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Public Debt, High Inflation and Economic Depression: A Survival Analysis Approach

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Public Debt, High Inflation and Economic Depression: A Survival Analysis Approach

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Dedication

To my parents and grandparents.

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Abstract

This dissertation investigates the association between public debt, high inflation and economics depression by using the survival analysis approach.

In the first chapter, I use non-parametric survival analysis to check the association between high debt episodes and high inflation episodes. Both the Kaplan-Meier estimate and the Nelson-Aelon analysis indicate that the existence of an overlapping high inflation episode is correlated with longer duration of high debt episodes. The reverse is also true: the existence of a high debt episode overlap is associated with longer duration of high inflation episodes. Whether or not the country is a member of the OECD does not matter for the survival experience.

In the second chapter, I use the Cox PH model, the exponential model and the Weibull model to analyze the survival experience of high debt episodes and high inflation episodes. I first consider the duration of high debt episodes (HDEs). Across all specifications, HDE duration depends on two variables: (1) the existence of an overlap with a high inflation episode; (2) the growth rate of GDP. These effects are not independent. At high GDP growth, an HIF overlap is associated with a shorter HDE. At low GDP growth, however, an HIF overlap is associated with a longer HDE. I also examine the duration of HIFs, using HDE overlap and GDP growth as regressors. Interestingly, the results are mixed and there is no evidence of interaction between HDE overlap and GDP growth.

Debt of the public sector has been associated with low, or even negative, growth. This position is most closely associated with Reinhart and Rogoff (2010). In the third chapter, we analyze the duration of economic crises, in the form of depressions, to see

if there is an association with consecutive years of high public-debt-to-GDP ratios. We find that high debt is positively correlated with the duration of depressions while inflation is negatively correlated with such duration.

The last chapter uses parametric survival analysis to analyze the duration of economic crises to see if they have an association with the occurrence of consecutive years of high public-debt-to-GDP ratios. By comparing the results from five parametric models, including the exponential, the Weibull, the log-normal, the log-logistic, and the generalized gamma regression, all of which have the accelerated failure time (AFT) metric, I find that there is a positive association between high debt episodes and the length of economic crises. All of the results are consistent with each other throughout the models. The main results were not refuted by adding covariates including economic factors, political factors, cultural factors and financial crises as controls.

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Chapter 1

Survival Analysis of Episode of High Debt and High Inflation: A Non-parametric Approach

1.1 Introduction

One aftermath of the great recession is rising levels of public debt. For example, the public debt to GDP ratio for all the G7 countries increased between the year 2006 and 2016. One problem with the sustained high level of sovereign debt is it may hamper long run economic development since it not only shows dangerous signals of the government fiscal sustainability, but may reduce the private investment and lower productivity of the country in the long run. Even though empirically and theoretically, no papers have provided strong evidence about whether the causality goes from high public debt to slower economic growth or vice versa, exploring the question of which factors are correlated with high debt is certainly interesting. At the same time, when the country begins to accumulate debt, it usually lasts for a while. According to Reinhart et al. (2012), once a public debt overhang has lasted five years, it is likely to last 10 years or much more (unless the debt was caused by a war). Thus the length of time countries have high debt overhang is a topic worth studying which has barely been explored before.

In this chapter, I focus on checking correlations between high public debt and high inflation while controlling other macroeconomic variables. There are other papers exploring the relation between inflation and public debt. For example, there are papers checking the effect of inflation on the choice of government debt structure

(Mandilaras and Levine (2001)) and there are also papers analyzing the impact of large nominal debt overhang on the temptation to inflate (Aizenman and Marion (2011)). However, very few of the previous studies have checked the impact of inflation on the duration of high debt episodes, or vice versa. More specifically, in this chapter, I focus on whether the existence of high inflation episodes interferes with the duration of high debt episodes, when controlling for covariates of interest. At the same time, I also check whether the mean and median time of high inflation episodes have been prolonged or shorten when the countries have high debt simultaneously. By doing these, this study offers some extensions to the existing empirical research regarding the relation between inflation and public debt. The dependent variables of interest are the time to occurrence of an event. They contain censored or truncated observations. The estimates of sample median and mean from using standard least square estimation will be biased. Thus I will use survival analysis.[1]

Section 2 will be the literature review. Section 3 introduces the source of data and section 4 introduces survival analysis. Section 5 analyzes whether the existence of high inflation affects the duration of high debt episodes. Section 6 analyzes the survival experience of high inflation episodes. Section 7 provides a conclusion for the chapter.

1.2 Literature Review

Different data and empirical methods were utilized to check the relation between public debt and inflation. The results are still controversial.

1.2.1 Effects of inflation on public debt

Mandilaras and Levine (2001) check whether people's expectations toward future inflation affect the amount of deflatable debt issued by the government. Their results

[1]The introduction regarding the methodology of survival analysis can be find in Appendix A.

prove that higher expected inflation induces a change in debt management: government issues less of inflation-sensitive securities in order to enhance its credibility[2]. Their study also shows that the increasing debt to GDP ratios are associated with issuance of less inflation-sensitive debt. Taking advantage of a broad new historical dataset on public debt including data on 44 countries across 200 years, Reinhart and Rogoff (2010) shows that there is no systematic relation between debt and inflation for advanced economies. However, in emerging market economies, high public debt coincides with a high inflation rate. Their study was done by simply checking the inflation rate among different quantiles of average (median) debt ratio. Reinhart and Sbrancia (2011) find the inflation rate, when combined with other regulations in the financial sector, contributes to a substantial debt reduction in advanced economies for the years between 1945 and 1970. However, other researchers find minor role of inflation on public debt (Giannitsarou and Scott (2008); Abbas et al. (2013)).

1.2.2 Effects of public debt on inflation

Aizenman and Marion (2011) check whether it is normal for the US government to print more money to pay off the debt, which would increase the inflation rate. They conclude that "eroding the debt through inflation is not farfetched". US can still increase inflation for about 5 percent for several years to reduce the high debt ratio since there are more foreign debt holders even though the current debt maturity period is shorter.

Cox (1985) shows that increases in federal debt fuel inflation when debt is measured using the market value. However, further studies done by Hafer and Hein (1988) shows that failure to capture the interest rate effects inherent in the market value measure was the main reason why there is a positive relation between federal

[2] They define the price index, foreign currency and short term maturity debt as "non-sensitive" debt since these types of debts can not be inflated away and investors enjoy returns that cannot be eroded by surprise inflation.

debt and inflation. When incorporating interest rates using the market series, the increases in federal debt don't cause higher rates of inflation.

While many studies are devoted to studying the factors that may affect the compositions of public debt, including both external debt and domestic debt, the factors that may help explain the duration of debt were less closely studied. Compared to the existing research, my paper is decidedly empirical. Instead of checking the genesis of debt, I check whether the existence of high debt is associated with the duration of high inflation and whether the existence of high inflation is correlated with the duration of high debt among countries in the world using data from the year 1950 to 2010.

1.3 Data

The data for inflation is transformed from the Consumer Price Index (CPI) based on:

$$Inflation_t = ln(CPI_t) - ln(CPI_{t-1})$$

CPI is available yearly between the year 1949 and 2014 including 155 countries in the world provided by International Financial Statistics (IFS). To be categorized as experiencing a high inflation episode, the country needs to have an inflation rate higher than a threshold value for at least four consecutive years. The threshold value used in this chapter is 10 percent[3]. The selection of the threshold value is arbitrary. One fact is high inflation rates differ across countries based on their different backgrounds and experiences with inflation. A moderate inflation rate can range from 3 percent to 30 percent. That is, for countries with an inflation targeting policy which sets the inflation rate around 2 percent, 4 percent would represent a high inflation rate. Moreover, I define variable *HIF* to be a dummy variable that equals 1 when the country is in the high inflation episode; otherwise it equals 0.

[3]The 20 percent threshold is also used as comparison.

The other main variable of interest is *the debt to GDP ratio.* This data is from historical public debt database (HPDD) which covers almost all International Monetary Funds (IMF) countries (174 countries) and spans a long time period (Abbas et al. (2011)). For a country to be classified as experiencing high debt episode, the country should have a debt to GDP ratio higher than the threshold value for at least *4* consecutive years. In this chapter, we concentrate on 50 percent as the threshold value.[4] Again, the selection of threshold value is arbitrary. The 90 percent threshold is used in the paper of Reinhart and Rogoff (2010) and is now viewed as the benchmark. The use of the 90 percent threshold would reduce the sample, so most of my results use the 50 percent threshold value. Furthermore, I define *HDE* to be a dummy variable with its value equals 1 when the country is in high debt episode; otherwise, it equals 0.

Table 1.1 shows the incidence of HDE and HIF across countries.[5] Based on Table 1.1, we have 131 counts of HIF_{10} in total. Within these 131 HIF_{10}, 32 of them overlap with HDE_{90}[6], that is, during the time when these 32 countries experienced a high inflation episode, they also experienced a high debt episode, for at least one year in common. 60 HIF_{10} overlap with an HDE_{50}; within the sample, I have 25 counts of HIF_{20} in total and 14 of these HIF_{20} come across an HDE_{90}. A total of 23 HIF_{20} overlaps with an HDE_{50}.

The numbers in the column titled "*Unc_HIF*" are the total number of *HDE* regardless of the existence of HIF. For example, we have 72 HDE_{90} and 144 HDE_{50}

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[4]Results for the 90 percent threshold are also shown in the Appendix

[5]Both HIF_{20} and HIF_{10} are called high inflation episode, the only difference is they use different threshold values. HIF_{10} refers to the period when a country has inflation rate higher than 10 percent for four consecutive years; For HIF_{20}, we used 20 percent instead of 10 percent as the threshold value. Similarly, the only difference between HDE_{90} and HDE_{50} is threshold value of 90 percent for the former instead of 50 percent in the latter in the definition of high debt episode.

[6] Apparently, when we combine value 12 from table 2, value 18 from table 3, value 2 from table 34, and the sum of 12, 18 and 2 is 32, confirming that fact that we have 32 episodes that the HIF_{10} and HDE_{90} happens at the same time.

in our sample. Meanwhile, the numbers in the row titled "*Unc_HDE*" are the total number of *HIF* regardless of the existence of the *HDE.* For example, there are 25 HIF_{20} and 131 HIF_{10} in our sample regardless of the occurrence of high debt episode.

Table 1.2, 1.3 and 1.4 show more detailed information on the sequence of the HIF and HDE. Table 1.2 shows us the frequency when the high inflation episode began before the high debt episode and they have at least one year in common. For example, there are 12 times in our sample when HIF_{10} happened before the HDE_{90}. Table 1.3 show the frequency when the high debt episode happened and then was followed by the high inflation episode, conditional on the fact that the two episodes have at least one year in common. For example, there are 18 times when HDE_{90} began first and then was followed by HIF_{10}. There are only 2 times when the HIF_{10} and HDE_{90} began at the same year, as shown by Table 1.4 column 3 and row 2. Adding the above mentioned "12", "18", and "2" together, I get 32 which is the total number of counts that HIF_{20} come across HDE_{90}, regardless of the sequence.

When I analyze the survival experience of the high debt episode, I trim my data set by discarding all observations for which *HDE* is missing. That is, I focus on high debt episode in such a way that *HDE* is either 1 or 0. This does not guarantee that we have data for inflation, so inflation may contain missing values for debt. Now define the variable *YHIF* as a *subset* of *HDE*, those that have at least one year in common with an *HIF*. To be more specific, $YHIF = HDE = 1$ for every year of the *HDE* if an *HDE* overlaps with an *HIF* for at least one year. Otherwise, $YHIF = 0$ and $HDE = 1$ for the corresponding high debt episode. *YHIF* is constant within any episode defined by *HDE.*

Similarly, when I check the survival experience of the high inflation episode, I trim the data set by discarding all observation for which *HIF* is missing. The covariate of interest *YHDE* is defined as a *subset* of *HIF*, those that have at least one year in common with an *HDE*. More specifically, $YHDE = HIF = 1$ for every year of the *HIF* if *HIF* overlaps with an *HDE* for at least one year; Otherwise, $YHDE = 0$ and $HIF = 1$ for the corresponding high inflation episode. *YHDE* is constant within any episode defined by *HIF*.

In defining *YHIF*, as long as *HDE* contains at least one value for *HIF*, we ignore any missing values for *HIF* in the same HDE, which implicitly assumes that those missing value for *HIF* are all zero. By doing so, we may underestimate the number of *HDEs* for which $YHIF = 1$ since there exists the possibility that had we have the data for inflation, *YHIF* may switch from $YHIF = 0$ to $YHIF = 1$. A similar problem also shows up for *YHDE*.

There are 144 counts of HDE_{50} over 110 countries. More specifically, 81 countries experienced HDE_{50} once, 24 countries experienced an HDE_{50} twice, and five countries experienced a high debt episode three times.[7] Based on Table 1.5, among the 144 high debt episodes, 40 of them were censored at the end of sample year and 104 of them exited the *HDE* normally (also called failed); At the same time, among the 144 *HDEs*, 95 of them never come across high inflation episode. For the remaining 49 episodes which did overlap with a high inflation episode, 11 of them were censored.

There are 131 counts of HIF_{10} over 94 countries, 67 countries experienced an HIF_{10} once, 20 countries experienced an HIF_{10} twice, 5 countries experienced an HIF_{10} 3 times,[8] Nigeria experienced an HIF_{10} 4 times and Iran experienced an HIF_{10} 5 times. Meanwhile, base on Table 1.6, among the 131 high inflation episodes, 60 of them overlaps with a high debt episode and among the 60 episodes, only one of them is censored. For the remaining 71 episodes which never overlap with a high debt episode, only two of them were censored. The longest duration of HIF_{10} is 29 years which happened in Colombia from the year 1971 to 1999.

[7] These five countries are UK, Grenada, Ireland, Mauritius and Netherlands.

[8] These five countries are Ghana, Jamaica, Kenya, Paraguay and Congo.

1.4 Survival Analysis

Survival analysis is the study of the duration of the time to the occurrence of an event, and how potential factors may impact it. It is frequently applied in clinical trial and less frequently used in economics. The variable of interest is duration, which is random and non-negative discrete or continuous. To begin with survival analysis, I convert yearly data into the duration format.[9] First, in Section 5, the criterion variable is the duration of the high debt episode. I thus define the entering event to be the year when HDE begins with $t = 1$; The corresponding exit event (which is usually called "failure" in survival analysis) is the time when HDE ends. So the exit event represents the fact that the country is getting out the high debt episode, which is a good news. For example, Netherlands, one member of the OECD countries, experienced three episodes of HDE_{50} shown in Figure 1.1 (a) . The first episode began in 1950 which is the same year when my sample year starts and it ended around 1969. This is an example of "left truncation" since we don't know when the HDE began. The second episode of HDE_{50} happened between 1982 and 2005. The third episode began around 2008 and it never ended until the end of my sample in 2012. The third episode is counted as "right censoring". Thus the three HDE episodes happened within Netherlands contribute three lengths of records for my data.[10] For Netherlands, year 1950, 1982, 2008 are reset to be time $t = 1$ in my sample. With the gap between 1950 and 1969 to be 20, the duration of first HDE is 20 years as shown in Figure 1.1 (b).

[9] Detailed introduction regarding the specific terminologies used in survival analysis is in Appendix A.

[10] In this chapter, I allow for multiple entries for the same countries, and I assume that any two episodes are independent to each other.

1.5 Survival Experience of High Debt Episode

In this section, we check potential factors that are associated with the duration of *HDEs* with special attention to the existence of high inflation episodes (*YHIF*).[11]

1.5.1 Kaplan-Meier Statistic

In this part, we use the Kaplan-Meier statistics to compare the survival experience across the covariate of interest at a qualitative level. The Kaplan-Meier estimate, one of the most frequently used nonparametric estimates, calculates the survival probability directly based on all the observations available, including both censored and uncensored, while making no assumption about the functional form of the survival function $S(t)$. $S(t)$ is the probability of surviving beyond time t where t refers to the t^{th} year countries were in the high debt episode. First of all, it ranks the order of each observed survival time t_j. Then it calculates the probability of surviving, conditional on the fact that they have survived for the observed survival time t_j. The survival function for each survival time is the multiplication of all the conditional probabilities of surviving that ever happened until the specific survival time. The Kaplan-Meier statistics give the probability of remaining in the high debt episode at each observed survival time t_j. Mathematically, the Kaplan-Meier estimator at time t is given by

$$\hat{S}(t) = \prod_{j|t_j \leq t} (\frac{n_j - d_j}{n_j})$$

with n_j representing the number of countries that were experiencing a high debt episode at time t_j. In other words, n_j is the number of countries at risk at year t_j; d_j is the number of countries that were in the last year of a high debt episode at year t_j (the total number of exit at year t_j).

[11] Results shown in the following sections only use HDE_{50} and HIF_{10} since by doing this, I can keep relatively larger sample size.

Table 1.7 shows the detailed output for the Kaplan-Meier estimate. The minimum requirement for a high debt episode is 4 years. So the corresponding observed minimum duration of HDE in the Table 1.7 is 4 years. According to Table 1.7, when time equals 4, the number of countries that were experiencing a high debt episode is 144 (noted in the column titled "No. at Risk" in the Table 1.7). It represents 144 countries in our sample were still experiencing a high debt episode at time $t = 4$. At the same time, the total number of countries that were in the last year of HDE is 4 (listed in the column "No. Exited") with six episodes of high debt being censored (listed in the column "Censored") . Thus the estimated value of the survival function at time 4 is $\hat{S}(4) = 1 - \frac{4}{144} = 0.972$. The remaining total number of episodes at risk is 134 which is shown the next row when $t = 5$.

At $t = 5$, total number of high debt episodes that were in the last year is 6. Thus the corresponding survival probability is $(1 - \frac{6}{134})$. The estimated survival function is the successive multiplication of the estimated conditional probabilities up to time t, so $\hat{S}(5) = (1 - \frac{6}{134}) * 0.972 = 0.929$. The probability of continuing to have high debt at $t = 5$, is still as high as 92.9 percent.

The last line of Table 1.7 tells us that the observed maximum duration of an HDE in our sample is 43 years, which happened in Egypt and Guyana. These two countries have debt data available between the year 1970 and 2012. Throughout the whole time when data are available, they were experiencing high debt and the data was censored at year 2012. Thus, these two countries may actually have more than 43 years of high debt if more years of data were available.

The last three columns of Table 1.7 also provide information on the standard error and confidence interval for the Kaplan-Meier estimates. The standard error of Kaplan-Meier estimate is calculated based on the following equation (Greenwood (1926) formula):

$$\widehat{Var}\{\hat{S}(t)\} = \hat{S}^2(t) \sum_{j|t_j \leqslant t} \frac{d_j}{n_j(n_j - d_j)}$$

However, the standard error used for the 95% confidence interval is the asymptotic variance proposed by Kalbfleisch and Prentice (2002a)[12]:

$$\hat{Var}\{ln(-ln\hat{S}(t))\} = \frac{1}{(ln\hat{S}(t))^2}\sum_{t_j \leq t}\frac{d_j}{n_j(n_j - d_j)}$$

Table 1.8 presents side-by-side comparison of the Kaplan-Meier estimates between $YHIF = 0$ and $YHIF = 1$. Apparently, the numbers in the column for $YHIF = 1$ are larger than those in the column with $YHIF = 0$. For example, when time equals 24, the estimate for $YHIF = 1$ is 40.3 percent, while the value for $YHIF = 0$ is only 26.2 percent. Thus, HDEs overlapping with an HIF have a relatively higher probability of remaining in the high debt episode compared to the other group.

Figure 1.2 visualizes results from Kaplan-Meier analysis.[13] The four panels of Figure 2 show the whole sample (Figure 1.2 (a)), and the samples split by *YHIF* and *OECD* (Figure 1.2(b)-1.2 (d)). The numbers above the curves in the graphs are the number of censored observations at the time point. All the curves have negative slopes representing the fact that they have decreasing probabilities of surviving over time, but the speeds of descent change for different time slots. For example, the survival curve in Figure 1.2 (a) is flatter after it reaches 50 percent probability of surviving, representing a slower speed of decreasing.

In Figure 1.2 (c), the red solid curve ($YHIF = 1$) always lies above the blue dashed line ($YHIF = 0$) with a relatively bigger gap when the time is around 19. Meanwhile, it takes less than 15 years for the blue dashed line (lower line) to reach the 50 percent survival probability, while it take more than 20 years for the red solid line (upper line) to reach the same level. Thus, on average, the HDEs with $YHIF = 1$ lasts longer than the other group.

[12] When using Greenwood's formula, the 95% confidence interval generated would be outside the range of $S(t)$. The alternative idea from Kalbfleisch and Prentice (2002a) can solve this problem.

[13] When estimated survival curve doesn't go to zero, the observations with the largest duration were censored.

Comparison between the OECD and non-OECD member is in Figure 1.2 (d). The red solid line (OECD member) is on top of the blue dashed line (Non-OECD). Thus OECD members have higher probability of remaining in the HDE once they enter into the high debt episode compared to non-OECD members, which also implies the average duration of HDE for OECD members is longer than the non-OECD countries. However, since the two lines entangle with each other after a certain some point, formal statistical tests are needed to verify the above statement.

Furthermore, when we categorize the countries into four different groups as shown in Figure 1.2 (b), the lines cross with each other. The four groups are

1) $OECD = 1$ & $YHIF = 1$

2) $OECD = 1$ & $YHIF = 0$

3) $OECD = 0$ & $YHIF = 1$

4) $OECD = 0$ & $YHIF = 0$.

The group which contains non-OECD and $YHIF = 0$ has the lowest probability of surviving compared to all the other groups.

1.5.2 Median and Mean Survival Time

The median survival time is the minimum time at which 50 percent of the subjects are expected to survive. That is:

$$\widehat{t_{50}} = min\{t_i | \widehat{S}(t_i) \leqslant 0.5\}$$

$\widehat{S}(t_i)$ was used in place of $S(t)$, representing the fact that Kaplan-Meier estimate is utilized to get an estimate of the median survival time $\widehat{t_{50}}$.

The mean survival time μ_T is defined as following:

$$\mu_T = \int_0^{t_{max}} \widehat{S}(t)dt$$

where t_{max} represents the maximum observed survival time.

Based on Table 1.9, the estimated median survival time for HDE_{50} is 18 years when we consider all the countries as one group. The median survival time for HDE_{50} with $YHIF = 0$ is 15 years and 22 years for HDEs with $YHIF = 1$ while the median survival time is 17 for non-OECD members and 20 for OECD members.

According to Table 1.10, the restricted mean for the group without HIF is 18.504 and 23.18 for the other group. Similarly, the restricted mean survival time[14] for OECD members is 21.902 years and 19.03 years for non-OECD members. This relative long average duration of public debt ratio is consistent with the results from Reinhart et al. (2012) that "Once a public debt overhang has lasted five years, it is likely to last 10 years or much more". In their paper, their average duration of debt overhang episodes was 23 years in the advanced economies using the data available as early as 1800.

1.5.3 Comparison of Survival Functions

In this part, we test whether the observed differences in Figure 1.2 are statistically significant. In other words, we check the equality of survivor functions across two groups. Several nonparametric statistical tests can be utilized. We will focus on the results from the Log-Rank test and the Wilcoxon test.

Log-Rank Test The Log-rank test is one of the tests that allow us to compare the overall equality of the two survival functions, instead of a specific time point.[15] Generally speaking the test is done by comparing the expected versus the observed number

[14]Since some of the observation are right censored, the restricted mean will underestimate the true mean, thus I also attach the result of extended mean. The extended mean is computed by extending the Kaplan-Meier Product limit survivor curve to zero by using an exponentially fit curve and computing the area under the entire curve. Please refer to Page 121 Mario Cleves and Marchenko (2016) for detailed explanation.

[15]For an explanation, see Appendix C.

of exits for each group and then combining these comparisons over all observed exit times.

Upper part of Table 1.11 lists the results of the test for the survival experience across the two groups: HDEs ovlapping with and without an HIF. The null hypothesis is: $h_1(t|YHIF = 1) = h_2(t|YHIF = 0)$ which assumes the *hazard functions* for the two groups are the same[16]. The column for "Event Observed" refers to the number of failures observed. For countries without an HIF overlap, there are 66 observed exits and 38 exits observed for countries with an HIF. The “Events Expected” refers to the total number of events that would be observed if the two groups share the same survival function[17]. Based on upper part of Table 1.11, the difference between the events observed and events expected is large enough to produce a significant p-value=0.054 for the χ^2 test with one degree of freedom, thus rejecting the null hypothesis within 90 percent confidence interval. This result confirms with the graphical view from Kaplan-Meier estimate and the survivor experiences are different for HDEs with $YHIF = 0$ and $YHIF = 1$.

Lower part of Table 1.11 shows the log-rank test result between the OECD and non-OECD countries. The p-value is 0.2958 which is not statistically significant. The difference between the number of expected events and events observed are not big enough to give us a statistically significant result which in turn tells us that the survival experience is not different between OECD and non-OECD countries.

Wilcoxon test In this section, we test the equality of survivor function using the Wilcoxon test. A detailed discussion regarding the difference between the Wilcoxon test and the Log-Rank test can be seen in Appendix C. Generally speaking, the two ways of test are similar to each other. The only difference is they put different weights

[16]See Appendix A.1 for a definition and discussion of the hazard function, and its relationship to the survival function.

[17]Based on Equation C.1 in Appendix C .

when forming an overall test statistic. The Log-rank test emphasizes the differences between functions at larger values of time while Wilcoxon test puts more weight on smaller values of time. The results for the Wilcoxon test are shown in Table 1.12. The p-value for the chi-square test equals 0.018. The difference between the $HDEs$ with and without an HIF is statistically significant within 95 percent confidence interval. Results from both of the Wilcoxon test and the Log-Rank test are consistent with each other. When we check the results for OECD and non-OECD members, the p-value is 0.233. Thus we fail to reject the null hypothesis and the survival experience is similar whether or not the country is OECD member.

1.5.4 The Nelson-Aalen estimate

Nelson (1972) and Aalen (1978) use a different nonparametric method to reveal the survival experience of the subject of interest. Instead of estimating the survival function directly, they estimate the *cumulative hazards function* $H(t)$ which is the total amount of risks that have been accumulated up to time t.[18] Theoretically, the $H(t)$ is convertible with $S(t)$: $H(t) = -ln\{S(t)\}$.[19] The Nelson-Aalen estimator of $H(t)$ can be retrieved by using the following formula:

$$\widehat{H}(t) = \sum_{j|t_j \leqslant t} \frac{d_j}{n_j}$$

where n_j is the number of countries at risk at time t_j, d_j is the number of exits at time t_j. Intuitively, the Nelson-Aalen estimates the hazard at each distinct time of exit t_j as the ratio of the number of exits d_j that occurred at time t_j to the total number of countries exposed to risk n_j. The cumulative hazard at time t_j is the sum of all the

[18]Please refer to Equation A.1 in appendix A for more detailed explanation.

[19]Generally speaking, the survival functions from Nelson-Aalen estimates are always greater than or equal to the Kaplan-Meier estimator. By using the Taylor series expansion, we can see that $\frac{d_j}{n_j} \leq -ln(1 - \frac{d_j}{n_j})$. However if the size of the risk sets n_j relative to the number of events $d_j$is larger, the Nelson-Aalen and the Kaplan-Meier estimators of the survival function will be similar to each other.

hazards up to time t_j. For example, when $t = 4$, the total number of countries at risk is 144 shown in the column named “Beg. Total” in the Table 1.13. Four countries are in the last year of the HDE_{50}, shown in the column titled “Exit”. Thus the cumulative hazard at time $t = 4$ is $\hat{H}(4) = \frac{4}{141} \approx 0.028$. When time $t = 5$, the total number of countries at risk is 134 (144-4-6=134). The reduction of observation from time 4 to time 5 comes from two different parts: one is from the exiting (we observe four episodes exited HDE), the other part is from censoring (the number of episodes censored at time $t = 4$ is 6 shown in the column titled “Censored”). Similarly, the $h(t|t = 5) = \frac{6}{134}$ with 6 representing 6 observations were in the last year of HDE when $t = 5$. Thus the Nelson-Aalen cumulative hazard estimator at this time point equals to $\frac{6}{134} + 0.028 = 0.073$. The standard error is also shown in the column “Std. Error” of Table 1.13 and it is estimated is based on $\widehat{Var}\{\widehat{H}(t)\} = \sum_{j|t_j \leqslant t} \frac{d_j}{n_j^2}$.

Figure 1.3 shows the graphical results of the Nelson-Aalen estimates. As before, there are 4 panels. As the time passes, the probability of exiting the high debt episodes increases. However, the speeds of exiting are different among different country groups. Figure 1.3 (c) shows the comparison between the country groups with $YHIF = 1$ and $YHIF = 0$ -the blue dashed line (countries without an HIF) lays above the red one (countries with an HIF). So countries coming across an HIF, on average, have longer durations of HDE than the other group. This result is consistent with the result from Kaplan-Meier estimate.

The comparison between OECD and non-OECD members is in Figure 1.3 (d). The two lines which represent two different country groups are closely attached to each other at the beginning of analysis time and then separate with red solid line laying below the blue dashed line (Non-OECD member). When time is around 26 years, the two lines attach to each other again. The later part of the red solid line (OECD members) may not be accurate since the number of observations dropped quickly and only a few observation left.

1.5.5 Discussion

In this section, we used the Kaplan-Meier estimate and Nelson-Aalen estimate to check whether the existence of a high inflation episode is associated with the duration of high debt episode. In general, the existence of a high inflation episode coincides with longer duration of high debt. Meanwhile, whether the country is a member OECD does not appear to be important to the duration of HDEs. One possible channel to explain these results is that government prints new money, which creates high inflation, to pay off the debt burden. Then the country remains an HDE for longer period and will exit when they default or there is an economic boom.

1.6 Survival Experience for High Inflation Episode

1.6.1 Incidence

In this section, I turn things around and check whether the existence of a high debt overlap (*YHDE=1*) leads to a longer span of a high inflation episode (*HIF*) and whether the survival experience is different between OECD and non-OECD members. The methods used for analysis are the same as the previous section with the dependent variable now being the duration of an *HIF*.

1.6.2 Kaplan-Meier Analysis

The results in Figure 1.4 provide the results of the Kaplan-Meier analysis. The comparisons between groups $YHDE_{50} = 1$ and $YHDE_{50} = 0$ show that red solid line ($YHDE_{50} = 1$) lays above the blue dashed line ($YHDE_{50} = 0$). The distances between the two lines change over time and resemble the shape of an oval. A detailed examination reveals that it takes the group with high debt more than *seven* years to reach the 50 percent probability of surviving while it takes less than $five$ years for the group without high debt to reach the same level. Thus the HIFs with $YHDE = 1$ on average, have longer duration comparing to the other groups. On the other hand, the comparison between OECD and non-OECD members tells a different story. The two lines which represent two country groups entangle together which means that there is not much difference in the survival probability between these two groups. Formal statistical tests are needed to verify the conclusion.

Results from the Log-rank test are shown in Table 1.14.[20] The test p-value is 0.0019. Thus the durations of HIF are different depending on whether the country experience high debt. The existence of HDE_{50} interferes with the survival time of HIF_{10}. The lower panel of Table 1.14 provides test result for the survival experience between OECD and non-OECD members: p-value for the Chi-square test is 0.917. We failed to reject the null hypothesis and show that the survival experience is similar between OECD and non-OECD members. The tests from the Wilcoxon test (Table 1.15) shows similar stories to the Log-rank test.

Mean and Median Survival Time According to Table 1.16, the estimated median survival time for HIF_{10} is 6 years for the whole sample. The estimated median survival time for HIFs with $HDE_{50} = 0$ equals 5 years, and the median survival time equals 8 years for the other group. Table 1.17 provides the mean survival times for high inflation episodes. The average duration of HIF_{10} for the group with $HDE_{50} = 0$ is 6.878 years and 9.763 years for the other group with $YHDE_{50} = 1$.

Among the 103 episodes of defined high inflation, 14 of them happened within the OECD countries and the other 86.4% of the observation happened within the non-OECD countries. The median survival time is not much different between OECD and non-OECD members, as shown in Table **??**. The mean survival time for the OECD countries is around 8.714 years while it is 8.629 for non-OECD members (Table 1.17). These results are consistent with the Log-rank test and Wilcoxon test and we can

[20] Among the 124 episodes of HIF_{10}, three of them were censored, thus were excluded in this test.

conclude that whether the country is a member of OECD or not makes no difference in the duration of HIF_{10}.

1.6.3 Nelson-Aalen Estimate

Figure 1.5 shows the graphical results of Nelson-Aalen estimates for HIF_{10}. Firstly, I group the observation by the value of YHDE. The blue dashed line ($YHDE = 0$) lays above the red solid line ($YHDE = 1$) when $t \leq 21$. The total number of observations dropped quickly when $t > 21$, thus the comparison may not be accurate. Based on figure 1.5a, the hazard rate of exiting the HIF faced by the countries without high debt is higher compared to the other group. It takes the group within HDE less than six years to have the cumulative hazard rate to exceed one while it takes the other group more than 11 years to achieve the same level of cumulative hazard rate. Thus Nelson-Aalen estimates also show that it is easier for the countries without high debt to exit the high inflation episodes.

Figure 1.5b visualize Nelson-Aalen estimators for HIF_{10} by dividing the group into OECD and non-OECD countries. The red dashed line (OECD countries) entangles with the blue line (Non-OECD members) for most of analysis time. Thus it is hard by itself to tell from Nelson-Aelon analysis whether OECD member have longer duration of HIF_{10} comparing to the non-OECD members.

1.6.4 Discussion

In this section, we use nonparametric analysis to check about whether the existence of high debt episode is associated with the duration of high inflation. My results indicate that the existence of high debt is associated with longer duration of high inflation episodes. This result is consistent with the result found above for the duration of high-debt episodes.

1.7 Conclusion

This chapter uses non-parametric survival analysis to examine the association between high inflation episodes and high debt episodes. I focus on just two factors that may have association with the duration of each kind of episode. To be more specific, I first investigated the duration of the high debt episodes using the Kaplan-Meier estimator and Nelson-Aelon estimator. In examine the estimated survival function (Kaplan-Meier) and the estimated cumulative hazard function (Nelson-Aelon) for my whole sample. Then, I divide the sample by two different groupings. The first grouping is by whether or not the HDE had at least one year in common with a high inflation episode. The second grouping is whether or not the episode was in a country that is a member of the OECD. Then I check whether the median and mean survival times of the HDE are different when we group by either criterion.

My results indicate, on average, the existence of high inflation episodes is correlated with longer duration of high debt episodes. When the government faces high debt, the most convenient way to pay it off is print money, which activity incurs inflation. However, inflation will not solve the high debt problem ultimately. But it temporarily releases the burden and make the country stay in high debt for longer time. In the end, the government may default which will ruin the country's reputation in the international funds market or government can negotiate new ways to pay off the debts gradually, or a fortuitous economic boom would help the countries get rid the debt burden. But our data doesn't include information about how the countries exit high debt episodes. More research is necessary to address this question.

Secondly, the existence of high debt is associated with larger mean and median survival time of high inflation episodes. During the years when the country have continuous high inflation, the existence of high debt makes the high inflation more sustainable since there is a necessary for the government to print money to pay off the debt. And whether the country is the OECD member doesn't matter for the high inflation episode. Other possible explanation of why it happens is relevant to expectation. Knowing that the country has both high debt and high inflation, concerned investors have an expectation on inflation. This expectation will eventually lead to continuous increase in the price level in the real life even though the government may not actually do that. Meanwhile, high debt level represents a bad shape of the economic growth.

Table 1.1 Overlapping Episode

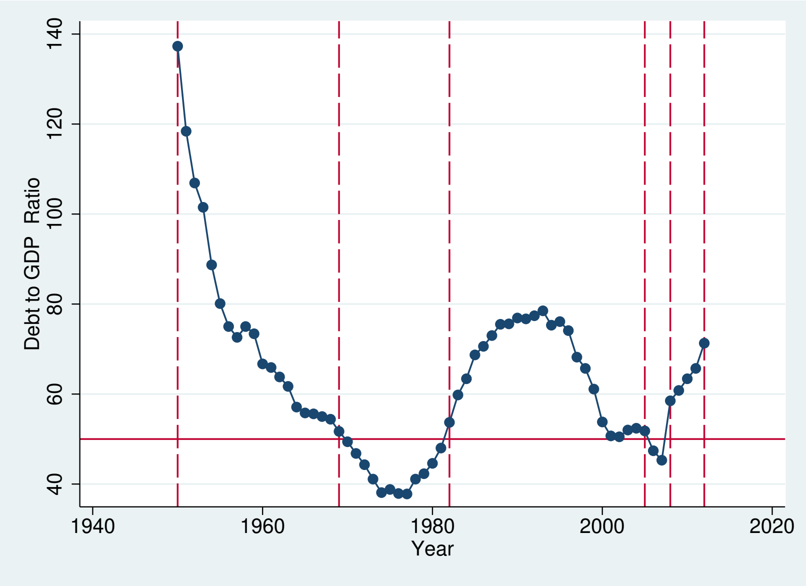
	Unc_HIF	HIF_{20}	HIF_{10}
Unc_HDE		25	131
HDE_{90}	72	14	32
HDE_{50}	144	23	60

Table 1.2 HIF First

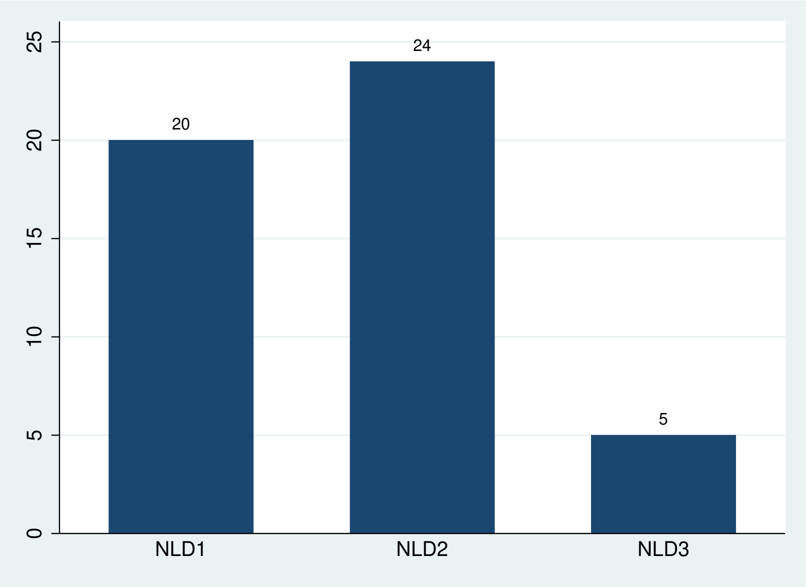
	HIF_{20}	HIF_{10}
HDE_{90}	5	12
HDE_{50}	6	19

Table 1.3 HDE First

	HIF_{20}	HIF_{10}
HDE_{90}	7	18
HDE_{50}	14	36



(a)Netherlands' Debt Ratio



(b)Duration for HDE in Netherlands

Figure 1.1 Survival Time for Netherland

Table 1.4 Simultaneous

	HIF_{20}	HIF_{10}
HDE_{90}	2	2
HDE_{50}	3	5

Table 1.5 Summary of Statistics for HDE_{50}

Event/ Group	YHIF=1	YHIF=0	Total
Exited	38	66	104
Censored	11	29	40
At Risk	49	95	144

Table 1.6 Summary of Statistics for HIF_{10}

Event/ Group	YHDE=1	YHDE=0	Total
Exited	59	69	128
Censored	1	2	3
At Risk	60	71	131

Table 1.7 Kaplan-Meier Output for HDE_{50}

Time	No. at Risk	No. Exited	Censored	Survivor Probability	Standard Error	[95% Conf. Int.]	
4	144	4	6	0.972	0.014	0.928	0.990
5	134	6	3	0.929	0.022	0.872	0.961
6	125	7	0	0.877	0.028	0.809	0.922
7	118	4	1	0.847	0.031	0.775	0.898
8	113	5	0	0.810	0.034	0.733	0.866
9	108	11	0	0.727	0.038	0.644	0.794
10	97	6	0	0.682	0.040	0.596	0.754
11	91	3	2	0.660	0.041	0.573	0.733
12	86	2	1	0.644	0.041	0.557	0.719
13	83	6	0	0.598	0.043	0.509	0.675
14	77	3	0	0.574	0.043	0.486	0.653
15	74	4	2	0.543	0.043	0.455	0.624
16	68	1	0	0.535	0.043	0.447	0.616
17	67	1	1	0.527	0.044	0.439	0.608
18	65	5	2	0.487	0.044	0.399	0.569
20	58	5	0	0.445	0.044	0.358	0.528
21	53	2	1	0.428	0.044	0.342	0.512
22	50	6	1	0.377	0.043	0.293	0.461
23	43	2	1	0.359	0.043	0.276	0.443
24	40	5	1	0.314	0.042	0.234	0.397
25	34	3	1	0.287	0.041	0.209	0.369
26	30	0	2	0.287	0.041	0.209	0.369
27	28	1	0	0.276	0.041	0.199	0.359
28	27	0	1	0.276	0.041	0.199	0.359
29	26	3	0	0.244	0.040	0.170	0.326
31	23	3	3	0.213	0.039	0.142	0.293
32	17	0	1	0.213	0.039	0.142	0.293
33	16	3	2	0.173	0.038	0.106	0.253
34	11	1	0	0.157	0.038	0.092	0.238
35	10	1	1	0.141	0.037	0.079	0.222
36	8	0	1	0.141	0.037	0.079	0.222
37	7	1	1	0.121	0.037	0.061	0.203
38	5	0	1	0.121	0.037	0.061	0.203
40	4	0	1	0.121	0.037	0.061	0.203
41	3	0	1	0.121	0.037	0.061	0.203
43	2	0	2	0.121	0.037	0.061	0.203

Table 1.8 KM Output for HDE_{50} Comparison Between YHIF

Time	Survivor Probability	
	$YHIF = 0$	$YHIF = 1$
4	0.958	1.000
8	0.773	0.878
12	0.594	0.735
16	0.468	0.653
20	0.354	0.592
24	0.262	0.403
28	0.245	0.334
32	0.184	0.263
36	0.103	0.197
40	0.103	0.158

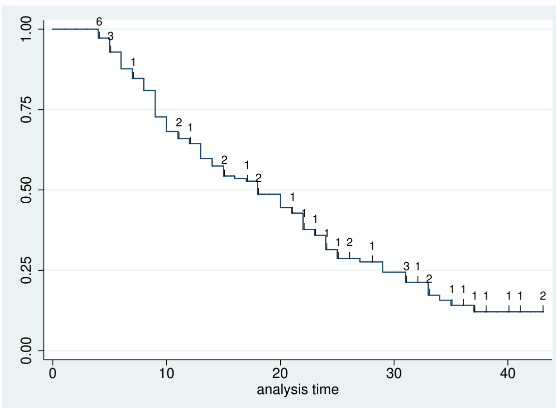
Table 1.9 The Median Survival Time for HDE_{50}

Group By YHIF

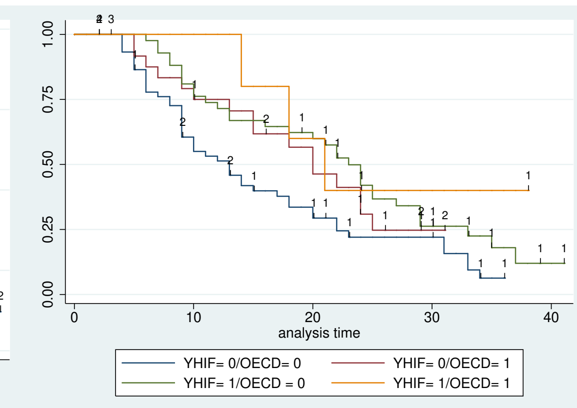
YHIF	No. of Subjects	50%	Std. Err.	95% Conf. Interval	
0	95	15	1.818	11	20
1	49	22	1.450	18	25
total	144	18	2.026	14	22

Group By OECD

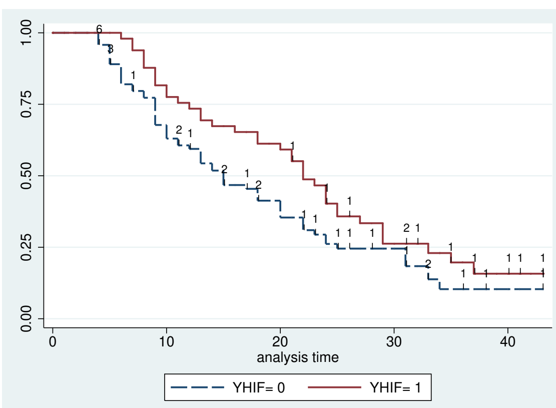
OECD	No. of Subjects	50%	Std. Err.	[95% Conf. Interval]	
0	106	17	2.389	12	22
1	34	20	1.599	15	25
total	140	18	2.109	14	22



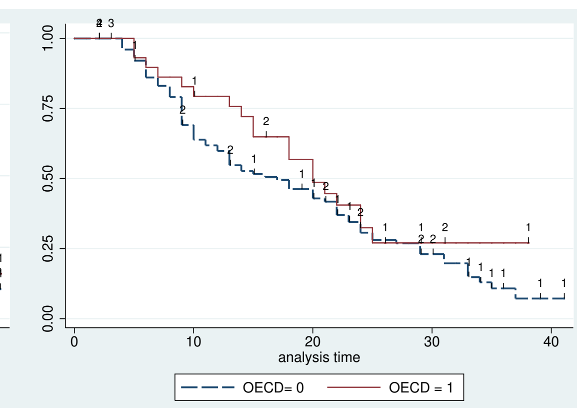
(a) Whole Sample



(b) Group by OECD and YHIF



(c) Group by YHIF



(d) Group by OECD

Figure 1.2 Kaplan-Meier Survival Esimtate

Table 1.10 The Mean Survival Time For HDE50

Group By YHIF

YHIF	No. Of Subjects	Restricted Mean	Std. Err.	[95% Conf.	Interval]
0	95	18.504	1.387	15.786	21.221
1	49	23.180	1.750	19.750	26.611
Total	144	20.200	1.101	18.041	22.359

YHIF	No. Of Subjects	Extended Mean
0	95	20.464
1	49	26.844
total	144	22.667

Group By OECD

OECD	No. Of Subjects	Restricted Mean	Std. Err.	[95% Conf.	Interval]
0	106	19.030	1.191	16.695	21.364
1	34	21.902	2.196	17.599	26.206
Total	140	19.674	1.062	17.592	21.756

OECD	No. of subjects	extended mean
0	106	20.151
1	34	29.758
Total	140	21.253

Table 1.11 Log Rank Test for Equality of Survivor Functions

Group By YHIF

YHIF	Events Observed	Events Expected
0	66	56.59
1	38	47.41
Total	104	104
$\chi^2(1)$=3.72		
$Pr > \chi^2 = 0.054$		

Group by OECD

OECD	Events Observed	Events Expected
0	80	75.74
1	19	23.26
Total	99	99
$\chi^2(1)$=1.09		
$Pr > \chi2 = 0.2958$		

Table 1.12 The Wilcoxon Test for Equality of Survivor Functions

Group by YHIF

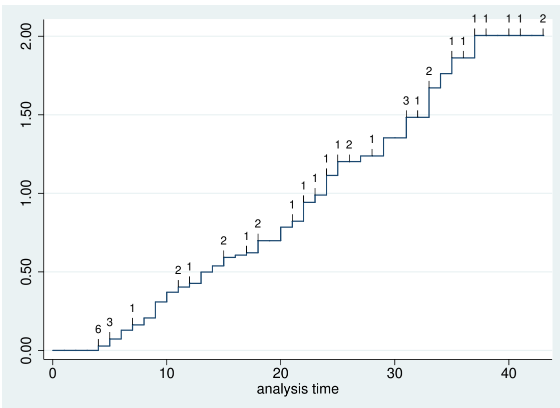
YHIF	Events Observed	Events Expected	Sum of Ranks
0	66	56.59	1018
1	38	47.41	-1018
Total	104	104	0
$\chi^2(1)$=5.60			
$Pr > \chi^2 = 0.018$			

Group by OECD

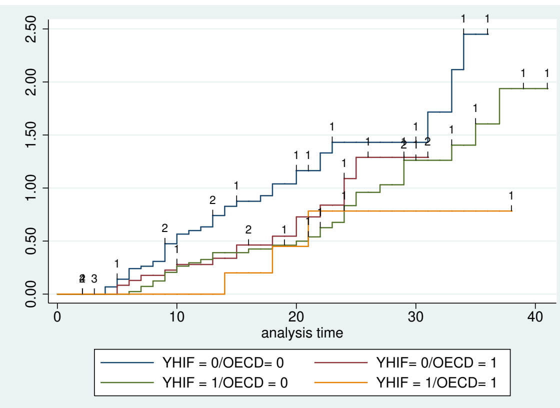
OECD	Events Observed	Events Expected	Sum of Ranks
0	80	75.74	416
1	19	23.26	-416
Total	99	99	0
$\chi^2(1)$=1.42			
$Pr > \chi^2 = 0.233$			

Table 1.13 Nelson-Aalen Estimate for HDE_{50}

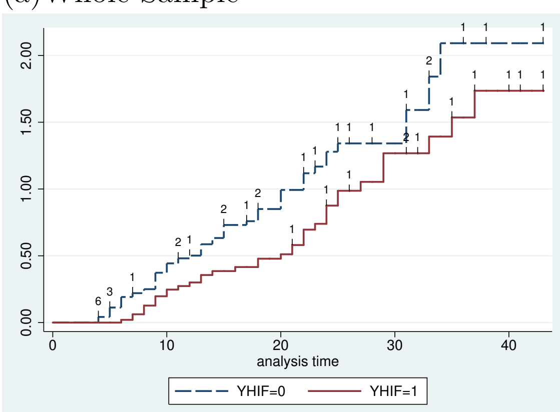
Time	Beg. Total	Exit	Censored	Nelson-Aalen Cum. Haz.	Std. Error	[95% Conf. Int.]	
4	144	4	6	0.028	0.014	0.010	0.074
5	134	6	3	0.073	0.023	0.039	0.135
6	125	7	0	0.129	0.031	0.080	0.207
7	118	4	1	0.163	0.036	0.106	0.249
8	113	5	0	0.207	0.041	0.141	0.304
9	108	11	0	0.309	0.051	0.223	0.427
10	97	6	0	0.370	0.057	0.274	0.501
11	91	3	2	0.403	0.060	0.301	0.540
12	86	2	1	0.427	0.062	0.321	0.568
13	83	6	0	0.499	0.069	0.381	0.654
14	77	3	0	0.538	0.072	0.413	0.700
15	74	4	2	0.592	0.077	0.458	0.765
16	68	1	0	0.607	0.079	0.471	0.782
17	67	1	1	0.622	0.080	0.483	0.800
18	65	5	2	0.699	0.087	0.547	0.892
20	58	5	0	0.785	0.095	0.619	0.996
21	53	2	1	0.822	0.099	0.650	1.041
22	50	6	1	0.942	0.110	0.749	1.186
23	43	2	1	0.989	0.115	0.787	1.243
24	40	5	1	1.114	0.128	0.889	1.396
25	34	3	1	1.202	0.138	0.960	1.505
26	30	0	2	1.202	0.138	0.960	1.505
27	28	1	0	1.238	0.142	0.988	1.551
28	27	0	1	1.238	0.142	0.988	1.551
29	26	3	0	1.353	0.157	1.078	1.699
31	23	3	3	1.484	0.174	1.179	1.868
32	17	0	1	1.484	0.174	1.179	1.868
33	16	3	2	1.671	0.205	1.314	2.126
34	11	1	0	1.762	0.224	1.373	2.262
35	10	1	1	1.862	0.246	1.438	2.412
36	8	0	1	1.862	0.246	1.438	2.412
37	7	1	1	2.005	0.284	1.519	2.647
38	5	0	1	2.005	0.284	1.519	2.647
40	4	0	1	2.005	0.284	1.519	2.647
41	3	0	1	2.005	0.284	1.519	2.647
43	2	0	2	2.005	0.284	1.519	2.647



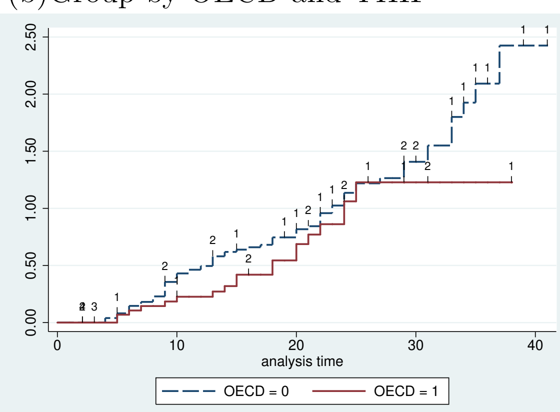
(a)Whole Sample



(b)Group by OECD and YHIF

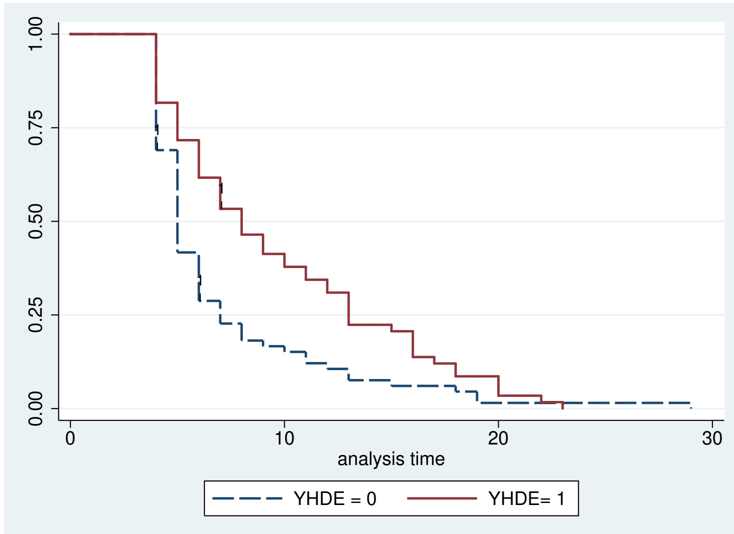


(c) Group by YHIF

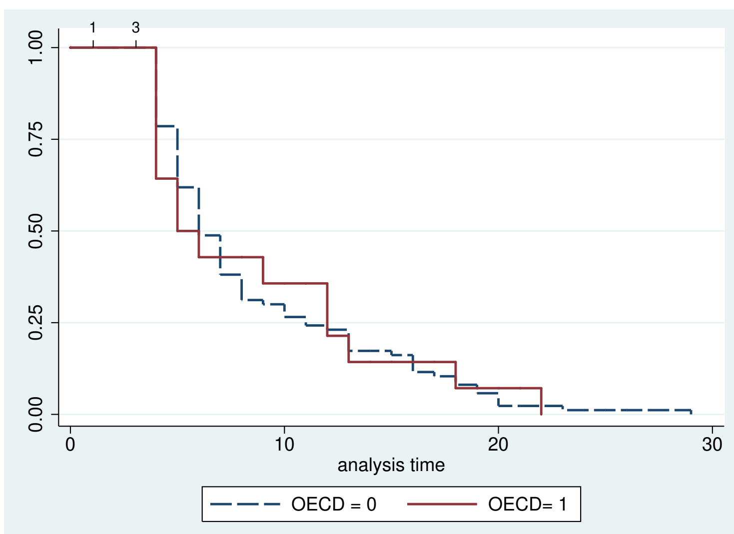


(d)Group by OECD

Figure 1.3 Nelson-Aalen Cumulative Hazard Estimates



Group by YHDE



Group by OECD

Figure 1.4 Kaplan-Meier Survival Estimate for HIF_{10}

Table 1.14 Log-Rank Test for HIF_{10}

Group by $YHDE_{50}$

$YHDE_{50}$	Events observed	Events expected
0	69	53.94
1	59	74.06
Total	128	128

$\chi^2(1) = 9.66$
$Pr > \chi^2 = 0.0019$

Group by OECD

$OECD$	Events observed	Events expected
0	85	85.32
1	14	13.68
Total	99	99

$\chi^2(1) = 0.01$
$Pr > \chi^2 = 0.917$

Table 1.15 Wilcoxon Test for HIF_{10}

Group by YHDE

$YHDE_{50}$	Events Observed	Events expected	Sum of Ranks
0	69	53.94	1495
1	59	74.06	-1495
Total	128	128	0

$\chi^2(1) = 12.59$
$Pr > \chi^2 = 0.0004$

Group by OECD

$OECD$	Events Observed	Events expected	Sum of Ranks
0	85	85.32	-87
1	14	13.68	87
Total	99	99	0

$\chi^2(1) = 0.21$
$Pr > \chi^2 = 0.649$

Table 1.16 The Median Survival Time for HIF_{10}

Group by $YHDE_{50}$

$YHDE_{50}$	No. of Subjects	50%	Std. Err.	95% Conf. Interval	
0	71	5	0.216	5	6
1	60	8	0.952	6	10
Total	131	6	0.376	5	7

Group by OECD

OECD	No. of Subjects	50%	Std. Err.	95% Conf. Interval	
0	89	6	0.458	6	7
1	14	5	1.247	4	12
Total	103	6	0.471	5	7

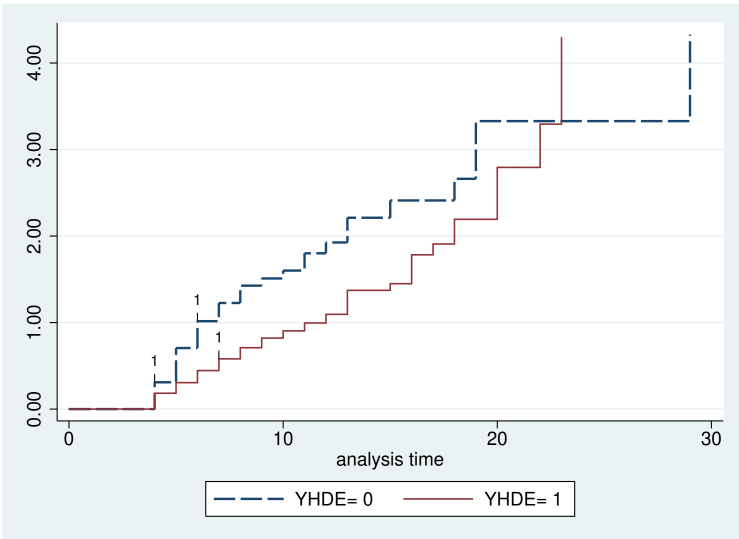
Table 1.17 The Mean Survival Time for HIF_{10}

Group by $YHDE_{50}$

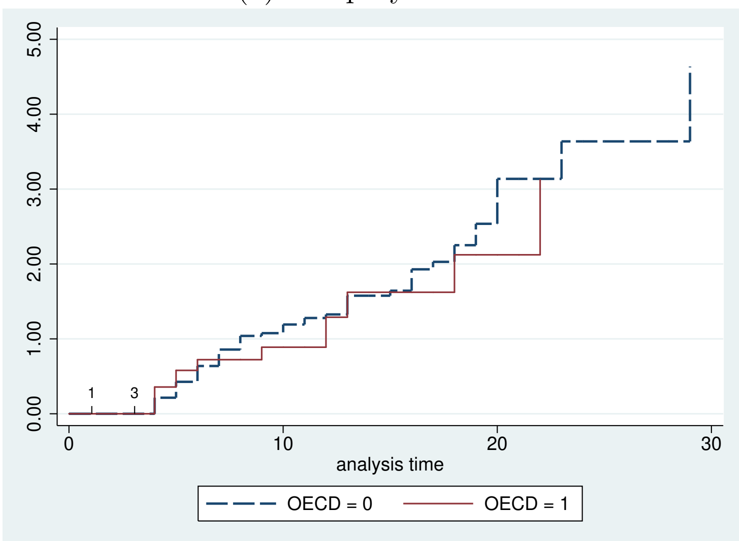
HDE_{50}	No. of Subjects	Mean
0	71	6.878
1	60	9.763
total	131	8.213

Group by OECD

OECD	No. of Subjects	Mean
0	89	8.629
1	14	8.714
Total	103	8.641



(a)Group by YHDE



(b)Group by OECD

Figure 1.5 Nelson-Aalen Estimate for HIF_{10}

Chapter 2

High Debt and High Inflation: A Semi-Parametric and Parametric Approach

2.1 Introduction

When it comes to questions regarding public debt, there are many papers analyzing reasons that may lead to high debt within a country and what policies government may enact to reduce the debt burden. These studies emphasize changes in the debt from year to year. Or they analyze the impact one event or one policy has by comparing debt before and after the event or the application of regulations. For example, in the paper "Growth in a time of debt", Reinhart and Rogoff (2010), using yearly data, find that there is no systematic relation between high debt levels and inflation for advanced economies as a group. However, high public debt levels do coincide with higher inflation in the developing economies. Inspired by their paper, an analysis of the relation between public debt and inflation rate is approached from a different angle. By utilizing survival analysis, we identify potential factors that may have association with the duration of public debt. That is, instead of checking the yearly change in the level of public debt ratio, we emphasize the number of consecutive years countries have sustained a high debt level. Particularly, I focus on the association between high inflation episodes and the duration of high debt episodes.

The recent accumulation of US government debt has raised concerns over the government's financial sustainability and its impact on economic performance (Reinhart and Rogoff (2010)). When governments experience financial crisis, they tend to print money to pay off the debt (Aizenman and Marion (2011)). This extra supply of money erodes the real value of money and diminishes government' s refinance ability in the future (Mandilaras and Levine (2001)). If we ignore the causality between public debt and high inflation, the existence of a high inflation episode should be associated with longer duration of existing high debt episodes since the high inflation makes the high debt level sustainable for more years without committing default. At the same time, the existence of high debt episodes should be associated with longer duration of existing high inflation episodes since as long as there is a high debt level, the government tends to print more money as one way to reduce the real burden of that debt.

In this chapter, I attempt an identification of high debt episodes to obtain insight on how different macroeconomic conditions impinge upon the sustainability of government debt and consecutive years of high inflation. More specifically, I evaluate whether countries experiencing high inflation during a high debt crisis will endure longer period of high debt. I utilize both semi-parametric and parametric survival analysis. Later, I reverse the implied causality and check factors that are associated with the duration of high inflation episodes. This is an extension to my previous work in Chapter 1 which uses non-parametric survival analysis to evaluate the survival experience of high debt episodes and high inflation episodes.

The conclusion for the analysis is that the magnitude and sign of the association between high inflation episodes and high debt episodes depends on the value of the GDP growth rate. The existence of high inflation episodes is associated with either longer or shorter duration of high debt episode: it depends on the value of the GDP growth rate. However, high GDP is associated with the longer high inflation episode. Duration of the HDE is not related to whether the country is a member of OECD. These results are consistent with each other in all the models used including the Cox PH model, exponential distribution and Weibull distribution.

In section 2.2, I describe the data; In section 2.3 and 2.4, the dependent variable of interest is the duration of high debt episodes. In section 2.5, duration of high inflation episodes is the dependent variable. Section 2.6 concludes this chapter.

2.2 Data

The main variable of interest is high government debt. High debt is defined relative to GDP: letting D be the nominal value of public debt and Y be nominal GDP, if the ratio $\frac{D}{Y}$ is above the threshold value of 50 percent, we say that the debt ratio is "high". Often we simply say that debt is "high".[1] Instead of checking yearly data, we focus on *episodes*: a high debt episode (HDE) is defined to be at least four consecutive years of a high debt ratio. Data on debt is derived from the historical public debt database (HPDD) covering almost all IMF countries (174 countries) and spanning several decades (Abbas et al. (2011)). The indicator variable HDE takes the value 1 when a country is in a high-debt episode, and 0 otherwise.

The other main variable of interest is high inflation. Inflation is considered high when it is over 10 percent. A high-inflation episode (HIF) is four or more consecutive years of high inflation. The data for inflation is derived from the Consumer Price Index (CPI). International Financial Statistics (IFS) provides yearly data on CPI covering 155 countries in the world since 1948. The indicator variable HIF takes the value *one* when a country is in a high-inflation episode, and 0 otherwise.

The indicator variable $YHIF$ defines a subset of HDE. It equals 1 whenever the country experiences HIF during any year that the country is in HDE. Even if HIF has only 1 year in common with the HDE, YHIF will take the value one for the entire corresponding HDE. Otherwise, $YHIF$ would take the value zero.

[1] We also use threshold value of 90 percent which reduces the sample size by a large amount. The use of 50 percent is reasonable based on Hansen (2015): he found a threshold value of 44 percent for the debt to GDP value in the US economy.

Symmetrically, the indicator variable $YHDE$ defines a subset of HIF. It equals 1 for the corresponding HIF whenever the country comes across an HDE during any year that the country is in HIF, even if HDE only has 1 year in common with the HIF. Otherwise $YHDE = 0$.

The dummy variable "OECD" equals 1 if the country is a member of the OECD; Otherwise, it equals 0.

2.3 Semi-Parametric Analysis for High Debt Episodes

2.3.1 The Cox Proportional Hazards Model

In this section, we explore potential factors that are associated with the duration of high debt episodes using the Cox proportional hazards model. In particular, we are interested in its association with high inflation episodes. The dependent variable of interest is the duration of HDE_{50} which contains information as to the number of years it takes for each high debt episode to be terminated. In my data, the minimum value of the dependent variable is 4 years since the minimum requirement for the definition of HDE_{50} is 4 consecutive years.

Unlike the Kaplan-Meier analysis used in the previous chapter, which is useful for comparing survival curves in discrete groups, the Cox proportional hazards model allows us to analyze the effect from several covariates that can vary over time. In this part, the Cox model can help determine if the existence of high inflation episodes and other control variables of interest affect the hazard rate for an HDE. The hazard rate $h(t|x_j, x_{-j})$ refers to the probability that a country will get out of the high debt episode in the next instant in the t^{th} year of the high debt episode, given the value of covariate x_j and all other control covariates x_{-j}. Our main control variable is *YHIF*, the indicator for an overlapping high-inflation episode. If we can demonstrate that the hazard rates are different between the country groups with and without a high inflation episode, we can infer that the existence of a high inflation episode is associated with the duration of the high debt episodes.

Based on Cox (1972), for a randomly picked HDE in the sample, the hazard rate equals:

$$h(t|x_j, x_{-j}) = h_0(t)exp(XB) \tag{2.1}$$

where X is a matrix of the covariates and B is the coefficient vector to be estimated. Equation 2.1 is expressed as the product of function $h_0(t)$ and $exp(XB)$. $h_0(t)$ is the so called baseline hazard function which is the hazard rate when all the covariates equal zero ($X = 0$). We assume the baseline hazard function is a function of time with unknown functional form and the only requirement is $h_0(t) \geq 0$. For the base group (when $X = 0$), the probability of getting out of the high debt burden at time t is $h_o(t)$, and we make no assumption about how the value of $h_0(t)$ changes over time[2]. The functional form $exp(XB)$ is chosen to insure that $h(t|x_j, x_{-j}) > 0$. By making the above two assumptions, the probability of getting out of the high debt burden $h(t|x_j, x_{-j})$ at the t^{th} year of a high debt episode is proportional to the baseline hazard.

The coefficient vector B is estimated by maximizing the log of the partial maximum likelihood function (noted as PL) with respect to each element of B. A more detailed explanation as to how we obtain the PL function is provided in Appendix D.2.

Our baseline results are shown in Table 2.1 using the exact marginal approximation to deal with ties.[3] The numbers reported are the elements of $B = (\beta_1, \beta_2, \ldots \beta_j)$.

[2]Compared to parametric regression model, this is actually one advantage of the semi-parametric analysis. One main reason is the parametric analysis may provide incorrect $\widehat{\beta}_x$ if the assumption about the baseline hazard is wrong. However, if we are able to make correct assumptions about the functional form of $h_0(t)$, we can have better estimate of $\widehat{\beta}_x$.

[3]In estimations of the Cox model, one has to decide how to handle ties. "Ties" refer to the fact that in our sample, several countries exit the high debt episode within the same year of the analysis time, making the exact time of exiting unclear. Here, I use the Exact Marginal calculation for adjusting ties (Kalbfleisch and Prentice (2002b) page 104, 130-133). There are different methods for dealing with ties including Exact Marginal, Exact Partial, and the Breslow/Efron method; all are based on different approximations used to settle ties. The Exact Marginal calculation provides

It is customary, however, to focus on e^{β_j} because e^{β_j} is the *relative hazard rate* and has a direct interpretation. Based on Table 2.1 Column (1), the probability for a randomly picked country to exit a high debt episode at the t^{th} year is:

$$h(t|YHIF, OECD) = h_0(t)exp(-0.594\,YHIF - 0.437\,OECD)$$

For the dummy variable $YHIF$, we have two groups of hazard rates:

$$h(t|YHIF = 1, OECD) = h_0(t)exp(-0.594 - 0.437OECD)$$

$$h(t|YHIF = 0, OECD) = h_0(t)exp(-0.437OECD)$$

The relative hazard ratio between the two groups is equal to:

$$\begin{aligned} \frac{h(t|YHIF=1,OECD)}{h(t|YHIF=0,OECD)} &= exp(-0.594) \\ &= 0.552 \end{aligned}$$

Countries with a high inflation overlap in general only have 55.2 percent probability of exiting the high debt episode compared to countries without an HIF overlap. The coefficient for covariate "YHIF" is statistically significant within 99 percent confidence interval. This result does not change in a big degree when we add more controls to the regression, as shown in other columns of Table 2.1. This means that high debt episodes last longer when they overlap with a high inflation episode.

The coefficient for covariate OECD is -0.437 in Column (1) of Table 2.1. The hazard ratio of $e^{-0.437} \approx 0.646$ shows the hazard rate is 35.4 percent lower if the country is a member of OECD. The result is statistically significant within 90 percent confidence interval. However, statistical significance is gone when we add other control variables.

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a better fit for the estimates but it is time consuming (Borucka (2014)). Exact marginal calculation assumes those countries which exit the high debt episode at the same year may not exit at the same time of the year. Our knowledge is limited by how precisely the data is measured. Appendix E provides detailed explanation about the difference between different approximations used.

GDP has been shown by many economists to have a significant association with the debt ratio of country (Easterly (2001); Abbas et al. (2011)). When we add *ln(GDP)* as one of the controls as shown in Column (2) of Table 2.1, the coefficient is 0.117. This indicates with every one percent unit increase in GDP, the probability of getting out of a high debt episode increases by $exp^{0.117} \approx 1.124$ percent. However, this coefficient is not statistically significant.

In Column (3) of Table 2.1, I add the growth rate of GDP[4] (Noted as g.GDP) as the alternative control instead of $ln(GDP)$. The coefficient for the GDP growth rate is 3.312 and it is statistically significant within 99 percent confidence interval. The comparison between $ln(GDP)$ and GDP growth rate provide some evidence that what matters for the duration of high debt episode is not the absolute value of GDP itself, but the growth rate of GDP. Hazard ratio for GDP growth rate equals $e^{3.312} = 27.440$ showing that when the GDP growth rate increases by one unit, it increases the probability of getting out of the high debt episode for the country in a large degree (*27* times) . This agrees with researches that GDP is the engine that drives the country out of the burden of a debt trap.

Further investigation is done in Column (4) of Table 2.1, which adds an interaction term between YHIF and the GDP growth rate. The coefficient for the GDP growth rate turns out to have no statistical significance but the interaction term is statistically significant within 99 percent confidence interval.

The hazard rate function for the regression in Column (4) can be written as:

$$h(t|\text{g.GDP}, YHIF, OECD) = exp(\beta_1 YHIF + \beta_2 OECD + \beta_3 \text{g.GDP} + \beta_4 YHIF * \text{g.GDP})$$

Take the log of the above equation, we have:

$$ln\{h(t|\text{g.GDP}, YHIF, \text{OECD})\} = \beta_1 YHIF + \beta_2 OECD + \beta_3 \text{g.GDP} + \beta_4 YHIF * \text{g.GDP}$$

[4] GDP Growth rate$_t$=$ln(GDP)_t - ln(GDP)_{t-1}$

The difference between the group with and without inflation can be shown in the following expression (a is GDP growth rate):

$$\begin{aligned} & ln(h|YHIF = 1,\ \text{g.GDP} = a,\ \text{OECD}) - ln(h|YHIF = 0,\ \text{g.GDP} = a,\ \text{OECD}) \\ &= (\beta_1 + \beta_2 OECD + \beta_3 a + \beta_4 a) - (\beta_2 OECD + \beta_3 a) \\ &= \beta_1 + \beta_4 a \end{aligned}$$

The estimated hazard ratio between the two groups with and without an high inflation episode is $e^{\beta_1+\beta_4 a}$ whose value is changing depending on the value of a. When the growth rate of GDP equals to 0.2, the hazard ratio is 1.670. When the country's GDP growth rate equals 20 percent, the country with YHIF=1 has 67 percent *higher* probability of getting out of the high debt episode compared to the group with $YHIF = 0$.[5] However, when the GDP growth rate equals 2 percent, the hazard ratio changes to 0.492, which means that the country with HIF has 51 precent *lower* probability of getting out of the high debt episode compared with the group without an HIF. Table 2.2 demonstrates that the effect of YHIF shown in the form of hazard ratio is only positive at high rates of GDP growth. When the GDP growth rate is around 2 percent, the existence of YHIF are shown to be associated with longer duration of the HDE.

Column 3 and 4 of Table 2.2 show the 95 percent confidence interval of hazard ratio calculated based on the following equation:

$$\hat{\beta}_1 + \hat{\beta}_4 a \pm z_{1-\alpha/2} \hat{SE}(\hat{\beta}_1 + \hat{\beta}_4 a)$$

$$where$$

$$\hat{SE}(\hat{\beta}_1 + \hat{\beta}_3 a) = \begin{bmatrix} \hat{Var}(\hat{\beta}_1) + a^2 \hat{Var}(\hat{\beta}_4) \\ +2a\hat{Cov}(\hat{\beta}_1, \hat{\beta}_4) \end{bmatrix}^{0.5}$$

[5] 2% is the mean of GDP growth rate.

In our regression,

$$\hat{Var}(\hat{\beta}_1) = 0.064$$
$$\hat{Var}(\hat{\beta}_4) = 4.664$$
$$\hat{Cov}(\hat{\beta}_1, \hat{\beta}_4) = -0.253$$

For example, the 95 percent confidence interval of the hazard ratio when the growth rate of GDP equals 2.1 percent is $(0.317, 0.764)$ with the estimated hazard rate to be 0.492. All the hazard rates within the within 95 percent confidence interval are less than *one*. When GDP growth rate is around 12.6 percent, the hazard ratio between *YHIF=0* and *YHIF=1* is approximately 1. So the 12.4 percent can be called the *marginal growth rate of GDP* whose value impinge the relation between high debt episodes and high inflation episodes.

Postestimation of the baseline cumulative hazard function Based on the regression results in Table 2.2 Column (4), we can generate the baseline survival function and the baseline cumulative hazard function over time. First, we examine the estimated survival function. The blue dashed line in Figure **??** is the probability of survival for the base group when all the covariates equal zero. There is a general trend of decreasing probability of survival over time. However, there is a huge gap between the two groups. The country group with $YHIF = 1$ has a higher probability of survival.

$$\begin{aligned} \hat{S}_1(t) &= \hat{S}_0^{exp(\hat{\beta}_1)} \\ &= \hat{S}_0^{0.427} \end{aligned}$$

where $S(t)$ is the survival function, that is "the probability that a subject selected at random will have probability of surviving longer time than t". Then we examine the estimated cumulative hazard function. The green dashed line in Figure2.1 (b) depicts the cumulative baseline hazard function changing over the analysis time with YHIF=0. It is generated based on the estimated value of $\hat{B}$:

$$\hat{H}_0(t) = \sum_{j:t_j \le t} \frac{d_j}{\sum_{j \in R_j} exp(X_j \hat{B})}$$

where d_j represents the number of countries that were in the last year of a high debt episode at the t_j observed failure time.[6] The blue solid line represents the cumulative hazard for the country group with $YHIF = 1$ and all the other covariates equal zero, which is obtained by using the following equation:

$$\hat{H}_1(t) = \hat{H}_0(t) * exp(\hat{\beta}_1)$$

which is based on the assumption that hazard rates are constant across the two country group. When the GDP growth rate is equal to zero, the relative hazard rate in between equals $exp(\hat{\beta}_1)$ which is $exp(-0.852) = 0.427$. So,

$$\hat{H}_1(t) = \hat{H}_0(t) * 0.427$$

The gaps existing between the dashed green and solid blue lines show that the existence of $YHIF = 1$ does tell us a different story for the duration of the high debt episode.

The Alternative Approximation to Deal With Ties Table 2.3 shows the estimated coefficients of the Cox-PH model utilizing the four different approximations available dealing with ties. Ties, in the sample, refer to the fact that in the t_j observed time of failure, several countries were in the last year of high debt episode simultaneously and we have no information delineating the exact order of countries exiting the high debt episodes. And there are four different methods used to approximate the actual scenario. For detailed information regarding the difference between the four methods, please refer to Appendix E.

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[6] When all the values of covariates equal zero, the estimator of $H_0(t)$ in the above equation is the same as Nelson-Aalen estimator which is expressed as:

$$\hat{H}(t) = \sum_{j:t_j \leq t} \frac{d_j}{n_j}$$

where n_j is total number of countries that were in the high debt episode, at the t_j observed failure time.

Column (1) of Table 2.3 employs the Breslow approximation. Robust standard error is used to correct for the heteroskedasticity in the sample. Column (2) uses Efron's approximation. Column (3) use the exact-marginal calculation and the last column depicts the conditional-partial calculation. Though different approximations show us slight difference in the impact of the covariates, the overall results are consistent with one another. The coefficient for $YHIF$ is around -0.8 and statistically significant within 99 percent confidence interval. The coefficients for the GDP growth rate are not statistically significant, but all the interaction terms are shown to be significant within 99 percent confidence interval and its value is around 6.5 which implies that the GDP growth rate has a positive effect in helping the country to exit the high debt episode. Meanwhile, when the GDP growth rate is zero, the hazard ratio between the country group with and without YHIF is around 44.9 percent.

More importantly, Table 2.4 shows the calculated marginal GDP growth rates for each of the Cox PH models. The largest value is 0.135 and the smallest is 0.122 which means that in the Cox model, the economy should maintain at least 12.2 percent GDP growth rate so that the existence of an high inflation episode is associate with shorter duration of high debt episodes.

In this section, we have shown that high inflation episodes are correlated with perpetuated high-debt episodes while high real growth does the opposite. My research does not check the causality and there are several channels through which GDP growth and the duration of high debt episodes are related. First of all, nominal GDP (Y), is in the denominator of our latent dependent variable $\frac{D}{Y}$. When Y increases, the debt-to-GDP ratio decreases. One the other hand, public expenditure such as medical care expense usually increase with Y which would lead to higher nominal debt. This effect is contrary to the first effect. Based on the results, real growth is associated with a shorter duration of HDE. Thus my result provides evidence that GDP growth rate can drive the economy out of the debt burden quicker.

At the same time, the association between *YHIF* and duration of high debt episodes can be explained by the sovereignty held by the government to print money. There are arguments regarding whether the government can escape high debt via high inflation. In the case of the US, higher inflation can help erode the real value of debt since most of the debt is in fixed nominal terms and held by people who reside outside of the country. Thus based on the current results, we can say, high inflation, if not strong enough to help the country get out of the high debt burden, seems to allow countries to remain in high debt episodes longer, without having debt default, since the countries are cushioned due to the eroded value of existing debt.

2.3.2 Robustness Check for the Proportional Hazards Model

In this section, I perform robustness checks for the Cox PH model. In this model, the basic assumption is the hazard rates across covariates are proportional to each other. In this part, I will use different methods to check the validity of this assumption.

The first method is the so called graphical comparison. Based on the PH assumption $h(t|x_j, x_{-j}) = h_0(t)exp(XB)$, we can derive that $S(t|x_j, x_{-j}) = S_0(t)^{exp(XB)}$ [7]. Additional manipulation of the survival function allows us to determine an equal distance between $-ln[-ln\{S(t|x_j, x_{-j})\}]$ and $-ln[-ln\{S_0(t)\}]$ for x_j which is a dummy variable:

$$-ln[-ln\{S(t)\}] = -ln[-ln\{S_0(t)\}] - XB$$

If the proportional-hazards assumption holds, the plotted curves of $-ln[-ln\{\hat{S}(t)\}]$ versus $ln(t)$ should be parallel between the two values of the dichotomous covariate x_j with an equal distant of β_j. According to Figure 2.2, the lower dashed blue line represents the country group with $YHIF = 0$ while the upper red solid line represents

[7] Since $h(t|x_j) = h_0(t)exp(x_j\beta_x)$, $H(t|x_j) = \int_0^t h(t|x_j)d_t = \int_0^t h_0(t)exp(x_j\beta_j)d_t = exp(x_j\beta_j)\int_0^t h_0(t)d_t = exp(x_j\beta_j)H_0(t)$. Also since $S(t) = e^{-H(t)}$, $S(t) = e^{-exp(x_j\beta_j)H_0(t)} = e^{-H_0(t)exp(x_j\beta_j)} = S_0(t)^{exp(x_j\beta_j)}$

$S(t) = e^{-H(t)} = e^{-H_0(t)exp(\beta' X_i)} = \{e^{-H_0(t)}\}^{exp(\beta' X_i)} = S_0(t)^{exp(\beta' X_i)}$

the country group with $YHIF = 1$ while controlling for GDP growth rate. These two lines shown are considered parallel which suggests that the PH assumption holds..

Scaled Schoenfeld Residual In the second method, I will utilize the scaled Schoenfeld residuals.[8] Based on Grambsch and Therneau (1994), the effect of the covariate on the surviving experience of subjects can be separated into two parts, the first is constant over time (a) and the second part can be expressed as a function of time $g_j(t)$ with the constant coefficient b.

$$\beta_j(t) = a + bg_j(t)$$

with $g_j(t)$ being a function of time t. Based on the PH assumption, $\beta_j(t)$ should be constant over time. Thus, the PH assumption holds only if $b = 0$.

Additionally, Grambsch and Therneau (1994) proved the scaled Schoenfeld residual $\hat{r^*}$ has the following property:

$$E(r_j^*(t)) \cong bg_j(t) \tag{2.2}$$

Thus, the scaled Schoenfeld residuals retrieved from the original Cox model can be used to check whether the PH assumption holds given different specifications of the function $g_j(t)$. We follow the literature and choose to include $g_j(t) = t$, $g_j(t) = ln(t)$, $g_j(t) = \hat{S}_{KM}(t)$ and $g_j(t) = rank(t)$. $\hat{S}_{KM}(t)$ is the value of estimated Kaplan Meier statistic corresponding to each observed time of failure. $rank(t)$ is a new series of numbers which is generated by placing the observed failure time t and assigning the order of time t as the new number. If the PH assumption is correct, the plot should have a zero slope fitted line. In other words, there should be no discernible pattern in the graph.

The scatterplots of the scaled Schoenfled residuals in Figure 2.3 are quite supportive of the results in the score test. Figure 2.3 shows the plots for the scaled Schoenfled

[8] The detailed description necessary to retrieve Schoenfeld residual mathematically can be found in Appendix F.

residuals for the dummy variable YHIF versus $g_j(t)$. We notice two bands of points in each of the four plots. The upper band shows subjects which have *YHIF=1* and the lower band corresponds to the subject with *YHIF=0*. There is no discernible pattern in the plots and the dots appear to be "randomly" scattered about zero. Accordingly the LOWESS smooth (locally weighted scatter plot smoothing) has a roughly zero slope. In summary, all four plots show no evidence of violating the proportional hazard assumption.

The third method is similar to the second one. The difference is it utilized a statistical test since visual comparison used may not be persuasive enough. Based on Equation (2.2), Grambsch and Therneau (1994) derived a general least square estimator of coefficients $\hat{b}$ and we use the score test to determine if $HO : b = 0$. Rejection of the null hypothesis indicates a deviation from the proportional hazard assumption. The results of the score test with the different specifications of $g_u(t)$ are shown in Table 2.5. The results indicate there is no evidence of the hazards being non-proportional in all the covariates under control. All the $p - values$ are shown to be statistically insignificant including the global test in the bottom row. Thus, we fail to reject the null hypothesis and our assumption the shape of hazard is the same across covariates is supported.

Overall Goodness of Fit The Cox-Snell residual can be utilized to check the overall goodness of fit of the model. If the PH assumption holds, CS_i should be distributed as a censored sample from a standard exponential distribution whose hazard function equals to one and the corresponding cumulative distribution function is a straight 45 degree line passing through the origin. If the Cox PH model fits well, a plot of the cumulative hazard based on the Cox-Snell residuals should be a line with 45-degree slope passing through the origin.

The test result is shown in Figure 2.4. The cumulative hazard of the Cox-Snell residuals moves around the 45 degree line. Noted in Mario Cleves and Marchenko (2016), "...even if we have a well-fitting Cox model, some variability about the 45 degree line is still expected...". The blue line fluctuates closely around the red line, so the model appears to be working as predicted.

2.4 Parametric Analysis for High Debt Episodes

In this section, I use parametric analysis to check the survival experience of high debt episodes. There are many different models among parametric analysis which differ from each other by the assumptions used regarding the distribution of the baseline hazard function. By applying an assumption that the shape of the baseline hazard to be monotonic over time, we use the exponential and Weibull model to analyze potential factors affecting the duration of high debt. Meanwhile, the exponential and Weibull model are the only two models having both proportional hazard and "accelerated failure time" (AFT) interpretation. However, it does not matter whether the results are shown in relative hazard ratio or AFT metrics, they are the same model. Even though there are differences in the coefficients estimated, they are transformable to each other.[9] As noted by David Hosmer Jr. (2008), the parametric accelerated failure time model gives an analysis of censored survival time data that is easy to interpret.

Even though their results can be directly compared with the Cox model, the method used parametric analysis to exploit information in the data is different from the Cox model. First of all, the Cox model compares survival experience of the countries in the high debt episode only during times when there were countries exiting the high debt episode, while parametric analysis do not simply rely on such comparisons. Instead it incorporates all information available each year whenever the data

[9] The coefficients estimates are different from each other. For the Weibull model, we have $\beta_{AFT} = \frac{-\beta_{PH}}{\lambda}$.

are available. Secondly, the Cox model (also called semi-parametric analysis) does not specify the complete baseline hazard function and we only estimate the relative hazard rate between different values of covariates while parametric model can specify the baseline hazard function[10].

Based on Equation (2.1), the Cox PH model can be written as:

$$h(t|x_j, x_{-j}) = h_0(t)exp(XB) \tag{2.3}$$

with $h_0(t)$ being left unspecified. In the parametric analysis, we make assumptions regarding the functional form of $h_0(t)$.

If we assume $h_0(t) = exp(a)$ for some constant value of a, Equation (2.3) is called the exponential model with unknown parameters (a, B) to be estimated.

If we assume $h_0(t) = \lambda t^{\lambda-1} exp(b)$ for some constant values of λ and b, Equation (2.3) is called the Weibull model with unknown parameters (λ, b, B) to be estimated.

We can compare the coefficient estimated from the exponential model and Weibull model with the Cox model to ascertain which of the models is a better fit. A better model should produce coefficients similar to the coefficients in the Cox model. The direct comparability with the Cox model is also one of the most appealing features of the Weibull and exponential models[11].

Another important feature of the parametric model is its accelerated failure time (AFT) interpretation. For a time-to-event variable t, which is the duration of the high debt episode in this part, an accelerated failure time model has the following expression:

$$T = exp(XB) \times \epsilon \tag{2.4}$$

The above equation characterizes the relation between t and the covariate vector X. Since time t must be positive, we assume the error component ϵ to be positive at all

[10]Please refer to Appendix D.2 for detailed explanation regarding mathematical derivation of coefficients.

[11]Mario Cleves and Marchenko (2016) page 236

times. Making mathematical manipulation to Equation (2.4), we have:

$$ln(T) = XB + ln(\epsilon) \tag{2.5}$$

The assumption regarding the distribution of ϵ in Equation (2.5) determines whether the model is the exponential model or Weibull model. If ϵ follows the exponential distribution, Equation (2.5) is called the exponential regression model. When the $ln(\epsilon)$ follows the Weibull distribution with two parameters (β_0, λ) where λ is called the shape parameter., the model is called the Weibull regression model[12].

2.4.1 Exponential Model

Under the exponential model specification, the baseline hazard function is constant over time. The corresponding survival function from the AFT metric for the exponential model is

$$S(t, X, B) = exp(-\frac{T}{exp^{XB}}) \tag{2.6}$$

where B is the vector of coefficients estimated from the model in the form of AFT. By setting $S(t, X, B) = 0.5$, we can obtain the median survival time for the high debt episode:

$$T_{50}(X, B) = -exp(XB) \times ln(0.5)$$

Thus, the ratio of the median survival time for a dichotomous covariate x_1 is:

$$TimeRatio = \frac{T_{50}(x_1 = 1, B)}{T_{50}(x_1 = 0, B)} = \frac{-exp(\beta_1 * 1)exp(\beta_{-1}x_{-1})ln(0.5)}{-exp(\beta_1 * 0)exp(\beta_{-1}x_{-1})ln(0.5)} = e^{\beta_1}$$

with β_1 being the coefficient for covariate x_1 and β_{-1} being coefficients for all other control variables.

[12] David Hosmer Jr. (2008) page 245.

The hazard function[13] corresponding to Equation (2.6) is[14]:

$$h(T, X, B) = e^{-(XB)}$$

and the hazard ratio for the dichotomous covariate x_1 is:

$$HR = e^{-\beta_1}$$

Table 2.6 and 2.7 demonstrate the regression results for the exponential model in two different forms. Table 2.6 shows the results in the form of relative hazards rate and Table 2.7 shows the results in the form of accelerated failure time. For example, the hazard ratio $e^{-\beta_1}$ for the covariate $YHIF$ is equal to 0.663 based on results shown in Table 2.6 Column (1). It is statistically significant within a 95 percent confidence interval. It tells us that the country group with YHIF faces a lower probability of ending the high debt episode. This is the case when I only control two variables of interest.

Similarly, based on Table 2.7 Column (1), the median duration of a high debt episode for non-OECD member countries with YHIF equals:

$$\hat{T}_{50}(YHIF = 1, OECD = 0) = -e^{2.893+0.411} * ln(0.5)$$

The median duration of the high debt episode for a non-OECD member without YHIF equals to:

$$\hat{T}_{50}(YHIF = 0, OECD = 0) = -e^{2.893} * ln(0.5)$$

Thus the estimated time ratio between the two categories of countries is:

$$\hat{TR}(\frac{YHIF = 1}{YHIF = 0}) = e^{0.411} = 1.508$$

[13]Noted that coefficient estimated based on the AFT and PH metrics are different from each other. In exponential model, $\beta_{AFT} = -\beta_{PH}$, In this part, all the B refers to coefficients estimated based on AFT metrics. Based on Equation 2.3, the hazard ratio for a dichotomous covariate should be $HR = e^{\beta_{PH}}$.

[14]$exp(a)$ is counted as constant part of estimated coefficients.

The median duration of high debt episode among countries groups with $YHIF = 1$ is estimated to be 1.508 times the median duration of high debt episode among countries with $YHIF = 0$. The estimated 95 percent confidence interval is :

$$(e^{-0.0088} \leq TR \leq e^{0.830}) = (0.991 \leq TR \leq 2.294)$$

The median duration of HDE with YHIF=1 is estimated to be between 0.991 and 2.294 times of those episodes without HIF within 95 percent confidence interval. Similarly, the median duration of HDE for OECD members with HIF equals to 28.06. The estimated time ratio for the dichotomous covariate $OECD$ equals:

$$\hat{TR}(\frac{OECD = 1}{OECD = 0}) = 1.49$$

The survival time for OECD members is about 1.49 times that of the non-OECD members. However, the regression results are not statistically significant. Also shown in Table 2.6 Column (1), the hazard ratio between OECD and non-OECD members is 0.672 with no statistical significance. Other columns in Table 2.6 and 2.7 show us results of the regression model when we add controls for different forms of GDP. Even when we add controls, the results are consistent with each other. Column (2) adds the logarithm of yearly GDP for the country, Column (3) adds the annual growth rate of GDP. The last column of the Table includes the annual growth rate of GDP and its interaction with the dummy variable YHIF. All the regressions in Table 2.6 show that YHIF has a multiplicative effect on the duration of the high debt episode.

The coefficient for YHIF represented in the last column of Table 2.7 equals 0.563 and it is statistically significant within 95 percent confidence interval; The coefficient for the interaction term is -3.929 and it is statistically significant within 95 percent confidence interval demonstrating the impact of YHIF changes depending on the value of the annual growth rate of GDP. When GDP growth rate equals to 0.019 (mean value of GDP growth rate), the estimated time ratio for the country group with HIF versus country groups without HIF is:

$$\hat{TR}(\frac{YHIF=1}{YHIF=0}|\text{g.GDP}=0.019, OECD) = e^{0.563-3.929*0.019} = 1.630$$

Thus, the regression results show what really matters is the growth rate of GDP instead of the absolute value of GDP. Also, based on the above equation, when the GDP growth rate equals 13.6 percent, the time ratio for the dummy variable *YHIF=1.* When the GDP growth rate is higher than the marginal value, the time ratio for $YHIF$ is smaller than 1; otherwise, it is larger than 1. This confirms with the previous results that with higher GDP growth rate, YHIF is associated with the shorter duration of HDEs; with relative longer GDP growth rate, YHIF is associated with the longer duration of HDEs.

The estimated time ratio comparing the duration which differs by one percentage point in *GDP growth rate* is

$$\hat{TR}(\frac{g.GDP+0.01}{g.GDP}|OECD, YHIF=0) = e^{-1.608*0.01} = 0.984$$

$$\hat{TR}(\frac{ln(GDP)+0.01}{ln(GDP)}|OECD, YHIF=1) = e^{-1.608*0.01-3.929*0.01} = 0.946$$

Thus, the duration of high debt episode for subjects with *one* percentage point higher GDP growth rate is shorter than those episodes with a smaller GDP growth rate.

When we make comparisons between the results from the Cox model and the exponential model, we find inconsistencies between coefficients estimated. For example, the hazard ratio for YHIF is 0.552 in the Cox model in Table 2.1 Column 1 while it is 0.663 in the exponential model based on Table 2.6 Column 1. These inconsistencies may suggest a constant baseline hazard assumption is wrong. We, therefore, choose another specification that allows $h_0(t)$ to change over time.

2.4.2 Weibull Model

The Weibull model assumes the baseline function to be:

$$h_0(t) = \lambda t^{\lambda-1} exp(b)$$

with λ being the so called ancillary shape parameter. This is also estimated from the model. If $\lambda = 1$, then the hazard function is constant, the Weibull model is the same as the exponential model. When $\lambda > 1$, the Weibull hazard is monotonically increasing; when $\lambda < 1$, the Weibull hazard model is monotonically decreasing.[15] The hazard rate in the Weibull model changes over time and can be written as[16]:

$$\begin{aligned} h(t) &= h_0(t)exp(x_j\beta_x) \\ &= \lambda t^{\lambda-1}exp(\beta_0 + x_j\beta_x) \\ &= \lambda t^{\lambda-1}exp(XB) \end{aligned}$$

In the following part, I estimate the Weibull model in both the PH metric and the AFT metric.

Consider the the AFT metric. adjusting Equation 2.4, we have

$$\epsilon = exp(-XB) \times T \tag{2.8}$$

and e^{-XB} is called the acceleration parameter since its value determines whether the covariates speed up or slow down the duration of the episode. Meanwhile, we can retrieve the median survival time for the subjects by equating its survival function to 0.5, so that we can have:

$$T_{50}(x, \beta, \lambda) = [-ln(0.5)]^{\frac{1}{\lambda}} e^{\beta_0 + x_j\beta_x}$$

Then for the dummy variable $YHIF$, the time ratio at the median survival time is:

$$\hat{TR}_{YHIF} = \frac{t_{50}(YHIF = 1, \beta, \lambda)}{t_{50}(YHIF = 0, \beta, \lambda)} = e^{\beta_{YHIF}}$$

[15] $exp(b)$ is the scale parameter and $exp(b) = exp(\beta_0)$ with β_0 being the constant part of the coefficient estimated from the model

[16] The hazard ratio for the dichotomous covariate x_1 is:

$$HR = e^{\beta_1}$$

with β_1 to be the coefficient estimated from the PH metric. The survival function corresponding to the proportional hazard assumption is:

$$S(t|x_j) = exp\{-exp(\beta_0 + x_j\beta)t^{\lambda}\} \tag{2.7}$$

Results from the Weibull distribution are summarized in Tables 2.8 and 2.9. Table 2.8 illustrates hazard rates estimated from proportional hazard and Table 2.9 delineates the coefficients estimated from the AFT metric.

Table 2.8 and 2.9 both show four different regressions to analyze whether the existence of a high inflation episode has an association with the duration of a high debt episode. The hazard ratio for $YHIF$ shown in Table 2.8 tells us that it indeed matters. The average magnitude of hazards rate is around 0.5 and it is statistically significant within 99 percent confidence interval. For those countries experiencing a high inflation episode during the time when they have a high debt episode, they face a lower risk of exiting a high debt episode and have a longer duration of high debt episode. In other words, it is harder for the countries to eliminate the high debt. This is the case when we only control only two variables of interest including *YHIF* and *OECD.*

In the last column of Table 2.9, we control for YHIF, OECD and GDP growth rate, the time ratio for the median survival time for the dummy variable YHIF when GDP growth rate equals 0.019 is:

$$\hat{TR}_{YHIF} = e^{0.451-2.075*0.019} = 1.509$$

The country group with HIF has 1.509 times longer survivor time compared to the country group without HIF. Meanwhile, the marginal GDP growth rate in this model equals 21.73 percent. If GDP growth rate is higher than 21.73 percent, those duration of HDEs with *YHIF=1* is shorter while the opposite is also true.

For the same model, the hazard ratio for covariate OECD is 0.660, and it is not statistically significant. When we control for $ln(GDP)$, $ln(GDP)$ is not statistically significant. However, when we control for GDP growth rate, it is statistically significant. The following two equations show the time ratio for the median survival time when GDP growth rate increase by 0.01.

$$\hat{TR}(\frac{g.GDP + 0.01}{g.GDP}|OECD, YHIF = 0) = e^{-0.718*0.01} = 0.993 \quad (2.9)$$

$$\hat{TR}(\frac{g.GDP + 0.01}{g.GDP}|OECD, YHIF = 1) = e^{-0.718*0.01-2.057*0.01} = 0.973 \quad (2.10)$$

In general, countries with higher GDP growth rate have shorter duration of HDE. As shown in Equation 2.9, when $YHIF = 0$ and controlling for OECD, the median survival time of high debt episode for countries with one percentage point higher GDP growth rate is 0.7 percent shorter than those episodes with smaller GDP growth rate. When $YHIF = 1$ shown in Equation 2.10, the magnitude of impact is larger and the median survival time is 2.7 percent shorter.

The estimation of $ln(\lambda)$ in the Weibull model are shown to be statistically significant within 99 percent confidence interval, thus we can reject the null hypothesis that $\lambda = 1$ and conjecture that the Weibull model is a better fit compared to the exponential model. Additionally, all the four columns of Table 2.9 have $\lambda > 1$ which means that the hazard function is monotonically *increasing* over time. Meanwhile, the estimated hazard ratios from the Weibull model are closer to the hazard ratios from the Cox PH model compared with the hazard ratios estimated from exponential model. For example, the hazard ratio for YHIF for Weibull model in Table 2.8 Column 4 equals 0.475, and it equals 0.569 in Table 2.6 Column 4 for the exponential model. For the Cox PH model, it equals 0.427 based on Table 2.1 Column 4. So the Weibull model provides a better fit.

2.5 Survival Experience for High Inflation Episode

In this section, I turn things around, and check potential factors that are associated with the duration of high inflation episode (HIF). The dependent variable of interest is the duration of HIF_{10} as is defined earlier. The main variable of interest is the dummy variable $YHDE$ which equals 1 whenever the country comes across an HDE

during any year that the country is in HIF. Three survival analysis models are utilized including the Cox PH model, the exponential model, and the Weibull model.

2.5.1 The Cox PH Model

The baseline results from the Cox PH model are shown in Table 2.10 using the exact marginal approximation to deal with ties. Similar to the previous part, the Cox PH models have no intercept and the coefficients shown should be exponentiated to represent the ratio of the hazards for unit change in the covariate. For example, the coefficient for $YHDE$ is -0.461 in the Column (1) in Table 2.10. Then the relative hazard ratio between the high inflation episodes with $YHDE = 1$ and $YHDE = 0$ is:

$$\begin{aligned} \frac{h(t|YHDE=1, OECD)}{h(t|YHDE=0, OECD)} &= exp(-0.461) \\ &\approx 0.631 \end{aligned}$$

The coefficient is statistically significant within 95 percent confidence interval. Thus HIFs with $YHDE = 1$ in general only have 63.1 percent probability of exiting the high inflation episode compared to HIFs with $YHDE = 0$. The magnitude of the coefficient and level of significant do not change in a big degree when I add more controls to the regression, as shown in other columns in Table 2.10.

The coefficient for $OECD$ is -0.0263, the hazard ratio equals $e^{-0.0263} \approx 0.974$ shown in column 1 in Table 2.10. However, the coefficient is not statistically significant. Even when we add other alternative controls, the results do not change. Thus whether the country is a member of the OECD does not matter for the duration of high inflation episode.

Two different types of indicator for economic development are included as additional controls. The first is $ln(GDP)$, its estimated coefficient is -0.249, the hazard ratio equals $e^{-0.249} \approx 0.780$. And it is statistically significant within 95 percent confidence interval. This suggests that higher GDP is associated with longer duration of the defined high inflation episode. The alternative choice is the GDP growth rate.

Its coefficient is 4.519, the hazard ratio is $e^{4.519} \approx 91.74$, and the coefficient is statistically significant within 99 percent confidence interval. Adding the interaction term to the regression in column (3) changed this result. As shown in column (4), when the interaction term between GDP growth rate and YHDE were added, the coefficient for GDP growth rate itself becomes insignificant, as is the interaction term itself., This result is different compared to our result in the previous part. GDP growth rate does not impinge the association between the high debt overlap and duration of the high inflation episodes.

2.5.2 Exponential Model

Table 2.11 demonstrates the regression results for the exponential model in the relative hazard metric and Table 2.12 shows the coefficients from AFT metric. The numbers shown are the relative hazards for the model. For example, in the Column (1) in Table 2.11, the listed hazard rate for $YHDE$ is 0.735 and there is no statistical significance. Even though the magnitude of $YHDE$ remains stable around 0.6-0.7, the statistical significance does not always hold. The statement that existence of high debt episode is associated with longer duration of the high inflation episode which can be confirmed in other models in this chapter can not be made here.

In Column (1), the hazard rate for OECD equals 0.951, however it is not statistically significant. The magnitude of the hazard rate and level of significance for OECD does not change even when I add other alternative controls. This shows that whether or not the country is a member of OECD do not matter for the duration of the high inflation episode.

The variable $ln(GDP)$ is used as a additional control in Column (2). The hazard rate for $ln(GDP)$ in column (2) equals 0.894 but it is not statistically significant. Alternatively, when I use the GDP growth rate as additional control, the hazard rate for GDP growth rate equals 45.254 and it is statistically significant within 99 percent confidence interval.

However, when I add an additional control - the interaction terms between GDP growth rate and YHDE, the statistical significance in the GDP growth rate changed. It is now no longer statistically significant but the interaction term is statistically significant within 95 percent confidence interval. In Column (4) Table 2.12, the time ratio for YHDE is based on the formula $e(0.463 - 5.092 * g.GDP)$. When the GDP growth rate is higher than 9.09 percent, the existence of HDE is associated with shorter duration of HIF. However, if the GDP growth rate is lower than 9.09 percent, the existence of HDE is associated with longer duration of HIF.

2.5.3 Weibull Model

The regression result from Weibull model for the HR metric is included in Table 2.13 and the AFT metric is included in Table 2.14 and the numbers provided in Table 2.14 are coefficients. When we only include $YHDE$ and $OECD$ as controls, the coefficient for $YHDE$ shown in Column (1) equals 0.257. Thus the time ratio at the median survival time is:

$$\hat{TR}_{YHDE} = \frac{t_{50}(YHDE = 1, \beta, \lambda)}{t_{50}(YHDE = 0, \beta, \lambda)} = e^{\beta_{YHDE}} = e^{0.257} = 1.293$$

and it is statistically significant within 95 percent confidence interval. For countries experiencing a high inflation episode, coming across a high debt episode does matter for the duration of high inflation episode. When we add GDP growth rate and its interaction term with *YHDE,* the estimated marginal GDP growth rate is $\frac{0.356}{2.991} =$ 12.229 percent which means that when GDP growth rate is high than the 12.229 percent, time ratio for *YHDE* is smaller than one; otherwise, time ratio is larger than one. This means when GDP growth rate is higher than 12.229 percent, those HIFs with *YHDE=1* has a shorter duration compared to the other group; when the GDP growth rate is lower than 12.229 percent, those HIFs with *YHDE=1* has a longer duration compared to the other group. The significant confidence interval is only around 90 percent.

Similar to the previous part, *OECD* does not matter for the duration of high inflation episodes. None of the estimated coefficients are shown to be statistically significant. The GDP growth rate matters for the duration of high inflation and its impact is correlated with the existence of high debt episodes.

In this section, we used three models to check potential factors that are associated with the duration of the high inflation episodes. Results from these three models are quite consistent with each other. Based one the estimated results from Weibull model in last column of Table 2.13, $ln(\hat{\lambda})$ equals 0.559. So the ancillary shape parameter $\hat{\lambda} = 1.57$ which means the hazard function is monotonically increasing over time instead of constant over time. Weibull model is a better fit compared with the exponential model. Comparing the exponential model and Weibull model with the Cox PH model also reveals that the hazard rates estimated from the Weibull model are more similar to the Cox PH model.

2.6 Conclusion

This chapter first investigates the potential factors that have association with the duration of high debt episodes. Our results demonstrate that the association between high inflation episodes and high debt episodes relies heavily on the GDP growth rate. In general, higher GDP growth rates are associated with shorter high debt episode. However, when the GDP growth rate is lower than some value, the existence of high inflation episodes is associated with longer duration of high debt episodes. Whether the country is a member of the OECD or not doesn't matter for the duration of the HDEs. This result is consistent across the Cox PH model, the exponential model, and the Weibull distribution.

This chapter also investigates potential factors that have an association with the duration of high inflation episodes. In general, high inflation episodes with $YHDE = 1$ have longer duration if GDP growth rate is lower than some threshold value. This effect is not stable across different models used. Whether the country is an OECD member or not does not matter for the duration of high inflation episodes.

I only show results for a threshold value of 50 percent for high debt episodes and 10 percent for high inflation episodes. The threshold value of 90 percent for the high debt episode could also be used. However, it would reduce the sample size significantly and so any results would be very imprecisely estimated.

Table 2.1 The Cox Proportional Hazards Model-HDE_{50} Exact Marginal

VARIABLES	(1)	(2)	(3)	(4)
YHIF	-0.594***	-0.545**	-0.523**	-0.852***
	(0.219)	(0.224)	(0.222)	(0.253)
OECD	-0.437*	-0.661**	-0.397	-0.417
	(0.263)	(0.330)	(0.264)	(0.264)
$ln(GDP)$		0.117		
		(0.107)		
g.GDP			3.312***	1.392
			(0.956)	(1.330)
YHIF*g.GDP				6.823***
				(2.160)
Observations	2,296	2,296	2,289	2,289

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.2 Estimated HR for YHIF with Fixed GDP Growth Rate

g.GDP	HR	95% CIE	
–0.062	0.279	0.180	0.434
0.021	0.492	0.317	0.764
0.054	0.617	0.397	0.957
0.09	0.788	0.508	1.224
0.2	1.670	1.075	2.592
0.75	71.183	45.851	110.509

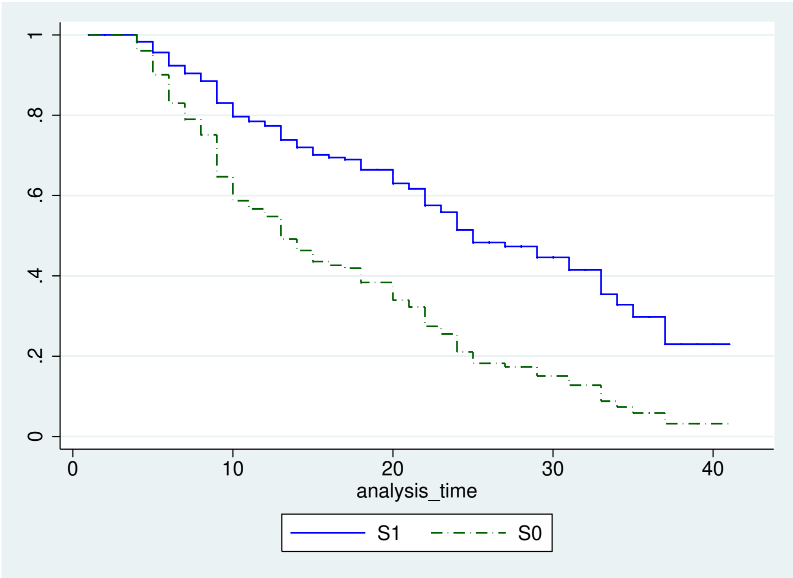
Table 2.3 The Cox Model Using Different Approximation

	(1) Breslow	(2)Efron	(3) Exact-Marginal	(4)Exact-Partial
YHIF	-0.809***	-0.841***	-0.852***	-0.884***
	(0.233)	(0.241)	(0.253)	(0.262)
OECD	-0.408	-0.418	-0.417	-0.436
	(0.249)	(0.256)	(0.264)	(0.271)
g.GDP	1.308	1.380	1.392	1.441
	(1.061)	(1.146)	(1.330)	(1.396)
YHIF*g.GDP	5.986***	6.509***	6.823***	7.270***
	(1.503)	(1.571)	(2.160)	(2.412)
Observations	2,289	2,289	2,289	2,289

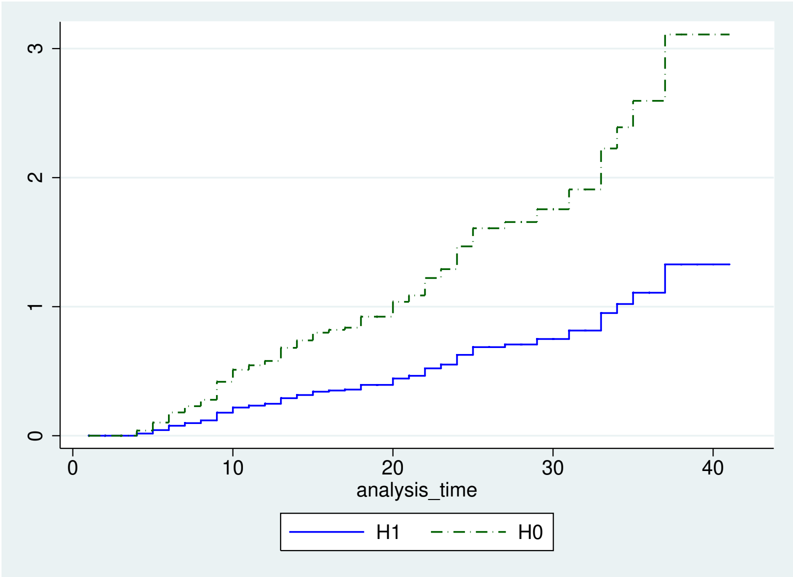
Robust standard errors in parentheses
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 2.4 Cutoff GDP Growth Rate

	(1) Breslow	(2)Efron	(3) Exact-Marginal	(4)Exact-Partial
Cutoff g.GDP	0.135	0.129	0.125	0.122



(a) Survival Probability Over time



(b) Cumulative Hazards Function

Figure 2.1 Post Estimation of Cox PH Model

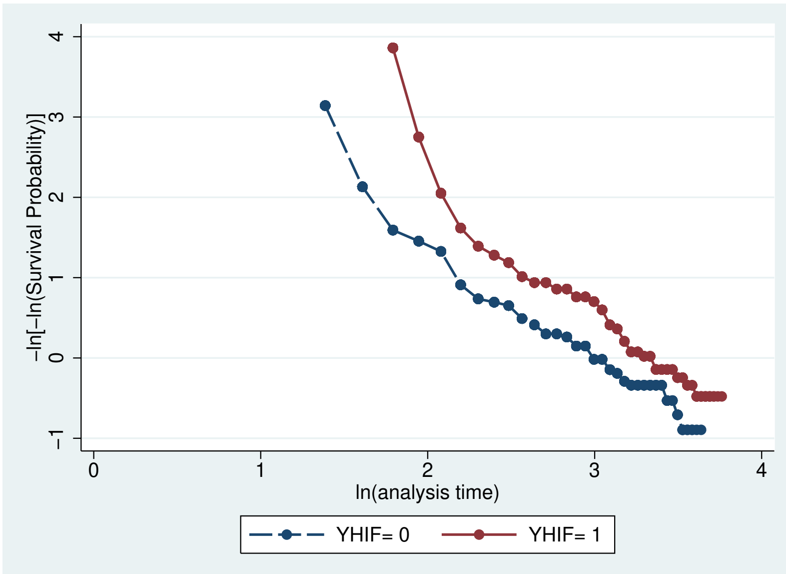
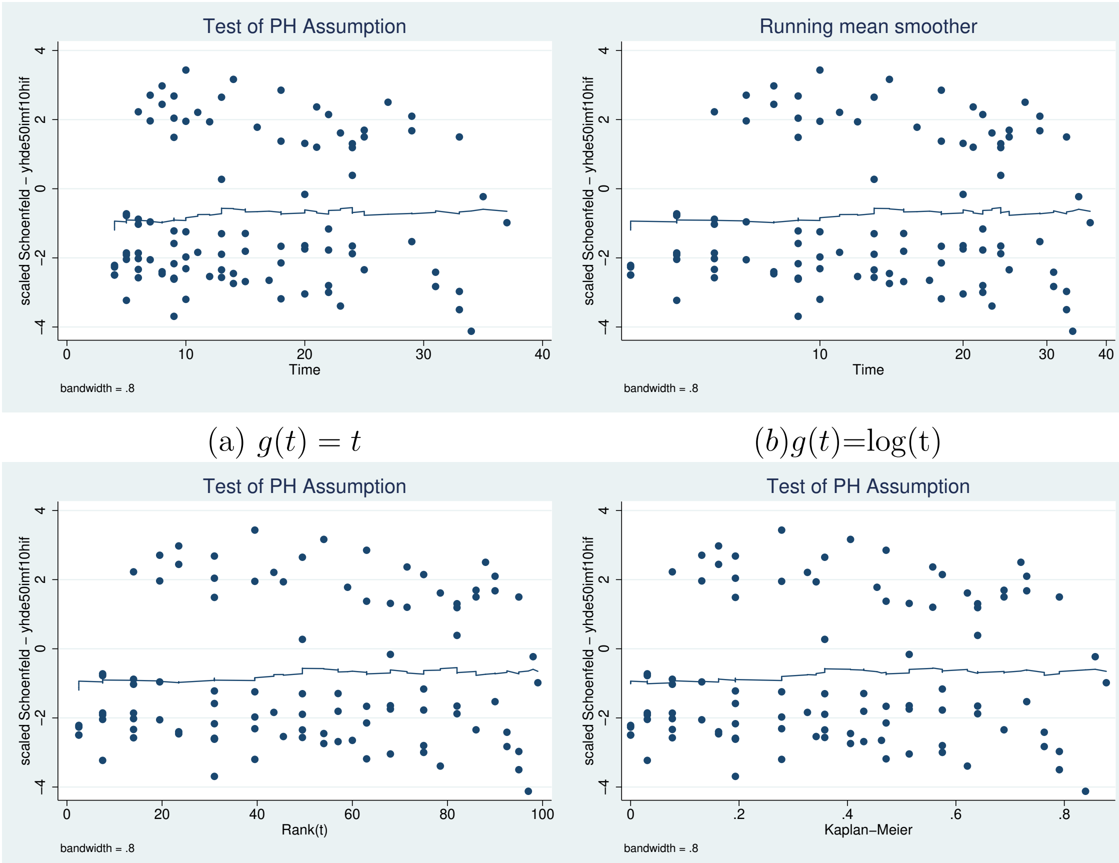


Figure 2.2 Test for the PH Assumption for YHIF

Table 2.5 Score Test and $p-$values for the Test of Proportional Hazards Assumption for HDE_{50}

		$g(t) = t$		$g(t) = ln(t)$		$g(t) = \hat{S}_{KM}(t)$		$g(t) = rank(t)$	
Covariate	df	$chi2$	p	chi2	p	chi2	p	chi2	p
YHIF	1	0.29	0.588	0.92	0.341	0.70	0.403	0.80	0.372
OECD	1	0.50	0.478	1.02	0.313	0.78	0.376	0.96	0.327
g.GDP	1	0.36	0.549	0.03	0.860	0.16	0.689	0.09	0.763
YHIF*g.GDP	1	0.08	0.7832	0.01	0.906	0.00	0.948	0.00	0.990
Global	4	1.24	0.8720	1.093	0.748	1.73	0.785	1.89	0.756



(a) $g(t) = t$ (b)$g(t)$=log(t)

(c) Rank of Time (d) Kaplan-Meier Estimate

Figure 2.3 Scatterplot of Scaled Schoenfeld Residuals for HDE_{50}

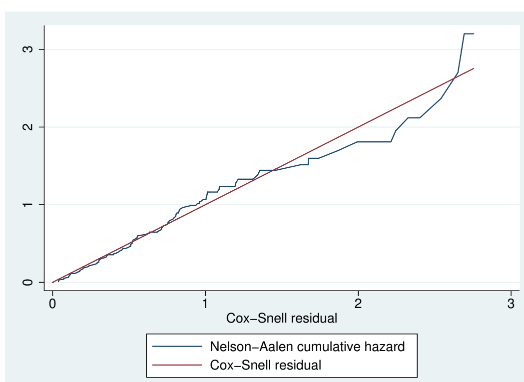


Figure 2.4 Overall Fitness of the Model-HDE_{50}

Table 2.6 Exponential Model for HDE_{50}-Hazard Ratio

VARIABLES	(1)	(2)	(3)	(4)
YHIF	0.663*	0.697*	0.695*	0.569**
	(0.142)	(0.151)	(0.149)	(0.132)
OECD	0.672	0.521**	0.700	0.682
	(0.176)	(0.167)	(0.184)	(0.179)
ln(GDP)		1.151		
		(0.122)		
g.GDP			21.648***	4.994
			(17.523)	(5.709)
g.GDP*YHIF				50.853**
				(80.400)
Constant	0.055***	0.017***	0.049***	0.053***
	(0.008)	(0.016)	(0.007)	(0.008)
Observations	2296	2296	2289	2289

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.7 Exponential Model for HDE_{50}-AFT

VARIABLES	(1)	(2)	(3)	(4)
YHIF	0.411*	0.361*	0.363*	0.563**
	(0.214)	(0.217)	(0.215)	(0.232)
OECD	0.398	0.653**	0.357	0.382
	(0.261)	(0.320)	(0.262)	(0.262)
ln(GDP)		-0.140		
		(0.106)		
g.GDP			-3.075***	-1.608
			(0.809)	(1.143)
g.GDP*YHIF				-3.929**
				(1.581)
Constant	2.893***	4.055***	3.011***	2.936***
	(0.143)	(0.896)	(0.152)	(0.150)
Observations	2296	2,296	2,289	2,289

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.8 Weibull Model for HDE_{50}-Hazard Ratio

VARIABLES	(1)	(2)	(3)	(4)
YHIF	0.542***	0.565***	0.564***	0.475***
	(0.117)	(0.124)	(0.123)	(0.112)
OECD	0.649	0.523**	0.673	0.660
	(0.170)	(0.169)	(0.177)	(0.173)
ln(GDP)		1.120		
		(0.115)		
g.GDP			13.283***	3.272
			(11.226)	(3.943)
g.GDP*YHIF				29.934**
				(48.996)
Constant	0.007***	0.003***	0.007***	0.008***
	(0.003)	(0.003)	(0.003)	(0.003)
$ln(\lambda)$	0.534***	0.527***	0.513***	0.502***
	(0.080)	(0.080)	(0.082)	(0.083)
Observations	2,296	2,296	2,289	2,289

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.9 Weibull Model for HDE_{50}-AFT

VARIABLES	(1)	(2)	(3)	(4)
YHIF	0.359***	0.337***	0.343***	0.451***
	(0.126)	(0.128)	(0.129)	(0.142)
OECD	0.254	0.383**	0.237	0.252
	(0.155)	(0.195)	(0.158)	(0.160)
ln(GDP)		-0.0668		
		(0.0614)		
g.GDP			-1.549***	-0.718
			(0.534)	(0.735)
g.GDP*YHIF				-2.057**
				(1.015)
Constant	2.915***	3.466***	2.977***	2.935***
	(0.0845)	(0.517)	(0.0913)	(0.0916)
$ln(\lambda)$	0.534***	0.527***	0.513***	0.502***
	(0.0801)	(0.0800)	(0.0819)	(0.0827)
Observations	2,296	2,296	2,289	2,289

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.10 The Cox Proportional Hazards Model-HIF_{10}

VARIABLES	(1)	(2)	(3)	(4)
YHDE	-0.461**	-0.616***	-0.449**	-0.584**
	(0.208)	(0.225)	(0.208)	(0.228)
OECD	-0.0263	0.283	-0.0429	-0.0298
	(0.292)	(0.333)	(0.292)	(0.292)
$ln(GDP)$		-0.249**		
		(0.126)		
g.GDP			4.519***	1.508
			(1.222)	(2.518)
g.GDP*YHDE				3.906
				(2.849)
Observations	854	854	853	853

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.11 Exponential Model for HIF_{10}-Hazard Ratio

VARIABLES	(1)	(2)	(3)	(4)
YHDE	0.735	0.699*	0.760	0.630**
	(0.150)	(0.148)	(0.155)	(0.138)
OECD	0.951	1.087	0.967	0.980
	(0.276)	(0.352)	(0.281)	(0.285)
$ln(GDP)$		0.894		
		(0.105)		
g.GDP			45.254**	1.620
			(53.774)	(3.094)
g.GDP*YHDE				162.755**
				(381.558)
Constant	0.140***	0.361	0.124***	0.138***
	(0.022)	(0.363)	(0.021)	(0.023)
Observations	854	854	853	853

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.12 Exponential Model for HIF_{10}-AFT

VARIABLES	(1)	(2)	(3)	(4)
YHDE	0.308	0.358*	0.275	0.463**
	(0.204)	0.212	(0.203)	(0.220)
OECD	0.051	-0.084	0.034	0.020
	(0.290)	0.324	(0.291)	(0.291)
$ln(GDP)$		0.112		
		(0.118)		
g.GDP			-3.812***	-0.482
			(1.188)	(1.910)
g.GDP*YHDE				-5.092**
				(2.344)
Constant	1.965***	1.020	2.086***	1.978***
	(0.160)	(1.006)	(0.021)	(0.166)
Observations	854	854	853	853

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.13 Weibull Model for HIF_{10}-Hazard Ratio

VARIABLES	(1)	(2)	(3)	(4)
YHDE	0.637**	0.522	0.645**	0.536***
	0.129	0.117	0.131	0.120
OECD	0.929	1.363	0.957	0.964
	0.269	0.456	0.278	0.280
$ln(GDP)$		0.741**		
		0.094		
g.GDP			67.311***	1.838
			87.852	4.222
g.GDP*YHDE				186.627*
				507.371
Constant	0.024	0.268	0.021***	0.024***
	0.009	0.285	0.008	0.009
$ln(\lambda)$	0.562***	0.599***	0.566***	0.559***
	(0.0720)	(0.0720)	(0.0727)	(0.0726)
Observations	854	854	853	853

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.14 Weibull Model for HIF_{10}-AFT

VARIABLES	(1)	(2)	(3)	(4)
YHDE	0.257**	0.357***	0.249**	0.356***
	(0.116)	(0.121)	(0.115)	(0.129)
OECD	0.0423	-0.170	0.0247	0.021
	(0.165)	(0.182)	(0.165)	(0.166)
$ln(GDP)$		0.165**		
		(0.0676)		
g.GDP			-2.391***	-0.348
			(0.748)	(1.313)
g.GDP*YHDE				-2.991*
				(1.573)
Constant	2.125***	0.723	2.200***	2.135***
	(0.0947)	(0.582)	(0.0980)	(0.100)
$ln(\lambda)$	0.562***	0.599***	0.566***	0.559***
	(0.0720)	(0.0720)	(0.0727)	(0.0726)
Observations	854	854	853	853

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Chapter 3

Negative Growth and High Debt: A Survival Analysis Approach

3.1 Introduction

In their paper "Growth in a Time of Debt", Carmen Reinhart and Kenneth Rogoff found an inverse association between high public debt and the growth rate of real GDP per capita across countries (Reinhart and Rogoff (2010)). This paper triggered a new round of discussion regarding debt policy and its effect on economic activity. Although it has been criticized for its data omissions and coding errors (Herndon et al. (2014)), later work has been inconclusive as to the main result. It has been difficult to demonstrate strong evidence for the causality going from high debt to economic growth or vice versa (Panizza et al. (2013)). Even though the discussions have been inconclusive, both empirically and theoretically, the negative association emphasized by Reinhart and Rogoff's paper has had an impact on the direction of research and public policy.[1] The relation between government debt issuance and the GDP growth rate is a topic worth exploring more fully.

In this chapter, we approach this question from a different perspective. First of all, we equate an economic crisis to a protracted fall in real output per capita, which we call a "negative growth episode" or, simply, a depression. Instead of investigating the yearly change of the GDP growth rate and public debt (or average of five year yearly

[1] In February 2010, George Osborne, the soon-to-be British Chancellor of the Exchequer, cited the results of Reinhart and Rogoff (2010) as he called for austerity policies on government spending.

data), which are the common approaches in the recent work, we focus on the length of negative growth periods across different countries. Then we use survival analysis to analyze the *duration* of economic crises and their relationship to episodes of high public debt-to-GDP ratios.[2] The empirical methodology includes non-parametric, semi-parametric, and parametric survival analysis. All the regressions show consistent results: high debt is positively correlated with long duration of negative growth episodes – countries with high debt ratios seem to be those for which it is also harder to get out of a depression. Causality, however, is difficult to establish. Does the high debt lead to a lengthening of the depression? Or, do long depressions lead countries to borrow more relative to their GDP? While causality is important, we think it is important to begin the discussion about duration of crises and existence of debt.

This chapter is organized as follows. In the next section we provide a review of the recent literature. In Section 3.3 we describe our data. We begin our analysis to see if the existence of high-debt episodes are systematically related to the duration of negative growth episodes in Sections 3.4 and 3.5. There, we use the non-parametric method of Kaplan-Meier. In Section 3.6 we study the same problem using the Cox Proportional Hazards semi-parametric technique. Our last approach is parametric. Section 3.7 uses three different parametric models to investigate the effect of debt on depression duration. Section 3.8 concludes the chapter.

3.2 Literature Review

The paper by Panizza et al. (2013) surveys different theoretical models that bring together public debt[3] and economic growth. The standard crowding-out effect can explain why high debt has a negative impact on economic growth. This negative

[2]In Breuer and McDermott (2018), the authors also look at episodes of debt and depression, but do not analyze duration. That paper describes coincidence of events and timing of entry and exit.

[3]All the terms “debt” mentioned in this paper refer to “debt-to-GDP ratio”.

effect can be exaggerated if high debt levels increase investors' uncertainty towards future governmental policies. Keynesian models, on the other hand, can explain why expansionary fiscal policies may have a positive effect on economic activity. Theoretical models, then, can only give ambiguous results about the effect of expanding debt on economic activity.

The empirical literature has found no consensus on the relationship between high debt levels and slower economic growth. As noted, the work of Reinhart and Rogoff has been especially influential in rekindling this area of research (Reinhart and Rogoff (2010); Reinhart et al. (2012)). They provided basic evidence that average and median GDP growth are substantially lower when public debt is above 90 percent of GDP. Their results did not go unchallenged; questions arose relative to the nature of the threshold itself and to the nature of causality.

One challenge is to verify whether there exists a common threshold beyond which public debt has a negative impact on the GDP growth. Different approaches have been employed to verify whether the relation between public debt and GDP growth rate is nonlinear. One way to determine this is by including a quadratic term in the regression. Checherita-Westphal and Rother (2012) checked the causality for the twelve European countries using this approach together with fixed effects and system GMM. Their results suggest an inverted U shaped curve between debt and growth. Another strand of the literature allows the threshold debt ratio - in a regression of output growth on the debt ratio – to be determined endogenously by a grid search. Cecchetti et al. (2011) found a threshold close to original threshold of .90 using this method. Others identified lower thresholds.[4] The work of Bruce Hansen (see, for example, Hansen (2015, 1999); Caner and Hansen (2001)) has been a large influence

[4] See, for example, Égert (2015), who finds a threshold of .20 for central government debt and .60 for general government debt. Baglan and Yoldas (2016) also find a low threshold (.18) in their two-regime analysis, but they note that it is quite imprecisely estimated.

on this approach.[5]

Causality is difficult to establish, or even to conceptualize. That is, debt and economic activity are potentially endogenous variables in a complex system. It is common in the literature to lag the debt variable as a way to reduce the possibility of reverse causality. Some economists have used instrumental variables to identify causality. One example is Woo and Kumar (2015) who employ internal instruments (lag of covariates) and the system GMM estimator to address possible endogeneity. However, system GMM may not be suitable for datasets with relatively small numbers of cross-sectional units. Checherita-Westphal and Rother (2012) study twelve euro-area countries over the period 1970-2008. They instrument the debt-to-GDP ratio of country i at time t with the average debt-to-GDP ratio in the other eleven countries at time t. The authors find a non-linear hump-shaped relationship between debt and growth. However, their analysis has been criticized for not satisfying the exclusion restriction for the instrumental variable.[6] Panizza and Presbitero (2014) use another instrumental variable - an interaction between foreign currency debt and movements in the exchange rate. Their analysis relies on the fact that changes in the exchange rate have a direct impact on the debt-to-GDP ratio if foreign currency debt is part of public debt. Their results do not support the negative causality between high debt and GDP growth for advanced economics.

Another area of concern is the proper definition of public debt. Gross debt, net debt, explicit debt or implicit government debt have been used in the literature. While gross debt is more reliable to collect, the use of net debt[7] provides more accurate information about the government's liability. Collecting this information is more

[5] See also Caner et al. (2010), who find .77 for all countries and .64 for developing countries; Elmeskov and Sutherland (2012) who find two thresholds, one of about .40 and one close to .66.; and Minea and Parent (2012), who find that the effect of debt on growth actually becomes *positive* at a threshold of 1.15.

[6] Panizza et al. (2013), p. 9.

[7] Net debt is gross debt excluding the asset held by government.

challenging and there is no clear-cut standard for net debt across countries.

Finally, there have been investigations into the nature of outside influences that may change the nature of the relation between growth and debt. Kourtellos et al. (2013) provided evidence suggesting the impact of debt depends on the development of democracy and the negative effect is limited to low-democracy countries. Other work suggests that the negative relation between high debt and GDP growth is not evident in OECD and other industrialized countries (see Breuer and McDermott (2018) , Panizza and Presbitero (2014), and Puente-Ajovín and Sanso-Navarro (2015)).

Our approach differs in two ways from this literature. First, we use an episodic approach. We do so to redirect attention to economic crises, not just reductions in the rate of growth. Second, we use survival analysis to investigate the duration of the crises. Our work analyzes a different question from that addressed by recent work.

3.3 Data

The definition of a "negative growth episode" (NGE) or "depression" is based on the work of Breuer and McDermott (2013). They define episodes of economic depression based on two criteria: the country must experience a cumulative decline in per capita output of 20 percent or more[8] that is sustained for at least four years.[9] This definition is necessarily arbitrary; there is no unambiguous way to measure what is meant by a depression. We focus on severe economic downturns because this seemed to be the motivating force behind the original hypothesis of Reinhart and Rogoff: that too much debt leads to a crisis, not just a softening of the rate of growth. The episodes are constructed from the Penn World Table, version 9.0. In this data, there are 141 depressions in 104 countries from the year 1950 to 2014. Within the sample period,

[8]More precisely, the criterion is a natural log difference of .20 or more.

[9]For a detailed description of the construction of data, see Breuer and McDermott (2013).

four countries[10] experienced three NGEs; 29 countries experienced two NGEs and 71 countries experienced only one NGE. The dummy variable NGE takes the value 1 whenever a country is in a depression and 0 otherwise. The average length of the negative growth episode is 10.54 years.[11]

A "high debt-to-GDP episode" (HDE) is defined to be a period lasting at least four consecutive years during which the debt-to-GDP ratio is at least 90 percent. As noted above, this definition comes from the paper by Reinhart and Rogoff (2010) that has become a benchmark for many papers. It, too, is arbitrary and some authors have tried lower cut-off points, or endogenous thresholds (Cecchetti et al. (2011)). We keep the 90 percent threshold to make our work comparable to other studies. For public debt data, we use the data in the IMF's Historical Public Debt Database (Abbas et al. (2011, 2013)). Between the years 1950 and 2012, there are 66 HDEs in 132 countries that have available data on public debt. Within the sample period, 73 of the countries never experienced an HDE; 52 of them experienced one HDE; And seven of them experienced two HDEs.[12] We define HDE to be a dummy variable that takes the value 1 if a country is in a high debt episode and 0 otherwise.

One thing to keep in mind is that the first thing we do is trim our data set by discarding all observations for which NGE (and y) are missing. Therefore, in our data, NGE is always either 0 or 1, but HDE may also contain missing values. This is important when we define the key indicator variable ovl. This dummy defines a *subset* of depressions, those that *overlap* with a high debt-to-GDP ratio episode. By "overlap" we mean that they have at least one year in common. In other words, $ovl = NGE = 1$ for every year of the NGE if an HDE overlaps with the NGE, even

[10] These four countries are Zimbabwe, Democratic Republic of the Congo, Guinea-Bissau and Lebanon.

[11] The longest NGE in our sample is Niger, 43 years long between 1963 and 2005. The second longest is 27 years. If Niger is excluded from the sample, the average length of an NGE is 10.30.

[12] They are Belgium, Israel, Singapore, Sri Lanka, Egypt, Jamaica, and Togo.

if it is just for one year. If an HDE does not overlap with the NGE, then during that depression, we have $NGE = 1$ but $ovl = 0$. The ovl dummy is constant within any episode defined by NGE.

In defining ovl, however, as long as the NGE contains at least one value for HDE, we ignore any missing values for HDE in that same NGE. This amounts to assuming that missing values (when there are some non-missing values) for HDE are 0. This, in turn, means that we are underestimating the number of NGEs for which $ovl = 1$, since if we had the data on HDE some of our $ovl = 0$ observations might switch to $ovl = 1$. The reverse could *not* happen: ovl could never switch from 1 to 0. This might bias our results, although the direction is not clear: it is not a case of simple measurement error. For this reason, we also construct the variable $ovl2$, which is the same as ovl but is constructed only after we remove all the NGEs for which the data for HDE is missing. This reduces our sample size by about a third, but provides a check on our results.

We use two other covariates as controls. The variable dev is an indicator variable to distinguish industrialized, developed countries ($dev = 1$) from those are still developing ($dev = 0$). Industrialized economies are defined as those with gross national income (GNI) per capita of $\$12,476$ or more in 2015; otherwise, they are defined as developing countries.[13]

Our only continuous variable is the inflation rate inf, which is defined as the log difference in the CPI.[14] We include inflation since there is a presumption that governments tend to be expansionary in terms of monetary policy when the debt burden becomes too great. It is interesting to see (1) if the relationship between high debt and depression is affected by inflation; and (2) if inflation has any independent effect on the duration of depressions.

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[13]This criteria, and the data, comes from the World Bank.

[14]The CPI data is from *International Financial Statistics* of the International Monetary Fund.

3.4 Survival Analysis

We use *survival analysis* – often called *duration analysis* or *event history analysis* – to analyze the time to the occurrence of the event, defined to be the year that the economy *exits* the negative growth episode. The term "survival" suggests that the current state is desirable. While often true in medicine, where the current state is living, it is much less so in economics, where the current state may well be bad, like unemployment or, as in our case, depression. It is natural then that the act of exit was often considered a bad thing, and was equated to "failure" and its probability called a "hazard". In our study, the exit is unequivocally good and survival is bad.

There are several reasons why ordinary least-squares (OLS) linear regression is not appropriate. First of all, the OLS model assumes the residuals are normally distributed. But this may not be true for the distribution of the time-to-event. For example, the instantaneous risk of exiting the NGE may be constant over time. More importantly, the time-to-event may not be distributed symmetrically, in which case the results from the linear regression model are biased. Another reason is the existence of right-censoring in the data. Right-censoring refers to the fact that some countries were still in a negative growth episode in the last year of available data, so we really do not know when the event – the exit – occurs. This happens frequently in duration data, and OLS cannot deal with it effectively. Survival analysis is the most efficient way to analyze episodic data.

To do survival analysis, we need to convert the yearly data into duration format. The analysis time t is rescaled from the yearly data and converted based on the calendar year in which the country entered the NGE. For example, Argentina had an NGE between 1997 and 2003. The algorithm for converting to analysis time t works like this. Year 1997 is set as the entering year ($t = 1$) for the NGE; Year 2003 is set as the time of the exit event for the NGE ($t = 7$); the duration of this NGE is considered to be 7 years.

We begin our analysis of the duration of depression with the Kaplan-Meier method.

3.5 Non-Parametric Analysis

3.5.1 The Kaplan-Meier Statistic

The most common non-parametric method of duration analysis is that of Kaplan and Meier (Kaplan and Meier (1958)). The object is to estimate the *survival function* $S(t)$ without making any assumptions about its functional form in nature. The survival function gives the probability that the episode will last to time t, which is also the probability that the episode will end right after time t. The Kaplan-Meier estimator, also called the product-limit estimator, allows the researcher to compare the survival experience across covariates that do not vary over time. In our case, we are mainly interested in using *ovl* to divide the sample into two groups to see there are systematic differences in their exit behavior.

We have several depression episodes in the data. Let the observed exit times be called $t_1, t_{2,}....t_j$. At any time t_j, one or more countries can exit. The Kaplan-Meier estimate of the survival function at time t is given by the equation:

$$\hat{S}(t) = \prod_{j|t_j \leq t} (\frac{n_j - d_j}{n_j})$$

The variable n_j is the number of episodes "at risk" at time t_j – that is, the total number of episodes that are still ongoing at time t_j – and d_j is the number that exit *naturally* during t_j. In particular, the value d_j excludes those episode that are censored at time t_j.[15] The estimate $\hat{S}(t)$ is the multiplication of all the conditional probabilities of surviving at each observed t_j for $t_j \leq t$.

Table 3.1 shows the detailed output for the Kaplan-Meier estimate for all of the NGEs in our data. That is, we carry out the analysis with all of the 141 NGEs,

[15] Censoring is only about *right censoring*, which refers to an NGE that ends due to the fact that no further data is available. If there were more years of data available, the NGE might be longer.

whether or not there was any data for HDE.

Since the minimum requirement for the negative growth episode is four years, the observed minimum exit time[16] t is 4 as shown in the second row of Table 3.1. When time equals 4 – that is, at any time within the 4th year after the start of the NGE – a total of 141 NGEs are still ongoing (listed in the column titled "No. At Risk "). Over the course of Year 4, 14 of them exit the depression state and are noted as "No. Exited" in the table. In the same row, there is a 1 in the column titled "Net Lost" representing the one right-censored episode. So the corresponding estimated survival probability is $\hat{S}(4) = \frac{141-14}{141} \approx 0.9007$. When the time is 5, the "No. At Risk" is 126 which is retrieved by $141-14-1$, and the number that exit is 24. The corresponding survival probability is $\hat{S}(5) = 0.9007 * (\frac{126-24}{126}) \approx 0.7291$. We interpret this number as follows: the probability of remaining in an economic depression at $t = 5$ is 72.91 percent.

The last row of Table 3.1 has time 43, which means that the observed maximum duration of an NGE in our sample is 43 years. The last three columns of Table 3.1 provide information on the standard error and confidence interval of the Kaplan-Meier estimates.[17] As time goes on, the estimated survival probability declines. All depressions must end eventually. Figure 3.1 shows how the estimated probability $\hat{S}(t)$ falls over time. There we see that the depression that lasts for 43 years is, indeed, an outlier.

We are most interested in whether or not depressions that overlap with high-debt episodes last longer than those that do not. To address this question, we cannot use the sample that appears in Table 3.1 and Figure 3.1 since some of the 141 NGEs

[16]Since the first year of NGE is set as the entering year, when the country exits at the fourth year, the corresponding duration is calculated as 4.

[17]The standard error is calculated based on the Greenwood (1926) formula $\hat{Var}\{\hat{S}(t)\} = \hat{S}^2(t)\sum_{j|t_j \le t} \frac{d_j}{n_j(n_j-d_j)}$. The 95 percent confidence interval is the asymptotic variance proposed by Kalbfleisch and Prentice (2002b).

have no data for ovl. We have 99 depressions for which $ovl = 0$ or $ovl = 1$.[18] Figure 3.2 (a), shows the differences in $\hat{S}(t)$ when we split our sample according to ovl: whether or not the country simultaneously experienced a high-debt episode in at least one year of its existence. It is apparent that the speeds at which countries exit the episodes are different depending on whether $ovl = 1$ or $ovl = 0$. The dashed, red line in Figure 3.2 (a) represents the group of NGEs for which $ovl = 1$; the solid, blue line represents the group of NGEs for which $ovl = 0$. The slopes of both lines are negative, but the concavities are different. The red line lies above the blue line across the majority of the analysis time, which means for each observed time t_j, the probability of remaining in an economic depression for the group with high debt is greater than the other group. A closer examination reveals it takes less than 8 years for the group without a high debt episode to reach the 50 percent survival probability while it takes more than 14 years for the group with high debt to obtain the same level. We investigate timing in greater detail below.

There is far less evidence that the level of development, in and of itself, is associated with duration of depression. The dashed, red line in Figure **??** represents the industrialized countries ($dev = 1$) while the solid, blue line represents the group of developing countries ($dev = 0$). Industrialized countries, in general, have shorter NGEs and recover more quickly from depression, but the difference is not striking. In fact, it is hardly perceptible.

3.5.2 Median and Mean Survival Time

The median and mean survival times are the main ways to characterize the information regarding duration in the context of the Kaplan-Meier model.

[18] Recall that if the NGE had *at least one non-missing* value for HDE we treated all the missing values as 0. See below for an alternative approach using $ovl2$ which excludes NGEs unless they have full data for HDE.

The *median* survival time is defined as:

$$\hat{t}(50) = min\{t_j | \hat{S}(t_j) \leq 0.5\}$$

This is the year after which exactly 50 percent of the episodes are expected to remain in an economic depression. To analyze median survival times, we again divide our sample into two groups. As before, we first split the sample according to whether or not there was an overlap with a high-debt episode (ovl). Then we divide according to per capita income level (dev). This two-way classification gives us four distinct groups. The results for the median times are shown in Table 3.2 (a). The rows titled "$\hat{t}\,(50)$" show the estimated median survival times for the groups, while the rows labeled "CI" give the confidence intervals around those estimates. We also show N, the number of negative growth episodes in each of the four groups. The last column (ovl) and row (dev) give the information for the two groups considered without respect to the other.

There are 99 NGEs for which ovl is either 0 or 1. The overall median survival time was 10 years. Of these, 63 never overlapped with a high-debt episode ($ovl = 0$: Panel A, Col. (4)). For them, the median survival time is 8 years. For the groups that *did* overlap with a high-debt episode ($ovl = 1$), the median survival time of the economic downturn is *almost twice as long*, 15 years (Panel B, Col. (4)). Only about a third of the countries had overlapping high-debt episodes (36 of 99), but those that did experienced a significantly longer time in the depression, according to the confidence intervals.

The state of development makes a big difference for the effect of ovl on survival. If $dev = 0$, then median survival times are almost twice as high for episodes that coincide with a high-debt episode (15 vs. 8); for $dev = 1$ they are virtually the same (6 vs 7).[19] In other words, the overall effect noted in Col. (4) is due almost entirely

[19] We note, however, that we have very few observations – only 4 - in the cell for $ovl = dev = 1$.

to the cohort of developing-country negative growth episodes.

The unconditional comparison between industrialized ($dev = 1$) and developing ($dev = 0$) countries also reveals a difference in median survival time, although it is not so precisely estimated (Panel C of Table 3.2 (a)). Industrialized countries' survival time is 6 years on average, compared to 10 years for developing countries. There is considerable common range in confidence intervals, however, so it is not clear if the difference is significant. Depression is far more common in developing countries, though. Of the 99 NGEs in this sample, 79 of them occurred in developing countries.[20]

The effect of ovl on the marginal effect of dev is enormous. When $ovl = 1$ the condition of being a developing country causes the median survival time to be over two times higher: 15 vs. 7 (Panel B). For $ovl = 0$, the effect is much smaller: 8 vs. 6 (Panel A).

The *mean* survival time μ_T is defined as

$$\mu_T = \int_0^{t_{max}} \hat{S}(t)dt$$

where t_{max} is the observed maximum survival time. As shown in Table 3.2 (b), the grand mean duration of an NGE is 11.59 years.

The basic message of Table 3.2 (b) is similar to that of Table 3.2 (a). The presence of a high-debt episode sometime during a depression is associated with a longer mean exit time (15.28 vs. 9.38). The level of development also matters for mean exit time: industrialized countries tend to exit earlier, but the association is not as striking (12.00 vs. 10.04). Compared to the median, the *difference* between the mean duration for $ovl = 1$ and $ovl = 0$ is less apparent.

If we use our small sample, in which we discard all NGEs unless there are no missing observations for HDE, the results are very similar although, as might be

[20] If we use the full 141 country sample, the point estimates are even closer: 6 ($dev = 1$) and 9 ($de = 0$). Their confidence intervals also show a large common range.

expected, across all categories survival times are shorter. While the sample size falls from 99 to 69, as we discussed in Section 3.3, we believe that the overlap measure based on this sample, which we call $ovl2$, will miss fewer overlaps than will ovl. The main result is preserved: for $ovl2 = 0$, the median survival time is 7, while for $ovl2 = 1$, the median survival time is 15. Mean duration is 7.84 for $ovl2 = 0$ and 13.46 for $ovl2 = 1$.

3.5.3 Tests of Significance

High debt appears to have a clear, inverse association with the duration of depressions. There are two common tests of the significance of the difference in estimated survival functions: the Log-Rank test and the Wilcoxon tests. Let $S_0(t)$ be the survival function for countries that experience an overlap with high debt; and let $S_1(t) = S_0(t)^{\psi}$ be the survival function for the group that did not have an overlap. Then the null hypothesis is that $\psi = 1$ against the alternative that $\psi < 1$.

The test statistic is the ratio:

$$R = \frac{\left(\sum (d_{0i} - e_{0i})\right)^2}{\sum v_{0i}} \tag{3.1}$$

In equation (3.1), d_{0i} stands for the number of exits at time t_i in the group that did overlap with high debt and e_{0i} and v_{0i} are, respectively, the expected value and variance of d_{0i} under the assumption that the two groups have the same distribution.[21] If true, the ratio R has the χ^2 distribution. The Wilcoxon test puts more weight on the observations that are early in the sample. That is, the numerator becomes $\left(\sum \omega_i (d_{0i} - e_{0i})\right)^2$ and the denominator becomes $\sum \omega_i^2 v_{0i}$, where the ω_i are the weights and decline according to the estimated survival function.

The results of these tests are shown in Table 3.3. The column entitled “Events Observed” shows a total of 24 exit events observed. Of these exits, 35 NGEs experi-

[21]It can be shown that under this condition, d_{0i} has the hypergeometric distribution. See Mario Cleves and Marchenko (2016) page 125.

enced an HDE overlap, while 59 NGEs did not overlap with an HDE.[22] The column entitled "Events Expected" provides the expected total number of events observed if the two country groups shared the same survival function. The difference between observed and expected events are large enough that the $p - value \approx 0.00$ from a χ^2 test for both of the Log-Rank test and Wilcoxon test. Thus, we reject the null hypothesis that they are the same, and conclude that the hazard functions are different for the two groups.

Lower part of Table 3.3 provides testing results after we separate the sample into two groups: industrialized countries ($dev = 1$) and developing countries ($dev = 0$). Based on the $p - value$ shown, we fail to reject the null hypothesis for industrialized countries, but do reject for developing countries. In other words, high debt does appear to be associated with longer economic crises in developing countries, but not in industrialized countries. This is an important result: while the duration of depressions is significantly correlated with high public-debt episodes, this phenomenon is primarily a statement about developing economies. Not only are there almost 4 times as many depressions in developing countries (75 vs 19), the relation between debt episodes and depression length is only significant for that group.

3.6 the Cox Proportional Hazards Model

3.6.1 The Basic Model and Results

We now shift the focus to the *hazard function* $h(t)$ for the remainder of the chapter. The hazard function is the instantaneous rate of "failure" – exiting the NGE – at time t, given that the country has been in the NGE for t years. It is defined to be the negative of the rate of change in the survivor function over time: $h(t) = -\frac{\dot{S}(t)}{S(t)}$. The non-parametric analysis of Kaplan and Meier is convenient for comparing survival

[22]The total number of NGEs is 94 which is 5 episodes less than 99 because five of the episodes in the sample are censored.

functions between groups for covariates with unchanging values over time. The Cox Proportional Hazards (PH) model (see Cox (1972)) allows us to analyze the relative effect of several covariates on survival time simultaneously. Moreover, these covariates can change over time.

The hazard function is assumed to have the following form:

$$h(t|x_i, x_{-i}) = h_0(t)exp(XB) \tag{3.2}$$

where B is a coefficient vector and X is a matrix of covariates, including a variable of interest x_i (here, the high-debt episode overlap) and other control covariates x_{-i} (here, the level of development and inflation). We call $h_0(t)$ the "baseline hazard function" because it represents the probability of exiting the NGE at time t when all the covariates are equal to zero, given that the country has been in the economic downturn for t years. The $h_0(t)$ function can have almost any functional form with respect to time. The only requirement is that $h_0(t) \geq 0$ since the hazard rate must be nonnegative. The functional form of $exp(XB)$ also guarantees that the hazard function is nonnegative over time. By defining the hazard function in such a way, the hazard rates are all proportional to $h_0(t)$ across different values of covariates. The fact that we do not need to make assumptions regarding the functional form of $h_0(t)$ is one clear advantage of the Cox PH model compared to the parametric models that we study in the next section and it is also the reason why the Cox PH model is called a semi-parametric model.

The coefficients in (3.2) are estimated by maximizing the log form of the partial likelihood function, which is independent of the baseline hazard function. Table 3.4 shows the estimates $\hat{B}$.[23] In this table, we control for three covariates; in addition

[23] We use the Efron approximation to the exact-marginal method to deal with tied failures. Even though the recorded data shows that some countries exited the NGE at the same t, they in fact must exit one by one – and we do not know the exact order. For more details, refer to Kalbfleisch and Prentice (2002b).

to ovl and dev, we also control for the rate of inflation inf, which varies each year, unlike the other two controls.

If the estimated coefficient of an indicator variable – like ovl or dev – is larger than zero, it means that being in that group increases the probability of exit from the depression. If negative, it reduces the probability of exit. Consider the meaning of the coefficient on ovl, which is -0.780. Ignoring inf for the time being, the estimated probability for a country to exit an NGE at time t is:

$$h\left(t|ovl, dev\right) = h_0\left(t\right) exp\left(-0.780ovl + .083dev\right)$$

Since ovl is a dummy variable, we have:

$$h\left(t|ovl = 1, dev\right) = h_0\left(t\right) exp\left(-0.780 + .083dev\right)$$

$$h\left(t|ovl = 0, dev\right) = h_0\left(t\right) exp\left(.083dev\right)$$

It follows that the *relative hazard ratio* between the two groups is:

$$\begin{aligned} \hat{H} \equiv \tfrac{h(t|ovl=1,dev)}{h(t|ovl=0,dev)} &= exp\left(-0.780\right) \\ &= 0.458 \end{aligned}$$

Regardless of the level of development, the hazard rate is lower for the group with $ovl = 1$. The probability of getting out of the negative growth episode for the group with an HDE overlap is only 45.8 percent of depressions that do not coincide with any high debt episodes. As we found earlier in the Kaplan-Meier analysis, the duration of NGEs is usually longer for countries experiencing high debt. The coefficient is statistically significant within the 1 percent confidence interval.

The income indicator variable dev appears to have no influence on the relative hazard ratio. Although the estimated coefficient is positive at .083, it is small in magnitude and not significantly different from zero. This result also accords with what we found with the Kaplan-Meier analysis.

In the second column of Table 3.4 we add the inflation rate, inf. One of the advantages of the Cox PH model is that it can handle time-varying covariates like inflation. The effect of inflation on the exit chance is *positive* and significant at the 1 percent level. Importantly, adding inf into the regression does not change the sign or significance of ovl. The coefficient for inf is 0.497, so that increasing the inflation rate by one unit, would be associated with an increased probability of exiting the NGE of 64.4 percent. That is, $e^{0.497} \approx 1.644$. A unit change in inflation, however, is a huge increase since we measure inflation as the difference in the natural log of the CPI. That means that a unit change is an increase by a factor of 2.718 in the CPI.

A more realistic exercise is to ask what happens if the inflation rate increases by 10 percentage points, the estimated hazard ratio is:

$$\hat{H}(\Delta inf = .1) = e^{0.1 \times 0.497} = 1.051$$

the corresponding probability of exiting the NGE increases by about $five$ percent. This is an interesting result. It is consistent with the story that when countries are in depression, it helps to exit by expanding the money supply, which can lead to inflation.

In the last column of Table 3.4 we replace the dummy variable *ovl* with the yearly debt-to-GDP ratio (*dgdp*) as an alternative measure of indebtedness. The coefficient for *dgdp* in column (3) is 0.003, which means that if the debt ratio were higher by 1 point (e.g. from 55 to 56) the probability of exiting the depression (the hazard rate) would be only a bit larger:

$$\hat{H}(\Delta dgdp = 1) = e^{0.003} \approx 1.003$$

The hazard ratio for one unit change in *dgdp* is close to one and it is not statistically significant. This result is consistent with the results found by Reinhart and Rogoff (2010). A minor change in the debt level does not make an appreciable difference in the probability of leaving a depression. Only if the debt to GDP ratio reaches a threshold or higher (90 percent in this chapter), will it show connections with negative economic activity. We also tried other thresholds. When the threshold equals 70 percent, the coefficient for *ovl* is not statistically significant; however, if the threshold equals 80 percent, the coefficient for *ovl* changes to be statistically significant within a five percent confidence interval. These results are shown in Appendix I.

When we control for *dgdp* instead of *ovl*, we now find that the level of development does matter. The hazard ratio between industrialized and developing countries is:

$$\begin{aligned} \frac{h(t|dev=1,\text{dgdp})}{h(t|dev=0,\text{dgdp})} &= exp(0.654) \\ &= 1.923 \end{aligned}$$

Industrialized countries, on average, have almost double the probability (92.3 percent higher) of getting out of negative economic growth compared to developing countries for each observed year. This number is much larger than the corresponding values in Columns (1) and (2). This result is also statistically significant at the 5 percent level. This may be the result of omitted variable bias. That is, if *ovl* matters and we omit it, the coefficient on *dev* will be biased upward if *ovl* and *dev* are negatively correlated. In fact, a simple OLS regression of *dev* on *ovl* produces a negative coefficient that is significant at the 1 percent level.

The results from our small sample (with only NGEs for which HDE data is complete) are very similar to those shown in Column(2) of Table 3.4. In fact, they are stronger: *ovl* has a larger inverse effect on the hazard rate and its significance rises to .1 percent. For this sample, the probability of exit is only 28 percent as large as countries with $ovl = 0$ (in contrast to the large sample, where the relative probability was 45 percent). The effect of inflation inf is likewise greater in absolute value and more precisely estimated. Even *dev* becomes significant at the 5 percent level when included with *ovl* – and its significance rises to .1 percent when *dgdp* replaces *ovl*.

3.6.2 Testing the Proportional Hazards Model

If the basic assumption of the PH model is satisfied, the hazard ratio – the ratio of the hazard rate between two groups – should be constant over analysis time. In our case, this means that the probability of exiting the depression for the group that *does* overlap with a debt episode should be in the same proportion – at all exit times – to the probability of exit for the group that *does not* overlap. The reason is that the component of these probabilities that depends on time $h_0(t)$ – the baseline hazard – is common to both and cancels out when the ratio is taken.

There are three common methods to test this hypothesis: a graphical, visual analysis of survival functions; a visual analysis of Schoenfeld residuals, and a formal test using Schoenfeld residuals. These tests are not straightforward, so we leave the details to Appendix G. Here, we simply note that the tests indicate that it is reasonable to accept the proportionality of hazards for our data.

Having established that the PH assumption is reasonable, we can assess the goodness of fit of our model. We rely on a visual test using the Cox-Snell residual method. Leaving the details to an appendix (see Appendix H), the test suggests that our model does fit the data well.

3.6.3 Discussion

In this section, we used the Cox Proportional Hazards model to test the association between high debt episodes and depressions when controlling for the level of development and inflation. We showed that depressions that overlap with an HDE tend to be longer than those that do not. In fact, the hazard rate for the NGE with an HDE overlap is only about half of that for NGEs without an HDE overlap. This result is consistent with the results of the previous section where we used the Kaplan Meier estimate. We also uncovered an effect that is quite robust in our data: higher inflation is associated with shorter duration of negative growth episodes. On the other hand, whether the country is industrialized or developing has a weak positive effect on the duration of an NGE. This is most apparent when we use the small sample.

3.7 Parametric Analysis

In this section, we use parametric models to analyze the survival experience of negative growth episodes. The main difference between the parametric model and Cox PH model concerns the specification of the baseline hazard function $h_0(t)$. In the Cox PH model (3.2), no assumption is made about $h_0(t)$. Parametric models, however, do require a particular form for the baseline hazard function.

Compared to the Cox PH model, parametric forms have certain advantages. When the impact of a covariate is strong or the effect of a covariate has a strong time trend, parametric models can be a sound alternate to the Cox model, since, if the specification of the hazard function is correct, the parametric model will give more efficient estimates (Klein and Moeschberger (2006)). If it is not correct, however, it is best to use the Cox model. Meanwhile, conditional on choosing the correct form for the baseline hazard function, this allows us to estimate the hazard function itself and not just the relative hazard between two states of the world as defined by the covariates in the model.

Three parametric models – that is, forms for $h_0(t)$ – are analyzed in this section. They are the *Weibull*, the *exponential*, and the *Gompertz* models. While there are others, these three fit easily into the Proportional Hazard (PH) form – or "metric" — which makes the results comparable with the Cox PH model. In fact, we will compare the hazard ratios between the Cox and parametric models and reject the parametric model if these hazard ratios differ by too great a margin.

Assume that baseline hazard function $h_0(t)$ has the following form:

$$h_0(t) = \lambda t^{\lambda-1} exp(\beta_0) \tag{3.3}$$

so that the complete hazard function has the form:

$$h(t) = h_0(t) e^{\beta_1 x} = \lambda t^{\lambda-1} e^{\beta_0} e^{\beta_1 x} = \lambda t^{\lambda-1} e^{(\beta_0+\beta_1 x)} \tag{3.4}$$

where x is the covariate of interest, but there may be more than one covariate. Covariates may change over time, as in the Cox PH model. This hazard function, and the associated survival function $S(t)$ and density function $f(t) = S(t)\,h(t)$, form the *Weibull Model*.

We estimate three parameters: the "shape" λ, the "scale" β_0, and the coefficient β_1. If $\lambda = 1$, we have the *Exponential Model*, which is unrealistic in many situations, including ours, since it means that the hazard function is constant over time. We have already seen that the survival of depressions declines over time, so that the hazard increases. This suggests that $\lambda > 1$, which we test below.

If the hazard function has the form:

$$h(t) = exp(\gamma t)e^{\beta_0}e^{\beta_1 x} \tag{3.5}$$

with γ being constant, we have the *Gompertz Model*.

One advantage of parametric analysis is that the parameters can be estimated using regular maximum likelihood. The maximum likelihood estimates of all three PH metric forms are displayed in Table 3.5. The top part of Table 3.5 provides estimated shape parameters for the Weibull, exponential and Gompertz models while the lower part of Table 3.5 provides the estimated hazard ratios.

The results in the top two rows of Table 3.5 show the estimates of λ and γ in the Weibull and Gompertz models – the exponential model simply imposes the value $\lambda = 1$ on the Weibull model. For the Weibull model, the Wald test is for $H_0 : ln(\lambda) = 0$ – which is equivalent to testing whether or not $\lambda = 1$ — and it provides a test statistic of 8.13. We can, therefore, reject the null hypothesis and conclude that the hazard function is not constant over time. Furthermore, $\hat{\lambda} = 2.016 > 0$, so the hazard rate is monotonically increasing over time. When we examine the results for the Gompertz model, we come to the same conclusion: the Wald statistic is 5.46 and the estimate $\hat{\gamma} = 0.073 > 0$, meaning that the estimated hazard rate is also monotone increasing. This is evidence that the Weibull and Gompertz models fit the data better than the exponential model, which is no surprise, since the exponential model specifies that the baseline hazard function is independent of time. This is not reasonable, given our earlier results.

We now discuss the hazard ratios e^{β_1}, which are comparable to those we found for the Cox model. For the Weibull model, the relative hazard rate between the group with $ovl = 1$ and that with $ovl = 0$ is 0.391 and it is statistically significant at one percent. In words, countries that experienced an overlap with a high-debt episode were only 39.1 percent as likely to exit the negative growth episode. The lower probability of exiting represents a relatively longer duration of the negative growth episode for the group with an overlap with an HDE. For the Gompertz model, the story is very similar: the corresponding hazard ratio is 0.425, and it is also significant. For the exponential model, the hazards rate for $ovl = 1$ is relatively higher at 0.621 and it is statistically significant only within $five$ percent. The results, then, are consistent across all the three models in Table 3.5. All indicate that consecutive years of high public debt-to-GDP ratios do coincide with longer duration of NGEs. These results provide support for the hypothesis in the other papers that high levels of public debt are correlated with the longer economic downturns.

Recall that the result from the Cox PH model, Table 3.4 column (2) shows that the coefficient for ovl is -0.899. And the corresponding hazard ratio is $exp^{-0.899} = 0.407$. Thus the Cox PH Model gives results that are very similar to those of the Weibull model and the Gompertz model, but not the Exponential model.

The level of development appears to make no difference in the relative probability of escaping a depression. The hazard rate between industrialized and developing countries is around 1.3 in all three models, but they are not statistically significant.

Inflation does matter, as in our other models. The hazard ratio is 1.775 for the Weibull model and it is statistically significant within the one percent confidence interval. If inflation rates increased by *ten* percentage points, the hazard rate would increase by 5.906 percent.[24] The results are similar for the other two models. A high inflation rate coincides with shorter duration of the economic depression.

To distinguish between the Weibull and Gompertz, we use the Akaike information criterion (AIC) which is defined as:

$$AIC = -2 \times ln(L) + 2(k + c)$$

where k is the total number of covariates in the model, c is the number of the distributional parameters and L is the estimated log likelihood. As reported in Table 3.5, the Weibull Model provides the lowest AIC of 118.048. Thus, the Weibull Model is selected to be best among the three parametric models considered.

Finally, we repeated this analysis with our small sample, using $ovl2$. Again, the results are similar, but *stronger* in that the magnitudes of the hazard ratios are more pronounced and they are more significant. In the case of the Weibull distribution, the hazard ratio falls to .280 for $ovl2$ (from .391 for ovl). In our small-sample Cox results, that hazard ratio was also .280. The hazard ratios for both dev and inf are both over 2.00. As in our other results, inf remains highly significant, and even dev becomes marginally significant with a $p - value$ of .042. The estimated shape parameter λ is above 2.0 and its log is highly significantly different from zero. The Gompertz results are similarly stronger.

3.8 Conclusion

This chapter investigates whether consecutive years of high public debt-to-GDP ratios are systematically associated with the duration of negative growth episodes (depressions). It provides another way to look at the debt-growth nexus stimulated by the work of Reinhart and Rogoff. Compared to the current literature, which analyzes the

[24] The effect is $e^{\beta * \Delta\pi} = \left(e^{\beta}\right)^{\Delta\pi} = 1.775^{.10} = 1.05906$, where $\Delta\pi$ is the change in the inflation rate .

relationship using yearly data, we focus on *episodes* when the countries experience consecutive years of GDP decline. The dependent variable of interest is the duration of long-term negative growth episodes instead of the magnitudes of changes in the GDP growth rate. Using survival analysis, we demonstrate that episodes of high public-debt ratios are associated with longer negative growth episodes – and may cause them to be longer.

We also find that a higher inflation rate is associated with a shorter duration of negative growth episodes. On the other hand, somewhat surprisingly, whether the country is industrialized or not does not seem to matter as much for the length of time they remain in economic depression – although the numbers of developing countries that experience depression is much greater than the number of industrial countries that fall into depression.

These results are consistent among all the survival analysis models we used: the Kaplan-Meier non-parametric model, the Cox Proportional Hazards regression model, and the parametric models: the exponential model, the Weibull model, and the Gompertz model.

The relationship between public debt and economic activity is complicated. It is natural to want to understand the nature of causality between the government's debt policy and the state of the economy, but there may be no single exogenous process that can explain the relationship. Under certain conditions, the high debt may cause slow growth; under others, the slow growth – caused by a third variable – may lead to a higher debt path. In this chapter we have shown that there is good reason to believe that prolonged periods of debt above a particular threshold are associated with longer economic depressions. This may provide a rationale for moderating the issuance of government debt in periods of economic uncertainty.

Table 3.1 Kaplan-Meier Estimate for NGE

Time	No. At Risk	No. Exited	Net Lost	Survival Probability	Standard Error	[95% Conf.Int.]	
4	141	14	1	0.901	0.025	0.838	0.940
5	126	24	2	0.729	0.038	0.647	0.795
6	100	13	0	0.634	0.041	0.549	0.708
7	87	11	2	0.554	0.042	0.468	0.632
8	74	11	0	0.472	0.043	0.386	0.552
9	63	3	0	0.449	0.043	0.365	0.530
10	60	12	0	0.359	0.041	0.280	0.440
11	48	4	0	0.330	0.040	0.252	0.409
12	44	3	0	0.307	0.040	0.232	0.386
13	41	5	0	0.270	0.038	0.198	0.346
14	36	4	0	0.240	0.037	0.172	0.314
15	32	6	0	0.195	0.034	0.133	0.266
16	26	1	0	0.187	0.034	0.127	0.257
17	25	5	0	0.150	0.031	0.096	0.215
18	20	1	0	0.142	0.030	0.090	0.207
19	19	4	0	0.112	0.027	0.066	0.172
21	15	3	0	0.090	0.025	0.049	0.146
22	12	2	0	0.075	0.023	0.038	0.128
23	10	2	0	0.060	0.021	0.028	0.109
24	8	4	0	0.030	0.015	0.010	0.070
25	4	1	0	0.023	0.013	0.006	0.059
27	3	2	0	0.008	0.008	0.001	0.038
43	1	1	0	0	.	.	.

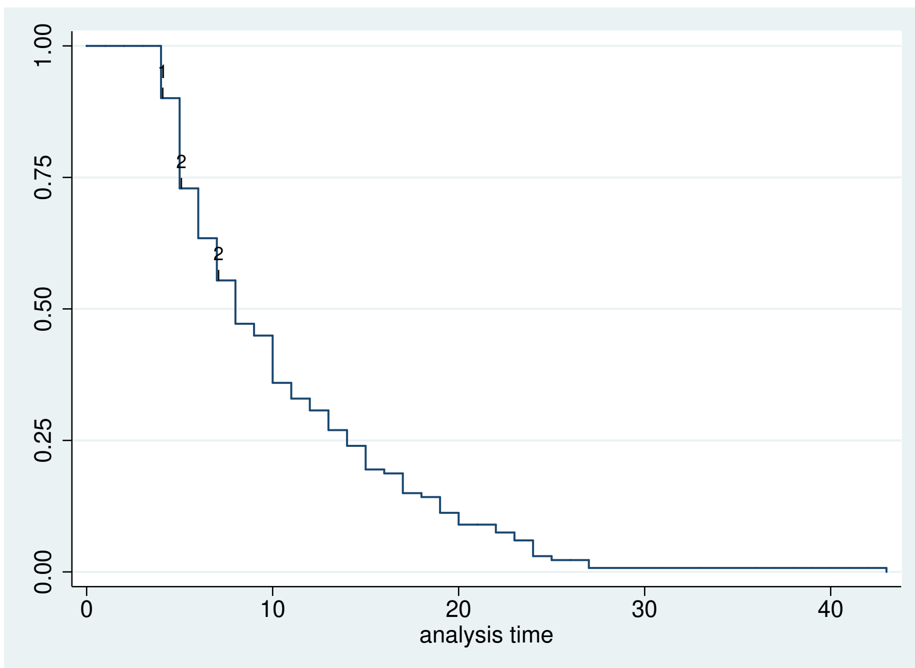
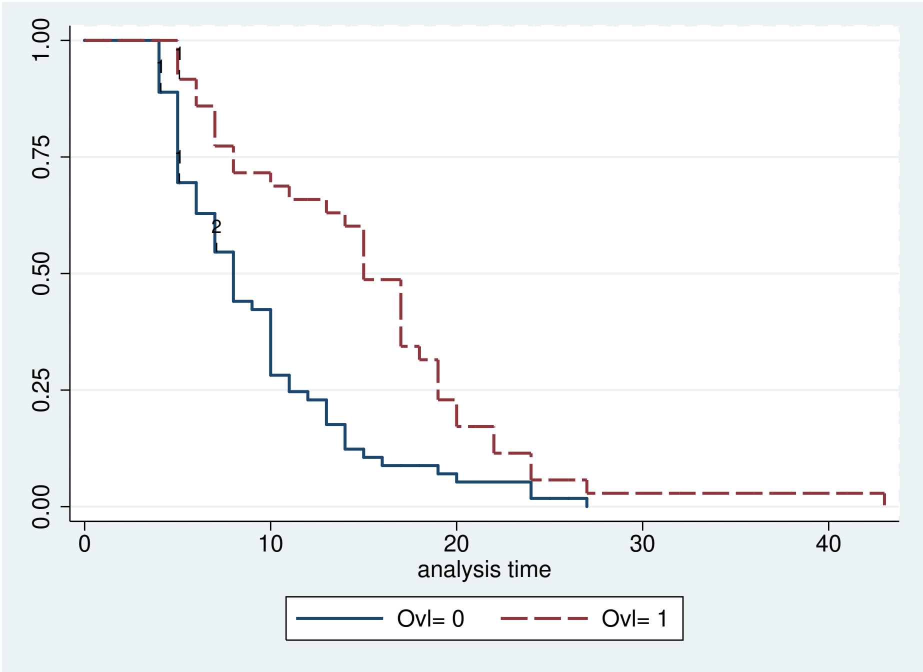
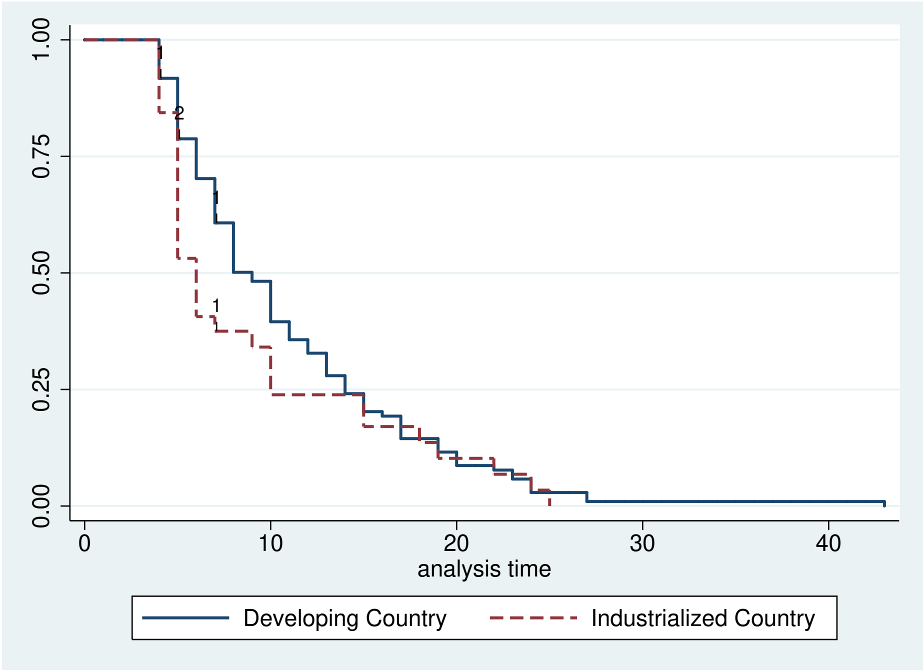


Figure 3.1 Kaplan-Meier Estimate: All Depressions



(a) Group by "Ovl": High Debt Episode



(b) Group by Income Level

Figure 3.2 Kaplan-Meier Estimates Comparison Between Groups

Table 3.2 Median and Mean Survival Times

(a) Median Survival Times

	(1)	(2)	(3)	(4)
		$dev = 0$	$dev = 1$	*Total (ovl)*
	$\hat{t}\,(50)$	8	6	8
A: $ovl = 0$	CI	$[7, 10]$	$[5, 10]$	$[6, 10]$
	N	47	16	63
	$\hat{t}\,(50)$	15	7	15
B: $ovl = 1$	CI	$[11, 17]$	$[5, ..]$	$[11, 17]$
	N	32	4	36
	$\hat{t}\,(50)$	10	6	10
C: *Total (dev)*	CI	$[8, 13]$	$[5, 10]$	$[8, 11]$
	N	79	20	99

(b) Mean Survival Times

	(1)	(2)	(3)	(4)
		$dev = 0$	$dev = 1$	*Total (ovl)*
	μ_T	9.47	9.22	9.38
A: $ovl = 0$	CI	$[7.95, 10.98]$	$[6.22, 12.22]$	$[8.03, 10.74]$
	N	47	16	63
	μ_T	15.57	13	15.28
B: $ovl = 1$	CI	$[12.83, 18.31]$	$[5.97, 20.03]$	$[12.72, 17.85]$
	N	32	4	36
	μ_T	12.00	10.04	11.59
C: Total (dev)	CI	$[10.41, 13.60]$	$[7.17, 12.92]$	$[10.19, 13.00]$
	N	79	20	99

Table 3.3 Significance Tests for Equality of Survivor Function

(a) Full Sample

ovl	Events Observed	Events Expected
0	59	43.09
1	35	50.91
Total	94	94.00
Log-Rank Test		Wilcoxon Test
$\chi^2(1) = 13.80$		$\chi^2(1) = 15.15$
$Pr > \chi^2 = 0.002$		$Pr > \chi^2 = 0.0001$

(b) Broken Down by Country Group

Industrialized Countries ($dev = 1$)		
ovl	Events observed	Events expected
0	15	13.94
1	4	5.06
Total	19	19
Log-Rank Test		Wilcoxon Test
$\chi^2(1) = 0.36$		$\chi^2(1) = 0.68$
$Pr > \chi^2 = 0.548$		$Pr > \chi^2 = 0.408$

Developing Countries ($dev = 0$)		
ovl	Events observed	Events expected
0	44	30.14
1	31	44.86
Total	75	75
Log-Rank Test		Wilcoxon Test
$\chi^2(1) = 13.64$		$\chi^2(1) = 14.41$
$Pr > \chi^2 = 0.002$		$Pr > \chi^2 = 0.0001$

Table 3.4 The Cox Proportional Hazard Model

Variables	(1)	(2)	(3)
ovl	-0.780***	-0.810***	
	(0.227)	(0.263)	
dev	0.083	0.305	0.654**
	(0.264)	(0.292)	(0.292)
inf		0.497***	0.603***
		(0.186)	(0.188)
dgdp			0.003
			(0.002)
Exits	94	76	75
N	1007	800	751

Standard Errors in Parentheses
*** $p < 0.01$, **$p < 0.05$, *$p < 0.10$

Table 3.5 Parametric Regression Models

	Weibull		Exponential		Gompertz	
	$\hat{\lambda} = 2.016$ Wald = 8.13		$\lambda = 1.00$		$\hat{\gamma} = 0.073$ Wald = 5.46	
Covariate	*HR*	*SE*	*HR*	*SE*	*HR*	*SE*
ovl	0.391***	0.099	0.621**	0.152	0.425***	0.112
dev	1.425	0.413	1.226	0.354	1.392	0.406
inf	1.775***	0.296	1.479**	0.238	1.641***	0.264
AIC	118.048		160.733		138.165	

*** $p < 0.01$; **$p < 0.05$, *$p < 0.1$
HR: Hazard Ratio; SE: Standard Error

Chapter 4

Negative Growth and High Debt: An AFT Parametric Survival Analysis Approach

4.1 Introduction

The general objective of this chapter and Chapter 3 is to analyze the duration of economic crises and its association with the occurrence of consecutive years of high public-debt-to-GDP ratios. In contrast to Chapter 3, five parametric survival analysis models are utilized in this chapter. Moreover, in the previous chapter, I focused only on the Kaplan-Meier estimate, Cox PH model, and the parametric models that have the proportional hazard (PH) metric for the analysis. In this chapter, I use more models and direct my attention to the use of those parametric models that have the accelerated failure time (AFT) metric, including the exponential, Weibull, log-normal, log-logistic and generalized gamma regression. Models in the *PH* metric, which are used in the previous part, are useful as an analog to Cox PH model, but little attention is paid to the actual failure time. The *AFT* metric emphasizes duration time itself which is especially important when we have time-varying covariates.

Compared to the Cox PH model, full parametric models involve stronger assumptions. Thus, there exists a danger of misspecification. But for a finite sample, the loss in precision from the Cox PH model can be large (Cox and Oakes (1984)). As a result, it is useful to use the parametric model, in addition to semi-parametric models, to analyze economic crises. Based on Cox and Oakes (1984), one advantage of the full parametric model is that it can provide more efficient parameter estimates if

1) parameter values are largely different from zero 2) a strong time trend exists in covariates 3) the follow-up of the observation depends on covariates. A comparison among the five parametric models is made and two criteria are used to determine which model provides the most efficient estimation.

The chapter is organized in the following way. Section 2 introduces the source of the data. Section 3 compares the regression results using the five parametric models which have the AFT metric. Section 4 adds different covariates to the log-normal regression model and section 5 shows the regression results when using different threshold values for the definition of *HDE*. And section 6 concludes the chapter.

4.2 Data

The dependent variable of interest is the duration of a "negative growth episode" (NGE). It is generated based on the work of Breuer and McDermott (2013) and constructed from the Penn World Table, version 9.0. These NGEs are severe economic downturns and defined to be those episodes when the country has experienced a cumulative decline in per capita output of 20 percent or more for at least four consecutive years. [1]

We wish to test whether a "high debt-to-GDP episode" (HDE), which is defined to be a period lasting for at least four consecutive years during which the debt-to-GDP ratio is at least 90 percent, is associated with an NGE. The data for public debt is from the IMF's Historical Public Debt Database (Abbas et al. (2011, 2013)). The indicator variable *ovl* equals 1 when $NGE = 1$ and at least one year of the NGE overlaps with an HDE; otherwise, it equals 0.

Two other initial covariates of interest are *dev* and *inf*. Variable *dev* is an indicator variable showing the level of development for the country. For industrialized countries, $dev = 1$; and for developing countries, $dev = 0$ (Source of data: World Bank). The

[1] Please see Chapter 3 for more detail.

variable *inf* is the inflation rate from International Financial Statistics. Please refer to the previous part of the dissertation for a more detailed explanation of these two variables.

4.3 Parametric Models: AFT Metric

In the previous chapter, three full parametric models, the Weibull, the exponential, and the Gompertz models that have the PH metric were discussed. In this section, I will show five parametric models that have the AFT metric including Weibull, exponential, log-normal, log-logistic, and generalized gamma regression models. The Weibull and exponential models are the only two models which have both the PH and AFT metrics; the log-normal, log-logistic, and generalized gamma regression models only have the *AFT* metric; the Gompertz model does not have AFT metric, so it is omitted in this section. For a detailed discussion regarding the difference between the five models, please refer to Appendix J.

Let the observed exit time to be called t_j (the observed duration of the negative growth episode), then t_j can be expressed as the product of a positive component exp^{XB} and an error term ϵ_j which only takes positive values:

$$T_j = exp^{XB} * \epsilon_j \tag{4.1}$$

where B is the coefficient matrix. Thus Equation (4.1) is one convenient and plausible way to characterize the distribution of t_j. And it can be linearized by taking the natural log of each side of the equation, so that we have the following equation:

$$\ln(T_j) = XB + ln(\epsilon_j) \tag{4.2}$$

Depending on what is assumed about the distribution of ϵ_j, we have different types of AFT models.

With algebraic manipulation of Equation (4.1), we can get $\epsilon_j = exp^{-XB} * T_j$. The term $exp(-XB)$ is called the "acceleration parameter" (Mario Cleves and Marchenko

(2016)) since its value determines the effect of covariates on the duration of survival episodes. In AFT metrics, the effects of covariates are shown to decelerate or accelerate the speed of exiting NGEs.

If *exp(-XB)>1,* the country is expected to exit the NGE quicker; and if *exp(-XB)<1*, it takes more time for the country to exit the NGE; if $exp(-XB) = 1$, time passes normally. The results for the five models are shown in Table 4.1.

4.3.1 Exponential model

In the exponential model, we assume that

$$\epsilon_j \sim Exponential\{exp(\beta_0)\}$$

That is, ϵ_j is assumed to be distributed as exponential with mean $exp\,(\beta_0)$.

Table 4.1 Column (1) provides the estimated coefficients for the exponential model. These coefficients correspond to B in Equation (4.1). Therefore, the acceleration parameter for *ovl* is $exp(-0.477) = 0.621 < 1$. For those NGEs that overlap with an HDE ($ovl = 1$), they have longer durations in general compared to those with $ovl = 0$. The effect of *ovl* is to slow down the time to exit the NGE. At the same time, the estimated impact of inflation is to accelerate the time to exit the NGEs with $exp(0.391) = 1.478 > 1$. However, the estimated coefficient of *dev* is not statistically significant and the corresponding acceleration parameter is really close to *one*.

Parametric models written in the AFT metric are convertible to the PH metric for both the exponential model and the Weibull model. For the exponential model, the hazard ratio equals $\exp(-\hat{\beta}_{AFT})$ with $\hat{\beta}_{AFT}$ being the coefficient estimated using AFT metric and $\hat{\beta}_{AFT} = -\hat{\beta}_{HR}$ where $\hat{\beta}_{HR}$ is the coefficient estimated using the proportional hazard metric. This can be confirmed by comparing the values in Table 5 in Chapter 3 with the results in Table 4.1 in this chapter. For example, the hazard ratio for variable *ovl* in Table 5 equals 0.621. The estimated coefficient for *ovl* in Table 4.1 for the exponential model in this part equals 0.477 where $exp(-0.477) = 0.621$.

When using the exponential model, we explicitly assume the hazards are constant over time which is probably unrealistic. Thus, in the following parts, I will use alternative assumptions regarding the distribution of ϵ_j.

4.3.2 Weibull model

In the Weibull model, we assume that

$$\epsilon_j \sim Weibull\{\beta_0, \lambda\}$$

That is, ϵ_j follows the Weibull distribution with two parameters (β_0, λ) where λ is called the shape parameter and as shown in the second row of Table 4.1, $\hat{\lambda} = 2.016$.

Based on the B coefficients estimated for the Weibull model in Table 4.1 Column (2), the effect of *ovl* is to slow down the time to exit the *NGEs* with the term $exp(-0.481) = 0.618 < 1$ and it is statistically significant within 1 percent confidence interval. At the same time, the impact of inflation is to accelerate the time to exit and it is statistically significant within the *5 percent* confidence interval. The estimated coefficient of *dev* again, is not statistically significant with the corresponding acceleration parameter $exp(0.224) \approx 1.251$.

As mentioned above, coefficients estimated from the *PH* metric are transformable with the coefficients estimated in the AFT metric with $\beta_{AFT} = \frac{-\beta_{PH}}{\lambda}$ for Weibull model. And the corresponding hazard ratio equals $\exp(-\hat{\lambda}\hat{\beta}_{AFT})$. This can be verified by comparing the results in Table 5 in the Chapter 3 with the results in Table 4.1 in this part. For example, the hazard ratio for *ovl* in Table 4.1 equals $exp(-2.016 * 0.481) \approx 0.379$ which is very similar to the value listed in Table 5 in the Chapter 3, which equals *0.391.*

4.3.3 Log-normal Regression

If $\epsilon_i \sim \text{lognormal}(\beta_0, \sigma)$ with two parameters β_0 and σ, Equation (4.2) is called the log-normal regression.

The esitmation of $\sigma$are listed in the second row of the Table 4.1 and $\hat{\sigma} = 0.485$. Since the log-normal regression model will turn out to have the best fit, in Table 4.2, we explicitly set out not only the coefficents, but also the acceleration parameters for each individual variable. Coefficient for *ovl* as shown in Table 4.2 equals 0.491. Its acceleration parameter $exp(-0.491) = 0.612$ is smaller than 1. Compared to the group without high debt, the NGEs with an HDE overlap move at a slower speed to exit the NGEs. The coefficient for *inflation* is -0.208, the corresponding acceleration parameter $exp(0.208) = 1.231$. An increase in inflation is associated with shorter duration of NGEs and the effect is statistically significant within the *5 percent* confidence interval. The coefficient for *dev* is -0.211 with $exp(0.211) = 1.235$. Higher income levels are associated with shorter duration of NGEs, but it is not statistically significant, which means whether the country is an industrialized country or not does not matter for the duration of NGEs.

4.3.4 Log-logistic Regression

If $\epsilon_i \sim loglogistic(\beta_0, \gamma)$, then ϵ_j has log-logistic distribution with two parameters β_0 and γ, and Equation (4.2) is called the log-logistic regression. In our model, $\hat{\gamma} = 0.280$ as shown in second row of Table 4.1.

For the log-logistic model, the way of interpretation is different from the other models. Whether the effect of the covariate is accelerating or decelerating time can be shown by using the median survival time. First of all, we set the survival function for the log-logistic model

$$S(t_j|X_j) = [1 + \{exp(-\beta_0 - x_j\beta_x)t_j\}^{\frac{1}{\gamma}}]^{-1}$$

to be equal to *0.5* and solve for t, the median survival time t_{50} for the log-logistic model is:

$$t_{50}(x, \beta, \gamma) = exp(\beta_0 + x_j\beta_x)$$

Then the time ratio (TR) at median evaluated at $ovl = 1$ versus $ovl = 0$ is:

$$\hat{TR}(t_{50}) = exp(\hat{\beta}_{ovl}) = 1.726$$

The exponentiated coefficient is the acceleration factor on the time scale. The median survival time for the NGEs with high debt is 1.726 times of the other group. Experiencing a high debt episode is associated with longer duration of the NGEs. The coefficient for *Inflation* is -0.183, $\hat{TR}_{inf}(t_{50}) = exp(-0.183) = 0.833$. An increase in inflation is associated with shorter duration of the NGEs.

4.3.5 Choosing among parametric models

Since there are many parametric models, the next question is which one provides the most appropriate parametric model? The more frequent used standard in choosing among different models is the Akaike information criterion (AIC): the best model should be one with the lowest value of AIC which is given by:

$$AIC = -2 \times ln(L) + 2(k + c)$$

In this expression, k is the total number of covariates in the parametric model, c is the number of the distributional parameters and L is the estimated log likelihood. That is, $c = 1$ for exponential model, $c = 2$ for Weibull, log-normal and log-logistic models, $c = 3$ for generalized gamma model. Our tests indicate that lognormal regression model provides the lowest AIC 106.497.

Another way of making the selection is based on the results of the generalized gamma model. For Equation (4.1), if $\epsilon_j \sim GenGamma(\beta_0, \kappa, \sigma)$, we have the generalized gamma regression with three parameters β_0, κ, σ, resulting in:

$$E\{ln(t_j|x_j\} = \beta_0 + x_j\beta_x + E(u_j)$$

where $u_j = ln(\epsilon_j)$, $E(u_j) = \frac{\sigma\Gamma(\gamma)}{\sqrt{\gamma}\Gamma'(\gamma)} + ln(\gamma)$ with $\gamma = |\kappa|^{-2}$ and $\Gamma()$ being the gamma function. The hazard function of the generalized gamma model is flexible and it can have many different shapes. If $\kappa = 1$, the distribution is the same as the Weibull function with the shape parameter $\lambda = \frac{1}{\sigma}$; if $\kappa = \sigma = 1$, the distribution is the same as the exponential distribution; if $\kappa = 0$, it is the lognormal distribution. So the generalized gamma model is often used as one standard to help us to choose the most appropriate parametric model. Based on the regression results from Table 4.1, $\hat{\kappa} = -0.186$. The Wald test statistic for the hypothesis that $H_0 : \kappa = 0$ is $z = -0.53$ and significance level is 0.594 based on the estimation result. We fail to reject the null hypothesis and the log-normal model is the one which offers us the most appropriate regression results. Meanwhile, when checking the 95% confidence interval for κ, it ranges between $(-0.872, 0.500)$ which rules out the value of 1 but includes 0. Thus, the results of generalized gamma regression suggests the log-normal model provides a better fit when compared with the Weibull model. Both the AIC standard and the generalized gamma regression point to the lognormal regression model to be the best among all survival analysis models used.

4.3.6 Post estimation of log-normal regression model

Figure 4.1 presents the predicted log-normal *hazard function*s, which provide the instantaneous probability of exiting the NGEs. Note that the predicted hazard function $h(t|x)$ can be obtained even for models which have only the AFT metric; however, the $h(t|x)$ may not have a *PH* decomposition and cannot be written in the *PH* form $h_0(t)exp(XB)$. Figure 4.1 provides hazard functions of four different groups based on whether they belong to an industrialized country or an emerging market economy and whether the NGEs encounter HDE while assuming the inflation rate is at its mean. The general shapes of the hazard functions for developing and industrialized countries are the same in that the hazard functions are increasing first, reaching peak, and then decreasing. The differences exist in: industrialized countries take more time to reach the peak turning point and the slope of the negative part is much flatter. In the following section, more analysis will be done using the log-normal regression.

4.4 More Covariates in the Log-normal Regression Model

4.4.1 Description of Additional Covariates

In this part, I use the log-normal regression model to see if our original results are robust to the addition of covariates in four categories: 1) economic variables, 2) political variables, 3) financial crisis variables, and 4) cultural variables. The selection of these covariates is similar to those used by Breuer and McDermott (2013). The intention is to control for other variables that have been shown to be important to long run economic development. The 20 additional covariates used are all listed below:

1. Civil Liberties: This index ranges from *1* to *7* with *7* representing the largest degree of freedom. Source: Freedom house

2. Democracy: This index ranges from *1* to *10* with *10* denoting the highest degree of institutional democracy. Source: Polity IV

3. Constraint on the Executive Power: This index ranges from *1* to *7* with *7* representing the largest degree of constraint. Source:Polity IV

4. Civil War: This is a dummy variable with its value equal to *1* when the negative growth episode encounters civil war. Source: Correlates of War Database.

5. Autocracy: This index ranges from *1* to *10*, higher value represents higher degree of the autocracy within the country. Source: Polity IV

6. Political Rights: This index ranges from *1* to *7* with *7* representing the largest degree of political rights. Source: Freedom house

7. Ethnic fractionalization: This indicator shows the degree of concentration for different ethnolinguistic groups within a country, ranges from *0* to 1. Source: Alesina et al. (2003)

8. Religious fractionalization: This indicator denotes the degree of concentration for different religious groups within the country, ranges from *0* to 1. Source: Alesina et al. (2003).

9. Trust: This value is based on the answer to the question "Generally speaking, would you say that most people can be trusted or that you need to be very careful in dealing with people?" Higher value of *Trust* means more proportion of people selected "most people can be trusted. Source: Question *A165*, *World Values Survey.*

10. Ethnic Polarization: This is a measure of concentration of ethnolinguistic groups within a country; If there are two groups within a country, its value reaches the maximum. Source: Montalvo and Reynal-Querol (2005)

11. Religious Polarization: Religious polarization is a measure of concentration of religious groups within a country; If there are only two groups within a country, its value reaches the maximum. Source: Montalvo and Reynal-Querol (2005)

12. Attitude toward Government Responsibility: This value ranges from 1 to 10 with 1 representing "People should take more responsibility" and *10* representing "The government should take more responsibility". Response is averaged over individuals across all the available waves for each countries. Source: Question *E037 World Values Survey.*

13. Confidence in the Justice and Court System: Confidence is a index with the value ranging from 1 to 4 based on the answer to the question "how much confidence do you have in the justice system". The value 1 represents "no

confidence at all" and 4 represents "a great deal". Source:Question *E069_17 World Values Survey*.

14. Latitude: Latitude is the distance from the equator (Porta et al. (1998)).

15. Banking Crisis: This is a dummy variable which equals 1 when the events such as "bank closure, merger, or takeover by the public sector or larger scale government assistance" happen. Source: Laeven and Valencia (2008)

16. Currency Crises: This is a dummy variable which equals 1 when the country experiences "the nominal depreciation of the currency of at least 30 percent that is also at least a 10 percent increase in the rate of depreciation compared to the year before". Laeven and Valencia (2008)

17. Openness: This value is the ratio of sum of import and export to the total GDP. It is from Penn World Table 7.0.

18. Liberalization: This is an indicator ranges between 0 and 1, and it is retrieved from Wacziarg and Welch (2008).

19. Population: it is in millions and collected from United Nations.

20. The natural logarithm of real GDP per capita: It is from Penn World Table 9.0.

4.4.2 Robustness Check

To maintain degrees of freedom, for each regression, I only add one new covariate each time. The results for the log-normal regression model shown in Tables 4.3, 4.4, 4.5, and 4.6 are to be compared with the results in Table 4.1 to verify whether the association between *ovl* and the country's resilience ability from economic depression has changed after adding these different controls.

Comparing results in Table 4.3, 4.4, 4.5, and 4.6 with the results in Column (1) of Table 4.1, I have found that the association between *NGE* and *HDE* remains even though many different controls were added. I, thus, obtain the conclusion that the existence of high debt episodes is associated with longer depressions, whereas higher inflation is associated with shorter depressions. The magnitude of the estimated coefficients for *ovl* are similar to the results in Column (1) of Table 4.1. The association between debt level and the resilience of the country is not hampered by other factors existing in the world.

Table 4.3 presents the estimation results when I insert economic variables, one at a time: *openness, liberalization, population*, and *ln*(*Real GDP)*. Variable *openness* is defined as $\frac{Import+Export}{GDP}$ which is gathered from Penn World Table 7.0 and the estimated coefficient is -0.003. The corresponding acceleration parameter equals $exp(0.003) \approx 1.003$ which approximates one but higher than 1, meaning the higher degree of exposure to world trade accelerates the speed the country to exit the economic depression. The Row titled "No. Subjects" equals 75 which represents that we have 75 HDEs within the regression, and 69 in the Row titled "No. Exits" represents we have 69 HDEs that exited the episode normally, and the remaining $75 - 69 = 6$ HDEs were censored.

Another covariate *liberalization*, an indicator ranges between *0* and *1*, is retrieved from Wacziarg and Welch (2008). Compared to the covariate *opennness*, *liberalization* measures not only the degree of trade openness, but also takes into account monetary and fiscal policies together with the existence of state monopoly within the country. So *liberalization* reflects the degree of economic freedom within the country. I expected the association between the trade liberalization and duration of NGEs to be negative. That is, higher degree of liberalization should be coincident with better performance of the economy (Wacziarg and Welch (2008) and Bekaert et al. (2005)). The estimated result turns out to be as I expected, the coefficient equals -0.435, with the corresponding acceleration parameter to be 1.545, showing that higher degree of economic liberalization is associated with shorter duration of economic depression in general. The impacts of the two variables mentioned above are evident within 95 percent confidence interval.

The impact of *population* on economic growth is controversial. Based on the study of Peterson (2017), lower population growth in industrialized countries would create economic problems while high population growth in developing countries would have negative impact on economic development. In this study, *population* shows no association with the duration of NGEs with its estimated acceleration parameter approximating 1 and there is no statistical significance. The estimated coefficient for output per capita in the form of *ln*(*Real GDP per capita)* is 0.042 and the corresponding acceleration parameter is $exp(-0.042) = 0.959$. Thus, the magnitude of the negative association is close to one and there is no statistical significance.

Table 4.4 presents the results when controlling for the political environment, including civil liberties, democracy, constraint on executive power, an indicator for civil war, autocracy, and political rights. The coefficient for civil liberties equals -0.076 and the corresponding acceleration parameter is 1.078. It is statistically significant within *10 percen*t confidence interval. A higher degree of civil liberties is associated with shorter duration of *NGEs*. Meanwhile the coefficient for autocracy equals 0.035 and it is statistically significant within 5 percent confidence interval. This means higher autocracy is associated with lower resilience ability for the country to exit the NGEs. These results are the same as many of the current researches which associate better economic performance with better institutional quality(Chong and Calderon (2000),Butkiewicz and Yanikkaya (2006)). Meanwhile Table 4.4 also show that *democracy* etc have no effect.

Table 4.5 examines the impact of cultural factors, including l*atitude, ethnic fractionalization, religious fractionalization* and *trust*. Besides that, we also use ethnic polarization, religious polarization, attitude toward government responsibility and confidence in the justice and court system. The data for the cultural variables are limited. Some variables only have 1 observation for each country across the sample period. Thus, I assume that the cultural factors are constant for each country by using averages of all available observation for each country.

Variable *latitude* is the distance from the equator (Porta et al. (1998)) to the country of analysis. It helps capture the initial conditions of human capital and perhaps ethnic and climatic difference. The coefficient for *latitude* is -0.052, and the corresponding acceleration parameter is $exp(0.052) = 1.053$. However, there is no statistical significance. Thus, we fail to prove that latitude is associated with the resilience ability of the country to get out the negative growth episode.

Actually, except for *ethnic fractionalization* and *attitude*, all the other cultural variables included show no statistical significance within our regressions. Although not significant, all of the coefficients are positive. *Ethnic fractionalization*, *religious polarization*, *ethnic polarization* and *religious polarization* are ways of examining social conflict. The difference between fractionalization and polarization is the value of the fractionalization increases as we have more number of groups within the country while the value of polarization maximizes when we have only 2 groups within one country. The coefficient for *ethnic fractionalization* is 0.634 and the acceleration parameter is 0.530. Thus a higher value of *religious fractionalization* is associated with longer duration of *NGEs*. Thus this provides some evidence that higher potential social conflict has a negative impact on the ability of the country to get out of a depression. This result is similar to Easterly and Levine (1997) who find that *ethnic fractionalization* in Africa explains a large part of the difference in the public policies used which have an impact on the economic growth of the country.

Three variables including *trust, confidence in the justice system* and *the attitude toward government responsibility* are generated based on the survey answers from the World Value Survey. Trust is a form of social capital. Low trust is proved to be associated with lower growth. Confidence in the justice and court system reflect the reliability that property rights and liberties can be protected. Attitude towards government responsibility reflect degree of preference toward government welfare. The coefficient for "*attitude*" equals 0.154 with 10 percent confidence interval. It means that if the people in the country believe that government should take more responsibility, then the *NGEs* would be longer.

In Table 4.6, two types of crises including currency crises and bank crises are examined to determine if they affect the resilience of a country's ability to escape from depression. *bank crises* is a dummy variable, its value equals 1 when either closure, merger or government intervention occurs to the financial institutions. This variable is from Laeven and Valencia (2008). Both of the crises mentioned fail to find any association with the duration of NGEs.

4.5 Other Threshold Value for *HDE*

In this part, I do a robustness check by running the the log-normal regression model again using two alternative threshold values for the definition of the high debt episode, *70 percent* and *80 percent.* The results are shown in Table 4.7. The coefficient for *ovl70* equals *0.178* and there is no statistical significance; Coefficient for *ovl80* equals *0.357* and it is statistically significant within *one percent* confidence interval. The coefficient for *ovl* when we use 90 percent as threshold value shown in Table 4.2 equals 0.491. Thus, when we use higher threshold value *of 80 percent,* the magnitude of the coefficient is similar to when we use the threshold value of *90* percent. The key claim from Reinhart and Rogoff (2010) is that countries having debt above 90 percent of GDP have poor economic performance, compared to others. In this chapter, a threshold value for high debt is thus supported; it may differ among countries, but appears to be over *80* percent.

4.6 Conclusion

This chapter investigates whether consecutive years of high public-debt-to-GDP ratios are systematically associated with the duration of negative growth episodes (depressions) using parametric analysis, including the exponential model, Weibull model, log-normal model, log-logistic model and generalized gamma model, all of which have the form of accelerated failure time (AFT) metric. Using these five full parametric analysis models, I demonstrate that episodes of high public-debt ratios are associated with longer negative growth episodes; a higher inflation rate is associated with shorter duration of negative growth episodes. Regardless of a country is being industrialized or not, it does not matter for the length of time the country remains in an economic depression. These results are consistent among all the survival analysis models I used. Comparison among different models shows the lognormal model is the most desirable.

To add further robust checks, I employ 20 different covariates, including economic factors, political factors, cultural factors and financial crises factors, one-at-a-time, to the original lognormal regression model. My main results were not refuted by adding these covariates as additional controls.

Table 4.1 Parametric Regression Model-AFT Metric

	Exponential		Weibull		Log-normal		Log-logistic		Generalized Gamma	
	$\lambda = 1$		$\hat{\lambda} = 2.016$		$\hat{\sigma} = 0.485$		$\hat{\gamma} = 0.280$		$\hat{\sigma} = 0.479$	
									$\hat{\kappa} = -0.186$	
Covariate	Coefficient	SE	Coefficient	SE	Coefficient	SE	Coefficient	SE	Coefficient	SE
ovl	0.477*	0.245	0.481***	0.122	0.491***	0.121	0.546***	0.125	0.481***	0.122
dev	-0.204	0.289	-0.224	0.143	-0.211	0.142	-0.232	0.152	-0.224	0.143
inf	-0.391**	0.161	-0.195**	0.094	-0.208**	0.091	-0.183**	0.075	-0.195**	0.094
$\hat{\beta}_0$	2.258***	0.184	2.423***	0.095	2.161***	0.090	2.136***	0.090	2.120***	0.117
AIC	160.733		118.048		106.497		109.146		108.209	

*** $p < 0.01$; **$p < 0.05$, *$p < 0.1$

SE: Standard Error

Table 4.2 Acceleration Parameter for the Lognormal Model

Covariate	Lognormal	
	Coefficient	Acceleration parameter
ovl	0.491***	0.612
dev	-0.211	1.235
inf	-0.208**	1.231
$\hat{\beta}_0$	2.161***	0.115
No. Subjects	80	
No. Failures	74	
*** $p < 0.01$; **$p < 0.05$, *$p < 0.1$		

Table 4.3 Log-Normal Regression with Economic Factors

Coefficients	$X = openness$	$X = liberalization$
ovl	0.531***	0.536***
inf	-0.212**	-0.184**
dev	-0.100	-0.022
X	-0.003**	-0.435**
No. Subjects	75	77
No. Exits	69	69
$\hat{\sigma}$	0.465	0.456

Coefficients	$X = population$	$X = ln(Real\ GDP)$
ovl	0.497***	0.520***
inf	-0.207**	-0.211**
dev	-0.206	-0.296
X	0.00017	0.042
No. Subjects	81	81
No. Exits	76	76
$\hat{\sigma}$	0.485	0.486

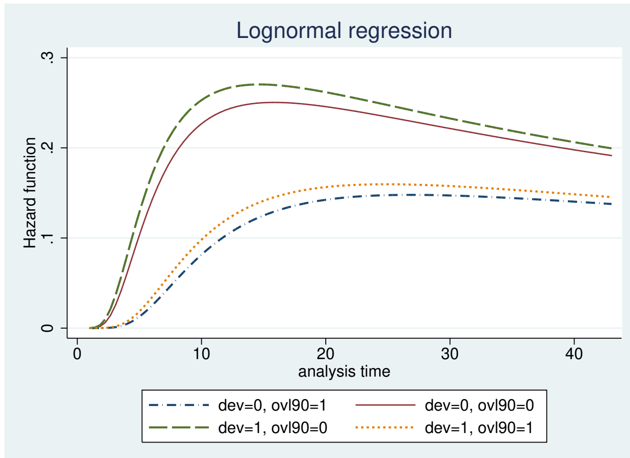


Figure 4.1 Hazard Function From Log-normal Post Estimation

Table 4.4 Log-normal Regression Results with Political Factors

Coefficient	X = *civil Liberties*	X = *democracy*	X = *cons.exec*
ovl	0.527***	0.524***	0.524***
inf	-0.201**	-0.201**	-0.203**
dev	-0.077	-0.108	-0.119
X	-0.076*	-0.025	-0.035
No. Subjects	76	75	75
No. Exits	68	67	67
$\hat{\sigma}$	0.460	0.464	0.464

Coefficient	X = *civil War*	X = *autocracy*	X = *political rights*
ovl	0.567***	0.534***	0.549***
inf	-0.217**	-0.193**	-0.196**
dev	-0.174	-0.174	-0.079
X	0.053	0.035**	-0.040
No. Subjects	77	75	76
No. Exits	70	67	68
$\hat{\sigma}$	0.469	0.464	0.464

Table 4.5 Log-normal Regression Results with Cultural Factors

Coefficient	X = *latitude*	X = *ethnic frac.*	X = *religious frac.*
ovl	0.586***	0.492***	0.583***
inf	-0.211**	-0.167*	-0.194**
dev	-0.084	0.020	-0.092
X	-0.052	0.634**	0.030
No. Subjects	77	75	76
No. Exits	69	68	68
$\hat{\sigma}$	0.478	0.461	0.481

Coefficient	X = *trust*	X = *religious polar.*	X = *ethnic polar.*
ovl	0.385**	0.599***	0.619***
inf	-0.198**	-0.230*	-0.233**
dev	-0.186	-0.223	-0.237
X	0.890	0.106	0.122
No. Subjects	38	65	65
No. Exits	36	58	58
$\hat{\sigma}$	0.453	0.443	0.443

Coefficient	X = confidence	X = *attitude*
ovl	0.408**	0.347*
inf	-0.108	-0.212**
dev	-0.289	-0.224
X	0.416	0.154*
No. Subjects	36	38
No. Exits	34	36
$\hat{\sigma}$	0.447	0.444

Table 4.6 Log-normal Regression Results with Financial Crisis

Coefficient	X = *currency crises*	X = *bank crises*
ovl	0.590***	0.585***
inf	-0.207**	-0.223**
dev	-0.088	-0.083
X	-0.109	0.238
No. Subjects	76	76
No. Exits	68	68
$\hat{\sigma}$	0.477	0.474

Table 4.7 Lognormal Model with Other Thresholds

Covariate	*ovl70*	*ovl80*
ovl	0.178	0.357***
	(0.134)	(0.124)
dev	-0.282*	-0.262*
	(0.158)	(0.149)
inf	-0.238**	-0.223**
	(0.105)	(0.0978)
$\hat{\beta}_0$	2.267***	2.196***
	(0.120)	(0.0999)
$\hat{\sigma}$	0.532***	0.510***
	(0.044)	(0.043)
No. Subjects	81	81
No. Exits	76	76

*** $p < 0.01$; **$p < 0.05$, *$p < 0.1$
Note: Standard errors in parentheses.

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Appendix A

Terminology for Survival Analysis

Survival analysis is the modeling of data where the outcome variable of interest is the time to occurrence of an event of interest. We usually call the event of interest to be the "failure event" even though it may not necessary to be a real failure. For example, survival analysis was applied to check the probability of default on debt on a given future time period for individual credit card applicants (Bellotti and Crook (2009)). In this case, the failure event refers to the action that the applicants default their credit cards. There are also papers using survival analysis to check the duration of fiscal crises (Ekanayake (2016)). In this scenario, the failure event is the *end* of the fiscal crises. In my paper, one of the outcome variables of interest is the duration of the high debt episode. The fail event refers to the last year of high debt episode. Thus I call the corresponding "failure" in my paper to be "exit". Correspondingly, the onset of the risk is the first year when the country starts a high debt episode. Compared to other statistical methods, survival analysis yields good estimates of parameters when the data includes observations with censoring or truncation.

A.1 Survival Function and Hazard Function

Survival analysis has some unique statistical terms. If we have a non-negative random variable T to represent the time to the exit event, the cumulative distribution function can be retrieved:

$$F(t) = Pr(T \leq t)$$

F(t) shows the probability that a subject, selected at random, has a survival time less than or equal to time t. The derivative of function $F(t)$ is the probability density function $f(t)$ telling us the probability of surviving at time t. These two descriptive statistics are widely used in the standard economic models. However, survival analysts talk more about the reverse cumulative distribution function-the survival function which is the probability of surviving beyond time t, denoted:

$$S(t) = Pr(T > t)$$

$S(t)$ is the probability that there is no exit event before time t. For example, in my paper, $S(t)$ refers to the probability that the duration of high debt episode will be longer than time t. When $t = 0$, then $S(0) = 1$ and the value of $S(t)$ decreases towards zero as time t goes to infinity. The survival functions are monotone, non-increasing. This is to say that eventually every object is going to fail (die); what matters is how long it takes. The sum of survival function and its corresponding cumulative distribution function is one:

$$S(t) = 1 - F(t)$$

Survival analysis has three different forms - nonparametric, semi-parametric and parametric. The difference among them are depending on how we assume the form of survival function and how covariates affect the survival experience.

Another important statistical value frequently used in survival analysis is the *hazard function*, $h(t)$, which is the instantaneous rate of failure. It is the limiting probability that the failure event occurs in a given interval, conditional upon the subject having survived to the beginning of that interval, divided by the width of the interval.

$$h(t) = lim_{\Delta t \to 0} \frac{Pr(t + \Delta t > T > t | T > t)}{\Delta t} = \frac{f(t)}{S(t)}$$

The hazard rate ranges from zero to infinity. When the value is zero, it represents no risk at all and when it equals to infinity, it represents a failure at that moment is going to happen for sure. The corresponding cumulative hazard function $H(t)$ measures the total amount of risks that have been accumulated up to time t.

$$H(t) = \int_0^t h(u)du \tag{A.1}$$

Applying the above definition to my paper and assuming that $t = 4$, the value of $F(4)$ is the probability that a country experiencing a high debt ratio, has a high debt ratio lasting for less than or equal to four years. The value of $f(4)$ is the possibility of a country to remain in the state of high debt ratio after four years of experiencing high debt ratio. The value of *S(4)* is the possibility of the country to have the high debt ratio lasting for more than four years. Finally $h(4)$ is the possibility that the country exits the high inflation episode after 4 years of being in that episode.

A.2 Median and Mean

The median time, T, statistically is the halfway point. Half of the population has a value larger than T and half of the population has a value smaller than T. In the survival model, the median exit time $\tilde{\mu}_t$ is the $50th$ percentile of the exit time distribution with $t_{50} = \tilde{\mu}_T$. It represents the moment beyond which 50% of subjects are expected to survive:

$$S(\tilde{\mu}_T) = 0.5$$

The presence of censored observations makes it impossible to get the mean or median survival time using standard methods. For example, in order to get median survival time using the standard method, we need to sort the data and find the middle one as the median. With the existence of censored data, it is impossible to make sure that the order is correct. Instead, we can obtain the median by calculating the survival probabilities. In the case of no censored observations, the median survival time is the middle observation of the ranked survival times.

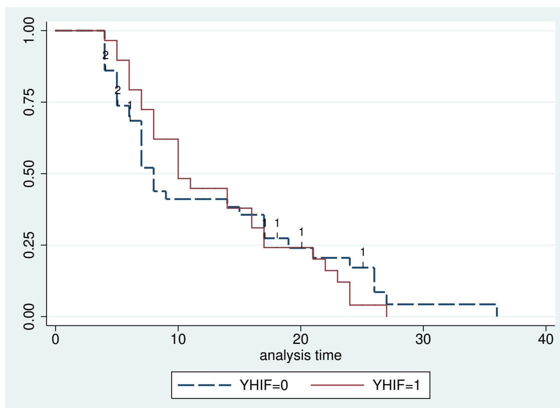
The mean time to exit, μ_t, is defined as:

$$\mu_T = \int_0^\infty tf(t)dt$$

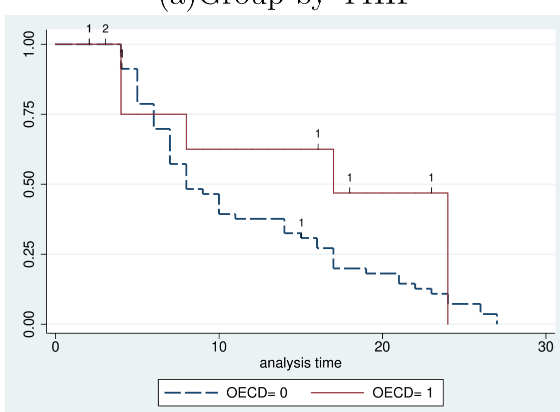
Appendix B

Results for HDE_{90} with HIF_{10}

This part shows the analysis results when using the threshold value of 90 percent for HDE.

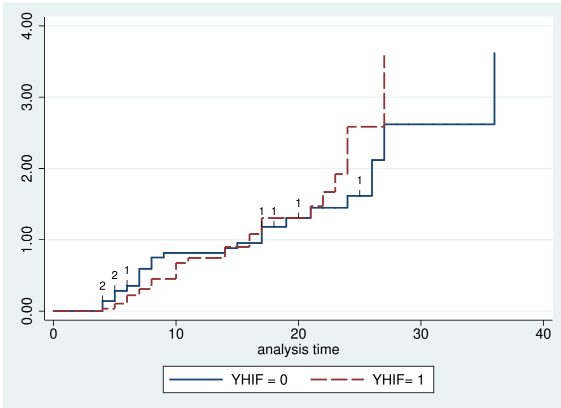


(a)Group by YHIF

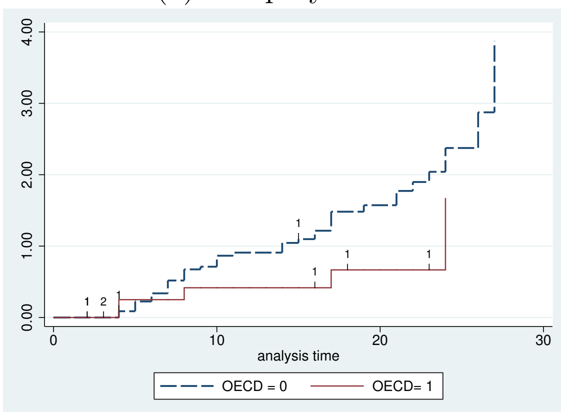


(b)Group by OECD

Figure B.1 Kaplan-Meier Survival Estimates for HDE_{90}



(a)Group by YHIF



(b)Group By OECD

Figure B.2 Nelson-Aalen Cumulative Hazard Estimates for HDE_{90}

Table B.1 Log-Rank test for HDE_{90}

(a)Group by HIF_{10}

$YHIF_{10}$	Events observed	Events expected
0	35	35.38
1	28	27.62
Total	63	63
$\chi^2(1) = 0.01$		
$Pr > \chi^2 = 0.917$		

(b)Group by OECD

$OECD$	Events observed	Events expected
0	56	52.33
1	5	8.67
Total	61	61
$\chi^2(1) = 2.09$		
$Pr > \chi^2 = 0.148$		

Table B.2 Median and Mean Survival Time for HDE_{90}

(a)Median Survival time for HDE_{90}

$YHIF_{10}$	No. of subjects	50%	Std. Err.	[95% Conf.	Interval]
0	43	8	0.653	7	17
1	29	10	1.076	8	16
Total	72	10	1.091	7	14

(b)Mean Survival time for HDE_{90}

$YHIF_{10}$	No. of subjects	Mean	Std. Err.	[95% Conf.	Interval]
0	43	12.849	1.517	9.877	15.821
1	29	13.086	1.288	10.561	15.611
Total	72	12.889	1.016	10.899	14.880

Table B.3 Median and Mean Survival Time for HIF_{10}

Median Survival time for HIF_{10}

$YHDE_{90}$	No. of subjects	50%	Std. Err.	[95% Conf.	Interval]
0	101	5	0.301	5	6
1	30	8	0.913	6	11
Total	131	6	0.376	5	7

Mean Survival time for HIF_{10}

$YHDE_{90}$	No. of subjects	Mean	Std. Err.	[95% Conf.	Interval]
0	101	7.634	0.502	6.651	8.617
1	30	10.167	0.979	8.248	12.085
Total	131	8.213	0.456	7.319	9.108

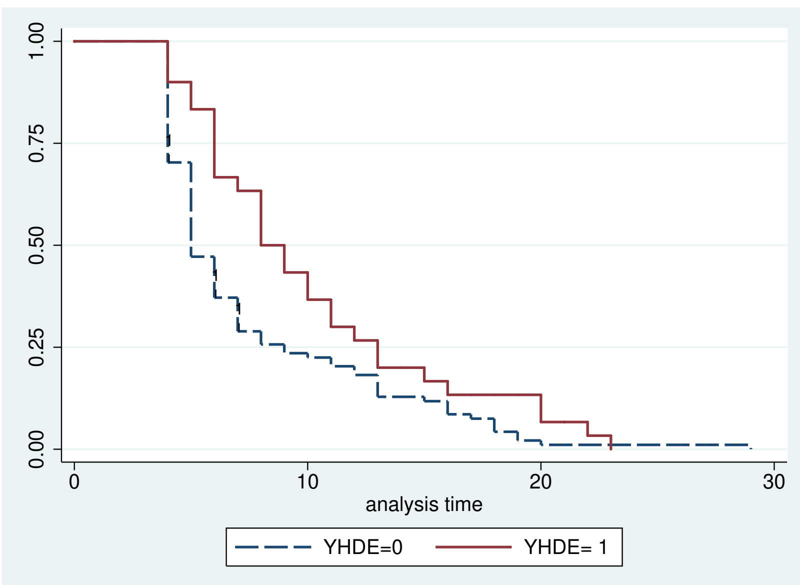


Figure B.3 Kaplan Meier Estimate for HIF_{10}

Table B.4 Log Rank Test for HIF_{10}

$YHDE_{90}$	Events observed	Events expected
0	98	87.43
1	30	40.57
Total	128	128

$\chi^2(1) = 5.22$

$Pr > \chi^2 = 0.022$

Appendix C

Log-Rank Test and Wilcoxon Test

In this section, we present a brief discussion regarding to the difference between the Log-rank test and Wilcoxon test. Both tests can be used to test for the equality of survivor functions across different groups for the Kaplan-Meier estimate; however, they are different in a minor way, which turns out to have different powers for testing the equality. To begin the discussion, we make up stories regarding the frequency of events happened in two different groups as shown in Table C.1. Suppose that we have two group of observations, group 1 and group 0. The total number of subjects at risk at observed survival time t_i is n_i, and there are n_{1i} subjects at risk in group 1 and n_{0i}subjects at risk in group 0. And among the n_{1i} subjects in group 1, there are d_{1i} exit events observed and the remaining $n_{1i} - d_{1i}$ are called the "Not Exit Events"; Similarly, among the n_{0i} subjects at group 0, there are d_{0i} exit events and $n_{0i} - d_{0i}$ "Not Exit Events". The total number of deaths within both groups is d_i.

The total number of "Exit Event" is obtained by assuming that the survival function is the same in each of group 1 and group 0. For example, the estimator for group 1 is

$$\hat{e}_{1i} = \frac{n_{1i} d_i}{n_i} \tag{C.1}$$

Then the estimator for the variance of d_{1t} is defined as follows:

$$\hat{v}_{1i} = \frac{n_{1i} n_{0i} d_i (n_i - d_i)}{n_i^2 (n_i - 1)}$$

The general form for the test statistics is defined as a ratio of weighted sums over the observed survival times:

$$Q = \frac{[\sum_{i=1}^{m} w_i(d_{1i} - \hat{e}_{1i})]^2}{\sum_{i=1}^{m} w_i \hat{v}_{1i}}$$

The value of w_i is weight which depends on the specific test. Under the null hypothesis that the survival functions are the same across two groups and the survival experience is independent to each other, the p-value can be obtained by using the chi-square distribution with one degree of freedom ($p = pr(\chi^2(1)) \geq Q$).

When the value of $w_i = 1$, the test is often called the *Log-rank te*st which puts more weight on the larger values of time. When the weights are equal to the number of subjects at risk at each survival time, $w_i = n_i$, this test is called the *Wilcoxon test.* It puts more weight on differences between the survival functions at smaller values of time.

Table C.1 Test of Equality of Survival Functions in Two Groups

Event/Group	1	0	Total
Exit Events	d_{1i}	d_{0i}	d_i
Not Exit Events	$n_{1i} - d_{1i}$	$n_{0i} - d_{0i}$	$n_i - d_i$
At Risk	n_{1i}	n_{0i}	n_i

Appendix D

Computation of Survival Analysis

D.1 Maximum Likelihood

The use of maximum likelihood is to choose parameters to maximize the likelihood that matches the model for the observed data. When the disturbances of the model are not normally distributed, the most efficient estimators are the maximum likelihood estimators, at least asymptotically. Survival analysis uses maximum likelihood to get the parameters of interest.

The first step is to build up the maximum likelihood function L, which is an expression that shows the probability of observed data under the model.

In the following explanation, I will use the the duration of high inflation episode as an example. When we have $F(t, \beta, x)$ with $t = 5, x = 1$,[1] the value of this cumulative distribution function gives the probability of an OECD country to have a high inflation episode less than or equal to 5 years. Thus the value of $S(5, \beta, 1)$ gives the proportion of an OECD country expected to have at least 5 years of continuous high inflation.

When the observation is right censored, we know nothing about the exact time when the high inflation episode is going to end. However, we know about the probability of surviving for t years when the observation is censored. Thus we can use $S(t, \beta, x)$ to approximate the probability. Meanwhile, to get the maximum likelihood function, we need the probability of the moment surviving when the observation is

[1] x is dummy variable and $x = 1$ if the country is a member of the OECD countries; $x = 0$ if not.

not censored. $f(t, \beta, x)$ gives the probability when the survival time is exactly t.

Under the assumption of independent observations, the full likelihood function is obtained by multiplying the respective contributions of observed triplets, a value of $f(t, \beta, x)$ for a non-censored observation and a value of $S(t, \beta, x)$ for censored observation.

$$[f(t, \beta, x)]^c \times [S(t, \beta, x)]^{1-c}$$

c equals to 0 if the data is censored and equals to 1 if not censored.

Under the assumption that the observations are assumed to be independent, the likelihood function is the product of the expression in the above equation over the entire sample and is shown as follows:

$$l(\beta) = \prod_{i=1}^{n} \left\{ [f(t_i, \beta, x_i)]^{c_i} \times [S(t_i, \beta, x_i)]^{1-c_i} \right\}$$

To obtain the parameter of interest, β, we maximize the log-likelihood function:

$$L(\beta) = \sum_{i=1}^{n} \{c_i ln[f(t_i, \beta, x_i)] \times (1 - c_i)[S(t_i, \beta, x_i)]\} \tag{D.1}$$

D.2 Proportional Hazards Models-Likelihood Calculation

Based on Cox(1972), the hazard rate at time $t$for a subject whose covariate is x is given by:

$$h_i(t) = h_0(t) * exp\{\beta_1 x_{i1} + \beta_2 x_{i2} + ... + \beta_k x_{ik}\} \tag{D.2}$$

$h_0(t)$ is called the baseline hazard function which can be any complicated function of t as long as $h_0(t) \geq 0$ and its functional form with time is not specified in the model. The remaining part of the hazard function $h_i(t)$ involves an exponential function of covariates and it does not depend on time. Thus the hazard ratio between any two subject at t^{th} analysis time is:

$$\begin{aligned} \frac{h_i(t)}{h_j(t)} &= \frac{h_0(t) * exp\{\beta_1 x_{i1} + \beta_2 x_{i2} + ... + \beta_k x_{ik}\}}{h_0(t) * exp\{\beta_1 x_{j1} + \beta_2 x_{j2} + ... + \beta_k x_{jk}\}} \\ &= exp\{\beta_1(x_{i1} - x_{j1}) + ... + \beta_k(x_{ik} - x_{jk})\} \end{aligned} \tag{D.3}$$

And β in the above equation is to be estimated from the maximum likelihood estimation. Now assume that we have no tied event, which means that each episode of high debt episode in the sample has a at different time slot.

The probability that a country gets out of the high debt episode is calculated by dividing the hazard for the country to exit HDE by the sum of all the hazards for all the countries who are under high debt episode. And the partial likelihood function is the product of the likelihoods for all the countries in the sample assuming that the HDE events for the countries are independent of each other:

$$PL = \prod_{j=1}^{n} L_j \tag{D.4}$$

PL represents "partial likelihood" and L_j is the likelihood for the j^{th} event.

If an event occurred at 5^{th} month, the L_j could be written as

$$L_j = \left(\frac{h_0(5)exp\{\beta x_i\}}{h_0(5)exp\{\beta x_i\} + h_0(5)exp\{\beta x_{i+1}\} + ... + h_0(5)exp\{\beta x_n\}} \right)^{c_i} \tag{D.5}$$

with $c_i = 1$ if the country exits the HDE and $c_i = 0$ if censored (HDE ended just because it is the last year of the sample). $h_0(t)$ can be cancelled out in the above equation.

$$L_i = \left(\frac{exp\{\beta x_i\}}{exp\{\beta x_i\} + exp\{\beta x_{i+1}\} + ... + exp\{\beta x_n\}} \right)^{c_i} \tag{D.6}$$

The hazards included in the denominator are only those individuals who are at risk at the i^{th} event (or censoring) time. And the entire likelihood function can be expressed very concisely as

$$PL = \prod_{i=1}^{n} \left(\frac{exp\{\beta x_i\}}{\sum_{t_j \geq t}^{n} exp\{\beta x_j\}} \right)^{c_i} \tag{D.7}$$

The partial maximum likelihood estimate of β can be obtained by maximizing the log form of above equation with respect to β.

Appendix E

Handling Tied Data

In the above explanation, we assume that we know the exact ordering of the failure events and the failure events happened one by one. However, in the real life, we frequently come across scenarios when we have ties. For example, in our sample, in the 6^{th} year of analysis time, different countries were in the last year of the HDE. That is, based on the limitation of data, we have no information about the exact timing of which country exited HDE earlier compared to other counties. However, in the Cox regression, the ordering of the failure event is important to the calculation of partial likelihood function. When we face ties, we need special treatment. In the following paragraph, I will use an example to better show you the different approximation methods used.

Based on the Table E.1, the second and third patient have the same survival times. The partial likelihood function is $L(\beta) = L_3(\beta)4(\beta)L_5(\beta)$ which is based on the failure time shown in the sample at time 3, 4, 5. Failure happened at time 2 is censored data, so it is not included in the partial likelihood calculation. And $L_4(\beta)$

Table E.1 An Example of Ties for Cox PH model

Patient	Failure time	Failure or Censored	x
1	2	0	x_1
2	3	1	x_2
3	3	1	x_3
4	4	1	x_4
5	5	1	x_5

and $L_5(\beta)$ can be written as:

$$L_4(\beta) = \frac{e^{x_4}}{e^{x_4} + e^{x_5}}$$

$$L_5(\beta) = \frac{e^{x_5}}{e^{x_5}} = 1$$

Now the calculation of $L_3(\beta)$ needs some special treatment based on different assumption used. In the following paragraphs, four different approximations are introduced.

The Exact-Marginal calculation Since $L_3(\beta)$ =P[observe two failure at time 3]=P[A_2+A_3] assuming failure event happened to patient 2 and 3 are independent to each other. Thus we can have:

$$L_3(\beta) = P[A_2] + P[A_3]$$

Both patient 2 and 3 failed at time 3. However, they did not really fail at the exact same time, we were just limited to the information we have. Thus we can assume $P[A_2]$ is the probability that patient 2 failed first and then comes the patient 3. Then we can have

$$P[A_2] = \frac{e^{x_2\beta}}{e^{x_2\beta} + e^{x_3\beta} + e^{x_4\beta} + e^{x_5\beta}} \times \frac{e^{x_3\beta}}{e^{x_3\beta} + e^{x_4\beta} + e^{x_5\beta}} \tag{E.1}$$

$$P[A_3] = \frac{e^{x_3\beta}}{e^{x_2\beta} + e^{x_3\beta} + e^{x_4\beta} + e^{x_5\beta}} \times \frac{e^{x_2\beta}}{e^{x_2\beta} + e^{x_4\beta} + e^{x_5\beta}} \tag{E.2}$$

The above method is called the exact-marginal method for the estimation of β.

Breslow's approximation The exact-marginal is computationally intensive. So we can use the Breslow's approximation (Breslow (1974)) to deal with ties:

$$\frac{e^{x_3\beta}}{e^{x_3\beta} + e^{x_4\beta} + e^{x_5\beta}} \approx \frac{e^{x_3\beta}}{e^{x_2\beta} + e^{x_3\beta} + e^{x_4\beta} + e^{x_5\beta}}$$

so

$$P[A_2] = \frac{e^{x_2\beta}}{e^{x_2\beta} + e^{x_3\beta} + e^{x_4\beta} + e^{x_5\beta}} \times \frac{e^{x_3\beta}}{e^{x_2\beta} + e^{x_3\beta} + e^{x_4\beta} + e^{x_5\beta}}$$

Meanwhile:

$$\frac{e^{x_2\beta}}{e^{x_2\beta}+e^{x_4\beta}+e^{x_5\beta}} \approx \frac{e^{x_2\beta}}{e^{x_1\beta}+e^{x_2\beta}+e^{x_3\beta}+e^{x_4\beta}+e^{x_5\beta}}$$

$$P[A_3] = \frac{e^{x_2\beta}}{e^{x_2\beta}+e^{x_3\beta}+e^{x_4\beta}+e^{x_5\beta}} \times \frac{e^{x_3\beta}}{e^{x_2\beta}+e^{x_3\beta}+e^{x_4\beta}+e^{x_5\beta}}$$

$$L_3(\beta) = \frac{2e^{x_2\beta}e^{x_3\beta}}{(e^{x_2\beta}+e^{x_3\beta}+e^{x_4\beta}+e^{x_5\beta})^2}$$

Efron's Method Efron's method is also an approximation to the Exact marginal calculation. Compared to the exact marginal method, it uses the weight to adjust the subsequent risk set. In our example, patient 2 and 3 failed at time 3 . With $P[A_2]$ representing the possibility that patient 2 failed earlier than patient 3, based on Equation E.1, it assumes that the possibility that patient 3 failed is out of the total risk set of $e^{x_3\beta}+e^{x_4\beta}+e^{x_5\beta}$. With $P[A_3]$ representing the possibility that patient 3 failed earlier than patient 2, based on Equation E.2, it assumes that the possibility that patient 2 failed is out of the total risk set of $e^{x_2\beta}+e^{x_4\beta}+e^{x_5\beta}$. Efron use average of two risk sets which is

$$(e^{x_3\beta}+e^{x_4\beta}+e^{x_5\beta}+e^{x_2\beta}+e^{x_4\beta}+e^{x_5\beta})/2 = \frac{1}{2}(e^{x_3\beta}+e^{x_2\beta})+e^{x_4\beta}+e^{x_5\beta}$$

as approximation. Thus:

$$P[A_2] = \frac{e^{x_2\beta}}{e^{x_2\beta}+e^{x_3\beta}+e^{x_4\beta}+e^{x_5\beta}} \times \frac{e^{x_3\beta}}{\frac{1}{2}(e^{x_2\beta}+e^{x_3\beta})+e^{x_4\beta}+e^{x_5\beta}}$$

$$P[A_3] = \frac{e^{x_3\beta}}{e^{x_2\beta}+e^{x_3\beta}+e^{x_4\beta}+e^{x_5\beta}} \times \frac{e^{x_2\beta}}{\frac{1}{2}(e^{x_2\beta}+e^{x_3\beta})+e^{x_4\beta}+e^{x_5\beta}}$$

$$L_3(\beta) = \frac{2e^{x_2\beta}e^{x_3\beta}}{(e^{x_2\beta}+e^{x_3\beta}+e^{x_4\beta}+e^{x_5\beta})\{\frac{1}{2}(e^{x_2\beta}+e^{x_3\beta})+e^{x_4\beta}+e^{x_5\beta}\}}$$

The Exact-Partial calculation Now we assume patient 2 and 3 did fail at the same time and treat this problem as multinomial problem. Now two failures are to happen at the same time among patients 2, 3, 4, 5, the possibilities are:

- 2 and 3 fail
- 2 and 4 fail
- 2 and 5 fail
- 3 and 4 fail
- 3 and 5 fail
- 4 and 5 fail

The conditional probability that 2 and 3 failed is :

$$L_3(\beta) = \frac{e^{x_2\beta}e^{x_3\beta}}{e^{x_2\beta}e^{x_3\beta} + e^{x_2\beta}e^{x_4\beta} + e^{x_2\beta}e^{x_3\beta} + e^{x_2\beta}e^{x_5\beta} + e^{x_3\beta}e^{x_4\beta} + e^{x_3\beta}e^{x_5\beta} + e^{x_4\beta}e^{x_5\beta}}$$

This method is called the exact-partial calculation.

Appendix F

Computation of Schoenfeld Residuals

According to Appendix **??**, the partial likelihood is given by the following expression:

$$PL = \prod_{i=1}^{n} \left(\frac{\exp\{\beta x_i\}}{\sum_{j=1}^{n} Y_{ij} \exp\{\beta x_j\}} \right)^{c_i} \tag{F.1}$$

Assuming that there is no tied times and excluding the censoring data, we can retrieve Equation (F.2):

$$l_p(\beta) = \prod_{i=1}^{m} \frac{e^{x_{(i)}\beta}}{\sum_{j \in R(t_{(i)})} e^{x_j \beta}} \tag{F.2}$$

The corresponding log partial likelihood function is

$$L_p(\beta) = \sum_{i=1}^{m} \left\{ x_{(i)} - ln \left[\sum_{j \in R(t_{(i)})} e^{x_j \beta} \right] \right\} \tag{F.3}$$

The maximum partial likelihood estimator can be achieved by differentiating the right hand size of equation F.3 with respect to β_k, and set the equation equals zero. The derivative with respect to β is

$$\begin{aligned} \frac{\partial L_p(\beta)}{\partial \beta_k} &= \sum_{i=1}^{m} \left\{ x_{ik} - \frac{\sum_{j \in R(t_i)} x_{jk} e^{x'_j \beta}}{\sum_{j \in R(t_i)} e^{x'_{jk} \beta}} \right\} \\ &= \sum_{i=1}^{m} \left\{ x_{ik} - \bar{x}_{w_{i,k}} \right\} \end{aligned} \tag{F.4}$$

where

$$\bar{x}_{w_i k} = \frac{\sum_{j \in R(t_i)} x_{jk} e^{x'_j \beta}}{\sum_{j \in R(t_i)} e^{x'_j \beta}}$$

The estimator of the Schoenfeld residual for the i^{th} subject on the k^{th}covariate is obtained from the Equation (F.4) by substituting the partial likelihood estimator of the coefficient, $\hat{\beta}$,

$$\hat{r}_{ik} = x_{ik} - \hat{\bar{x}}_{w_i k}$$

where $\hat{\bar{x}} = \frac{\sum_{j \in R(t_i)} x_{jk} e^{x'_j \hat{\beta}}}{\sum_{j \in R(t_i)} e^{x'_j \hat{\beta}}}$ is the estimator of the risk set conditional mean of the covariate. The sum of Schoenfeld residuals equal to zero. Intuitively $\hat{r}_{ik}$ is the difference between the covariate value for the failed observation and the weighted average of the covariate values (weighted according to the estimate relative hazard form the Cox proportional hazards model. Grambsch and Thermeau (1994) suggest that scaling the Schoenfeld residuals by an estimator of its variance yields a residual with greater diagnostic power than the unscaled residuals. The vector of scaled Schoenfeld residual is the product of the inverse of the covariance matrix times the vector of residuals, namely

$$\hat{r}_i^* = \left[\widehat{Var}(\hat{r}_i)\right]^{-1} \hat{r}_i \qquad \text{(F.5)}$$

The elements in the covariance matrix $\hat{Var}(\hat{r}_i)$ are a weighted version of the usual sum-of-squares matrix computed using the data in the risk set. For the i^{th} subject, the diagonal elements in this matrix are:

$$\widehat{Var}(\hat{r}_i)_{kk} = \sum_{j \in R(t_i)} \hat{w}_{ij} (x_{jk} - \hat{\bar{x}}_{w_{ik}})^2 \qquad \text{(F.6)}$$

And the off diagonal elements in the matrix are

$$\widehat{Var}(\hat{r}_i)_{kl} = \sum_{j \in R(t_i)} \hat{w}_{ij} (x_{jk} - \hat{\bar{x}}_{w_i k})(x_{jl} - \hat{\bar{x}}_{w_i, k})$$

where $\hat{w}_{ij} = \frac{e^{x'_j \hat{\beta}}}{\sum_{l \in R(t_i)} e^{x'_l \hat{\beta}}}$

However, there is an approximation for F.5 which is:

$$\hat{r}_i^* = m\widehat{Var}(\hat{\beta})\hat{r}_i$$

with m being the number of observed uncensored events.

Deleted Link test from accessing the adequacy of the proportional hazard function. Link Test: It is a test based on re-estimation. If our original model is correctly specified, adding additional variables will contribute little or no explanatory

Table F.1 Link Test for HDE_{50}

	Haz. Ratio	Robust Std. Err.	z	P>z	[95% CIE]	
$\hat{\beta}_1$	3.392802	3.11362	1.33	0.183	0.561564	20.49829
$\hat{\beta}_2$	1.33929	1.687839	0.23	0.817	0.1132789	15.83436

power to the original model. In this test, we first retrieve $\hat{\beta}_x$ from the Cox proportional model and plug $\hat{\beta}_x$ into the equation: $LRH = \beta_1(x\hat{\beta}_x)+\beta_2(x\hat{\beta}_x)^2$. If $x\hat{\beta}_x$ is the correct specification to the original model, we should have $\beta_1 = 1$ and $\beta_2 = 0$. According to Table F.1, the coefficient for $\hat{\beta}_2$ is not statistically significant. So it verifies that the coefficient on the squared linear predictor is insignificant.

Appendix G

Testing the PH Assumption

According to the Cox PH model, the ratio of the hazard functions between any two groups should be independent of time. Although both hazards would have increased after, say, 10 years, their ratio should be the same as in the first year. Here, we check whether this PH condition is valid in our data.

If the PH model is correct, the survival function of one group (say, $ovl = 1$) is related to the survival function of the other group ($ovl = 0$) as follows:[1]

$$S_1(t) = S_0(t)^{\exp(\beta_{ovl})} \tag{G.1}$$

This means that $\ln S_1 = \ln S_0 \exp(\beta_{ovl})$, where we note that $\ln S_1 < 0$ and $\ln S_0 < 0$ since survival functions are positive but less than 1. Multiply both sides of the log expression by -1 (to make each side positive) and take the natural log a second time to yield:

$$\ln[-\ln S_1] - \ln[-\ln S_0] = \beta_{ovl} \tag{G.2}$$

In words, Equation (G.2) says that there should be an equal distance of β_{ovl} between the two transformed survival functions over analysis time, assuming all the other covariates x_{-i} are the same in the two groups.

In Figure G.1 we plot the transformed Kaplan-Meier survival functions against the natural log of time. The top line is for $ovl = 1$ and the bottom line for $ovl = 0$.

[1] To see this, note that $h(t|x_i, x_{-i}) = h_0(t)exp(x\beta_x)$ implies that the cumulative hazard can be written as
$H(t|x_i) = \int_0^t h(t|x_i, x_{-i})d_t = \int_0^t h_0(t)exp(XB)d_t = exp(XB)\int_0^t h_0(t)d_t = exp(XB)H_0(t)$. By definition, $S(t) = e^{-exp(XB)H_0(t)} = e^{-H_0(t)exp(XB)} = S_0(t)^{exp(XB)}$.

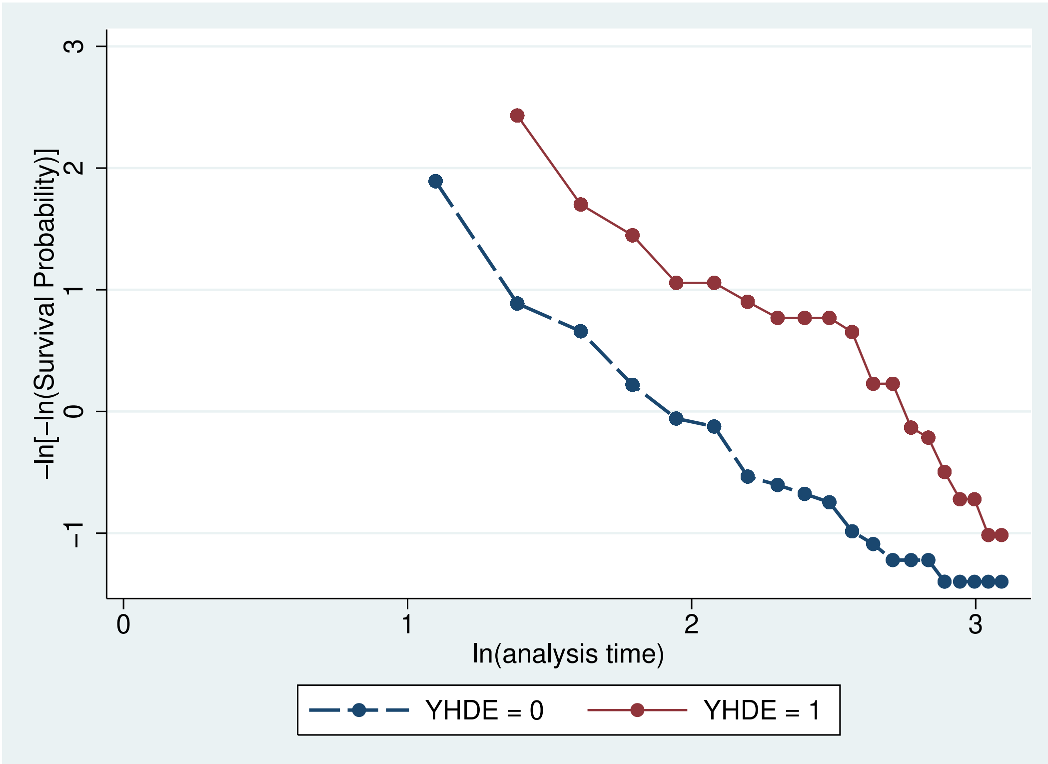


Figure G.1 Transformed Survival Functions

The two lines are roughly parallel to each other, which suggests that the PH condition holds.

There are two other methods that rely on the s*caled Schoenfeld residuals*[2]. If the Cox PH assumption is correct, Grambsch and Therneau (1994) showed that these residuals, scaled in a certain way, should have a zero slope when graphed with respect to failure time. Our visual inspection of the residuals (not shown) confirms a lack of a relationship between the residuals and the failure date, t.

There is also a formal test based on the *Schoenfeld residuals*. According to the Cox model, the coefficient of the covariate x on the survival experience should be

[2]A Schoenfeld residuals is the difference between the covariate value of a failed observation and a weighted average of covariate values over subjects at risk of failure at that time. The weights come from the estimates of the Cox model. See Grambsch and Therneau (1994) and Mario Cleves and Marchenko (2016).

Table G.1 Test of the PH Assumption

Covariate	df	$g(t)=t$		$g(t)=ln(t)$		$g(t)=\hat{S}_{KM}(t)$		$g(t)=rank(t)$	
		$chi2$	p	$chi2$	p	$chi2$	p	$chi2$	p
ovl	1	1.21	0.271	0.97	0.324	0.96	0.326	1.13	0.289
dev	1	0.15	0.701	0.81	0.367	0.90	0.342	0.82	0.364
inf	1	0.00	0.976	0.00	0.951	0.00	0.945	0.00	0.952
Global	3	1.59	0.662	2.21	0.530	2.31	0.511	2.40	0.493

constant in time. Assume that the coefficient is given by:

$$\beta(t) = a + bg(t) \tag{G.3}$$

where a and b are constants, and $g(t)$ is some function of time. If the PH assumption holds, then $b = 0$. Grambsch and Therneau (1994) proved that the mean of the approximation of scaled Schoenfeld residuals has the form:

$$E(r(t)) \cong bg(t)$$

where $r\,(t)$ are the Schoenfeld residuals. If we specify the $g\,(t)$ function we can use regression tests of the residuals to see if $\hat{b} = 0$. Table G.1 shows the results of the tests using four common specifications of the function $g\,(t)$. In the table, $\hat{S}_{KM}\,(t)$ refers to the Kaplan-Meier estimate of the survival function and $rank\,(t)$ is the rank of each failure. All the *p-values* in Table G.1 are higher than 0.1, so we fail to reject the null hypothesis. The main results in Table G.1 show no evidence of violation of the proportional hazard assumption.

Appendix H

Goodness of Fit

If the Cox PH model fits the data well, the true cumulative hazard function $H_0(t)$ has an exponential distribution with a hazard rate of 1. Although the Cox model does not assume a particular form for the baseline hazard function, we can back out an estimate of the baseline cumulative hazard $\hat{H}_0$ conditional on $\hat{\beta}$ that was estimated in fitting the Cox model to our data. Now define the Cox-Snell residual for the jth observation to be:

$$CS_j = \hat{H}_0(t_j) \exp\left(X_j \hat{\beta}\right) \tag{H.1}$$

The better the model fit, the closer the estimated cumulative hazards rate of the Cox-Snell residuals themselves would be to 45-degree line passing through the origin. We use the Nelson-Aalen method to estimate the cumulative hazard rate of the Cox-Snell residuals. That is, we treat the CS residuals as the time variable and the original data as exit or "failure" data, and then generate the Nelson-Aalen cumulative hazard function and plot it against the CS residuals. The result is shown in Figure H.1.

Our estimate of the cumulative hazard of the Cox snell residuals (solid line) moves fairly closely around the dashed 45-degree line. Some variability around the 45 degree line is normal even if we have a well fitted model. Thus, the Cox-Snell residuals suggest that the Cox model fits the data well.

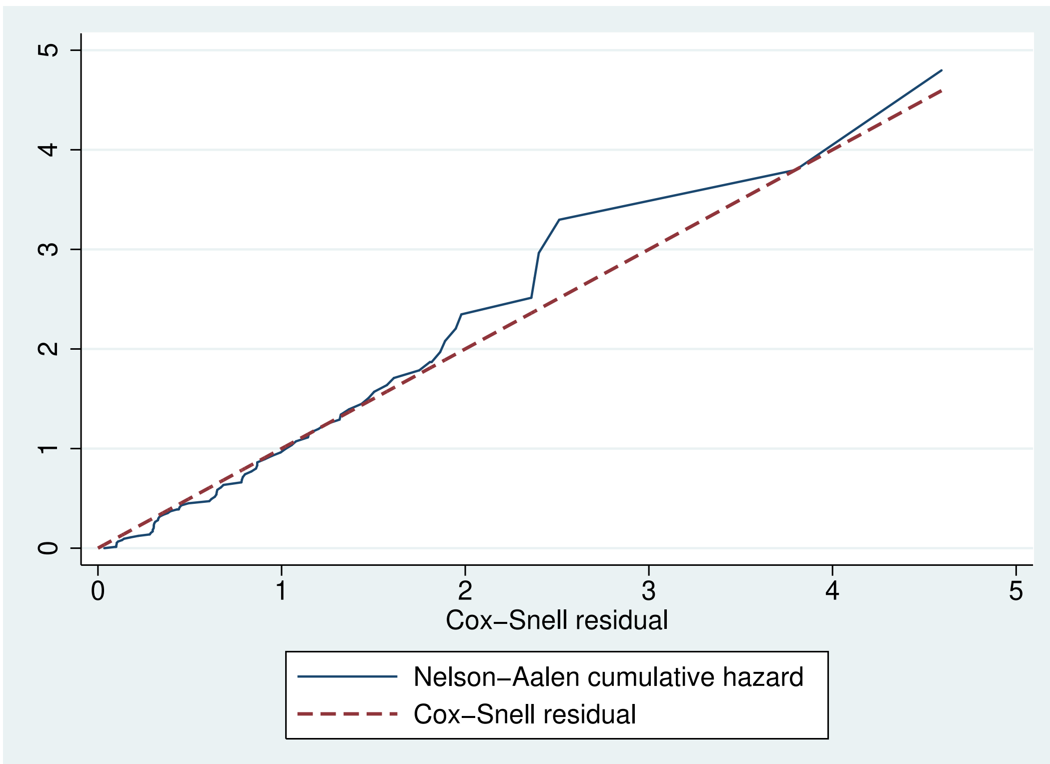


Figure H.1 Goodness of Fit

Appendix I

Alternative Thresholds for Defining High-Debt Episodes

Here, we show the regression results of the Cox PH model using different threshold standards for the definition of *ovl*, which is associated with the definition of HDE. HDE_{80} uses the threshold for debt-to-GDP of 80 percent; HDE_{70} uses the threshold of 70 percent. The results are shown in Table I.1. When we use HDE_{80}, the hazard ratio for *ovl* is $e^{-0.572} = 0.564$ and it is statistically significant at the 10 percent level; when we use HDE_{70}, the hazard ratio for *ovl* is $e^{-0.224} = 0.799$ and it is not statistically significant.

This provides support for the idea that the effect of debt is non-linear. Only high-debt episodes above a certain threshold will affect or be associated with difficulty in quitting the negative growth episode. Interestingly, the association of inflation and duration is considerably larger in the cases of lower debt thresholds.

Table I.1 Cox Model with HDE_{70} and HDE_{80}

Coefficient	HDE_{80}	HDE_{70}
ovl	−0.572*	−0.224
	(0.250)	(0.258)
dev	0.351	0.422
	(0.293)	(0.297)
inf	0.507***	0.552***
	(0.188)	(0.192)
Exits	76	76
N	800	800

Standard Errors in Parentheses
*** $p < 0.01$, **$p < 0.05$, *$p < 0.10$

Appendix J

The Five AFT Parametric Models

In this part, I provide more detailed about the five parametric models that have the AFT metrics: the Weibull, exponential, log-normal, log-logistic and generalized gamma regression models and how they are different from each other.

Equation (4.2) in the text is:

$$\ln(T_j) = XB + \ln(\epsilon_j) \tag{J.1}$$

It can be expressed as:

$$\ln(T_j) = XB + \ln(\epsilon_j) = x_j\beta_x + \beta_0 + u_j \tag{J.2}$$

where $u_j = \ln(\epsilon_j)$

J.1 Exponential Model

In the exponential model, we assume that

$$\epsilon_j \sim Exponential\{\exp(\beta_0)\}$$

ϵ_j is assumed to be distributed as exponential with mean $exp(\beta_0)$. And [1]it turns out that:

$$E\{\ln(T_j)|x_j\} = x_j\beta_x + \beta_0 + \Gamma'(1)$$

where $\Gamma'(1) \approx -0.577$ is the negative of Euler's constant.

[1] u_j follows the extreme value distribution which is the limiting distribution (as $n \to \infty$) of the minimum of a large number of unbounded i.i.d random variables. In my paper, durations of the episode are values bounded below by *zero*. Thus the limiting distribution here is also called Gumbel distribution or the log-Weibull distribution.

J.2 Weibull Model

In the Weibull model, we assume that

$$\epsilon_j \sim Weibull\{\beta_0, \lambda\}$$

So ϵ_j follows Weibull distribution with two parameters (β_0, λ) where λ is called the shape parameter. Now:

$$E\{\ln(T_j)|x_j\} = \beta_0 + \frac{\Gamma'(1)}{\lambda} + x_j\beta_x$$

J.3 Log-normal Regression

If $\epsilon_i \sim \text{lognormal}(\beta_0, \sigma)$ with two parameters β_0 and σ, Equation (4.2) is called the log-normal regression[2]. In this case:

$$E\{\ln(T_j)|x_j\} = \beta_0 + x_j\beta_x + E\{u_j\} \tag{J.3}$$

where u_j follows a standard normal distribution with mean zero and standard deviation of σ, it follows that:

$$E\{\ln(T_i)|x_i\} = \beta_0 + x_j\beta_x \tag{J.4}$$

J.4 Log-logistic Regression

If $\epsilon_i \sim \textit{loglogistic}(\beta_0, \gamma)$, we assume that ϵ_j has log-logistic distribution with two parameters β_0 and γ, and Equation (4.2) is called the log-logistic regression. Meanwhile, u_j in Equation (J.2) follows a logistic distribution with mean 0 and standard deviation $\pi\gamma/\sqrt{3}$. As a result, $E\{\ln(t_i)|x_i\}$ is the same as equation (J.4) for the log-normal regression model.

---

[2]The corresponding cumulative distribution function is $F(\epsilon) = \Phi(\frac{\ln\epsilon - \beta_0}{\sigma})$. $\Phi()$ is the cumulative distribution function from a standard normal distribution.

J.5 Generalized Gamma Regression

If $\epsilon_j \sim GenGamma(\beta_0, \kappa, \sigma)$, we have the generalized gamma regression with three parameters β_0, κ, σ, resulting in:

$$E\{\ln(T_j|x_j\} = \beta_0 + x_j\beta_x + E(u_j)$$

where $E(u_j) = \frac{\sigma\Gamma(\gamma)}{\sqrt{\gamma}\Gamma'(\gamma)} + \ln(\gamma)$ with $\gamma = |\kappa|^{-2}$ and $\Gamma()$ being the gamma function. This model's hazard function is flexible in that it can have many different shapes. So this model is often used as one standard to help us to choose the most appropriate parametric model.

Appendix K

Odds Ratio

Odds of some events refer to the likelihood that the event will take place. The Odds ratio is the ratio of the odds of an event occurring in one group to the odds of it occurring in another group.

One advantage of using the log-logistic regression is its slope coefficient can be expressed as an odds-ratio which is independent of time. The odds ratio is a measure of relative effect. In our paper, it can help the comparison of survival experience of the group with ($ovl = 1$) relative to the group without HDE ($ovl = 0$). According to Cox and Oakes (1984), the log-logistic model is the only AFT model which has a proportional odds ratio that is independent of time. To get the odds-ratio, we begin by writing the survival function of the log-logistic regression:

$$S(t_j|X_j) = [1 + \{\exp(-\beta_0 - x_j\beta_x)t_j\}^{\frac{1}{\gamma}}]^{-1} \tag{K.1}$$

The odds of surviving for at least time t_j is:

$$\frac{S(t_j|x_j)}{1 - S(t_j|xj)} = \exp(\frac{-(ln(t_j) - \beta_0 - x_j\beta_x)}{\gamma})$$

Based on this equation, the odds ratio at time t, evaluated at $ovl = 1$ and $ovl = 0$ is

$$OR(t, ovl = 1, ovl = 0) = \frac{\exp[\frac{-(ln(t) - \beta_0 - \beta_{YHDE} * 1 - x_{-i}\beta_x)}{\gamma}]}{\exp[\frac{-(ln(t) - \beta_0 - \beta_{YHDE} * 0 - x_{-i}\beta_x)}{\gamma}]} = \exp(\beta_{ovl}/\gamma) \tag{K.2}$$

Based on Table 4.1, $\hat{\gamma} = 0.280$. Then for the dummy covariate *ovl*, the odds-ratio equals $exp(0.546/0.280) = 7.029$. The odds of survival beyond time t for NGEs with high debt is 7.029 times that of the NGEs without high debt episode, this holds for all time t. For the continuous covariate inf, the odds ratio equals $exp(-0.183/0.280) = 0.520 < 1$, one unit increase in inflation is associated with lower odds of surviving beyond time t.