder, 1969; Hartshorn & Boren, 1990). They allow students to discover their own facts instead of having the teacher tell them the answers (Eggen & Kauchak, 2007). The hands-on tasks in this unit encouraged engagement, required fewer repeats of the directions, and showed students
more on task. Almost everyone came to Day 5 with their homework done, because they wanted to play the games. Not one student was off task during the lesson or the activity period, because they were using the materials, playing games, and trying to beat each other. During the reflection period that day, students compared their results to their predictions from the homework. Discussion revolved around how probability is based on chance, and even if the math shows one expectation, reality may not match it in the end.

Having prior knowledge on a subject allows students to make concrete links to the content they are currently working with (Ertmer & Newby, 2013; Bresser & Holtzman, 2018). The opening activities either activated this prior knowledge or created a base to build their new knowledge on. For example, Day 2 worked with poker hands. Students who were familiar with cards and the game of poker already were able to focus on the math and start working with the probabilities. Students who were unsure about cards spent more time learning what was in a deck instead of attempting the math being asked of them. This demonstrates that having a connection to the material prior to working with it facilitates the ability to transfer the knowledge and work more deeply with the content (Hoffer, 2012; Ertmer & Newby, 2013). When students were able to connect different activities from this unit, they recognized what was being asked and were able to apply what they learned to try more rigorous problems.

Constructivism supports risk taking in a safe space in order to encourage the use of problem solving and critical thinking skills (Joseph et al., 2000; Hoffer, 2012; Heuser, 2002; Bresser & Holtzman, 2018). Students need a safe space to make mistakes, to try multiple approaches, and to discover their own answers to the questions (Mighton, 2003; Hoffer, 2012; Joseph et al., 2000). At the start of this study, students were scared to be
wrong in front of their peers and worried about what others thought of their solutions and grades. As they progressed through the workshops, students become more comfortable with sharing their answers and ways of thinking, took more risks with their approaches to the content, became less worried about what other students would say in response to their explanations. As students became more willing to try new approaches, the more difficult problems began making sense and students were more willing to try the level 4 questions on the final assessment.

Finally, collaboration is important to constructivism as it leads to communication, internalization, and solidifies comprehension (Dewey, 1938; Heuser, 2002; Piaget & Inhelder, 1969; Vygotsky, 1968). As students explain their thinking to others they are able to make sense of the material in their own minds (Piaget & Inhelder, 1969). Working together promotes engagement and cooperation that leads to internalization of the material (Hoffer, 2012; Vygotsky, 1968). Discussions and reflections with peers stimulate deeper thinking about the process and purpose of the mathematics (Dewey, 1938; Bresser & Holtzman, 2018; Heuser, 2002). This study showed examples of this as students became more comfortable with the workshop model. They began working together more to find solutions and to check each other’s work for errors. When they were engaged with the topic, discussions about math and decision making were unprompted within the class. For example, when working on conditional probability, students were interested in the drug test results of airline pilots. They began discussions about why companies would use these tests and how they could give so many false positives. This led students to use critical thinking and decision making skills even though they weren’t part of the original lesson plan.
Constructivist learning focuses on students building their own knowledge, working together to deepen understanding, and completing rich tasks that promote the use of higher-level thinking skills (Dewey, 1938; Eggen & Kauchak, 2007; Legnard & Austin, 2012; Schiro, 2013). This study showed that the combination of these elements kept students on task and engaged in the material throughout the unit. All students were able to demonstrate proficiency on the post-test within three attempts, and many attempted and were successful with the level 4 questions. Students were able to retain the material and use it correctly, even after two snow days and a weeklong vacation.

Math Workshop

The completion of this study and analysis of results showed overall consistencies with the research about Math Workshop with elementary and middle school students. Math Workshop reduces anxiety by developing higher levels of self-esteem and encouraging communication (Ashcraft, 2002; Slavin, 1980; Dabkowska & Sosnovski, 2016). This framework also aids retention by increasing attentiveness and time on task through a constructivist approach (Dabkowska & Sosnovski, 2016; Mighton, 2003; Slavin, 1980; Cai, Moyer, & Grochowski, 1999).

As everyone’s first experience with the Math Workshop framework, it took time for students to become comfortable with the process. The students in this study were less scared to take chances at the end of this unit. They were more comfortable working in groups. Their confidence grew as they took more risks and began talking about their thinking with their peers. The more comfortable they grew with the workshop process, the more students worked together to find their own errors and to discuss deeper aspects of the mathematics. Their problem solving increased as they communicated about their
thinking, and tried different ways to solve new problems (Dabkowska & Sosnovski, 2016). The fact that more students attempted the level 4 questions on the post-test than the pre-test shows that they were less anxious about getting answers wrong and more comfortable with thinking at that level (Ashcraft, 2002; Slavin, 1980).

Similar to the studies in elementary and middle schools, students were generally attentive during this research study (Mighton, 2003). Students were engaged in the tasks and had math centered discussions when they found the tasks relevant to their lives. The opening activities activated knowledge and allowed students to make predictions using new content (Hoffer, 2012; Bresser & Holtzman, 2018; Heuser, 2002). The reflection period allowed students to test those predictions, look for errors in their thinking, build connections with other activities, and consolidate their learning (Hoffer, 2012; Heuser, 2002). The connections that students made led to more buy-in on future tasks, keeping students on pace and engaged in the material. As time on task increased, so did engagement and discussion about the mathematics. Students started unprompted discussions on the usefulness of what they were learning in the real world, leading to the use of critical thinking and problem solving skills.

Dabkowska and Sosnovski (2016) saw that Math Workshop aided retention in students, and the findings in this study support this. Fewer questions were left blank on the post-test than the pre-test. This showed that students knew the material, or at least felt more confident that they did on the first try. The fact that students were able to demonstrate proficiency within three attempts demonstrated that students were prepared for this assessment. The discussions on Day 8 involved material from previous days.
working with insurance. Remembering and using content and information over the course of multiple classes also supports the idea of retention.

The only finding inconsistent with these studies was that many students didn’t feel they learned better with the Math Workshop framework. Seven students agreed that they learned better with this method, seven disagreed, and six saw no difference in learning between Math Workshop and Dewey’s (1938) traditional method. As the educator, I saw deeper learning and success on the daily tasks and assessments. These students have been learning math in a traditional way for 10-12 years. This framework was new to them and required the use of higher-level thinking skills. Students were encouraged to struggle and had to build their own confidence as they progressed through the unit. This level of discomfort may have led them to believe they learned worse using this method, but the data doesn’t support those beliefs.

21st Century Skills

Around the world there is a call for students to leave high school with problem solving and critical thinking skills (Voogt & Roblin, 2012). With information accessible through the internet, businesses and universities need students to be able to use that information in novel ways (Voogt & Roblin, 2012). The Math Workshop framework develops these skills by encouraging students to explore new content through exploration, complete rigorous tasks to find answers to questions, and invent their own solutions while learning new mathematical material (Hoffer, 2012; Heuser, 2002; Bresser & Holtzman, 2018). The productive struggle requires students to attempt to work with the information with little guidance, strengthening perseverance and the use of higher-level thinking skills.
Active exploration builds problem solving and critical thinking (Ertmer & Newby, 2013). Math workshop encourages students to do this exploration through rich tasks and productive struggle (Hoffer, 2012; Warshauer, 2014). With ill-defined tasks, students must invent their own solutions, and then share them out while explaining what they did (Heuser, 2002; Bresser & Holtzman, 2018; Hoffer, 2012). In this study, students self-reported their perceived use of thinking skills. Analysis and problem solving had high responses throughout the study, with problem solving only having one day with fewer than 10 students reporting its use. Students were seen making decisions, explaining their reasoning, and looking for errors with their peers in the video recordings. As the unit progressed, they appeared more comfortable using these skills as they began unprompted conversations at levels beyond the material being taught. Their interest in the topics helped prompt students to want to dig deeper into the material.

The tasks used in this study encouraged problem solving because students were not always given every piece of the information before working with the content (Heuser, 2002; Hoffer, 2012). This caused students to use problem solving and decision making more while learning the material, and to report them more on the surveys and in their journals. Since the tasks required the use of higher-level thinking skills, students were more prepared to apply them on the post-test than they were on the pre-test. The fact that more students not only attempted, but were successful with the level 4 questions on the post-test shows that practice with the complex problems throughout the unit made them more comfortable with their use in novel examples.

Critical thinking and problem solving are skills that are necessary to be successful in the 21st century (P21, 2016; Voogt & Roblin, 2012). Using constructivism tenants,
Math Workshop encourages the use of these in students (Hoffer, 2012; Heuser, 2002; Bresser & Holtzman, 2018). The students in this study not only demonstrated the use of these skills, but their comfort using them in new situations grew with experience and practice.

**Conclusions**

While the Math Workshop framework for teaching math may present more work for the teacher, it is a benefit to the students in their learning. Math Workshop increased the use of the higher-level thinking skills that are expected of students in the 21st century. Even though students did not report a statistically significant increase in use of these skills, there is evidence to the contrary in the video recordings. This framework increased engagement and overall level of understanding for the students, as shown in deeper discussions and on task behavior. Lastly, this approach to teaching mathematics increased the confidence level and willingness of students to try novel problems.

The opening activities, rich tasks, mini lessons, and reflection periods that make up the Math Workshop framework were successful in increasing the use of 21st century skills in the students of this study. The tasks required of the students each day encouraged the use of problem solving and critical thinking for the students to be successful (Hoffer, 2012; Heuser, 2002; Bresser & Holtzman, 2018). Times of productive struggle gave students the chance to use these skills without risk, knowing they would get feedback and instructions after attempting to answer the questions (Bresser & Holtzman, 2018; Hoffer, 2012; Schiro, 2013; Warshauer, 2014). Students reported out their perceived use of thinking skills, and *problem solving* was the second highest overall reported skill, followed by *decision making*. While there was no
statistical significance to show an increase in the use of these skills, the video recordings and success on the post-test disagree. We spent time going over the meaning of each thinking skill before starting this unit, but overall students were unfamiliar with some of the names of the higher-level thinking skills. This could have led to false reporting of their use as students did not understand what they were using for skills. For example, specifying was one of the lowest reported skills, but many of the tasks and two of the level 4 questions on the post-test required the use of this skill to be successful. If students were unfamiliar with the term, this could have caused the difference in data from the self-reporting and the qualitative evidence of its use.

Math Workshop also increased the level of engagement and understanding of the students. Participation increased when lessons were relevant, and the conversations demonstrated a desire to learn more as well as an understanding of the information presented to them. Collaboration gave students the chance to defend and explain their reasoning before sharing their understanding with the entire class (Vygotsky, 1968; Bresser & Holtzman, 2018). Students were on task when they were engaged with the material, leading to assignments being turned in on time (Slavin, 1980). Keeping up with the work, trying each rigorous task, and participating in the mathematical discussions led to better retention and deeper understanding of the material (Dabkowska & Sosnovski, 2016; Ertmer & Newby, 2013). This was proven when the students demonstrated proficiency on their post-test within three attempts after two snow days and a weeklong vacation.

Finally, the level of confidence increased in these students as they progressed through the action research study. At first, students avoided the more complex tasks and
waited for someone to give them answers. As they became more comfortable with the process and felt safe in the environment, they were more willing to take risks and try the problems in front of them (Hoffer, 2012; Slavin, 1980). The more activities that we did and reflected on, the higher self-esteem grew and the lower anxiety became (Slavin, 1980; Ashcraft, 2002). This was seen as students began to ask each other for help, shared out their answers, and attempted the level 4 questions on the post-test. As students learned that mistakes were acceptable and an important part of the learning process, they became more willing to try new things and look for their own errors (Bresser & Holtzman, 2018; Mighton, 2003).

Upon the completion of the unit students were asked if they learned better with Math Workshop or the traditional method of teaching and their answers were mixed. Seven students supported each method, while six students claimed they did not see a difference between their learning. While this does not match my previous experience teaching this unit, or the level of success on the post-test, it is what the students reported. This difference could come from the fact that Math Workshop is new to all of them. This sample is made of students who have learned math in the traditional way for 10-12 years. They were put into situations where they did not know if their answers were correct right away and were asked to use thinking skills they were not used to. This could have caused discomfort during the unit, explaining why students reported they did worse overall.

Overall, the Math Workshop framework increased the use of higher-level thinking skills, reduced the number of attempts to demonstrate proficiency, and had a positive effect on students’ attitudes/feelings toward mathematics. For one of the traditionally harder units, more students completed the level four work successfully than I have seen
previously, showing they did learn more deeply and retained that knowledge. They were more engaged with an abstract concept through the use of hands-on and relevant tasks. I saw a change in confidence, participation, and understanding, but if students do not feel those changes, then the framework has not affected them personally. With more exposure to this framework for learning math, these results may become more concrete.

**Recommendations**

Based on the results of this study, further research is necessary in order to make executive decisions about the widespread implementation of Math Workshop within District A. I would recommend that students be exposed to this type of teaching more often. Teachers at Monarch High School should create other units using this framework. The more that students experience a productive struggle, the more comfortable they will become. The more practice that they have with using critical thinking skills, the easier it will be to recognize their use. If we work with more students on a wider range of topics, the data gathered will be able to support future decisions more accurately. Answers to the same survey questions in this study may change after more exposure to this style of learning. Once more data is collected within Monarch High School, the updated results will be shared with the entire district. If the results remain positive, implementation in other schools will be addressed and more data collected within the different populations. The greater variety in samples, the more this data can be useful outside of District A.

The Math Workshop framework can be adjusted and the tasks can be reframed to focus on other skills. Upon implementation of this approach on another unit, more of the tasks need to focus on problem solving. Instead of choosing tasks that seem interesting and match the content, thinking skills should be taken into consideration while making
the choice. I would also recommend that the reflection piece be broadened. Students liked this part of the workshop model overall, but much of the reflection during this study was on thinking skills and feelings. With reflection on their work and what they learned, students may be able to better see the progress that they are making.

**Implementation Plan**

Doing research is not much use if the results are not shared and used to improve future practice. I plan to share my findings with the math teachers and the principal of Monarch High School in a department meeting this fall. These people are all aware of this study and are excited to hear how it went. We will use the results of this study to incorporate parts of Math Workshop into our building curriculum. We are hoping to implement some units next year and work together this fall to decide who will implement with which topics. Once we make decisions on which pieces to implement and continue to examine, we will share this with the math department of District A. If things continue to go well, our findings could be evocative to other teachers as an example of how constructivist methods might bring positive effect on their classrooms as well.

The information will also be presented to the math curriculum committee for the district. I am part of this committee, and we are in the process of updating the math curriculum, so the team is excited to hear the results and discuss implementation in other schools. If the data continues to be positive and in support of Math Workshop, the units will be shared across the district and added to the curriculum as a whole, ensuring implementation across the district. Professional development opportunities will be created to help teachers understand the framework, learn the different ways to approach it, and practice designing interesting units for their students.
Reflection on Action Research

From start to finish, this study took a lot of work. Deciding what to research was the easy part as it was introduced to me at the perfect time. Doing the literature review gave me a better understanding of constructivism and how it is reflected in the Math Workshop framework. Designing the workshop lessons was interesting and exciting, as I was able to look for activities I thought my students would find relevant. Developing the unit showed me that the workshop model does require more time and effort than just reteaching the same way I always have, but the results also proved it was worth it to keep working on it.

Several outcomes were as expected when I designed this study, including the increase in engagement and generally a better performance on the post-test. Students liked the interactive lessons, even when they were resistant to the change at first. They were willing to do what I asked and are excited to hear the results of the overall study that they were a part of. Even the students who were not actual participants in the study benefitted from this experience, and I will teach this unit with the workshop framework again in the future.

On the other hand, there were a few things that I did not expect as outcomes from this study. Students took longer than I thought to get comfortable with making mistakes. They are so used to having to be correct all the time and following specific steps, that trying different approaches and being wrong was scary. I did find that they more interesting the task, the lower this discomfort levels seemed, but I thought that they would be into the framework sooner than they actually were. Had we practiced with the
Math Workshop framework before the completion of this study, their comfort may have come faster and the data could have been stronger.

After reviewing the data, I realized the we did not do a lot with problem solving explicitly. To improve this study, I would include tasks that encouraged practicing this skill. I would look more at the thinking skills while choosing the tasks instead of upon the completion of the study. If I were to redo this study, I would choose a smaller unit to introduce the concept of Math Workshop. This would allow students time to make the adjustments to their expectations of what a math class should be, and then introduce this larger unit as the focus of the research.

I was delighted to see students enjoying mathematics. I often hear complaints that math is hard, boring or useless, and those greatly diminished during the implementation of the Math Workshop framework. Students were able to see and explain the value that the math they were learning had in the real world. Students also had fun during this study. Deep conversations began when discussing the content of some of the problems, students tried to outsmart the expected value when playing games, and I heard “I liked that today,” more than I ever have in 11 years of teaching.

On a professional level, the increase in the use of critical thinking skills was important. Just seeing students using them independently by the end of the unit encouraged me to share the results with my peers. Hearing students enjoy discussing math and sharing their deeper understandings showed that continuing to study the implementation of Math Workshop at the high school level is important.
Limitations

Every study has limitations because there is no way to control every aspect of the design, implementation and participation. There were a few limitations within the design of this study that could not be helped at the time. Maine is a very uniform state when considering race and ethnicity. Some areas are changing with the addition of refugees, but the town that houses Monarch High School is not one of them. Although my sample represented the general population of the school, it may not represent populations with different demographics.

Another design limitation was timing. Classes are only an hour long and there are many things that need to be accomplished along with the teaching. After taking attendance, meeting with the students on other topics, checking in with students who stopped in to do other work, and setting up materials, each class period had about 45 minutes of actual work time. When you leave 5-10 minutes at the end for reflection, this time drops to 35-40 minutes to get in the other three aspects of the workshop model. Longer classes may make the activities less hectic and give more time for students who felt rushed.

Timing was also an issue with the time of year in Maine. I had to work around the completion of the previous unit, winter in Maine, and the upcoming February vacation. While I planned for snow days in the original design, the closer we got to vacation the less likely they seemed to be coming. I took the opportunity to add the insurance actuary task that enriched their understanding of expected value. As a storm approached, I adjusted the schedule again to allow for only one review day and to let students take the assessment before vacation. The storm ended up being large and we
missed two days of school and left early on another one. This shortened one class period even more, and pushed the test to be taken after a week-long vacation. While weather is always going to be a concern when planning in the winter, the results of the posttest may have looked different without the vacation. In order to improve the results of this study, I would move it to a different time of year. Weather and vacations are less likely to impact the school calendar in the fall.

The final limitation that I feel was the lack of experience with the workshop model for both the students and myself. While I knew what I planned to teach, I was not sure what to expect from the students and had to make adjustments as we went. The students’ lack of experience with this model showed up in discomfort with being wrong and taking longer than expected to jump into the process. If they were introduced to the idea of Math Workshop before the implementation of this study, those kinks could have been worked out ahead of time instead of on the go.

**Recommendations for Further Research**

Research is a recursive process. The results of one study prompt the questions and ideas for the next one. This study shows the potential for Math Workshop in improving thinking skills and changing feelings about math in high school students. I recommend that other teachers try this with the same and different content. This will provide a wider array of students and levels of math to show if there is a difference overall. It will also allow us to bounce ideas off of each other to improve our tasks and make them richer for the students. The more practice that students get with the process, they more comfortable they will feel with it. That level of comfort could alter the answers to some of the questions and would give a stronger picture of the data. Different
teachers may get different results as well. The diversity in teaching styles could show other aspects to the results that have not yet been considered.

To expand this study, I would include more units in my teaching and expose more of my students. I also would do more preparation with looking at the required thinking skills involved in the tasks that I chose. This would allow me to see if problem solving can be increased when it is required to do daily tasks like the use of critical thinking did in this study.

Summary

This dissertation in practice examined multiple effects that the Math Workshop framework for teaching mathematics had on teaching high school students. Opening activities, mini lessons, worktime, and reflection were used to build connections and deepen their understanding of the mathematical content. The study demonstrated evidence that supported the literature on Math Workshop, even at the high school level.

Overall, students were more engaged with content that made abstract mathematical content more concrete. They had unprompted discussions about topics that contributed to the use of higher-level thinking skills. Students were able to demonstrate proficiency on the post-test, with many able to successfully complete the level 4 questions. Student confidence increased as they became more comfortable with making mistakes and learning the process.

The study showed that when students have more practice with higher-level thinking skills, they are more comfortable using them on the assessments. At the beginning of the study, students wanted me to give them formulas so they could work
with the thinking skills they were used to. By the end of the study, they were willing to take risks and work at the level that the teachers of District A are looking for.
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Appendix A: Marzano Taxonomy

Table A.1: Marzano’s Taxonomy

<table>
<thead>
<tr>
<th>Complexity</th>
<th>Level of Difficulty</th>
<th>Process</th>
<th>Useful Verbs, Phrases, Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6</td>
<td>Self System Thinking</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Examining Importance</td>
<td>The student can analyze how important specific knowledge is to them.</td>
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<tr>
<td></td>
<td></td>
<td>Examining Efficacy</td>
<td>The student can examine how much they believe they can improve their understanding of specific knowledge.</td>
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<td></td>
<td></td>
<td>Examining Emotional Response</td>
<td>The student can identify emotional responses associated with a piece of knowledge and determine why those associations exist.</td>
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<td></td>
<td></td>
<td>Examining Motivation</td>
<td>The student can examine their own motivation to improve their understanding or competence in specific knowledge.</td>
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<td></td>
<td>5</td>
<td>Metacognition</td>
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<td></td>
<td></td>
<td>Specifying Goals</td>
<td>The student can set specific goals relative to knowledge and develop a plan for accomplishing the goal.</td>
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<td></td>
<td>Process Monitoring</td>
<td>The student can self-monitor the process of achieving a goal.</td>
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<td>Monitoring Clarity</td>
<td>The student can determine how well they understand knowledge.</td>
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<td></td>
<td></td>
<td>Monitoring Accuracy</td>
<td>The student can determine how accurately their understanding of knowledge is and defend their judgments.</td>
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<td>4</td>
<td>Knowledge Utilization</td>
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<tr>
<td></td>
<td></td>
<td>Investigating</td>
<td>investigate; research; find out about; take a position on; what are the different features of; how &amp; why did this happen; what would have happened if The student generates a hypothesis and tests the assertions and opinions of others to test the hypothesis.</td>
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<td></td>
<td></td>
<td>Experiencing</td>
<td>experiment; generate and test; test the idea that; what would happen if; how would you test that; how would you determine if; how can this be explained? based on the experiment, what can be predicted The student generates and tests a hypothesis by conducting an experiment and collecting data.</td>
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<td></td>
<td>Problem-Solving</td>
<td>solve; how would you overcome; adapt; develop a strategy to; figure out a way to; how will you reach your goal under those conditions The student can accomplish a goal for which obstacles exist.</td>
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<td></td>
<td>Decision-Making</td>
<td>decide; select the best among the following alternatives; which among the following would be the best; what is the best way; which of these is most suitable The student can select among alternatives that initially appear to be equal and defend their choice.</td>
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<td></td>
<td>3</td>
<td>Analysis</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Specifying</td>
<td>make and defend, predict, judge, decide; what would have to happen, develop an argument for; under what conditions The student can make and defend predictions about what might happen.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Generalizing</td>
<td>what conclusions can be drawn; what inferences can be made; create a general principle, generalization or rule; trace the development of; form conclusions The student can infer new generalizations from known knowledge.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Analyzing Errors</td>
<td>identify errors or problems; identify issues or misunderstandings; assess; critique; diagnose; evaluate; edit; revise The student can identify and explain logical or factual errors in knowledge.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Classifying</td>
<td>classify, organize; sort; identify a broader category; identify different types or subcategories The student can identify, organize, and subcategorize categories to which information belongs.</td>
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<tr>
<td></td>
<td></td>
<td>Matching</td>
<td>categorize; compare &amp; contrast; differentiate; discriminate; distinguish; sort; create an analogy or metaphor The student can identify similarities and differences in knowledge.</td>
</tr>
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<td>2</td>
<td>Comprehension</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Symbolizing</td>
<td>symbolize; depict; represent; illustrate; draw; show; use models, diagram chart The student can depict critical aspects of knowledge in a pictorial or symbolic form.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Integrating</td>
<td>describe how or why; describe the key parts of; describe the effects; describe the relationship between; explain ways in which; paraphrase; summarize The student can identify the critical or essential elements of knowledge.</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>Retrieval</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Executing</td>
<td>use; demonstrate; show; make; complete; draft The student can perform procedures without significant errors.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Recalling</td>
<td>exemplify; name; list; label; state; describe; show; what, where, when The student can produce information on demand.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Recognizing</td>
<td>recognize (from a list); select from (a list); identify (from a list); determine if the following statements are true The student can determine whether provided information is accurate, incorrect, or unknown.</td>
</tr>
</tbody>
</table>
Appendix B: Consent Letter

Dear Parent or Guardian,

My name is Elizabeth Ross. In case you do not know, I am your child’s Algebra II teacher. I am currently working toward my doctorate in education and will be conducting my research study this semester in my Algebra II courses. I am conducting this study to see what impact Math Workshop has on higher-level thinking skills. Specifically, I am interested in the types of problems that students can solve and if they are willing to go for level 4s on their math assessments. I am also planning to collect some limited data from my students and am asking for your child’s participation in this research.

Your child’s participation will involve a few anonymous online surveys, practice math problems, daily journal entries, and a final assessment on the probability unit. Participation in this study will not create any extra work for the student. In fact, all students will do the exact same things in class, no matter your decision on this form. The only difference in students who do and do not participate will be whether I use their school work and math journal entries when analyzing data. Each student will be given a notebook where they will do practice problems and answer reflection questions. At the completion of the study, notebooks will be collected. They will be assessed for the purposes of meeting Measurement Topics and then all identifying information will be removed before data is analyzed for my research. The journals of students who have not given consent will not be used after Measurement Topic information has been gathered. I
will also be recording classes with a camcorder. The purpose of this is only to give me a chance to observe the lessons when I am not teaching. I am the only person who will see these videos. All information gathered in this study will be kept confidential and will only be reported out in a statistical analysis with no specific connections made to individuals. There will be no identifying information about the students or school included in the report written after the study has been completed.

Your decision whether your child participates in this study or not will not interfere with their course grade, relationship with the instructor, or the assignments that will be completed. You or your child may choose to withdraw from the study at any time without penalty. The results of the research study may be published, but your child’s name will not be used. Data collection will be kept confidential and will not be shared with anyone. I will destroy all data within one year of completing the study.

Feel free to contact me with any questions that you may have at the above email address. If you would prefer to talk on the phone, you can call Monday through Friday from 7:00 - 7:30 AM or 2:30 - 3:00 PM. Do not sign this form until your questions have been answered to your satisfaction.

Sincerely,

Elizabeth Ross
To be completed by the Parent or Guardian: You are making a decision whether or not your child’s classwork and mathematics journal entries can be used in Miss Ross’ doctoral research study during their Algebra II class. Please circle an option below.

I give consent for my child to participate in the referenced study.

I do NOT give consent for my child to participate in the referenced study.

Child’s Name:_________________________________________ Date:________________________

Parent/Guardian Name:_________________________________________

Parent/Guardian Signature:____________________________________

To be completed by the Student: You are making a decision whether or not your
classwork and mathematics journal entries can be used in Miss Ross’ doctoral research
study during their Algebra II class. Please circle an option below.

YES. I want to be in the study. I understand the study will be done during class time. I understand that, even if I say “yes” now, I can change my mind later.

NO. I do not want to be in the study.

Your Name:_________________________________________ Date:________________________

Your Signature:________________________________________
Appendix C: Pre-test

Table C.1: Pre-test

<table>
<thead>
<tr>
<th>Level</th>
<th>Question</th>
</tr>
</thead>
</table>
| 2     | 1. Circle the compound events in the following list:  
Roll a die and spin a spinner  
Pick two cards  
Pick two students from a school  
Roll two dice and add them  
Pick two boys from a class  
Pick one girl to drive |
<p>| 2     | 2. How do you find the probability of something? |
| 3     | 3. What is the probability of rolling a three and then a four on a die? |
| 3     | 4. What is the probability of picking a heart and then a spade from a deck of cards if you do not replace the first card? |
| 4     | 5. What effect does putting the card back in the deck have on #4? |
| 4     | 6. Fred has 3 black socks, 2 yellow socks, and 5 white socks. If he picks two socks, what is the chance they are the same color? How do you know this? |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7. What is expected value?</td>
</tr>
<tr>
<td>3</td>
<td>8. You flip a coin. If you get heads, you win $5. If you get tails, you lose $3. What is the expected value?</td>
</tr>
<tr>
<td>4</td>
<td>9. Why would an insurance company use expected value?</td>
</tr>
</tbody>
</table>
| 4 | 10. Which game would you play and why (use math to support your answer)?
  a. Pick a card for $2. If you get an Ace you win $50.
  b. Roll a die for $2. If you get an even number, you win $5. |
| 4 | 11. Matt says the expected value of the following game is $2. What did he do wrong? The game costs $2 to play. If you roll an even number, you win $3. If you roll an odd number, you win $1. |
| 2 | 12. What makes an event independent or dependent? |
| 2 | 13. What is the formula for conditional probability? |
| 3 | 14. There are 250 students. 100 are girls and 150 are boys. 25 girls play soccer. 35 boys play soccer. What is the probability of picking a soccer player if I only pick from the girls? |
| 3 | 15. Explain how to solve #14 using the conditional probability formula. |
| 4 | 16. Martha says that answer to #14 is 10%. What did she do wrong? |
Appendix D: Observation Form

Table D.1: Observation Form

<table>
<thead>
<tr>
<th>Date:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Time</th>
<th>Notes</th>
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</tbody>
</table>
Appendix E: Journal Questions

Day 1

1. What did you do today?
2. How did you feel about it?
3. Why do you think you felt this way?
4. Did you have any thoughts or questions?
5. I used the following thinking skills today

   Recognizing  Symbolizing  Matching  Generalizing
   Recalling    Classifying  Specifying  Investigating
   Executing    Analyzing Errors  Problem Solving  Experimenting
   Decision Making  Integrating

Answer the following on a scale of 1-5
1=Strongly Disagree
2=Disagree
3=Neutral
4=Agree
5=Strongly Agree

6. This activity was interesting:
7. This activity relevant to my life:
8. This activity was relevant to the Measurement Topics:
9. I understood what we did today:
Day 2/3/6/9/10/11

1. What did you do today?
2. How did you feel about it?
3. Why do you think you felt this way?
4. Did you have any thoughts or questions?
5. Which MT(s) did this activity connect to?
6. I used the following thinking skills today - Use your sheet. These are labeled 1-14.

Answer the following on a scale of 1-5
1=Strongly Disagree  2=Disagree  3=Neutral  4=Agree  5=Strongly Agree

7. This activity was interesting:
8. This activity relevant to my life:
9. This activity was relevant to the Measurement Topics:
10. I understood what we did today:
11. I understand today’s connection to the MTs that I will be tested on:

Day 4

1. What did you do today?
2. How did you feel about it?
3. Why do you think you felt this way?
4. Did you have any thoughts or questions?
5. Which MT(s) did this activity connect to?
6. I used the following thinking skills today - Use your sheet. These are labeled 1-14.

Answer the following on a scale of 1-5
1=Strongly Disagree  2=Disagree  3=Neutral  4=Agree  5=Strongly Agree

7. This activity was interesting:
8. This activity relevant to my life:
9. This activity was relevant to the Measurement Topics:
10. I understood what we did today:
11. I understand today’s connection to the MTs that I will be tested on:

12. What do you like about this way of learning so far?
13. What don’t you like?
14. Is this similar to your previous math classes?
15. What is different/the same?
16. What is working for you?
17. What is not working for you?
Day 5

1. Were your choices right for A, B, and C?
2. Why do you think they were or were not?
3. What would happen if I added 5 purple blocks to game 4?
4. After playing the games however you wanted how much money did you end up with?
5. How does this compare with what your homework and our class experiment told you to expect?
6. What did you do today?
7. How did you feel about it?
8. Why do you think you felt this way?
9. Did you have any thoughts or questions?
10. Which MT(s) did this activity connect to?
11. I used the following thinking skills today - Use your sheet. These are labeled 1-14.

Answer the following on a scale of 1-5
1=Strongly Disagree  2=Disagree  3=Neutral  4=Agree  5=Strongly Agree

12. This activity was interesting:
13. This activity relevant to my life:
14. This activity was relevant to the Measurement Topics:
15. I understood what we did today:
16. I understand today’s connection to the MTs that I will be tested on:

Day 8 (Over Day 7 and 8)

1. What did you do over the past two days?
2. How did you feel about it?
3. Why do you think you felt this way?
4. Did you have any thoughts or questions?
5. Which MT(s) did this activity connect to?
6. I used the following thinking skills today - Use you sheet. These are labeled 1-14.

Answer the following on a scale of 1-5
1=Strongly Disagree  2=Disagree  3=Neutral  4=Agree  5=Strongly Agree

7. This activity was interesting:
8. This activity relevant to my life:
9. This activity was relevant to the Measurement Topics:
10. I understood what we did today:
11. I understand today’s connection to the MTs that I will be tested on:
Final

1. What did you do during this unit?
2. How did you feel about it?
3. Why do you think you felt this way?
4. Did you have any thoughts or questions?
5. Which MT(s) did this unit connect to?
6. I used the following thinking skills - Use your sheet. These are labeled 1-14.

Answer the following on a scale of 1-5
1=Strongly Disagree  2=Disagree  3=Neutral  4=Agree  5=Strongly Agree

7. This unit was interesting:
8. This unit relevant to my life:
9. This unit was relevant to the Measurement Topics:
10. I understood what we did by the end of the unit:
11. I understand the unit’s connection to the MTs that I was tested on:

12. Any comments, concerns, ideas, or anything you want to share about this unit?
Appendix F: Day 1 Survey

1) Do you usually go for 4s in math? Why or why not?

2) Do you like math? Why or why not?

3) Do you think math is easy? Why or why not?

4) Please check the following if you know what it means in relation to thinking skills.

- Recognizing
- Symbolizing
- Matching
- Generalizing
- Recalling
- Classifying
- Specifying
- Investigating
- Executing
- Analyzing Errors
- Problem Solving
- Experimenting
- Decision Making
- Integrating

5) Please check the following if you have uses one of these skills this year in math.

- Recognizing
- Symbolizing
- Matching
- Generalizing
- Recalling
- Classifying
- Specifying
- Investigating
- Executing
- Analyzing Errors
- Problem Solving
- Experimenting
- Decision Making
- Integrating
Appendix G: Day 8 Survey

1) Would you try the 4 on this test? Why or why not?

2) Do you like math? Why or why not?

3) Have your feelings changed during this unit?
   Yes  No

4) Do you think math is easy? Why or why not?

5) Have your feelings changed during this unit?
   Yes  No

6) Please circle the following if you have used any of these skills THIS UNIT in math:

   Recognizing  Symbolizing  Matching  Generalizing
   Recalling    Classifying   Specifying  Investigating
   Executing    Analyzing Errors  Problem Solving  Experimenting
   Decision Making  Integrating
Appendix H: Final Survey

1) Did you try the 4 on this test? Why or why not?

2) Do you like math? Why or why not?

3) Have your feelings changed during this unit? Yes No

4) Do you think math is easy? Why or why not?

5) Have your feelings changed during this unit? Yes No

6) Do you like learning math this way versus a typical math class?
    Yes No

7) Do you think you learned better than usual math units? Worse? What helped or hurt?

8) What was your favorites lesson? Why?

9) Do you prefer learning with activities like this, or how I usually teach math?

10) Please circle the following if you have used on of these skills THIS UNIT in math:
    Recognizing Symbolizing Matching Generalizing
    Recalling Classifying Specifying Investigating
    Executing Analyzing Errors Problem Solving Experimenting
    Decision Making Integrating

11) Is there anything else that you think I should know about this unit, how you feel about it, to ways to help you?
## Table I.1: Post-test

<table>
<thead>
<tr>
<th>Level</th>
<th>Question</th>
</tr>
</thead>
</table>
| 2     | 1. Circle the compound events in the following list:  
Roll a die and pick a card  
Pick two cards with replacement  
Pick two students from a school |
|       | Roll two dice and multiply them  
Pick two students from two different classes  
Pick one boy to drive |
<p>| 2     | 2. How do you find the probability of something? |
| 3     | 3. What is the probability of rolling a five and then a five on a die? |
| 3     | 4. What is the probability of picking a heart and then a five from a deck of cards if you do not replace the first card? |
| 4     | 5. What effect does putting the card back in the deck have on #4? |
| 4     | 6. Fred has 4 black socks, 3 yellow socks, and 6 white socks. If he picks two socks, what is the chance they are the same color? How do you know this? |</p>
<table>
<thead>
<tr>
<th></th>
<th>7. What is expected value?</th>
</tr>
</thead>
<tbody>
<tr>
<td>8. You flip a coin. If you get heads, you win $3. If you get tails, you lose $5. What is the expected value?</td>
<td></td>
</tr>
<tr>
<td>9. Why would an insurance company use expected value?</td>
<td></td>
</tr>
<tr>
<td>10. Which game would you play and why (use math to support your answer)?</td>
<td></td>
</tr>
<tr>
<td>a. Pick a card for $1. If you get an Ace you win $5.</td>
<td></td>
</tr>
<tr>
<td>b. Roll a die for $2. If you get an even number, you win $5.</td>
<td></td>
</tr>
<tr>
<td>3/4 11. Matt says the expected value of the following game is $2. What did he do wrong? The game costs $3 to play. If you roll an even number, you win $3. If you roll an odd number, you lose.</td>
<td></td>
</tr>
<tr>
<td>12. Give an example of an independent event. A dependent event.</td>
<td></td>
</tr>
<tr>
<td>13. What is the formula for conditional probability?</td>
<td></td>
</tr>
<tr>
<td>14. There are 250 students. 160 are girls and 90 are boys. 25 girls play soccer. 35 boys play soccer. What is the probability of picking a soccer player if I only pick from the boys?</td>
<td></td>
</tr>
<tr>
<td>15. Explain how to solve #14 using the conditional probability formula.</td>
<td></td>
</tr>
<tr>
<td>16. Martha says that answer to #14 is 14%. What did she do wrong?</td>
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</table>
Appendix J: Measurement Topics

Table J.1: Measurement Topic 1

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
<th>Taxonomy</th>
</tr>
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<tbody>
<tr>
<td>4.0</td>
<td>In addition to the 3.0 knowledge, infers or applies beyond what is taught.</td>
<td></td>
</tr>
<tr>
<td>3.0</td>
<td>Is skilled at using the rules of probability to compute probabilities of compound events</td>
<td>Executing</td>
</tr>
<tr>
<td>2.0</td>
<td>Knows the terms: compound events, expected value</td>
<td>Recalling</td>
</tr>
<tr>
<td>1.0</td>
<td>With help….has the 2.0 content</td>
<td></td>
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</table>

Table J.2: Measurement Topic 2

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
<th>Taxonomy</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>In addition to the 3.0 knowledge, infers or applies beyond what is taught.</td>
<td></td>
</tr>
<tr>
<td>3.0</td>
<td>Is skilled at calculating expected values.</td>
<td>Executing</td>
</tr>
<tr>
<td>2.0</td>
<td>Knows the terms: compound events, expected value</td>
<td>Recalling</td>
</tr>
<tr>
<td>1.0</td>
<td>With help….has the 2.0 content</td>
<td></td>
</tr>
</tbody>
</table>

Table J.3: Measurement Topic 3

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
<th>Taxonomy</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>In addition to the 3.0 knowledge, infers or applies beyond what is taught.</td>
<td></td>
</tr>
<tr>
<td>3.0</td>
<td>Is skilled at using probability to evaluate outcomes of decisions.</td>
<td>Classifying</td>
</tr>
<tr>
<td>2.0</td>
<td>Knows the terms: compound events, expected value</td>
<td>Recalling</td>
</tr>
<tr>
<td>1.0</td>
<td>With help….has the 2.0 content</td>
<td></td>
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Table J.4: Measurement Topic 4

<table>
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<tr>
<th>Score</th>
<th>Description</th>
<th>Taxonomy</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>In addition to the 3.0 knowledge, infers or applies beyond what is taught.</td>
<td></td>
</tr>
<tr>
<td>3.0</td>
<td>Understands when two events A and B are independent.</td>
<td>Executing</td>
</tr>
<tr>
<td>2.0</td>
<td>Knows the term: conditional probability Knows the conditional probability formula</td>
<td>Recalling</td>
</tr>
<tr>
<td>1.0</td>
<td>With help….has the 2.0 content</td>
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</table>
Appendix K: Lesson Outlines

Table K.1: Lesson Outlines

<table>
<thead>
<tr>
<th>Day</th>
<th>Opener</th>
<th>Mini-Lesson</th>
<th>Worktime</th>
<th>Reflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Day 1 Survey</td>
<td>Mini-Lesson: What is the purpose of a pre-test?</td>
<td>Pre-test</td>
<td>Journal</td>
</tr>
<tr>
<td>2</td>
<td>What is the probability of selecting a boy in this class right now?</td>
<td>Go over a deck of cards, poker hands, and how to deal poker.</td>
<td>Poker Activity</td>
<td>Reflect on answers compared to reality Journal</td>
</tr>
<tr>
<td>3</td>
<td>What is the probability of selecting two girls from this class?</td>
<td>Review Simple and Compound Events and Independent and Dependent Events Quick review of the difference in finding independent and dependent probabilities</td>
<td>Worksheet for 20 minutes Go over student rules Give actual rules</td>
<td>How did you do making a rule? Journal</td>
</tr>
<tr>
<td>4</td>
<td>Given four situations, make a choice and explain why</td>
<td>Expected Value formula Do one example together</td>
<td>Find the expected value for the other three games</td>
<td>Did your choice match? Journal</td>
</tr>
<tr>
<td>5</td>
<td>Fire Insurance</td>
<td>Go over homework</td>
<td>Game Day</td>
<td>How did you do? Journal</td>
</tr>
</tbody>
</table>

143
<table>
<thead>
<tr>
<th>Day</th>
<th>Activity</th>
<th>Description</th>
<th>Person</th>
<th>Notes</th>
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<tbody>
<tr>
<td>Day 6</td>
<td>Life Insurance Problem</td>
<td>Insurance expected value process</td>
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<td>Journal</td>
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<tr>
<td>Day 7</td>
<td>Health Insurance</td>
<td>Explain Actuary Packet</td>
<td>Person 1</td>
<td>Journal</td>
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<tr>
<td>Day 8</td>
<td>N/A</td>
<td>Review Actuary Packet</td>
<td>Person 2</td>
<td>Survey</td>
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<td>Day 9</td>
<td>Pilot Table</td>
<td>Conditional Probability</td>
<td>Voting Table</td>
<td>Journal</td>
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<tr>
<td>Day 10</td>
<td>Practice a table problem</td>
<td>Tree diagram review and example</td>
<td>Packet</td>
<td>Journal</td>
</tr>
<tr>
<td>Day 11</td>
<td>Practice a tree diagram problem</td>
<td>Venn diagram review and example</td>
<td>Paper</td>
<td>Journal</td>
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<tr>
<td>Day 12</td>
<td>NONE</td>
<td>As needed</td>
<td>Practice Test</td>
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