Using a Nondispersive Wave Propagation for Measuring Dynamic Fracture Initiation Toughness of Materials: Experimental and Numerical Based Study

Ali Fahad Fahem

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USING A NONDISPERSI VE WAVE PROPAGATION FOR MEASURING DYNAMIC FRACTURE INITIATION TOUGHNESS OF MATERIALS: EXPERIMENTAL AND NUMERICAL BASED STUDY

by

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ABSTRACT

Fracture mechanics has been a subject of great interest in the engineering community for decades. During this period, fracture parameters such as Stress Intensity Factor (SIF), J-integral, and Crack-Tip Opening Displacement (CTOD) have been developed and used to characterize the fracture properties of most engineering materials under quasi-static loading condition. Usually, these properties are obtained experimentally by using standard methods such as ASTM E399, E1820 or E1920 to evaluate the stress intensity factor $K_{IC}^{static}$, elastic-plastic toughness $J_{IC}^{static}$ and crack tip opening displacement (CTOD) respectively. Conversely, most critical engineering applications are subjected to a sudden or high strain rate which could cause a dynamic fracture event. In this case, the quasi-static methods are insufficient in accurately determining the dynamic fracture parameters in materials. In light of this, three projects related to develop a new experiment method to measure dynamic initiation toughness are presented in this document.

First, a novel experimental and numerical approach is proposed to determine the dynamic fracture initiation toughness of materials based on cylindrical specimen subjected to the nondispersive torsional wave. Cylindrical tubular specimens with a full spiral crack on the surface are subjected to dynamic torsion loading using a torsional Hopkinson bar apparatus. The torsion load creates predominantly a tensile stress perpendicular to the spiral $v$-groove of the specimen that causes mode I fracture. The torque applied to the specimen and the time of fracture are measured.
Stereo Digital Image Correlation is used to measure the time at which the crack propagation initiated. A 3D format of the dynamic interaction integral method is utilized to calculate three component of dynamic stress intensity factors by using auxiliary and actually fields. Using the torque and the time of fracture as input, commercial FE package, ABAQUS, is applied to analyze an entire model of the spiral crack body and extract the dynamic fracture parameters. The result shows that the spiral crack-torsional loading configuration indeed generates a mode I fracture. Three alloyed alloys; Al 7050-T6, Al 2024-T3, and Al 6061-T6, were considered, and the results were consistent, repeatable and in good agreement with the results in the literature. For the three materials, the dynamic fracture initiation toughness \( K_a \) was higher than the corresponding quasi-static fracture toughness \( K_c \). Following are The advantages of this method: avoid the axial inertia load effects; avoid friction force effect; and reduce the wave dispersion phenomena effect.

Secondly, A solution is proposed to obtain a geometry factor for a Mode I stress intensity factor of a cylindrical specimen with spiral crack subjected to torsion. Cylindrical torsion specimens, solid and tubular, with a spiral crack on the surface, were subjected to pure torsion. The torque at fracture was measured and used as input for finite element analysis to extract the stress intensity factor at the corresponding fracture load by using a numerical solution of interaction integral method. From the fracture intensity factor obtained from the FE, and the geometry of the specimen the geometry factor for different crack depth was calculated inversely. Finally, following Benthem’s asymptotic solution approach, the geometry factor for cylindrical samples with a spiral crack on the surface is presented in a standard form. The proposed model was verified by testing a polycarbonate cylindrical specimen and comparing the existing fracture intensity value of different
materials in the open literature. The proposed formulas are in good agreement with the standard methods with a maximum difference of about 1.7%.

In overall, the results show that the spiral crack torsional loading configuration at the inclined angle, 45°, indeed generates a pure mode I fracture, and the results are consistent, repeatable and in good agreement with the results in the literature. For the dynamic fracture initiation toughness $K_{td}$ was higher than the corresponding quasi-static fracture toughness.
PREFACE

The present work addresses a new method to characterize the dynamic initiation fracture toughness of materials, Mode-I. Material types presented herein include Aluminum alloys 2024-T3, 6061-T6, 7072-T6, and Polycarbonate (PC). One-dimension wave propagation theory, FE method, and a full-field measurement (3D digital image correlation) are presented.

Chapter 1 provides a general background on the importance of dynamic fracture mechanics. Commonly used experimental approaches in the study of dynamic fracture mechanics with their current limitations are discussed. The challenges and the progress in the spiral crack and Torsional Spilt Hopkinson Bar (TSHB) are also presented and addressed concisely.

Chapter 2 presents a thorough investigation of the investigation of dynamic initiation fracture toughness of mode-I by using nondispersive wave under intermediate and high loading rate. The application of the one-dimension formula of wave propagation and high-speed photography in conjunction with stereovision digital image correlation is highlighted.

Chapter 3 presents the application of Benthem solution on the spiral crack. The geometry factor of a spiral crack under pure torsional load is developed and shown in this section. A finite element performed, and asymptotic solution of circumference crack is
converted to spiral crack, and the geometry factor is extracted. A new formula of spiral crack related to a quasi-static far field load and crack geometry is highlighted.

Chapter 4 presents a thorough study conducted with a concise summary of the present work and recommendations for future research.
# Table of Contents

**Acknowledgements** ........................................................................................................ iii

**Abstract** ........................................................................................................................... iv

**Preface** .......................................................................................................................... vii

**List of Tables** .................................................................................................................. xi

**List of Figures** ................................................................................................................ xii

**Chapter 1: Introduction** ................................................................................................. 1

1.1 **Hopkinson Bar for Dynamic Fracture Toughness Tests** .............................................. 1

1.2 **Initiated Of Dynamic Fracture Loading Test** ................................................................. 3

1.3 **Dynamic Fracture Mechanics** ..................................................................................... 6

1.4 **Objective of Present Study** .......................................................................................... 12

1.5 **List of Reference** ........................................................................................................ 13

**Chapter 2: Mode-I Dynamic Fracture Initiation Toughness Using Nondispersive Wave** ........................................................................................................................................ 18

2.1 **Abstract** .................................................................................................................... 18

2.2 **Introduction** .............................................................................................................. 19

2.3 **Material and Specimen Geometry** ............................................................................. 22

2.4 **Torsional Hopkinson Bar (THB)** ............................................................................. 23

2.5 **Experimental Data Analysis** .................................................................................... 25

2.6 **High-speed Imaging and Stereo-digital Image Correlation** ...................................... 28

2.7. **Interaction Integral for 3-D Spiral Crack** ................................................................. 31
LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Properties of material under quasi-static condition</td>
<td>23</td>
</tr>
<tr>
<td>2.2</td>
<td>VIC-3D Stereo-DIC analysis parameters</td>
<td>30</td>
</tr>
<tr>
<td>2.3</td>
<td>Calibration system parameters obtained of the stereo cameras setup used</td>
<td>31</td>
</tr>
<tr>
<td>2.4</td>
<td>Benchmark verification of FE model</td>
<td>42</td>
</tr>
<tr>
<td>2.5</td>
<td>Initiation of dynamic fracture toughness</td>
<td>52</td>
</tr>
<tr>
<td>3.1</td>
<td>Verification of the result with literature work</td>
<td>76</td>
</tr>
<tr>
<td>3.2</td>
<td>SIFs and configuration factors of solid and tube bars</td>
<td>79</td>
</tr>
<tr>
<td>3.3</td>
<td>Dimensions of a Polycarbonate torsional specimen in (mm)</td>
<td>85</td>
</tr>
<tr>
<td>3.4</td>
<td>Spiral crack Polycarbonate specimens results under pure torsion load</td>
<td>90</td>
</tr>
<tr>
<td>3.5</td>
<td>Comparison spiral crack and 3PB laboratory work</td>
<td>90</td>
</tr>
<tr>
<td>3.6</td>
<td>3PB results of Polycarbonate fracture mechanics</td>
<td>92</td>
</tr>
<tr>
<td>3.7</td>
<td>Benchmark comparison works</td>
<td>92</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

Figure 1.1 Schematic illustration of split Hopkinson pressure bar (SHPB) .................... 1

Figure 1.2 Schematic of compact comparison (CC) specimen subject to 1D compression wave ................................................................. 5

Figure 1.3 Fracture mechanics modes ................................................................... 7

Figure 1.4 Dynamic fracture properties of materials charcutiers depended on events ...... 8

Figure 1.5 Initiating propagation, and arrest fracture toughness of epoxy ................. 10

Figure 2.1 Schematic of spiral v-notch torsion specimen (SNTS) ............................ 24

Figure 2.2 Schematic of Torsional Split Hopkinson Bar (TSHB) and specimen (Dimensions in mm) ........................................................................ 26

Figure 2.3 The experimental setup used in this work: (A) High-speed cameras, (B) Specimen, (C) Signal condition and amplifier, (D) Input bar,(E) Clamping system, (F) Store bar and (G) Twist angle measurement ................................. 26

Figure 2.4 Typical shear strain plot, stored, dynamic, and static released ................ 27

Figure 2.5 Schematic of specimen under torsional load ........................................... 28

Figure 2.6 Specimen geometry and typical speckle pattern and the corresponding grayscale value ........................................................................... 30

Figure 2.7 3-D schematic of a partition of spiral crack, pointwise volume integral domain, and q-function ...................................................................... 34

Figure 2.8 Finite element model of cylindrical spiral specimen along with torsional Hopkinson bar ........................................................................... 42

Figure 2.9 Typical incident transmitted and reflected shear waves (Alu. 2024-T3) .... 44

Figure 2.10 Typical incident and transmitted torques (Alu. 6061-T6) ...................... 44

Figure 2.11 Typical stereo DIC result of a full field displacement at the crack edge- Alu. 2024-T3 ....................................................................................... 46
Figure 2.12 Typical incident and effective impulse torque waves ................................. 46
Figure 2.13 FE Result of dynamic stress intensity factors of Al. 7075-T6 (t = t_f) ...... 47
Figure 2.14 Typical full-field displacement data of Alu. 6061-T6 measured by using 3D-DIC ................................................................. 48
Figure 2.15 Typical shear strain data of Alu. 2024-T3 measured by using 3D-DIC ...... 49
Figure 2.16 Fracture initiation in Al. 2024-T3 SNTS .................................................... 49
Figure 2.17 Typical FE Result of Dynamic Stress Intensity Factors of A) Alu. 6061-T6, B) Alu. 2024-T ................................................................. 51
Figure 3.1 Solid and tube bars with a spiral crack under pure torsion load ............... 65
Figure 3.2 3-D schematic of a partition of spiral crack and volume integral domain .... 68
Figure 3.3 One slice of element configuration around crack front ................................ 74
Figure 3.4 Stability and convergence of a finite element model ............................. 76
Figure 3.5 Finite element model of a full spiral crack ................................................. 77
Figure 3.6 Similarity of f-term between the (a) circumferential and (b) spiral crack ...... 79
Figure 3.7 Configuration factor of a spiral crack on a solid bar .............................. 82
Figure 3.8 Configuration factor of a spiral crack on a tube bar ............................... 83
Figure 3.9 Actual and schematic of solid specimen with a full spiral crack .......... 84
Figure 3.10 Experimental setup of torsional specimen test ....................................... 86
Figure 3.11 Tubular polycarbonate specimen with Aluminum core ..................... 87
Figure 3.12 Three-Point Bending Experimental Setup ............................................. 89
Figure 3.13 Spiral crack and 3PB experimental results .......................................... 91
CHAPTER 1
INTRODUCTION

1.1 HOPKINSON BAR FOR DYNAMIC FRACTURE TOUGHNESS TESTS

The Split Hopkinson Pressure Bar (SHPB) apparatus was developed in 1914 by Bertram Hopkinson [1–3]. The Hopkinson bar was established more than 100 years ago based on elastic wave propagation in a bar concept. It is used to test and measure the mechanical properties of materials under a wide range of high strain rate \( (100\text{s}^{-1} \text{ to } 5000\text{s}^{-1}) \)[4]. Important information can be achieved from the SHPB and stress-strain curve with a different loading rate. In general, the pressure load is applied to one end and propagated to another end through the test specimen. The presser load generates a strain wave propagates in the longitudinal and radial direction as well, Fig. (1.1).

Figure 1.1: Schematic illustration of Split Hopkinson Pressure Bar (SHPB)
The strain gage (S.G.), which are cemented on the surface of input and output bars, were used with Elastic Modulus and one-dimension wave propagation theory to calculate the global stress-strain curve of the specimen response. The following assumptions are required for Hopkinson bar calculation[1,4–6]:

a) The maximum wave amplitude and propagates does not pass the elastic limit of the bars, even the specimen may deform and reach the plastic limit because the specimen has a smaller cross-section than the bars.

b) The waves distortion is neglected, and the global load is measured and used as the uniform load on the specimen by neglected the three dimensions effect (radial deformation), even the load on the cross section of the specimen is not uniform.

c) The small specimen is required to reduce the leak of equilibrium conditions during the loading time.

d) The friction force between the specimen and the bars are neglected. Although the investigates use lubrication to minimize the fraction force to achieve pure compression load, it is never vanishing.

The Hopkinson bar, later, was developed to another version: Tensile Spilt Hopkinson Pressure Bar (TSHPB) 1960[7] and Torsional Spilt Hopkinson Bar (TSHB) in 1971[8]. The torsional split Hopkinson bar (TSHB), which is used in this work, offers a reliable torsional impulse wave. The TSHB is a copied of the split Hopkinson pressure bar, and it is used to observe dynamic material behavior under impulse torsional load. The first torsional split Hopkinson bar appeared was develop by Lewis and Campbell [9]. They were
able to generate torsional impulse wave by using a tapered flange and epoxy to hold the stored bar.

The most advantage of the torsion wave is shown when compared to the compression wave. First, the compressive wave has geometric dispersion or a change in the loading pulse shape, and this phenomenon cannot be eliminated never, i.e., the far field load measured by strain gages is not equal to the wave that applied to the specimen. However, the torsional wave travels with an identical velocity, and there is no geometric dispersion of a propagating wave [10–12]. Additional problems in SHPB testing is a radial inertia effect which is developed due to the Poisson ratio effect. This expansion is impossible to avoid with the compression wave, but the radial inertia effect is absent from TSHB[11,13]. In other words, at lowest torsional mode, the torsional wave propagation has a constant amplitude and non-dispersion performance. Thus, the torsional wave propagates along the bar, and it can record at any position since it is free of Pochhammer-Chree oscillations [2,11,14].

1.2 INITIATED OF DYNAMIC FRACTURE LOADING TEST

Due to increasing a catastrophe of structures under the dynamic loading (impact loading, shock wave loading or stress wave), the crack’s specimens must be tested under high loading rate setup and understanding its behavior with different loading rates. In 1960, the crack specimen was tested by using the Charpy impact apparatus as a dynamic fracture test. In 1980 a Proposed standard method of dynamic fracture toughness by using Charpy impact test of pre-crack metallic materials showed up in an ASTM E24. The pioneered researchers lighted it up that the Charpy test does not have a physical relation between the
dynamic fracture toughness and hammer forces [2,15,16]. Thus, the researchers employed the Hopkinson bar apparatus to test the dynamic fracture toughness of the material; they called this setup Hopkinson Bar Loaded Fracture test[2,17–19].

The first time the Hopkinson bar loaded fracture test was used to test dynamics stress intensity factor of material in the 1970s [2,20]. Also, the dynamic fracture toughness terms were used first time by Costin and el at in 1977, The Costin-Duffy-Freund (CDF) theory showed up for fracture initiation toughness [20]. The CDF used the global stress wave load, that measure by strain gage, and quasi-static formula of SIF to estimate the fracture toughness at a fracture initiation time[21].

As demonstrated earlier, the fundamentals issues were developed in SHPB moving to Hopkins loaded fracture test, too. Furthermore, the one-dimension wave propagation assumption, stress equilibrium in a specimen cross-section, boundary contact (specimen-input and specimen-output), and pulse loading shape are affected the accurate of fracture toughness result.

The nonuniform specimen sandwich between the bars affected the one-dimension wave propagation assumption, and this assumption is not valid anymore in a specimen cross-section. For example, in the case of the CT specimen, the longitudinal wave converted to transverse wave and converted back to longitudinal wave again. The one-dimension wave in a specimen does not seem clear as in Fig. (1.2).

Then, that could affect the equilibrium stress condition on the specimen. With a 3PB simple, loss of contact in a few micrometers can be developed in a few milliseconds before the incident wave reaches the specimen that can be the root of a colossal error.
The equilibrium at the boundary area (contact area) can be valid but the equilibrium inside the specimen required three or four reflected waves to get equilibrium, and that depended on the period of specimen oscillation. Along this reflected waves time, the specimen has to stay contact with an input and output bars [22]. Thus, the stress and strain are not uniform in the specimen cross section, and most of the analysts neglected dynamic stress equilibrium inside the specimen, and they used only the dynamic load on the boundary [2].

The constant pulse shape is important to achieve the following points: keep the strain rate constant, improve equilibrium stress condition reduces the dispersive wave, and generate smooth loading function. These criteria can be achieved when the rise time be longer and longer wave period are used and let the fracture time larger than rise-time and in an equilibrium condition. However, in general, the impact speed has to be less than $2.5\,(m/s)$ [2].
After 1983, the new optical method, Digital Image Correlation (DIC) was developed and used as a direct and non-contact experimental measurement method [23]. With a modern and availability of high-speed comers [24], the DIC used to provided full-field displacement and strain data of the specimen surface that subjected to high loading rate [5,18,25]. Furthermore, for the experimental fracture test, the DIC method is used to estimate the COD and the crack initiation time. The DIC method opens the door to start over and investigate the dynamic behavior of materials and dynamic fracture toughness of material as well. Even though, the DIC provided new technology to measure the deformation and strain directly on the specimen, the non-uniform load in the specimen, and achieve equilibrium condition still a big problem with SHPB and T-SHPB.

1.3 DYNAMIC FRACTURE MECHANICS

The fracture mechanics approaches are developed during the last decade, and most of the early works were applied on a linear elastic material under quasi-static conditions[26,27]. A quick overview of fracture history is presented here for completeness. Very early, in 1920, depended on the result of Inglis [28], with a first low of thermodynamic, Griffith analyzed and developed the energy release rate of elliptical crack for linear elastic materials [29]. In 1956, Irwin modified Griffith solution by considering the elastic and plastic flow effect. Also, he introduces the factor that pointing the intensity of stress around the crack tip, and he called it the Stress Intensity Factor (SIFs) of materials [30]. In 1957, Williams introduced a polynomial series equation represent the stress distribution around the crack tip by using the concept of linear elastic fracture mechanics (LEFM). The first terms of that series are related to Irwin parameter of SIF (K). The stress
intensity factor (K) has a subscript I, II, or III depended on the fracture mode directions as following: opening mode ($K_I$), In-plane shear mode ($K_{II}$), and out of plane shear mode ($K_{III}$), Fig. (1.3)[31].

Figure 1. 3: Fracture mechanics modes

With all the above works the external load applied to the crack body assumed to be quasi-static. With a high loading rate (impact load, impulse wave, thermal shock, explosives) the materials behavior is significantly different and depended [32]. Theoretical, as most of the physical properties of materials are changed with dynamic loading conditions, the dynamic fracture mechanics is significant compared with a static.

Furthermore, the conservation of energy theory that was used for static fracture analysis requires to adjust in dynamic case [33]. The analysis of dynamic fracture mechanics is more challenged then the static and the equation of motion is used instead of
the equilibrium equation[34]. Thun, the fracture mechanics approaches are extended to time-depended to calculate the SIFs and plane strain fracture toughness of materials[35]. Unlike a quasistatic condition, the dynamic fracture properties of materials characterized depended on events as follows. Dynamic Initiation fracture toughness $K_{id} = f(T, \dot{K}_I)$ which is function of temperature $T$, and loading rate $\dot{K}_I = \left(\frac{K_{id}}{t_f}\right)$, where $K_{id}$ is dynamic initiation fracture toughness and $t_f$ is the initiation fracture time; Dynamic propagation toughness $K_{ID} = f(v, T, \dot{K}_I)$ which is function of Temperature, loading rate and crack velocity $v$; Dynamic crack arrest toughness $K_{ia} = f(v \to 0, T, \dot{K}_I)$ which is a propagation fracture toughness when the velocity of crack reaching zero, and it does not equal to dynamic initiation toughness, Fig. (1.4) [2,35].

Figure 1.4: Dynamic fracture properties of materials characterized depended on events
As mentioned earlier in section 1.1, the experimental works of dynamic fracture toughness started with Costin et al. 1977; they are establishing dynamic fracture initiation toughness of materials by using Tension Hopkinson Bar apparatus. They test Steel specimen with \(1(\text{in})\) diameter and a circumferential crack notch. One-dimension wave propagation theory was used to calculate the global stress as far-field stress applied on the specimen function of time. Also, the crack opening displacement (COD) was measured by using the optical device (cameras and light). They assumed the plastic flow is small; then they used the quasi-static formula of stress intensity factor Mode-I to calculate the dynamic initiation fracture toughness. They show that the dynamic values are consistent with the static values of fracture toughness, also they mentioned that the dynamic fracture toughness properties are more sensitive to loading rate [20].

Nishioka, 1982, et al., develop the relationship between the crack opening displacement (COD) and the tearing load to estimate dynamic fracture initiation and propagation toughness. They used a static load and three-point bending specimen. The inertia develop from the crack moving was neglected, and a quasi-static formula was used as a function of time[36].

Kalthoff et al., 1977, used a wedge-loaded double-cantilever-beam to investigated dynamic arrested, and propagation fracture toughness of materials by the generated small crack jump. They used shadow optical technique with the transmission materials, epoxy resin. The result shows that, when the crack length \(a\) is less than the critical value \(a_u\) \((a_o \leq a < a_o)\), the dynamic propagation fracture toughness is smaller than the static. When the crack length equal to the arrested crack length \(a = a_o\), the dynamic fracture arrest is
increasing more than the static. Finally, when the crack length longer than the critical values $a > a^*$, the dynamic propagation fracture toughness is larger than the static fracture toughness Fig. (1.5)[37].

![Figure 1.5: Initiating, propagation, and arrest fracture toughness of epoxy [37]](image)

In case crack body under impact loading the fracture initiation toughness is strong demonstrated by dynamic load effect (inertia effect) and the stress intensity factor proportional with the impact velocity of projector [37,38].

In 1987, Duffy, el at. tested the dynamic initiation fracture toughness of ceramic under pure mode-I and pure mode-III respectively. A circumferential-notch on a cylindrical rod specimen was tested with SHBP and TSHB respectively. The one-dimension theory
was used to calculate the far field load, and a quasi-static formula was used since the inertia load was neglected and plastic flow assumes small. The authors show that the dynamic initiation fracture toughness of Mode-I and Mode-III are higher than the static fracture toughness by 50%, i.e. \( K_{ID} = 1.5 K_f \) [39,40].

Shindo and Li, 1989, used elastodynamic approaches and Laplace transform technique in 1989. The internal and external circumferential edge crack subjected to torsional impact load was solved by [41]. Takashi, 1993, used a special arrangement of spilled Hopkinson bar to load three-point bending specimen. The dynamic stress intensity factor history evaluated by using dynamic finite element solution. The fracture initiation time indicated by using a strain gauge near the crack tip. He showed that the dynamic stress intensity factor calculated from the quasi-static equation is overestimated of true value than the dynamic stress intensity factor calculated from the dynamic finite element method [42].

Treqoning et al, 1992, used a standard 3PB specimen with impact system was used to investigate dynamic fracture toughness depended on CTOD data at the loading rate \( 1 \text{MPa} \sqrt{\text{m}/\text{s}} \leq \dot{K_1} \leq 10^6 \text{MPa} \sqrt{\text{m}/\text{s}} \). At low and intermediate rates, a quasistatic formula of SIF was used to estimate dynamic initiation toughness at initiation time. At high loading rate, the linear relation between the CTOD and SIF are assuming, and K-CTOD relations were developed. For both loading rate range, there is no change in fracture toughness related to the static test[43]. Wen et al, 1997, used a J-integral to calculate Dynamic stress intensity factor by using Laplace transform [44]. Weisbrod, 2000, used a single short beam to identify the dynamic fracture toughness of 3PB specimen. Experimental load profile with a quasi-static formula and FE model were used[45].
Jiang, 2004, Inertia model was used with the Hopkinson bar analysis. 3PB specimen fracture parameter was tested with a different loading rate as well as the inertia model[46]

1.4 OBJECTIVE OF PRESENT STUDY

The objective of the present study is to develop a new experimental method and associated theoretical formula to measure dynamic initiation toughness of metals $K_{id}$.

A fully v-notch spiral crack was used to investigate the dynamic fracture toughness of materials subjected to torsional impulse load. A torsional split Hopkinson bar apparatus was used in order to achieve this aimed. Experimental works and theoretical analysis accumulated are presented in the form of couple conference and journal articles, also presented herein forthcoming chapters as following detail:

- Characterization of the dynamic initiation fracture toughness ($K_{id}$) of materials subjected to intermediate torsional impulse load, by employing TSHB and fully surface spiral crack. A lower influence of concurrent inertia force was expected on the dynamic behavior of the materials. The finite element solution was used with the advantage of 3D-DIC and strain gauge experimental information to estimate dynamic fracture parameters.

- Develop a successful novel relationship between the far-field torsional load and local field load near the tip of a spiral crack configuration that can be used to estimate the stress intensity factor of mode-I under quasi-static load. The new formula validated through the experiment work and open literature resource. This formula with the torsional fracture load avoids the limitations associated with the classical fracture toughness experimental methods.
1.5 LIST OF REFERENCES


1990.


CHAPTER 2
MODE-I DYNAMIC FRACTURE INITIATION TOUGHNESS USING NONDISPERSSIVE WAVE

2.1 ABSTRACT

An experimental and numerical approach is proposed to determine the dynamic fracture initiation toughness of materials subjected to dynamic torsional load. A cylindrical tubular specimen with a full spiral surface crack is subjected to dynamic torsional load using a torsional Hopkinson bar apparatus. The torsion load creates predominantly tensile stress perpendicular to the spiral v-groove of the specimen, resulting in nominally Mode I conditions. The torque applied to the specimen is measured by strain gages attached to the bar and the time at which the crack propagation initiated is measured using stereo imaging and stereo digital image correlation. Using the measured torque and the time of fracture as input, a commercial FE package, ABAQUS, is utilized to analyze an entire model of the spiral crack body and numerically extract the dynamic fracture parameters. A 3D format of the dynamic interaction integral method is utilized to calculate the three components of the applied dynamic stress intensity factor. The result demonstrates that the spiral crack-torsional loading configuration indeed generates nominally Mode I conditions and can be used to study dynamic fracture initiation toughness. Three aluminum alloys; Al 7050-T6, Al 2024-T3, and Al 6061-T6, were experimentally studied. Experimental results are consistent, repeatable and in good agreement with literature data.
2.2 INTRODUCTION

Fracture mechanics has been a subject of great interest in the engineering community for decades. During this period, fracture parameters such as Stress Intensity Factors (SIFs), J-integral, Crack-Tip Opening Displacement (CTOD), Crack-Tip Opening Angle (CTOA) and the three-dimensional Crack Tip Displacement (CTD) have been developed and used to characterize the fracture properties of many engineering materials. Under quasi-static loading conditions, these properties typically are obtained experimentally by using standard methods such as ASTM E399 for the Mode I stress intensity factor, \( K_{IC}^{static} \); E1820 for elastic-plastic toughness, \( J_{IC}^{static} \); E1920 to evaluate CTOD [1–4]. These parameters are essential in the selection and judgment of materials best suited for a particular engineering design application.

Conversely, in many critical engineering applications, components are subjected to sudden or high strain loading which could result in dynamic fracture. Quasi-static methods are insufficient to accurately determine the dynamic fracture parameters in materials under extreme conditions [5]. In light of this, investigators have developed applied experimental methods to determine the dynamic fracture initiation toughness of materials subjected to extreme loading conditions.

Currently, there are two different traditional methods (with some modifications) that have been widely used to estimate the dynamic fracture toughness of materials: Charpy fixture with V-notched specimens, and the Hopkinson Pressure Bar apparatus. The Charpy test is a standard method to determine the amount of energy absorbed by a material during fracture. One of the limitations of the Charpy test is that the fracture strength can be measured only at an intermediate loading rate (10-100 /s). Also, there is no physical
relationship between the hammer load and fracture parameters. In this case, empirical equations are used to estimate fracture parameters [6] due to its simplicity. Despite its well-known theoretical weaknesses, the standard Charpy test is still popular in industry to characterize fracture toughness of materials at intermediate loading rates [7–9].

The Split Hopkinson Bar is a widely utilized method to characterize the dynamic behavior of materials at high strain rates, up to $10,000 \, \text{s}^{-1}$ [10]. Though Split Hopkinson Pressure Bars (SHPBs) are mainly employed to obtain the high strain rate constitutive response of materials [11], they have been modified and used to investigate the dynamic fracture toughness of materials [6,12,13]. Typically, the compression SHPB apparatus with three points bending and Brazilian disk specimens, have been used to obtain Mode I and mixed Mode I/II conditions at fracture, respectively. Due to the unavailability of a closed-form solution in the SHPB experiment, a quasi-static equation oftentimes is used to extract the fracture parameters. In other words, to calculate the dynamic fracture toughness, researchers usually use the plane strain quasi-static fracture mechanics equation, Eq. 2.1, by replacing the static load (P) with dynamic load (P(t)). [14,15]. In this form the technique can be used to estimate the fracture toughness of materials only if the time of fracture is sufficiently long to neglect inertia effects. To satisfy this condition and avoid transient effects, such experiments generally are performed at a low impact speed, which limits the application of this method to low rate loading conditions [6,15].

$$K_I(t) = \frac{S}{BW^{3/2}} f\left(\frac{a}{W}\right) \times \begin{cases} P \Rightarrow \text{Static} \\ P(t) \Rightarrow \text{Dynamic} \end{cases}$$

(2.1)
where $P$ is the load applied; $a$ is the crack length, $S$ is the span of the specimen, $W$ is the width of the specimen and $f \left( \frac{a}{W} \right)$ is the geometry correction factor.

Given the limitations noted above, investigators proposed an alternative method to measure the dynamic fracture toughness of materials. Truss in 1984 and Sweeney in 1985 used a cylindrical specimen with small v-notch crack inclined at 45° to its axis. They subjected the specimen to a pure torsion load. Since pure torsion load produces a principal tensile stress in a 45° plane with a spiral notch, the torsion load generates Mode-I (opening mode) conditions and thus can be used to determine the Mode-I fracture toughness of polymers [16,17]. Similarly, a torsional specimen with a full spiral v-notch crack at 45° to its axis was used by Wang and his group to determine the quasi-static fracture toughness of different materials, such as ceramics, metal, polymer, and concrete [18–20].

More recently, the potential of the technique for studying the dynamic fracture properties of materials when using a torsional Hopkinson bar has been studied [21–25]. A specific advantage of high rate torsional loading is the observation that torsional waves are non-dispersive, which allows the torsional wave to propagate along the bars without a change in its form. Due to this, in a torsional Hopkinson bar, strain gages can be placed at any position along the bar and reliable measurement can be measured. More importantly, the radial inertia does not affect the wave propagation [26–29], which makes it ideal to measure properties at low, intermediate and high strain rates while holding the dynamic equilibrium condition.

The objective of this work is to demonstrate the use of cylindrical specimen with a spiral crack subjected to dynamic torsional loading conditions to measure dynamic Mode-
I fracture properties of materials. To the authors’ knowledge, this is the first work using the spiral crack cylinder and a torsional Hopkinson bar to extract dynamic Mode-I fracture properties for materials. The method takes advantages of the non-dispersive wave propagation properties of torsional waves and the negligible axial inertia of a torsional wave at high strain rate. Dynamic experiments are performed using spiral-cracked cylindrical specimen using a Torsional Hopkinson Bar in conjunction with stereo digital image correlation (Stereo-DIC). The torque related to fracture initiation and the time at which the crack propagation initiated are measured and finite element simulations are performed to obtain the dynamic interaction integral method and extract fracture parameters.

2.3 MATERIAL AND SPECIMEN GEOMETRY

Spiral notched cylindrical torsion (SNT) specimens, with a spiral notch at 45° with respect to the longitudinal axis, are manufactured using aluminum alloys 2024-T3, 6061-T6 and 7050-T7651. These materials are common in aerospace and automobile applications. The as-received mechanical properties for all three materials are given in Table (2.1)[30]. Several 3.15mm thick tubular cylinder specimens with a 45° spiral v-notch groove were prepared from the as-received solid bars (see Figure 2.1). The spiral crack is machined on the outer surface of these specimens using a 4-axis lathe. The outer diameter, inside diameter and gage length of the specimens are 19.00 mm, 12.70mm and 59.66mm, respectively. The crack depth is 2.00 mm, and the crack ligament is 1.15 mm. The accuracy of spiral crack specimen dimensions was within ±0.01mm. The average grain size for Aluminum alloys 2024-T3, 6061-T6, and 7075-T6 used for the study is about 13.70 µm, 14.00 µm, and 31.70 µm respectively [31,32]. The total number of grains across a
1000.00\mu m thick crack ligament, in 2024-T3, 6061-T6, and 7075-T6 are 73, 72 and 32 respectively. For measuring the physical properties of materials, 8-10 grains are sufficient for representative volume element (RVE), and hence the as-manufactured specimen thickness and crack ligament are sufficient to extract continuum-level fracture parameters [33,34]. It is noted that Mode I fracture parameters are extracted from the finite element model based on the actual geometry of the specimen. Since previous studies of brittle material by Knauss and Ravi-Chandar [35,36] have shown that Mode I dynamic initiation fracture value has very small difference between plane-stress and plane-strain conditions. Chao et al. [37,38], the T-stress (higher-order term of William series) is decreasing as the loading rate increases. Thus, the affect by the three-dimensional stress state in the vicinity of the crack tip at initiation condition may not appreciably.

Table 2.1: Properties of material under quasi-static condition [30]

<table>
<thead>
<tr>
<th>Aluminum alloy</th>
<th>Density ( \rho ) (g/cc)</th>
<th>Modulus of Elasticity ( E ) (GPa)</th>
<th>Poisson's Ratio ( \nu )</th>
<th>Yield Stress ( \sigma_y ) (MPa)</th>
<th>Shear Modulus ( G ) (GPa)</th>
<th>Fracture Toughness ( K_c ) (MPa(\sqrt{m}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>2024-T3</td>
<td>2.78</td>
<td>73.10</td>
<td>0.33</td>
<td>324.00</td>
<td>28.00</td>
<td>32.00 (TL)</td>
</tr>
<tr>
<td>6061-T6</td>
<td>2.70</td>
<td>68.90</td>
<td>0.33</td>
<td>276.00</td>
<td>26.00</td>
<td>29.00 (TL)</td>
</tr>
<tr>
<td>7050-T6</td>
<td>2.81</td>
<td>71.70</td>
<td>0.33</td>
<td>503.00</td>
<td>26.90</td>
<td>27.50 (TL)</td>
</tr>
</tbody>
</table>

where TL is Orientation

2.4 **TORSIONAL HOPKINSON BAR (THB)**

Figure 2.1 illustrates a schematic of the experimental setup of THB used in this work. The full experimental setup, including the signal conditioning amplifier and the high-speed cameras, are shown in Fig. 2.3. Details of the THB and background are available in the literature [26,27,39].
Figure 2.1: Schematic of spiral V-notch torsion specimen (SNTS)
For the sake of completeness, a brief presentation is provided. The THB used in this work has a 2400.00mm long incident bar and a 2300.00mm long transmitter bar. Both bars are 25.40 mm diameter and manufactured from high-strength Grade 5 Titanium (ASTM B348). The bars are supported in a horizontal plane and are free to rotate around their central axis. An internal hexagonal groove is manufactured at the end of the incident and transmitter bar. The spiral notch specimen is sandwiched between the two bars via the hexagonal joint and a thin layer of JB-Weld epoxy. The epoxy is used around the hexagonal interface to reduce slip between the specimen and the bars. The assembly provides a reliable, consistent connection that can be used to load the samples at high loading rate.

During loading, a hydraulically driven rotary actuator, shown in Fig. 2.3, is used to apply and store shear strain in the 635.00mm portion of the incident bar located between the rotary actuator and the clamp system. The stored shear strain is suddenly released by breaking a brittle notched bolt installed in the clamping mechanism. During this time, half of the stored shear strain propagates towards the specimen through the incident bar, and half of the stored strain is released towards the clamp. Typical dynamically propagated and released strain signals are shown in Fig. 2.4. When the incident wave reached the specimen, some of the wave will transmit to the output bar through the specimen, and the rest will reflect back to the incident bar. The incident, transmitted and reflected shear strain data is acquired using strain gauges attached to the bars. Two-element 90-degree Rosette (MMF003193) strain gages are attached to both bars.

2.5 EXPERIMENTAL DATA ANALYSIS

Classical torsion theory and one-dimensional wave analysis are used to calculate
Figure 2.2: Schematic of Torsional Split Hopkinson Bar (TSHB) and specimen (Dimensions in \textit{mm})

Figure 2.3: The experimental setup used in this work: (A) High-speed cameras, (B) Specimen, (C) Signal condition and amplifier, (D) Input bar, (E) Clamping system, (F) Store bar and (G) Twist angle measurement
the torque applied to the specimen, \( T_s(t) \). The incident torque, \( T_i(t) \), is obtained as shown in Eq. (2.2):

\[
T_i(t) = \frac{GD^3 \pi}{16} \times \gamma_i(t)
\]  

(2.2)

where \( G \) is the shear modulus of the bar; \( D \) is the bar diameter and \( \gamma_i(t) \) is the incident wave. As shown in Fig. (2.5), \( T_i(t) \) is the torque at the input bar-specimen interface, and \( T_s(t) \) is the torque at the output bar-specimen interface and given as shown in Eqs. (2.3 and 2.4) [40];

\[
T_s(t) = \frac{GD^3 \pi}{16} \left[ \gamma_i(t) + \gamma_r(t) \right]
\]  

(2.3)
\[ T_2(t) = \frac{GD^3\pi}{16} \left[ \gamma_T(t) \right] \]  

(2.4)

where \( \gamma_i(t), \gamma_R(t), \gamma_T(t) \) are incident, reflected and transmitted shear strain, respectively.

In this work, the complete input bar, specimen, and output bar assembly is modeled with appropriate boundary conditions. More details regarding to the model is provided later in the in a finite element solution section.

![Figure 2.5: Schematic of specimen under torsional load](image)

2.6 HIGH-SPEED IMAGING AND STEREO-DIGITAL IMAGE CORRELATION

Full-field measurements of the specimen surface around the edge of the crack face were obtained using stereo digital image correlation (Stereo DIC or 3D-DIC). A typical speckle pattern around the crack edges with corresponding gray-scale histograms is shown in Fig. (2.6A). The gray-scale intensity depicted in Fig. (2.6B) shows a bell-shaped distribution of the intensity pattern without having saturated pixels; such a distribution is suitable for DIC measurements \[34\]. Two high-speed Photron SAX-2 cameras with two...
speckle pattern around the crack edges with corresponding gray-scale histograms is shown in Fig. (2.6A). The gray-scale intensity depicted in Fig. (2.6B) shows a bell-shaped distribution of the intensity pattern without having saturated pixels; such a distribution is suitable for DIC measurements [34]. Two high-speed Photron SAX-2 cameras with two sets of Tokina 100 mm lenses are used to record the surface deformation around the spiral crack edges at a rate of 200,000 frames per second with a resolution of 256X152 pixels², (8.11 pixel/mm). The images are processed using VIC-3D, a commercial digital image correlation software developed and distributed by Correlated Solutions, Inc. The parameters for the Stereo DIC system are shown in the Table 2.2. The calibration parameters of the stereo camera system are shown in Table 2.3 and Fig. (2.6d). Typical torque-time relationships, measured and calculated based on displacement fields on the specimen from the 3D DIC measurements, are plotted in Fig.(2.6C). The two measurements agree very well, with a maximum difference of less than 2.2%. The full field displacement $u, v$ and $w$ were also used to measure the crack edges opening displacement and to estimate the time at which fracture initiated.

In the next sections, an outline of the numerical solution methodology, that used to extract the fracture parameters, is shown. The interaction integral method used in this work is first discussed. Then, the extraction of stress intensity factor from the interaction integral method is explained. Finally, the complete finite element model is presented. In a finite element section, the geometry model, meshing technique, material properties, and boundary condition are explained, and for more details can be found in Appendix-A.
Figure 2.6: Specimen geometry and typical speckle pattern and the corresponding grayscale value

Table 2.2: VIC-3D stereo-DIC analysis parameters

<table>
<thead>
<tr>
<th>Image Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subset size (pixels X pixels)</td>
<td>25.00 X 25.00</td>
</tr>
<tr>
<td>Subset spacing (pixels)</td>
<td>5.00</td>
</tr>
<tr>
<td>Average Speckle size (Pixel X Pixel)</td>
<td>5.00 X 5.00</td>
</tr>
<tr>
<td>Interpolation</td>
<td>Optimized 8-tap</td>
</tr>
<tr>
<td>Grid Calibration Dot Spacing</td>
<td>5.00 mm</td>
</tr>
<tr>
<td>Calibration Score</td>
<td>0.02</td>
</tr>
<tr>
<td>Strain filter Size and Type</td>
<td>9.00 (Lagrange)</td>
</tr>
<tr>
<td>Software</td>
<td>Vic-3D</td>
</tr>
<tr>
<td>Stereo angle</td>
<td>≈ 14 (Degree)</td>
</tr>
<tr>
<td>Field of view (FOV)</td>
<td>21.00 mm</td>
</tr>
</tbody>
</table>
Table 2.3: Calibration system parameters obtained of the stereo cameras

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Camera 0</th>
<th>Camera 1</th>
<th>Relative position ($T_{x,y,z,a,b,y}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Result</td>
<td>SD*</td>
<td>Result</td>
<td>SD*</td>
</tr>
<tr>
<td>Center (x) (pixels)</td>
<td>496.7 02.0</td>
<td>499.19 02.0</td>
<td>$T_x =$ 167.0 (mm) .01</td>
</tr>
<tr>
<td>Center (y) (pixels)</td>
<td>511.1 03.7</td>
<td>516.47 03.8</td>
<td>$T_y =$ 01.9 (mm) .00</td>
</tr>
<tr>
<td>Focal Length (x)</td>
<td>5633 15.8</td>
<td>5628.1 15.8</td>
<td>$T_z =$ 17.4 (mm) .38</td>
</tr>
<tr>
<td>Focal Length (y)</td>
<td>5633 15.8</td>
<td>5628.5 15.8</td>
<td>$T_u =$ 00.1 (deg.) .00</td>
</tr>
<tr>
<td>Skew (deg.)</td>
<td>000.1 0.01</td>
<td>000.02 00.0</td>
<td>$T_\beta =$ 13.0 (deg.) .00</td>
</tr>
<tr>
<td>Kappa 1</td>
<td>000.1 0.00</td>
<td>000.13 00.0</td>
<td>$T_\gamma =$ 00.7 (deg.) .00</td>
</tr>
</tbody>
</table>

SD* (Standard Division)

2.7 INTERACTION INTEGRAL FOR 3-D SPIRAL CRACK

The interaction integral method is used to calculate the SIF. The J-integral method used in this work, was first developed as a measure of energy release rate for non-linear materials near the crack tip by Rice [41]. Particularly for the general dynamic case, the J-Integral formulation for non-growing crack is extended by adding the kinetic energy density ($T$) to the strain energy density ($W$) of material, as shown in Eq. (2.5.1 to 5.3) [42–44].

$$J = \lim_{\Gamma \rightarrow 0} \int_{\Gamma} \left( (W + T)n_i - \sigma_{ij} n_j \frac{\partial u_i}{\partial x_j} \right) d\Gamma$$  \hspace{1cm} (2.5.1)

Where

$$W = \int_{0}^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij}$$  \hspace{1cm} (2.5.2)
\[ T = \frac{1}{2} \rho \frac{\partial u_i}{\partial t} \frac{\partial u_i}{\partial t} \]  
(2.5.3)

For a 3-D curve (like spiral crack), the divergence theorem was applied to Eq. (2.5) to convert it from the line integral to area and volume integral.

As shown in Fig. (2.7), the segment of volume integral domain at a specific point on the crack front is extended from point \( a \) to point \( c \) through the volume center point \( b \). The general solution of \( J \)-integral of the volume segment on a spiral crack front is calculated as shown in previous studies [42,45–48],

\[ \bar{J}_{(s_{a},s_{c})} = \int_{s_{a}}^{s_{c}} [J(s)q(s)] ds = \bar{J}_1 + \bar{J}_2 + \bar{J}_3 \]  
(2.6.1)

Where:

\[ \bar{J}_1 = \int_V \left( \sigma_{ij} \frac{\partial u_i}{\partial x_j} - W \frac{\partial q_k}{\partial x_k} \right) dV \]  
(2.6.2)

\[ \bar{J}_2 = -\int_V \left( \frac{\partial W}{\partial x_k} - \sigma_{ij} \frac{\partial^2 u_i}{\partial x_j \partial x_k} \right) q_k dV \rightarrow (\text{Thermal strains effect}) \]  
(2.6.3)

\[ \bar{J}_3 = -\int_V \left( T \frac{\partial q_k}{\partial x_k} - \rho \frac{\partial^2 u_i}{\partial t \partial x_k} \frac{\partial u_i}{\partial x_k} q_k + \rho \frac{\partial u_i}{\partial t} \frac{\partial^2 u_i}{\partial t \partial x_k} q_k \right) dV \rightarrow (\text{Dynamic loading effects}) \]  
(2.6.4)

and

\[ T \frac{\partial q_k}{\partial x_k} \rightarrow \text{Flux of the Kinetic energy in the direction of the crack propagation} \]

\[ \rho \frac{\partial^2 u_i}{\partial t^2 \partial x_k} q_k \rightarrow \text{Represents the material acceleration} \]

\[ \rho \frac{\partial u_i}{\partial t} \frac{\partial^2 u_i}{\partial t \partial x_k} q_k \rightarrow \text{Identified the spatial gradient of the velocities} \]
Then, the mean value of the J-integral at point \( b \) (the middle of the volume segment) can be written as,

\[
J(s_b) = \frac{\int_a^c [J(s) q_s] ds}{\int_a^c q_s ds} = \frac{\int_{a-c}^{a} q_s ds}{A_q}
\]  

(2.7)

Where:

\( \bar{J}(s) \): A dynamic weighted average of J-integral over the crack front volume segment as shown in Fig. (2.7).

\( V \): As illustrated in Fig. (2.7), the volume enclosed by surfaces \( S^\pm, S_1, S_2, S_3, S_4 \)

\( S^\pm, S_1, S_2, S_3, S_4 \): The crack face surfaces, an upper surface, an outer surface, an inner surface, and bottom surface respectively, of the volume domain shown in Fig. (2.7).

\( \Gamma(s) \): Contour path around (s) point and perpendicular on the spiral crack front that swept along \( \Delta L \) to generate a volume integral domain \( V \).

\( q_s \): The smooth continuous weight function (unity at the surface close to the crack tip and vanish as the outer surface as shown in Fig. (2.7B))

\( u_i \): Displacement

\( \sigma_{ij}, \epsilon_{ij} \): Cauchy stress tensor and strain tensor

\( S \): Position along the crack front

\( \rho \): Material density

\( A_q \): Project area of the q-function.

For 3D elasto-dynamic problem, neglecting thermal effects, the J-integral can be written as,
\[
\bar{J}_{a-c} = \bar{J}_{act} = \bar{J}_1 + \bar{J}_3 = \int_V \left( \sigma_{ij} u_{i,1} - \frac{1}{2} \sigma_{ij} \varepsilon_{ij} - \frac{1}{2} \rho \dot{u}_i \dot{u}_i + \rho \dot{\bar{u}}_i \dot{u}_i u_{i,1} - \rho \dot{\bar{u}}_i \dot{u}_i u_{i,1} \right) q_{ij} \, dV \quad (2.8)
\]

Figure 2.7: 3-D schematic of a partition of spiral crack, pointwise volume integral domain, and q-function
On the basis of the dynamic J-integral formula, an auxiliary load field was added to the spiral’s crack front. The auxiliary J-integral, $\overline{J}_{\text{aux}}$, can be written as,

$$
\overline{J}_{\text{aux}} = \int_J \left( \sigma_{ij} u_{i,1}^{\text{aux}} - \frac{1}{2} \sigma_{ij} \varepsilon_{ij}^{\text{aux}} - \frac{1}{2} \rho \ddot{u}_{i,1}^{\text{aux}} \dddot{u}_{i,1}^{\text{aux}} + \rho \dddot{u}_{i,1}^{\text{aux}} \dddot{u}_{i,1}^{\text{aux}} - \rho \dddot{u}_{i,1}^{\text{aux}} \dddot{u}_{i,1}^{\text{aux}} \right) q_{ij} dV
$$

The auxiliary loading field Eq. (2.9) was added to the actual field load Eq. (2.8), thus the superposition J-integral around crack front can be written as,

$$
\overline{J}_{\text{Sup}}^{\text{Sup}} = \int_J \left[ \left( \sigma_{ij} + \sigma_{ij}^{\text{aux}} \right) \left( u_{i,1} + u_{i,1}^{\text{aux}} \right) - \frac{1}{2} \left( \sigma_{ij} + \sigma_{ij}^{\text{aux}} \right) \left( \varepsilon_{ij} + \varepsilon_{ij}^{\text{aux}} \right) - \frac{1}{2} \rho \left( \ddot{u}_{i,1} + \ddot{u}_{i,1}^{\text{aux}} \right) \left( \dddot{u}_{i,1} + \dddot{u}_{i,1}^{\text{aux}} \right) + \rho \left( \dddot{u}_{i,1} + \dddot{u}_{i,1}^{\text{aux}} \right) \left( u_{i,1} + u_{i,1}^{\text{aux}} \right) - q_{ij} dV \right. \\
\left. - \rho \left( \dddot{u}_{i,1} + \dddot{u}_{i,1}^{\text{aux}} \right) \left( u_{i,1} + u_{i,1}^{\text{aux}} \right) \right]
$$

Now, according to the definition, the dynamic interaction integral $\overline{J}_{\text{Int}}$ can be written as [49],

$$
\overline{J}_{\text{Int}} = \overline{J}_{\text{Sup}}^{\text{Sup}} - \overline{J} - \overline{J}_{\text{aux}}
$$

Now, substitute Eqs. (2.8, 9 and 10) into Eq. (2.11), yield to Eq.(2.12). Furthermore Eq. (2.12) can be simplified by assuming the auxiliary velocity is zero, $\dddot{u}_{i,1}^{\text{aux}} = 0$, [48]. Also, for a linear elastic system, $\dddot{u}_{i,1}^{\text{aux}} \times u_{i,1}^{\text{aux}} = \dddot{u}_{i,1} \times u_{i,1}$. Thus, the interaction integral does not depend on the material velocity. Thus, Eq. (2.12) can be simplified and written as Eq. (2.13).
As shown in Eq. (2.13), the kinetic energy term is eliminated from the dynamic interaction integral relation. It is also good to mention that, the dynamic J-integral that is available in Abaqus-implicit, use Eq. (2.13), and neglects the kinetic energy effect [50]. Vargas and Dodds [42–44], shows that for most impact responses, the inertia components of the J-integral contributes less than 0.1% of the total J and can be neglected from the analysis. In the case of torsional loading, the inertial effect is very minimal [26–29], and Eq. (2.13) can be used safely. It is worth to mention that the inaction integral method has some limitation (which is came from the J-integral) is that the method applied on a small-scale-yielding(SSY) condition[51]. In general, Eq. (2.13) can be written in three different modes that depend on the auxiliary loading field as,

\[
\bar{J}_{\text{Inter}} = \int_V \left[ \sigma_{ij} u_{i,j} + \sigma_{ij} u_{i,j}^{\text{aux}} + \sigma_{ij} u_{i,j}^{\text{aux}} + \frac{1}{2} \sigma_{ij} e_{ij} - \frac{1}{2} \sigma_{ij} e_{ij} - \frac{1}{2} \sigma_{ij} e_{ij} - \frac{1}{2} \sigma_{ij} e_{ij} \right] q_j \, dV
\] (2.12)

\[
\bar{J}_{\text{Inter}} = \int_V \left[ \sigma_{ij} u_{i,j} + \sigma_{ij} u_{i,j}^{\text{aux}} + \sigma_{ij} u_{i,j}^{\text{aux}} + \frac{1}{2} \sigma_{ij} e_{ij} - \frac{1}{2} \sigma_{ij} e_{ij} - \frac{1}{2} \sigma_{ij} e_{ij} - \frac{1}{2} \sigma_{ij} e_{ij} \right] q_j \, dV
\] (2.13)

As shown in Eq. (2.13), the kinetic energy term is eliminated from the dynamic interaction integral relation. It is also good to mention that, the dynamic J-integral that is available in Abaqus-implicit, use Eq. (2.13), and neglects the kinetic energy effect [50]. Vargas and Dodds [42–44], shows that for most impact responses, the inertia components of the J-integral contributes less than 0.1% of the total J and can be neglected from the analysis. In the case of torsional loading, the inertial effect is very minimal [26–29], and Eq. (2.13) can be used safely. It is worth to mention that the inaction integral method has some limitation (which is came from the J-integral) is that the method applied on a small-scale-yielding(SSY) condition[51]. In general, Eq. (2.13) can be written in three different modes that depend on the auxiliary loading field as,
Similar to Eq. (2.7), the result of Eq. (2.14) is justified along a 3-D segment by using a weighted function $q(s)$ as,

$$
\overline{J}_{\text{inter}}^{\alpha}(b,t) = \frac{\int \overline{J}^\alpha(s) q_s ds}{\int q_s ds} \quad (\text{no sum on } \alpha = I, II, \text{ and } III)
$$

(2.15)

Where: $\overline{J}_{\text{inter}}^{\alpha}(b,t) = [\overline{J}_{\text{inter}}^{I}(b,t), \overline{J}_{\text{inter}}^{II}(b,t), \overline{J}_{\text{inter}}^{III}(b,t)]^T$

The $\overline{J}_{\text{inter}}^{\alpha}(b,t)$ is the interaction integral of a unit virtual advance of a finite crack front segment for a specific mode at a specific point as a function of time. The discretized form of interaction integral for a three-dimensional domain is used in a finite element solution. The stresses, strains, and displacement were calculated with a standard Gauss quadrature procedure and all the integration point in each element inside the volume domain were assembled as shown in Eq. (2.16).

$$
\overline{J}_{\text{inter}} = \sum_{v} \sum_{\text{element}} \left[ \begin{array}{c}
\sigma_{ij}(t)\left(u_{ij}^{\text{true}}(t)\right)^\alpha + \left(\sigma_{ij}^{\text{true}}(t)\right)^\alpha u_{ij}(t) - \frac{1}{2} \sigma_{ij}(t)\left(\varepsilon_{ij}^{\text{true}}(t)\right)^\alpha - \\
-\frac{1}{2} \left(\sigma_{ij}^{\text{true}}(t)\right)^\alpha \varepsilon_{ij}(t)
\end{array} \right]_{q_s \det J} p \, w_p
$$

(2.16)

In this case $G.Q.P$ is a Gaussian quadrature integration point at each element, $w_p$ is respective weight function at each integration point, $[.....]_p$ are evaluated at Gauss points[52], and $\det J$ is determinant of Jacobian for 3D coordinates. The FE commercial software “ABAQUS Standard Dynamic-Implicit 2017” was used to solve Eq. (2.16).
Additional details for the numerical solution method are available in open literature, [52–54].

2.8 EXTRACTION OF STRESS INTENSITY FACTORS

In the case of isotropic linear elastic materials and infinitesimal deformation, the stress intensity factors are related to the corresponding J-integral as shown in Eq. (2.17) [49,50,55].

\[ J = \frac{1}{8\pi} K^T \cdot B^{-1} \cdot K \]  

(2.17)

Where:

\[ K = [K_I, K_{II}, K_{III}]^T : \] Stress intensity factor vector components (opening mode (Mode-I), in-plane shear mode (Mode-II), and out of plane shear mode (Mode-III) respectively).

\[ J = [J_{int}^I, J_{int}^{II}, J_{int}^{III}]^T : \] J-Integral components that related to three modes of fracture.

\[ B = [\text{Energy Factors}] : \] A second-order tensor depend on the directions and elastic properties of the material. It called the pre-logarithmic energy factor tensor [55].

The J-integral define in Eq. (2.18) is a general mixed mode relationship representing energy release rate on a crack. The integral interaction method was used again to separate the J-integral into the corresponding SIFs due to the different fracture modes. This method was introduced by Asaro and Shih [49,55].

The general equation Eq. (2.17), can be further expanded as Eq. (2.18),
\[ J = \frac{1}{8\pi} \left[ K_I B_{11}^{-1} K_I + K_{II} B_{22}^{-1} K_{II} + K_{III} B_{33}^{-1} K_{III} + \right. \\
\left. +2K_I B_{12}^{-1} K_{II} + 2K_I B_{13}^{-1} K_{III} + 2K_{II} B_{23}^{-1} K_{III} \right] \]  \hspace{1cm} (2.18)

The individual parameters can be obtained from the above relation following the procedure explained below. The procedure to obtain Mode-I is discussed in detail.

Eq. (2.18) can be rearranged by collecting like terms as,

\[ J = \frac{1}{8\pi} \left[ (K_I B_{11}^{-1} K_I + 2K_I B_{12}^{-1} K_{II} + 2K_I B_{13}^{-1} K_{III}) + \text{terms not include } K_I \right] \]  \hspace{1cm} (2.19)

Following a similar procedure, the J-integral for an auxiliary pure Mode-I \( J_{aux.} \) can be written as,

\[ J_{aux.}^I = \frac{1}{8\pi} \kappa_I^j B_{11}^{-1} \kappa_I^j \]  \hspace{1cm} (2.20)

Now, superposing the auxiliary field Eq. (2.20), onto the actual fields Eq. (2.19), the total field of J-integral can be written as,

\[ J_{total}^I = \frac{1}{8\pi} \left[ (K_I + \kappa_I) B_{11}^{-1} (K_I + \kappa_I) + 2(K_I + \kappa_I) B_{12}^{-1} K_{II} + 2(K_I + \kappa_I) B_{13}^{-1} K_{III} \right] + \text{terms not include } K_I \]  \hspace{1cm} (2.21)

The interaction integral for Mode-I can be written as Eq.(2.22) [49,54].

\[ J_{Inter.}^I = J_{total}^I - J - J_{aux.}^I = \frac{1}{8\pi} \left( (K_I + \kappa_I) B_{11}^{-1} (K_I + \kappa_I) - K_I B_{11}^{-1} K_I \right) \]
\[ - \kappa_I B_{11}^{-1} \kappa_I + 2(K_I + \kappa_I) B_{12}^{-1} (K_{II}) - \ldots. \]  \hspace{1cm} (2.22)
For homogeneous, isotropic and linear elastic materials and infinitesimal deformation, \( B_{\alpha\beta} \) is a diagonal matrix, and for plane strain condition can be written as [55],

\[
B_{11} = B_{22} = \frac{E}{8\pi(1-\nu^2)}, \quad \text{and} \quad B_{33} = \frac{E}{8\pi(1+\nu)}, \quad \text{and} \quad B_{12} = B_{13} = B_{23} = 0
\]  

(2.23)

Substituting Eq. (2.23) into Eq. (2.22), the final relation for Mode I can be written as,

\[
J^I_{\text{Inter.}} = \frac{B_{11}^{-1}}{8\pi} \left( K_I^2 + K_I \kappa_I + K_I^2 \kappa_I + K_I^2 - K_I^2 \right) = \frac{1}{4\pi} \kappa_I B_{11}^{-1} K_I
\]

(2.24)

Similar procedure can be used for Mode II and Mode III the \( J \)-integral for each mode can be obtained as,

\[
J^\alpha_{\text{Inter.}}(s) = \frac{1}{4\pi} \kappa_\alpha B_{\alpha\beta}^{-1} K_\beta \quad (\text{no sum on } \alpha = I, II, \text{and III})
\]

(2.25)

Where \( \kappa_\alpha \) is auxiliary stress intensity factors, and it can be assumed unity. Thus, in dynamic case, Eq. (2.25) can be rewritten as,

\[
K_\beta(t) = 4\pi B_{\alpha\beta} J^\alpha_{\text{int}}(t) \quad (\text{no sum on } \alpha = I, II, III)
\]

(2.26)

And the corresponding stress-intensity factor as a function of \( J \)-integral can be written as,

\[
K_I(t) = \frac{E}{10(1-\nu^2)} \sum_{i=1}^{s} J^I_{\text{Intre.}}(t)
\]

\[
K_{II}(t) = \frac{E}{10(1-\nu^2)} \sum_{i=1}^{s} J^{II}_{\text{Intre.}}(t)
\]

(2.27)

\[
K_{III}(t) = \frac{E}{10(1+\nu)} \sum_{i=1}^{s} J^{III}_{\text{Intre.}}(t)
\]
Where $J_{\text{inter}}^\omega$ are evaluated numerically from Eq. (2.15). The finite element model was generated to calculate the stress intensity factor at each point (in the middle of volume segment) along the spiral’s crack front line. At each plane, as shown in Fig. (2.8), five different volume segments around the crack front were generated to extract the fracture parameters. Since the J-integral is path-independent, the mean value at each plane is used as a final value to calculate the stress intensity factor at each middle point along the crack front.

2.9 FINITE ELEMENT ANALYSIS

The incident torque measured experimentally was used as input to the finite element model. The boundary conditions are applied in three steps. First, one end of the bar was fixed in three dimensions ($r, \theta, \text{and} z$). Second, the impulse torsional load was applied on the other end as a moment load. Finally, the crack tip and crack faces were fixed with respect to Z, Rx, and Ry[23,57]. The dynamic J-integral is calculated by using keyword *CONTOUR INTEGRAL subroutine program of a standard ABAQUS dynamic-implicit. The dynamic stress intensity factor was calculated at each node on the crack front. Since the linear elastic fracture mechanics and SSY condition were used to extract the fracture parameter, a isotropic liner elastic constitutive model was used to with a finite element model ($\sigma_{ij} = D_{ijkl} \varepsilon_{kl}$), Where $\sigma_{ij}$ is Cauchy stress tensor, $D_{ijkl}$ is a fourth order elastic tensor, and $\varepsilon_{kl}$ is a total elastic strain tensor [50].

As a benchmark, quasi-static model of a quarter spiral crack was developed, and the fracture toughness of different materials was calculated and compared with the existing value in the literature. As shown in Table 2.4, the finite element model is in excellen
agreement with available data in the literature [18].

Figure 2.8: Finite element model of cylindrical spiral specimen along with torsional Hopkinson bar

Table 2.4: Benchmark verifications of the FE model

<table>
<thead>
<tr>
<th>Materials</th>
<th>$K_c$ $(MPa\sqrt{m})$</th>
<th>Difference %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FE model</td>
<td>Ref. [18]</td>
</tr>
<tr>
<td>Alu.7475-T651</td>
<td>47.60</td>
<td>47.30</td>
</tr>
<tr>
<td>Steel A302B</td>
<td>54.20</td>
<td>54.90</td>
</tr>
<tr>
<td>Ceramic</td>
<td>02.00</td>
<td>02.10</td>
</tr>
</tbody>
</table>
2.10 RESULTS AND DISCUSSION

Typical incident, transmitted and reflected wave signals measured, in this experiment, are shown in Fig. (2.9). The shear wave travels in the Titanium-G5 bar at a velocity of 3152.00 m/s. Once the incident wave reached the specimen at 1100.00 $\mu$s, part of the incident wave was transmitted to the output bar through the SNT specimen, and the rest of incident was reflected from the interface between the input bar and the specimen. The reflected wave signal has two local maxima as shown in Fig. (2.9). The first maxima, point (1), could be associated with the reflection of the wave at the interface due to materials and geometries different (Impedance mismatch). The second maxima reflection point (2) is believed to be associated with a crack initiation in the specimen. The noticeable drop of transmitted waves at the same time with a rapid increase of the reflected wave can evidently show the crack propagation initiation instance. However, the exact time at which crack propagation initiated is challenging to specify based on only the wave signals, and high-speed imaging is used in this work as discussed later.

2.11 DYNAMIC STRESS-STATE EQUILIBRIUM VERIFICATION

For reliable dynamic fracture initiation toughness and valid Hopkinson torsional experimental results, one of the fundamental assumptions that must be held during the test is stress equilibrium at two sides of the specimen (incident-specimen, and specimen - transmitted interfaces). The torques applied on both sides of the spiral crack specimen, $T_1$ and $T_2$ are shown in Fig (2.10). The equilibrium time is about $\sim$10 $\mu$s, and the two torques are in a good agreement, indicating that the specimen was subjected to a pure torsional load.
Figure 2.9: Typical incident, transmitted and reflected shear waves (Alu. 2024-T3)

Figure 2.10: Typical incident and transmitted torques (Alu. 6061-T6)
2.12 DYNAMIC FRACTURE INITIATION TIME DETERMINATION \( (t_f) \)

In order to calculate the fracture initiation toughness accurately, identifying the fracture initiation time is critical. Stereo- digital image correlation data was used to provide more quantitative information on the crack initiation’s time. The data acquisition (DAQ) device was also used to synchronize the wave signals with the corresponding images so that the load, time and location of the crack initiation can be easily identified.

Figure (2.11) show the in-plane \( (v) \) displacements of two points across the crack front line on the surface of the specimen. It is clear that both displacements have a distinct feature at about 210\( \mu \text{s} \). The corresponding time in the incident-reflected signals is shown in Fig (2.9). It indicates that the fracture is initiated at the second maxima in the reflected signal discussed earlier, which also matches with the image at which the crack propagation becomes visible. In all three materials tested, the crack initiation time \( t_f \) is higher than the rise time \( t_o \), the fracture initiation at a constant strain rate and a dynamic equilibrium condition.

2.13 DYNAMIC FRACTURE INITIATION TOUGHNESS \( (K_{Id}) \)

As discussed earlier, using the incident torque, Eq. (2.2), Fig. (2.12), and initiation fracture time as an input, the dynamic initiation fracture toughness \( K_{Id} \) is determined numerically using dynamic energy release rate theory. Though the interest is on the opening mode, for completeness the three modes of dynamic fracture intensity factor \( K_I(t), K_{II}(t), \) and \( K_{III}(t) \) are calculated. As shown in the Fig. (2.13), the opening mode (Mode I) is at least one order magnitude higher than the other two modes. As expected, Mode II is almost zero and Mode III is within the range of the numerical error.
Figure 2.11: Typical stereo DIC result of a full field displacement at the crack edge- Alu. 2024-T3

Figure 2.12: Typical incident and effective impulse torque waves
As shown in Fig. (2.13), the Mode I stress intensity factor value is maximum and almost constant in the first quarter of the specimen, \( \approx 5.625 \) to \( 11.25 \) \( mm \) from the loading edge. In the high-speed image, it was observed that this area is the region at which the crack is initiated. Fig. (2.14) shows a full field displacement of Alu. 6061-T6 at two different time scales. Two points perpendicular to the crack tip on both sides of the crack edges were chosen to estimate the crack edges displacement (CED) and crack mouth opening displacement (CMOD) to evaluate the initiation fracture time as shown in Fig. (2.14). The displacement components values at the upper edge (black point) of the specimen denoted as \( 0 \left( U_0, V_0, W_0 \right) \), and the displacement components values at the lower edge (red point) indicated as \( 1 \left( U_1, V_1, W_1 \right) \) were used to measure the crack mouth opening displacement (CMOD), as shown in Eq. (2.28.1-3)[58,59]:

Figure 2.13: FE result of dynamic stress intensity factors of Al. 7075-T6 \( \left( t = t_f \right) \)
\[ CMOD(t) = ECD_0(t) - ECD_1(t) \]  

(2.28.1)

\[ ECD_0(t) = \sqrt{U_0^2(t) + V_0^2(t) + W_0^2(t)} \]  

(2.28.2)

\[ ECD_1(t) = \sqrt{U_1^2(t) + V_1^2(t) + W_1^2(t)} \]  

(2.28.3)

As shown in Fig. (2.15), a full field shear strain around the crack edge of Alu. 2024-T3 are measured by using 3D-DIC. A transmitted shear waive reach the maximum value of \( \sim 0.25\% \). Fig. (2.16) shows in-situ specimen image immediately after the fracture time \( (t > t_f) \). The figure shows that the crack is initiated in the middle section of the crack front. This is a significant behavior of spiral crack subjected to pure torsion. Fig. (2.16) shows a typical final fractured spiral specimen. From the figure, it is evident that, the
maximum opening displacement is developed at the middle location of the gage length. Thus, the numerical result data of dynamic stress intensity factors at the middle nodes, Fig. (2.13), are extracted, averaged, and plotted as a function of time up to the fracture initiation time $t_f$ for each material.

![Wave propagation direction](image)

**Figure 2.15**: Typical shear strain data of Al. 2024-T3 measured by using 3D-DIC

![Fracture initiation in Al. 2024-T3 SNTS](image)

**Figure 2.16**: Fracture initiation in Al. 2024-T3 SNTS
The dynamic stress intensity factors of Al 6061-T6 for Mode I, II, and III are shown in Fig. (2.17A). As shown in the figure, the Mode I appear to be developed immediately after the loading wave reached the crack, however Mode II and III seems to have a delay about 20 µs. Also, the Mode I was increasing constantly until the initiation, however the Mode II and III are almost constant. It is clear that the Mode I fracture is the driving factor. For comparison, the stress intensity factor at crack initiation for Alu. 6061-T6, is presented below.

\[
\begin{align*}
K_I(t) &= 37.80 \text{MPa}\sqrt{\text{m}} \\
K_{II}(t) &= -3.70 \text{MPa}\sqrt{\text{m}} \\
K_{III}(t) &= 3.90 \text{MPa}\sqrt{\text{m}}
\end{align*}
\]

Note that the total J-integral is 18.67 KJ/m\(^2\) and the weight of stress intensity factor of each mode comparing to the total energy release rate was evaluated as presented below.

\[
\begin{align*}
K_I &\rightarrow 93.30\% J_{\text{total}} \\
K_{II} &\rightarrow 0.80\% J_{\text{total}} \\
K_{III} &\rightarrow 5.80\% J_{\text{total}}
\end{align*}
\]

As shown above, the Mode-I has much more effect and controls the value of total energy around the crack tip and plays a great role in the crack propagation initiation. Hence, the Mode -I stress intensity factor at the fracture time can be consider as the dynamic fracture initiation toughness of the materials tested in this work.

A very similar behavior was seen in the dynamic stress intensity factor for Al 2024-T3 specimen as shown in Fig. (2.17B). As expected, the Mode I component is predominantly the driving feature for the fracture of the specimen. The dynamic initiation
fracture toughness, the stress intensity factor at the time of initiation, for Al 2024-T3 specimen is $K_I = 38.20 \pm 0.1 \text{MPa}\sqrt{m}$. This value is slightly higher than the quasi-static value of $30.20 \text{MPa}\sqrt{m}$. Furthermore, the fracture initiation time of Al 2024-T3 is longer than Alu. 6061-T6, since the specimen geometry is slightly different. Consistently, similar results are observed in Alu. 7075-T651, and without repeating the process only the final value is presented in Table 2.4.

![Figure 2.17: Typical FE result of dynamic stress intensity factors of A) Alu. 6061-T6, B) Alu. 2024-T](image)

The summary of the dynamic crack initiation toughness for all the materials considered is shown in Table 2.5. As shown in the table, the dynamic initiation fracture toughness is higher than the quasi-static value. Furthermore, the dynamic initiation toughness values obtained are in a good agreement with the literature values and agrees well with the general understanding that the dynamic fracture toughness is at least 40% higher than the quasi-static fracture toughness [60,61].
Table 2.5: Initiation of dynamic fracture toughness

<table>
<thead>
<tr>
<th>Materials</th>
<th>( t_f ) µs.</th>
<th>( K_{Id}^{Dyna.} ) MPa(\sqrt{m} )</th>
<th>( K_{Ic}^{Static} ) MPa(\sqrt{m} )</th>
<th>( \frac{K_{Ic}^{Dyna.}}{K_{Ic}^{Static}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alu.2024-T3</td>
<td>210.00</td>
<td>38.20</td>
<td>30.20 (T-L)</td>
<td>1.23</td>
</tr>
<tr>
<td>Alu.6061-T6</td>
<td>076.50</td>
<td>37.80</td>
<td>29.00 (T-L)</td>
<td>1.31</td>
</tr>
<tr>
<td>Alu.7075-T6</td>
<td>120.50</td>
<td>40.20</td>
<td>27.50 (T-L)</td>
<td>1.45</td>
</tr>
</tbody>
</table>

*Alu. 2024-T3 properties from [62]*

2.14 CONCLUSION

A new approach to estimate the dynamic fracture initiation toughness of materials without inertia effect is proposed. A cylindrical tubular specimen with a spiral crack at 45° on the surface is used to study the dynamic fracture toughness of materials. To demonstrate the method, three Aluminum alloys, 2024-T3, 6061-T6, and 7075-T6, are tested at room temperature. The specimens were subjected to dynamic torsional loading using a torsional Hopkinson bar apparatus. The incident strain signal is measured, and the torque applied on the specimen is analyzed using one-dimension wave theory. The time at which the crack propagation initiated is measured using stereo-digital image correlation. Using the torque measured and the time of crack initiation as input, a three-dimension full-size model is developed in ABAQUS to extract the fracture parameters. The 3D dynamic interaction integral method is used to calculate the stress intensity factors numerically based on energy release rate theory. The Mode I dynamic fracture toughness of all Aluminum alloys subjected to a loading rate between \( 180 \text{GPa}\sqrt{m}/\text{s} \leq \dot{K}_I \leq 494 \text{GPa}\sqrt{m}/\text{s} \) is found to be higher than the quasi-static value \( (i.e. K_{Ic}^{Dyna.} \cong (1.4 \mp 0.15) K_{Ic}^{Static}) \). In addition, the following summary can be stated about the proposed method:
Due to the advantage of the torsional wave being non-dispersive and the axial inertia is negligible, the proposed method is ideal to investigate the dynamic fracture toughness of materials at high loading rate.

Due to its unique geometry and loading condition, the proposed method can be adapted to any material and size, by which it avoids the limitation of the plane strain condition required in other standard methods.

The method can further extend to mixed mode loading by changing the angle of the spiral crack.

Since the spiral crack configuration does not have a closed form solution or analytical relation, the interaction integral formula was used to calculate the stress intensity factor numerically.

2.15 LIST OF REFERENCES


CHAPTER 3

GEOMETRY FACTORS FOR MODE I STRESS INTENSITY FACTOR
OF A CYLINDRICAL SPECIMEN WITH SPIRAL CRACK SUBJECTED
TO TORSION

3.1 ABSTRACT

A solution is proposed to obtain a geometry factor for a Mode I stress intensity factor of a cylindrical specimen with spiral crack subjected to torsion. Cylindrical torsion specimens, solid and tubular, with a spiral crack on the surface, were subjected to pure torsion. The torque at fracture was measured and used as input for finite element analysis to extract the stress intensity factor at the corresponding fracture load by using a numerical solution of interaction integral method. From the fracture intensity factor obtained from the FE, and the geometry of the specimen the geometry factor for different crack depth was calculated inversely. Finally, following Benthem’s asymptotic solution approach, the geometry factor for cylindrical samples with a spiral crack on the surface is presented in a standard form. The proposed model was verified by testing a polycarbonate cylindrical specimen and comparing the existing fracture intensity value of different materials in the open literature. The proposed formulas is in good agreement with the standard methods with a maximum difference of about 1.7
3.2 INTRODUCTION

The Stress Intensity Factor (SIF) $K$ developed by Irwin [52] relates the stress state at a crack tip with the rate of crack growth and has been effectively used to establish fracture based failure criterion. Irwin’s SIF is the first term in William’s solution and works well for linear elastic fracture mechanics (LEFM) near to the crack tip i.e., singularity dominated zone [53]. The SIF has been effectively used to describe fracture in different modes, opening mode, $K_I$, in-plane shear mode, $K_{II}$, and out-of-plane shear mode, $K_{III}$, [54].

A wide range of standard specimen geometries and test methods are available in fracture mechanics handbooks [4]. These methods have been effectively used to measure the fracture toughness, $K_{IC}$, of most engineering materials under the quasi-static condition [3]. However, the size and a plain strain condition requirement have limited the standard methods to fracture toughness of some materials and geometries. For example, the methods can’t be used to measure the fracture toughness of materials don’t have enough volume to make thick samples [5,20,55–57]. Recently, a cylindrical specimen with a spiral crack on the surface has been proposed to measure the mode I fracture toughness of materials. Note that a cylinder specimen subjected to pure torsion will generate principal stress on the surface of the specimen along its 45° from the axis. Hence, a spiral crack specimen, notched at 45° with respect to its axis, subjected to torsion is equivalent to a tensile load of equal magnitude perpendicular to the face of the crack [58] and can be used to study mode I fracture. Importantly, due to its geometry and loading condition, a cylindrical specimen with spiral crack always satisfy the plain strain condition [19]. In addition, under dynamic loading conditions, the torsional load has less inertia effect than tension or compression.
wave, and such a geometry could be ideal to measure the fracture toughness of materials under dynamic loading condition [24,25,27,59,60].

The early work on fracture toughness of materials using cylindrical specimen with a spiral crack under torsion are by Truss and Sweeney [18,58]. Truss [58], conducted torsion tests under superposed hydrostatic pressure on notched solid cylindrical specimens to investigate the fracture toughness of tough polyethylene. The specimen has a small v-notch at 45°. The stress intensity factor for a semi-elliptical surface crack in a sheet subjected to tension developed by Irwin [61] is adapted to extract the fracture toughness. Sweeney [18] used similar polymer material and used a razor blade to make a small crack at 45°. The stress distribution at the crack tip is estimated from a Nadai’s approximate equation of shear stress. Later, Wang [19–21] used cylindrical specimen with spiral crack on the surface to measure the Mode I fracture toughness of different materials. In these works, a full revolution spiral crack with constant crack depth were tested, and a finite element method was used to extract the fracture parameters.

Though a cylindrical specimen with spiral crack has many potential benefits, the unavailability of direct mathematical relation, between the fracture load (torque) and fracture parameters, makes the analysis cumbersome. As discussed above, due to unavailability of a closed form solution, the fracture parameters of materials from a cylindrical specimen with spiral crack subjected to pure torsion is extracted from finite element solution. In this work, a closed form solution, is proposed for mode I fracture toughness of cylindrical specimen with spiral crack (CSSC) subjected to torsion. The configuration factor for CSSC is developed by extending the Bentham’s asymptotic solution of cylinder specimen with circumferential crack. To the best of the authors’
knowledge, there is no such formulation in the literature. The proposed solution is verified with existing results in the literature and experimental results from standard methods.

3.3 THEORETICAL FORMULATION APPROACHES

In the standard fracture toughness methods, for finite geometry, usually, a configuration factor $Y$, is used in addition to the square root inverse singularity relation to experimentally extract the stress intensity factor and fracture toughness of materials using the well-known relation described in Eq. (3.1);

$$K = \sigma_o \sqrt{\pi c Y} \quad (3.1)$$

Where $\sigma_o$ is far-field stress; $c$ is crack depth $Y$ is configuration factor and $K$ is the stress intensity factor. A configuration factor for a specific geometry can be determined numerically using techniques such as boundary collection method, boundary stress correction method, Finite Element Methods, Least Square Method, and Asymptotic Approximation [61]. The proposed problem, a cylindrical specimen with spiral crack subjected to pure torsion as shown in Fig. (3.1), is similar to Bentham’s problem of circumferential crack with far-field torsional load. The main different is that the Bentham problem is circumference crack and results Mode III fracture, and the proposed problem is a spiral crack and generate Mode I fracture [62]. Two different geometries, solid and tubular cylindrical specimen are considered as shown in Fig (3.1), where $2h$ is the spiral pitch $\left(2\pi r_o\right)$, $r_o$ is the external radius, $a$ is the crack ligament, $c$ is the crack depth, $t$ is the tube thickness, $r_i$ is the internal radius, $\alpha = 45^\circ$ is the spiral angle, and $\beta_{sp}$ revolution angle of the spiral crack around the bar.
In general, for mode-I fracture, the stress intensity factor $K_I$ can be calculated from the following relation Eq.(3.2):

$$K_I = \tau_{\text{max}} \sqrt{\pi (a, c, \psi)} Y(a, c, \psi)$$  \hspace{1cm} (3.2)

Where $a$, $c$, and $\psi$ are the characteristic dimensions that can be measured. The $\delta$ and $\ell$ are crack ligament and crack depth respectively, and $\psi$ either bar radius or tube thickness.

i.e. $\psi = \begin{cases} r_o & \text{outer radius for solid specimen} \\ t & \text{wall thickness for tubular specimen} \end{cases}$
And, Y is configuration factor, depends on the geometry and far-field loading condition, \( \tau_{max} \) is maximum shear stress at the section far from the crack and can be calculated as Eq.(3.3):

\[
\tau_{max} = \frac{T r_o}{J}
\]  

(3.3)

Where \( T \) is the applied torque, and \( J \) is the polar moment of inertia and can be calculated as \( J = \frac{\pi}{32} D_o^4 \) for solid and \( J = \frac{\pi}{32} \left( D_o^4 - D_i^4 \right) \) for tubular samples [63]. By rearranging Eq. (3.2), the configuration factor can be written as Eq.(3.4):

\[
Y(a, c, \psi) = \frac{K_I}{\tau_{max} \sqrt{\pi(a, c, \psi)}}
\]  

(3.4)

For any given \( \tau_{max} \), a finite element method can be used to calculate the stress intensity factor \( K_I \) of a cylindrical specimen with spiral crack at different aspect ratio \( (c/r_o) \) and \( (c/l) \) for solid and tube bars respectively. The finite element method was used to perform the interaction integral method that was used to extract the stress intensity factor. Thus, the result along Eq. (3.4) can be used to calculate the configuration factor \( Y(a, c, \psi) \).

3.4 NUMERICAL SOLUTION METHODOLOGY

A numerical solution and procedure for extracting fracture parameters such as Stress Intensity Factor (SIF), Energy Release Rate (G), and J-Integral are available in open literatures with extensive details [35–41]. However, for a completeness, a brief description
of the method followed in this paper is presented in the following section. First, the details of interaction integral skim are presented. Then, the extraction of stress intensity factor based on interaction integral method is explained. Finally, the finite element modeling is highlighted.

3.4.1 Interaction Integral for 3-D Spiral Crack

The J-integral method, used in this work, was developed as a measure of elastic-plastic fracture parameter at the crack tip by Rice [42]. The J-integral was formulated based on the domain energy integral methodology and can be used for linear and nonlinear elastic materials and for static and dynamic loading conditions as well [3]. Furthermore, the J-Integral can be related to strain energy release rate (G), in a special case, for an isotropic linear elastic material and infinitesimal deformation.

For a special case of a 3-D curve (like Spiral line), the body force and thermal load are small, and can be neglected. Furthermore, assume the crack faces are traction free, crack-front curvature was neglected and only a mechanical quasi-static loading condition are applied, Fig. (3.2). The 3D J-Integral of volume segment domain on a spiral crack front can be expressed as Eq. (3.5), similar to Eq. (2.5.1) in chapter 2, [43–45].

$$\overline{J}(s) = \int_{V} \left( \sigma_{ij} u_{j,i} - W \delta_{ij} \right) q_{s} dV \quad (3.5)$$

The $\overline{J}(s)$ is not constant along the volume segment $\Delta L$, thus the approximation of pointwise integral at point $(s = b)$ is adjusted by divided Eq. (3.5) on the weight function as described in Eq. (3.6). The details of weight function $q(s)$ is available in a literature, for example, you can see Vargas and Dodds’ works [43].
\[ J(b) = \frac{J(s)}{q(s) ds} \]  

Figure 3.2: 3-D schematic of a partition of spiral crack and volume integral domain

The interaction integral method, which founded on the J-integral formula[23], was used to estimate the stress intensity factor for each mode individually [44] as briefly discussed below. An auxiliary load field was added to the spiral’s crack front volume segment as Eq. (3.7):

\[ \overline{J}^{aux}(s) = \int_V \left[ \sigma_{ij}^{aux} u_{j,i}^{aux} - \left( \frac{1}{2} \sigma_{jk}^{aux} \varepsilon_{jk}^{aux} \right) \delta_{ii} \right] q_j dV \]  

The auxiliary load field Eq. (3.7) was added to the actual field load Eq. (3.5), thus the total J-integral in a volume segment around crack front is written as Eq. (3.8),
\[
\bar{J}^{\text{total}}(s) = \int \left[ (\sigma_{ij} + \sigma_{ij}^{\text{aux}})(u_{j,1} + u_{j,1}^{\text{aux}}) - \frac{1}{2}(\sigma_{jk} + \sigma_{jk}^{\text{aux}})(\varepsilon_{jk} + \varepsilon_{jk}^{\text{aux}}) \delta_{ij} \right] q, dV \tag{3.8}
\]

According to the definition, the interaction integral \( \bar{J}_{\text{inter}}(s) \) of a volume segment can be written as Eq. (3.9), [36]:

\[
\bar{J}_{\text{inter}}(s) = \bar{J}^{\text{total}}(s) - \bar{J}(s) - \bar{J}^{\text{aux}}(s) \tag{3.9}
\]

Thus, Eq. (3.9) can be further written as,

\[
\bar{J}_{\text{inter}}(s) = \int \left[ \sigma_{ij} u_{j,1} + \sigma_{ij}^{\text{aux}} u_{j,1} - \frac{1}{2}(\sigma_{jk} \varepsilon_{jk}^{\text{aux}} + \sigma_{jk}^{\text{aux}} \varepsilon_{jk}) \delta_{ij} \right] q, dV \tag{3.10}
\]

Eq. (3.10) can be written in three different modes that depend on the direction of the auxiliary loading as,

\[
\bar{J}_{\text{inter}}^{\alpha}(s) = \int \left[ \sigma_{ij}^a (u_{j,1}^a + (\sigma_{ij}^{\text{aux}})^a u_{j,1}) - \frac{1}{2}(\sigma_{jk}^{\alpha} \varepsilon_{jk}^{\text{aux}} + \sigma_{jk}^{\text{aux}} \varepsilon_{jk}) \delta_{ij} \right] q^a, dV \tag{3.11}
\]

Eq.(3.11) applied for different volumes domain, and the mean value of interaction integral \( \bar{J}_{\text{int}}^{\alpha}(s) \) is divided on the weighted function \( q(s) \) to get the interaction integral of pointwise point \( b \) inside the volume segment domain [36,43,44,49,64]:

\[
J_{\text{inter}}^{\alpha}(b) = \bar{J}_{\text{inter}}^{\alpha}(s) \left. \right|_{\text{Ad.}} \quad \text{(no sum on } \alpha = I, II, \text{ and III)} \tag{3.12}
\]

Where \( J_{\text{inter}}^{\alpha}(b) = \left[ J_{\text{inter}}^{I}(b), J_{\text{inter}}^{II}(b), J_{\text{inter}}^{III}(b) \right]^T \)

Where \( J_{\text{inter}}^{\alpha}(b) \) is the interaction integral for a unit virtual advance of a finite crack front segment for specific mode. The discretized form of J-Integral for three-dimensional domain that can be used with a finite element solution is described in Eq. (3.13). The
standard Gauss quadrature procedures including all the integration point in a volume domain were assembled as:

\[
\left(J_{\text{int.}}(s)\right)_\alpha^p = \sum_{\text{all elements}} \sum_{p=1}^{m} \left[ \left( \sigma_{ij} \left( u_{\text{aux}}^{\text{aux}} \right)_{i,j} \right)_\alpha^p + \left( \sigma_{ij}^{\text{aux}} \right)_\alpha^p u_{j,i} - \sigma_{ij}^{\text{aux}} \left( e_{\text{aux}}^{\text{aux}} \right)_\alpha^p \delta_{ij} \right] q_{ij}^p \det J \right] w_p \quad (3.13)
\]

\( m \) is Gaussian integration point at each element, in this case. \( w_p \) is the respective weight function at each integration point \([......]_p\) and are evaluated at Gauss points[35], and \( \det J \) is determinant of Jacobian for 3D coordinates.

3.4.2 Extraction of Stress Intensity Factors

The stress intensity factors \( K_I, K_{II} \) and \( K_{III} \) in LEFM characterize the effect of a far field load on the stress and strain fields at the crack tip. Furthermore, in this work, for an isotropic linear elastic materials and in infinitesimal deformation, stress intensity factors is related to the energy release rate as[36,49,65],

\[
J = \frac{1}{8\pi} K^T \cdot B^{-1} \cdot K \quad (3.14)
\]

Where:

\[
K = \begin{bmatrix} K_I, K_{II}, K_{III} \end{bmatrix}^T : \quad \text{Stress intensity vectors components (opining mode (Mode-I), in-plane shear mode (Mode-II), and out of plane shear mode (Mode-III) respectively).}
\]

\[
J = \begin{bmatrix} J_{III}^{I}, J_{III}^{II}, J_{III}^{III} \end{bmatrix}^T : \quad \text{J-Integral components that related to three modes of fracture.}
\]

\[
B = \begin{bmatrix} \text{Energy Factors} \end{bmatrix} : \quad \text{A second-order tensor depended on the directions and elastic properties of material. It called the pre-logarithmic energy factor tensor[49].}
\]
The J-integral define in Eq. (3.14) is a total energy release rate for the crack in a mixed mode and general materials. The interaction integral method was used again in separating the individual interaction integral to the related SIFs caused by different fracture modes. This method was introduced by Asaro and Shih [36,49] and is adapted in this work. The general relation Eq. (3.14), is further expanded as,

$$J = \frac{1}{8\pi} \left[ K_I B_{i1}^{-1} K_I + K_{II} B_{22}^{-1} K_{II} + K_{III} B_{33}^{-1} K_{III} + 2K_I B_{i2}^{-1} K_{II} + 2K_I B_{i3}^{-1} K_{III} + 2K_{II} B_{23}^{-1} K_{III} \right]$$

(3.15)

The procedure to obtain of the Mode-I parameter is shown below. By collecting terms that has mode I, Eq. (3.15) can further rearranged as,

$$J = \frac{1}{8\pi} \left[ (K_I B_{i1}^{-1} K_I + 2K_I B_{i2}^{-1} K_{II} + 2K_I B_{i3}^{-1} K_{III}) + \text{terms not include } K_I \right]$$

(3.16)

Following similar procedure, let the crack model subjected to pure mode-I auxiliary load. then, rewrite Eq. (3.14) for an auxiliary pure Mode-I J-integral, $J^I_{aux.}$, and related auxiliary SIF as:

$$J^I_{aux.} = \frac{1}{8\pi} \kappa_I B_{i1}^{-1} \kappa_I$$

(3.17)

Now, by superposing the auxiliary field Eq. (3.17), onto the actual fields Eq. (3.16), the total field of J-integral can be written as,

$$J^I_{total} = \frac{1}{8\pi} \left[ ((K_I + \kappa_I) B_{i1}^{-1} (K_I + \kappa_I) + 2(K_I + \kappa_I) B_{i2}^{-1} K_{II} + 2(K_I + \kappa_I) B_{i3}^{-1} K_{III}) + \right. \left. \text{terms not include } K_I \right]$$

(3.18)

And, the interaction integral for mode I can be written as[36,45],

$$J^I_{inter.} = J^I_{total} - J - J^I_{aux.} = \frac{1}{8\pi} \left[ (K_I + \kappa_I) B_{i1}^{-1} (K_I + \kappa_I) - K_I B_{11}^{-1} K_I - \kappa_I B_{i1}^{-1} \kappa_I + \right. \left. 2(K_I + \kappa_I) B_{i2}^{-1} K_{II} - \right. \left. \text{terms not include } K_I \right]$$

(3.19)
For a special case of homogeneous, isotropic and linear elastic materials and in infinitesimal deformation, $B_{\alpha\beta}$ is diagonal matrix, and for plane strain condition can be written as [49],

$$B_{11} = B_{22} = \frac{E}{8\pi (1-\nu^2)}, \quad \text{and} \quad B_{33} = \frac{E}{8\pi (1+\nu)}, \quad \text{and} \quad B_{13} = B_{13} = B_{23} = 0$$  \hspace{1cm} (3.20)

Substituting the $B_{\alpha\beta}$ relation given in Eq. (3.20) into Eq. (3.19) leads to,

$$J^I_{\text{inter.}} = \frac{B_{11}^{-1}}{8\pi} \left( K_i^2 + K_i K_I + K_I K_i + K_i^2 - K_i^2 - K_i^2 \right) = \frac{1}{4\pi} \kappa_i B_{11}^{-1} K_i$$  \hspace{1cm} (3.21)

Similar procedure can be followed to obtain relation for Mode II and Mode III conditions as,

$$J^{\alpha}_{\text{inter.}}(s) = \frac{1}{4\pi} \kappa_\alpha B_{\alpha\beta}^{-1} K_\beta \quad (\text{no sum on } \alpha = I, II, \text{ and } III)$$  \hspace{1cm} (3.22)

where $\kappa_\alpha$ is auxiliary stress intensity factors, and it can be assumed unity. The solution of interaction integral Eq. (3.19) shows a liner system of three modes of fracture and they lead to Eq. (3.22). Equation (3.22) can be rewritten as,

$$K_\beta = 4\pi B_{\alpha\beta} J^{\alpha}_{\text{inter.}} \quad (\text{no sum on } \alpha = I, II, III)$$  \hspace{1cm} (3.23)

Where $J^{\alpha}_{\text{inter.}}(s)$ are evaluated numerically from Eq. (3.13). The finite element model was generated to calculate the stress intensity at each pointwise (nodes) along the spiral’s crack front line. At each location (nodes), the five different paths around the crack front were generated to extract the fracture parameter as shown in Fig. (3.3F). Since, the $J$-integral is path-independent, the mean value at each node is used as a final value of stress intensity factor at each point along in a three-dimensional crack front [72].
The SIF \( (K_i) \) can be evaluated by applying Eq. (3.20) into Eq. (3.23) and can be written as:

\[
K_i = \frac{E}{10(1-\nu^2)} \sum_{i=1}^{5} J_{intre}^i; \quad K_{II} = \frac{E}{10(1-\nu^2)} \sum_{i=1}^{5} J_{intre}^{II}; \quad K_{III} = \frac{E}{10(1+\nu)} \sum_{i=1}^{5} J_{intre}^{III}.
\]

(3.24)

3.4.3 Finite Element Modeling

A commercial finite element software, ABAQUS-SAE, is used to perform and calculate the stress intensity factors \( K_i, K_{II}, \) and \( K_{III} \) from Eq. (3.24) with the assumption of linear elastic fracture mechanics (LEFM). Isoperimetric Hexahedron element with a quadratic function is used to perform a numerical solution of J-Integral given in Eqs. (3.13), which is recommended for a three-dimensional fracture mechanics simulation. The structural elements C3D20R are used to mesh the solid model and solution processing under static loading condition. The element has totally 20 nodes, 8 nodes at corners and 12 mid-side nodes as shown in Fig. (3.3A). At the crack tip, the first ring, the three nodes of the element collapse down to the same points to generate wedge element as shown in Fig. (3.3B). These nodes at the crack tip are tied together. Thus, in this shape, the element called singular elastic wedge element since it contains the singularity term of \( 1/\sqrt{r} \) \cite{65,66}. The brick elements are regenerated to create spider-web configuration with a great refinement around the crack front. The advantage of spider mesh is to smooth transition performance and concentrated mesh that be used to evaluate a J-Integral. The total elements around the crack tip are 2024 elements, where 46 elements in a radial direction and 44 elements in a circumferential direction as shown in Fig. (3.3C and 3D). The remote area from the tip is meshed with coarser mesh elements as shown in Fig. (3.3E). That area has
a total of 132 elements. The total number of elements in one planar is 2156. In Overall 362,208 elements is used to mesh wholly the model and most of the elements concentrated at the middle part of the model Fig. (3.4D-F).

Figure 3.3: One slice of element configuration around crack front

Full details of a 3D model of a cylindrical specimen with a spiral crack under torsional loading was created as shown in Fig. (3.4). In this configuration, there is no symmetry along the crack section, and hence a full-scale of the specimen is modeled as shown in Fig. (3.4). The geometry has a gauge diameter of 20.3 mm and a spiral crack pitch is 63.75 mm.
Also, there are 5mm space between the notch ends and the stress-concentration zone at each end. The spiral crack was generated by a fall revolve shell around the cylinder at 45°. Reference points on the center of each face of the specimen, D (loaded point) and E (fixed point), are created on which the boundary conditions (BC) and the loads are applied. On the fixed point, at $z = 0$, translation and rotation motions are set to be zero ($u = v = w = R_x = R_y = R_z = 0$), and at the loading surface, $z = L$, a uniform torsion load is applied ($T_z = T; and u = v = R_x = R_y = 0$) [67–69]. It is important to remark that; the direction of the torsion load should follow the direction of the spiral crack to generate the opening mode crack. Additional boundary conditions are required at the crack tip, crack face and a center line of the bar to avoid any bending or buckling effect and to reduce nonlinear error. The model divided to 168 planar. The 82-planar concentrated at the middle section of the crack ($z = 60 \rightarrow 80 \text{mm}$) Fig. (3.4).

Mesh sensitivity, convergence, and stability of the finite element model are tested by varying the number of elements [37]. The stress intensity factors and J-integral are calculated numerically and compared with literature value, as shown in Fig. (3.5), [19]. The result indicates that, a minimum $4 \times 10^4$ elements in the middle section of the model as shown in Fig. (3.4B and D), is required to get stable performance and accurate result of fracture parameters.

Different materials, different far-field loading and different dimensions. The finite element model is shown to be within less than 6% different. Once the finite element parameters are verified, and the mesh sensitivity analysis is performed, the finite element analysis of the spiral crack is repeated sufficient times for different configuration factors $(c/r_o)$ and $(c/t)$ for solid and tube bars respectively.
Figure 3.4: Stability and convergence of a finite element model

Table 3.1: Verification of the result with literature work [19]

<table>
<thead>
<tr>
<th>Materials</th>
<th>T (Nm)</th>
<th>Radius (mm)</th>
<th>Crack Depth (mm)</th>
<th>$K_I$ (FEM) $(MPa\sqrt{m})$</th>
<th>$K_I$ [19] $(MPa\sqrt{m})$</th>
<th>different %</th>
<th>$K_{IC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alu.7075</td>
<td>720.000</td>
<td>12.7</td>
<td>5.08</td>
<td>23.4</td>
<td>20.1</td>
<td>5.49</td>
<td>No</td>
</tr>
<tr>
<td>Alu.7075</td>
<td>1279.23</td>
<td>12.7</td>
<td>7.62</td>
<td>47.7</td>
<td>47.3</td>
<td>1.00</td>
<td>Yes</td>
</tr>
<tr>
<td>Steel-A302B</td>
<td>816.570</td>
<td>10.15</td>
<td>7.62</td>
<td>56.9</td>
<td>55.2</td>
<td>3.03</td>
<td>Yes</td>
</tr>
<tr>
<td>Ceramic</td>
<td>054.000</td>
<td>8.5</td>
<td>5.08</td>
<td>2.08</td>
<td>2.2</td>
<td>5.25</td>
<td>Yes</td>
</tr>
</tbody>
</table>

In all these repeated cases, the far field load and the material properties were kept constant, and only the crack depth, $c$, was varying each time. The SIFs are extracted as a function of aspect ratio (crack depth/effective thickness) and used to obtain the configuration factor.
Figure 3.5: Finite element model of a full spiral crack
Moreover, the finite element model is verified and compared with experimental results from the open literature. As shown in a Table (3.1).

3.5 GEOMETRY CONFIGURATION FACTORS

By substituting Eq. (3.3) into Eq. (3.4), the configuration factor for both solid and tubular specimens can be written as shown in Eq. (3.25);

\[ Y^s(a, c, r_o) = \frac{r_o^3 K^s_{II}}{2T} \sqrt{\frac{\pi}{(a, c, r_o)}} \]  \hspace{1cm} (3.25a)

\[ Y^t(a, c, t) = \left( r_o^3 - r_i^4 \right) \frac{K^t_{II}}{2T} \sqrt{\frac{\pi}{(a, c, t)}} \]  \hspace{1cm} (3.25b)

Where, \( T \) is the torque applied, \( K^s_{II} \) and \( K^t_{II} \) are the stress intensity factor of a solid and tube bars respectively. These factors can be calculated numerically as discussed in the earlier section. The remaining in Eq. (3.25), are geometry parameters. Using Eq. (3.25) with the numerical result of SIFs, the geometry factor can be calculated. The results of Mode I stress intensity factor and configuration factor as a function of aspect ratio, for the solid and tubular specimen, are shown in Table (3.2).

The configuration factor Eq. (3.25), further simplified to a standard form similar to Bentham configuration factors. Bentham’s configuration factor for circumferential crack with far-field torsion loading is expressed in two terms, \( f \) and \( G \) terms. The \( f \)-term depends mainly on the relation between the crack ligament and bar radius and it can be written in three different forms as shown in Eq. (3.26).
Table 3.2: SIFs and configuration factors of solid and tube bars

<table>
<thead>
<tr>
<th>Repeat No.</th>
<th>( \frac{c}{r} )</th>
<th>( K'_1 ) ( MPa \sqrt{m} )</th>
<th>( Y'(a,c,r_o) ) Eq. (5a)</th>
<th>Repeat No.</th>
<th>( \frac{c}{t} )</th>
<th>( K'_1 ) ( MPa \sqrt{m} )</th>
<th>( Y'(a,c,t) ) Eq. (5b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>1.9107</td>
<td>0.899</td>
<td>1</td>
<td>0.12</td>
<td>1.5842</td>
<td>0.96939</td>
</tr>
<tr>
<td>2</td>
<td>0.15</td>
<td>2.354</td>
<td>0.9055</td>
<td>2</td>
<td>0.16</td>
<td>1.7788</td>
<td>0.94264</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>2.6454</td>
<td>0.8801</td>
<td>3</td>
<td>0.24</td>
<td>2.2542</td>
<td>0.97047</td>
</tr>
<tr>
<td>4</td>
<td>0.31</td>
<td>3.2014</td>
<td>0.8487</td>
<td>4</td>
<td>0.31</td>
<td>2.6052</td>
<td>0.97622</td>
</tr>
<tr>
<td>5</td>
<td>0.4</td>
<td>3.5062</td>
<td>0.8248</td>
<td>5</td>
<td>0.47</td>
<td>3.3121</td>
<td>1.01336</td>
</tr>
<tr>
<td>6</td>
<td>0.6</td>
<td>4.0454</td>
<td>0.777</td>
<td>6</td>
<td>0.63</td>
<td>4.1302</td>
<td>1.09436</td>
</tr>
<tr>
<td>7</td>
<td>0.8</td>
<td>4.4336</td>
<td>0.7375</td>
<td>7</td>
<td>0.87</td>
<td>5.799</td>
<td>1.31036</td>
</tr>
<tr>
<td>8</td>
<td>0.92</td>
<td>4.5585</td>
<td>0.7149</td>
<td>8</td>
<td>0.95</td>
<td>6.3779</td>
<td>1.37982</td>
</tr>
</tbody>
</table>

Figure 3.6: Similarity of \( f \)-term between the (a) circumferential and (b) spiral crack

The characteristic dimension mentioned in Eq. (3.26) is the dimension can be measured and used with Eq. (3.2), [62];

\[
f_1(a,\psi) = \sqrt{1 - \frac{a}{\psi}} \quad ; \text{the characteristic dimension is crack ligament } (a) \quad (3.26a)
\]

\[
f_2(a,\psi) = \frac{a}{\psi} \quad ; \text{the characteristic dimension is crack depth } (c) \quad (3.26b)
\]

\[
f_3(a,\psi) = \frac{a}{\psi \left( 1 - \frac{a}{\psi} \right)} \quad ; \text{the characteristic dimension is maximum radius } (r_o) \quad (3.26c)
\]
On the other hand, the \textit{G-term} is more sensitive to crack depth and the effect of a far-field torsional loading on the crack tip [62]. The \textit{G-term} does not have a closed form solution and was obtained by curve fitting. Similarly, the configuration factor for spiral crack expressed in Eqs. (3.25) can be written in \( f \) and \( G \) terms as Eq. (3.27):

\[
Y_i(a,c,\psi) = f_i(a,\psi) \times G(c,\psi) \quad (\text{no sum on } i = 1, 2 \text{ and } 3)
\]  

(3.27)

Since the circumferential crack and the spiral crack have the same crack ligament \((a)\), to radius ratio \((r)\), as shown in Fig. (3.6). Also, the \textit{f-term} does not depend on the crack revolution angle. The same \textit{f-terms}, Eqs (3.26), can be used. Substitute Eq. (3.27) and Eq. (3.26) into Eq. (3.25), the \textit{G-term} can be obtained from the numerical value of SIF as Eq. (3.28):

\[
G^s(c, r_o) = \frac{1}{f_i(a, r_o)} \frac{r_o^3 K_i^j}{2 T} \left( \frac{\pi}{\sqrt{x}} \right) \quad (\text{no sum on } i) \rightarrow \text{For Solid bar}
\]  

(3.28a)

\[
G^t(c, t) = \frac{1}{f_j(a, t)} \left( r_o^3 - \frac{r_i^4}{r_o} \right) \frac{K_i^j}{2 T} \left( \frac{\pi}{\sqrt{x}} \right) \quad (\text{no sum on } j) \rightarrow \text{For tube bar}
\]  

(3.28b)

As shown above in Eq. (3.28), the \textit{G-term} depends on the geometry and loading but doesn’t depend on materials properties. Hence a general solution can be obtained for any materials at a given geometry and loading condition.

Mode I stress intensity factor \( K_i \) at a different aspect ratio \( (c/r_o) \) for solid and \( (c/t) \) for the tubular cylindrical specimen with spiral crack subjected to torsion computed numerically are shown in Table (3.2). The aspect ratio ranges from shallow crack to deep.
crack. Please note, the \textit{f-term} has three different from and it depends on the characteristic dimension that be choice by analyst while the \textit{G-term} is unique.

Finally, the variation of two terms of Mode I configuration factors of a spiral crack \( Y_1 \), Eq. (3.27), and \textit{G-term} Eq. (3.28) as a function of aspect ratio for cylindrical specimen, for the solid and tubular specimen, are shown in Figs. (3.7 and 8), respectively. As a measurable element result, the trend of a configuration term (\textit{f and G}) in Fig. (3.7 and 8) is similar to the exact solution of Benthem for a cylindrical specimen with circumferential cracks [62].

For solid specimen Eq. (3.29),

\begin{align*}
K_{1s}^s &= \tau_{\text{max}} \sqrt{\pi a Y_1^s (a, r_o, c)} \quad \text{where} \quad Y_1^s (a, r_o, c) = \sqrt{1 - \frac{a}{a}} G^s \left( \frac{c}{r_o} \right) \quad \text{or} \quad (3.29a) \\
K_{1c}^s &= \tau_{\text{max}} \sqrt{\pi c Y_2^s (a, r_o, c)} \quad \text{where} \quad Y_2^s (a, r_o, c) = \sqrt{\frac{a}{a}} G^s \left( \frac{c}{r_o} \right) \quad \text{or} \quad (3.29b) \\
K_{1r}^s &= \tau_{\text{max}} \sqrt{\pi r_o Y_3^s (a, r_o, c)} \quad \text{where} \quad Y_3^s (a, r_o, c) = \sqrt{\frac{a}{a}} \left( 1 - \frac{a}{a} \right) G^s \left( \frac{c}{r_o} \right) \quad (3.29c)
\end{align*}

Where

\begin{align*}
G^s \left( \frac{c}{r_o} \right) &= 0.917 \left[ 1 + 1.25 \left( \frac{c}{r_o} \right) - 9.085 \left( \frac{c}{r_o} \right)^2 + 31.807 \left( \frac{c}{r_o} \right)^3 - \\
&\quad -45.65 \left( \frac{c}{r_o} \right)^4 + 24.49 \left( \frac{c}{r_o} \right)^5 \right] \quad (3.29d)
\end{align*}
Finally, a regression curve fitting is performed to obtain a polynomial equation of the $G$-term that covers all the range of aspect ratio. The new Mode I fracture solution for $a$ and $c$ is given by

$$ K_I = \max \sqrt{\pi a Y_1^r(a,t,c)} $$

where

$$ Y_1^r(a,t,c) = \sqrt{1 - \frac{a}{t} G^r \left( \frac{c}{t} \right) } $$

or

$$ G = \max \pi a Y_1^r(a,t,c) $$

For tubular specimen Eq. (3.30),

$$ K_I = \max \sqrt{\pi a Y_1^r(a,t,c)} $$

where

$$ Y_1^r(a,t,c) = \sqrt{1 - \frac{a}{t} G^r \left( \frac{c}{t} \right) } $$

or

$$ G = \max \pi a Y_1^r(a,t,c) $$
Figure 3.8: Configuration factor of a spiral crack on a tube bar

\[ K_I' = \tau_{\text{max}} \sqrt{\pi c} Y_2' (a, t, c) \]  where \[ Y_2' (a, t, c) = \sqrt{\frac{a}{t}} G' \left( \frac{c}{t} \right) \]  or \[ \left(3.30b \right) \]

\[ K_I' = \tau_{\text{max}} \sqrt{\pi r} Y_3' (a, r, c) \]  where \[ Y_3' (a, r, c) = \sqrt{\frac{a}{t}} \left(1 - \frac{a}{t} \right) G' \left( \frac{c}{t} \right) \]  \[ \left(3.30c \right) \]

Where:
\[
G'(\frac{c}{t}) = 0.88 \left( 1 - 1.239\left(\frac{c}{t}\right) + 36.013\left(\frac{c}{t}\right)^2 - 194.261\left(\frac{c}{t}\right)^3 + 
468.875\left(\frac{c}{t}\right)^4 - 516.386\left(\frac{c}{t}\right)^5 + 216.034\left(\frac{c}{t}\right)^6 \right)
\]

(3.30d)

3.6 EXPERIMENTAL VALIDATION

In addition to aluminum samples tested and used to develop the model presented above, polycarbonate material is used to verify the proposed formulation. Both three-point bending and spiral crack specimens were machined from the as-received polycarbonate solid cylinder. First, cylindrical specimens with a spiral crack at 45° form the axis were tested under pure torsion loading, and the maximum torque at the onset of fracture was obtained. The fracture toughness of the material was calculated using Eqs. (3.29 and 3.30). On the other hand, a three-point bending test, according to ASTM 1280, is conducted to measure a fracture toughness. The result was compared with the result from a cylindrical specimen with spiral crack according to this work.

3.6.1 Spiral Crack with a Pure Torsional Load Approach

Figure 3.9: Actual and schematic of solid specimen with a full spiral crack
As shown in Fig. (3.9), $L$ is the total length of the specimen; $L_s$ is uncracked length that is used to minimize the effect of the stress-concentration zone at the v-notch groove ends; $L_g$ is the gage length (spiral pitch); $L_D$ loading length (twisting applied); $L_E$ fixed end; $D = D_g$ is the specimen diameter (gauge diameter). Even the tubular specimen has the same dimensions, it has also; $D_i$ internal diameter; and $t$ thickness. Finally, $T$, $\theta$ are torque and angle of twist respectively.

Six cylindrical specimens with spiral crack were prepared. The specimens were fabricated with different aspect ratios ($c/r_o$), where $c$ is the crack depth and $r_o$ is the radius of the cylinder. All of them have a v-notched of helix angle of $45^\circ$ at the center of the bar as shown in Fig. (3.9). A 4-D milling machine and Mico-Engraving V-groove cutter tools (the tip has V-shape at $60^\circ$, and a diameter of $127 \mu m$) were used to generate a v-notch groove on the spiral path on the surface along the gauge length. Finally, stainless-steel razor blade was used to make the final sharp artificial cracks. The dimensions of the specimens are shown in Table (3.3).

Table 3.3: Dimensions of a Polycarbonate torsional specimen in (mm)

<table>
<thead>
<tr>
<th>Specimen #</th>
<th>$L$ mm</th>
<th>$D$ mm</th>
<th>$L_g$ mm</th>
<th>$D_g$ mm</th>
<th>$L_D,E$ Mm</th>
<th>$L_s$ mm</th>
<th>$\beta_{SP.}$ Deg.</th>
<th>$c$ mm</th>
<th>$d_i$ mm (if tube)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specimen#1</td>
<td>140.0</td>
<td>25.4</td>
<td>79.756</td>
<td>25.4</td>
<td>25</td>
<td>5</td>
<td>360</td>
<td>4.6</td>
<td>12.7</td>
</tr>
<tr>
<td>Specimen#2</td>
<td>140.0</td>
<td>25.4</td>
<td>79.756</td>
<td>25.4</td>
<td>25</td>
<td>5</td>
<td>360</td>
<td>6.0</td>
<td>12.7</td>
</tr>
<tr>
<td>Specimen#3</td>
<td>140.0</td>
<td>25.4</td>
<td>79.756</td>
<td>25.4</td>
<td>25</td>
<td>5</td>
<td>360</td>
<td>6.5</td>
<td>12.7</td>
</tr>
<tr>
<td>Specimen#4</td>
<td>140.0</td>
<td>25.4</td>
<td>79.756</td>
<td>25.4</td>
<td>25</td>
<td>5</td>
<td>360</td>
<td>7.5</td>
<td>12.7</td>
</tr>
<tr>
<td>Specimen#5</td>
<td>140.0</td>
<td>25.4</td>
<td>79.756</td>
<td>25.4</td>
<td>25</td>
<td>5</td>
<td>360</td>
<td>8.0</td>
<td>12.7</td>
</tr>
<tr>
<td>Specimen#6</td>
<td>140.0</td>
<td>25.4</td>
<td>79.756</td>
<td>25.4</td>
<td>25</td>
<td>5</td>
<td>360</td>
<td>7.0</td>
<td>12.7</td>
</tr>
</tbody>
</table>

The test was performed under angular displacement control at a rate of ($1 \text{ deg./min}$). The torque, and the normal load were recorded through the load cell of Test Resources machine. Digital microscope, VHX-5000, was used to measure the fracture surface of the
broken specimen. In general, a brittle fracture was witnessed in all specimens as indicated by the fracture surface shown in Fig. (3.10). The stress intensity factor (SIF) and the fracture toughness were calculated from the proposed equation, Eq. (3.29 and 3.30) and are tabulated in a result section, Table (3.4). Since the torque load transfers to the specimen through the friction force between the specimen and the grips surface and to reduce the sliding effect. For a tubular spiral crack specimen case, the Aluminum core was added to ends of specimen (along of \( L_D, \) and \( L_E \)), Fig. (3.11).

3.6.2 Three-Point Bending Approach

A range of three-point bending specimens, with different aspect ratios\( (c/w) \), was fabricated from a 25.4 mm polycarbonate solid bar.

Figure 3.10: Experimental setup of torsional specimen test
The final dimensions of the 3PB specimen shown in Fig. (3.12A) are adjusted according to ASTM 1280-01 [70]. An initial notch crack was made in the mid-section of the specimen with the handsaw and then a stainless-steel razor blade was used to make the final artificial sharp crack as shown in Figs. (3.12C and D). The specimen geometry and dimension required for plain strain condition was verified with the standard dimensions criteria as shown in Eq. (3.31).

\[
B, a \geq 2.5 \left(\frac{K_{lc}}{\sigma_{ys}}\right)^2 \quad 0.45 \leq \left(\frac{a}{W}\right) \leq 0.55
\]

(3.31)

Figure 3.11: Tubular polycarbonate specimen with Aluminum core
Where $\sigma_{ys}$ is the yield strength of the polycarbonate material at room temperature. The three-point Bending experiment was conducted at a loading rate of 1 mm/min. The experiment was repeated three times, and the load-displacement data was recorded each time. The stress intensity factor was calculated using the standard formula Eq. (3.32) [3,5].

$$K_I = \left[ \frac{P_i S}{B \sqrt{W^3}} \right] f\left(\frac{a_i}{W}\right)$$

(3.32)

Where:

$$f\left(\frac{a_i}{W}\right) = \frac{3(a_i/W)^3[1.99 - (a_i/W)(1 - (a_i/W))(2.15 - 3.93(a_i/W) + 2.7(a_i/W)^2)]}{2(1 + 2(a_i/W))(1 - (a_i/W))^{3/2}}$$

Where $P$ is the load at fracture, $a$ is the crack depth, $S$ is the effective specimen span, $W$ is specimen height and $B$ is the width. As expected, a brittle fracture was observed in all three specimens as clearly seen in the fracture surface shown in Fig. (3.12E). The fracture toughness $K_{Ic}$, was identified at the Pop-in point ($P_i = P_Q$) for each test. The mean value of fracture toughness of Polycarbonate and the standard deviation are shown in result section, in Table (3.5).

3.7 RESULTS AND DISCUSSION

3.7.1 Spiral Crack Specimen

The six CSSC specimens are performed at room temperature, and the result is listed in Table (3.4). At each test, the fracture torque at fracture is extracted. The $f$ and $G$ terms
are calculated using Eqs. (3.26a and 3.29d). These values along the torque at fracture are used to calculate SIF of a cylindrical bar $K_{lc}$ according to Eq. (3.29a).

![Figure 3.12](image)

Figure 3.12: Three-point bending experimental setup

The mean value of these tests is $K_{lc} = 3.814 \text{MPa} \sqrt{m}$ with a standard deviation of $\bar{\sigma}_{K_{lc}} = 0.06 \text{MPa} \sqrt{m}$ as shown in Table (3.4) of polycarbonate bar with 25.4mm diameter.
3.7.2 Three-point Bending Specimen

The fracture loaded $P_o$ and other dimensions are given in Table (3.6). As shown in the table, the mean value of the fracture toughness is $K_{ic} = 3.878 \text{MPa} \sqrt{m}$ with a standard deviation of $\sigma_{K_{ic}} = 0.03 \text{MPa} \sqrt{m}$.

Table 3.4: Spiral crack Polycarbonate specimens results under pure torsion load

<table>
<thead>
<tr>
<th>($\frac{c}{r}$)</th>
<th>Material</th>
<th>T (Nm)</th>
<th>$c$ (mm)</th>
<th>$a$ (mm)</th>
<th>$G$ (Eq. 29d)</th>
<th>$f_1$ (Eq.3.26a)</th>
<th>$K_{ic}$ (Eq.29a)</th>
<th>$K_{ic}$ Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.47</td>
<td>Poly.</td>
<td>113.090</td>
<td>6.0</td>
<td>6.74</td>
<td>1.117</td>
<td>0.767</td>
<td>3.886</td>
<td></td>
</tr>
<tr>
<td>0.51</td>
<td>Poly.</td>
<td>110.609</td>
<td>6.5</td>
<td>6.20</td>
<td>1.148</td>
<td>0.821</td>
<td>3.914</td>
<td></td>
</tr>
<tr>
<td>0.55</td>
<td>Poly.</td>
<td>104.400</td>
<td>7.0</td>
<td>5.70</td>
<td>1.181</td>
<td>0.740</td>
<td>3.810</td>
<td></td>
</tr>
<tr>
<td>0.59</td>
<td>Poly.</td>
<td>101.550</td>
<td>7.5</td>
<td>5.20</td>
<td>1.218</td>
<td>0.768</td>
<td>3.780</td>
<td></td>
</tr>
<tr>
<td>0.64</td>
<td>Poly.</td>
<td>94.6620</td>
<td>8.0</td>
<td>4.70</td>
<td>1.273</td>
<td>0.800</td>
<td>3.741</td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>Poly.</td>
<td>43.2170</td>
<td>11.0</td>
<td>1.70</td>
<td>1.488</td>
<td>0.867</td>
<td>3.754</td>
<td></td>
</tr>
</tbody>
</table>

*SD is a Standard Deviation

The average value from the two-independent experiment is shown in Table (3.5). As clearly observed, the spiral crack with a proposed mathematical formula predicted the fracture toughness of material very well compared with the standard method with the different of less than 1.7%.

Table 3.5: Comparison spiral crack and 3PB laboratory work

<table>
<thead>
<tr>
<th>Material</th>
<th>Torsion Test $K_{ic}$ Eq. (3.29a)</th>
<th>3PB Test $K_{ic}$ Eq. (3.32)</th>
<th>%Different</th>
<th>$K_{ic}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polycarbonate</td>
<td>3.814 MPa $\sqrt{m}$</td>
<td>3.878 MPa $\sqrt{m}$</td>
<td>1.7</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Furthermore, the fracture toughens obtained from the spiral, and Three-Point Bend tests as a function of aspect ratio are plotted Fig. (3.13). Clearly, the Fig. (3.13) show that the spiral crack specimen is size independent. With a full spiral crack orientation, the plane strain condition is valid even the aspect ratio is higher.

![Graph showing fracture toughness as a function of aspect ratio](image)

Figure 3.13: Spiral crack and 3PB experimental results

The proposed method is verified by comparing results for different materials such as Aluminum, Steel, Ceramic, and Concrete in the literature [19,71]. These materials have been tested, and the fracture toughness was extracted using finite element method. The fracture toughness was calculated from the proposed method, using the torque at fracture and the geometry as input. The table (3.7) shows that the results from the proposed method have a differences of less than 6% compared with other results in the literature.
### Table 3.6: 3PB results of Polycarbonate fracture mechanics

<table>
<thead>
<tr>
<th>$(\frac{c}{W})$</th>
<th>Material</th>
<th>P (N)</th>
<th>c (mm)</th>
<th>W(mm)</th>
<th>S(mm)</th>
<th>B(mm)</th>
<th>$K_{ic}$ (Eq. 3.29a)</th>
<th>$K_{ic}$ (Average)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.45</td>
<td>Poly.</td>
<td>872.89</td>
<td>10</td>
<td>22</td>
<td>71</td>
<td>11.3</td>
<td>3.893</td>
<td>3.878 (MPa(\sqrt{m})) (SD=0.03)</td>
</tr>
<tr>
<td>0.46</td>
<td>Poly.</td>
<td>771.93</td>
<td>10</td>
<td>21.9</td>
<td>75</td>
<td>10.9</td>
<td>3.835</td>
<td></td>
</tr>
<tr>
<td>0.55</td>
<td>Poly.</td>
<td>350.03</td>
<td>9.3</td>
<td>16.8</td>
<td>62</td>
<td>8.2</td>
<td>3.906</td>
<td></td>
</tr>
</tbody>
</table>

### Table 3.7: Benchmark comparison works

<table>
<thead>
<tr>
<th>Materials</th>
<th>Fracture Load</th>
<th>$(\frac{c}{r})$</th>
<th>$r$ (mm)</th>
<th>$K_{ic}$ (MPa(\sqrt{m}))</th>
<th>$G(c/r)$ (Eq. 3.29d)</th>
<th>$f_1$ (Eq. 3.6a)</th>
<th>$K_{ic}$ Eq. (3.29a) (MPa(\sqrt{m}))</th>
<th>$K_{ic}$ ASTM MPa(\sqrt{m})</th>
<th>Different %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al.7475-T7</td>
<td>0.009000 rad.</td>
<td>0.598</td>
<td>12.70</td>
<td>51.30</td>
<td>1.226</td>
<td>0.948</td>
<td>47.741</td>
<td>48.3</td>
<td>1.16</td>
</tr>
<tr>
<td>St. A302B</td>
<td>0.004680 rad.</td>
<td>0.751</td>
<td>10.15</td>
<td>55.80</td>
<td>1.483</td>
<td>1.285</td>
<td>56.952</td>
<td>54.9</td>
<td>3.74</td>
</tr>
<tr>
<td>Ceramic</td>
<td>0.000702 rad.</td>
<td>0.060</td>
<td>8.500</td>
<td>2.205</td>
<td>0.962</td>
<td>0.217</td>
<td>2.0860</td>
<td>2.20</td>
<td>5.20</td>
</tr>
<tr>
<td>Concrete</td>
<td>T= 46.9 Nm</td>
<td>0.130</td>
<td>20.30</td>
<td>3.500</td>
<td>0.979</td>
<td>0.350</td>
<td>3.3000</td>
<td>N/A</td>
<td>5.60</td>
</tr>
</tbody>
</table>
3.8 CONCLUSIONS

A novel closed-form solution for Mode I fracture toughness of materials from a cylindrical specimen with spiral crack subjected to pure torsion load is developed by using finite element methods and Bentham equations. A full 3D model of a cylindrical specimen with spiral crack subjected to far-field torsion load is developed, and the stress intensity factor is extracted based on interaction integral method. The geometry factor is then obtained from the SIF and the torque at fracture for a given geometry. Further, the geometry factor is divided into two characteristics parameters, similar to Bentham equations, and the corresponding function is obtained through polynomial fitting. The proposed method is later used to get the fracture toughness of materials just utilizing the torque at fracture and specimen geometries. The fracture toughness of different materials is obtained and compared with standard methods and found to be accepted, less than 2% different. From the result, the following conclusions can be drawn:

a) A spiral crack is independent on the geometry of the specimen, i.e., plane strain condition is valid.

b) The result form the proposed method is in good agreement for different materials.

c) The method works for both shallow and deep cracks and can be used to test fracture toughness of range of materials.

d) The fracture torque was measured by a load cell at a great accuracy. However, the crack dimensions are critical to the accuracy of the method and measuring the crack depth was a challenge.

e) The method can be extended in the case of dynamic loading condition.
3.9 LIST OF REFERENCES


[38] User AS. Abaqus 6.14. Dassault Systèmes Simulia Corp, Provid RI, USA 2014;Dassault S.


CHAPTER 4

SUMMARY OF THE PRESENT WORKS AND RECOMMENDATIONS FOR FUTURE RESEARCH

4.1 SUMMARY

Dynamic initiation fracture toughness of Aluminum alloys subjected to non-dispersive wave propagation with a high loading rate was used to study the Mode-I of fracture as the first part of this research work. Far-field loading signal was used to measure the fracture load and a 3D-DIC, full-field measurements, was conducted to investigate the initiation fracture time, and a numerical solution of interaction integral was used to extract the dynamic stress intensity factor. Aluminum alloys; Al. 6061-T6, 2024-T3 and 7075-T651 specimens with the same geometry, dimension, spiral angle, crack depth and crack ligaments, were subjected to high loading torsional impulse load. The following comments summarize the highlights points;

- It was clearly shown that mode-I fracture is significant during dynamic fracture response of a cylindrical specimen with spiral groove at $45^\circ$.
- Due to the nature of torsional wave propagation, the torsional wave is non-dispersive and has less axial inertia, the load remains the same along the length
of the bar. Hence, torsional loading is ideal for investigating of the dynamic fracture initiation results accurately.

- Since there is no exact solution for cylinder specimen with a spiral crack under torsion loading, a numerical-experimental approach was used to solve the problem. The value of a dynamic stress intensity factor as a function of time was calculated numerically.

- It was clearly observed that the boundary conditions and the load applied are the main important parameter of the numerical simulation since they need to be updated depending on the specimen’s dimensions. This was explained numerical solution section.

The second part of the present document was dedicated to developing an exact solution for the static fracture of a spiral crack under pure far-field torsional load. Bentham’s asymptotic solution of circumferential crack was adapted and used in spiral crack configuration. A new formula of Mode-I fracture of spiral crack under pure torsional load is presented and verified by testing Polycarbonate materials and using results from a standard ASTM formula. The results obtained from the new formula can be summarized as the following:

a) A spiral crack is independent on the geometry of the specimen, i.e., plane strain condition is automatically satisfied. The method works for both shallow and deep cracks and can be used to test fracture toughness of a range of materials.

b) The fracture torque was measured by a load cell at great accuracy. However, the crack dimensions are critical to the accuracy of the method and measuring the crack depth was a challenge.
4.2 RECOMMENDATIONS

Regarding the non-dispersive dynamic stress wave and a spiral crack specimen described in this work, the following areas can be potential future research topics.

- The approach detailed in this document is a general methodology that can be employed to study the dynamic initiation fracture toughness of different materials.

- The dynamic, cohesive fracture of epoxy material that is used in composite bonding can be investigated with the proposed method. A two half of spiral crack specimen made from a strong material can be filled with an appropriate epoxy. In this case, the epoxy can be tested under pure torsional load.

- The dynamic adhesive fracture of materials can be investigated using the proposed method. In this case, a specimen made from two pieces and glued together, for example, metal-metal, metal rubber, metal-epoxy, or composite-composite, specimen can be used.

- The proposed static formula of stress intensity factor of spiral crack presented in chapter 3, can be extended for the dynamic fracture condition.

- The main limiting challenge in the experimental work conducted in this work was the loading rate generated by the current climbing system. Develop a new clamping mechanism, such as a magnetic clamping system, that can to generate a higher loading rate would help to investigate at high loading rate.

- The Interaction integral setup analysis coding can be built with python software and can be integrated with the digital image correlation software for better data analysis.
• It would be ideal if a pre-crack is generated by using the torsional fatigue system and investigate the effect of artificial crack on the fracture toughness value at different loading rate
BIBLIOGRAPHY


3D Simulia Abaqus, User Manual 2016. Dassault Systèmes Simulia Corp, USA.


APPENDIX A

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