# The Effects Of Systemic Functional Linguistics And Gradual Release Of Responsibility On Student Self-Efficacy And Engagement In Mathematics

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# The Effects of Systemic Functional Linguistics and Gradual Release of Responsibility on Student Self-Efficacy and Engagement in Mathematics

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## DEDICATION

To my wife, whom has been ever so loving, patient and supportive;

To my daughter, whom I love dearly;

To my parents, whom have made this possible;

To Dr. Todd Lilly, whom has been my guide;

To my colleagues, whom I can share experiences and grow professionally;

And, to my students who give me inspiration.

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## ABSTRACT

This paper addresses the problem of practice that secondary mathematics students do not feel confident in communicating mathematically. The researcher has placed an emphasis on math language by using a systemic functional linguistic approach to teaching mathematics. Building student confidence and engagement in mathematics is the main focus of the methods presented in this action research study. The researcher wants to investigate how systemic functional linguistics and gradual release of responsibility affects student self-efficacy and engagement in secondary mathematics. A mixed-methods design uses researcher field notes, student surveys and interviews, and student work to examine the levels of self-efficacy and engagement among students in the researcher's classroom. Preliminary findings show that students initially resisted SFL approaches, but gained more confidence as the study progressed. Student responses indicate that GRR strategies aided in building self-efficacy. The researcher includes reflections and implications for future research so that the study can be replicated by other educators.



*Keywords:* action research, math language, systemic functional linguistics, self-efficacy, engagement, gradual release of responsibility, group work.

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# CHAPTER 1: INTRODUCTION

## Overview Dissertation in Practice (DP)

One may think that students that do well in mathematics are good with numbers. Of course, numbers are a large part of mathematics, but to be truly successful there is much more to it than calculations. Teachers must also show young mathematicians how to interpret their results - a practice done through multiple modes of communication. There is a set of vocabulary, symbols, operations, and notations unique to the subject that is important to be able to use to communicate mathematics (Caglar, 2003). These unique items are referred to as the lexis of mathematics. Many students need help in developing the skill needed to use math lexis effectively (Cooke & Buchholz, 2005). Math and language are intertwined (de Oliveira & Cheng, 2011).

Systemic functional linguistics (SFL) is an analytical approach introduced by Michael Halliday. The major premise of SFL is that language is a process people use to make meaning (Halliday, 1978). The process involves making language choices that describe thoughts or to make meaning of situations. Text made from the choices can be analyzed to determine connections or misconceptions (Halliday & Matthiessen, 2004). SFL is also referred to as the social semiotic system (Lemke, 1990). There are three semiotic systems in mathematics that are used to create meaning (de Oliveira & Cheng, 2011):

1. Natural language and text that is used to describe the problem.

2. Symbols and notation that are used to solve problems.

3. Visuals such as charts, graphs, and diagrams used to show results.

Learners should be able to internally process mathematics and communicate it interpersonally and textually using multiple modes (Caglar, 2003). "Math language" is filled with unique vocabulary terms, symbols, graphical representations, and formats in which students are uncomfortable using (Borasi, Siegel, Fonzi, & Smith, 1998). By examining students' use of math language, teachers can develop a better understanding of how students are making meaning in their learning (Schleppegrell, 2010).

## Purpose of the Study

The purpose of this study is to apply an SFL approach to teaching mathematics and examine students' understanding of math language by analyzing how they communicate mathematics through conversations with peers, writing, and one-on-one interviews. The researcher implemented Gradual Release of Responsibility (GRR) instructional techniques to deliver content. This approach allowed the teacher researcher to guide students to develop reading and writing comprehension in mathematics. The researcher has designed an action research study with intent to give students maximum exposure to the lexis of mathematics. The purpose of this study is to find the effects that increasing math language exposure, using SFL in conjunction with GRR, has on student engagement and self-efficacy in a secondary Algebra 1 classroom.

### Problem of Practice (PoP) Statement

In developing as a teacher, the researcher has reflected on ways to improve teaching methods in efforts to improve student self-efficacy and engagement. "It is important to remember that the goal of any action research project is a desire to make things better, improve some specific practice, or correct something that is not working as well as it should" (Mertler, 2014, p. 39). The Problem of Practice (PoP) the researcher has identified among his Algebra 1 students is that they are not confident in using the language of mathematics. The PoP investigated in this action research study is the lack of engagement due to the inability to communicate mathematically and the resulting lack of student self-efficacy in secondary mathematics students. Put simply, the researcher has noticed that the average student often lacks the ability to express themselves concerning the topics covered in class.

Students may be able to perform the mathematical operations and use the formulas, but have difficulty interpreting the results and sometimes fail in their ability to adequately explain the mathematics they are doing. This is especially evident in application problems, i.e. word problems involving reading comprehension, and assignments that require the student to persevere in thinking or interpret results. It is also evident when students are required to express their results using different modes. As a result, students are less confident in doing assignments due to lack of understanding the lexis, not necessarily the mathematical process. When students are told that they need to know how to use math language correctly, i.e. the syntax, sentence structure, and text, students often do not understand why this is important.

Many students complain that language is for English class (Smith & Angotti, 2012). Some do not even attempt the assignments, not necessarily because of neglect, but again, because they simply just do not understand how to get started. The researcher has noticed that many times when steps are modeled and guided the students are able to complete the problem. However, getting started without any help sometimes is an issue.

Successfully completing word problems is a major barrier in learning mathematics for students (Boonen, Reed, Schoonenboom, & Jolles, 2016). The ability to solve word problems reflects the students' ability to apply what they are learning to a real-world situation. This also applies to analyzing graphs and tables. This is a staple in South Carolina State Math Standards. "To solve a word problem, the students must derive the relevant information from the text to set up a calculation problem for retrieving the missing information" (Korpershoek, Kuyper, & Van Der Werf, 2014, p. 1013). When students do not know the proper use of the math language, they will likely have trouble understanding the directions, which can lead to decreased engagement. "Research shows that the difficulties experienced by many students in solving word problems arise not from their inability to execute computations, but from difficulties in understanding the problem text, identifying solution-relevant components and the relations between them, and making a complete and coherent representation of the situation described in the problem" (Boonen et al., 2016, p. 57).

The students' inability to persevere in problem-solving also stems from the problem of practice. Students seem to give up very quickly and easily when they encounter something in math that they do not understand immediately (Wilburne & Dause, 2017). Often times it is due to lack of understanding of what is being asked of them to accomplish. John Dewey thought that students learned through experience. In Dewey's laboratory schools children raised plants in a garden. When plants were failing, the students looked to their textbooks for answers. As a result, the students were more vested in picking the right type of seed and soil and were more interested in knowing the right ways to prepare the soil and fertilize the plants (Watras, 2012). When proper planning takes place and thoughtful assignments relevant to the students' lives are given, students may be more likely to persevere in finding answers and less likely to give up when they hit a stumbling block.

Teachers typically conduct a lesson and bombard students with factoid after factoid, vocabulary word after vocabulary word, and formula after formula. Students sometimes learn these concepts for a brief period of time, but if they do not use it for something that they see as interesting or useful, then they will likely forget much of the information taught. "When the person formed a meaning of something in the environment by undergoing the results of an action, he or she could shape later activities to make those experiences more fruitful" (Watras, 2012, p. 163).

## Research Question

In this study, the researcher attempts to give students more exposure to the math language that they need to learn and use. The research question is based on both William Bagley and John Dewey's ideologies and principles that children learn by observing (Bagley), but then by doing (Dewey). The researcher applied the GRR instructional model in class and analyzed student text and interpersonally conversations using an SFL approach. The researcher made daily field notes to report the findings. Self-efficacy was monitored and assessed through student surveys and teacher observations. Engagement was measured by student participation, behavior, and involvement during discussions. The overarching research question that will guide the study is the following: **"What effect does systemic functional linguistics in conjunction with the gradual release of responsibility have on student self-efficacy and engagement in secondary mathematics?"**

## Purpose Statement

As mentioned in the PoP, the researcher has noticed through conversation and observation that many high school students are not confident in doing math. Some of the researcher's students believe that if they are not a "math person," then it is acceptable for them to be mediocre at math. An overarching observation is that this attitude fosters a belief that learning math language is not necessary to actually do the mathematics. Many students perceive that mathematics is quantitative and overlook the qualitative aspects, including the importance of text in mathematics. This study focuses on self-efficacy and engagement, but future implications may lead to studies in math achievement, as student engagement has been linked to student achievement (Dotterer & Lowe, 2011). More detailed discussions of the research question will take place in Chapter 3 of the DiP.

## Rationale for the PoP

While there are studies that show that higher reading comprehension skills positively affect math achievement (Korpershoek et al., 2014), few studies show the effect that systemic functional linguistics in conjunction with gradual release instructional methods may have on math engagement and self-efficacy in a secondary school setting. Students must develop an understanding of content vocabulary in order to master content (McAdams, 2011). Large class sizes and time management often tempt teachers to adopt direct instructional strategies. Direct instruction has been highly scrutinized as of late, often misinterpreted to as lecture-based teaching (Magliaro, Lockee, & Burton, 2005). Bagley, a key contributor of traditional direct instruction methods, or *Essentialism*, felt that school subjects were a means to guide teachers in shaping students' behavior (Watras, 2012). Hence, teachers use traditional ways of teaching because it feels comfortable and seems an effective way to manage time and student behavior.

While direct instruction has its advantages, a mixture of teacher-centered and student-centered instruction may broaden the reach to students (Gordon, Barnes, & Martin, 2009). Student-centered learning techniques requires that the students are more active in the learning process (Judi & Sahari, 2013). While some students do well with teacher-centered instruction, studies have also shown that many students also benefit from student-centered techniques. Moreover, students have even more success when working as a team (Nicoll-Senft, 2009). GRR allows the instructor to first model for students with direct instruction techniques. Then the strategy shifts from teacher-centered instruction to student-centered learning by slowly and systematically turning over learning responsibility to the students as they work together to solve problems. Before the end of the learning period, each learner is able to work individually to show complete competence without the help of the instructor (Fisher & Frey, 2008). As mentioned previously, the researcher will also use an SFL approach to analyze the students' language choices and meaning-making in the content. If the student does not identify with the content, then the student's level of engagement may decrease (Freitas & Zolkower, 2009).

**Causes of the PoP**

Sometimes communicating mathematically can be a real challenge for the student. The researcher has identified two main factors that may contribute to the inability to communicate mathematically: reading level and lack of opportunities. In order to successfully communicate in math, and in particular, to analyze math text, advanced reading skills are needed. Some students may understand the math, but since they are not accustomed to using math language on a regular basis, their ability to solve contextualized problems may be decreased if they do not possess advanced reading skills (Korpershoek et al., 2014). The different reading levels of students must be placed in consideration. Plans to show how this consideration will be made will be addressed later in Chapter 3 of the DiP.

When a student is presented with a word problem or any text, including charts, graphs, and diagrams that contain math language, and the student does not understand the lexis, then the performance can be affected (Kovarik, 2010). Developing the ability to use the mathematical symbols and language is crucial to the success of the math student (Borasi et al., 1998).

**Area of Specialization (Literature Review)**

For the sake of this study, the researcher has used books and the search engine Education Source and ERIC to search for and find reliable peer-reviewed resources to help build background knowledge and understanding of SFL and GRR. The researcher will explain systemic functional linguistics and its role in teaching mathematics. The researcher will also discuss the four major components of GRR: modeling (I do), guided practice (we do it together), collaborative learning (you do it together), and independent assessment (you do it alone). Each of these components will be dissected and discussed in detail. Topics included in the literature review will include think-aloud models, guided practice techniques, direct instruction, student-centered instruction, and team-based learning.

## Barriers to the Study

The researcher understands that there are barriers in the study in which he must prepare. The researcher anticipated cultural barriers in the study. Differentiated instruction techniques are important to give learners of different cultures an opportunity to be successful (Mainini & Banes, 2017). The researcher also addresses socioeconomic barriers. Team-based learning is a vital component to GRR, so the researcher must be knowledgeable in fostering effective group work among students of different interests and backgrounds.

### Cultural and Socioeconomic Barriers

The language barrier will be an obstacle for the problem of practice as the focus is on math language development, with the primary natural language being English. ESL students may not feel comfortable and thereby reluctant to participate (Kalyanpur & Kirmani, 2005). Accommodations were necessary for those that lack the language skills of others as the researcher anticipated that these students may be at a disadvantage throughout the study. In order to get an accurate study the researcher made sure that students wrote substantially and used terminology correctly. There is one ESL student in the study that required accommodations for language needs. Due to the additional layer of needs and the complexity this presented to the study, the researcher did not include data from this student. However, the researcher made sure that this student's learning needs were met and that the study did not place the student at a disadvantage or negatively affect the learning process for this student.

### Team-based Learning Barriers

Throughout their academic careers, students reach many different skills levels, have different preferred methods of learning, and a variety of strengths and needs (Mainini & Banes, 2017). Each student needs individualized attention, but engaging in team-based learning can lead to increased engagement. The researcher is careful to research the most effective methods to implement group assessments. This is one way that the researcher plans to differentiate instruction.

## Overview of DP

This chapter has introduced the problem of practice (PoP) that will be studied using action research methods. The problem of practice investigated in the study is the lack of sufficient focus on math lexis, which results in low student self-efficacy and decreased engagement in secondary mathematics students. The purpose of this study is to find how SFL and GRR affects math students’ self efficacy and engagement in secondary mathematics.

Chapter Two: "Literature Review" inspects prior research on the problem of practice and discusses the purpose of the literature review in more detail. Chapter Three: "Methodology" discusses the purpose of the study, the methods of the action research and the research design in more detail. Once the methods are discussed, Chapter Four: "Findings" presents the findings and interprets the results of the study. Chapter Five: "Conclusions and Implications for Future Research" summarizes and discusses the major points of the study. Upon interpreting the findings of the study, the researcher concludes the DiP by discussing an action plan and suggestions for future research.

# CHAPTER 2: LITERATURE REVIEW

## Introduction

The previous chapter introduced the PoP that many students perceive math to be quantitative and that learning the math lexis is not necessary in performing the quantitative requirements in which mathematics is so well known (Thompson & Rubenstein, 2000). By failing to adequately focus on math language, students do not have the proper background knowledge needed to understand how to communicate mathematically, which can lead to low math achievement (Dunston & Tyminski, 2013). The researcher has seen that many incumbent students avoid using vocabulary terms, have limited understanding of math notation, and lack abilities in writing sentences or statements using math vocabulary and symbols. In other words, students have difficulty in the syntax, sentence structure, and text of mathematical statements. Writing, in general, is an arduous task. Students may know how to perform quantitative tasks well, but their lack of math literacy sometimes keeps them from being able to start the problem on their own or interpret the results. In this chapter, the researcher will explore previous research on this topic to find links between SFL and GRR and its effects on self-efficacy and engagement.

For the sake of this study, the researcher has used books, the university library, and the search engine ERIC to search for and find reliable peer-reviewed resources to help build background knowledge and understanding of math lexis, systemic functional linguistics, GRR, self-efficacy and student engagement. Efforts have been made to limit the search to recent articles, especially when referencing past studies. In this literature review, the researcher only focused on articles that related to the problem of practice and proposed research question. The researcher has organized the literature review in two main parts. First, the goal is to give the reader an in-depth exposure to the review of literature that are related to the PoP. The main constructs in this study are math lexis, self-efficacy, and student engagement. The researcher chose to use an SFL approach to analyze students' self-efficacy in communicating mathematics. The researcher adopted a GRR instructional strategy to deliver content.

The strategy used in this research was to identify key concepts in the PoP by creating a fishbone diagram. Once these key concepts were identified, key word searches were conducted in the ERIC search engine. Article annotations helped organize the literature and citations were included in the outline under each subheading. The researcher made detailed notes while reading the literature and identified common themes and threads across studies. This literature review highlights the common themes and threads discovered through prior research to better support the PoP.

Low levels of math language understanding can lead to low levels of perseverance in solving word problems (Dunston & Tyminski, 2013). Many students reflect negative attitudes toward word problems in math, possibly because of unfamiliar text (Smith & Angotti, 2012). Their lack of confidence in math language could contribute to this perception (Phillips, Bardsley, Bach, & Gibb-Brown, 2009). It is the educator's responsibility to help students to connect math language and other text with their understanding of the math concepts (Dunston & Tyminski, 2013). The ultimate goal in mathematics is connecting the math to real-world applications, which implies that students need to be able to problem-solve given a certain situation (Popovic & Lederman, 2015). GRR can provide a way for learners to interact with each other while practicing their skills using math lexis across multiple modes.

There are many factors that can contribute to students' inability to communicate mathematically. One of these factors is the students' reading ability. Students with low reading levels may have difficulty with the reading comprehension that is needed in solving word problems (Darling, 2013). Knowing how to use math language and symbols are a crucial part to the students' ability to internally process and to communicate mathematically both interpersonally and textually (Korpershoek, Kuyper, & Van Der Werf, 2014). It is the instructor's responsibility to provide students with the proper amount of opportunities to use math lexis during class. Unfortunately, many teachers do not place appropriate emphasis on using correct syntax (Dunston & Tyminski, 2013). As a result, students may overlook the importance of syntax and sentence structure in communicating mathematically (Falle, 2004). In fact, many of the researchers' students do not even read the directions and jump right into solving the problems. An SFL approach may provide a way for the researcher to effectively analyze student self-efficacy in math language.

In this chapter the researcher will discuss the PoP in more detail through extensive review of the literature on this topic. First, this review discusses the construct of math language, its importance, and students' attitudes toward communicating mathematically. Using previous studies, the review will provide an in-depth description of SFL and its role in analyzing student self-efficacy in communicating what they know. Next, the review will elaborate more on another key component of the study – the use of GRR to foster student engagement. During the exploration of the problem of practice, the review will discuss the historical perspectives that guide the study. These include relevant key individuals whose ideas and theories have contributed to the strategies that pertain to SFL and GRR.

## Purpose of the Review

This literature review provides the theoretical framework by which the researcher made decisions in the study. The literature review allows the researcher to use previous research to make decisions that will increase the effectiveness of this action research study (Mertler, 2014). Examining literature on relevant topics also helps to make connections between this study and others that are similar (Johnson, 2008). The researcher conducted an extensive review of the lexis of mathematics, SFL, and GRR to increase his knowledge of these key components to the study. The studies included in this review helped guide the researcher in making decisions in designing the methods of research, which will be discussed in detail in Chapter 3: "Methodology."

## Key Concepts

While researching the PoP, reoccurring themes arose in the literature. We will begin by discussing math language, its importance in mathematics, and student and educator perceptions on math language. Then, the review will discuss the role of SFL in mathematics. Finally, the researcher will discuss student self-efficacy and engagement through the examination of GRR instructional model.

Historical perspectives will drive this study. Key individuals such as John Dewey (1938) and William Bagley (1938) have influenced the methods that will take place. It is important to contrast these views to help explain decisions that are made during the study. Dewey (1938) believed that students learned by doing and Bagley (1938) believed that students needed guidance and structure in their learning (Watras, 2012). Comparing curriculum ideologies such as Dewey's (1938) learner-centered and Bagley's (1938) teacher-centered classrooms is important because GRR combines the two instructional methods. The researcher is a teacher-participant in the study. The researcher designed the lessons, created the assignments, and monitored student progress. Therefore, the researchers' role as curriculum leader will affect the study. This literature review will thoroughly discuss all of these key concepts.

## Lexis of Mathematics

Math is filled with vocabulary words, symbols, notation, charts, graphs, and many other representations in which the student must be able to read and decipher in order to be successful (Phillips et al., 2009). Many scholars agree that it is the job of the math teacher to place emphasis on math language and require the students to know and learn this material (Korpershoek et al., 2014). Placing emphasis on vocabulary can sometimes be a difficult task as students are not often familiar with the words. "Math terms are not situated in everyday conversations or discussions because these words are rarely included as dialogue in the latest Hollywood productions and not generally found in novels, newspapers, or social media" (Dunston & Tyminski, 2013, p. 40).

## Reading and Mathematics

According to Halliday (2014), "any situation can be characterized in terms of field, tenor, and mode" (p. 33). *Field* refers to what is happening in the situation. In mathematics, this would refer to the symbols, operations, or mathematical action taking place. *Tenor* refers to who is taking part in the activity (Halliday, 2014). The researcher and the students are the tenor in this study. Acknowledging the tenor is vital. Taking the square root of a negative integer will have different meaning to a middle school pre-algebra student as opposed to a more advanced algebra student. While a novice math student may conclude that $\sqrt{-1}$ has no real solution, a more advanced student will know that $\sqrt{-1} = i$, where *i* represents an imaginary number. Both students are correct. However, the more advanced student will understand that the definition of an imaginary number is there is no real solution, but mathematical processes can take place, whereas the novice student will not likely make that connection. *Text* is any meaning-making event, including written language, spoken language, pictures, graphs, or even non-verbal cues (Knapp & Watkins, 2005). *Mode* refers to the role that language, or text, plays in the situation (Halliday, 2014).

Mathematics may be interesting from a linguistics perspective being that it is a combination of texts, or *multimodal* (Hughes, 2009). For instance, those studying mathematics are combining native language, including the syntax, sentence structure, and punctuation, with the lexis of mathematics, which contains its own syntax, sentence structure, and punctuation. It takes extensive knowledge of mathematical syntax to understand function notation. $F(x)$, read, "F of x," refers to a function (or set of solutions) whose name is "F" and represents the outputs of all possible inputs, which are called "x." One might also note that in one field, parenthesis directs the use of multiplication. However, this is not true in this field. Also, often times, mathematics requires the combination of written and visual text in the mode of a chart, table, or graph (Joutsenlahti & Kulju, 2017.

By placing disproportionate amounts of emphasis on math language and computational processes, teachers can be counterproductive in reaching what many educators and curriculum developers feel is the ultimate goal in curriculum standards – using math skills to solve real-world problems (Popovic & Lederman, 2015). Often times this standard is met through the use of word problems (Kotsopoulos, 2007). When teachers place computational skills over math lexis, students' perceptions of vocabulary, symbols, and text likely will be that learning language and syntax is of little importance (Orten, 2004). Some students already view mathematics as somewhat of a foreign language (Kotsopoulos, 2007), and the student needs to be fluent in reading math text, writing notation, using proper syntax, understanding symbols, and knowing how to pronounce words in order to process the field internally (Phillips et al., 2009). It is possible that some students have low math achievement due to their inability to understand the field (Dunston & Tyminski, 2013). A student can know how to perform calculations and use a formula, but if they do not understand the field or syntax, they will have difficulty determining where to begin (Korpershoek et al., 2014). The ability to communicate mathematically is directly tied to students' ability to understand math concepts (Dunston & Tyminski, 2013).

Teaching math language is not easy. Students typically struggle with it, especially those that have low reading skills (Korpershoek et al., 2014). However, it is the educators' responsibility to be persistent in encouraging the correct use of math lexis. It is important to use math terminology frequently so that students hear the language. By relentlessly teaching students mathematic sentence structure and syntax, and exposing tenor to multiple modes, educators can improve students' abilities to communicate mathematically (Phillips et al., 2009).

Word problems seem to be every math students' nemesis. "Regardless of the math level being taught, students often react anxiously or negatively to [word] problems. When students encounter word problems, they either claim that they have never been able to understand [word] problems or state that they do not know where to begin" (Darling, 2013, p. 178). Lack of math lexis understanding may be a major cause to low student self-efficacy in solving word problems (Darling, 2013). To better combat this issue, it is essential that educators are aware of the tenor (Tyler, 2013). One study shows that providing age-appropriate reading materials that show how math is used are beneficial in helping students make meaning (Borasi, Siegel, Fonzi, & Smith, 1998). Word problems can be constructed so that it acknowledges tenor in efforts to decrease anxiety in students and gives intrinsic motivation to solve problems (Darling, 2013).

There are words used in natural language that take on a completely different meaning in mathematics (Njoroge, 2003). *Prime* in natural language would mean "most important" or "main," but in math it has no such meaning. In fact, there are multiple uses for the word "prime". *Prime* numbers are numbers that can only be divided by itself and one. *Prime* also refers to a symbol ($A' = $ "$A\ prime$") used to denote a transformation or a second coordinate or equation. The phrase "less than" can also take on different meanings. "Three *less than* some number" implies that subtraction is needed $(x - 3)$, and the algebraic expression is written in reverse order from the natural language. However, "three is *less than* some number" implies an inequality $(3 < x)$. Inputting the word "is" changes the *mood* of the statement. "Mood is the major interpersonal system of the clause; it provides interactants involved in dialogue with the resources for giving or demanding a commodity, either information or goods-&-services" (Halliday, 2014, p. 97). In natural language, the word "is" is present tense indicative of *be*, which communicates factual information. In mathematics, the word "is" represents *equal to*, and therefore, represents a symbol (=). However, in this case, when combined with "less than," it represents the *inequality* (<). These are just a few examples of the complex lexis of mathematics.

**Techniques in Teaching the Lexis Of Mathematics**

There are a variety of methods that can be used to help students increase their abilities to use math language correctly. A qualitative study by Dunston & Tyminski (2013) shows how the use of graphic organizers such as the Frayer Model, developed by Nancy Frayer, and the Four Squares Model gives students a way to associate vocabulary terms in multiple ways such as with comparisons, pictures, and key words, allowing students to become comfortable with multiple modes. Frayer intended her model to give students an opportunity to "go far beyond learning mere definitions of words; instead, they develop a far deeper understanding of concepts" (Wanjiru & O-Conner, 2015, p. 203). Their research found that knowledge of math vocabulary is necessary for math achievement (Dunston & Tyminski, 2013). For struggling readers, different genres of text, such as a teacher-made video or a link to a YouTube video may go a long way in helping them see how to communicate mathematically. Likewise, computer-based tutorials can be a good way to supplement face-to-face instruction (Korpershoek et al., 2014). Regardless of methods used, math educators should be aware of the importance of teaching math lexis as well as the computational aspects of the curriculum, as being fluent in reading texts and communicating to others is essential to being able to understand the concepts (Phillips et al., 2009).

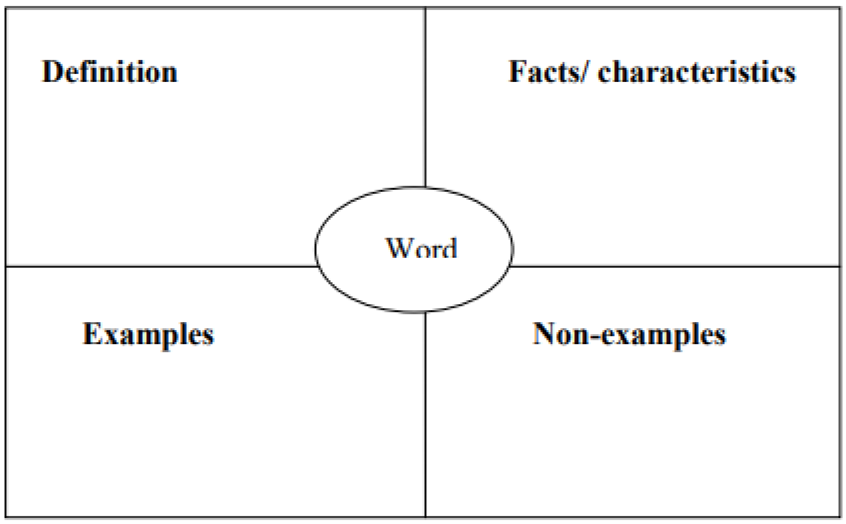



**Figure 2.1 The Frayer Model**

In order to build skills in math language, it is essential to allow students opportunities in communicating mathematics (Kotsopoulos, 2007). Falle (2013) conducts a study to help teachers understand the tenor by holding one-on-one conversations about the math topics being covered in class. The two topics covered were "how to calculate a square root" and "how to find the area of a rectangle." The teacher-researcher met with the students and verbalized math problems to gauge the level of understanding in using math symbols and vocabulary. It is important to listen to students' conversations and how they are using the terminology. The research by Falle (2013) found that occasionally meeting with students one-on-one may help the educator have a more accurate idea of their students' abilities to communicate mathematically.

Combining literacy and mathematics is an overarching theme in reviewing the literature. Korpershoek et al. (2014) compared math and reading ability to see if there was a correlation. They measured math ability of 1,446 high school students by three math-related tests and reading ability by three reading-related tests. The results showed that math and reading ability share a positive correlation with grades in mathematics (Korpershoek et al., 2014). However, many math teachers are not trained to combine these two aspects of learning. Another study by Phillips et al. (2009) researched how professional development effected math teachers' ability to teach math lexis. The research consisted of two phases. In the first phase, the researchers used a constructivist approach and allowed the teachers choice in their professional development. The teachers attended workshops, which they collaborated with reading and literacy specialists on developing strategies to teach mathematics. In the second phase, the teachers implemented the training in their classrooms. The findings showed that professional development is needed in improving the math educators' confidence, literacy knowledge, and even enthusiasm so that they can better reach students who struggle with math language (Phillips et al., 2009).

Teachers can only expect students to be comfortable in communicating mathematics if they give them the opportunity to do so (Wanjiru & O-Conner, 2015). Bell (1993) discusses ways that educators can get their students to talk about math. Teachers can gather information on the tenor and use relevant material that will peek the students' interest.  Using themes can also be beneficial in helping students link the math to the real world. Asking open-ended questions or asking questions that have multiple correct responses will improve dialogue in the classroom. Acknowledging every student response is critical in building confidence. This goes along with creating positive, non-threatening classroom environments that foster conversations led by students instead of teachers (Bell, 1993).

Students, and even some math teachers, often have the misconception that learning math lexis is not as important as performing mathematical calculations (Dunston & Tyminski, 2013). However, studies discussed in this section have shown that higher levels of math language understanding can lead to higher achievement. When students do not understand the lexis, they can have an increased sense of anxiety and lower self-confidence which can cause their thinking about the math to be stagnant. Many are waiting for the "first step" because they do not understand the field. Teachers must emphasize the importance of using math language correctly, including text, syntax, sentence, and multiple modes, and implement this in their instructional strategies. Educators must also be trained to combine math and literacy skills in their lessons so that students will begin to associate math as being a combination of semiotic systems (Phillips et al., 2009).

## Systemic Functional Linguistics (SFL) in Mathematics

"Because all language use contributes to the construal of the social contexts in which it occurs, a functional theory of language enables us to identify linguistic choices that realize particular kinds of contexts … Rather than seeing language as a set of rules, the functional linguistic perspective sees the language system as a set of options available for construing different kinds of meanings" (Schleppegrell, 2004, p. 4). Since math language is such an integral part in the learning of mathematics, it may be helpful to analyze how the language is being used. An SFL approach to teaching mathematics examines how the curriculum influences the students' language choices in communicating mathematics (Halliday & Hasan, 1976; Halliday, 1994). As stated earlier, it is important to highlight that words are not the only form of language and text (Knapp & Watkins, 2005). SFL acknowledges several complex systems, including ever-changing fields and modes, working in conjunction to make meaning for a given tenor.

In this study, the researcher is particularly interested in ways the students use language to meet their learning needs in mathematics. Choices in language are influenced by the field, the tenor, and the mode for a particular situation (Huang, Berg, Siegrist, & Chanaichon, 2017). The *mood* may also have an effect on language (Halliday, 2014). For example, a student may use different language choices when speaking to a close friend than with the principal of the school. In math, this means that a student must learn to how to use words not commonly used in every day conversations in order to effectively communicate mathematics among their peers. Notation, vocabulary, and use of symbols are primary avenues in making meaning in mathematics (Freitas & Zolkower, 2009).

An SFL perspective of the language choices, sentence structure, and context are essential in analyzing student understanding of the course content (Halliday, 1976; Lemke, 1990). Although a single idea can be expressed using multiple modes, i.e. chart, graph, or equation, each representation takes on its own meaning (Lemke, 2004). "The overt purpose of the teaching and learning of mathematics is the development of mathematical powers in the pupil. To put it another way, this is the development of the mathematical subject, which is achieved through employing text and engaging in mathematical conversation in which the symbolic comes to dominate" (Ernest, 2004, p.

80). Students’ language choices and the *tone* of conversation can reflect self-efficacy in mathematics.

### Self-Efficacy and Engagement

Academic engagement is measured by the degree in which students participate in learning activities. This includes cognitive engagement, behavioral engagement, and their overall abilities to connect to the material (Appleton, Christenson, & Furlong, 2008). There are several factors that contribute to student engagement in mathematics. These factors include effort, rewards for participating or consequences for not participating in an activity, level of cognitive ability, and level of persistence (Miller, Montalvo, Ravindran, & Nichols, 1996). Another factor is teacher support. When students feel that the teacher believes in them, it builds a strong learning environment that promotes engagement (Liu et al., 2018). Self-efficacy is the belief that one has in one-self in performing a task (Rowan-Kenyon, Swan, & Creager, 2012). "Student self-perceptions about this ability interact with learning or performance goals to influence the degree of involvement, willingness to attempt, and intensity of perseverance in working through challenging assignments" (Perry & Steck, 2015, p. 128). Students with higher self-efficacy, or self-confidence, may be more likely to persevere in solving math problems as well as engage in more activities in mathematics (Zeldin & Pajares, 2001).

## Gradual Release of Responsibility (GRR)

There have been many concepts that shape the way teachers present information to their students. Over the last decades and even centuries, we have seen educational views shift from learning by observation with teacher-centered models to learning by discovery with student-centered models (Harasim, 2012). Many educators argue which model is the most effective, which is why the researcher has chosen to use a model that blends the teacher-centered and student-centered pedagogy. The Gradual Release of Responsibility (GRR) is an approach that emphasizes the use of modeling during direct instruction, thinking aloud with students, and heavy group collaboration during practice. The idea is that the teacher converts the classroom from teacher-centered instruction to student-centered learning. By the end of instruction, the learner assumes most or all of the responsibility (Pylman, 2016).

During instruction, students must learn how to think using higher-ordered reasoning and logic, as well as develop the brain to problem-solve (Bernadowski, 2016). Modeling, a key component of GRR, is crucial in guiding students how to use information that they know to solve problems. To solve problems, students must be able to identify the field, develop a plan to execute, and make adjustments as necessary until a solution is found (Meyer, 1998). "The ultimate goal is learner independence" and each student must be responsible for learning (Clark, 2014, p. 29). Responsibility is handed over gradually, like taking the training wheels off of a bicycle. This is done in four distinct steps:

1. Focus Lesson: "I do it."
2. Guided Instruction: "We Do It"
3. Collaborative Learning: "You Do It Together"
4. Independent Assessment: "You Do It Alone" (Fisher & Frey, 2008).

Table 2.1 shows the four stages of GRR and the teacher activities found in each stage. The researcher used the four stages as a framework for lesson planning. It is important to note that the stages can be revisited as student data give insights into when they are ready to move to the next stage of instruction (Fisher & Frey, 2008).

**Components to the Model**

The Gradual Release of Responsibility (GRR) was first introduced to education in 1983, mainly in reading and literature classrooms (Clark, 2014). The model relies heavily on direct instruction through modeling immediately followed by guided instruction. The model then shifts to student-centered instruction through collaborative group learning. The instructor gradually breaks groups down until students are able to perform the task individually.

**Table 2.1 The Gradual Release of Responsibility**

| Stage | Teacher Activity |
| --- | --- |
| **"I Do It"**<br>**Direct Instruction**<br>**Teacher-centered instruction** | Establishes learning objectives<br>Models<br>Think-aloud models |
| **"We Do It"**<br>**Guided Practice**<br>**Semi-Teacher-centered instruction**<br>**Substantial Teacher Assistance** | Working closely with students<br>Whole-group/ small instruction<br>Checking for understanding<br>Additional modeling |

| | |
|---|---|
| **"You Do It Together"**<br>**Group Collaboration**<br>**Semi-Student-Centered Instruction**<br>**(Minimal Teacher Assistance)** | Moves from group to group<br>Facilitates learning<br>Provides support<br>Clarifies |
| **"You Do It Independently"**<br>**Independent Assessment**<br>**Student-centered instruction** | Provides feedback<br>Evaluates understanding |

Doug Fisher and Nancy Frey (2008) are two key contributors to GRR. They describe the model as having four distinct stages: (1) "I Do It;" (2) "We Do It;" (3) "You Do It Together;" and (4) "You Do It Independently." In the beginning stages of instruction the learning responsibility belongs to the instructor. However, as instruction progresses, knowledge and experience are transferred and the responsibility is handed over to the students (Duke & Pearson, 2002).

The first stage, "I Do It," consists of teacher modeling through direct instruction. Modeling coincides with behaviorists ideals as the main role for the instructor is to show the student how to perform a task and the main role for the learner is to observe the response to the stimulus (Harasim, 2012). Once the teacher has modeled the strategy, he then provides opportunities for the students to practice the strategy with their peers. The instructor serves as a guide, offering assistance as needed. Planning of the model is essential to the effectiveness of the instruction, and teachers should be aware of each phase of GRR (Ensar, 2014).

During the first stage, is important to provide a concrete example for the students to follow. This can be done through demonstrations such as a "think-aloud" (Fisher & Frey, 2008). During a think-aloud exercise, the instructor demonstrates his thinking by talking out loud as he approaches the problem. As the teacher is thinking aloud, the students are engaged in the process by observing and thinking of questions that need to be answered. The students are witnessing the approach of the expert (Ensar, 2014). The think-aloud technique works as a gateway to the mind of the teacher (Buehl, 2009). The students can see how the instructor creates his thoughts about the topic and how he begins his solution-forming process. Connections are established as the students take note of how the instructor confronts an obstacle or why he may change his point of view. All the while, the instructor leaves room for the students to also think with him during the process as they make inferences (Fisher & Frey, 2008).

The second stage of GRR, "We Do It," consists of guided instruction as the students work as a whole or in small groups. During this phase, the instructor monitors the level of understanding of each student. The instructor then decides whether to continue instruction or return to the previous stage. This is also the phase that the instructor fields questions and clears up misunderstandings. After analyzing student understanding, the instructor will either return to the previous phase or proceed to the third phase of instruction.

The third stage of the model, "You Do It Together," requires that students work in small groups to complete a task. The task should reflect the model given by the instructor and discussed in the "think-aloud" activity. One tactic is to assign specific jobs to each member of the group, and then have the members collaborate to reach a final product (Fisher & Frey, 2008).  As the instructor senses understanding among the groups, the level of support decreases gradually and the students are encouraged to work together with minimal help from the instructor. Eventually, the instructor relinquishes all responsibility on the student when they are ready for the final phase - to perform the task individually (Ensar, 2014).

The final phase of instruction, "You Do It Independently," provides the student with the opportunity to show what they have learned. In this phase the student should be able to accomplish the task without any help from the instructor or their peers (Fisher & Frey, 2008). The idea is that through gradual release the student becomes an independent learner who is able to demonstrate the model initiated in the focus lesson (Echevaria, Vogt, & Short, 2007).

**Cases Studies**

This section will analyze how the Gradual Release of Responsibility works by taking a look at two case studies. These particular case studies were chosen for two reasons: (1) to show how the Gradual Release of Responsibility looks when implemented in a classroom; (2) to identify challenges that can occur when using GRR. Solutions to the challenges will be addressed following the case studies.

**Case Study One**

In a study by Nash-Ditzel (2010), five college students of average skill-level were selected to participate in a 10-week research project. All five of the participants received reading support in high school. The goal of the study was to transform the students into self-regulating readers. The participants were selected from a community college in New Jersey. The study took place over two semesters at the college and the participants agreed to attend classes and turn in assignments regularly and on time.

While teaching the participants reading strategies, the researcher aimed to build metacognition through making personal connections to text, discussing sentence structure, and asking questions about, drawing inferences from, and summarizing text. Metacognition, is defined by Cross and Paris (1988) as “the knowledge and control children have over their own thinking and learning” (p. 131). The instructor first introduced the text and conducted a think-aloud activity with the participants. Think-aloud models are considered the "I Do It" phase in GRRM (Fisher & Frey, 2008). The instructor placed emphasis on which strategies to use while reading the text. Then, the participants worked in pairs or small groups to practice the strategies while reading text. Finally, the students were assessed individually to measure their reading skill (Nash-Ditzel, 2010).

The participants were asked on two occasions to conduct think-aloud sessions as they read. During the think-aloud, the students were asked to stop while they were reading and express what they were thinking. Think-aloud protocols are commonly used in literary classrooms to show students how to think about text while reading to improve reading comprehension (Davey, 1983).The first think-aloud protocol did not yield positive results as the readers' thoughts often strayed from the material or failed to make connections that the author intended. However, according to Nash-Ditzel (2010), after re-modeling, all five of the participants began making proper connections during the second think-aloud. All five participants also showed improvement in their reading comprehension scores from the first protocol to the second protocol.

The most important finding to the study is that all students showed some ability to choose proper strategies to link background knowledge to the text. The students progressed from what seemed like forced think-aloud protocol to voluntary think-aloud practice strategies. This may suggest that the think-aloud strategy showed the students how to reflect on their learning. Learning to reflect on one's *own* learning develops metacognitive regulation (Jaleel & Premachandran, 2016).

The researcher in this study encountered challenges, especially during the first think-aloud protocol. One challenge was that the students had difficulty making appropriate connections to the text. Nash-Ditzel (2010) notes that the students quickly stray off-topic and seem to have difficulty making the connection that the author intends for them to make. This may be due to the potential lack of structure a think aloud can present, especially for a novice reader. I also noticed that the instructor only initiated three phases of the process, skipping what GRRM labels as the "We Do It" phase. This may be a source for confusion if students have not had enough time to ask questions or practice with the instructor.

### Case Study Two

In a study by Young, Stokes, and Rasinski (2017), researchers collected data over a five-day period from a group of theater students. On the first day, the instructor read the script aloud to model fluent reading and to help students comprehend the text. The researcher believes that after the read-aloud, the students will have a better understanding of how to read their parts. During the read-aloud, the students are only asked to write down questions and think carefully about the text. The students are asked to raise their hand briefly if they have developed a question, but they are told to hold their questions for a later time in the instruction process. A word wall is created of important text after the read-aloud.

The next day, the students select their scripts, dividing them into small groups based on their roles. The instructor begins giving learning responsibility to the students as the groups begin reading their parts. The instructor gives feedback. On the third day, the students begin rehearsing individual roles while the teacher engages individuals in discussions about their characters.

On the fourth day, the students participate in a dress rehearsal. The students work to establish tone while reading the text. The instructor offers less guidance and allows students to correct themselves and their peers. After the dress rehearsal the students re-tell the story in their own words. This practice helps students to remember chronological events in the script. The students perform their act in front of a live audience on the last day of the study. Following the performance, the students are asked to reflect on how they performed.

A common challenge with GRR is that instructors can choose to release students before they are ready. Since the instructor read aloud only once before releasing responsibility to the students, the instructor may notice that students may struggle with grasping tone during the dress rehearsal. Another challenge may be that discussions among small groups may be unproductive, unorganized, or even chaotic (Dougherty Stahl, 2009). Also, the teacher can only be with one small group at a time to offer feedback and guidance. Some instructors may choose to stay with one group longer than another because of comprehension concerns or behavior issues. Also, groups may have one or two members dominating the discussion.

## Barriers to Implementation

As with all learning models, the instructor will face challenges in implementation. This section aims to look at past research to aid in dealing with those challenges. Upon researching GRRM, the reoccurring challenges involved structure during think-aloud protocols, transitioning between phases, and effective group work.

### Structure

One of the main challenges that threaten GRR, and many other constructivist approaches, is the perceived lack of structure. "Often teachers must overcome personal fear about their control over large class sizes and challenging teaching situations as they move from teacher-centered to student-centered instruction. These teachers, as much as students, deserve large amounts of collaboration and support" (Clark, 2014, p. 31). Students with more advanced comprehension skills tend to make connections that are more in line with the content; likewise, students with novice comprehension skills tend to activate background knowledge that is not connected to the content (Seipel, Carlson, & Clinton, 2017). This is one reason that think-aloud protocols actually help to keep structure intact.

During the think-aloud, Fisher & Frey (2008) recommend that the instructor ask the students to refrain from asking questions. Young, Stokes, and Rasinski (2017) asked their students to only briefly raise their hands if they had a question, but they did not allow them to actually ask the question. Instead, the students wrote the question down. This helps to keep structure during the think-aloud. After all, the purpose of the think-aloud is to give students an opportunity to observe the thought process of an expert (Ensar, 2014).

### Phase Transitions

Those implementing GRR may experience problems transitioning from one phase to another. Teachers often expect students to apply learning too quickly (Clark, 2014). Teachers must be experts in knowing the tenor (Maloney & Saltmarsh, 2016). It is completely acceptable to cycle back and re-teach or re-model missed concepts if students are not ready to work on their own. According to Clark (2014), this is a healthy practice in meeting the needs of students.

Transitions can also be confusing to the student at times, especially if inadequate time is spent in an earlier phase. Sometimes instructors omit phases altogether. Classroom confusion is usually a result of omitting the "We Do" phase or the "You Do It Together" phase. This means that the students have not had enough guided practice or group practice (Pearson & Gallagher, 1983). If this is the case, simply cycle back and repeat the needed phase before initiating "You Do It Independently."

## Historical Perspectives (SFL and GRR)

The following section discusses the historical perspectives and key contributors for the components to this action research study. The key individuals discussed in this section are Michael Halliday, John Dewey, and William Bagley. SFL began several decades ago by Michael Halladay and has continued to develop. John Dewey and

William Bagley had conflicting views on educational methods, but GRR combines the two ideologies. Both individuals were key contributors many years ago and their ideas are still being used in many classrooms today.

**Systemic Functional Linguistics (Halladay)**

Michael Halliday, a key contributor to SFL, was a linguist who began developing the functional model in the 1960's. He argued that "language is a resource for making meaning. As one learns the language, so too one learns its culture, its values, its social practices" (Christie, 2018, p. 142). Functional linguistics refers to the choices that people make in how they make meaning of written text, verbal conversations, or other forms of communication (Halliday, 1978).

SFL analyzes how systems in language are used together to convey meaning to the reader. "It is necessary to involve students in the analysis of language so that they can learn how language achieves communicative objectives by seeing it playing useful roles in situations where culture and context are key issues" (Montes et al., 2014, p. 106). As mentioned throughout this chapter, SFL uses *field, tenor,* and *mode* to analyze text. It is important to recognize the impact these systems play in making meaning of text. Again, *Field* refers to the subject matter in the text or the situation at hand. *Tenor* "refers to who is taking part, to the nature of the participants, [and] their statuses and roles" (Halliday & Hasan, 1989). *Mode* refers to how the text is delivered (Montes et al., 2014).

**Learner-Centered Instruction (Dewey)**

Progressivism, or learner-centered ideologies, can be traced all the way back to the late 1500’s in Europe. Many have contributed to this ideology over the years (Schiro, 2013). Among those contributors was John Dewey (Keskin, 2014). John Dewey (1938) believed that schools were institutions where children could be stimulated both intellectually and socially (Dewey, 2013). Progressivism is based on the principle that curriculum should be prepared with the students' interest in mind (Keskin, 2014). Dewey (1938) believed that experiences were what drove learning. "An experience occurred when someone tried to do something to the environment" (Watras, 2012, p. 162). Dewey's (1938) views opposed those of traditional education where students were expected to remain docile while reading textbooks and memorizing facts the teacher had deemed important. Instead, he viewed education as an opportunity to let children experience what they were learning.

Another principle of progressive education is that students can use their experience in daily life to gain knowledge permanently. "These experiences are purposeful, and should be rich and 'educative' though they can come from any number of sources within the classroom and community, not just textbooks or traditional content areas" (Hogan & Bruce, 2013, p. 2). Dewey believes that the school and classroom should be democratic and students should have freedom in what they learn (Keskin, 2014). This requires that teachers know their students so that they can use their interests in developing the curriculum (Tyler, 2013). Discovery-based learning is at the forefront of progressivism. However, some would argue that it only works if organized efficiently (Krahenbuhl, 2016).

**Teacher-Centered Instruction (Bagley)**

Essentialists, or traditionalists, led by the philosophies of William Bagley (1938) would argue that students are novices and their "discoveries" need to be guided by experts in the field. "Acting in this capacity, they are likely to 'discover' many things.

Unfortunately, many of these ‘discoveries’ will simply be untrue, incorrect, and unless addressed quickly, will become not merely obstacles to avoid, but misconceptions stored in long-term memory that are even more difficult to rectify” (Krahenbuhl, 2016, p. 101).

William Bagley (1938) endorsed the essentialist movement in 1938, complaining that Dewey’s (1938) progressive schools were the reason that the United States was falling behind in comparison with other countries with respect to education (Watras, 2012). Traditionalists who follow Bagley (1938) believe that the major disciplines shape the curriculum; the students are thought of as minds that can possibly contribute further to the disciplines (Schiro, 2013). Unfortunately, many look at advocates of Bagley (1938) and teacher-centered pedagogy as being anti-student (Krahenbuhl, 2016). This has caused curriculum “wars” among the disciplines that educators are still fighting to this day (Schiro, 2013).

## Theoretical Perspectives

This section will focus on two theoretical perspectives that are outstanding in regards to GRR: the *progressivist/ constructivist* view, and the *essentialist/ behaviorist* view. These theories and ideologies are linked to the key individuals mentioned in the previous section. The progressivist view emphasizes the importance of the child and determining the students’ interests so that they can be used in instruction. The essentialist view emphasizes the academic disciplines and how they have accumulated over the centuries into what defines our universe and how it operates (Tyler, 2013). In this section I will discuss these views in detail and then show how it influences GRR methods and instruction.

## Theoretical Framework (GRR)

The following paragraphs describe the theoretical framework of the Gradual Release of Responsibility Model. First, we will look at behaviorism, constructivism, progressivism, and essentialism, as GRR is a mixture of these learning theories.

## Behaviorism

The behaviorist approach was developed in the early 1800's and was one of the first learning theories. Behaviorists defined learning as the transfer of knowledge from the expert teacher to the student (Harasim, 2012). Learning is responding appropriately to a stimulus (Ertmer & Newby, 2013). The instructor is to determine cues and organize practice as to lead students toward a desired response. The instructor also arranges environmental conditions so that students can respond in the appropriate manner (Gropper, 1987). Developed during a time when the scientific method was still relatively new, "behaviorism provided a theory of learning that was empirical, observable and measurable" (Harasim, 2012, p. 10).

One advantage of the Behaviorist Learning Theory is that learners work toward a clear, organized goal (Mergel, 1998). The approach is highly effective in demonstrating a task and using lower-leveled thinking skills such as memorizing (Ertmer & Newby, 2013). Disadvantages are that learners are sometimes limited only to what can be observed and some behaviors may be difficult to explain or model. The theory may also be weak in higher-leveled thinking skills (Schunk, 1991).

## Constructivism

The Constructivist Learning Theory, based on research by Jean Piaget and Lev Vygotsky, shifts the responsibility of learning from the instructor to the student

(Harasim, 2012). "Humans create meaning as opposed to acquiring it" (Ertmer & Newby, 2013, p. 55). The learner is more than a just sponge for knowledge - the learner contributes to the knowledge by being an integral part in the process (Duffy & Jonassen, 1991). The instructor's role is to show the learner how to construct knowledge and communicate it with others (Cunningham, 1991). Learners are asked to interpret rather than just recite facts (Ertmer & Newby, 2013). Learning is created through conversations and experiences (Harasim, 2012).

Advantages of the constructivist approach are that learners can apply tools and information in real-world settings (Ertmer & Newby, 2013). Students are also able to use problem-solving skills to manipulate information (Harasim, 2012). While constructivist teaching styles allow for student exploration, sometimes educators practicing this theory may lack structure in their instruction. Without structure, a task may be performed in numerous ways, making it difficult to determine the field. (Mergel, 1998).

**Progressivism**

The learner-centered ideology is based on John Dewey's (1938) constructivist theory, or progressivism. Student growth is not about the conclusions, such as the final grades or what they can remember – it is about the process the student goes through during the learning phase (Hogan & Bruce, 2013). Those practicing learner-centered ideology feel that focus should lie in developing skills in teamwork, communication, ability to solve problems, analyze situations, and question themselves (Noddings, 2013). Some of the key components to the learner-centered classroom are dialogue, sense of community, and participation (Hogan & Bruce, 2013).

In a typical class period, teachers dominate about 80% of the discussion with lectures and only about 25% of the students are likely to participate in the discussion (Elsass & Bigelow, 2016). Freire (2013) suggests that the teacher and student can learn from each other through dialogue. Dialogue is important because it gets students to be exposed to other viewpoints and, more importantly, understand other viewpoints. *Dialogue* differs from *discussion*; dialogue is not predetermined whereas discussions can be catered to lead students to a planned conclusion (Elsass & Bigelow, 2016). Dialogue "involves educators providing specific instructions, guidance, exercises and opportunities, and feedback involves facilitating meaningful interactions and exchanges with fellow students, friends, mentors, and other educators" (Ivancevich, Gilbert, & Konopaske, 2009, p. 197).

In a learner-centered environment, the instructor must let the child discover the knowledge, but getting the student to persevere through the difficult times can be a daunting task (Bruner, 2013). The learner-centered instructor creates curriculum based on the tenor. The teacher provides choices for the student to have multiple ways to explore the topic (Schiro, 2013). The students are encouraged to work together. The learner-centered teacher encourages cooperation over competition (Keskin, 2014).

**Essentialism**

The teacher-centered classroom is on the other side of the spectrum. The academic discipline is the focal point of instruction and the student is thought of as a mind (Schiro, 2013). This does not mean, however, that the teacher does not care about the student. While the constructivist approach does offer much to education in the form of pedagogy, giving free reign to the student can be unproductive if execution is inefficient. Scholar academics would argue that students are novices and lack experience to apply their skills effectively without some guidance (Krahenbuhl, 2016). These views coincide nicely with the views of William Bagley (1938). Units typically follow a structured field that students complete based on the teacher's instructions. Bruner (2013) gives an example of this format in his study involving a fifth-grade social studies class. He found that a unit should consist of six components: 1) offer information on historical background and key individuals; 2) offer questions that the students should think about during the unit; 3) give materials and assignments that the students can use to test their knowledge; 4) model exercises such as puzzles and games; 5) use multiple genres such as documentaries or other instructional videos on the topic; and 6) offer supplemental materials such as topic-related articles or other books that go into more detail on the task at hand.

## Conclusion

This chapter discussed SFL and GRR and historical perspectives that are relevant to this action research study. Chapter 3 "Methodology" will discuss the action plan and methods to collect data. Chapter 4 "Findings" will provide a description and analysis of the data obtained in this study and how it pertains to the research question. Chapter 5 "Conclusion and Suggestions for Future Research" will outline the next steps, including how the results will be shared with others, as well as ideas for improvements when replicating the research study.

# CHAPTER 3: METHODOLOGY

## Introduction (Overview)

This chapter will discuss the purpose of the study and the researcher's role in detail. The researcher will revisit the problem of practice and discuss the research design. A summary will conclude the chapter. In the following pages the researcher will highlight the types of data that are used in the study. The goal of the action research study is to address the research question:

> **RC: "What effect does systemic functional linguistics in conjunction with the gradual release of responsibility have on student self-efficacy and engagement in secondary mathematics?"**

This study focuses on the researchers' classroom and instructional practice; it is not designed to make generalizations to the population as a whole.

## Positionality Statement

It is important to acknowledge that students are of many different backgrounds and cultures. The researcher was aware not to let the researcher's own personal background and culture affect the results of the study. In other words, the researcher acknowledged that his students may be of different backgrounds and culture than his own. In order to completely evaluate the data gathered during this study, the researcher needed critical reflection. According to Howard (2003), critical reflection should examine how race, culture, and socioeconomic status affect learning and understandings. Unlike many of his students, the researcher grew up in a small rural town in the southeast United States in a middle class home. As a child, the researcher had the resources needed to be successful in school. Many of the participants in this study are considered economically disadvantaged and may not have resources at home. The students' cultures and values may also be different as 50% of the participants are of different race and ethnicity than the researcher. The students in this action research are in an Algebra I CP course and the majority of students are in their first year of high school. However, nine students are retaking the course.

As mentioned, the students come from a wide range of family backgrounds. Some of them may not have adults at home that push them or check their school work. “When parents show an interest in their children’s schoolwork, and are willing to assist them with homework, and hold them accountable for the completion of homework assignments, children are more likely to apply themselves and perform better in school” (Ndebele, 2015, p. 74). The researcher must take into account these students’ situations and know that this will affect the study. Other students may have a lower standard for what they consider a “good” grade than the researcher. Students who historically struggle in math may be satisfied with a merely a passing grade and thus be less likely to push themselves to do much better than that. This may have an even greater effect for any student in the study that may perceive to be exceeding expectations. For instance, a student who is satisfied with a passing grade, but currently holds an average grade of 75, may have decided to not participate in an assignment or may have elected to not prepare for a major assessment due to the fact that they are already above their expectations. This has an effect on the study. Although these aspects of the study have likely kept the study from reaching its full potential, the researcher must be aware of the biases he may hold based on differences in expectations and personal background.

## Planning for Action Research

The researcher developed a plan that incorporates tenor and supports students' self-regulated learning. Assignments were carefully designed to supplement face-to-face instruction and give students an opportunity to use math language on a daily basis in efforts to build student self-efficacy in persevering and completing assignments. In planning for the action research process the researcher has taken three things into consideration:

1. The research has chosen a problem of practice in which he has a personal interest.
2. The researcher feels the problem of practice is important.
3. The researcher is aware of the amount of time the research will take place (Mertler, 2014).

## Action Research Design

The research will take place for six weeks, which will equate to two full Algebra I CP units. Since the research question seeks to measure student engagement and self-efficacy, the researcher decided a mixed-methods research design would best fit the action research. Furthermore, the researcher determined that math engagement can best be measured with student participation and behavior during the data collection period. The researcher conducted surveys and interviews to gauge students' thoughts on the instructional techniques throughout the implementation of the design. The researcher is a full participant in the study. The researcher instructed the students, created the assignments, implemented GRR methods, recorded field notes, collected and interpreted the data, and reflected upon the results.

The researcher recorded daily field notes, held interviews with students, and conducted two student surveys - once at the beginning of the data collection period and once at the end. The results are interpreted with charts and graphs to analyze the data. The charts and graphs used include tables and bar graphs. The researcher will reflect on the data and a rationale will be provided for the next steps in instruction in Chapter 5: "Conclusion and Suggestions for Future Research."

There are 52 student-participants from two Algebra I CP classes. The researcher feels this sample size is large enough and there is a mixture of race and ethnicity among the participants. There are 30 male and 22 female participants. The students are of the following ethnicities: 26 Caucasian, 19 African-American, 2 Mixed-Raced, 2 Hispanic, 1 Asian, and 2 Pacific Islander. The age range of the participants is between 14-19 years of age. Thirty students are fourteen years of age; fourteen students are fifteen years of age; four students are sixteen years of age; four students are older than sixteen years of age. One student is considered ELL. Special accommodations are required for this student. Due to the specific challenges this presents, the researcher has chosen to not include this student in the results of the study.

Academic High School (AHS) (pseudonym) is located in the low country of South Carolina. AHS has 1,711 students and consists of grades 9-12. Of these students 44% are Caucasian, 47% African American, 5% Hispanic, and 2% mixed race, 1% Asian, and 1% Hawaiian Native / Pacific Islander. The socioeconomic makeup of this school is 49% economically disadvantaged with around 42% receiving free lunch and about 8%

receiving reduced lunch prices. According to the principal, the student to teacher ratio is 33:1 in core subject area classrooms. AHS reports that 70.7% of students passed the Algebra 1 End-of-Course Examination in 2017. This is slightly below state average. Also, 71% of students received a rating of Platinum, Gold, or Silver in Applied Mathematics on the ACT WorkKeys assessment. However, students scored below average in reading. This is slightly above the state average. Students at AHS scored below state average in all sections of the ACT - Composite, English, Reading, Mathematics, Science, and Writing - in 2017. Fifty-seven percent of students who attend AHS live in poverty.

**Ethical Considerations**

The goal of action research is to improve instruction for all students (Dana & Yendol-Hoppey, 2014). When conducting research of any kind it is important to make sure that it is ethical. Maintaining integrity is critical to the success of the study (Daniel, 2016). The researcher followed ethical protocol to maintain integrity throughout the study. First, permission was obtained from the principal to conduct research at the school. Then, the researcher obtained approval of the IRB to conduct the research. The IRB board ruled this research to be "exempt." Finally, a district research approval packet was completed and submitted to gain permission from the school district. The purpose of the research, along with the plans for the results, was shared with all stakeholders. This includes teachers and staff members who can use the study results to make future teaching decisions. More of this will be discussed in Chapter 5: "Conclusion and Suggestions for Future Research."

District guidelines for conducting research were followed as well as the regulations of the Federal Educational Rights and Privacy Act (FERPA). This included keeping all information regarding students' test scores private and confidential. It is important that confidentiality is used when reporting test scores and the participants were assured that their scores would be kept private. Students' names were not disclosed. Before the study began, permission was received from the participants and their parents to conduct the study. An informed consent form that explains the nature of the study was issued and all participation was made voluntary. The informed consent form disclosed all information regarding the intentions of conducting the study and there was no deception of any kind used in gaining permission from the parents or student volunteers. Information disclosed included in the consent form was the time period in which the research will take place (six weeks), the sample size, and the outline of the research.

## Developing an Action Research Plan

The researcher has noticed that students' perceptions of what is important in mathematics is mostly quantitative. However, much of mathematics is actually qualitative or non-computational. Learning symbols, notation, and vocabulary is essential to the success of the students in Algebra I. This is the case for many courses in mathematics, so this study is not to be limited to Algebra I. When students choose to focus solely on the quantitative aspects of the course, many do not understand how to interpret results using multiple modes or even the field. In this course, the interpretations are very important as the state standards and objectives require the students to interpret the results. Many students focus on the mathematical "solution" as the answer and think they have completed the problem, but in fact, they have much more to accomplish. The goal is to get them to understand the solution to a problem, not just be able to calculate it. It is the process, not the end-result that is important (Dewey, 1938).

The purpose of this study is to incorporate SFL and GRR into Algebra I to see how it affects student self-efficacy and engagement. The intentions are to address the problem of practice by analyzing mathematical statements and student language choices when they communicate mathematics, and using GRR as an instructional means to give the students more opportunities to engage in mathematical discussions with their peers. The study aims to create more avenues for students to actively communicate math lexis – language that otherwise would not be used in everyday conversations with peers. The researcher wants to examine the extent the role that SFL plays in students' abilities to express mathematics internally, interpersonally, and textually. In this study, students received traditional direct instruction, but were also given opportunities to work and communicate with peers in student-centered learning activities. “Reading literacy is generally conceived as the ability to understand and use written documents containing both verbal and pictorial information, for example texts, pictures, charts, and tables while math literacy is the ability to mathematize real-world situations and appropriately use mathematics in problem contexts” (Korpershoek et al., 2014, p. 1014). To focus on this concept, the instructor initiated daily think-aloud models to show students how to analyze problems in mathematics. Then, students were asked to present solutions to problems in more than one mode. Choices include procedures, graphs, written paragraphs, charts, or tables. On three separate occasions, students were asked to write text about a topic in mathematics. The researcher used Appendix N to accomplish this task. The students were limited to using topics covered in Algebra 1 CP. The length of the responses, student use of math language, as well as their level of understanding were analyzed for student self-efficacy.

Mentioned in Chapter One: "Introduction," an anticipated barrier is that some of the students were at a disadvantage because of low reading levels. Scaffolding was very important in assisting students in completing the tasks assigned. “Cognitive scaffolding allows learners to reach places that they would otherwise be unable to reach” (Frederick, Courtney, & Caniglia, 2014, p. 22). The researcher gradually introduced more scaffolding in writing assignments as the collection period progressed. The instructor also used the think aloud models to scaffold critical thinking skills.  A word wall was created throughout the study. The researcher added relevant text to the wall as the students encountered it in the study. The word wall included vocabulary, symbols, and diagrams to help aid the students throughout the study.

**Quantitative Design vs. Qualitative Design**

Qualitative research designs allow the researcher to use methods such as interviews, surveys, and journals to gather data (Dana & Yendol-Hoppey, 2014). This is considered an advantage in action research because data is gathered from the study participants in their natural setting. Quantitative research uses statistical data for saving time and resources and allows the use of control and treatment groups which gives clear objectives and makes the study easier to replicate (Daniel, 2016). A disadvantage to qualitative research is that it often limits findings to a particular group and, therefore, is difficult to generalize. Not quantifying the data makes it difficult to explain the findings, “Since the approach is characterized by feelings and personal reports, it is believed that the approach cannot give reliable and consistent data when compared to using quantifiable figures" (Daniel, 2016, p. 93). By combining qualitative and quantitative research, the researcher was able to use quantitative measures with the participant's feelings and opinions to give the research more validity. However, there were some potential issues that the researcher faced by using both quantitative and qualitative research. First of all, time was a factor in the study. The researcher collected data for six weeks. While collecting qualitative data, the researcher made sure the data were reliable. He used daily field notes and assigned numerical values to engagement levels. He also assigned numerical values to represent student understanding when analyzing student work. The researcher included student responses to surveys as they were important to show how scores were assigned. Student work was also included to show implementation of SFL and students' levels of self-efficacy and engagement.

## Plan for Collecting Data

As plans were made for the action research study, choices were made in collecting data and effectively analyzing the data from the study. There were two types of statistics used in action research – descriptive statistics and inferential statistics. Both quantitative and qualitative data was used to measure engagement and self-efficacy. This data was in the form of student work, surveys, interviews, and participation levels. One of the challenges to the action research was determining how to interpret the data. Questions like "what has the researcher learned about the children?" need to be asked in order to piece the data together and make it meaningful (Dana & Yendol-Hoppey, 2014). Descriptive statistics were important in determining the students' level of engagement. Median and average grades were useful descriptive statistics. Likert scales were also useful in gathering opinions from the students on the methods used in class. The researcher understands that inferential statistics such as a t-tests can be used to determine if there was any statistical significance to the action research study that can be applied to the population (Mertler, 2014). However, since this study was not designed to generalize to the population, t-tests will not be included in the study. As a result, the researcher only used descriptive statistics. “Descriptives and related graphical representations of our results help us determine whether we have the distribution of scores that we expected” (Carr, 2008, p. 45). Once the data was gathered, the data was analyzed in many ways. Creating charts and visual representations not only helped to analyze and interpret, but also played an important role in presenting the data. These charts will be discussed at length in Chapter Four: “Findings”.

Formative and summative assessments were a part of the data collection process. Formative assessments were mainly in the form of observation and questioning during the face-to-face instruction as well as records of student conversations. The researcher recorded formative data in daily field notes. This data was useful in the reflection process discussed in Chapter 5: “Conclusions and Suggestions for Future Research”. The researcher also considered the student work to be formative assessments.

Summative assessments were also used to collect data. There were three group summative assessments and four individual summative assessments in the study. Since the researcher is interested in student engagement, the scores of summative assessments were only discussed to show improvements in student self-efficacy and engagement. The researcher recorded the participation levels and group effectiveness during the group assignments and conducted interviews and surveys to gather information on individual assessments.

## Plan for Analyzing Data

In order to effectively analyze the data, two types of data analysis were utilized – formative data analysis and summative data analysis. Formative data analysis was ongoing throughout the action research study and occurred as the data was collected. Summative data analysis occurred after the data was collected (Dana & Yendol-Hoppey, 2014). In order to analyze the formative data, the researcher will reflect on field notes made during the data collection period. Since the study is interested in student math engagement and follows a mixed-methods action research design, measures of central tendency such as mean and median played a large part in the analysis of the data. The researcher decided which central tendencies were most appropriate as outliers had an effect on the outcomes. In order to better analyze the data, the researcher has included graphical organizers and visuals such as frequency tables, charts, and bar graphs. Upon creating and analyzing each of these statistical representations, the researcher reflected upon which representations make the most sense of the data. It was useful to show different representations in different situations, such as trends in those that do or do not participate in the math journals (only focusing on those students that agree to participate in the study).

During the data collection period, records were kept of assignments the students were asked to complete. These assignments can be found in the Appendix in chronological order, with exception to the math journals found in Appendix N as they were assigned throughout the study. Engagement levels of students were recorded daily. This data was tracked throughout the action research study so that trends, such as improvement in understanding, could be analyzed.

## Plan for Developing an Action Plan

Teachers must develop practices that are meaningful to their students and teacher reflection is a key component to evaluating performance. “Reflection-in-action is an ongoing process that is predicated on continually thinking about one’s actions and then modifying them accordingly” (Howard, 2003, p. 200). Without observation and critical thought or reflection, then actions cannot be sustained over time or truly successful (Guillaumier, 2016). To help with the reflection process, the researcher asked:

1. What was learned from the study?
2. Was the researcher able to answer the research question?
3. How did the research design work with addressing the research question?
4. What can the researcher do better next time?

No action research study is perfect and effective teachers acknowledge their errors and make improvements (Howard, 2003). Reflection encourages teachers to make educational decisions based on research as opposed to making decisions based on impulse (Farrell & Jacobs, 2016). Once the data was gathered, and upon analyzing the data, possible factors that may have affected the data were considered. These possible factors include, but are not limited to, socio-economic factors and student skill level in reading and writing. Incorporating the findings with existing research helped to provide a stronger base of understanding to use for the next cycle of research. It also helps to identify professional development needs (Mertler, 2014). “(Dewey) viewed reflection as a special form of problem solving steeped in scaffolding of experiences and events that should be viewed as an active and deliberate cognitive process” (Howard, 2003, p. 197).

## Summary and Conclusion

The researcher has noticed that a common misconception among high school algebra students is that math is completely quantitative in nature. Students associate math with numbers, and vocabulary and language with courses such as English or History, and this sometimes leads students to believe that learning math lexis is not necessary to actually do the math. This type of thinking can cause learners to only focus on the calculations and not give quality interpretations. Research in Chapter 2: "Literature Review" has shown that the language of math is very important to understand in order to grasp the concepts in secondary math classes (De Oliveira & Cheng, 2011). It does little good if a student has found answers, but cannot explain what they mean. The researcher designed a mixed-methods study that focuses on student understanding and engagement. The researcher has also researched and developed an action plan which implements SFL and GRR strategies in teaching mathematics.

Data was collected for six weeks and the results carefully recorded and analyzed in many ways using charts, tables, and graphs. Chapter 4: "Findings" will present the results of the study and discuss the research question in even more detail. In Chapter 5: "Conclusions and Suggestions for Future Research," the researcher will reflect on the results and discuss plans for future research, including how the researcher will share with others in hopes that other educators can improve the instruction in their classroom.

# CHAPTER 4: FINDINGS

## Introduction

The purpose of this research study is to evaluate student self-efficacy and engagement in the teacher-researcher's Algebra I students. The PoP investigated is the lack of student self-efficacy in communicating mathematics. The researcher has implemented a combination of SFL and GRR instructional approaches in efforts to improve students' abilities to understand and communicate math language. The study aims to answer the research question: **"What effect does systemic functional linguistics in conjunction with the gradual release of responsibility have on student self-efficacy and engagement in secondary mathematics?"**

The teacher implemented a mixed-methods action research design. Qualitative data includes student surveys and interviews as well as researcher field notes. Quantitative data includes Likert scales used to measure student feelings and self-efficacy. The researcher modeled to students how to analyze math problems using an SFL approach. The goal was to show students how to make meaning of math language so that they may make improvements in their abilities to communicate math both verbally and in writing. Student language choices, the use of math language, syntax and sentence structure were all examined to determine the level of understanding and math lexis efficacy of each student. The researcher has included teacher models as well as student work as examples to show how the SFL approach was implemented.

The researcher followed the GRR instructional model to deliver material. Students were given models and allowed time to practice with the teacher-researcher before asked to complete assignments in small groups. Finally, the students were assessed individually. Engagement levels were monitored and recorded. Test scores were also included to show understanding. Students were surveyed to gather data on the effectiveness of the instructional techniques.

The study took place over six weeks and covered two units in Algebra I. The curriculum units of study in this action research are solving linear equations in one variable and graphing linear equations in two variables. The student learning objectives included solving one-step and multi-step linear equations, proportions, literal equations, using slope and rate of change, and graphing and writing linear equations using intercepts and/ or slope and coordinates. Students were required to perform procedures, use multiple representations, and interpret results. As mentioned, an SFL approach highlighted the use of math language in these units as a means to make meaning of the math and increase student self-efficacy and engagement.

## Findings

In this section, the researcher will display the data collected in the six-week instructional period. The collection period consisted of two major units of study: Solving Linear Equations and Graphing Linear Equations. The researcher has organized the data into several categories: a pre-study survey, math journals, teacher models, guided practice, group assessments, independent assessments, and a post-study survey.

**Pre-Study Survey**

The teacher-researcher collected data through a variety of surveys, interviews, field notes, student work and writing samples. To gather data of student thoughts on self-efficacy, group work, and math language, the researcher conducted a survey prior to collecting data. Table 4.1 shows the results. The categories were weighted so that the researcher could calculate a weighted mean, which was used to quantify the data and make comparisons later in the study.

The researcher noticed that prior to the data-collection period the students exhibit some confidence in their ability to do mathematics. The students also fairly agree that math involves some amount of reading skill. However, the majority of students do not enjoy doing word problems and have not placed a major emphasis on learning math vocabulary. A majority of students also display low confidence in their ability to learn math symbols and vocabulary.

Nearly 73% of students in this study feel that observing a teacher model helps them to learn mathematics. Another 20% remained neutral on this topic. Therefore, only 7% percent of students feel that they do not benefit from teacher models in mathematics. Similar results were found when asking students about their thoughts on group assignments. Over 70% of students expressed that group assessments helped to build their confidence before an individual assessment. Higher weighted means ($\mu>3$) indicate that students are in more agreement with the statement. Lower weighted means ($\mu<3$) indicate that students disagree with the statement.

**Table 4.1 Pre-Data Collection Survey**

| Question | Strongly Disagree (1) | Somewhat Disagree (2) | Neutral (3) | Somewhat Agree (4) | Highly Disagree (5) | Weighted Mean (μ) |
|---|---|---|---|---|---|---|
| **I believe that I am good at math.** | 11.36% | 6.82% | 34.09% | 27.27% | 20.45% | 3.39 |
| **To do math, you must be a good reader.** | 4.44% | 13.33% | 24.44% | 28.89% | 28.89% | 3.64 |
| **I enjoy doing word problems in math.** | 31.33% | 22.22% | 31.11% | 11.11% | 4.44% | 2.36 |
| **In the past, I have put major emphasis on learning math. vocabulary** | 8.89% | 20% | 51.11% | 13.33% | 6.67% | 2.89 |
| **I have difficulty with** | 11.11% | 35.56% | 33.33% | 11.11% | 8.89% | 2.71 |

| | | | | | | |
|---|---|---|---|---|---|---|
| **learning math symbols and terminology.** | | | | | | |
| **I learn from watching a teacher model math problems.** | 2.27% | 4.55% | 20.45% | 29.55% | 43.18% | 4.07 |
| **I enjoy learning in groups.** | 6.67% | 4.44% | 15.56% | 35.56% | 37.78% | 3.93 |
| **Group assessments help me to build confidence before a test.** | 4.44% | 8.89% | 15.56% | 31.11% | 40% | 3.93 |

**Student Math Journals**

Students were given a list of math language terms, which were covered in a solving linear equations unit. As students were exposed to math language, the texts were posted on a word wall in the classroom. Texts include relevant vocabulary terms, which were often paired with a visual representation of the term. Math journals were conducted to allow students to communicate math freely through writing. These journals can be found in Appendix P. The students were encouraged to refer to the word wall when writing their journal entries.

The researcher noted that students were reluctant to participate in the first two math journals. The researcher included data from three journal entries during the study. Table 4.2 shows the journal grading rubric used to evaluate student responses. A student who scores level zero would make statements about the math but would not attempt to explain what they know. A student who score level one would attempt to make meaning, but has not quite grasped the concept. A student who scores level two will show adequate meaning for some concepts, but make assumptions with other concepts. For example,

> S: "To do the slope, you plot the coordinate and count rise over run. The rise is the number of units up or down, and the run is the number of units left or right."

**Table 4.2 Student Journal Grading Rubric**

| Level of Understanding | Criteria |
|---|---|
| **0** | No mathematical explanation |
| **1** | Student shows minimal mathematical understanding; lacks in depth knowledge or fails to explicitly make meaning; minor mistakes. |
| **2** | Student shows adequate mathematical understanding; shows explicit meaning, but also makes some assumptions |
| **3** | Student shows extensive mathematical understanding; makes no assumptions |

The assumption made in this case is that the reader understands how to plot a coordinate. Also, the student does not clearly explain how to determine when to count up, down, left, or right. Also, the researcher asked the student “What do you do when you are done counting rise over run?” A student who scores level three will not make these assumptions and explicitly communicate the meaning they are making in the mathematics.

The first journal entry assignment was very broad. The students were asked to write about a math topic of their choice from the first two weeks of the study. Forty-four students participated in the first journal activity. Thirty-eight students wrote at least three sentences. The mean number of sentences written by the students was 3.64 sentences. On average, students used approximately 7.25 math language text in their explanations. The mean level of understanding shown in the student writing was 0.77.

Responses in Table 4.3 show student responses evaluated at different levels of understanding. The teacher-researcher used the students’ choice of language to evaluate the level of understanding. Twenty students were evaluated as zero level of understanding; eighteen students showed a level of understanding rating: 1; six students showed a level of understanding rating: 2; and no students showed a level of understanding rating: 3.

The second journal assignment was designed as a daily journal assignment conducted for one week. This journal assignment was given during week four in the study. Students were given time at the end of the class period to write about the topics they learned that day. The researcher calculated the mean number of sentences written, mean math language usage, and mean level of understanding for each student that participated in the journal. The students were asked to write at least three sentences per day in the journal. Only thirty-five students participated in the activity. The students averaged 2.38 sentences per journal entry with an average of 5.04 math language references in each entry. The student mean level of understanding was 1.06. Many students merely either mentioned what they learned without developing any meaningful mathematical context to their response or they discussed an opinion such as "I like standard form because it is easy to use."

In the third journal entry, the researcher gave the participants three options. This journal entry occurred at the end of the study. Students had been given feedback on the previous two journals prior to writing the third. Much emphasis had been placed on students analyzing math language at the time of this journal. Forty-five students participated in this activity. The students wrote an average of 4.33 sentences and math language usage increased to 12.06 words per entry. The level of understanding also increased to a mean score of 1.86. Table 4.3 shows a comparison of the data in the three journal entries.

**Table 4.3 Student Math Journal Data**

| | Length (Sentences) | Math Language Usage (Words per entry) | Score Rating (0-3) |
|---|---|---|---|
| Journal 1 (Week 2) | 3.64 | 7.25 | 0.77 |
| Journal 2 (Week 4) | 2.38 | 5.04 | 1.06 |
| Journal 3 (Week 6) | 4.33 | 12.07 | 1.87 |

**Table 4.4 Student Journal Responses**

| Level | Journal 1 | Journal 2 | Journal 3 |
|---|---|---|---|
| 0 | "I **know** a constant is a variable that never changes." "A whole number means itself and nothing else. Nothing else can go through it. It cannot have holes." | "I learned about linear graphs and domain and range." "Standard form is the easiest way to write a linear equation." | "To graph this equation (1,3), you will use the x-int=1 and the y-int=3 and plot these points." "To graph $4x + 6y = 24$, you must first divide each intercept by 24 to get the point you need to plot." |
| 1 | "Real numbers is a way to classify numbers so that we can use them to solve operations." "Combining like terms is when you add or subtract all the terms in an equation. For | "The formula is $y = mx + b$." "If the x and y are on the same side of the equal sign, it is in standard form." "The slope is found by doing $\frac{rise}{run}$ to graph the points." | "Using the slope should tell you which way the line will form." "Convert to slope-intercept form, then plot." |

| | | | |
|---|---|---|---|
| | example, $2x - 4x - 1$ would be $6x - 1$." "The first quadrant is $(+,+)$, $2(-,+)$, $3(-,-)$, $4(+,-)$." | | |
| 2 | "A one-step equation with a fraction is supposed to be done a certain way. $Example$: $\frac{1}{5}x = 250$. Multiply by 5 on both sides." | "X-intercept is the number on the x-axis and y-intercept is the number on the y-axis where the line is." | "You have to make the slope -2 a fraction since there is an invisible 1 below, so make it $\frac{-2}{1}$ and you can do $\frac{rise}{run}$." "To be parallel is to be the exact same slope $(\frac{1}{2})$ and never cross. So the slope has to be $\frac{1}{2}$." |

| | | | |
|---|---|---|---|
| **3** | (No students received a rating of 3 for this assignment.) | "I learned how to do slope. $m = \frac{y_2 - y_1}{x_2 - x_1}$ Ex. (5,-10) and (3,20). $\frac{20-(-10)}{3-5} = \frac{30}{-2} = -15$." "The standard form of a linear equation is $Ax + By = C$. The x-intercept is found by solving $Ax = C$, for *x*. The y-intercept is found by solving $By = C$, for *y*." | "To plot (1,3) you would start at the origin and go right 1 and up 3 to plot the first point." "After plotting your point, go down two and right one since your slope is NEGATIVE 2!" |

The following are examples of student work that shows the level of student self-efficacy. Tone was important in analyzing self-efficacy. In Figure 4.1, the student used steps to explain how to convert a linear equation into slope-intercept form. She finished her explanation with the phrase, “There you go!” The student’s exclamation indicates that she felt confident in her work. The student in Figure 4.2 used the word “boom” to wrap up her journal. This also indicates a high level of confidence in the student.

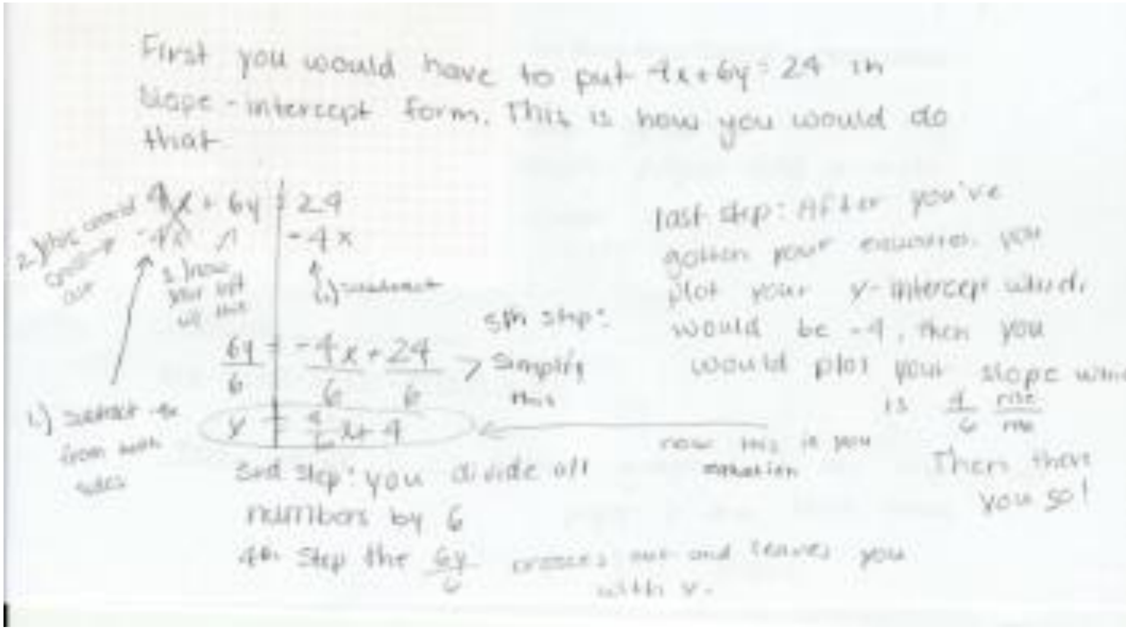
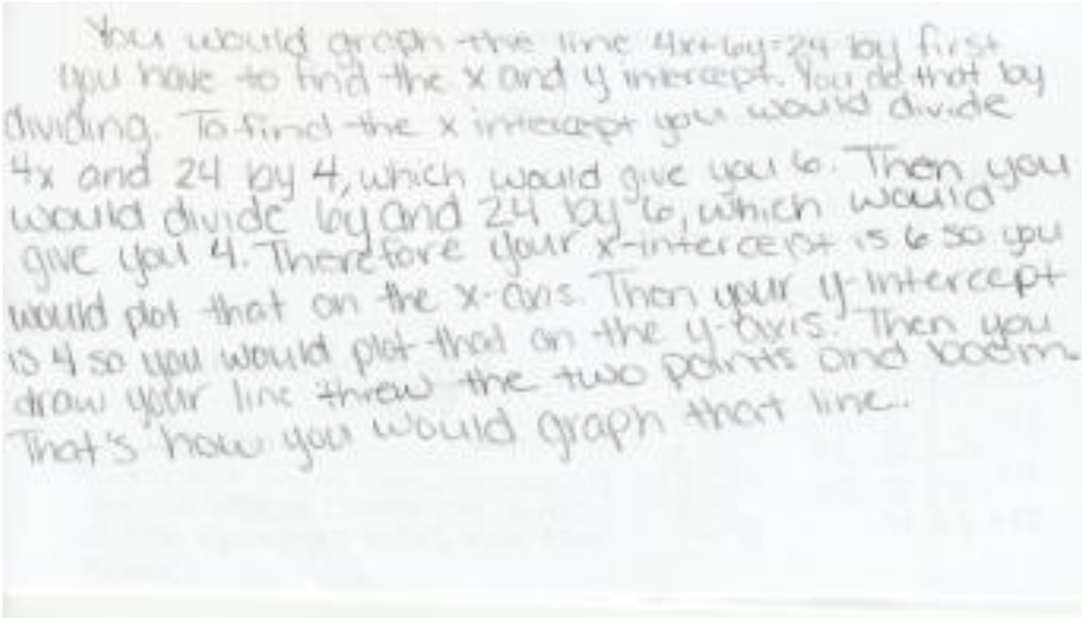



**Figure 4.1 Student Journal Entry A**

**Teacher Models**

The researcher began delivering new material with a think aloud model. During the think aloud model, the researcher talked the students through the problem, circling math language and assigning meaning to the text. The researcher used this opportunity to show the learners how an expert thinks through and perseveres in solving math problems. Figure 4.3 is an example of a think aloud activity during unit two. The researcher often referred to the word wall when encountering formulas and math vocabulary. Students were encouraged to perform the same analytical techniques in their work during guided practice.

During guided practice, the researcher took detailed field notes to record the interactions he was having with the participants and the interactions the participants were having with each other. The following comments are examples of what the researcher noted during guided practice:



You would graph the line 4x+6y=24 by first you have to find the x and y intercept. You do that by dividing. To find the x intercept you would divide 4x and 24 by 4, which would give you 6. Then you would divide 6y and 24 by 6, which would give you 4. Therefore your x-intercept is 6 so you would plot that on the x-axis. Then your y-intercept is 4 so you would plot that on the y-axis. Then you draw your line threw the two points and boom. That's how you would graph that line.

**Figure 4.2 Student Journal Entry B**

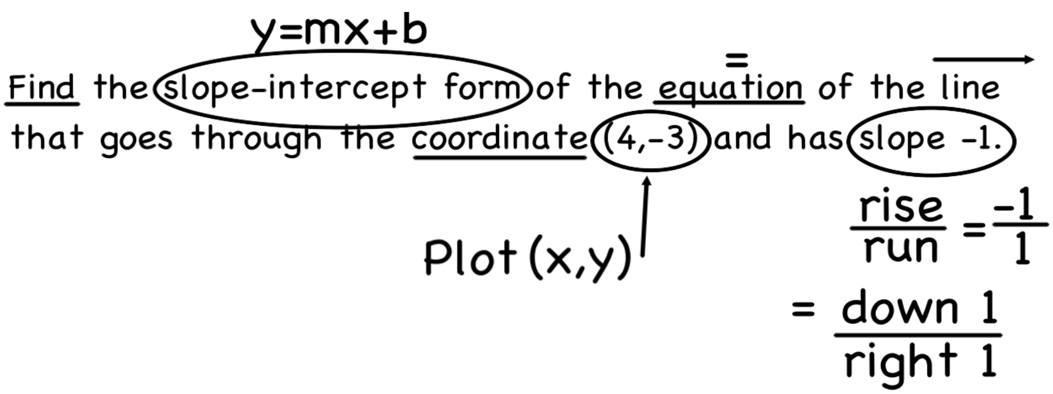



**Figure 4.3 Think Aloud Model**

S1: "So, multiply by the magic number (least common multiple), right?"

S2: "Student A is not communicating with us."

S3: "I'm lost."

S4: "So, I was thinking, $3x = 7$ is ϕ, right?"

S5: "It's a circle with a line through it (ϕ)."

S4: "What does that mean?"

S5: "No solution."

S6: "Whatever you have to do this side, you have to do to that side."

The student comments show the student use of math language during conversations. S1 mentions the “magic number.” The researcher noticed the reluctance of students to work with fractions, so he taught them how to multiply the equation by the least common multiple to change fractions into whole numbers. The students created a name for the least common multiple (LCM). They called the LCM the “magic number” because it is the tool that could use to “make fractions disappear.”

Comments like those made by S2 and S3 were cues that students were experiencing low levels of engagement or self-efficacy. The researcher made note of these concerns on field notes and was sure to address these concerns immediately by sitting with the group to scaffold the conversation or by giving a student one-on-one attention during guided practice.

The conversation between S4 and S5 shows students communicating meaning of mathematical symbols. S4 indicated that she had put some thought into what it meant for an equation to have “no solution.” She also knew the symbol for “no solution.” But, she ended her statement with a question (right?). This indicated that her self-efficacy was low, and in fact, she was incorrect as the equation did have a solution. Although the researcher was in the guided practice phase, he reverted back to the modeling phase to show the student what an equation with no solution would look like.

Figure 4.4 shows the thought process that a student went through to set up an equation written out in natural language. The student did the problem as the researcher circled text to help translate for the student. The student made meaning of the circled text as the researcher wrote on her paper. Then, the student solved the problem on her own. The researcher made note to the student that “less than” in math would cause the expression to be written in reverse order than it is written out in English text, just as discussed in Chapter 2: "Literature Review."

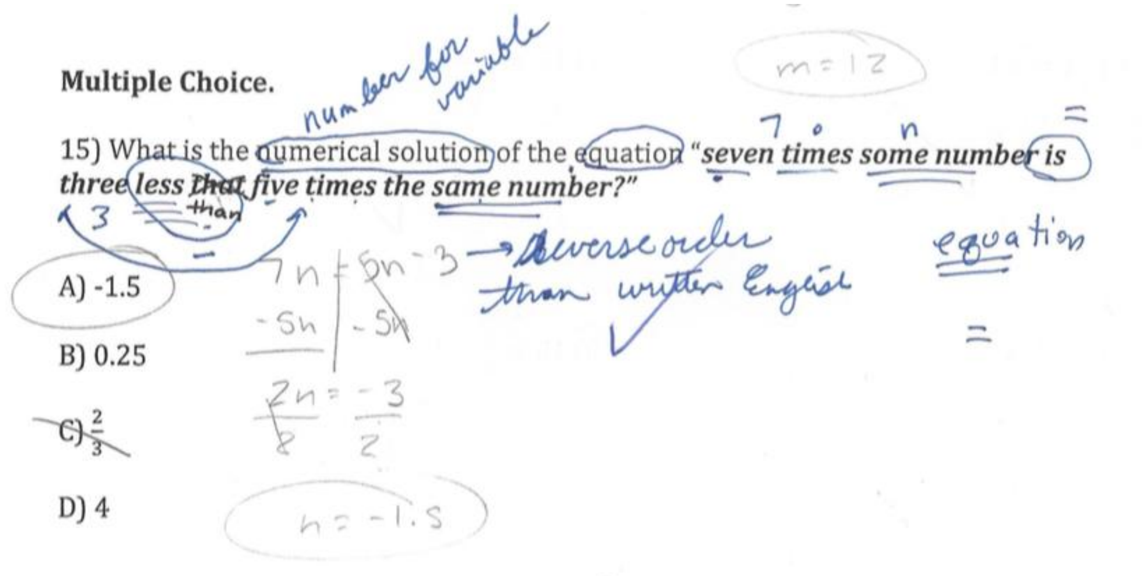



**Figure 4.4 Student Interview**

**Group Assessments**

Before each individual assessment, the researcher allowed the participants to work in groups to complete a task or assignment. He called the assessments “Group Quizzes.” The researcher recorded field notes during guided practice and group assessments. Figure 4.5 is an example of the daily field notes created by the researcher. The researcher used a daily seating chart to mark observations of student engagement and/or off-task behaviors. The researcher also made note of student conversations as he visited among groups. The field notes were a major instrument used to assess engagement in the classroom.

Multiple guided practice activities and one group assessment was given for each unit. Guided practice activities consisted of both applications and procedural questions and required students to use multiple representations. During the group assessment, each group was given one note card to represent a question they could ask the teacher. This technique increased student interaction among groups. The note card encouraged the students to discuss questions and talk about the math before reverting to the teacher-researcher for help. Each assessment was scored individually to allow students to ultimately make their own decisions.

The researcher noted that many students were reluctant to work together in the beginning, but as time passed, more interpersonal connections happened between students. During the first group assignment, students were heard saying, "An expression is how you express something - you do not use an equal sign. An equation you are trying to solve." The following application problem was given and a group used their one question to the researcher:

Uber company charges $2.10 plus $0.80 per mile. Pierce paid a fare of $11.70. **Write and solve an equation** to find the number of miles he traveled.

S: "$0.80m + 2.10$. Does that work?"

TR: "What does an equation need?"

S: "An equal sign."

TR: "So what do you set it equal to? Eighty cents per mile plus two dollars and ten cents equals what?"

S: "The total. $0.80m + 2.10 = 11.70$."

Another group asked about a procedure: $2x - 2 = 4x + 6$

S: "Do you subtract six on one side *and* add two on the other?"

TR: "What does an equal sign mean in math?"

S: "The same."

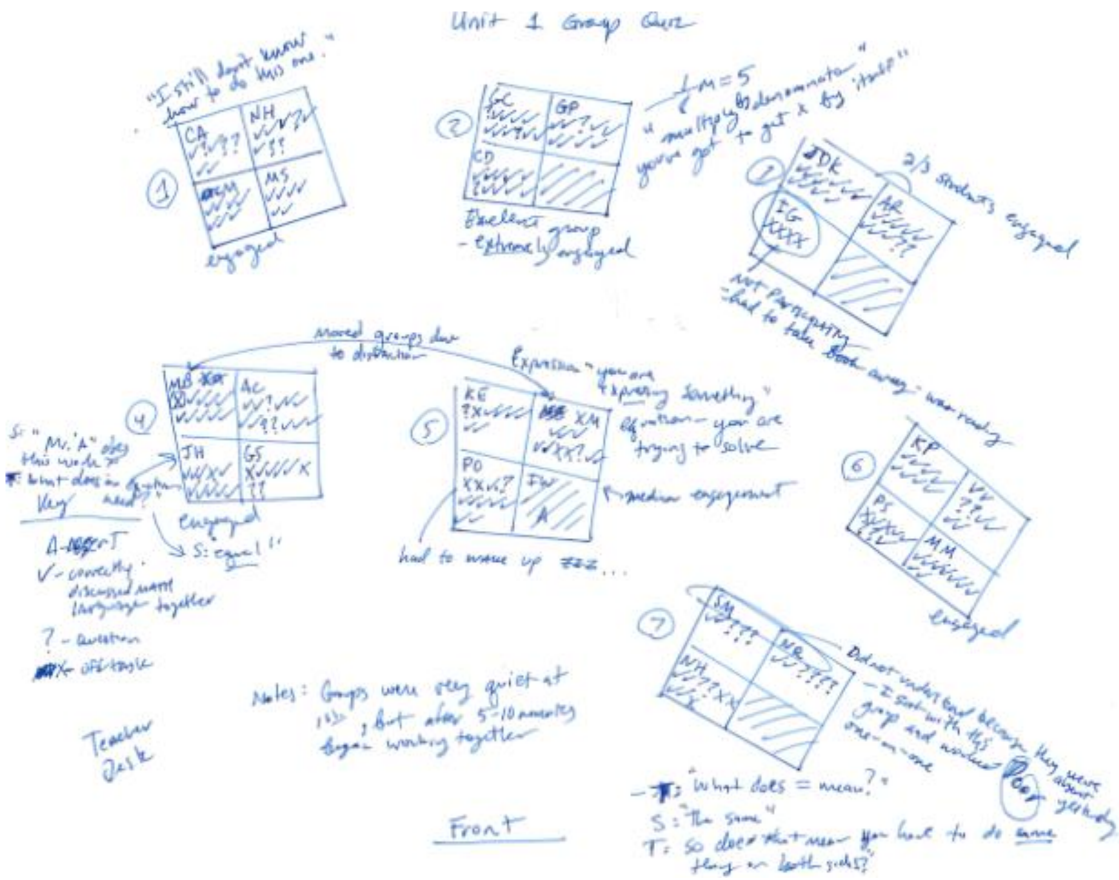



**Figure 4.5 Field Notes**

TR: "Did you do the same on both sides?"

S: "No. So, I just need to add two on both sides."

## Individual Assessments

The researcher included three individual assessments in the unit. There was a final assessment at the end of the first two units. There was also a midterm exam that covered both units. Students did not perform well at the end of unit one. So, at the end of unit two, the researcher chose to return to unit one to reassess the students. The researcher allowed students to work in groups for one day to review material for unit one. Then, the students were asked to complete the assessment individually the following day. Students knew ahead of time that they would be reassessed.

Student performance and engagement increased as the study progressed. While student achievement is not measured in this study, it is important to note the level of achievement as it may indicate levels of self-efficacy. Results of the unit tests were poor and may not have reflected the students' true knowledge. Many students exhibited high levels of self-confidence, so it was surprising to see poor averages. The researcher chose to perform a post-test to determine if students had made improvements over the course of the study. Improvements may indicate increases in the level of self-efficacy. The results of the individual assessments show gradual improvement over the course of the study, with 76% of students showing improvement from the Unit 1 individual assessment to the Unit 1 individual post-test assessment. Table 4.5 shows the results from the individual assessments. The midterm assessment boasted the highest mean scores.

**Table 4.5 Individual Assessments**

| ***Assessment*** | ***Mean Score (n=52)*** | ***Median Score (n=52)*** |
|---|---|---|
| ***Unit 1 Linear Equations*** | 66.5% | 66 |
| ***Unit 2 Linear Graphs*** | 68.7% | 75 |
| ***Unit 1 Post-test (Retest)*** | 69.4% | 69 |
| ***Midterm Assessment*** | 74.5% | 74 |

**Engagement**

The researcher kept a daily log of engagement per student. The researcher evaluated engagement levels every two weeks. A high percentage of students remained engaged throughout the study. Lower levels of engagement occurred during the middle weeks, possibly due to the results on the first individual assessment. Table 4.6 shows the results. The researcher used Miller et al., (1996) suggestions for measuring engagement. Each student was assigned a daily score using the following factors to measure and/or improve engagement levels:

1.Level of effort

2. Level of persistence

3. Level of cognitive ability

4. Giving rewards for participating or consequences for not participating

A rating was awarded to each student and then the researcher averaged the ratings at the end of each two-week period.

**Table 4.6 Student Engagement Levels**

| Week | Engagement Level (%) |
|---|---|
| **2** | 94.4 |
| **4** | 86.9 |
| **6** | 88 |

**Collection Period Post-Survey**

The researcher conducted a collection period post-survey to evaluate student self-efficacy and overall opinions on the study. The survey was conducted at www.surveymonkey.com. Figure 4.6 shows the students thoughts on the four stages of GRR. The data shows that almost half of the students say that guided practice is the most influential stage in GRR, with teacher models following close behind. This is interesting considering the high percentages of students favoring group assessments in the pre-survey. Figure 4.7 shows the participants thoughts on SFL techniques. Table 4.7 shows other data gathered in the post-survey.  The students felt that the word wall was the most helpful in learning how to communicate mathematically.

**Evidence of SFL and GRR**

The teacher researcher used GRR instructional model with an SFL approach in delivering content. Each lesson began with a think aloud model in which the researcher introduced math language. Several models were delivered to the students, followed by guided practice. During guided practice, the researcher visited every student to assess his or her understanding. The researcher used formative assessments to gage the students' level of understanding. Often times, the researcher reverted back to modeling to address student misconceptions. Students were asked to think-pair-share during guided practice to allow for peer collaboration. At least two class periods were devoted to a group assessment before each unit test, which was completed by each individual student without the help of the researcher or fellow classmates. Finally, each individual participated in an individual assessment in which they had little to no assistance from peers or the researcher. During the individual assessment, the researcher would answer questions pertaining to the directions, but would not offer guidance in completing the tasks.

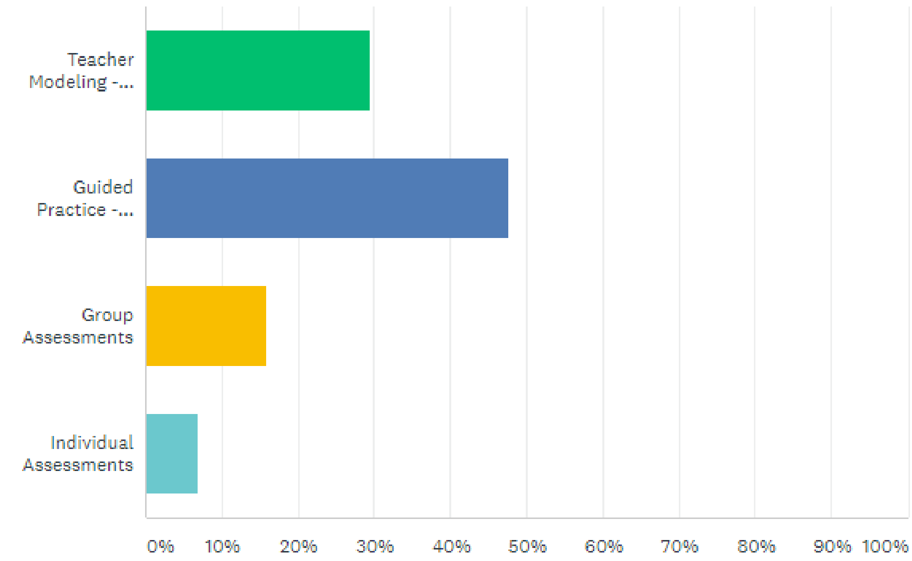



**Figure 4.6 Student Survey - Most Influential GRR Techniques**

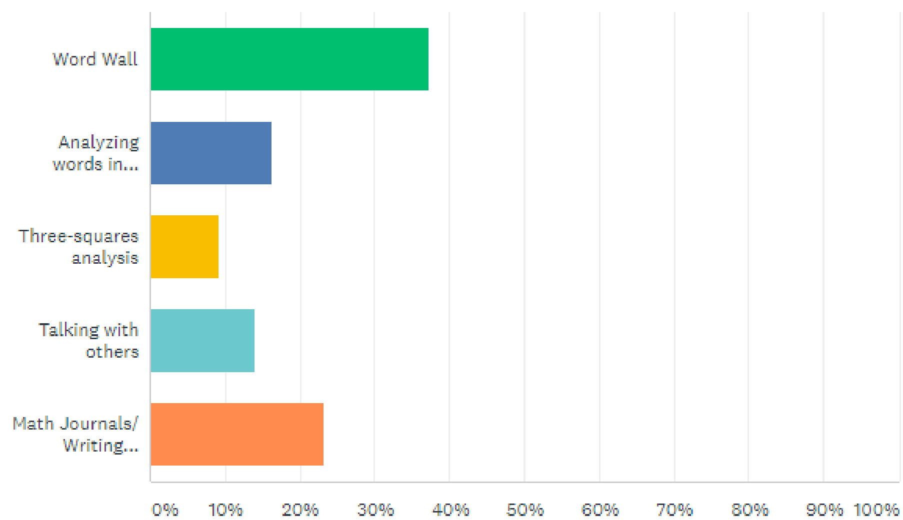



**Figure 4.7 Student Survey - Most Influential SFL Techniques**

**Table 4.7 Post-Collection Period Survey**

| Question | Strongly Disagree (1) | Somewhat Disagree (2) | Neutral (3) | Somewhat Agree (4) | Strongly Agree (5) | Weighted Mean |
|---|---|---|---|---|---|---|
| I am comfortable in communicating mathematics through writing. | 6.82% | 20.45% | 29.55% | 27.27% | 15.91% | 3.25 |
| I am comfortable in communicating mathematics verbally. | 0.00% | 9.09% | 36.36% | 31.82% | 22.73% | 3.68 |
| Knowing math language is important in completing math problems. | 2.27% | 4.55% | 20.45% | 36.36% | 36.26% | 4.00 |
| I place emphasis on learning math | 2.27% | 0.00% | 54.55% | 36.36% | 6.82% | 3.45 |

| | | | | | | |
|---|---|---|---|---|---|---|
| **vocabulary.** | | | | | | |
| **Group assessments help me to be more confident on individual assessments.** | 4.55% | 9.09% | 9.09% | 29.55% | 47.73% | 4.07 |
| **Teacher modeling and think alouds have made me more confident during guided practice.** | 0.00% | 6.82% | 22.73% | 22.73% | 47.73% | 4.11 |

According to Figure 4.8, over 80% of participants feel more confident now than before the study.

The researcher made a word wall to highlight math language covered during the study. The word wall consisted of the word and a picture or example of the word, but no definition was given. The word wall is pictured in Figure 4.9. The researcher implemented several writing assignments during the study. The student writing was analyzed for engagement, math language use, and level of understanding. The researcher made adjustments to the writing assignments during the course of the study in efforts to improve the quality of student work and student engagement. The researcher implemented a teacher-made "Three-Squares Analysis" to solve application problems.

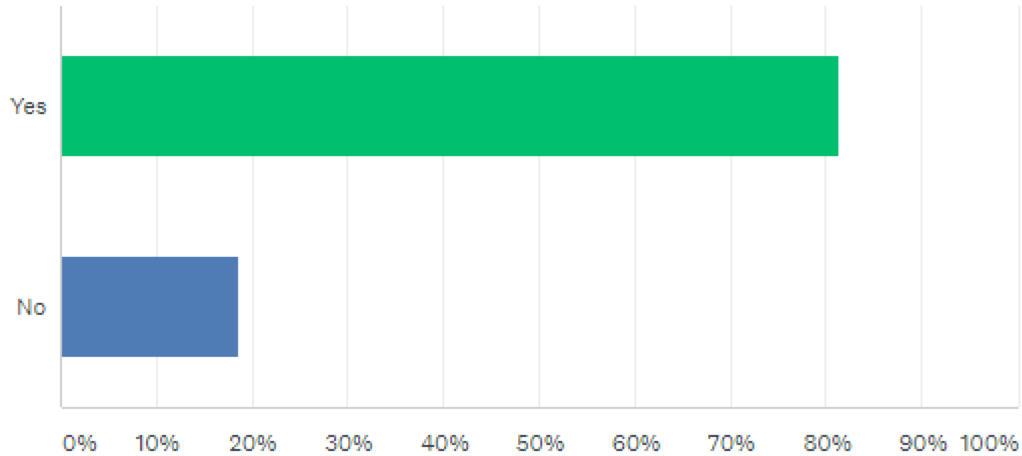



**Figure 4.8 Do You Feel More Confident in Doing Mathematics?**

The "Three Squares Analysis" is a researcher-created variation of the Frayer Model discussed in Chapter 2: "Literature Review." In the Three-Squares Analysis, the student:

1. Analyzes math language in a word problem.
2. Shows the procedure.
3. Interprets the results.

Table 4.8 shows the components of the "Three Squares Analysis" used in the study. This strategy was used to help students in organizing their thoughts and develop internal understanding and then textually communicate the findings.

During think aloud models and guided practice, the researcher analyzed math lexis (refer to Figure 4.3). The researcher circled and underlined math language, including recognizing nouns and verbs and expressed meanings textually as he read. This strategy was used to identify the field. The researcher encouraged the students to follow


**Figure 4.9 Word Wall**

**Table 4.8 Three Squares Analysis**

| 1 | 2 | 3 |
|---|---|---|
| Circle math language. Write numerical values of importance and assign meaning to them; label variables | Show procedures | Interpret the results in sentence format. |

this practice as they completed the assignments in Unit Two: Linear Graphs. Figure 4.10 is an example of student work and how the student attempted to follow the teacher model during an assignment. The modes varied between students. The student in Figure 4.10 chose to graph in order to find the missing x-value. Algebraic methods were also a choice. Students were encouraged to show both modes, although many students chose only one mode per response. However, it is evident that the student did use algebraic methods for another problem.

The student work in Figure 4.11 shows a three-squares analysis used to solve word problems. The researcher noticed the student wrote, “Don’t understand at all.” The researcher interviewed the student and asked probing questions, “Do you need a variable in an algebraic equation?” The student was able to complete the problem on her own.

The researcher also implemented SFL in his conversations with the participants. Placing emphasis on the math language helped scaffold problems and lead students to the solution. The following interview is an example:

Field: Graph a line that goes through (2,3) and has the slope of -2.

S: "What do I do with (2,3)?"

TR: "What is (2,3) called in math?"

S: "A coordinate - the x and y."

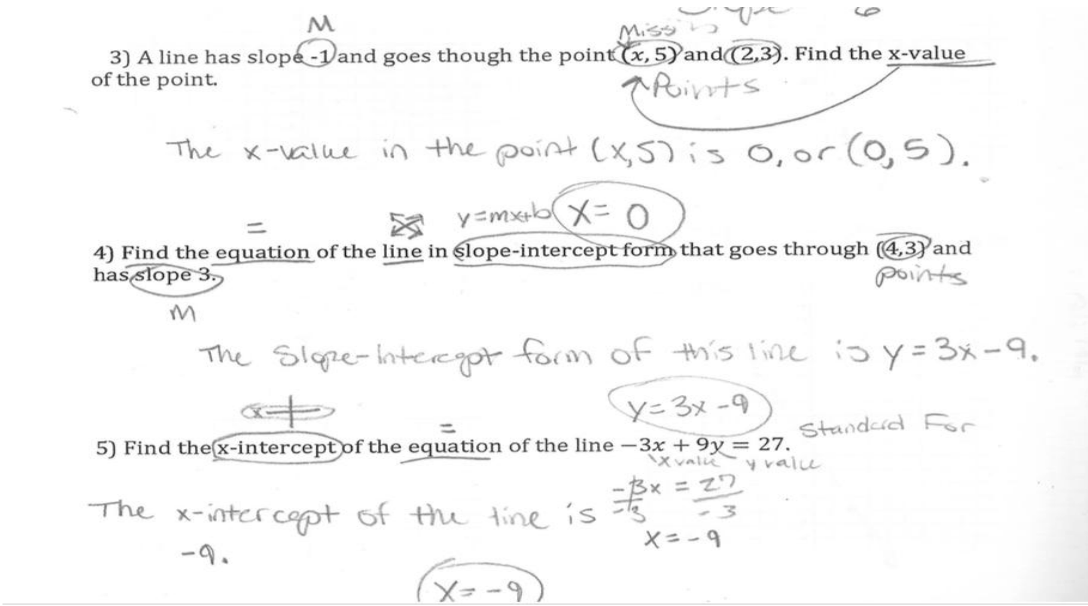
TR: "What do we do with coordinates?"

S: "We plot them."

TR: "Right. Then how do we get another coordinate?"

S: "The slope - OH YEAH! That's rise over run. I got it now."

Notice the student's tone in the last sentence. Her tone exhibits excitement and that a learning moment just occurred. The researcher interviewed all students who scored a zero to elicit more response. One student mentioned the terms "variable, operation, exponent, and combining like terms" in the journal, but did not elaborate. The researcher asked follow up questions:



3) A line has slope -1 and goes though the point $(x, 5)$ and $(2,3)$. Find the x-value of the point.

Points

The x-value in the point (x,5) is 0, or (0,5).

y=mx+b X= 0

4) Find the equation of the line in slope-intercept form that goes through (4,3) and has slope 3.

Points

M

The slope-intercept form of this line is y = 3x − 9.

y = 3x − 9

5) Find the x-intercept of the equation of the line $-3x + 9y = 27$.

x value y value

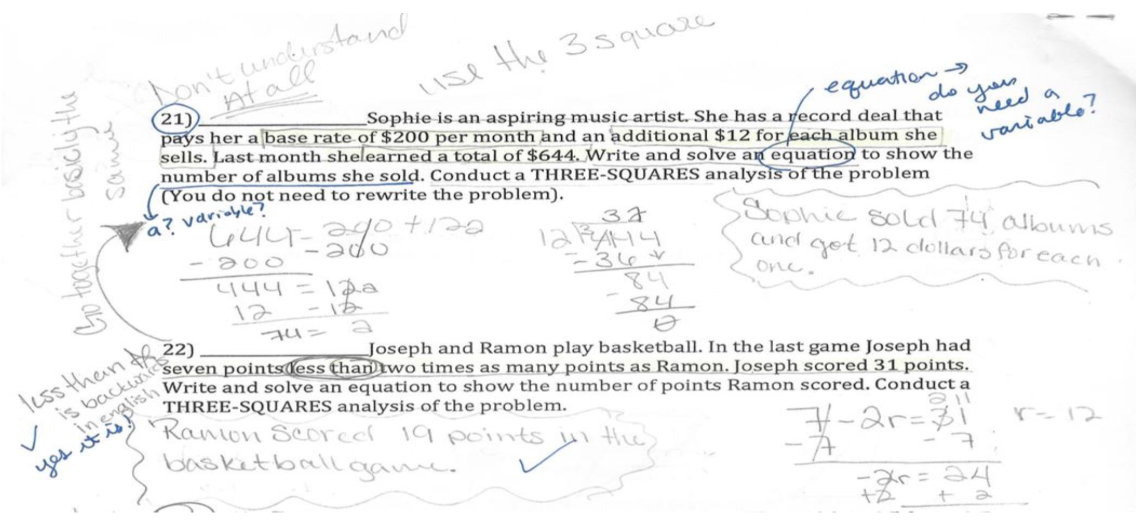
Standard For

The x-intercept of the line is −9.

−3x = 27
−3 −3
x = −9

(x = −9)

**Figure 4.10 Student Work Sample - SFL**



Don't understand At all

use the 3 square

equation → do you need a variable?

21) ________________ Sophie is an aspiring music artist. She has a record deal that pays her a base rate of $200 per month and an additional $12 for each album she sells. Last month she earned a total of $644. Write and solve an equation to show the number of albums she sold. Conduct a THREE-SQUARES analysis of the problem (You do not need to rewrite the problem).

a? variable?

Sophie sold 74 albums and got 12 dollars for each one.

Go together basically the same

22) ________________ Joseph and Ramon play basketball. In the last game Joseph had seven points less than two times as many points as Ramon. Joseph scored 31 points. Write and solve an equation to show the number of points Ramon scored. Conduct a THREE-SQUARES analysis of the problem.

less than is backwards in english

yes it is!

Ramon scored 19 points in the basketball game.

7 − 2r = 31

r = 12

**Figure 4.11 Student Work - Three Squares Analysis**

TR: "What is a variable?"

S: "An unknown number written as a letter."

TR: "What is an operation?"

S: "Addition, subtraction, multiplication, division."

TR: "How do we combine like terms?"

S: "If they got the same variable, then they together. The same exponent is together, too."

The short responses indicate either minimal understanding or the lack of confidence in interpersonal communication. The researcher sensed the student was having difficulty explaining "like terms" in expressions. A example was written:

TR: "Can you simplify this? $x^2 + y^2$."

S: "Yes. $xy^2$."

Now, the teacher-researcher knew that the student did not understand. The researcher then reverted back to the modeling phase in GRR and re-taught combining like terms using expressions that contained multiple variables and constants, then the researcher used expressions that contained exponents.

"Solve" is one of the words we encounter many times in mathematics. The researcher noticed that it is among the most misused words in this study. A student usually defines solving as, "to find the answer," as many students stated during the study. However, "to solve" takes on multiple meanings. For example, to solve a linear equation means to find the value for the unknown variable in the equation. To solve a system of linear equations means to find the intersection of two lines. To solve a quadratic equation means to find the x-intercept(s), or the intersection(s) of the parabola and the x-axis. During the journal, a student wrote, "I hate combining like terms because it takes so long to solve it." In this case the student was describing combining like terms in an expression. Expressions do not have an equal sign. Therefore, expressions cannot be solved, they must be simplified. The researcher used the word wall to refer to math language terms "equation" and "expression" during think-aloud models and reinforced, "We can solve equations, but we cannot solve expressions." The word wall was used consistently during the four stages of GRR as a reference for students. When a student was having difficulty communicating mathematically, the word wall was the primary resource for developing vocabulary.

The students began showing more understanding in their writing as the study progressed. By the third journal, students were putting more effort into their writing, using more math language, and levels of understanding significantly increased. The increase in performance may be a result of SFL and GRR, but it could also be the intentional decisions the researcher made in the assignment. In the third assignment, the researcher gave the students three specific choices in their writing, whereas in the previous two journals the students were given much more freedom in their writing. The students seemed to be more confident when they had more structure in the assignment.

The researcher noticed that students regularly referred to the word wall during assignments. Over 40% of students replied that the word wall is the aspect of SFL that helped them the most in their learning. Almost 25% of students said that math journals helped them the most. Nearly 20% stated that analyzing text in problems was the most helpful. This data indicates that these SFL techniques are all effective for some students and worth studying again in the future.

The results on the individual assessments were lower than expected considering the high levels of engagement and perceived self-efficacy. There are many factors that may have contributed to the low scores. These factors include, but are not limited to, the time of the study being early in the school year; the design of the test; and, the school was closed for an entire week during a hurricane. The results show a steady increase in average scores and it is not unreasonable to conclude that individual assessments may show higher averages if the study was replicated or conducted for a longer duration.

The level of rigor of the assessments could also have an effect on the performance levels. This study was conducted at a time where the researcher was collaborating to create common assessments in the department. The assessments were teacher-made assessments designed to prepare students for the state end-of-course examination. The assessments were given for the first time in this study and the researcher had not had an opportunity to reflect on the assessment and compare student data with other teachers in the department. The researcher will compare data and make curriculum decisions to assure the assessments are rigorous and fair. It is also possible that the students' scores may change over time as they get used to the level of rigor that these assessments present.

The data collection post survey shows that nearly 80% of students feel that group assessments help build their confidence before individual assessments. However, most students replied that guided practice is the most influential in their learning, followed by teacher modeling. This indicates that all three phases are important to helping students in building self-efficacy in mathematics. Nearly 70% of students felt that the researcher models helped.

**Addressing the Research Question**

Students resisted SFL techniques at the beginning of the study. It was difficult at first to get students to participate in analyzing the language in math problems. The first unit consisted of solving linear equations in one variable, which often lacked words, but had heavy use of math symbols and conceptual knowledge. The researcher focused on operations and variables in his approach to using functional linguistics. He also frequently referred to the word wall. The researcher implemented the three-squares analysis to solving word problems. Students participated in this technique, many expressing their satisfaction in the techniques' ability to help them organize their thoughts for word problems. During the second unit, more students were found analyzing text, possibly because there were more opportunities, as the problems contained more fields and modes. Although journals showed a decrease in the amount of sentences written and math language usage from Journal 1 to Journal 2, the quality of writing increased dramatically in Journal 3. The math language usage was highest at the end of the data collection period and mathematical understanding increased from journal entry to journal entry.

More research is needed, but the data in student writing and surveys may suggest that using an SFL approach was helpful in building confidence and increasing engagement in mathematics for some students. Over 80% of students reported that they felt more confident in doing mathematics at the end of the study than at the beginning. During an interview, a student said, "I like how you explained things in depth. The three-squares analysis helped me with my word problems. Math class always tries to throw you off when it comes to word problems. In the first box, you take the key things out of the question. In the second box, you solve the question. In the third box, you write your answer in a sentence. This helps to keep your thoughts organized."

The data also suggests that GRR instructional techniques may also have a positive effect on student self-efficacy and engagement. When surveyed, most students claim that group assessments give them confidence before a test. When asked about the efficiency of teacher models, students claim that teacher think aloud models help them to feel confident about their ability to perform mathematics. The researcher assigned student engagement scores over the course of the study. The average engagement score was at least 87% throughout the study. Data in this study suggests that both SFL and GRR approaches to teaching mathematics may be beneficial to students in this study and no data indicates that it is harmful to student self-efficacy, engagement, or learning.

## Conclusion

The researcher has performed a mixed-methods study and analyzed data from student work, surveys, and interviews to determine the impact an SFL and GRR approach has had on student self-efficacy and engagement. The researcher deliberately taught math as it were a language, emphasizing symbols, vocabulary, proper syntax, and sentence structure while allowing students to engage in verbal and textual conversations using natural language as well as other modes of communication, such as graphs, to make meaning of linear equations in algebra. While more research is needed, the data in this study indicate that an SFL and GRR approach to teaching mathematics may have an effect on student self-efficacy and engagement in secondary mathematics students. In Chapter 5: "Conclusion and Suggestions for Future Research," the researcher will reflect on the results of the study, discuss how the results will be used, and offer suggestions for improving the research.

# CHAPTER 5: CONCLUSION AND SUGGESTIONS FOR FUTURE RESEARCH

## Introduction

The researcher acknowledges that many high school Algebra I students have difficulty in communicating mathematics. Students often need help with the first step of problems, but then are more capable in completing mathematical tasks. Students in Algebra I at Academic High School do not boast high confidence levels in math lexis and this may affect their ability to communicate and participate in class. The researcher addressed the PoP by implementing a systemic functional linguistics approach to teaching mathematics by teaching students how to analyze the text in the problems. Content was delivered using the gradual release of responsibility instructional model. This study aims to gain insight into the research question: **"What effect does systemic functional linguistics in conjunction with the gradual release of responsibility have on student self-efficacy and engagement in secondary mathematics?"**

This study implements a mixed-methods research design. Data was collected using student interviews and surveys, field notes, and student work samples. Self-efficacy was measured using Likert scales and analyzing student interpersonal interactions. Engagement was measured by researcher observations on student participation and behaviors, student work samples, and the completion of assignments.

The data in the study support the PoP in that many students are uncomfortable with math language, especially in word problems. Data shows that understanding the field is a significant obstacle in doing the mathematics. The researcher found that the majority of students feel that teacher modeling and practicing in a group increases student self-efficacy, indicating that SFL and GRR instructional techniques may have been effective in addressing the PoP.

## The Importance of Reflection in Action Research

The reflective process is a crucial component of teacher education and professional development (Hebert, 2015). Mertler (2014) suggests that "the teacher-researcher becomes the missing link between the theoretical researchers and the practicing educators" (p. 245). Dewey (1933) viewed education as developing reflective practices that encourages the learning community to step out of their routines and engage in intellectual thought and action. Teachers are decision-makers and problem-solvers who are placed in unpredictable situations on a daily basis (Schon, 1987). In developing the action plan, the researcher thought of ways that he could adapt his teaching so that learners may increase their confidence levels and engagement in his classroom. Dewey (1933) also warns that falling into non-reflective, routine practice can result in teaching classes instead of teaching students. This is what motivated the researcher to identify a PoP in his own students and work to address it.

Action research is a form of evidence-based teaching as it involves collecting data from students and reflecting on the data to make informed decisions for future instruction (Farrell, 2012). Data and possible solutions are the two factors for all reflective practice (Dewey, 1933). After gathering and analyzing data, the teacher-researcher forms a plan of action for future inquiries. "[Reflective thinking] is focused, careful, and methodical: focused in that it centers on a particular object or situation, and is carried out with the aim of understanding an issue at hand" (Farrell, 2012, p. 362). Reflection is a key component to understanding one's own practice, but also to examine other factors in the study, such as "*who* was involved in the process, *what* led you to want to examine this aspect of your practice, *why* you chose to do what you did, *where* is the appropriate place to implement future changes, and *how* this has impacted your practice" (Mertler, 2014, p. 258).

Schon (1983) suggests that excellent teachers make adaptations to each individual student and situation. Teachers make instructional decisions every day as situations occur, many times without the luxury of thought or knowing whether the decision was the correct choice. This makes teaching unique to other occupations and extremely complex. Sharing research and experiences with colleagues is essential to professional growth and development (Mertler, 2014).

Prior to initiating the action research, the researcher reflected on a PoP. After careful thought, the researcher recognized that he needs to improve his practice in the area of student engagement. Since this is a broad topic, the research refined his PoP by identifying a possible origin for the PoP. The origin examined in this study is that students do not seem to be confident in communicating mathematics and this may be affecting engagement levels in the classroom. One of the most outstanding and consistent barriers to student engagement and perseverance in doing math in the researcher's Algebra I classroom is the lack of understanding math language. Upon analyzing the PoP, the researcher made the decision to implement methods of instruction that showed students how to analyze language and symbols in math in ways that they could make meaning of the problems. The researcher also knew that his students would need many opportunities to communicate mathematics, both through interpersonally and textually, in order to build self-efficacy.

Before researching and collecting data, the researcher met with colleagues to discuss his ideas. Not to his surprise, many of his colleagues shared with him that they were having similar experiences with their students. The researcher then went to the principal and shared this information. Being a former math teacher himself, the principal agreed that mathematics is very much a language, and learning how to use the language is a key component to doing the math. Once the researcher obtained support from his principal and district, he began his research.

The study took place at the beginning of the school year. The researcher shared the PoP with his students as he knew their thoughts would be most influential in the study. After all, tenor is a major component in SFL. Therefore, getting to know the students was a  Throughout the study the researcher reassessed the students to see if their thoughts and feelings had changed. While many of the students agreed that understanding math meant understanding math language, many students were reluctant to engage in meaningful conversations about mathematics. After several weeks of modeling and coaching, students slowly began to engage in SFL practices, circling words and equations, underlining phrases, and writing sentences to make meaning of the math language. During the study, the researcher continued to get to know his students and this may have had an effect on their engagement levels. The researcher feels it is possible that if this study is conducted at a different time in the school year, especially after a relationship has been built with students, that the results may be different. There are other acknowledgements and implications that the researcher will discuss later in this chapter.

## Role of the Teacher-Researcher as Curriculum Leader

Since this action research study requires students to collaborate in groups and to communicate mathematically, it is important to foster an environment that builds a strong community (Bambrick-Santoyo, 2012). To build trust within a learning community, the researcher practiced servant-leadership in the study. While traces of servant-leadership have been found in ancient civilizations, it has recently resurfaced with contributions by Robert Greenleaf (Van de Bunt-Kokhuis & Sultan, 2012). He described servant-leadership as making an effort to serve first and put the focus on the people. This differs from traditional leadership, which puts more focus on tasks and control in the organization. Servant leaders exhibit qualities such as listening to others, caring, understanding for others' feelings, awareness, giving quality feedback, and empowering others (Van de Bunt-Kokhuis & Sultan, 2012). The researcher deliberately used these qualities during the study when interacting with colleagues in order to enhance collaboration and to seek ways to improve his own teaching.

One way the researcher implemented servant-leadership was by making the curriculum and ideas available to others. He kept in close contact with the principal, other department leaders, and teachers during the study, discussing both the aspects that went well in the study as well as areas that may need improvement. It is important to share the results of this action research with teachers and administrators so that they can make future instructional choices that may be in the best interest of future students (Mertler, 2014).

As the curriculum leader in the classroom, the researcher was responsible for giving students a sense of direction (Sergiovanni, 2013). The students were given tasks which intentionally put them in positions to interact with their peers and communicate mathematically. Many students needed guidance and coaching in the early stages as they were unfamiliar with the expectations and how to provide productive, content-rich discussions with their peers. It was the researcher's job as facilitator to lead them in the right direction and to build a strong learning community so that they could develop trust in one another. If servant-leadership qualities are applied and the focus is on trust and building a learning community, then student engagement will likely increase (Bambrick-Santoyo, 2012). The researcher's goal as a teacher-facilitator was to guide the students along the way until they were able to guide themselves.

Finally, the researcher used suggestions by Wojner and Uden (2005) to  foster successful communication and to build trust. Prior to the study, the researcher learned as much about the tenor as possible so that he could set students up for success. He then used the curriculum to set mood and tone that fits the culture of the students. The researcher created an environment that promoted risk-taking and participation. The researcher acted as a facilitator, guiding productive discussions. The goal was to establish trust so that, gradually, the students were able to guide their own discussions and could engage in deeper conversations that showed a high level of understanding and knowledge of the field (Wojnar & Uden, 2005).

As curriculum leader, another goal was to build a strong learning community in the school. The researcher worked to be a valued staff member who believes in sharing and building relationships with other educators in efforts to increase student engagement.

He worked with administration to develop professional learning communities (PLC's) in his department. The goal of the PLC's is to increase rigor in all mathematics classrooms at AHS. It was agreed upon by peers that the idea of building trust among teachers and students is extremely important in achieving a high level of student engagement in classroom discussions and assignments. By putting the emphasis on serving others, the researcher can show students that he cares about their success and makes decisions with the students' best interest in mind.

The teacher-researcher serves as the active chairperson of the math department at AHS. As mentioned earlier, the researcher held daily discussions with administration as well as other teachers and instructional leaders to gain insight into their thoughts about the study. The researcher also regularly attended district level meetings to reflect and share on current practices with department chairpersons from other district-area schools. The researcher led department faculty meetings in which he shared the results of the study and gained feedback from department members. Several mathematics teachers showed interest in implementing SFL and GRR in their classrooms. The researcher provided these teachers with resources and guidance with the implementation. The researcher allowed two induction teachers to observe instruction of this study. Follow up conferences were conducted to reflect on the results in their classrooms.

**Action Plan**

This action research study will utilize a collaborative process between the teacher researcher and the student-participants. The first step in the collaborative process was to communicate to the students the researcher's vision by revealing and discussing the objectives of the study (Mertler, 2014; Senge, 2013). This set the stage as the participants must feel the need for the curriculum (Brubaker, 2004). According to Brubaker (2004), the inner curriculum is “what each person experiences as learning settings are cooperatively created” (p. 29). Every action was intentional and geared toward committing to the inner curriculum.

The researcher worked throughout the study to establish and maintain a learning community among himself and the participants. During the study, the collaborative process took place in the classroom as the students learned to build trust among each other through team learning (Senge, 2013). After the study, the collaborative process shifted toward sharing the results with other staff members in the efforts to improve instruction beyond the researcher's classroom (Mertler, 2014).

The researcher continued to reflect throughout the study. Once the researcher began collecting data, he analyzed it in multiple ways to make sense of the data. He reflected on themes and patterns to help explain the findings (Dana & Yendol-Hoppey, 2014). The researcher recorded the instructional decisions made so that he could return to them after the study to determine if those decisions should be made in the future. The researcher will discuss future implications later in this chapter.

The researcher played two roles in the study, the teacher and the researcher. As teacher, the researcher used his talents to motivate and encourage students as well as guide them to the instructional goal of building community. Making students feel a sense of belonging by showing them they are a part of a team through the curriculum is an important part of teaching (Brubaker, 2004). As the researcher, it is his responsibility to use data-driven instruction to make choices in lesson planning (Bambrick-Santoyo, 2012). This is one reason why analyzing and reflecting on the data is so vital to the action research study.

Finally, the data in the action research study was be made available to the student-participants. They reflected on what went well, what did not, and why these results occurred. The researcher will use feedback from the students when making future instructional decisions.

This study represents the first of three phases of school improvement in which the researcher plans to initiate. The three phases include classroom implementation, department implementation, and school implementation. The researcher serves as a member of the leadership team striving to improve school data for state report cards. One of the vital components of the school report card are the Algebra I End-of-Course examinations and ACT and SAT standardized tests. After classroom implementation, the researcher will meet with the math department to examine school data and discuss how this research study may affect student self-efficacy in other math subject areas. Finally, the third phase involves sharing with members of other departments to bring ideas to assist students in improving scores on other End-of-Course tests, advanced placements examinations, and college and career readiness goals.

## Implications for Future Research

While the DiP indicates that the PoP exists among math students in the researcher's classroom and that an SFL and GRR approach may have a positive impact on student self-efficacy and engagement in mathematics, there are several factors that must be acknowledged for future research.

This study was conducted at the beginning of the school year, prior to any personal relationships being built between the researcher and his students. While the researcher knew demographics, previous academic history, and other student data, part of the study included getting to know the students and building trust. If the study had been conducted later in the year, after a level of trust had already been established, the results may have changed.

Student ages may have affected the results of the study. A large percentage of students in this study were freshmen students fourteen to fifteen years of age. It will be interesting to see, if replicated with older students, if the study would yield different results. This study only included two Algebra I classes. Further research is necessary in other Algebra I classes to determine the effectiveness in implementing SFL and GRR. This is true not only for Algebra I, but in other mathematics subject areas.

Time was a factor in the study. In order to validate the effectiveness of SFL and GRR in mathematics classrooms, the researcher will need to replicate the study and examine students over a longer period of time. Since the school improvement goal is to improve student success on standardized tests, the study will need to be conducted several times over the duration of an entire course. This study did not aim to improve test scores of standardized tests, but the researcher hopes improvements in self-efficacy and engagement will lead to meeting other school improvement goals. This study can lead to future studies that address goals such as these.

The researcher felt that individual test averages were lower than expected. The students indicated that they were more confident in doing mathematics, and engagement level were very high, but the average test score was below average. The researcher acknowledges that it is possible that the test scores may improve if the study is conducted for longer than six weeks. Also, the researcher will use student data to create an item analysis to determine if there were any flaws in the design of the individual assessments. The assessment may need to be reevaluated for its validity. There may have been items on the assessments that should be moved to another time in the course when students are more skillful and fluent in the field.

Finally, this study marks the first time that the researcher intentionally implemented SFL and GRR instructional techniques in the classroom. It is possible that, as the researcher and others gain experience in using these strategies, results may change in future studies. With experience comes confidence and efficiency. As the researcher reflected throughout the study, he noted several changes that he would make in the future. First, the researcher felt that the students needed more scaffolding and guidance in their writing assignments. Some students struggled with engaging in interpersonal discussions about broad topics. For example, "Write about a topic you learned about in this class" may be too broad of an assignment. A more specific task such as "Write about what we learned today" may yield more student engagement.

The researcher also feels that giving students choices in textual interactions may lead to increased engagement. The researcher learned that scaffolding writing assignments was effective in generating more quality responses. Perhaps in future studies the researcher will not only have students respond in written language, but also offer opportunities to use other modes in their writing, such as charts and graphs. The last journal assignment yielded the highest results in sentence length, math language usage, and mathematical understanding. Compared to the other journal assignments, the students were guided more by the researcher being more specific in the writing assignment. It may be possible that the earlier journal entries may have been more productive if they followed a similar format.

The researcher felt it was powerful that students were creating their own language, such as the "magic number," to refer to math language. Although, the students knew how to find the LCM, using the "magic number" changed the mood from a math class to a more familiar setting. Student attempts to change mood and tone were effective in creating more trusting environments that promoted learning. The researcher invited the students to do so whenever they felt the need. The researcher now looks for ways to adjust mood and tone so that students feel more comfortable.

A vital component to GRR is the "You Do It Together" phase. The researcher noticed that giving assignments names such as "Group QUIZ" yielded higher levels of student engagement compared to group assignments that did not include the word "QUIZ," even though they were weighted the same. Practicing functional linguistics has even allowed the researcher to examine his own words in his assessments. The researcher also plans to implement future studies in group work to improve his effectiveness in implementing GRR strategies. Giving groups only one question per assignment did well in encouraging collaboration in the groups. This tactic was helpful in turning over the learning responsibility to the students.

The researcher found that SFL in mathematics is more than merely creating opportunities for students to communicate mathematics. It involves the careful analysis of sentence structure and language usage in mathematic statements and problems. SFL in mathematics is much more than examining natural language choices. Since mathematics is multimodal, it is necessary to examine other forms of text, including graphs, tables, and diagrams. The researcher learned that SFL strategies use field, tenor, mode, mood, and tone to examine student self-efficacy and engagement internally, interpersonally, and textually. As the study progressed, the researcher gained experience in this component of the study. It is a suggestion for future research to provide a rubric for assignments that includes SFL components in student work. Requiring students to implement SFL techniques in the field as opposed to *recommending* students to follow the teacher models may positively affect engagement and yield more accurate results.

This action research study has been a transformational process to both the researcher and the participants. The goal of action research is to examine practices and to use data to improve future instruction (Mertler, 2014). Being a novice in both SFL and GRR required the researcher to do extensive reading on prior research in order to implement these strategies effectively. Throughout the study, the researcher learned that SFL in mathematics is much more than analyzing word choices - SFL analyzes student language choices using many different systems of language. The researcher progressed from focusing on vocabulary and written text to emphasizing mathematical syntax, sentence structure, and creating graphs. All of these systems join together to form the lexis of mathematics. The researcher plans to use the results reflected upon in this chapter to make improvements and replicate this study again with other students. The researcher will also share this experience with other educators in hopes that it may improve their teaching.

In the beginning, the student-participants were reluctant to participate in SFL strategies, further validating the PoP. With persistence in implementation, the researcher observed high levels of engagement and received positive results in student feedback regarding strategies used in the study. Student journals improved over the course of the study, indicating increasing levels of self-efficacy and engagement. Individual assessments also slightly improved over time. Perhaps, most importantly, student responses to surveys reported that GRR techniques such as think-aloud models, guided practice, and group assessments, as well as SFL strategies such as the word wall and journal entries, all significantly contributed to building student self-efficacy and engagement in this Algebra I classroom.

## Conclusion

The DiP has addressed the researcher's PoP that students have difficulty in communicating mathematics. This stems from the lack of understanding of math language and low confidence levels. The researcher has implemented an SFL approach to teaching mathematics while using GRR instructional techniques in the classroom. The study addressed the research question: **“What effect does systemic functional linguistics in conjunction with the gradual release of responsibility have on student self-efficacy and engagement in secondary mathematics?"**

Chapter 1: "Overview of DiP" introduced the PoP, the  purpose of the study, and the research question. Chapter 2: "Literature Review" discussed math lexis, SFL, and GRR, as well as prior studies relevant to the DiP. Chapter 3: "Methodology" laid out the action plan to collect and analyze data. Chapter 4: "Findings" presented the results of the study. Chapter 5: "Conclusion and Suggestions for Future Research" reflected on the results of the study, how the results will be used, discussed factors that may have affected the results, and provided insight into changes that can be made for future research.

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## APPENDIX A: THREE SQUARES ANALYSIS

Directions:

Read the math problem. Identify the text that you can use in the problem. Circle and underline key math language words, symbols, and notation.

1) Identify all math language – vocabulary, symbols, numerical values, etc.

2) Show the procedure. Solve the problem algebraically, graphically, or show the thought process required.

3) Write a brief explanation of the results in at least one complete sentence.

**Three-Squares Analysis Chart**

| **1) List math language and the meanings found in the text.** | **2) Show procedures.** | **3) Interpret the results.** |
|---|---|---|

# APPENDIX B: UNIT 1 - GUIDED PRACTICE A

## I. Applications

1) The formula a = 46c gives the floor area $a$ in square meters that be wired using $c$ circuits.

a) Solve $a = 46c$ for $c$.

b) If a room is 322 square meters, how many circuits are required to wire this room?

2) The formula $C = 2\pi r$ relates the circumference $C$ of a circle to its radius $r$.

a) Solve $C = 2\pi r$ for $r$

b) If a circle's circumference is 15 inches, what is its radius? Leave the symbol $\pi$ in your answer.

## II. Solve for the given variable.

4) $-2 = 4r + s$ for $s$.

5) $xy - 5 = k$ for $k$.

6) $\frac{m}{n} = p - 6$ for n.

7) $P = A + B + C$, for $B$.

8) $A = \frac{a+b}{2}$, for $b$.

9) $ax + by = c$, for $y$.

**Solve.**

10) $d = \frac{c}{\pi}$, for $\pi$

11) $a = \frac{1}{2}bh$, for $b$.

12) $5t - 2r = 25$, for $r$

13) $K = \frac{1}{2}mv^2$, for $m$.

**III. EOC Prep. Circle the best answer choice.**

14) Which of the following is the correct method for solving $2a - 5b = 10$ for $b$?

A) Add *5b* to both sides, then divide both sides by 2.

B) Subtract *5b* from both sides, then divide both sides by 2.

C) Divide both sides by *5*, then add *2a* to both sides.

D) Subtract *2a* from both sides, then divide both sides by -5.

15) The formula for the volume of a rectangular prism is $V = lwh$. Matthew wants to make a cardboard box with a length of seven inches, and width of five inches, and a volume of 210 cubic inches. Which variable does Matthew need to solve for in order to build his box? Explain your choice.

A) *V*

B) *l*

C) *w*

D) *h*

**APPENDIX C: UNIT 1 - GUIDED PRACTICE B**

I. Evaluate. **A=2** **B=3** **C=-6**

___________1) B – C ____________2) 2A + 3B – 4C

3) Nikolas runs eight miles each week.

a) Write an expression for the number of miles he runs in *n* weeks.

b) Find the number of miles Nikolas runs in 5 weeks.

II. Solve. Circle your final answer.

4) $-5 + z = 8$

5) $-\frac{m}{8} = -5$

6) $\frac{7}{8}x = 42$

7) $15 = 3 - 4x$

8) $\frac{2}{3} - \frac{x}{5} = \frac{4}{5}$

9) $15 = \frac{c}{3} - 2$

Solve. Circle your final answer.

10) $2x - 2 = 4x + 6$

11) $3(x + 1) = 2x + 7$

12) $9x - 8 + 4x = 7x + 16$

13) $3(2x - 5) = 2(3x - 2)$

III. Perform a Three-Squares Analysis.

14) Uber company charges \$2.10 plus \$0.80 per mile. Pierce paid a fare of \$11.70.

**Write and solve an equation to find the number of miles he traveled.**

**Equation: ____________________________**

**Solution: ___________________**

**Multiple Choice.**

15) What is the numerical solution of the equation "***seven times some number is three less that five times the same number?***

A) -1.5

B) 0.25

C) $\frac{2}{3}$

D) 4

## APPENDIX D: UNIT 1 - GROUP ASSESSMENT A

**Station 1**

Directions: Do a Three-Squares analysis of the problems below.

1) The ratio of the height of a bonsai ficus tree to the height of a full-size ficus tree is 1:9. The bonsai ficus is six inches tall. What is the height of the full-size tree?

2) At one factory, the ratio of defective light bulbs produced to total light bulbs produced is about 3:500. How many light bulbs are expected to be defective when 12,000 are produced?

**Station 2**

Directions: Do a Three-Squares analysis of the problems below.

1) Four gallons of gasoline weigh 25 pounds. What is the unit rate in pound per gallon?

2) Fifteen ounces of gold cost $6058.50. Find the unit rate in dollars per ounce.

3) The tropical giant bamboo can grow 11.9 feet in 3 days. What is the rate of growth in inches per hour? *Round your answer to the nearest hundredth and show that your answer is reasonable.* (1 foot = 12 inches)

**Station 3**

Directions: Write the problem and solve each proportion. Please show your procedure and box in your final answer.

1) $\frac{v}{6} = \frac{1}{2}$ 2) $\frac{2}{5} = \frac{4}{y}$ 3) $\frac{2}{h} = -\frac{5}{6}$ 4) $\frac{3}{10} = \frac{b+7}{20}$

5) $\frac{5t}{9} = \frac{1}{2}$ 6) $\frac{2}{3} = \frac{6}{q-4}$

**Station 4**

Directions: Write the problem and perform a Four-Squares analysis.

1) On a certain day, the exchange rate was 60 U.S. Dollars for 50 euro. How many U.S. dollars were 70 euro worth that day?

2) An environmental scientist wants to estimate the number of carp in a pond. He captures 100 carp, tags all of them, and releases them. A week later he captures 85 carp and records how many have tags. His results are shown in the table below. Write and solve a proportion to estimate the number of carp in the pond.

| Status | Number Captured |
|---|---|
| Tagged | 20 |
| Not Tagged | 65 |

**Station 5**

Directions: Solve the proportion.

1) $\frac{9}{2} = \frac{5}{x+1}$ 2) $\frac{x-1}{3} = \frac{x+1}{5}$ 3) $\frac{m}{3} = \frac{m+4}{7}$ 4) $\frac{5}{2n} = \frac{8}{3n-24}$

**Station 6**

Directions: Solve the problems below. Show your procedure.

1) A particular shade of paint is made by mixing 5 parts red paint with 7 parts blue paint. To make this shade, Steeley mixed 12 quarts of blue paint with 8 quarts of red paint. Did Steeley mix the correct shade? Explain using math and complete sentences.

2) Jeremy, Noah, and Ciara are film animators. In one 8-hour day, Jeremy rendered 203 frames, Noah rendered 216 frames, and Ciara rendered 227 frames. How many more frames per hour did Noah render than Jeremy?

**Station 7**

Directions: Show your procedure and select the best answer. Circle significant math language and assign meaning with writing.

1) One day the U.S. dollar was worth approximately 100 yen. An exchange of 2,500 yen was made that day. What was the value of the exchange in dollars?

**a) $25**

**b) $400**

**c)$2,500**

**d) $40,000**

2) Jasmine walks at a speed of 4 miles per hour. She walks for 20 minutes in a straight line. Approximately what distance does Jasmine walk?

**a) 0.06 miles**

**b) 1.3 miles**

**c) 5 miles**

**d) 80 miles**

3) A shampoo company conducted a survey a found that 3 out of 8 people use their brand of shampoo. Which could be used to find the expected number of users *n* in a city of 75,000 people?

**a)** $\frac{3}{8} = \frac{75{,}000}{n}$ **b)** $\frac{3}{75{,}000} = \frac{n}{8}$ **c)** $\frac{8}{3} = \frac{n}{75{,}000}$ **d)** $\frac{3}{8} = \frac{n}{75{,}000}$

4) A statue is 3 feet tall. The display case for a model of the statue can fit a model that is no more than 9 inches tall. Which of the following scales below allows for the tallest model of the statue that will fit in the display case?

**a) 2:1**

**b) 1:1**

**c) 1:3**

**d) 1:4**

## APPENDIX E: UNIT 1 - GROUP ASSESSMENT B

Evaluating expressions.

1) ____________________a) Find the expression of the perimeter of the triangle with sides (4 + x), 3x, and (6 – 2x).

____________________b)What would be the perimeter of the triangle if x = 5ft?

2) ____________________Evaluate $5x - 7\ for\ x = 4,6,8, and\ 10$. Organize your answers in a table.

| Input | Output |
|---|---|
| 4 | |
| 6 | |
| 8 | |
| 10 | |

3) ____________________If $x - 1 = 7 - 2(x + 3)$, then what would x equal?

4) ____________________If $\frac{2a}{3} + \frac{1}{5} = 5, what\ is\ the\ solution\ for\ a$?

5) _______________$r - 2s = 14$, solve the formula for *s*.

6) ____________________Today's temperature is three degrees cooler than yesterday's temperature. Write an expression for the temperature today.

7) ____________________If Jack is three times older that his sister Judy, which of the following expressions represents Jack's age if Judy is $j$ years old?

a) $3 \div j$

b) $3j$

c) $3 - j$

d)$\frac{1}{3}j$

8) ____________________If $0 = 6n - 36$, then find the value of $n - 5$.

9) ____________________ If $8n + 22 = 70$, find the value of $3n$.

10) ____________________Parker needs 108 signatures for his petition. So far, he has 27. Write and solve an equation to determine how many more signatures he needs.

11) The sum of the measures of two angles is $180°$. One angle measures $3a$ and the other angle measures $2a - 25$.

A) ____________________Find $a$.

B) ____________________Then find the measure of each angle.

12) ____________________Solve $\frac{2}{3}n = 4n - \frac{10}{3}n - \frac{1}{2}$.

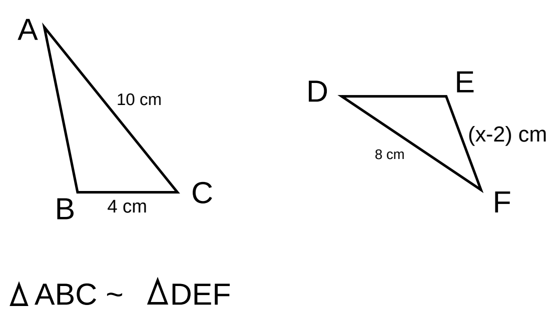
13) ______________________Solve $-3r - 8 = -5r - 12$.

14) ______________________Solve $C = \frac{360}{n}\ for\ n$.

15) ______________________Solve the proportion. $\frac{1}{3} = \frac{x}{x-6}$

16) ______________________A recipe calls for a casserole to use 2 cups of rice. The recipe makes 6 servings of casserole. How many cups of rice will you need to make 10 servings of casserole?

17) ______________________Find side length EF.




18) ______________________A map has a scale of 3 cm: 75 miles. If Charlotte and Charleston are 7 cm apart, what would be the distance in miles?

19) ______________________Dain can run a marathon in six hours. How many miles per hour did Dain run? (1 marathon = 26 miles)

20) ______________________A painting is 2 inches tall and 3 inches wide. If the painting is increased to a size of 15 inches wide, how tall would it be?

21) ______________________A house cat can run about 30 miles per hour on average. How many feet per minute can a house cat run? (5280 feet = 1 mile).

22) _____________________Sophie is an aspiring music artist. She has a record deal that pays her a base rate of $200 per month and an additional $12 for each album she sells. Last month she earned a total of $644. Write and solve an equation to show the number of albums she sold. Conduct a THREE-SQUARES analysis of the problem (You do not need to rewrite the problem).

23) _____________________Joseph and Ramon play basketball. In the last game Joseph had seven points less than two times as many points as Ramon. Joseph scored 31 points. Write and solve an equation to show the number of points Ramon scored. Conduct a THREE-SQUARES analysis of the problem.

24) _____________________The perimeter of a rectangle is 34 feet. The width is 6.5 feet. Write and solve an equation to show the length of the rectangle.

25) _____________________Which values of P and Q does the equation have infinitely many solutions?

$$Px + Q = 46x + 23$$

A) P= -46 and Q=-23
B) P=-46 and Q = 23
C) P=46 and Q=23
D) P=46 and Q = -23

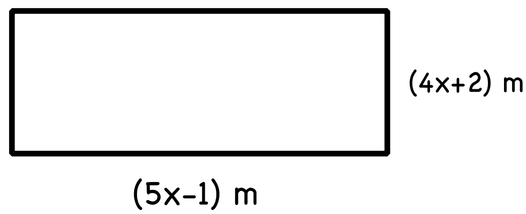
26) _____________________Which value for A will give a solution x=4?

$$Ax + 5 = 17$$

## APPENDIX F: UNIT 1 - INDEPENDENT ASSESSMENT

__________________1) Solve $5x - 9 = 3x + 7$.

__________________2 a) Find the expression for the perimeter of the rectangle below.




b) Find the perimeter in meters if $x = 10$ meters.

__________________3) Solve $C = \frac{5}{9}(F - 32)$ for $F$.

4) Select **all** equations that $x = 3$ is a solution. There may be more than one selection. Circle all that apply. Partial credit can be given if you show work or explain your reasoning.

A) $x + 4 = 7$

B) $\frac{x}{9} = 3$

C) $5 - x = 2$

D )$8x = -24$

E) $x - 10 = 7$

5) Shequan works at Foot Locker. He gets $40 per day as wages and $4.50 for every pair of shoes he sells in a day. His daily earnings is $112. Conduct a THREE-SQUARES analysis. **Write and solve an equation to show the number of shoes Shequan sells in one day.**

__________________6) Solve $-11f = 7(1 - 2f) + 5$.

__________________7) Solve $9e + 4 = -5e + 14 + 13e$.

8) Which values of A and B will result in no solution $\{\phi\}$? Circle one. Explain your reasoning.

$$Ax - 3 = 4x - B$$

A) $A = -4, B = 3$ **Reasoning:**

B) $A = -4, B = -3$

C) $A = 4, B = 3$

D) $A = 4, B = -3$

__________________9) Solve $2f + \frac{5}{3} = \frac{4}{9}$.

10) How many solutions does the following equation have? Justify your reasoning.

$$4(y - 30) = 4y - 120$$

A) None

B) Infinitely Many **Justify:**

C) One

11) Which value of $n$ for $3n + 5 = 32$ makes a true statement? Circle one.

A) $n = 6$

B) $n = 8$

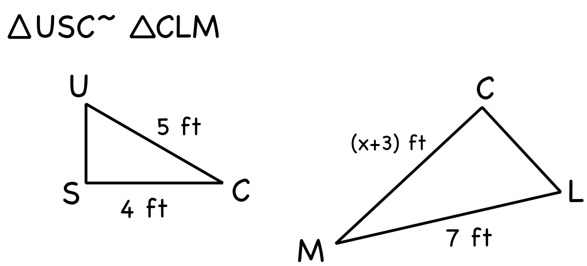
C) $n = 9$

D) $n = 12$

__________________12) Solve $2 - \frac{1}{2}n = 3n + 16$.

__________________13) Chef Morgan is cooking for Sunday brunch. She knows that the ratio of pancakes to people is 22:8. She has to cook for 128 people. How many pancakes should she make?

14) __________________A) Use the similarity statement to solve for x.



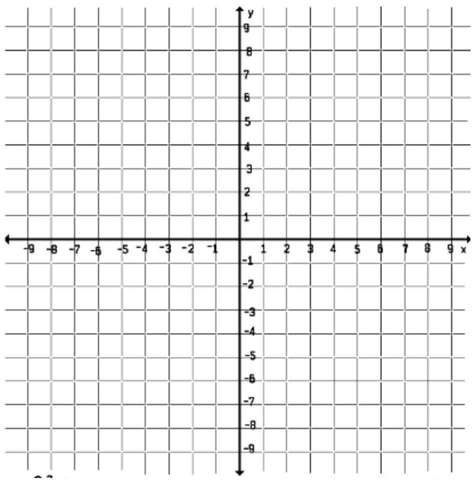

__________________B) What is the length of side CM?

__________________15) What value for A gives a solution $x = -2$ in the equation

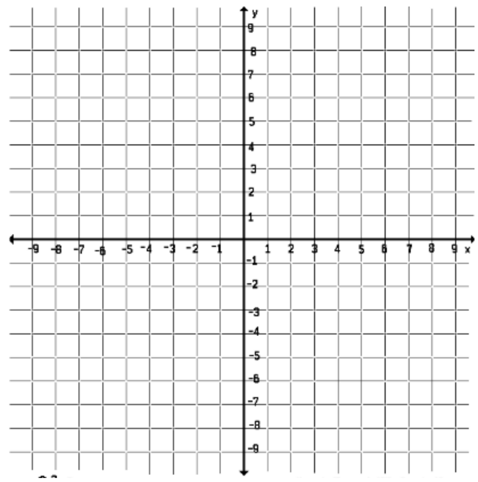
$Ax - 4 = 10$?

## APPENDIX G: UNIT 2 - GUIDED PRACTICE A

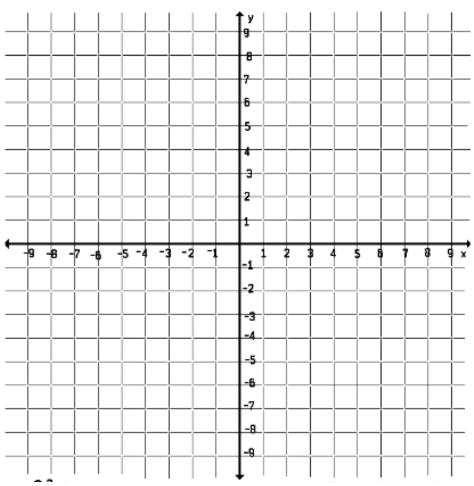
1) Graph $x = -6$.



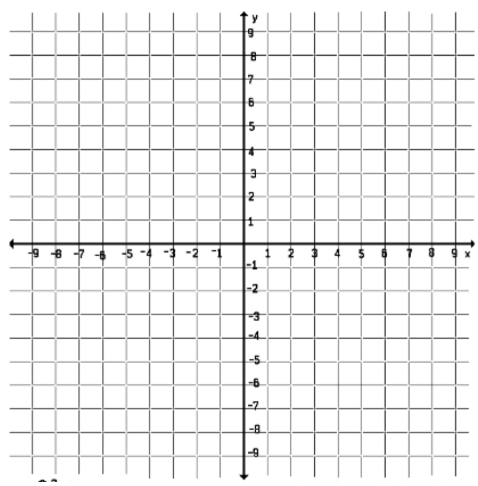
2) Graph $y = 3$.



3) Graph $7x + 3y = 21$.



4) Graph $6y = 2x - 12$



5) Find the rate of change in the table. 6) Put the line $\frac{3}{4}x = \frac{1}{2}y - 6$ in standard

7) Find the slope of the line $4x + 2y = 10$.

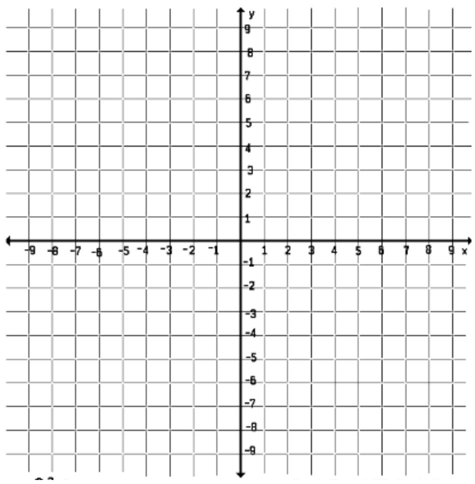
| X | 0 | 2 | 4 | 6 |
|---|---|---|---|---|
| Y | 25 | 45 | 65 | 85 |

8) Find the slope of the line that goes through (2,7) and (4, 4).

9) Find the slope of the line that has x-intercept = -5 and y-intercept = 3.

10) Find the y-intercept of the line $-3x + 9 = 3y$.

11) Graph the line $y = 4x - 3$.



12) Write the equation $3x + 2y = 8$ in slope-intercept form. Label the slope and y-intercept.

13) A caterer charges $200 fee plus $18 per person served.

a) Write an equation that represents the cost as a function of the number of guests. Define the variable.

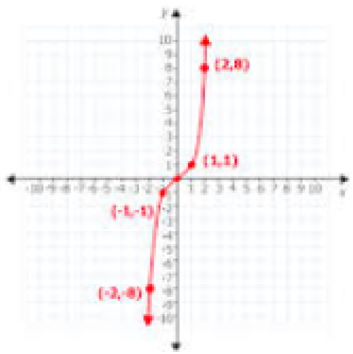
b) Identify the slope and y-intercept and **describe what they mean in the context of the problem.**

c) Find the cost of catering an event for 200 guests.

## APPENDIX H: UNIT 2 - GUIDED PRACTICE B

1) Identify the linear functions. Circle all that apply.

A)

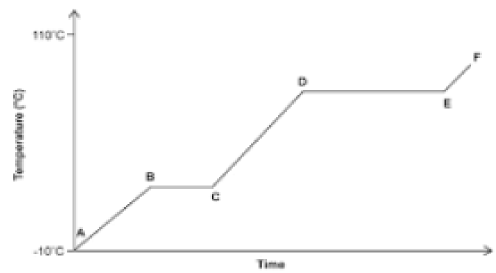



B) $\{(0,6), (3, 12), (6, 18), (7, 21)\}$

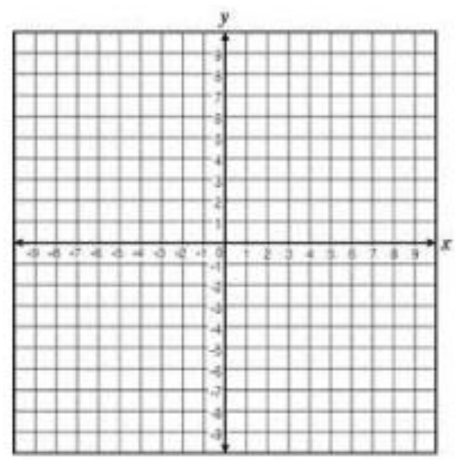
C)

| X | 5 | 8 | 14 | 20 | 23 |
|---|---|---|---|---|---|
| y | -7 | -5 | -1 | 3 | 5 |

D)




2) Find the x-intercept and the y-intercept of the linear function $-8x + 12y = 24$. Then, graph the function.



Justify (Show work):

x-intercept: ____________________

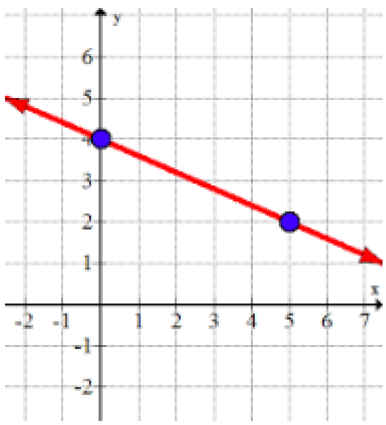
y-intercept: _____________________

__________________________3) Write the linear equation $\frac{3}{4}y = \frac{2}{3}x - 6$ in STANDARD FORM. Then, find the x-intercept.

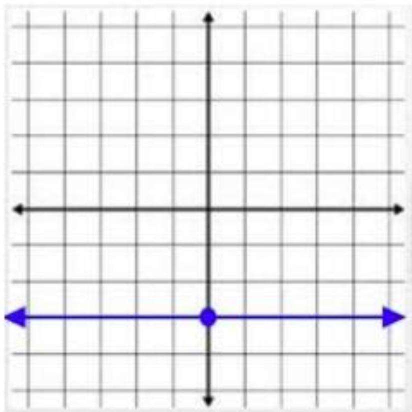
________________4) Find the slope of the line that goes through the coordinates (6,-4) and (-4,-6).

5) Find the slopes of the lines graphed below:

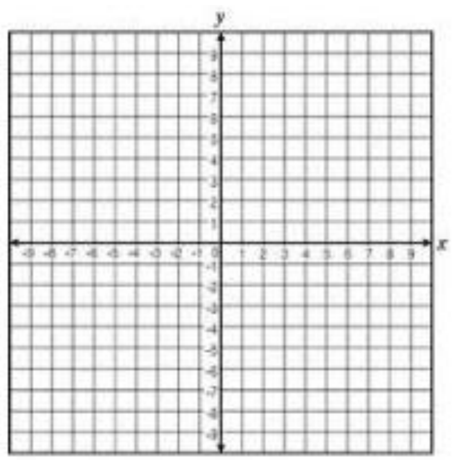
____________A)



______________B)



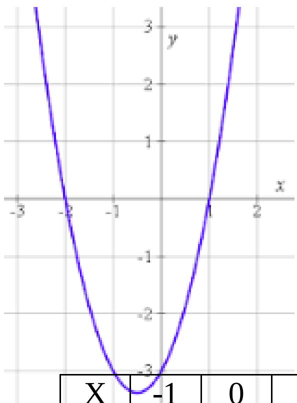
_________________6) Find the area of the triangle formed by the line $x = -3$, the line $y = -4$, and the line $-4x = 12 + 3y$.



## APPENDIX I: UNIT 2 - GROUP ASSESSMENT A

1) Identify the linear functions. Circle all that apply.

A)




B) $\{(-3,-2),(0,-1),(3,0),(6,2),(9,4)\}$

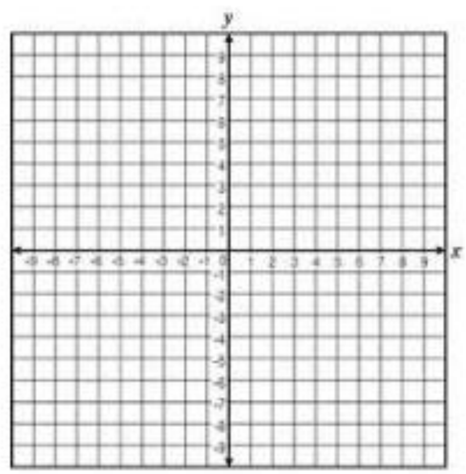
C)

| X | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| y | -3 | -2 | -1 | 0 | 1 |

D) $\frac{1}{2}x + 6y = 12$

2) Find the x-intercept and the y-intercept of the linear function

$5x - 3y = 15$. Then, graph the function.



Justify intercepts (show work):

x-intercept: ____________________

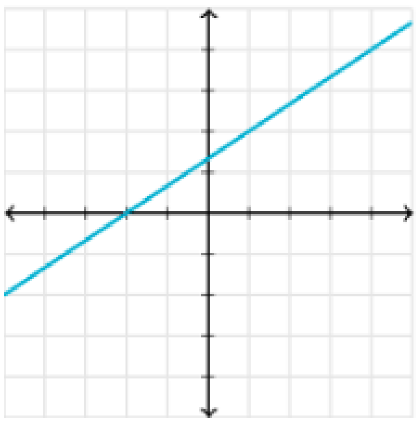
y-intercept: ____________________

________________________3) Write the linear equation $\frac{2}{3}y = \frac{1}{6}x - 4$ in STANDARD FORM.

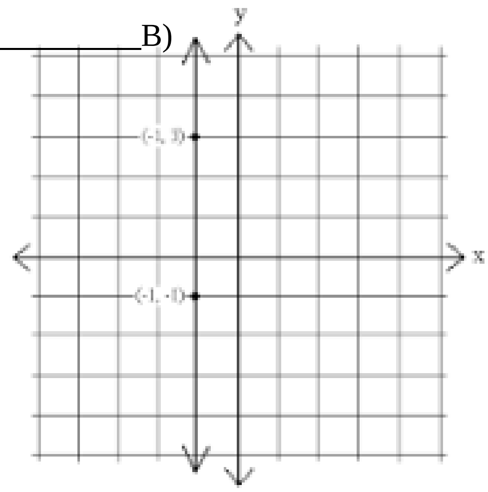
_______________4) Find the slope of the line that goes through the coordinates (0, -3) and (5, -5).

5) Find the slopes of the lines graphed below:

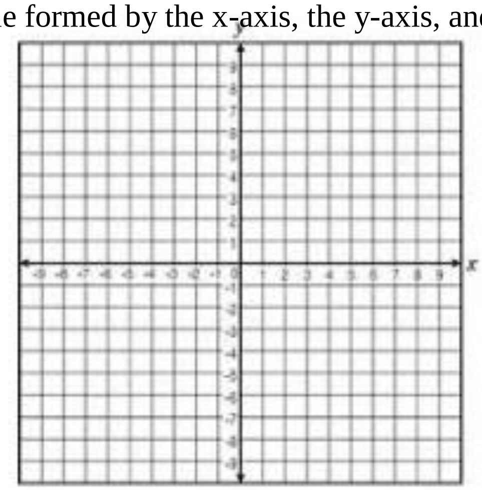
___________A)



_____________B)




________________6) Find the area of the triangle formed by the x-axis, the y-axis, and the line $4x - 2y = 12$.



________________7) Find the slope of the line that contains the coordinates (5, -7) and (6, -4).

## APPENDIX J: UNIT 2 - GROUP ASSESSMENT B

1) Determine whether the order pairs are linear. Write "LINEAR" or "NOT LINEAR."

Justify your reasoning.

A)

| X | Y |
|---|---|
| -4 | -3 |
| 4 | 3 |
| 8 | 5 |
| 16 | 10 |
| 20 | 12 |

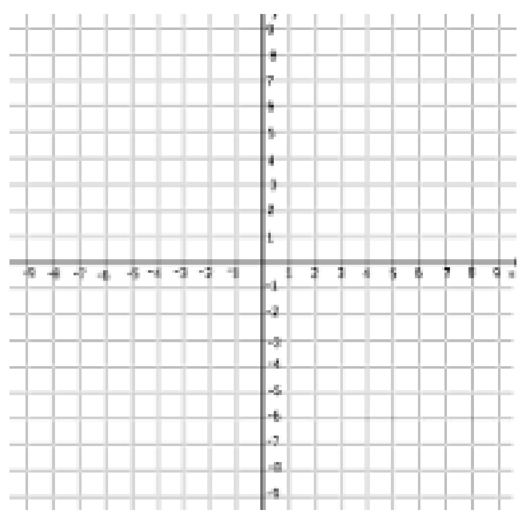
Justify:

B) $\{(-2, 1), (0, 3), (2, 5), (4, 7), (5, 9)\}$

Justify:

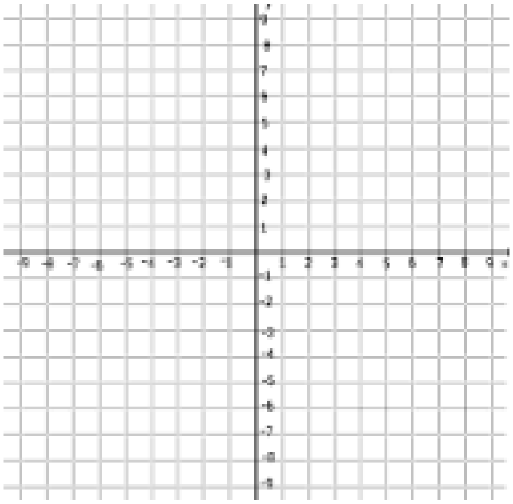
2) Determine whether the equation is linear.  If so, graph the equation. If not, write "NOT LINEAR."

$$-x = \frac{1}{3}y - 1$$

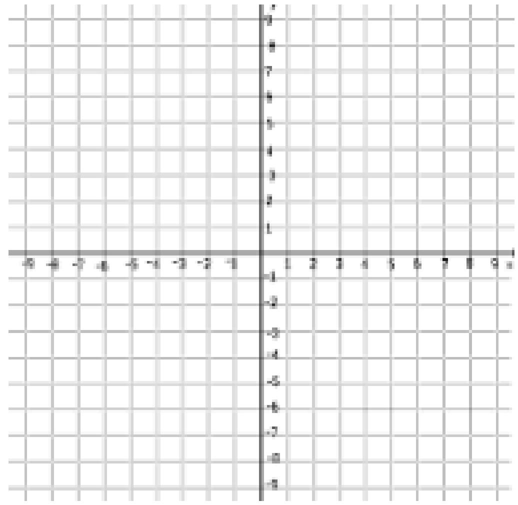


3) Write the equation $\frac{3}{5}y = 1 - \frac{1}{2}x$ in STANDARD FORM.

4) Graph the equation $x + 2y = -10$.



5) Graph the equation $y = -\frac{1}{4}x + 5$



6) Find the slope of the line that contains the coordinates $(9, -10)$ and $(5, 6)$.

7) Find the slope of the line described by $3x - 6y = 18$.

8) Find the value of $y$ so that the points lie on the line with the given slope.

$(-3, y)$ and $(0, 2), m = \frac{1}{3}$

9) Which function has the same y-intercept as $y = \frac{1}{4}x - 8$?

A) $2x + 4y = -16$

B) $x + 4y = -8$

C) $-\frac{1}{2}x + y = 8$

D) $x - \frac{1}{2}y = 4$

10) Graph the equation $y - 5 = -3(x - 1)$.

11) A line contains the coordinates $(-3,4)$ and $(6,-2)$. What are the slope and y-intercept?

A) slope = $-\frac{3}{2}$; y-intercept= 2

B) slope = $-\frac{2}{3}$; y-intercept = 2

C) slope = $-\frac{2}{3}$; y-intercept = 3

D) slope = $\frac{2}{3}$; y-intercept = 2

12) Write an equation of a line in SLOPE-INTERCEPT FORM that is parallel to $y = -x - 2$ and passes through the coordinate $(2, 4)$.

13) Write an equation of a line in SLOPE-INTERCEPT FORM that is perpendicular to $-4x + 3y = 12$ and passes through the coordinate $(2, 2)$.

14) Which linear equation are parallel to $y = \frac{1}{2}x + 1$? **CIRCLE ALL THAT APPLY!**

A) $y = -2x$

B) $y = \frac{1}{2}x + 2$

C) $4x - 2y = 6$

D) $y - 4 = \frac{1}{2}(x + 5)$

E) $y = 2 - \frac{1}{2}$

## APPENDIX K: UNIT 2 - INDIVIDUAL ASSESSMENT A

1) Find the rate of change in the table.

| X | Y |
|---|---|
| -4 | 0 |
| -2 | 5 |
| 4 | 20 |

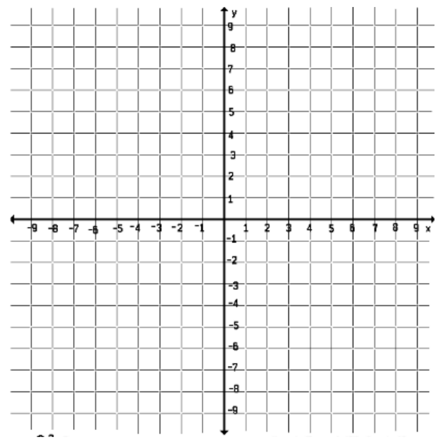
2) Find the slope of the line that passes through (-6,3) and (6,1).

3) A line has slope -2 and goes through the point (x,5) and (2,3). Find the x-vlaue of the point.

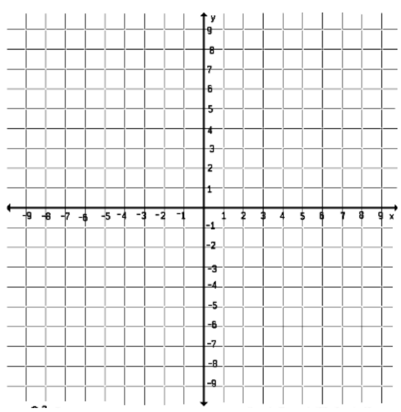
4) Find the equation of the line in slope-intercept form that goes through (4,3) and has slope 3.

5) Find the x-intercept of the equation of the line $-3x + 9y = 27$.

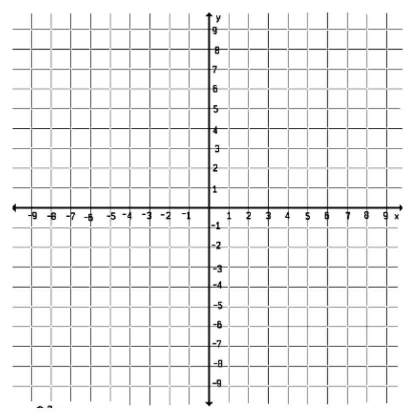
6) Find the equation of the line in slope-intercept form that goes through (9, -1) and (6, 2).



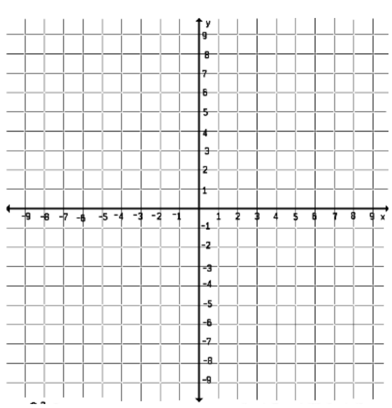
7) Graph $4x - 2y = 10$.



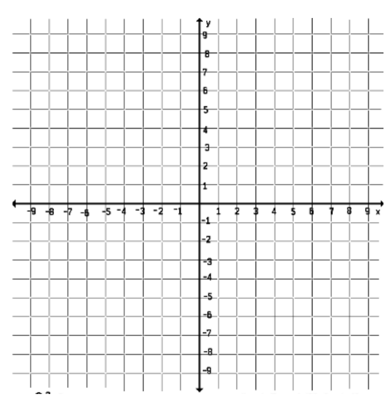
8) Graph $y = -\frac{3}{4}x + 6$



9) Graph $4x = 2y + 6$



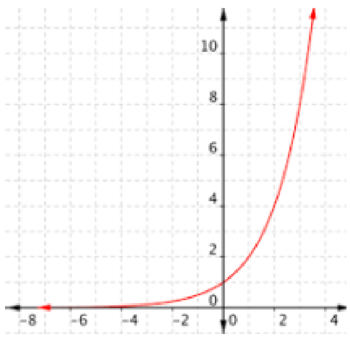
10) Graph $\frac{1}{2}y = -x + 3$



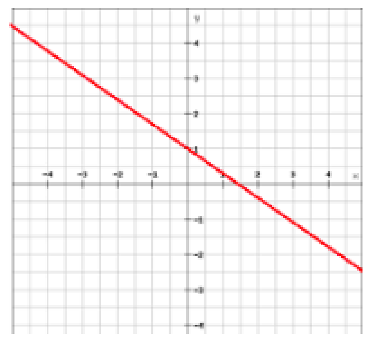
## APPENDIX L: UNIT 2 - INDIVIDUAL ASSESSMENT B

1) Identify the linear graphs. Circle all that apply.

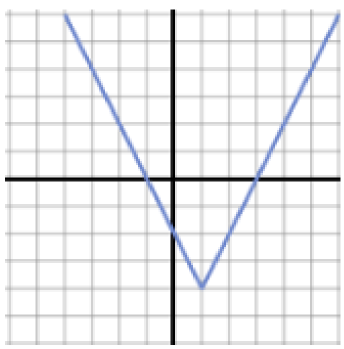
A)



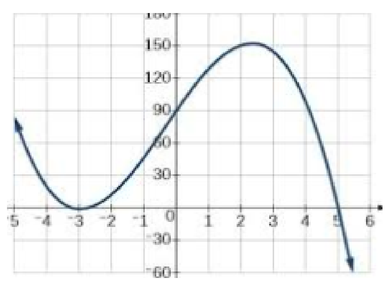

B)



C)



D)




2) Determine whether the order pairs are linear. Write "LINEAR" or "NOT LINEAR."

Justify your reasoning.

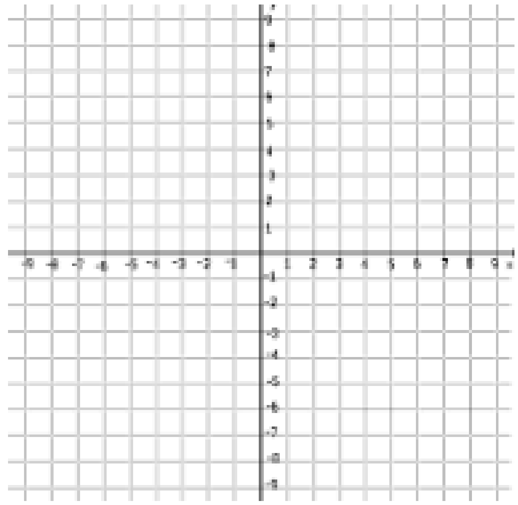
A)

| X | Y |
|---|---|
| 0 | -3 |
| 4 | 0 |
| 8 | 3 |
| 12 | 6 |
| 16 | 9 |

B) $\{(-4, 13), (-2, 1), (0, -3), (2, 1), (4, 13)\}$

3) Determine whether the equation is linear.  If so, graph the equation. If not, write "NOT LINEAR."

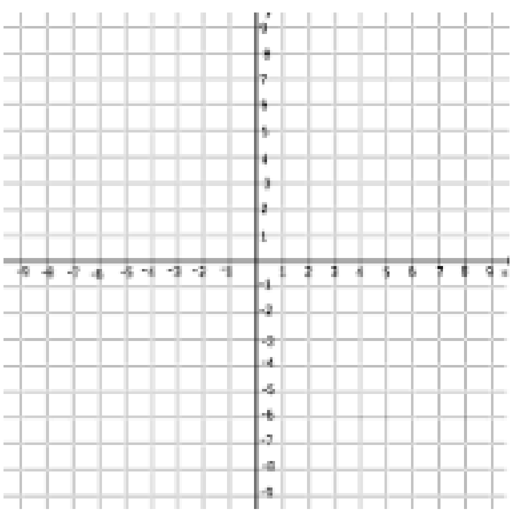
$x = 2y + 4$



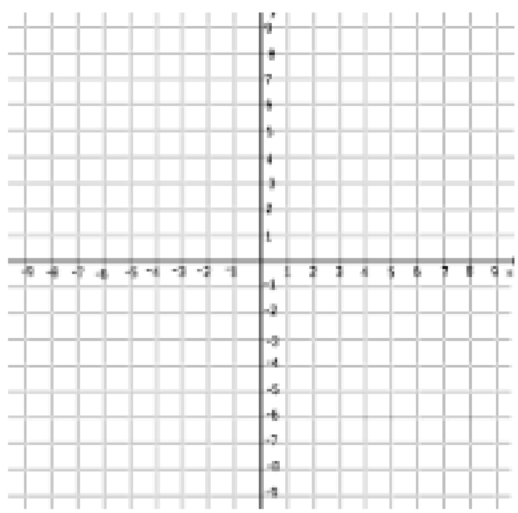
4) Find the x-intercept and the y-intercept of the equation $3x - 7y = 21$.

5) Write the equation $\frac{2}{3}y = 4 - \frac{1}{2}x$ in STANDARD FORM.

6) Graph the equation $-3x + 4y = -12$.



7) Graph the equation $y = -\frac{1}{2}x + 7$



________8) Find the slope of the line that contains the coordinates $(5, -7)$ and $(6, -4)$.

________9) Find the slope of the line described by $6x - 5y = 30$.

________10) Find the value of $x$ so that the points lie on the line with the given slope.

$(x, 2)$ and $(-5, 8), m = -1$

11) Which function has the same y-intercept as $y = \frac{1}{2}x - 2$?

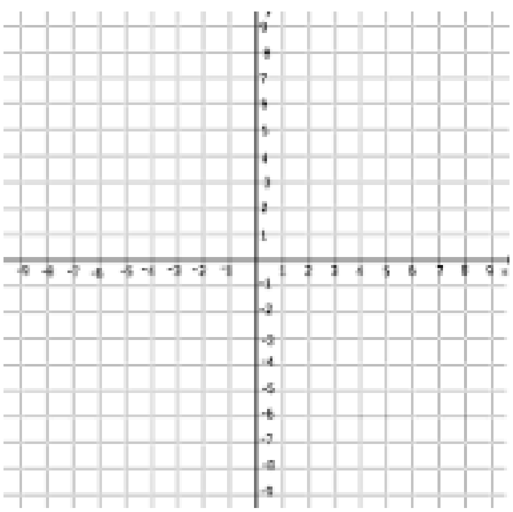
A) $2x + 3y = 6$

B) $x + 4y = -8$

C) $-\frac{1}{2}x + y = 4$

D) $\frac{1}{2}x - 2y = -2$

12) Graph the equation $y - 1 = -(x - 3)$.



13) A line contains the coordinates $(4{,}4)$ and $(5{,}2)$. What are the slope and y-intercept?
A) slope = -2; y-intercept= 2

B) slope = $-\frac{1}{2}$; y-intercept = -2

C) slope = -2; y-intercept = 12

D) slope = 12; y-intercept = -2

14) Write an equation of a line in SLOPE-INTERCEPT FORM

that is parallel to $y = 2x + 3$ and passes through the coordinate $(1, 7)$.

15) Write an equation of a line in SLOPE-INTERCEPT FORM that is perpendicular to

$5x + 2y = 10$ and passes through the coordinate $(3, -5)$.

16) Which linear equation are parallel to $y = -3x + 2$? **CIRCLE ALL THAT APPLY!**

A) $y = -3x$

B) $y = \frac{1}{3}x + 2$

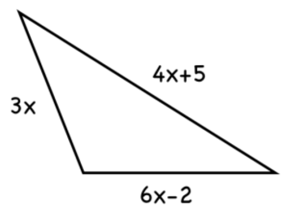
C) $3x + y = 6$

D) $y - 9 = 3(x + 2)$

E) $y = 2 - 3x$

## APPENDIX M: UNIT 1 POST-TEST - GROUP ASSESSMENT

1) A) Find the expression for the perimeter of the triangle.




B) What is the perimeter if $x = 4$ ft?

2) Which is the solution for $3x - 6 = -3x + 6$?
   A) All real numbers
   B) No Solution
   C) $x = 2$
   D) $x = 0$

3) What is the solution for *A* and *B* so that the equation yields no solution?

$$Ax + 5 = -3x - B$$

A) $A = -3, B = 5$

B) $A = -3, B = -5$

C) $A = 3, B = 5$

D) $A = 3, B = -5$

4) Solve the equation $4x + 3 = 5x - 2$

5) What is the solution to the equation $\frac{2}{3}x - 5 = 7$?

6) If $3d - (9 - 2d) = 5$, find the value of $3d$.

7) a) Solve the equation $\frac{1}{2}x - \frac{4}{5} = \frac{3}{10}$ b) Solve $6x - 4 = 2(3x - 2)$

8) Solve the proportion $\frac{8}{x+10} = \frac{1}{12}$

9) Season football tickets for Clemson Tigers football games are $250 for a season-long parking pass and $75 per home game. The total cost for the season ticket is $775. Perform a three-squares analysis to write an equation and solve for the number of home games that season.

10) Three packs of markers cost $9.00 less than 5 packs of markers. Which equation best represents this situation?

A) $5x + 9 = 3x$

B) $3x + 9 = 5x$

C) $3x - 9 = 5x$

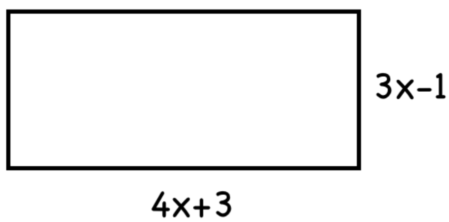
D) $9 - 3x = 5x$

11) A rectangle has length 5 inches and width 3 inches. If a similar rectangle has a width of 15 inches, what is its length?

12) The ratio of boys to girls in math class is 5:2. If there are 8 girls in the class, how many boys are there?

## APPENDIX N: UNIT 1 POST-TEST - INDIVIDUAL ASSESSMENT

1a) Find the expression for perimeter of the rectangle.




1b) What would be the perimeter if $x = 10$ ft?

2) If $x - 3 = 4 - 2(x + 5)$, then $x =$?

A) -3 B) -1 C) 1 D) $\frac{3}{2}$ E) $\frac{11}{3}$

3) What value of *n* makes the equation below have no solution?

$$2x + 2 = nx - 3$$

A) -2

B) 0

C) 2

D) 3

4) Solve the proportion $\frac{20}{x} = \frac{4}{x-5}$

5) Four times a number is two less than six times the same number minus ten. What is the number?

6) The scale on a map is 1 inch: 500 miles. If two cities are 875 miles apart, how far apart are they on the map?

7) Solve the equation $\frac{5}{6}x - \frac{2}{3} = 1$

8) Caleb and Ramon are beginning an exercise program to train for football. Ramon weighs 150 pounds and hopes to gain 2 pounds per week. Caleb weighs 195 pounds and hopes to lose 1 pound per week. Perform a three squares analysis to answer the following question:

If the plan works, how many weeks will it take for the boys to weigh **the same**?

9) Solve $-2(x-1) + 5x = 2(2x-1)$ for $x$.

10) Solve $6 - 2x - 1 = 4x + 8 - 6x - 3$.

## APPENDIX O: MIDTERM - INDIVIDUAL ASSESSMENT

Directions : Select the best answer for each question. Do NOT leave any answers blank.

1) If $3x - 6 = 18$, which is a solution for $x$?

A) 4

B) 6

C) 8

D) 24

2) Solve the equation $-\frac{1}{2}x = 10$.

A) $x = -20$

B) $x = 20$

C) $x = -5$

D) $x = 5$

3) If $2x + 4 = 16$, find the value of $x - 3$.

A) 3

B) 6

C) 7

D) 10

4) Which is an expression that describes "*eight less than some number n?*"

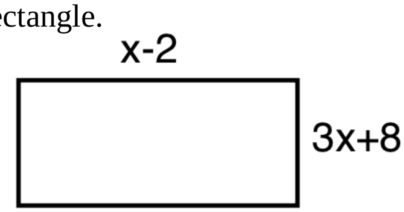
A) $n - 8$

B) $8 - n$

C) $n(8 - n)$

D) $\frac{8}{n}$

5) Find the expression for the perimeter of the rectangle.




A) $4x + 6$

B) $8x + 12$

C) $2x - 6$

D) $4x + 10$

6) Solve the linear equation $2(5x - 2) = 8x$.

A) -2

B) -1

C) 1

D) 2

7) Solve the proportion $\frac{x}{6} = \frac{2}{3}$.

A) $x = 3$

B) $x = 4$

C) $x = 9$

D) $x = 12$

8) The ratio of girls to boys in Algebra class is 3:5. If there are 12 girls, how many boys are there?

A) 7

B) 7.2

C) 20

D) 60

9) Solve the equation $\frac{2}{3}x - 7 = 3$ for $x$.

A) -6

B) 1

C) 8

D) 15

10) Solve the proportion $\frac{f+3}{12} = \frac{7}{2}$.

A) 39

B) 40.5

C) 43.5

D) 45

11) Alan is saving to take an ACT prep course that cost $350. So far, he has saved $180, and he adds $17 to his savings each week. How many more weeks must he save to be able to afford the course?

A) 2

B) 10

C) 11

D) 31

12) What value of A and B gives infinitely many solutions?

$$4x + B = Ax - 3$$

A) $A = 4, B = 3$

B) $A = 4, B = -3$

C) $A = -4, B = 3$

D) $A = -4, B = -3$

13) Solve $\frac{1}{2}x - \frac{3}{4} = \frac{1}{2}$.

A) $-\frac{1}{2}$

B) $\frac{5}{2}$

C) $\frac{7}{2}$

D) 4

14) Solve $-2(x + 2) = -2x + 1$

A) No Real Solution

B) Infinitely Many Solutions

C) $x = 0$

D) $x = 5$

15) Charleston Power Yoga charges $63 starting fee plus $12 per class. West Ashley Yoga charges no starting fee and $15 per class. How many classes will the cost be the same at both Yoga companies?

A) 3

B) 5

C) 21

D) 27

16) Find the slope of the line that goes through $(-9, -5)$ and $(3, -7)$.

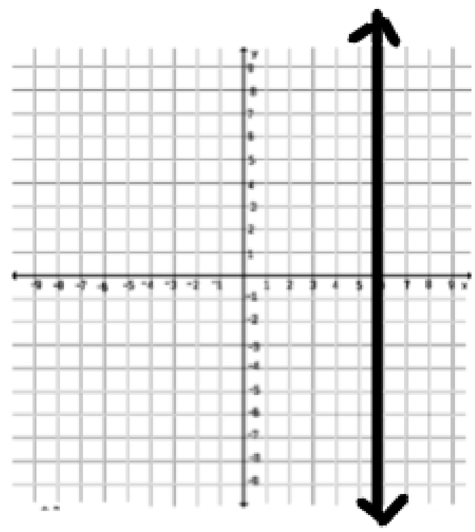
A) $-6$

B) $-\frac{1}{6}$

C) $\frac{1}{6}$

D) 6

17) Identify the slope of the line shown.



A) $-6$

B) 0

C) 6

D) *undefined*

18) Find the slope of the line $3x + 4y = 12$.

A) $-\frac{3}{4}$

B) $\frac{3}{4}$

C) 3

D) 4

19) Identify the x-intercept of the linear equation $5y = 4x - 20$.

A) $-5$

B) $-4$

C) 4

D) 5

20) Write the equation of the line in Slope-Intercept form that has slope $-2$ and passes through $(-2, -3)$.

A) $y = -2x - 1$

B) $y = -2x + 1$

C) $y = -2x - 3$

D) $y = -2x - 7$

21) Write the equation of the line in Point-Slope form that has slope $-\frac{2}{3}$ that passes through the coordinate $(-3,1)$.

A) $y - 3 = -\frac{2}{3}(x + 1)$

B) $y - 1 = -\frac{2}{3}(x - 3)$

C) $y - 1 = -\frac{2}{3}(x + 3)$

D) $y + 1 = -\frac{2}{3}(x - 3)$

22) Write the equation of the line in Slope-Intercept form that passes through $(5,1)$ and $(8,7)$.

A) $y = 2x - 9$

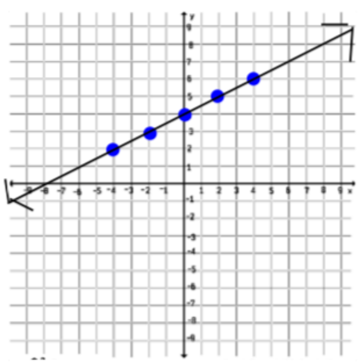
B) $y = -2x + 11$

C) $y = \frac{1}{2}x - \frac{3}{2}$

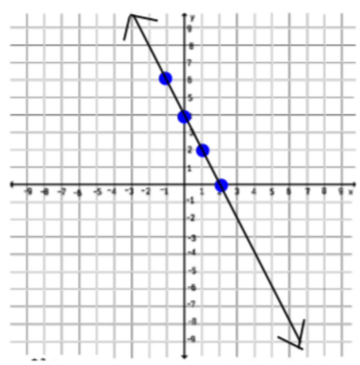
D) $y = -\frac{1}{2}x + \frac{7}{2}$

23) Identify the graph of the linear equation $y = -\frac{1}{2}x + 4$.

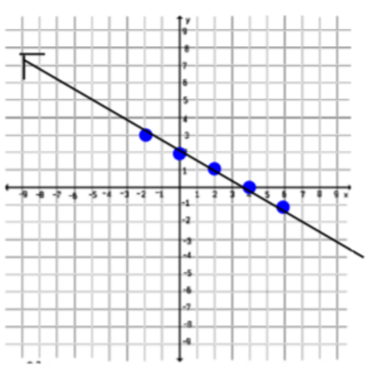
A)



B)



C)



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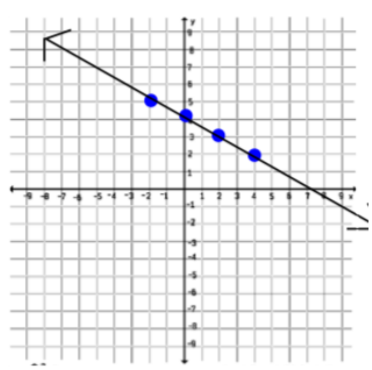
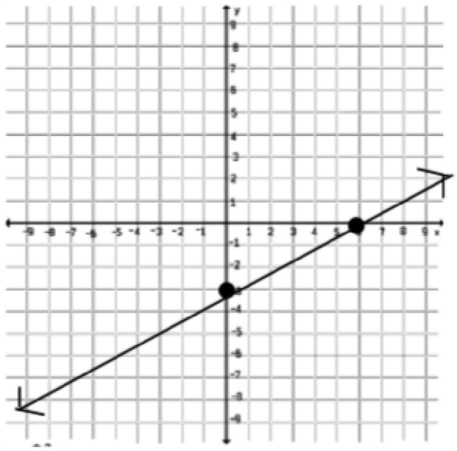
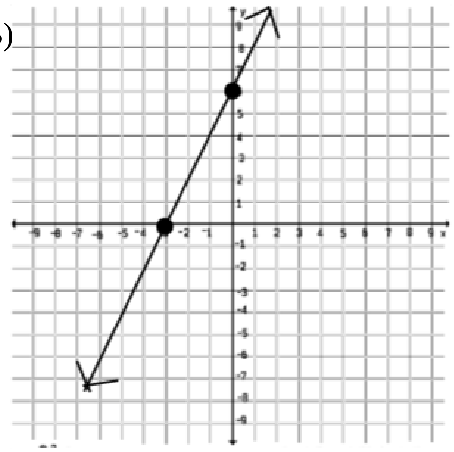
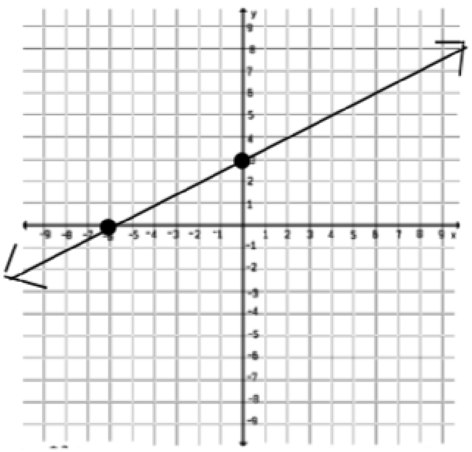
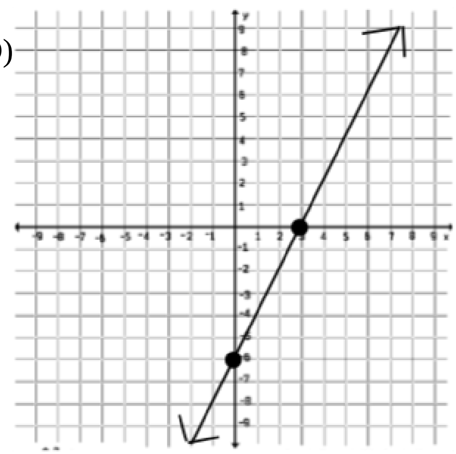


24) Which is of the following is the graph of the linear equation $-3x+6y=18$?

A) 

B) 

C) 

D) 

25) Write an equation of the line in Slope-Intercept form that is parallel to $y=-3x-4$ and passes through $(2,-7)$.

A) $y=-3x-13$

B) $y=-3x+13$

C) $y=-3x-1$

D) $y=-3x-4$

## APPENDIX P: MATH JOURNALS

**Journal 1 - Week 2**

Directions: In three or more sentences, describe, in detail, a topic that you have learned in Unit One that you feel most confident in explaining to another student. Refer to the word wall for math language words.

**Journal 2 - Week 4**

Directions: Please write a brief summary of what you accomplished today in class. Write at least three sentences explaining how to do the math that you learned. You must use at least one math language term in your explanation. You are also welcome to ask questions in your journal.

**Journal 3 - Week 6**

**Directions: Choose one of the following. Use three or more sentences to communicate to another student about how to perform the task.**

Task 1:

Explain in words how to graph a line that goes through (1,3) and has slope -2.

Task 2:

Explain in words how to graph the line $4x + 6y = 24$.

Task 3:

Explain in words how to find graph a line parallel to $y = \frac{1}{2}x - 4$.