Continuous Tow Path Generation for Constant and Variable Stiffness Composite Laminates on Single and Double Curved Surfaces

Caleb R. Pupo

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CONTINUOUS TOW PATH GENERATION FOR CONSTANT AND VARIABLE STIFFNESS COMPOSITE LAMINATES ON SINGLE AND DOUBLE CURVED SURFACES

by

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DEDICATION

I would like to dedicate this work to the Almighty God, to my unconditionally supportive family, to my beloved, and to my friends. A special feeling of gratitude to my loving parents, Caleb R. Pupo Sr. and Mery C. Correa whose words of encouragement and constant sacrifices pushed me to become the person I am today. My sisters Elibeth A. Pupo and Kathya M. Pupo have never left my side and are always supportive of me.

I also dedicate my work to my beloved Mattson Carpenter who have supported me through this process. I will forever be grateful for your love and support.

Last but not least, I dedicate this work to my closest friends Timothy Shelley and Carlos Restrepo for being there for me through my entire college career. It has been an amazing experience, my brothers.
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ABSTRACT

A novel approach for the generation of steered tow paths on curved shell surfaces is presented. The approach is used to translate optimal theoretical analysis expressed from ply to ply as fiber angle distributions for steered laminates into discrete tow paths on planar, single and double curved surfaces. By adapting the multi mesh approach earlier developed by van Tooren and Elham [1–3] to the new tow path generation code. Two meshes are utilized, a coarse mesh which is generated during the optimization and contains the optimal fiber angle distributions, and a denser mesh obtained from tessellating the surface to better follow the curvature of the 3D geometry. The coarse mesh is referred to as the manufacturing mesh (MM) and it is used to define fiber angle fields using Lagrangian interpolation applied to the nodal fiber angle values by mathematically manipulating the MM elements into isoparametric space. The dense mesh is referred to as the tessellation mesh (TM) and it is used to orthogonally project the interpolated tow paths onto the tessellated surface. The tow path planner (TPP) initiates by introducing seed points, which are the defined starting locations of tow paths. Different seed point locations yield different tow path distributions on the surface, and therefore different seed point propagation strategies are introduced in the TPP. Finally, the software is validated through the implementation of various fiber angle distributions generated by hand, and the coupling between TopSteer and the new TPP is validated through a simple 3D optimization.
PREFACE

Before you lies the thesis manuscript “Continuous tow path generation for constant and variable stiffness composite laminates on single and double curved surfaces”. The project was undertaken at the request of GKN – Fokker Aerospace at the University of South Carolina. The project’s focus was on answering the question “How to generate continuous tow paths on single and double curved shell surfaces through a theoretical design generated by TopSteer for which fiber angle distribution per ply is given as a continuous angle field, discretized using the manufacturing mesh concept?” The research was difficult but conducting extensive investigation has allowed me to answer the question identified. Although further investigation is necessary to develop a robust TPP software, the work presented in this manuscript sets up a solid foundation for future work.
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LIST OF SYMBOLS

\( w_i \)  Weights in the spline function.

\( N(x) \)  Spline function.

\( P_i \)  Control points in the spline function.

\( \phi \)  RBF.

\( \Phi \)  Multivariable RBF.

\( \lambda_j \)  A real number in the RBF.

\( \varphi(x) \)  Lagrangian interpolation function.

\( N_i \)  Shape function.

\( R \)  Rodrigues’ rotation matrix.

\( I \)  Identity matrix.

\( V_z \)  Skew-symmetric cross-product of \( \hat{\theta} \).

\( \hat{\theta} \)  Vector orthogonal to vector \( \vec{V}_{avg_{un}} \) and z-axis unit vector.

\( f_{rot} \)  Rotated vector \( f \).

\( \vec{V}_{avg_{un}} \)  Averaged unit normal vector of a surface.

\( \vec{x} \)  Position vector of point \( x \).

\( proj_{V} \)  Projected vector onto surface \( V \).

\( \xi \)  \( x \) coordinate of a point in isoparametric space.

\( \eta \)  \( y \) coordinate of a point in isoparametric space.

\( \alpha, \beta, \gamma \)  AFP machine head angles of rotation.
LIST OF ABBREVIATIONS

AFP ................................................................. Automatic Fiber Placement
ATL ........................................................................ Automatic Tape Laying
CM ...................................................................... Calculation Mesh
CS ........................................................................ Constant Stiffness
CPMFM ......................................................... Composite Part Manufacturing and Fiber Modeler
ELSG ...................................................................... Edge Limited Seed Generation
FE ........................................................................ Finite Element
FMM ...................................................................... Fast Marching Method
MBSG .................................................................... Median Based Seed Generation
MCG ....................................................................... Machine Code Generator
MM ........................................................................ Manufacturing Mesh
NURBS ........................................................... Non-Uniform Radial Basis Spline
RBF ....................................................................... Radial Basis Function
TM ......................................................................... Tessellation Mesh
TPP ....................................................................... Tow Path Planner
VS ........................................................................ Variable Stiffness
CHAPTER 1. INTRODUCTION

Path planning is not a new concept in manufacturing. Subtractive manufacturing uses path planning for the generation of parts. The machining approach uses a block of material in which chips are subtracted to finally obtain the part desired. Similarly, the path planning approach is also used in additive manufacturing, e.g. filament winding, automatic fiber placement (AFP), and automatic tape laying (ATL), to generate parts based on a tool path in which material is laid onto a geometry. Filament winding is the additive manufacturing approach mainly utilized in the manufacture of cylindrical structures such as pressure vessels. Basically, the process winds filaments under tension over a rotating mandrel [4–6]. AFP is the manufacturing approach of laying tows or courses of tows on a surface to obtain the designed composite laminate. Finally, ATL is the manufacturing approach of laying tapes, which is similar to AFP, but tapes are significantly wider than tows. During the layup procedure on 3D surfaces, tows are preferred as they can deform further than tapes, thus, increasing radius of curvature on the tow and minimizing defects in the out of plane direction such as wrinkling [7–14].

Composite laminates are usually manufactured with constant stiffness (CS) plies. CS plies are those in which the fiber direction in each of the plies in the laminate stays constant. Over the years different optimization approaches have been developed to create high performance composite laminates, better known as variable stiffness (VS) laminates.
VS laminates allow the change in fiber direction within the ply. The development of VS laminates required a post-processing software to translate a continuous fiber angle field into discrete tow paths. Therefore, Jahangir [3, 15] developed a software which can generate tow paths given a discrete fiber angle distribution on planar surfaces with and without holes. The inherent gap and overlap defects from steered plies were alleviated through the development of multiple seeding algorithms and gap recognition algorithms to minimize gaps in steered plies.

The purpose of this manuscript is to document the development of a new tow path generation methodology for 3D single and double curved surfaces. The methodology couples an optimized discrete fiber angle distribution on curved shell geometries to tool paths. By adapting the multi mesh approach and the isoparametric space, the generation of tow paths on 3D surfaces is accomplished and presented. During the tow path generation of 3D CS plies, tows become steered as curved geometries introduce unwanted curvature to the tow, thus generating steered tows, which introduce unwanted gaps and overlaps. Thus, various seeding strategies are required and implemented to populate a surface with minimum gaps. To better understand the new methodology, the following breakdown is presented:

Chapter 2 presents a literature review of the general AFP tow placement approach. Then, multiple tow path generation methodologies are studied to further understand the complexity of tow path placement on 3D surfaces. Furthermore, the fiber steering optimization approach for constant thickness plies on 2D and 3D surfaces through the fiber angle optimization software known as TopSteer is reviewed because the results are used in
the tow path planning software. Finally, existing seed point propagation strategies are studied to better manufacture these steered laminates with minimal gaps and overlaps.

**Chapter 3** develops the theoretical generation approach and governing equations of tow trajectory generation on 3D surfaces. The discretization of an arbitrary surface is introduced and simplified equations for tow path generation are presented.

**Chapter 4** implements the theoretical approach in a series of examples of planar and curved surfaces. The discretization of the surfaces is presented and the multi mesh approach is applied for the manipulation of the surfaces. Finally, multiple seed point generation algorithms are presented in 3D.

**Chapter 5** couples the theoretical tow path trajectories generated to the AFP machine. The discretized tow paths are treated for violations to the AFP machine head angles and minimize manufacturing time.

**Chapter 6** concludes the work developed with recommendations and future software expansion opportunities.
CHAPTER 2. LITERATURE REVIEW

This chapter is focused on researching the various approaches utilized in the literature to generate tool path trajectories. First, the general AFP tow placement approaches used by the major software developers is presented. Then, various mathematical approaches to generate tow paths including meshes and meshless methods are introduced. Moreover, an overview of the in-house developed optimizing software for laminate lay-up for planar and curved surfaces is shown. Finally, previous work by Jahangir [15] on seed point propagation strategies is presented, and all of these topics are introduced to aid with the development of the tow path generation software for 3D shell surfaces.

SECTION 2.1. GENERAL AUTOMATIC FIBER PLACEMENT APPROACH

The general AFP tow placement on planar and curved geometries can be achieved through various approaches. In general, industry utilizes the developed software module by CATIA called the composite part manufacturing and fiber modeler (CPMFM) as well as the iCPS Ingersoll developed module for fiber placement called CPS2. The CPMFM by CATIA utilizes the method of draping material over a surface. This method divides the 3D geometry into different 2D sections and generate cut-outs for placement on the 3D geometry. The CPS2 by Ingersoll utilizes the non-uniform rational basis spline (NURBS) method. NURBS is commonly used in computer graphics to generate curves and surfaces.
NURBS is a meshless approach which provide the flexibility to design shapes using various spline functions. This is a modular approach in that sense. The NURBS curve \( N(x) \) is defined as shown in (1).

\[
C(x) = \frac{\sum_{i=1}^{n} N_i(x)w_iP_i}{\sum_{i=1}^{n} N_i(x)w_i}
\]  

(1)

Where \( N_i(x) \) is the spline function, \( w_i \) is the weights, and \( P_i \) are the control points.

SECTION 2.2. TOW PATH GENERATION METHODOLOGIES

This section presents the tow path generation methodologies developed that could be implemented in the TPP software. The meshless approach provides the B-splines approach and the radial basis functions (RBFs) approach. The splines approach uses a sequence of points denoted as control points, which the path attempts to follow as close as possible [4, 13, 17, 19–21]. The RBFs approach is an interpolation method which can be used on meshless and mesh methods. An RBF is a real-valued function which gives value to each point defined based on its distance with respect to the origin or center point, this distance is usually defined on a Euclidean space [4, 20, 22–25]. Interpolation functions utilize a mesh and nodal values to construct new data points within the space between given data points, in the case of a mesh the new data points are constructed through the element’s surface. Finally, the fast marching method (FMM) utilizes a mesh and the Eikonial equation to predict the position of a wave at a given time [12, 13, 17, 26, 27].
SUB-SECTION 2.2.1. SPLINES

Splines are utilized in meshless methods because it only requires a set of control points which are used to generate a curve. The curve attempts to connect these control points by defining a parametrical equation which can accomplish such task. A curved which accomplishes the task of connecting all the control points is called an interpolating curve, and a curve which passes near the control points is referred to as approximating curve as shown in Figure 2.1.

*Figure 2.1 Splines: a) Interpolating curve and b) approximating curve*

The generation of splines results in bad connectivity and low resolution when large amounts of control points are used in a given area. This is due to the fact that splines can be defined in different degrees. The highest degree that is generally used for splines is 3 degrees. This is usually used in computer graphics as it constructs smooth curves. These polynomial curves are of the general form:

\[ a + bx + cx^2 + dx^3 = y \]  \hspace{1cm} (2)

If a set of points are given into (2), the line calculated must pass through these points, for example:

*point 1: (−1,2)*
point 2: (0,0)

point 3: (1, –2)

point 4: (2,0)

\[
\begin{bmatrix}
1 & -1 & 1 & -1 \\
1 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 \\
1 & 2 & 4 & 8
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c \\
d
\end{bmatrix}
= 
\begin{bmatrix}
2 \\
0 \\
-2 \\
0
\end{bmatrix}
\]

Thus, solving for \(a, b, c,\) and \(d\) and substituting those values into (2) would yield an equation that passes through the control points given.

**SUB-SECTION 2.2.2. RADIAL BASIS FUNCTIONS**

RBFs are functions utilized as a meshless interpolation approach generally used to interpolate in higher dimensions. An RBF is usually noted as \(\phi: \mathbb{R}^+ \to \mathbb{R}\). Now to get the RBF, \(\phi\), to a multivariate function, a new function is defined as follow.

For each \(x = (x_1, x_2, ..., x_n)\) in \(\mathbb{R}^n\):

\[
\Phi(x) = \phi \left( \|x\|_2 \right)
\]

(3)

Where \(\|x\|_2\) is the Euclidean distance of the point \(x\) from the origin.

\[
\|x\|_2 = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}
\]

(4)

During an interpolation, the function \(f: \mathbb{R}^n \to \mathbb{R}\) has a finite number of points \(x_j \in \mathbb{R}^n\) and all of them are known as centers. These centers then generate multiple RBFs by shifting \(\phi\) to the different data points. Thus, the function \(\phi \left( \|x - x_j\|_2 \right)\) has the same shape as the RBF \(\Phi\) but shifted. The goal is to approximate the function \(s\), which is a linear combination of the basis functions mathematically shown in (5) [4, 20, 22–25].
Although RBFs are very useful in the interpolation of higher dimensions, the functions generated at the various data points overlap in certain areas. The overlap of these functions could potentially generate discontinuities at certain points, which would induce discontinuities in tow path trajectory applications.

**SUB-SECTION 2.2.3.  INTERPOLATION FUNCTIONS**

Interpolation is the mathematical approach utilized to generate new data points between a discrete set of data points. The Lagrangian interpolating function is a polynomial with degree equal to the number of data points (n) minus 1. This polynomial passes through the n points given by (6) [12, 17, 28–30].

\[
\varphi(x, y) = \sum_{i=1}^{n} \varphi_i N_i(x, y)
\]

Where \( N_i(x, y) \) is of the form (7) for a square element,

\[
N_1(x, y) = \frac{1}{4}(1 - x)(1 - y)
\]

\[
N_2(x, y) = \frac{1}{4}(1 + x)(1 - y)
\]

\[
N_3(x, y) = \frac{1}{4}(1 + x)(1 + y)
\]

\[
N_4(x, y) = \frac{1}{4}(1 - x)(1 + y)
\]

When utilizing interpolating polynomials, the amount of data points to be interpolated between nodal points n introduces a tradeoff. The more data points used in the interpolation the greater the function’s oscillation, but the data points will show a better fit of the function.
SUB-SECTION 2.2.4. FAST MARCHING METHOD

The method is based on the Eikonal Equation, which is mostly utilized in the propagation of a wave. Thus, the position of the wave can be calculated when it propagates. The process starts by generating a reference curve on the surface. To start the propagation, all of the points on the reference path contain a time value of 0, then the points are propagated through the mesh at a defined speed as shown in Figure 2.2 [12].

![Figure 2.2 Fast Marching Method solving sequence](image)

Figure 2.2 Fast Marching Method solving sequence

Figure 2.3 shows a triangular mesh element in which the time values of points $A$ and $B$ noted as $T_A$ and $T_B$ respectively are known, thus one calculates the time value of point $C$ noted as $T_C$.

Using (8),(9), and (10) one is able to calculate the time step at $C, T_C$ [26, 27].

\[
\theta = \arcsin \left( \frac{T_B - T_A}{f \cdot AB} \right) \tag{8}
\]

\[
h = BC \cdot \sin(\beta - \theta) \tag{9}
\]

\[
T_c = h \cdot f + T_B \tag{10}
\]
Figure 2.3 Triangular element breakdown to calculate time steps

Having calculated the time value and the distance from the reference curve for every node, an offset curve can be drawn at a proper distance. Moreover, to obtain an exact parallel curve an infinite reference curve must be defined. Figure 2.4 shows the difference in the result of not using infinitely defined reference curves [12, 26].

Figure 2.4 Parallel propagation of a) Not extended reference curve and b) extended reference curve

SECTION 2.3. CONTINUOUS FIBER STEERING OPTIMIZATION APPROACH FOR CONSTANT THICKNESS PLIES

This section provides an overview of the in-house developed code TopSteer [1, 31]. Which optimizes fiber angles at nodal positions to obtain optimized constant stiffness and/or variable stiffness laminates using continuous fibers. The optimizer utilizes failure
criterion as the objective function for optimization by employing a failure criteria specified by the user which includes buckling based optimization.

**SUB-SECTION 2.3.1. 2D TOPSTEER**

The 2D TopSteer software developed [1] first introduced the multi mesh approach on planar surfaces. The approach consists of the definition and coupling of a calculation mesh (CM) which is a dense FE mesh used to capture the stress concentrations in a given geometry, and the manufacturing mesh (MM) which is a much coarser mesh used to discretize the design variables, fiber angles, thus reducing the number of design variables and computational time. Figure 2.5 shows a) the MM with the design variables, i.e. nodal values of the fiber angles and b) the superposed MM onto the CM. This figure shows an example of the mesh density difference. Note that the MM consists of quadrilateral elements and the CM is composed of triangular elements. The triangular elements for the CM are utilized because the refinement of the mesh then becomes simpler. Also, van Tooren and Elham [2] developed their own finite element (FE) solver using these triangular elements and symbolic integration to obtain the element’s stiffness matrices in MATLAB® which resolves the need for numerical integration and the associated inaccuracy.

The optimization begins with a given CS laminate defined in TopSteer and utilizing the optimizer *fmincon* embedded in the MATLAB® software a converged optimized fiber angle distribution is exported to the post processor to generate discrete tow paths on the planar surface as shown in section 2.4.
SUB-SECTION 2.3.2. 3D TOPSTEER

The 3D TopSteer composite laminate optimization software is an extension of the 2D version. As the continuation of 2D TopSteer, 3D TopSteer employs the multi mesh approach by using the MM to discretize the design variables, and the CM to discretize the displacement field in the geometry. Although 3D TopSteer is an extension of 2D TopSteer, the software differs in many aspects. First, 3D TopSteer optimizes composite laminates outside of the MATLAB® environment and into the Python environment. van Zanten et al. [42] uses ABAQUS® to generate 3D shell surfaces, the MM, and the CM. The MM is made of quadrilateral elements as previous, but the CM switches from triangular to quadrilateral elements. During the analysis, ABAQUS® is utilized as the FE solver, which is no longer a symbolic calculation but a numerical calculation, thus bringing forth inaccuracies in the element’s stiffness matrices calculated. During the analysis, the geometry is defined, the MM and the CM are generated and material properties, i.e. fiber...
angles, at the MM element nodes are prescribed. Then, these fiber angles are interpolated at the CM element’s centroids to move into the stress analysis of the geometry. The interpolation of the fiber angle onto CM elements is possible by mapping the CM element’s centroid on the MM elements, and thereafter introducing the isoparametric space. The MM elements and the mapped CM element centroids are moved into isoparametric space by mathematically manipulating the 3D MM elements to the xy-plane, thus enabling the use the isoparametric mapping introduced by Hua. [32] The isoparametric space is utilized to facilitate the interpolation of the fiber angles at CM element centroids. After the CM element centroids are assigned a fiber angle, the analysis is performed, failure indices are exported, and the failure criteria is evaluated. The design variables at the MM nodes are updated if the maximum values of the failure index in the laminate are not converged, then the process is repeated until the maximum values of the failure index in the laminate is obtained. Figure 2.6 shows the flowchart of 3D TopSteer.

![Figure 2.6 TopSteer 3D optimizer flowchart](image-url)
SECTION 2.4 TOW PATH PLANNING: SEED POINT PROPAGATION STRATEGIES

The starting positions for the generation of tow paths on 3D surfaces is critical, as the placement of the starting position determines the curvature the part will induce on the tow path and the tow paths thereafter. Jahangir [3, 15] defines a tow path generation software for steered tows which minimizes the gaps induced by steering by applying different seed point generation algorithms. The two generation strategies presented are the median based seed generation (MBSG) and the edge limited seed generation (ELSG). The MBSG approach utilizes a reference tow and perpendicularly propagates a seed point from the middle of the reference tow path, as shown in Figure 2.7.

![Figure 2.7 Demonstration of MBSG](image)

The ELSG approach generates seed points on the edge of the geometry by utilizing a reference tow path and shadow tows. A shadow tow is a generated tow which can be used to see if the tow generated is usable or not. A tow is usable if the curvature constraint is not violated. If the tow is not usable then it is discarded and a new seed point along the edge is generated. The process continues in a clockwise manner until the tows are
populated on the right side of the geometry, and then the software switches to counterclockwise to populate the remaining surface. Figure 2.8 shows a demonstration of the approach on a continuous planar surface.

Furthermore, an image based post-processing module is also introduced in the software which allowed for the generation of tow paths in recognized gaps. The software creates a white and black figure with tow paths denoted in black, and gaps in white. In the center of these gaps seed points are generated, tow paths are calculated using these seed points, and the process is repeated until the plate is populated. Figure 2.9 shows the process on a planar steered surface where a) is the post processed plate showing gaps in white and tows in black, b) shows the seed points generated in blue, c) shows the generated tows on throughout the place, and d) shows the populated plate.

Figure 2.8 Demonstration of ELSG
Figure 2.9 Demonstration of the image based post-processor a) post processed plate, b) seed points generated in the middle of recognized gaps, c) tows created using seed points generated, and d) populated steered plate

SECTION 2.5. SUMMARY: WHAT IS THE TOW PATH GENERATION APPROACH BEST SUITED FOR THE SOFTWARE?

After reviewing the literature for the possible tow path generation approaches, the most effective and modular approach for this specific software would be interpolation functions, specifically Lagrangian. The Lagrangian interpolation approach is effective because it uses a mesh and nodal values as inputs, which is the result of the optimization from the 2D and 3D TopSteer optimizers. Lagrangian interpolation functions guarantee $C^0$ continuity and can be utilized as shown by van Zanten et al. [42] through the adaptation of
isoparametric transformations. The use of isoparametric transformations ease the interpolation of tow paths in 3D shell structures by mathematically manipulating the problem into a 2D space. Finally, the adaptation of the multi mesh approach to the tow path planning software would decrease computational time by decreasing the number of design variables as presented in the TopSteer optimizers.
CHAPTER 3. THEORETICAL BACKGROUND

This chapter presents the theoretical approaches developed for the generation of tow trajectories. A generic framework to generate tow trajectories is introduced as the basis for the tow path planning tool. Then, an arbitrary surface is discretized through various methods for different applications in the tow path generation framework. Next, the mathematical manipulation of elements is presented. Moreover, the isoparametric space is introduced for the generation of tow paths, and thereby introducing the reverse isoparametrization of elements and tow path points. Thereafter, the tow path planning design variable referred to as seed point is introduced and its contribution to the resulting ply is presented in later chapters. Similarly, the interpolation functions to be used are derived to their simplest form, and the tow path is generated as a discretized collection of points calculated throughout the MM element surface. Finally, the tow path is processed to its final location on the surface of the TM elements and the final theoretical flowchart is illustrated.

SECTION 3.1. GENERIC FRAMEWORK

This section presents a generic framework representing the basis for the tow path planning tool development. Figure 3.1 shows a high level overview of the variables needed for tow path generation.
First, a surface is provided to place tow paths onto along with a fiber direction desired for the given ply. The surface could be provided as a mathematical equation or a discretization of sorts, and the fiber direction for the ply could be given as global fiber orientation, which would yield a constant stiffness ply or as a discretized nodal fiber angle distribution, which could yield variable stiffness plies [2, 4, 37, 38, 5, 10, 12, 21, 33–36]. Then, a starting location provided by the user is required to commence tow path growth. Given a location, an interpolation methodology must be applied to grow the tow path through the geometry. After an initial tow path is generated, multiple propagation strategies could be performed on the ply. Examples previously utilized are parallel tow, constant curvature, and mid-tow seed point propagation just to name a few [5, 12]. Once the surface is covered and the user is satisfied, the tow paths created are exported for AFP manufacturing.
SECTION 3.2. MULTI MESH APPROACH

From section 3.1 the first step is to define the surface on which material must be laid upon and a discretized fiber angle distribution. The surfaces defined are initially generated through CATIA®, therefore mathematical equations of the surfaces are not provided. Figure 3.2 shows the CATIA® generated single curved surface to visualize the importance of the multi mesh approach in the 3D tow path planning tool.

![Figure 3.2 CATIA® generated surface](image)

From the optimization, the tow path generation tool is given a fiber angle distribution embedded in the MM generated in ABAQUS® as the result of the optimization from TopSteer. Figure 3.3 shows the discretized actual surface using three MM elements.

As shown the MM does not represent 3D surfaces accurately, therefore a discretization representative of the original surface is necessary. Thus, a denser mesh referred to as the TM is introduced by employing the CATIA® tessellation module on curved geometries. In this manner, geometries as the one shown in Figure 3.2 can be represented accurately as presented Figure 3.4. The geometry must be accurately represented to manufacture the tow paths on the 3D surface as desired.
Thus, the need of multiple meshes introduces the multi mesh approach developed in the optimization of TopSteer. [1] The MM and the TM are interrelated in this manner, thus facilitating the generation and projection of tow paths from the MM to the TM.
SECTION 3.3. ROTATION AND TRANSLATION OF MM ELEMENT NODE COORDINATES FOR TOW PATH GENERATION

The MM element is a quadrilateral element in 3D space. To simplify calculations, the element is moved to 2D space since the 3D quadrilateral element can be warped. The process starts by calculating the unit normal vector of the element surface using (11).

\[ n_i = a_j \times b_k \quad i, j, k: 1 \cdots 4 \]  

Where \( a_j \) and \( b_k \) are defined as the sides of the quadrilateral which define an element. This results in 4 normal vectors, one at each vertex of the quadrilateral, i.e. at nodal locations. These vectors are then averaged and normalized as shown in Figure 3.5. Thus, ensuring that a single unit vector on a warped element is defined.

Then, the element is rotated such that the unit vector of the surface is parallel with the \( z \) global normal vector, thereby making the surface parallel with the \( xy \)-plane. To calculate the rotation matrix of the unit vector of the surface to the \( z \)-axis unit vector, Rodrigues’ rotation formula is presented in (12).

\[ R = I + \sin(\theta) V_z + (1 + \cos(\theta)) V_z^2 \]
Where $R$ represents the rotation matrix. The angle by which the unit vector $\vec{V}_{avg\ un}$ is rotated around unit vector $V_z$ is denoted by $\theta$. Where $V_z$ is the skew-symmetric cross-product of $\hat{\vartheta}$ shown in (13), and $I$ is the identity matrix.

$$V_z = \begin{bmatrix} 0 & -\hat{v}_3 & \hat{v}_2 \\ \hat{v}_3 & 0 & -\hat{v}_1 \\ -\hat{v}_2 & \hat{v}_1 & 0 \end{bmatrix} \quad (13)$$

To calculate the rotation matrix which rotates the vector $\vec{V}_{avg\ un}$ around the $z$-axis unit vector $\hat{k} = [0,0,1]$, the vector $\hat{\vartheta}$ must be orthogonal to $\vec{V}_{avg\ un}$ and the $z$-axis unit vector, and is calculated as follows:

$$\hat{\vartheta} = \frac{\vec{V}_{avg\ un} \times \hat{k}}{||\vec{V}_{avg\ un} \times \hat{k}||} \quad (14)$$

Since $\sin(\theta) = ||\vec{V}_{avg\ un} \times \hat{k}||$, (12) can be rewritten as (15).

$$R = I + V_z + \frac{(1 + \cos(\theta))}{\sin(\theta)}V_z^2 \quad (15)$$

Thus, the rotation matrix is applied to the MM elements’ unit normal and the seed points as follows:

$$f_{rot} = Rf \quad (16)$$

Figure 3.6 illustrates Rodrigues’ rotation applied to a general MM element denoted in blue with its unit vector denoted in yellow. The $z$-axis unit vector is shown in red, and finally the purple element is the rotated element parallel to the $xy$-plane.
Finally, the element is translated to the $xy$-plane by assigning a value of zero to the $z$-coordinate at the nodes. Figure 3.7 shows the progression with a single curved geometry discretized by three MM elements.
SECTION 3.4. MOVING TO AND FROM ISOPARAMETRIC SPACE

The isoparametric space is used to ease the interpolation through 3D shell elements. The rotated and translated MM element and a seed point in the MM element are moved into isoparametric space to generate tow paths use reverse isoparametric transformations. The fiber directions at the nodal positions are mapped onto the isoparametric element’s nodes, and one must pay attention to how the angles are defined. In this case, the angles are defined with respect to the global $x$-axis. Once the tow path is generated through the surface of the isoparametric MM element, the tow path created undergoes an isoparametric transformation to move from isoparametric space to global space. These mathematical procedures are derived in sections 3.4.2 and 3.4.1 respectively.

SUB-SECTION 3.4.1. ISOPARAMETRIC MAPPING FORMULAS AND APPROACH

Isoparametric mapping is the mathematical procedure of moving coordinates in natural space into global space. This procedure was introduced by Taig [39] in 1961 and the equations are shown in (17) and (18). The equation for the $z$-coordinate is not defined, thus the translation of the MM elements into the $xy$-plane.

\[
x(\xi, \eta) = \sum_{i=1}^{4} N_i(\xi, \eta)x_i
\]

\[
y(\xi, \eta) = \sum_{i=1}^{4} N_i(\xi, \eta)y_i
\]

Where $\xi$ and $\eta$ are the natural coordinates and as usual $x$ and $y$ are the global coordinates. The formulas derived by Taig [39] utilize a particular node numbering system.
for the quadrilateral element. Figure 3.8 presents the numbering system in the global coordinate quadrilateral element and natural coordinate quadrilateral element.

![Figure 3.8 a) Quadrilateral element in Isoparametric space and b) Generic quadrilateral element in global space](image)

The previously shown natural coordinate quadrilateral element employs (19), (20), (21) and (22) to calculate the Lagrangian shape functions.

\[
N_1(\xi, \eta) = \frac{1}{4}(1 - \xi)(1 - \eta) \quad (19)
\]
\[
N_2(\xi, \eta) = \frac{1}{4}(1 + \xi)(1 - \eta) \quad (20)
\]
\[
N_3(\xi, \eta) = \frac{1}{4}(1 + \xi)(1 + \eta) \quad (21)
\]
\[
N_4(\xi, \eta) = \frac{1}{4}(1 - \xi)(1 + \eta) \quad (22)
\]

Therefore, to calculate \(x\) and \(y\), substitute (19), (20), (21) and (22) into (17) and (18) to get (23) and (24)

\[
x = \frac{x_1}{4}(1 - \xi)(1 - \eta) + \frac{x_2}{4}(1 + \xi)(1 - \eta) + \frac{x_3}{4}(1 + \xi)(1 + \eta) + \frac{x_4}{4}(1 - \xi)(1 + \eta) \quad (23)
\]
\[
y = \frac{y_1}{4}(1 - \xi)(1 - \eta) + \frac{y_2}{4}(1 + \xi)(1 - \eta) + \frac{y_3}{4}(1 + \xi)(1 + \eta) + \frac{y_4}{4}(1 - \xi)(1 + \eta) \quad (24)
\]

Thus, tow paths generated in isoparametric space can be translated into global space by applying the Lagrangian interpolation to each of the points discretizing the tow path in
isoparametric space. Figure 3.9 shows an example of an arbitrary point in isoparametric space moved to global space on an arbitrary MM element.

**Figure 3.9** Point isoparametrically mapped onto an arbitrary quadrilateral element in global space

**SUB-SECTION 3.4.2. REVERSE ISOPARAMETRIC MAPPING FORMULAS AND APPROACH**

Reverse isoparametric mapping is the mathematical procedure of moving points in global space to natural space. This mathematical procedure is necessary to use simple Lagrangian interpolation in isoparametric space to generate tow paths in 2D. As shown in section 3.4.1, a relationship between the global space and the isoparametric space is defined in (23) and (24), but the system of equations cannot be solved without further conditions [32]. Hua developed a set of general solutions and cases for the reverse isoparametrization of a point in the xy-plane. To map the points from the global space into the isoparametric space (23) and (24) are to be solved for $\xi$ and $\eta$. Therefore, $a$, $b$, $c$ and $d$ are introduced in
(25), (26) and (27) as functions of global coordinates of nodal points to be used in the bilinear system of equations.

\[ d_1 = 4x - (x_1 + x_2 + x_3 + x_4) \]  
\[ d_2 = 4y - (y_1 + y_2 + y_3 + y_4) \]  
\[
\begin{bmatrix}
  a_1 & a_2 \\
  b_1 & b_2 \\
  c_1 & c_2
\end{bmatrix}
= 
\begin{bmatrix}
  1 & -1 & 1 & -1 \\
  -1 & 1 & 1 & -1 \\
  -1 & -1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
  x_1 & y_1 \\
  x_2 & y_2 \\
  x_3 & y_3 \\
  x_4 & y_4
\end{bmatrix}
\]  

Using (27) and (26), (23) and (24) can be rewritten as (28).

\[
\begin{bmatrix}
  b_1 & c_1 \\
  b_2 & c_2
\end{bmatrix}
\begin{bmatrix}
  \xi \\
  \eta
\end{bmatrix}
= 
\begin{bmatrix}
  d_1 - a_1\xi\eta \\
  d_2 - a_2\xi\eta
\end{bmatrix}
\]  

To solve the bilinear system of equations shown in (28), a series of conditions were defined by Hua, and to simplify the process in the conditions the determinant of a 2 by 2 matrix is defined as (29):

\[ r_s = \begin{vmatrix}
  r_1 & s_1 \\
  r_2 & s_2
\end{vmatrix} = r_1s_2 - r_2s_1 \]  

In (29), \( r \) and \( s \) are interchangeable variables which can be replaced by \( a, b, c \) and \( d \) from (27) to be used in the following set of conditions defined by Hua where the following inequalities must always be true.

\[ a_1 \neq b_1 \text{ and } a_2 \neq c_2 \]  

**List of conditions:**

1. \( a_1 = 0, a_2 = 0 \) – (28) is solvable because it becomes in a linear system, thus resulting in (31)

\[ \xi = \frac{d_c}{a_1d_2 + b_c} \quad \eta = \frac{b_d}{a_2d_1 + b_c} \]  

2. \( a_1 = 0, a_2 \neq 0 \) – (28) is not solvable unless conditions are defined on \( c_1 \)
a. $c_1 = 0$ – The solution for (28) then becomes (32)

$$\xi = \frac{d_1}{c_1} \quad \eta = \frac{b_d}{a_2d_1 + b_1c_2}$$  \hspace{1cm} (32)

b. $c_1 \neq 0$ – The solution for (28) then becomes (33)

$$0 = a_2b_1\xi^2 + (c_b - a_2d_1)\xi + d_c \quad \eta = \frac{d_1 - b_1\xi}{c_1}$$  \hspace{1cm} (33)

3. $a_1 \neq 0$ and $a_2 \neq 0$

a. $a_b \neq 0$ and $a_c \neq 0$ – The solution for (28) then becomes (34)

$$0 = a_b\xi^2 + (c_b - d_a)\xi + d_c \quad \eta = \frac{a_d - b_a\xi}{a_c}$$  \hspace{1cm} (34)

b. $a_b \neq 0$ and $a_c = 0$ – The solution for (28) then becomes (35)

$$\xi = \frac{a_d}{a_b} \quad \eta = \frac{a_1d_b}{c_1a_b + a_1a_d}$$  \hspace{1cm} (35)

c. $a_b = 0$ then $a_c$ must be 0 which yields the solution for (28) as (36)

$$\xi = \frac{a_1d_c}{b_1a_c + a_1a_d} \quad \eta = \frac{a_d}{a_c}$$  \hspace{1cm} (36)

4. $a_2 = 0$ and $c_2 \neq 0$ – (28) is not solvable unless conditions are defined on $b_2$

a. $b_2 = 0$ – The solution for (28) then becomes (37)

$$\xi = \frac{d_c}{a_1d_2 + b_1c_2} \quad \eta = \frac{d_2}{c_2}$$  \hspace{1cm} (37)

b. $b_2 \neq 0$ – The solution for (28) then becomes (38)

$$0 = a_1b_1\xi^2 + (c_b - a_1d_2)\xi + d_c \quad \eta = \frac{d_2 - b_2\xi}{c_2}$$  \hspace{1cm} (38)
The presented set of conditions derived by Hua [32] is shown graphically in the appendix A [33] to visually attempt to understand the meaning of the variables and the conditions applied onto them.

SECTION 3.5. ISOPARAMETRIC LAGRANGIAN INTERPOLATION FORMULA FOR TOW PATH GENERATION

The fiber angle field in the isoparametric space is approximated as a function of $\xi$ and $\eta$. The angle is calculated with:

$$\phi(\xi, \eta) = \sum_{i=1}^{n} N_i(\xi, \eta) \phi_i$$ (39)

Where $N_i$ are the general shape functions for linear Lagrangian interpolation. The node numbering from Figure 3.8 is maintained, and $i$ is the node number.

$$N_1(\xi, \eta) = \frac{\xi - \xi_2}{\xi_1 - \xi_2} \frac{\eta - \eta_4}{\eta_1 - \eta_4}$$ (40)

$$N_2(\xi, \eta) = \frac{\xi - \xi_1}{\xi_2 - \xi_1} \frac{\eta - \eta_3}{\eta_2 - \eta_3}$$ (41)

$$N_3(\xi, \eta) = \frac{\xi - \xi_4}{\xi_3 - \xi_4} \frac{\eta - \eta_2}{\eta_3 - \eta_2}$$ (42)

$$N_4(\xi, \eta) = \frac{\xi - \xi_3}{\xi_4 - \xi_3} \frac{\eta - \eta_1}{\eta_4 - \eta_1}$$ (43)

Since the quadrilateral element is moved to isoparametric space, the element becomes a square element, therefore (40), (41), (42) and (43) can be further simplified as (44), (45), (46) and (47) respectively.

$$N_1(\xi, \eta) = \frac{(1 - \xi)(1 - \eta)}{4}$$ (44)

$$N_2(\xi, \eta) = \frac{(1 + \xi)(1 - \eta)}{4}$$ (45)
\[ N_3(\xi, \eta) = \frac{(1 + \xi)(1 + \eta)}{4} \] (46)

\[ N_4(\xi, \eta) = \frac{(1 - \xi)(1 + \eta)}{4} \] (47)

These simplified shape functions are used in (39) to calculate the fiber angle at a point within the isoparametric element. The angle calculated is used to calculate the next point at a defined distance to finally accomplish a continuous fiber path through the surface of the isoparametric element.

SECTION 3.6. ORTHOGONAL PROJECTION FOR FIBER PLACEMENT FROM THE MM SURFACE TO THE TM SURFACE

The tow path generated lies on the MM, which as previously stated does not represent the curved geometry accurately. Therefore, an approach to place tow path trajectories generated on the MM surface onto the real surface is developed by using orthogonal projections. Orthogonal projection is the mathematical procedure of orthogonally projecting a vector into its components. Figure 3.10 is an example of a position vector of a point in space orthogonally projected onto a surface, and the normal axis of the surface.

Point \( O \) is the origin of the surface, and the position vector \( \vec{x} \) with respect to reference point \( O \) calculated as follow.

\[ \vec{x} = [x_O - x_i, y_O - y_i, z_O - z_i] \] (48)

Where \( x_O, y_O, \) and \( z_O \) are the \( x, y, \) and \( z \) coordinates of point \( O, \) and \( x_i, y_i, \) and \( z_i \) are the \( x, y, \) and \( z \) components of the point in space. After deriving \( \vec{x}, \) the orthogonal projection \( \text{proj}_v(\vec{x}) \) is calculated through (49).

\[ \text{proj}_v(\vec{x}) = \vec{x} - (\vec{x} \cdot \vec{u})\vec{u} \] (49)
Where \( \overrightarrow{u} \) is the normal unit vector of the surface the point in space is projected onto.

![Diagram](image)

**Figure 3.10 Orthogonal projection of an arbitrary point on a general surface**

**SECTION 3.7. SUMMARY**

In summary, a theoretical framework of the mathematical approach to generate tow paths on 3D surfaces is presented. The user inputs defined as a discretized surface, MM and TM, a fiber angle distribution prescribed on the MM nodes and seed points were introduced. The multi mesh approach was presented to generate tow paths usable for AFP manufacturing by interrelating the MM and the TM. First, the MM is treated through a series of mathematical manipulations to the MM elements to isoparametric space for simplified tow path generation in 2D space. For this purpose, the reverse isoparametric conditions and equations were presented such that the transformation from global to isoparametric space is possible. After a tow path is generated through the isoparametric element, the discretized tow path is moved from isoparametric space to global space using the isoparametric transformation equations presented. Finally, the 3D MM surface is to be orthogonally projected to place the generated tow paths on the TM surface from the MM surface.
CHAPTER 4. IMPLEMENTATION OF THEORETICAL APPROACH FOR TOW PATH GENERATION

This chapter presents the implementation of the theoretical approach presented in Chapter 3 into a functional tow path planning tool for fiber path placement on 3D surfaces in the Python environment. The surface is discretized through the MM and the TM, and the elements are manipulated such that interpolation of tow paths is simplified. The coarse discretization of the surface in the MM allows for the reduction of design variables or fiber angles at MM element nodes, thus minimizing computational time in the generation of tow paths; and the dense discretization of the surface in the TM allows for the placement of the MM generated tow paths onto the actual 3D surface. Then, different seed point algorithms are introduced to attempt maximum coverage of the surface. Furthermore, the tow path trajectory generation approach on the 3D MM surface is presented, and the 3D MM tow path placement on the TM surface is demonstrated. Finally, Figure 4.1 is the flowchart of the software developed to visualize each step. Where the input parameters are user defined surface discretized by the MM and TM, and the optimization results from TopSteer. Unit normal vectors are calculated along with rotation matrices. The seed points generated are then mapped onto the MM surface and translated to isoparametric space for tow path generation. Once the tow path is generated, the tow path is translated to global space and orthogonally projected onto the TM surface for AFP manufacturing. Each step is further explained in the following sections.
SECTION 4.1. SURFACE DISCRETIZATION

In 2D surfaces, simple meshes can be used to discretize a surface provided holes are not part of the geometry. A simple quadrilateral can represent the surface exactly. In 3D surfaces, it is not as simple to represent a surface. First, every surface can be approximated by a mathematical equation, although such equation may be troublesome to derive for some surfaces. In this section, arbitrary surfaces are not approximated by mathematical equations, instead, surfaces are discretized through the presented tessellations approach. Figure 4.2 shows an arbitrary single and double curved surface generated using the CATIA software.
Figure 4.2 CATIA® generated a) single curved surface and b) double curved surface

These surfaces can easily be exported and imported into any software as a .PRT file which is part file generated by CATIA®, or .STL file which is defined as standard tessellation language. CATIA® is utilized to tessellate the surfaces and they are exported through .STL files because the TPP employs these files as the means to import part into the software. Figure 4.3 shows the single and double curved tessellated representations of Figure 4.2 in the Python environment.

Figure 4.3 Tessellated a) single curved surface and b) double curved surface in the Python environment
SECTION 4.2. DESIGN VARIABLE DISCRETIZATION

The design variables are the nodal values of the discretized fiber angle distribution provided by the results from the performed optimization by the TopSteer software. The output of such optimization as a discrete continuous fiber angle field is contained within the coarse mesh known as the MM. Figure 4.4 shows an example of a MM with fiber angles prescribed at the bottom left MM element nodes, and its superposition onto the TM for a planar surface.

These angles are given in degrees and it is important to note that these are in-plane angles. Figure 4.5 shows how the single and double curved MM and their respective TM are superposed such that in the later steps, a projection of fibers generated on the MM can be placed on the TM surface.
SECTION 4.3. ORTHOGONAL PROJECTIONS IN THE TPP

Orthogonal projections as explained in section 3.6 show the projection of a point to a surface with its own coordinate system. The projection of tow paths points from the MM to the TM surface then utilizes the TM element’s own coordinate system as shown in Figure 4.6, therefore an additional step is performed to obtain the coordinates of the projected point with respect to the TM origin. Figure 4.6 shows an arbitrary tow path point on the MM surface which is essentially a point in space projected onto the TM surface in the following progression, a) vector driven from origin of the surface to point in space, b) orthogonally projected vector onto the surface, c) vector driven from global origin to origin of the surface and d) vector from global origin to orthogonally projected point.
Figure 4.6 Projection of a point in space to an arbitrary surface and obtaining the coordinates of the point with respect to the global coordinate system a) vector driven from origin of the surface to point in space, b) orthogonally projected vector onto the surface, c) vector driven from global origin to origin of the surface and d) vector from global origin to orthogonally projected point

SECTION 4.4. SEED POINT DEFINITION

Seed points are the initial positions required by the TPP to generate fiber trajectories on the surface. The distribution of these seed points determines the distribution of tow paths covering the surface. Figure 4.7 shows a single MM element with multiple seeding distributions and their respective tow path distribution responses. As shown, different seed point distributions result in different tow path distributions. The position of the seed point impacts the approximation of the function, and therefore the tow paths generated on the same surface and utilizing the same fiber angle distribution yield different results.
Figure 4.7 Tow path generated on a single MM element with seeding a) along the edges, b) through the middle, c) through the diagonal, and d) through the opposite diagonal

Thus, different seed point generation strategies are implemented to allow the user to obtain tow patterns that fit the requirements, for example a pattern with minimum gaps and overlaps. Jahangir [15] developed many seed point propagation strategies to minimize gaps, eliminate overlaps and maximize the coverage of planar surfaces on variable stiffness laminates. This chapter contains the seed point strategies currently adapted in the 3D TPP, from user defined seed points to evenly spaced seeding on the edges, evenly spaced seeding through the diagonals, and mid-line seed point propagation.

SUB-SECTION 4.4.1. USER DEFINED SEEDING

User defined seed points are starting positions defined by the user utilizing coordinates. These seed points can be placed on the xy-plane in the form \([x, y, 0]\), or they
can be placed specifically on an element surface in the form \([x, y, z]\). Once the seed points are provided, the TPP will orthogonally project the seed points onto the MM element surface using the approach presented in section 3.6. Figure 4.8 shows a) single curved and b) double curved MM with a set of user defined seed points denoted in red on the xy-plane, and their projections denoted in yellow, blue, and green to better visualize which projection belongs to which seed point.

![Figure 4.8 User defined seed points on a) single curved surface and b) double curved surface](image)

**SUB-SECTION 4.4.2. EVENLY SPACED SEEDING**

In some cases, for example constant stiffness laminas on planar surfaces, evenly spaced seeding using a seed point pitch equal to one tow width can result in complete coverage of the ply with minimal gaps and overlaps. On 3D surfaces, evenly spaced seeding at tow width distance apart would generate parallel tow paths, but inherently tows laid on 3D surfaces deform because curvature is induced by the surface itself. Moreover, evenly spaced seeding at tow width distance apart is still an effective seeding strategy which can be utilized as an attempt to achieve maximum ply coverage. Therefore, a
rectangular boundary around the entire geometry is constructed using the minimum and maximum $x$ and $y$ coordinates of all the nodes in the MM. Figure 4.9 shows the boundary drawn around an arbitrary surface.

Figure 4.9 3D geometry bounded by a generated 2D surface

Various seeding strategies can be applied to the resulting plane, i.e. evenly spaced seeding along the edges, horizontally, vertically, and diagonally through the plane. In Figure 4.10, $a$) presents the edge seeding on the single curved surface, $b$) is the edge seeding on the double curved surface, $c$) shows horizontally seeding through the single curved geometry, $d$) shows horizontally seeding through the double curved geometry, $e$) shows vertically seeding through the single curved geometry, $f$) shows vertically seeding through the double curved geometry, $g$) is an example of diagonally seeding through the surface of a single curved geometry, and $h$) is an example of diagonally seeding through the surface of a double curved geometry.
Figure 4.10 Seeding propagation strategies a) along the edges, c) horizontally, e) vertically, and g) diagonally through the geometry of a single curved surface. b) Along the edges, d) horizontally, f) vertically, and h) diagonally through the geometry of a double curved surface
SUB-SECTION 4.4.3. MID-LINE SEED POINT PROPAGATION

The mid-line seed point propagation strategy developed by Jahangir [15] and implemented in the 3D TPP is presented. The approach utilizes a single seed point to populate the surface with tow paths. The single seed point is used to generate a reference tow path and using the mid-point of this tow path a perpendicular seed point is generated. The algorithm then generates another tow path and using the perpendicularly generated seed point, and using the midpoint of the new tow path, another seed point is generated parallel to it. The process continues until the last generated seed point is outside of the geometric boundary. Then the algorithm moves in the opposite direction and the process is repeated. Figure 4.11 presents the mid-line seed point propagation strategy using a single seed point in the middle of the geometry. The blue seeds are propagated to the left of the geometry, while the red seeds are the propagation to the right of the geometry.

![Figure 4.11 Mid-line seed point propagation results on a) single curved surface and b) double curved surface](image)

\[\text{Figure 4.11 Mid-line seed point propagation results on a) single curved surface and b) double curved surface}\]
SECTION 4.5. TOW PATH GENERATION ON 3D MM

This section presents the approach to transform 3D MM elements to the isoparametric space by implementing Rodrigues’ rotation, translation to the xy-plane and transformation to isoparametric space discussed in section 4.5.2 to generate tow paths. Followed by isoparametric mapping to move tow paths from isoparametric space to global space in section 4.5.3. Finally, a gap and overlap evaluation algorithm is presented to estimate the geometric area covered by the tow paths on the MM.

SUB-SECTION 4.5.1. MANIPULATION OF 3D MM

The 3D MM element containing the seed point and the seed point itself are rotated and translated to the xy-plane with (12), (13) and (16) to be able to use the isoparametric space approach on the MM element such that the tow path generation is simplified. Figure 4.12 shows the sequence of the rotation and translation of the MM element and an arbitrary seed point.

![Figure 4.12](image)

*Figure 4.12 a) single MM element with seed point in the center, b) MM and seed point rotation, and c) MM and seed point translation to the xy-plane*
SUB-SECTION 4.5.2. GENERATION OF TOW PATHS IN ISOPARAMETRIC SPACE

With the MM element and the seed point on the \( xy \)-plane, the next step consists of moving the seed point to the isoparametric space. The reverse isoparametric mapping presented in section 3.4.2 is applied to the seed point to move it to the isoparametric space. Figure 4.13 shows an arbitrary MM element with a seed point in global and in isoparametric space.

![Figure 4.13 Seed point in a) global MM element and b) isoparametric element](image)

The isoparametric space enables the use of 2D Lagrangian interpolation, thus simplifying the calculation of tow paths on the MM surface. To generate a tow path a step size is defined such that after approximating the fiber direction at the seed point a point is calculated to discretize the tow path. The step size must be contained by the following bounds \( 0 < S_{\text{size}} < 1 \). Figure 4.14 presents the isoparametric element with a 45° angle
prescribed at each of the nodes and a step size of 0.1 to develop a tow path in the isoparametric space.

Figure 4.14 45° tow path through an isoparametric element

SUB-SECTION 4.5.3. ISOPARAMETRIC MAPPING OF THE TOW PATHS TO THE 3D MM

Once the tow path’s last point is calculated to be outside of the isoparametric MM element, the isoparametric mapping procedure is applied on the points discretizing the tow path utilizing (23) and (24) presented in section 3.4.1 to move from isoparametric space to global space. Figure 4.15 presents, a) the isoparametric tow path on the isoparametric element, and b) the translation and rotation of the tow path to the 3D MM element.
Figure 4.15 a) the isoparametric tow path on the isoparametric element, b) the translation and rotation of the tow path to the 3D MM element.

Figure 4.16 shows the presented framework on a single and double curved surface discretized by multiple MM elements. A single 90° tow path is generated by prescribing a 90° angle at each MM element node in both geometries and utilizing the presented tow path generation framework.

Figure 4.16 90° tow path on a) single curved surface and b) double curved surface
SECTION 4.6. TOW PATH’S ORTHOGONAL PROJECTION ONTO TM SURFACE

This section presents the projection of the 3D MM generated tow paths onto the TM surface. The procedure is performed employing the approach presented in section 3.6. With a tow path on the 3D MM surface as shown in Figure 4.16, an orthogonal projection is performed to each point in the discretized tow path to place the tow path onto the TM surface, which represents the original geometry. The projection of the 3D MM tow path generated through the single and double curved surfaces shown in Figure 4.16 are projected onto the TM in Figure 4.17.

![Images showing orthogonal projection of tow paths](image)

*Figure 4.17 Orthogonal projection of the 90° tow path onto a) single curved surface and b) double curved surface*

Finally, to manufacture the tow paths, the AFP machine requires the definition of tow path points in the form of, \([x, y, z, \alpha, \beta, \gamma]\) with respect to the coordinate system of the AFP machine. Therefore, each point discretizing the tow path generated by the TPP must be defined by these six values to proceed with the manufacturing step. Figure 4.18 shows
the angles which describe vector \( \mathbf{u} \) in 3D space, where \( u \) is a representative point in the tow path.

![Diagram of angles defining a vector in 3D space](image)

**Figure 4.18** Angles \( \alpha, \beta \) and \( \gamma \) defining a vector in 3D space

The \( x, y, z \), and \( \alpha \) values were calculated during the tow path generation step, where \( \alpha \) is the in-plain angle of the head equal to the fiber direction. The additional angles \( \beta \) and \( \gamma \), are calculated using (50) and (51).

\[
\mathbf{u}_2 = \mathbf{u} \cdot \mathbf{j} = \cos(\beta) \tag{50}
\]

\[
\mathbf{u}_3 = \mathbf{u} \cdot \mathbf{k} = \cos(\gamma) \tag{51}
\]

Where \( \mathbf{u}_2 \) and \( \mathbf{u}_3 \) are the \( y \) and \( z \) components of vector \( \mathbf{u} \), and \( \mathbf{j} \) and \( \mathbf{k} \) are the unit vectors of the \( y \) and \( z \) axis respectively.

**SECTION 4.7. POST-PROCESSING OF THE TOW PATHS FOR AFP MANUFACTURING**

This section performs a preliminary post-processing treatment to the generated tow paths to minimize the time in which the AFP machine code is generated from the tow paths provided. The approach uses the tow paths, the machine head angles presented in section 4.6 and a set of parameters to shorten the amount of points needed to discretize the tow path.
During the tow path generation, the user is expected to choose the smallest step size necessary to capture the behavior of the optimized lamina. Thus, tow paths generated will contain a multitude of points discretizing the tow path which are not necessarily essential for the manufacturing step but will most likely cause time delays in the post-processing step to generate machine code. The following condition is implemented to ensure the tow paths exported to the machine code generator (MCG) only contain the amount of points necessary to accurately manufacture the generated tow path on the 3D surface. If the difference in $\alpha$, $\beta$ and $\gamma$ from the first to the second point in the tow path is less than the default $2^\circ$, then the second point is deleted, and the third point is evaluated in the same manner. Note that the default difference between these angles can be updated by the user.

Then, the previous sequence is iteratively performed through the tow paths generated. The following shows a pseudo code showing the conditions to delete tow path points:

**Conditions:**

\[
\text{If } |\theta_i \pm \theta_{i+1}| < 2^\circ \text{ and } |eta_i \pm \beta_{i+1}| < 2^\circ \text{ and } |\gamma_i \pm \gamma_{i+1}| < 2^\circ : \\
\text{DELETE Point}_{i+1}
\]

These post-processed tow paths are then exported to the MCG to be post-processed.

**SECTION 4.8. SUMMARY**

The discretization of the geometry using the MM simplifies the calculation of tow paths by reducing the design variables. The use of the multi mesh approach facilitates the projection of tow paths from a coarsely discretized surface to a densely discretized surface. Multiple seeding algorithms have been presented and implemented to attempt maximum coverage of the surface. These seeding algorithms could be combined with user defined
seed points to further minimize gaps on the surface. The use of the isoparametric space was successfully adapted into the TPP to treat warped elements mostly encountered in double curved surfaces, and aid with simplifying the interpolation of the tow path through the curved geometries. In addition, the orthogonal projection approach made it possible to place the MM generated tow paths onto the TM surface to export for AFP machine manufacturing. Finally, a preliminary post processing step was presented to minimize AFP code generation time by utilizing the angles of the head and step between points as parameters in section 4.7.
CHAPTER 5. RESULTS

This chapter presents the results obtained from the developed software on 2D and 3D shell surfaces. The approach presented shows the development of tow path trajectories on each of the presented surfaces. First, the benchmark angle distributions in Figure 5.1 are utilized to validate the capabilities of the tow path generation tool on both 2D and 3D MM surfaces as presented in section 5.1 and section 5.2 respectively. The benchmark angle distributions in Figure 5.1 are referred to as a) the skinny-man b) the fat-man c) the flower, and the MM nodal values applied in a counter-clockwise direction starting at the bottom left node as [45°, 135°, 45°, 135°], [135°, 45°, 135°, 45°], and [90°, 90°, 45°, 135°], respectively. The tow paths generated on the 2D MM surface are projected to the 3D TM surfaces in section 5.3 to have the tow paths placed on the approximated real surface. Then, the tow paths generated and placed on the TM surface are post-processed such that the least amount of points discretizing each tow path is exported as presented in section 5.4. Finally, the results from a couple of optimizations on a planar and a single curved surface performed by TopSteer on both 2D and 3D shell surfaces is imported. Utilizing these input files tow paths are generated to ensure the coupling between the TopSteer optimizing code and the tow path planning code is fully functional in section 5.5. In conclusion, a summary of the chapter is presented in section 5.6.
SECTION 5.1. TOW PATHS ON THE 2D MM SURFACE

This section presents the generation of tow paths on a planar surface provided the benchmark angle distributions shown in Figure 5.1. Figure 5.2 shows a set of tow paths generated on a planar MM utilizing user defined seed points. The tow paths utilized a step size of .1. The step size was chosen to be small in order to capture the behavior expected using the benchmark angle distributions.

Figure 5.1 Benchmark angle distributions on a single MM element starting from the bottom left corner and moving counter clockwise a) the skinny-man [45°, 135°, 45°, 135°], b) the fat-man [135°, 45°, 135°, 45°], c) the flower [90°, 90°, 45°, 135°]

Figure 5.2 Skinny-man, fat-man and flower distributions on a planar surface
SECTION 5.2. TOW PATHS GENERATED ON A SINGLE MM ELEMENT PROJECTED TO CURVED GEOMETRIES

This section presents the orthogonal projection approach as performed in the TPP. The approach is performed with a single MM element containing within its boundaries the single and double curved surfaces shown in Figure 4.2. Tow paths are generated utilizing the benchmark angle distributions shown in Figure 5.1. Similar tow paths as shown in Figure 5.2 are generated for each surface utilizing the user defined seeding approach. A single MM element is used to further demonstrate that the discretization of the MM can be of its simplest form, and tow paths generated on such MM can still generate manufacturable tow paths in curved shells. The multi mesh approach enables the software to populate a 3D surface with tow paths utilizing a single MM element.

SECTION 5.2.1. SINGLE CURVED SURFACE

The single curved surface discretized utilizing the tessellation in CATIA and presented in Figure 4.3α is used as the TM onto which the tow paths generated shown in Figure 5.2 are projected. The tow paths are generated utilizing the benchmark angle distributions presented in Figure 5.1, a single MM element, and user defined seed points. The MM was generated in ABAQUS®, although since it is a single MM element, the user is also able to replicate the input files manually. Figure 5.3 shows the tow paths generated on the single MM and the progression to projected them onto the TM surface.
SECTION 5.2.2. DOUBLE CURVED SURFACE

The double curved surface discretized and presented in Figure 4.3b is utilized as the TM onto which the MM tow paths generated are projected on. The tow paths are generated on a single MM element utilizing the benchmark angle distributions presented in Figure 5.1. The MM is generated using ABAQUS® and the TM is generated using CATIA® as previously stated. Once again, the tow paths are generated with user defined seed points and Figure 5.4 presents a picture of the prior.
SECTION 5.3. POST-PROCESSING OF TOW PATHS FOR AFP MACHINE CODE GENERATOR

This section presents how the tow paths generated and projected onto the TM surface, i.e. Figure 4.16, are post-processed to ease the machine code generator (MCG). In this case, a 45° tow path projected onto the TM of a double curved surface is studied. Table 5.1 presents the length of the vector discretizing the tow path on the TM surface versus the same tow path after post-processing. From the pure numbers, one can observe the post-processing step can have a major impact in minimizing the time utilize to generate the AFP machine code.
Table 5.1 Comparison between vectors discretizing a 45° tow path generated and projected onto the double curved surface

<table>
<thead>
<tr>
<th>Tow Path Vector Length Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Before Processing</strong></td>
</tr>
<tr>
<td>24</td>
</tr>
</tbody>
</table>

Figure 5.5 shows the tow path before and after processing. The red tow path in Figure 5.5a contains 24 points discretizing the tow paths, Figure 5.5b shows the AFP machine tow path movement to manufacture the path, and Figure 5.5c shows both paths overlapping, which shows that the post processing does not affect the tow path behavior calculated during the generation step.

Figure 5.5 45° tow path on a double curved surface before processing, b) after processing, and c) superposed before and after processing

SECTION 5.4. COUPLING OF TOPSTEER WITH THE TPP

This section presents the results obtained from utilizing TopSteer optimization results to showcase the post-processor generated is compatible with the TopSteer software. First, the CoDeT panel [40], [41] which is a flat plate with holes and secondly a single
curved surface. The planar surface is subjected to tension and shear loading as studied by Gurdal et al. [34] and further reviewed by van Zanten et al. [42] The curved surface is subjected to circumferential tension loading as studied by van Zanten et al. [42]

SUB-SECTION 5.4.1. 2D PLATE WITH HOLES

The coupling with TopSteer is validated by generating tow paths with a given set of optimization results of the planar surface with holes studied in the CoDeT research. [40], [41] Figure 5.6 shows the geometry studied as well as the tow paths generated employing the user defined seed points with a given optimized angle distribution per ply from TopSteer.

![Image of tow paths](image)

*Figure 5.6 Tension shear loading optimization on a plate with holes*

SUB-SECTION 5.4.2. 3D SINGLE CURVED SURFACE WITHOUT HOLES

Figure 5.7 presents the test set up in ABAQUS®. As shown, the single curved plate is subjected to circumferential tension loading. [42]
Figure 5.7 Loading configuration in ABAQUS® for the single curved surface

The tow paths generated on the optimized single curved surface show the coupling with TopSteer was confirmed and finalized. Figure 5.8a shows the initial position of the tow paths before optimization, which is perpendicular to the loading direction. Figure 5.8b shows the optimized tow paths which are parallel to the loading condition.

Figure 5.8 a) Tow path direction before optimization and b) tow path direction after optimization
SECTION 5.5. SUMMARY

The chapter presented the results of the developed Tow Path Planning software with the adaptation of the multi mesh approach. The software was subjected to a diverse group of angle distributions, and tow paths were successfully generated through the MM surface and projected onto the TM surface. The benchmark angle distributions were applied to a planar MM element, and the tow paths generated were projected onto both the single and the double curved surface to showcase the benefits of the multi mesh approach adapted to the Tow Path Planning tool. Furthermore, the software was subjected to the coupling with TopSteer by optimizing the CoDeT plate and the single curved surface. [40], [41], [42] Finally, the post processing of tow paths for the manufacturing step was performed and the results are verified by comparison of the MCG path with the theoretical path.
CHAPTER 6. CONCLUSION AND FUTURE WORK

In conclusion, a novel approach to the tow path trajectory generation of steered laminates is presented. The TPP software developed handles planar, single and double curved surfaces. The framework is presented in detail by moving through the multiple mathematical manipulations needed to generate tow paths in all these different surfaces. The introduction and adaptation of the multi-mesh approach into the TPP is successfully performed, and the implementation of the isoparametric space to facilitate the interpolation of the tow path through the surface is verified. Also, a post processing step to optimize the generation of the AFP machine code is developed and verified. It is important to note that the tow path planning software can always be improved to generate better composite laminates as the manufacturing of composites is constantly evolving. The TPP software was developed in an open source language environment such that upgrades, and new optimization algorithms i.e. seed point algorithms, and the introduction of different interpolation functions can be implemented with minimal effort. Furthermore, the software currently needs to be validated by manufacturing one of the generated tow paths using the AFP machine. Finally, the software can be further developed by introducing an optimization routine to minimize gaps. This can be achieved by utilizing the density function approach utilized in the optimization of the laminate in junction with an image processing algorithm to detect gaps and prevent overlaps.
REFERENCES


curved fiber trajectories,” *Compos. Struct.*, 2015.


APPENDIX A

VISUALIZATION OF HUA’S GLOBAL ELEMENT CONDITIONS

Courtesy of van Zanten [43] the following figures show the representative elements that describe the conditions defined by Hua. [32]

**Condition 1:** \( a_1 = 0, \ a_2 = 0 \) – A representative element for this condition is shown in Figure A.1.

![Figure A.1 Representative quadrilateral for condition 1](image)

**Condition 2:** \( a_1 = 0, \ a_2 \neq 0, \ c_1 = 0 \) – A representative element for this condition is shown in Figure A.2.

![Figure A.2 Representative quadrilateral for condition 2](image)
**Condition 3:** \(a_1 = 0, a_2 \neq 0, c_1 \neq 0\) – A representative element for this condition is shown in Figure A.3.

![Figure A.3](image3.png)

*Figure A.3 Representative quadrilateral for condition 3*

**Condition 4:** \(a_1 \neq 0, a_2 \neq 0, a_b \neq 0, a_c \neq 0\) – A representative element for this condition is shown in Figure A.4.

![Figure A.4](image4.png)

*Figure A.4 Representative quadrilateral for condition 4*
**Condition 5:** \( a_1 \neq 0, a_2 \neq 0, a_b \neq 0, a_c = 0 \) – A representative element for this condition is shown in Figure A.5.

![Figure A.5 Representative quadrilateral for condition 5](image)

**Condition 6:** \( a_1 \neq 0, a_2 \neq 0, a_b = 0, a_c \neq 0 \) – A representative element for this condition is shown in Figure A.6.

![Figure A.6 Representative quadrilateral for condition 6](image)
**Condition 7:** \(a_1 \neq 0, a_2 = 0, b_2 = 0\) – A representative element for this condition is shown in Figure A.7.

![Figure A.6 Representative quadrilateral for condition 7](image)

**Condition 8:** \(a_1 \neq 0, a_2 = 0, b_2 \neq 0\) – A representative element for this condition is shown in Figure A.8.

![Figure A.6 Representative quadrilateral for condition 8](image)