2018

The Impact of Applying the First Two Steps of Polya's Four ProblemSolving Steps in an Advanced Mathematics Classroom

Christine A. Gray
University of South Carolina - Columbia

The Impact of Applying the First Two Steps of Polya's Four Problem-Solving Steps in an Advanced Mathematics Classroom

By

Christine A. Gray

Bachelor of Arts
Thiel College, 1987

Master of Education
Armstrong Atlantic State University, 2001

Master of Education
University of South Carolina, 2012

Submitted in Partial Fulfillment of the Requirements

For the Degree of Doctor of Education in

Curriculum and Instruction

College of Education

University of South Carolina

2018

Accepted by:

Leigh D'Amico, Major Professor

Suha Tamim, Committee Member

Jin Liu, Committee Member

Edwin Dickey, Committee Member

Cheryl L. Addy, Vice Provost and Dean of the Graduate School

DEDICATION

An entire village, including my wife, family, friends and colleagues encouraged me along the way. Thank you to my parents for always supporting me. This process started only a few weeks after the untimely passing of my mother. Family and friends supported my decision to continue even though my heart was heavy. I know you can see this mom, thank you for being my first teacher and my greatest role model. This one is for you!

ACKNOWLEDGEMENTS

I would like to express my deep gratitude to Dr. Leigh D’Amico, my research supervisor, for her patience, guidance, and encouragement. I would also like to thank Dr. Pat Read for listening and offering words of advice. My grateful thanks to Dora Fletcher who took the lead on all formatting issues and deadlines. Finally, a special thank you to Dora Fletcher, Bebe Cifaldi, Megan Deweese, Molly Lloyd, Sarah Owen, and Lawanda Dixon for sharing this experience from the beginning to the end. Thank you for keeping me together all the way to the finish line!

ABSTRACT

The teacher-researcher spent eight weeks as a participant observer in an advanced high school mathematics class in a small town in the southeast. The process of problem-solving was introduced to all twenty students from a combined IB Math SL and AP Calculus AB class. After the pretest, three students were selected, from under-represented groups, for in-depth observation, interviews, and collection of artifacts in order to determine the impact of problem-solving techniques. The work of these students, from underrepresented groups, was examined to gain insight into their experiences and thought processes surrounding problem-solving. The students answered questions about their attitude towards problem-solving. The students' attempts were marked for how far they were able go in solving the problem and the strategies that were utilized. The students were also interviewed to find out if their attitude was changing over the eight-week time period. Observations and interviews were coded and analyzed for themes and patterns. Three themes emerged related to the first two steps Polya's problem-solving strategies; the lack of problem-solving strategies introduced in previous math classes, the independence of the student, and the expectations of the student and/or their parents. The study attempted to examine the perspective of the students in using problem-solving techniques. The study represents an effort to gain new awareness about student learning that may have significant influence on teaching and learning problem-solving in an advanced mathematics classroom.

TABLE OF CONTENTS

LIST OF TABLES

CHAPTER 1
INTRODUCTION AND INSPIRATION

The teaching career of the teacher-researcher began in the middle of the math wars. This raised fundamental questions about the teaching of mathematics that frame my practice. “Is mathematics for the élite or for the masses? Are there tensions between excellence and equity? Should mathematics be seen as a democratizing force or as a vehicle for maintaining the status quo?” (Schoenfeld, 2004, p. 253). Drawing a circle in the sand in India is equivalent to drawing a circle on a blackboard in Australia. But who are the students drawing the circle, what is their background, and what is the context surrounding the reason for the circle to be drawn?

The purpose of public schools in the United States has been evolving since they were first instituted. Our nation is committed to public schools as democratic institutions, as places of learning in which people of all walks of life come to learn how to live together in a democratic society (Zeichner & Liston, 1996). Mathematics has also been evolving over this same time period. Initially, “one set of instructions from a school district in the 1890s instructed teachers that their students were to learn no more mathematics than would enable them to serve as clerks in local shops” (Schoenfeld, 2004, p. 256). Over the next century, there were many disputes and crises that changed the emphasis on mathematics. There were disputes over pure or applied mathematics, and crises involving world wars, Sputnik, and economic depression. Over the course of the last generation, there have been curriculum changes, textbook changes, technology changes, and demographic changes. All of these changes have had an impact on teaching mathematics in public schools.

The evolution of the public-school vision has moved from faith to citizenship to economic growth to equal opportunity (Labaree, 2011). In 1983, the National Commission on Excellence in Education produced a report entitled *A Nation at Risk.* The government wanted to focus on wealth production and thought it necessary to raise educational standards to achieve that goal. "*A Nation at Risk* invoked a crisis so far-reaching in its impact that it still governs the way we think about public education thirty years later" (Mehta, 2015, p.21). In 2002 the No Child Left Behind Act was signed into law. "The purpose of this title is to ensure that all children have a fair, equal, and significant opportunity to obtain a high-quality education and reach, at a minimum, proficiency on challenging state academic achievement standards and state academic assessments" (Labaree, 2011, p. 385).

Capitalism is dependent on its workforce. Labaree (2011) states, "Their consumer role focuses on the acquisition of education as a private good, for themselves and their children, an acquisition that can enhance their social opportunities in competition with others. From early in the history of American education, American families and individuals have looked on education as an important way to get ahead and stay ahead in a market society" (p. 390). Yet some are being left behind. "Perhaps the most fundamental shift in thinking that *A Nation at Risk* encapsulates is the elevation of the economic purposes of schooling over its many other purposes" (Mehta, 2015, p.23).

When public schools receive their state report cards, the main evaluation criteria are test scores in math and English. Our society has deemed these skills to be necessary in order to be a contributing member. Our schools try to produce a product that fits that description. Yet to be adequate with math skills is not the same as being a competent problem solver. Even advanced math students have trouble applying their math skills to a wide variety of complex problems. George Polya contributed to a wide range of math topics including analysis, number theory, geometry, probability, combinatorics, and graph theory (Alexanderson, 2000). Using Polya's seminal work, *How to Solve It*, a list of questions was generated to help students take the first step in problem-solving.

Alan Schoenfeld (2004) is a mathematician and researcher who has completed much research on mathematical teaching and learning. He is working with the Bill and Melinda Gates Foundation to create the Mathematics Assessment Project. The National Council of Teachers of Mathematics (NCTM) produced the *Handbook of Research on Mathematics Teaching and Learning*. The handbook was published in 1992 near the beginning of the teacher-researcher's teaching career and has influenced her teaching ever since. Building on the foundational work of George Polya, Alan Schoenfeld and the NCTM, this study explores the impact of Polya's questioning techniques for problem-solving in an advanced mathematics classroom in high school.

All three of these resources indicate that problem-solving skills can provide access for the underprivileged, be a bridge to the university, and provide a discerning electorate for our democracy (Grouws, 2003; Polya, 1985; Schoenfeld, 2004). This is why it is important to investigate whether the first two steps of George Polya's problem-solving techniques help students to engage in problem-solving. This study investigates the impact of how understanding the problem will help students solve the problem in an advanced mathematics class.

Problem of Practice

With teachers, schools, districts, and states being held accountable through state and federal test scores, more and more emphasis is being placed on students' ability to complete math problems correctly. The broad problem to be considered: if mathematics educators wish to develop problem-solving skills, they must teach students how to ask and answer questions on their own. Sometimes solving a math problem involves applying a simple skill or algorithm, which leads to the misconception that all math problems can be solved quickly and that there is only one correct way and one right answer (Pinkerton & Shafer, 2013). Students have a difficult time persevering in problem-solving, which is considered to be a cornerstone of the Eight Mathematical Practice Standards of the Common Core State Standards (National Council of Teachers of Mathematics, 2010). This also aligns with the National Council of Teachers of Mathematics Teaching Practices, supports the productive struggle in learning mathematics, and implements tasks that promote reasoning and problem-solving (National Council of Teachers of Mathematics, 2010). What do students do when faced with a math problem that they do not easily know how to solve? Frequently, they just give up. This study provides an opportunity to determine if questioning techniques help students understand the problem and hence persevere in solving it. The research question for this study is: *What impact will the application of Polya's first two steps of problem-solving have on underrepresented groups in an advanced high school mathematics classroom?*

Purpose

The primary purpose of this action research study is to investigate the impact of Polya's first two steps in problem-solving: understanding the problem and organizing a plan, to help students persevere in the problem-solving process. Given appropriate problem-solving exercises and appropriate resources, in what ways can the teacher facilitate or guide the students in working collaboratively and individually to successfully complete the task? The NCTM states, "Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in a productive struggle as they grapple with mathematical ideas and relationships" (National Council of Teachers of Mathematics, 2010, p. 2). On problem-solving, "Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem-solving and allow multiple entry points and varied solution strategies" (National Council of Teachers of Mathematics, 2010, p. 2). Understanding the problem may provide access for the students to successfully find one or more of the multiple entry points to the problem. Once the students gain a foothold on the problem, they may be able to persevere in solving the problem.

Framework

The evolution of the public-school vision has moved from faith to citizenship to economic growth and finally to equal opportunity (Labaree, 2011). Horace Mann's *Twelfth Annual Report*, published in 1848, maintained schools were to focus on citizenship. At the beginning of the twentieth century, Dewey and others introduced the progressive movement. Child-centered learning was a means of preparing students for their future social roles. *Brown versus the Board of Education of Topeka, Kansas* opened the doors to equal opportunity and social mobility. Labaree stated, 'In this decision, republican equality for citizens had turned into an equal opportunity for consumers' (p.387).

In 1983, the National Commission on Excellence in Education produced a report titled *A Nation at Risk.* Many of the problems identified in 1983 remain unaddressed, and stagnant student achievement continues to challenge educators and administrators. In the 1990s the reform effort centered around school choice, a battle between public and private or charter schools. Students were now educational consumers and education is to be considered a consumer good and now in the hands of the private sector (Labaree, 2011). In 2002 the No Child Left Behind Act was signed into law. This shows an equal interest in reforming standards as well as civil rights.

Related Literature Review

Problem-solving in a mathematics classroom has had varied roles over the years. Three roles have dominated the discourse: problem-solving as context, problem-solving as skill, and problem-solving as art. According to Stanic and Kilpatrick (1989), the teacher is responsible for choosing appropriate problems and for guiding the students from where they are to where they need to be in order to solve the problem. Society has had differing views of the worth of problem-solving over the past century. Polya tried his best to bridge the gap between the context, skill, and art of problem-solving in our society.

The *School Science and Mathematics* Journal published an article titled "Journey toward teaching mathematics through problem-solving" by Lynae Sakshaug and Kay Wohlhuter (2010). The article highlights the challenge of teaching mathematics through problem-solving since most teachers were taught through exercises. This is, therefore, a fundamental shift for educators. One of the five contexts evaluated in the results of this study involved the impact on students. Even though there were periods of frustration for both teachers and students, most teachers became more confident about their students' abilities to be problem solvers (Sakshaug & Wohlhuter, 2010). In the Common Core State Standards for Mathematics, the very first Standard for Mathematical Practice is that students should "understand problems and persevere in solving them" (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). The common practice in a mathematics classroom over the years has been to watch the teacher do a problem, do a problem with the teacher, and then try to do a problem on your own. Teaching mathematics through problem-solving is a chance to engage students more in the process and motivate them to pursue several valid attempts at solving the problem with little to no help along the way.

Teaching Children Mathematics, published by the National Council of Teachers of Mathematics featured an article focused on encouraging problem-solving among teachers. O'Donnell (2006) presents the experiences of three teachers and asks herself, "How effective am I in helping teachers understand problem-solving as presented in the NCTM Process Standards?" (p. 346). The study follows three teachers through an action research process and concluded, "It can help us as teachers realize what problem-solving is and the impact it can have" (O'Donnell, 2006, p. 351). This is not an easy task as evidenced by the numerous research studies focused on problem-solving. One might assume that advanced-level math students would be fairly confident and self-sufficient when it came to problem-solving, but they have been raised and immersed in a system that doesn't reward problem-solving skills. As such, the students believe they are good at math and become disheartened when the solution to a problem does not come quickly to them.

The Canadian Journal of Action Research published *Problem-based instruction: Getting at the big ideas and developing learners*, by Laura Inglis and Nicole Miller. A team of teachers from a large southern Ontario school district completed an action research project in grade 3 and grade 6. This study involved elementary and middle school students, so some of the techniques may or may not translate to older students. Inglis and Miller (2011) stated, “Completing this project provided us with concrete evidence to support our opinions and the research we had read about” (p. 11). “Observational analysis suggested a significant improvement in students’ ability to use math vocabulary in class, to fully explain their thoughts when writing solutions, and an increase in the sophistication of math strategy chosen for a given task” (p.8).

The Canadian Journal of Action Research published *Asking good questions: Promoting greater understanding of mathematics through purposeful teacher and student questioning*, by Susan Di Teodoro, Sharon Donders, Joy Kemp-Davidson, Peggy Robertson and Lori Schuyler. Once again, this study was completed in an elementary school setting and it needs to be determined as to whether it will translate to a high school setting. The researchers concluded, “Study results highlight the importance of student involvement in sorting and defining questioning criteria” (Di Teodoro, Donders, Kemp-Davidson, Robertson, & Schuyler, 2011, p. 18). Overall, the researchers felt both the quantity and quality of student questions increased. “They [the students] understood that questions were another avenue to better understanding” (Di Teodoro, Donders, Kemp-Davidson, Robertson, & Schuyler, 2011, p. 26).

Based on the need for more emphasis on problem-solving within classrooms (or with students) as determined through the literature, this action research plan focuses on structures of George Polya's four steps of problems solving outlined in his 1945 book, *How to Solve It: A new aspect of mathematical method*. These methods have been around for centuries, but Polya's book has become a model of simplicity and a collector's item for problem solvers. Polya's problem-solving techniques include: 1. understand the problem, 2. devise a plan, 3. carry out the plan, and 4. look back. The book gives many examples of the four stages and is cited by many textbooks as well as the NCTM. This study will focus on applying the first two steps of trying to understand the problem by using questioning techniques and devising a plan. Polya taught teachers to examine their approach to problem-solving "Much can be gained by taking the time to reflect and look back at what you have done, what worked, and what didn't. Doing this will enable you to predict what strategy to use to solve future problems" (p. xvi).

The NCTM *Executive Summary of the Principles to Actions* report states, "the standards-based education movement was an unprecedented initiative to promote systemic improvement in mathematics education" (National Council of Teachers of Mathematics, 2010, p. 1). With so much to work on the NCTM readily admits, "Too few students – especially those from traditionally underrepresented groups – are attaining high levels of mathematics learning" (National Council of Teachers of Mathematics, 2010, p. 2). This study will focus on underrepresented groups in an advanced mathematics classroom.

Research of a similar nature has not been done before. This particular research will add to existing literature and advance research in a high school mathematics classroom for underrepresented groups. In the review of theoretical and research literature, it will be noted that most of the research so far leads to the conclusion that students will benefit from the emphasis on problem-solving techniques. Even though the general consensus has been that emphasis on problem-solving benefits students, there are also challenges to this research. The biggest challenge has been the time it takes to implement the problem-solving techniques. It may take a while for the teacher and the students to learn how to agree, disagree, think about, talk about, and share their thoughts on solving a problem (Di Teodoro, Donders, Kemp-Davidson, Robertson, & Schuyler, 2011).

Summary

This action research study focuses on a mathematics class with an enrollment of 20 students. The teacher-researcher used a pre-assessment related to Polya's method to determine all students' current processes and skills related to problem-solving. Using the results of the pre-assessment, the teacher-researcher selected three students from underrepresented groups to participate in this study. The teacher-research integrated Polya's first two steps within an eight-week unit. During implementation, the teacher-research observed, conducted interviews, and followed the progress of the selected students through weekly assessments.

A post-assessment was administered to all students to explore progress and growth. The teacher-researcher specifically explored the impact on the selected students to determine the success of this approach with underrepresented students. This research may help advance ideas about how to teach mathematics through problem-solving.

CHAPTER 2

LITERATURE REVIEW

It has been my experience when public schools receive their report cards the main evaluation criteria are test scores in math and English even though other criteria, such as graduation rate play a part in the rating. It appears as though our society has deemed these skills to be necessary in order to be a contributing member. Therefore, our schools must produce a product that fits that description. To be adequate with math skills though is not the same as being a competent problem solver. Even advanced math students have trouble applying their math skills to a wide variety of complex problems (Polya, 1985). This is why it is important to investigate whether George Polya's problem-solving techniques will help students to persevere in problem-solving. It will be a small first step in advancing our democratic society.

"Polya was one of the most influential mathematicians of the 20th century" (Sury, 2014, p. 305). Sury describes Polya as having some innate ability not only to solve problems but to explain them in a way that others could understand. According to Sury, Polya was very critical of many textbooks for giving students the wrong idea about the presentation of mathematics. Too often students lose interest in the theoretical aspects of mathematics and never come to see any practical applications that may spark more interest. It was through two of George Polya's (1985, 1981) books, *How To Solve It: A New Aspect of Mathematical Method* and *Mathematical Discovery:*

On Understanding, Learning, and Teaching Problem-solving, that it was decided to focus this study on his problem-solving techniques. The teacher-researcher was introduced to the works of Polya during undergraduate studies and influenced by Polya again during graduate studies. So, the teacher-researcher is familiar with Polya's work and is confident in implementing hi strategies in the classroom.

Polya's (1985) four step process includes, "<u>First Principle</u>: *Understand the problem*, <u>Second Principle</u>: *Devise a plan,* <u>Third Principle</u>: *Carry out the plan,* and <u>Fourth Principle</u>: *Look back*" (p. xvi). Polya believed that the teacher needed to be trained not only in mathematics but in the process of how to teach problem-solving skills (Polya, 1981). This plan is discussed in more detail in the section on teacher beliefs and implementation challenges.

The approach to classroom instruction differs when observing a traditional approach to teach concepts versus a problem-solving approach. Traditionally, teachers worked examples, "that were short, out of context, and symbolic… with emphasis on procedural skills", and then, students follow the teacher's lead attempting to duplicate the results (D'Ambrosio, 2003, p. x). The problem-solving approach involves much more. The problems that are chosen "must have embedded in them the mathematics that is to be learned" (D'Ambrosio, 2003, p. xi). The problems must engage the students and the teachers must create an atmosphere in which all the students can be successful. Problem-solving is a scientific process that starts with understanding the problem and continues through evaluating and reflecting on the solution to the problem. Traditionally, research on math education has been narrowly focused on mathematical thinking and learning. More recent research shows a growing interest in, "exploring social aspects of mathematics" (Foote & Bartell, 2011, p.45). Mathematics is often considered objective and culture-free, but a new perspective is developing that we must examine the issues of equity and diversity (Foote & Bartell, 2011). Foote and Bartell followed 26 pre- and post-doctorate fellows. The authors and participants were teacher educators with k-12 experience and a research focus that included effective teaching practices for traditionally marginalized students. The findings showed that the life experiences of the researcher served to orient them to issues of equity and diversity in mathematics teaching and learning (Foote & Bartell, 2011). It will be important for the teacher-researcher to bear in mind race, gender, class, and religion when relating to these same issues with the student participants.

Problem of Practice

With teachers, schools, districts, and states being held accountable through state and federal test scores, more and more emphasis is being placed on students' ability to complete math problems correctly. Sometimes solving a math problem involves applying a simple skill or algorithm, which leads to the misconception that all math problems can be solved quickly and that there is only one correct way and one right answer (Pinkerton & Shafer, 2013). Students have a difficult time persevering in problem-solving which is a cornerstone of the Eight Mathematical Practice Standards of the Common Core State Standards (National Council of Teachers of Mathematics, 2010). This also aligns with the National Council of Teachers of Mathematics Teaching Practices which include: a) support productive struggle in learning mathematics and b) implement tasks that promote reasoning and problem-solving (National Council of Teachers of Mathematics, 2010). What do students do when faced with a math problem that they do not easily know how to solve? Most often they just give up. This study provides an opportunity to determine the impact of applying the first two of Polya’s (1985) four steps to problem-solving in advanced high school mathematics class. The entire class went through the same steps and procedures. The focus is on underrepresented groups. Case studies of the three participants help our understanding of how traditionally marginalized students solve problems in math class.

Purpose of Study

The primary purpose of this action research study is to determine the impact of applying the first two steps of Polya’s (1985) four steps to problem-solving in an advanced high school mathematics class. The teacher's responsibility in this study is to provide appropriate problem-solving exercises and use appropriate questioning to guide the students in working collaboratively and individually to set up the problem so they may persevere in completing the problem. The NCTM states, "effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships” (2010). On problem-solving, "effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem-solving and allow multiple entry points and varied solution strategies" (2010). The problems used in this study are contextualized by open-ended Advanced Placement Calculus problems using the Fundamental Theorem of Calculus.

The following sections summarize related literature including studies about Polya’s (1985) four steps of problems solving and the realities of trying to implement them. Many studies involved elementary and middle school students with a few studies focusing on high-school students. Robert Moses' Algebra Project is a prime example of a program implemented to encourage minority students to see mathematics as a gateway to a better life. "Math literacy is the key to 21st century citizenship" (Silva, Moses, Rivers & Johnson, 1990). Since this study involves advanced high school mathematics students, the research leads us to believe there is a need for more studies pertaining to high school students. Some of the implementation studies revolved around elementary pre-service teachers, which give a unique perspective on some of the pitfalls of trying to reorganize math content to focus on problems solving. According to Polya (1981), "Solving a problem means finding a way out of difficulty, a way around an obstacle, attaining an aim which was not immediately attainable…solving problems can be regarded as the most characteristically human activity" (p. ix). For this study, problem-solving is defined as "attaining an aim which was not immediately attainable" (p. ix). The four steps Polya (1985) endorses are defined as 1) understanding the problem, 2) devising a plan, 3) carrying out the plan, and 4) looking back. Polya (2002) stated, "Guessing comes to us naturally. Everybody tries to guess and does not have to be taught. What has to be taught is reasonable guessing. And especially what has to be taught is not to believe your own guesses but to test them" (p. 44).

This study looks at the impact of applying the first two steps of Polya's (1985) four steps to problem-solving in an advanced high school mathematics class. The students and teacher together developed the tactics of problem-solving, by applying Polya's (1985) four steps of problem-solving. Polya (1981) stated, "solving problems is a practical art…you can learn it only by imitation and practice" (p. ix). Problems with multiple access points from the students' prior learning were used to test the students' ability. Discussion about the steps taken ensued to gauge the students understanding. The students looked back on the work, whether they reached a solution or not, and reflected on what they learned from the problem-solving experience. By practicing the art of problem-solving together the class built a foundation of processes and base knowledge from which to draw upon for future problem-solving efforts. According to Polya (1982), "our knowledge about any subject consists of *information* and of *know-how*…what is *know-how* in mathematics? The ability to solve problems…requiring some degree of independence, judgment, originality, creativity" (p. xi). Polya considers it the duty of a high school math teacher to encourage methodical work in problem-solving (Polya, 1981). The following sections include a discussion of the research related to the historical and theoretical perspectives of mathematics and problem-solving, teacher beliefs about problem-solving, student challenges with problem-solving, implementation challenges for teachers, and assessment challenges.

Historical Perspective

The history of the role of problem-solving in mathematics education has changed over time. The following three themes: problem-solving as context, problem-solving as skill, and problem-solving as art will illustrate the varied roles of problem-solving in the mathematics classroom. In their seminal work, George Stanic and Jeremy Kilpatrick (1988) find themes for problem-solving throughout the history of mathematics.

Problem-solving as Context

One aspect of problem-solving involves the context for the use of problem-solving. Stanic and Kilpatrick (1988) identified five sub-themes: problem-solving as justification, problem-solving as motivation, problem-solving as a vehicle, problem-solving as recreation, and problem-solving as practice. "Historically, problem-solving has been included in the mathematics curriculum in part because the problems provide justification for teaching mathematics at all" (D'Ambrosio, 2003, p. 42). Problem-solving as motivation assumes that a skill introduced one day will be a stepping stone to solving a problem in the future. Problem-solving as a vehicle uses problems for the purpose of intuitively discovering techniques, concepts, and skills (Stanic & Kilpatrick, 1988). Problem-solving as recreation draws out the natural curiosity of the learner, which is also part motivation. If the learner is having fun with mathematics then the learner may pursue it more fully and the problems do not need to have an ulterior motive beyond having fun. Problem-solving as practice, to reinforce skills, is still the most widely used theme in the mathematics curriculum (Stanic & Kilpatrick, 1988).

Problem-solving as Skill

Even though in Polya's (1985) seminal work, *How to Solve It: A New Aspect of Mathematical Method*, he laid out a four-step plan for problem-solving, he confessed in interviews years later that his steps were not meant to be an algorithm for problem-solving (Polya, 2002). Yet the theme of problem-solving as skill has become more dominant than problem-solving as context. "Putting problem-solving in a hierarchy of skills to be acquired by students leads to certain consequences for the role of problem-solving in the curriculum" (Stanic & Kilpatrick, 1989, p. 43). The hierarchy difference between solving routine and non-routine problems leads to a select few advanced students being the only ones who ever reach the level of working non-routine problems. Therefore, some students are never exposed to non-routine problems.

Problem-solving as Art

This is where Polya's (1985) experience in learning and teaching mathematics led him to try to teach teachers how to approach problem-solving as an art, the art of discovery. Polya (1985) equates problem-solving to other practical arts, "like swimming, or skiing, or playing the piano" (p. ix). Stanic and Kilpatrick (as cited in D'Ambrosia, 2003) believed Polya recognized, "that techniques of problem-solving need to be illustrated by the teacher, discussed with the students, and practiced in an insightful, non-mechanical way" (p. 44). The teacher is the key. The teacher is responsible for choosing appropriate problems and for guiding the students from where they are to where they need to be in order to solve the problem. Stanic and Kilpatrick see "problem-solving as art as the most defensible, the fairest, and the most promising" (as cited in D'Ambrosio, 2003, p. 44). For the mathematics teacher, this presents the challenge of developing artistic intuition in students.

Society has had differing views of the worth of problem-solving over the past century. Horace Mann's *Twelfth Annual Report*, published in 1848, maintained schools were to focus on citizenship (Mann, 1848). If by citizenship Mann believes that students need to be productive members of their respective communities, then once again the ability to be a problem solver is very important. *Brown versus the Board of Education of Topeka, Kansas* (1954) opened the doors to equal opportunity and social mobility. Labaree (2011) stated, "in this decision, republican equality for citizens had turned into equal opportunity for consumer" (p.387). Therefore, problem-solving now takes on the importance of not just helping citizens be productive workers but also intelligent consumers. In 1983, the National Commission on Excellence in Education produced a report titled *A Nation at Risk.* The government wanted to focus on wealth production and thought it necessary to raise educational standards to achieve that goal. "*A Nation at Risk* invoked a crisis so far-reaching in its impact that it still governs the way we think about public education 30 years later" (Mehta, 2015, p.21). In the teacher-researcher's twenty-five years in public education, mathematics education has become a measuring stick for how well a school is performing yet standardized tests have not found a way to measure a student's problem-solving ability.

In 2001, the No Child Left Behind Act was signed into law. "The purpose of this title is to ensure that all children have a fair, equal, and significant opportunity to obtain a high-quality education and reach, at a minimum, proficiency on challenging State academic achievement standards and state academic assessments" (Labaree, 2011, p. 385). If these academic assessments do not measure open-ended problem-solving achievement, then how can the assessments provide the equality and opportunity society we want students to have. Measuring students and schools are still a work in progress even in the high-stakes testing venue of Advanced Placement exams.

The College Board offers examinations around the world to high school students wishing to test their proficiency at college level courses (College Board, 2017). Two of the three mathematics exams are Advanced Placement Calculus AB and Advanced Placement Calculus BC. These two exams offer students the opportunity to show their expertise in college level calculus. Scoring well on the exam could mean college credit for the student. On the Advanced Placement Calculus AB and BC exams, there are basic calculus free response questions that need to be graded by hand and not scanned through a machine. The College Board (2017) describes their grading process as "scored by thousands of college-faculty and expert AP teachers…thoroughly trained and moderated for fairness and consistency…[which] maintains accuracy of the scoring standards" (p. 2). The International Baccalaureate Organization [IBO] also assesses students around the world. There are over 4000 teachers around the world involved in the grading process for IBO exams which are also moderated for fairness and consistency (International Baccalaureate Organization, 2017). "Adopting new assessment techniques consistently ranks as one of the top concerns for teachers as they shift to teaching through problem-solving" (D'Ambrosio, 2003, p. 48)

Theoretical Perspective

At the beginning of the twentieth century, Dewey and others introduced the Progressive Movement (Schiro, 2013). This learner-centered ideology encouraged teachers to study children as they were. Educators should "watch them carefully, …listen to them intently, …collect data about them" as a means of preparing students for their future social roles (Schiro, 2013, p. 173).

> Lester Frank Ward emphasized that men had the ability to influence the social world in which they lived through their application of intelligence to the problems of their society and suggested that education through development of intelligence could influence society to be a more just and equitable place for people. (Schiro, 2013, p. 174)

Even in the 1880s and 1890s problem-solving was considered a priority so students could influence the social world they lived in. Stemhagen (2016) states, "deep and unresolved tensions in the philosophy of mathematics and the philosophy of mathematics education have made it difficult for promising pedagogies to be enacted" (p.

95). Stemhagen (2016) reflected on Dewey who suggested a philosophy of mathematics that would support social and moral development. He continues to espouse Dewey's philosophy of mathematics as one that will support democracy by using a student's life experiences as a means for solving problems (Stemhagen, 2016). Considering that Dewey's theories have been around for over 100 years and they still elicit strong positive and negative feelings about education, may mean the theories still have meaning in today's public schools. Stanic & Kilpatrick stated, "Dewey did not often use the term problem-solving, but it is clear that problems and problem-solving were crucial in Dewey's view of education and schooling" (as cited in D'Ambrosio, 2003, p. 45). In Dewey's lab school, math instruction supported democratic mathematical education which was a bit constructivist and a bit traditionalist (Stemhagen, 2016). Dewey endorsed a learner-centered ideology where the child's interests come first and the children learn by doing and experiencing (Schiro, 2013).

This is the same philosophy Polya (1981) endorses to become a better problem solver. "Learner-Centered educators view learning from a constructivist perspective" (Schiro, 2013, p. 135). In applying a portion of Polya's (1985) four steps of problem-solving, the students were afforded the opportunity to create their own meaning for a particular complex problem. The students brainstormed, investigated, and explored different methods for a solution. The students began individually and then shared experiences as they worked towards the goal of solving the problem. Students' ways of thinking may not match mine or any teachers, but if the teacher can place the students in the right setting or provide an intriguing stimulus then a connection may be made that will enhance the students' ability to solve the problem. This learner-centered process was cultivated throughout the study, so the students could use their prior knowledge and experience to solve problems.

This study follows a constructivist perspective while also leaning towards Dewey's goal of centering the learning around the students. "Democratic Mathematics Education promotes engaging in active inquiry and learning the power and limits of math as a promising path toward dissolution of the lines that keep society segmented" (Stemhagen, 2016, p. 108). Polya (2002) stated, “the most important action of learning is to discover it by yourself…what you discover by yourself will last longer and be better understood” (p. 42). So, Polya’s philosophy of problem-solving matches the learner-centered ideology and the social constructionist philosophy of education.

Teacher Beliefs and Implementation Challenges

It is the role of the teacher that will be discussed in this section. Grouws (2003) states the matter this way:

> For the vast majority of students, becoming mathematically proficient and developing problem-solving expertise involve studying mathematics under the guidance of a well-qualified teacher who selects problems that embody important mathematical ideas and then ensures that the mathematics involved in these problems is brought to the surface and made explicit. (p. 129)

The *School Science and Mathematics Journal* published an article that highlights the challenge of teaching mathematics through problem-solving when most teachers were taught through exercises (Sakshaug & Wohlhuter, 2010). A three-credit college course titled *Teaching Elementary School Mathematics* was the context for the study. The intent of the course was to help teachers become better problem solvers and help them incorporate problem-solving in their math lessons (Sakshaug & Wohlhuter, 2010). This is a fundamental shift for educators who were taught math in a traditional way that involved skill and drill practice with rote memorization. One of the contexts evaluated in this study involved the impact on students. With appropriate guidance, the students were able to solve complicated problems independently. Even though there were periods of frustration for both teachers and students, most teachers became more confident about their students' abilities to be problem solvers. "As teachers attempted to implement a problem-based instructional approach, often conflicting actions occurred" (Sakshaug & Wohlhuter, 2010, p. 407). Teachers encountered challenges such as successfully implementing the problem-solving approach on one problem and then not on the next one. Some teachers tried group work and then quickly got away from group work because it was hard to manage (Sakshaug & Wohlhuter, 2010). The teachers would journal about their experiences and acknowledge the importance of facilitating group interactions but were not sure how to go about doing that. The teachers found it difficult to give up control. The teachers also found that their own comfort level with the content made it difficult to gauge whether the methods the students were using were all appropriate. "Although questions asked by students are an important and productive part of every successful mathematics lesson, a teacher…can pose questions that …lead directly to the development of… understanding" (Grouws, 2003, p. 137).

Another implementation challenge is the textbook. A study was conducted over a fifteen-week problem-solving course requiring graduate students to choose a mathematical strand or a problem-solving strategy in order to create tasks for their classroom (O'Donnell, 2006). A group of teachers participating in the problem-solving course was given the assignment of finding problems in their textbook that could be used with their students. The teachers realized their adopted textbook did not contain satisfactory problems and they subsequently changed to a textbook with a spiraling format (O'Donnell, 2006). Not having an appropriate textbook was an unexpected outcome of the study. The teachers involved in this study found it necessary to model strategies to lessen the fear students have when it comes to solving complex math problems. The teachers concluded that problem-solving needs to be an interactive process, but teachers must be able to analyze the tasks and classify them into categories based on the cognitive demands of the task (O'Donnell, 2006). Since textbooks might not be the appropriate resource for problems, it is necessary for teachers to be able to share, create, and discuss problems with each other to build a library of resources.

Textbooks have changed over the years to the point where they no longer look like the books that were used by the parents of the students that teachers are currently teaching. The new textbooks do not follow the traditional presentation of examples then practice problems. Now there are colorful illustrations, assignments with fun names and topics from a variety of contexts (Schoenfeld, 2004). Schoenfeld (2004) thought this had a tendency to alienate parents who no longer felt comfortable trying to help their child with their homework.

In the Common Core State Standards for Mathematics, the very first Standard for Mathematical Practice is that students should “understand problems and persevere in solving them” (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). The common practice in a mathematics classroom over the years has been to watch the teacher do a problem, do a problem with the teacher, and then try to do a problem on your own. Polya (1981) endorses teaching mathematics through problem-solving as a chance to engage students more in the process and motivate them to make several valid attempts at solving the problem with selected guidance along the way. O'Donnell's (2006) article about becoming a better problem-solving teacher, presents the experiences of three teachers and asks, "how effective am I in helping teachers understand problem-solving as presented in the NCTM Process Standards?" (p. 346). The professor found opportunities to refine the course as she learned that her own perceptions needed to be analyzed. "I wondered why teachers were not using problem-solving as a foundation for their teaching and realized that my actions were being translated into their teaching practices and I need to practice the approach I hope the teachers adopt" (O'Donnell, 2006, p. 351). The study followed three teachers through an action research process and concluded, "it can help us as teachers realize what problem-solving is and the impact it can have" (O'Donnell, 2006, p. 351). My study will explore advanced level mathematics that currently has the scarcest amount of research. One might assume that advanced level mathematics students would be fairly confident and self-sufficient when it came to problem-solving, but they have been raised and immersed in a system that doesn't reward problem-solving skills. So, the students believe they are good at math and become extremely agitated or disheartened when the solution to a problem does not come quickly to them (Polya, 2002).

Inglis and Miller (2011) took a team of teachers from a large southern Ontario school district and completed an action research project in grade 3 and grade 6 that focused on problem-solving skills. Inglis and Miller (2011) stated, “completing this project provided us with concrete evidence to support our opinions and the research we had read about" (p. 11). This research included studies of new problem-solving strategies, engaging in reverse learning activities where students started with an exemplar and then graded their own work, and how to break problems down into steps. These teachers increased the frequency of utilizing work samples, using math exemplars, introducing vocabulary, scaffolding units, and providing real-life problem-solving opportunities (Inglis & Miller, 2011).

Di Teodoro et al., (2011) completed a study in an elementary school setting with four teachers, where they learned to define and ask deeper, more meaningful questions. The research question, "how does a focus on questioning in mathematics affect teacher and student abilities to ask good questions during problem tasks?" was answered with both quantity and quality of student questions increasing (p. 26). The researchers concluded, "study results highlight the importance of student involvement in sorting and defining questioning criteria" (p. 18). Overall, the researchers felt the students "understood that questions were another avenue to better understanding" (p. 26).

The research plan is to model a portion of George Polya's (1985) four steps of problems solving outlined in his book, *How To Solve It: A New Aspect Of Mathematical Method*. These methods have been around for centuries but Polya's (1985) book has become a model of simplicity and a collector's item for problem solvers. Polya's (1985) problem-solving techniques include: 1) understand the problem, 2) devise a plan, 3) carry out the plan, and 4) look back. The book gives many examples of the four stages and is cited by many textbooks as well as the NCTM. It will provide clear guidelines to the process the teacher-researcher will implement in the action research. Polya (1985) taught teachers and students to ask the following questions:

First Principle: *Understand the problem*

- Do you understand all the words used in stating the problem?
- What are you asked to find or show?
- Can you restate the problem in your own words?
- Can you think of a picture or diagram that might help you understand the problem?
- Is there enough information to enable you to find a solution?

Second Principle: *Devise a plan*

Strategies include:

- guess and check;
- look for a pattern;
- make an orderly list;
- draw a diagram;
- solve a simpler problem;
- use symmetry;
- use a model;
- consider special cases;
- work backward; and
- use direct reasoning.

Third Principle: *Carry out the plan*

All you need is care and patience, given you have the necessary skills. Persist with the plan you have chosen. If it continues not to work discard it and choose another. Don't be misled, this is how mathematics is done, even by professionals.

Fourth Principle: *Look back*

Much can be gained by taking the time to reflect and look back at what you have done, what worked, and what didn't. Doing this will enable you to predict what strategy to use to solve future problems. (Polya, 1985, p. xvi)

The NCTM has published many books, articles, and reports that will be used to further this action research project. The Executive Summary of the Principles to Actions report states, "the standards-based education movement was an unprecedented initiative to promote systemic improvement in mathematics education" (National Council of Teachers of Mathematics, 2010, p. 1). With so much to work on the NCTM readily admits, "too few students – especially those from traditionally underrepresented groups – are attaining high levels of mathematics learning" (National Council of Teachers of Mathematics, 2010, p. 2).

Two more challenges teachers face when trying to implement problem-solving strategies are time and understanding. Instructional time needs to be devoted to learning the strategies for productive problem-solving. Teachers must demonstrate the strategies and students must have the opportunity to utilize the strategies. With all of the pressure on schools to have students pass standardized tests, how does the teacher justify using class time to work on problem-solving if it takes time away from some other standard that will be tested? In a study of preservice teachers, some participants referred to external factors, such as administrative expectations as having an influence on their role in the classroom (Conner, Edenfield, Gleason, & Ersoz, 2011). There is no way to know how many of these preservice teachers will change their beliefs after the first few years of teaching. Will the new teachers be overwhelmed with the new expectations or will they have the support they need to follow through with their newfound respect for problem-solving? Will the standards become the priority or will the students be the priority?

Assessment Challenges

In the report, *A Nation at Risk* (1983), public education was described as "eroded by a rising tide of mediocrity that threatens our very future as a Nation and a people" (United States. National Commission on Excellence in Education). "Education is a primary concern for our country, and testing is a primary tool of education" (US Congress, 1992, p. iii). According to the US Congress (1992), there were several highlights in the report about the history of standardized testing including:

- Since their earliest administration in the mid-19th century, standardized tests have been used to assess student learning, hold schools accountable for results, and allocate educational opportunities to students.
- Throughout the history of educational testing, advances in test design and innovations in scanning and scoring technologies helped make group-administered testing of masses of students more efficient and reliable.
- High-stakes testing is not a new phenomenon. From the outset, standardized tests were used as an instrument of school reform and as a prod for student learning.
- Formal written testing began to replace oral examinations at about the same time that American schools changed their mission from servicing the elites to educating the masses. Since then tests have remained a symbol of the

American commitment to mass education, both for their perceived objectivity and for their undeniable efficiency.

- Although standardized tests were seen by some as instruments of fairness and scientific rigor applied to education, they were soon put to uses that exceeded the technical limits of their design. A review of the history of achievement testing reveals that the rationales for standardized tests and the controversies surrounding test use are as old as testing itself. (p. 110)

Some of the controversy still surrounding standardized testing is the intended use of the test results. “Tests became important tools for education policymakers, despite their apparently limited value to teachers, students, and principals” (US Congress, 1992, p. 110). These mass-produced tests are designed for quick and easy dissemination, scoring, and feedback. If problem-solving were added to a standardized test similar to an Advanced Placement exam, it would take numerous people to grade it and that may take more time and money before results would get back to the schools and the students. Maybe multiple choice is easier, but is it the right type of test to evaluate students' problem-solving skills?

Classroom assessments will need to be expanded when adapting your teaching to a problem-solving approach (Ziebarth, 2003). The teaching and learning focus shifts from skill mastery to deeper understanding of ideas. Assessment of students’ understanding has to come from a wide variety of sources including discussions, journals, group work, whole class work, individual work, portfolios, and projects (Ziebarth, 2003). Ziebarth (2003) went on to say the teacher’s role becomes that of an observer and listener instead of a lecturer and doer. Ziebarth's (2003) approach to resolving some practical issues for the teacher include:

- How can I effectively use more open-ended assessments techniques in my classroom?
- How can I help my students learn to accept a broader view of assessment?
- How can I assess my students as they learn in groups?
- How can I assemble all the assessment information to assign a letter grade, and how can this information be communicated to students and parents? (p. 181)

Student Challenges with Problem-solving

"The mark of powerful learning is the ability to solve problems in new contexts or to solve problems that differ from the ones one has been trained to solve. Competent Mathematicians have access to a wide variety of problem-solving strategies" (Schoenfeld, 2004, p. 262). In *The Math Wars,* Schoenfeld (2004) finds exclusive focus on the basics is just as unworkable to the extreme as exclusive focus on process. He contends that too many people have taken extreme positions and students are best served by using both methods separately and simultaneously as needed at the most appropriate developmental period of the student. Schoenfeld (2004) states:

> Research shows that students often fail to solve problems because they use the resources at their disposal (including time and what they know) inefficiently. For example, they would doggedly pursue inappropriate solution directions, thus depriving themselves of the opportunity to try other, potentially more effective methods. In contrast, more competent problem solvers would periodically take

> stock of their solution attempts and make decisions about what to pursue on the basis of those evaluations. Students would often stop working on problems, believing (on the basis of their prior experience with school mathematics) that problems should be solvable within a few minutes – and thus that a problem they failed to solve in short order was basically insolvable. Competent problem solvers persevered. They were also able to communicate the results of their mathematical work effectively, both orally and in writing. (p. 263)

Schoenfeld (2004) believes that students must do more than learning the relevant mathematical content, they must also think mathematically. Polya's (1985) strategies would align well with Schoenfeld's (2004). Polya believed students need to do problems and think about the means and methods they use in doing them (Polya, 1981). "Solving problems is practical art…you can learn it only by imitation and practice" (Polya, 1981, p. ix). When problem-solving is done well, students may learn a great deal. Schoenfeld (2004) thought this was a difficult design task:

> Structuring and supervising student interactions so that students can make progress on the problems, learn from each other, and know when they need more expert advice, is very hard…the logistical problems of supporting reform in substantive ways should not be underestimated. Teachers who had themselves been taught in traditional ways were now being asked to teach in new ways and not given much support in doing it. (p. 272)

Lee and Chen (2015) conducted a study on grade 7 geometry students in junior high school. The study was a nonequivalent pretest-posttest quasi-experimental design that combined Polya's (1985) problem-solving methods with multimedia demonstration.

Results revealed posttest performance in geometry reasoning of students receiving instruction based on Polya (1985) questioning was superior to the other group of students (Lee & Chen, 2015). The students receiving instruction based on Polya's (1985) questioning also expressed a stronger sense of participation and involvement (Lee & Chen, 2015). "Helping students to apply reasoning skills to problems of geometry requires that instructors develop the means to enhance the effectiveness of geometry instruction" (Lee & Chen, 2015, p. 1548). Polya's questioning prompts have proven to be an effective means of scaffolding thought processes in a range of disciplines (Lee & Chen, 2015). Polya's (1985) four steps of problem-solving are a learned process. Students must be taught how to use their resources effectively in order to become better problem solvers. "In the process of problem-solving when the problem-solving steps which Polya suggested are carried out successfully and efficiently, the students' problem-solving skills and achievements improve significantly" (Lee & Chen, 2015, p. 1550). Instruction in Polya's (1985) method is a teacher-centered approach meant to prompt students to elucidate their problem-solving ideas as well as the graphical relationships from the lesson. The questioning technique is a process of verbal guidance that can enhance learning motivation and activate thinking, which lead to the development of stronger conceptual links and deeper understanding (Lee & Chen, 2015). The study presents a framework for instruction of geometry reasoning by employing Polya's (1985) questioning prompts.

Summary

Problem-solving is a skill, a tool, and a process that will enable students to become productive citizens in our democratic society. The literature review highlights the history of problem-solving in US schools, and the underpinning theoretical learner-centered ideology and progressivism. Public schools are under a lot of pressure to perform which in part can be attributed to the policies and theories of the political and educational elite. Teachers bring their own traditions and judgments to the classroom, which can also be a hindrance to trying new techniques. The challenges of implementation include textbooks, assessment, time, and money that also make it difficult to have problem-solving be the focus of an advanced mathematics class. This study attempts to overcome the challenges and provide students with the opportunity to persevere in their problem-solving skills.

CHAPTER 3

METHODOLOGY

This chapter will outline the methodology the teacher-researcher used to answer the research question: *what impact will the application of Polya's first two steps to problem-solving have on an advanced high school mathematics classroom?* It also contains a description of the action research design, student participants, research site, data collection procedures, data analysis procedures, and ethical considerations. Action research allows the teacher to assume the role of researcher and participant (Mertler, 2014). This dual role allows the teacher-researcher to implement the treatment and observe the results while making needed adjustments if necessary.

Action Research Design

This study was designed to answer the question, "What impact will the application of the first two steps of Polya's four problem-solving steps have on an advanced mathematics classroom?" The teacher-researcher was especially interested in determining, "What is the impact on underrepresented groups?" The teacher-researcher used a mixed-methods action research design. Quantitative data was collected from the assigned problems using a scoring rubric from the College Board. Qualitative data was collected through a case study of underrepresented students within an advanced mathematics class. Three students were chosen for the case study based on results from a pre-assessment and students who were willing to participate.

The teacher-researcher began by taking the class roster and looking up the students' demographics, grades in their previous math classes and scores on their most recent standardized test. This general background information was for the purpose of being more familiar with the students in the classroom. The teacher-researcher administered a pre-assessment to determine the students' skills in applying problem-solving techniques to a mathematics problem. The teacher-researcher analyzed the approaches used by the students, looking for patterns. The data collected may contribute to the broader conversation of improving problem-solving skills for teaching and learning among all students. Based on the results on the pre-assessment, the teacher-researcher identified three students to follow throughout the implementation of Polya's questioning techniques. Each individual case was a unit of analysis for this study. The teacher-researcher identified themes and patterns within that case that were used in analysis.

Teacher-Researcher

The teacher-researcher teaches advanced math to mainly juniors and seniors in a suburban high school. These math classes are not required classes for graduation and therefore are considered electives. The classes are a blend of International Baccalaureate Mathematics Standard Level and Advanced Placement Calculus AB. Students may receive college credit for these classes if they perform well on the required assessments associated with each course. Typically, very bright and self-motivated individuals choose to take these classes. Student records show these students have usually passed previous math classes with A's and B's. Now they are in a class they find dynamic and challenging. The students need to have the skills to handle the challenge. The teacher-researcher wants to determine if Polya's questioning techniques will help the students become more adept at problem-solving.

The teacher-researcher collected data through a variety of resources. As mentioned previous, student records were used to determine demographics, grades in previous math classes, and most recent standardized test scores. A pre-assessment started the research process. Three students were selected based on the pre-assessment for intensive study. The teacher-researcher posted in the Google Classroom a set of commonly used questions and techniques to aid in problem-solving, then modeled those techniques and strategies throughout the lessons. The students were given problems to solve weekly throughout the six weeks of the unit of study which were assessed by the teacher-researcher on how successful the students were in using the strategies. The three selected participants were interviewed three times during the study, at the beginning, middle and end (Figure 3.1). These responses were coded by the teacher-researcher looking for patterns and themes. Mertler (2014) stated, "Its purpose is also to improve one's own professional judgment and give insight into better, more effective means of achieving desirable educational outcomes" (p. 13).

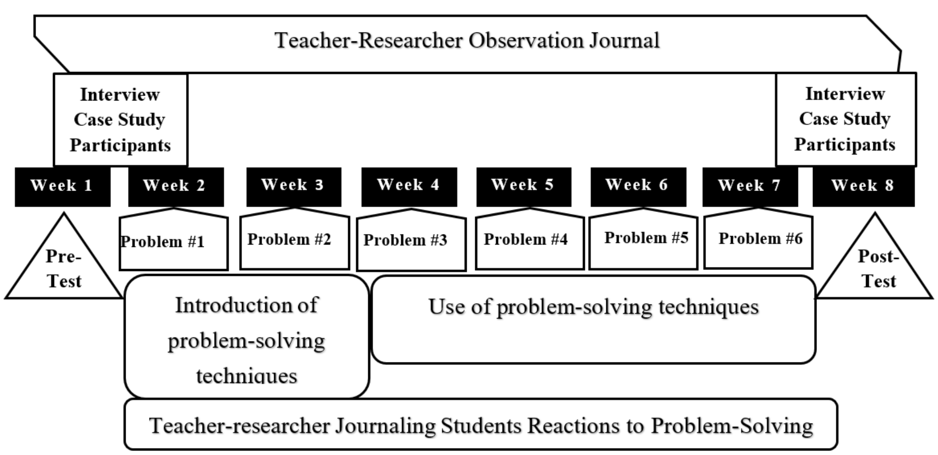

Figure 3.1 Research Trajectory Schemata

Setting

The classroom, school, school district, and community where this study was conducted would be easily recognizable if certain details about the students, their backgrounds, or experiences were recorded. In generalities, the town where the students and teacher-researcher lived and attended school was located within an hour of a larger town and a large city in a southeastern state. The economy is mainly driven by tourism and service-related industries, which dominate the extracurricular lives of many of the students.

The school district is one of the largest in the state with an enrollment over 21,000 students. The district is led by a superintendent and an eleven-member school board. There are six high schools in this district. Each high school has its own academic focus. The high school in this study has over 1300 students in grades nine through twelve and its academic focus is the International Baccalaureate Program. The students are basically evenly distributed between male and female and the school has a three-year average of 87% graduation rate within four years of initial enrollment.

The high school in this study has received county, state, and national recognition for academic standards and progress towards closing achievement gaps. The demographics of the high school have changed over the past ten years with an influx of Hispanic families to the community. The latest figures are about 54% white, 31% Hispanic, 11% African American, and 4% other. About 40% of the students qualify for free or reduced lunch. Of the approximately 40% who qualify for free or reduced lunch, the racial breakdown does not follow the school distribution as a whole. Those who qualify for free and reduced lunch are represented by 56% Hispanic, 33% African-

American and 11% white. The classes being used for this study are predominately white. The classes do not match the racial make-up of the school population.

The room in which the study will be implemented has an interactive white board at the front, white boards on the rest of the front wall and white boards on a side wall. Cabinets and shelving take up the other side wall. The back wall has no permanent fixtures. The walls around the room are covered with nameplates, each the size of a cinderblock on the wall, with the names of previous students from 2000 to present who have taken the challenge of advanced math classes. Some of the seats are grouped around four-person tables and some of the seating is individual desks in between the tables. The arrangement changes periodically to allow for group and individual work.

Student Participants

This action research study focused on students in a combined International Baccalaureate Mathematics SL class [IB Math SL] and an Advanced Placement Calculus AB class [AP Calc AB]. The students cover all the material for both syllabi and are expected to sit for both exams at the end of the year. Both exams place a premium on problem-solving skills in a conceptual context. The entire class participated in using the problem-solving techniques. All the students in the class received the same problem-solving emphasis. The students were disaggregated based on designation of an underrepresented group, previous math grades, standardized test scores, and the pre-assessment.

The advanced mathematics class in this study has 20 students. The demographic breakdown is 75% white, 20% Hispanic, and 5% Asian-American. There are eleven females and nine males. Four students are considered low socioeconomic status and ten have a high socioeconomic status, the other six would be considered middle class. Two of the students have Individual Education Plans (IEP). There are no English language learners that have not tested out of the English Language Learner program. Thirteen of the students are designated gifted and talented.

Demographic data, grades from previous math classes, and recent standardized test scores, were gathered to better understand the students in the class. The demographic data, grades, and standardized test scores are in the students' files. The teacher-researcher gathered this information prior to the onset of the study. By gaining information about their previous performance the teacher-researcher had a better understanding of the participants.

Intervention and Implementation

The first step was telling the students what the implementation process would be over the eight-week period. First a pretest was given on the Friday of week one. The teacher-researcher used the results of the pretest as the baseline to gauge whether students were using the problem-solving on the post-test. The intervention included modeling problem-solving strategies by the teacher-researcher, posting strategies in the Google classroom, then using practice problems once a week over six weeks.

The practice problems were given each Friday over the six-week period of the study. Given the number of problems on a typical IB or AP exam, the students knew to allot approximately fifteen minutes per problem. The first week the students were given twenty minutes to complete the practice problem. Each succeeding week the total time was reduced by one minute, so the students would get to feel the time crunch of an actual exam and still be able to apply the problem-solving strategies. On Monday the teacher-researcher distributed the practice problem from Friday to the students to discuss the strategies the students used and also additional strategies the teacher-researcher recommended. Throughout the week the teacher-researcher modeled and emphasized the strategies on a variety of problems, so the students could see the thought process and strategies in action.

Data Collection Procedures

This action research focuses on problem-solving techniques with advanced high school math students. The research question is: *what impact will the application of Polya's first two steps of problem-solving have on an advanced high school mathematics classroom?* Specifically, *what impact will the application of the problem-solving techniques have on underrepresented groups in the classroom?* Four primary data collection techniques were used: pre- and post-assessments, weekly assessments, observations, and interviews.

Assessments

All students took a pre-assessment and post-assessment to determine their skills related to Polya's problem-solving methods. The pre-assessment, post-assessment and weekly assessments were chosen for consistently having a low mean score on the AP Calculus AB Exam. The pre-assessment was question four from the 2003 AP Calculus AB Exam. It was graded according to the scoring guidelines published by the College Board and was inspected for problem-solving strategies. The teacher-researcher tallied the number of strategies each student attempted to use. If the teacher-researcher was unsure of the student's intentions, then she questioned the student about the work on their paper. The pre-assessment was administered on Friday of week one, prior to the unit of study. The post-assessment was administered at the end of the six-week study period. The post-assessment was question three from the 2016 AP Calculus AB Exam. It was also graded according to the scoring guidelines published by the College Board and inspected for elements of the problem-solving strategies. All the students had access to a graphic display calculator [GDC] when necessary to solve a problem.

There were eight AP Calculus AB questions used during the eight-week intervention. Table 3.2 shows the year, question number, mean and standard deviation for the questions, and mean and standard deviation for the free response in that year. The questions chosen were considered very difficult compared to other free response questions on the test. The fundamental theorem of calculus comprised the style of each question used during the intervention. The students practiced the most difficult type of question in order to be prepared during exam time to access that type of question and perform higher than the average.

Table 3.1 College Board Data for Each Intervention Question

Year	Question	Mean for the Question	Standard Deviation for the Question	Mean for the Year	Standard Deviation for the Year
2003	4	2.68	2.04	3.08	1.36
2004	5	2.63	2.44	2.97	1.45
2009	6	2.07	1.89	2.99	1.5
2010	5	1.75	1.99	2.81	1.55
2011	4	2.44	2.26	2.82	1.55
2012	3	2.67	2.58	2.97	1.58
2013	4	2.62	2.65	2.96	1.56
2016	3	2.74	2.53	2.96	1.58

At the end of week two through week seven the students took a weekly assessment on problem-solving. Weekly assessments were given by the teacher to understand student progress throughout the instruction. These included AP Calculus AB Exam questions from 2004, 2009, 2010, 2011, 2012, and 2013. These were also graded according to the scoring guidelines posted by the College Board. Data was gathered pertaining to the use of Polya's methods in the problem-solving process for each student in the class with special attention given to the three selected for the subgroup. The teacher-researcher identified circling, underlining, diagrams, notes on diagrams, adding figures to diagrams, and other notations as evidence the students were attempting to use one of the strategies. The data collected included the number of strategies used, whether the student made progress using the strategy and the score, out of nine points, that was achieved.

Interviews and Observations

A subgroup of three students from underrepresented groups were interviewed three times during the study. The underrepresented groups include, race, ethnicity, socio-economic status, Individual Education Program, special education status, English language learners, or gifted and talented. Once the three participants were selected, the teacher-researcher interviewed each one to gather baseline data on their feelings and attitudes towards problem-solving (Appendix B). The three students were interviewed again midway through the study after week four and a final interview was conducted after week eight.

The teacher-researcher recorded observations in a journal throughout the eight-week period of the study. Journal entries were based on verbal and non-verbal communication observed by the teacher-researcher with the students and between the students. These were observations of the entire class with a special interest in the three case study participants. The teacher-researcher used open and axial coding from the observations to find themes and patterns in the way the students use or discuss Polya's techniques to help in problem-solving.

Data Analysis

In order to answer the research question: *What impact will the application of the first two of Polya's four steps to problem-solving have on an advanced high school mathematics classroom*? several pieces of data were collected. Statistical analyses were used to analyze the pre- and post-assessments. Measures of central tendency and measures of variance allowed the teacher-researcher to understand overall class assessment scores and changes in scores between the pre-assessment and post-assessment. Measures of central tendency and measures of variance allowed the teacher-researcher to gauge changes in weekly assessment scores during the study. Subgroups based on underrepresented groups were explored to understand differences among subgroups of students in their assessment scores. Finally, correlation analysis was used to determine the relationship between the pre- and post-assessments. An informal survey of the students took place after completion of the actual AP Calculus AB exam and IB Mathematics SL exam to assess the students' comfort level using the strategies on a high stakes test.

The three students chosen from underrepresented groups were interviewed three times during the study, at the beginning, the middle, and the end. Open coding is defined as, "the analytic process through which concepts are identified and their properties and dimensions are discovered in data" (Strauss & Corbin, 1998, p. 101). Open coding was utilized to identify concepts in the interview transcripts. These concepts were broken into dimensions. The teacher-researcher categorized the dimensions for themes and patterns such as, math background, like/dislike problem-solving, understands English, understands context, made an initial attempt but could go no further, set up variables and made it past the first step, incorrect or correct solution, and showed improvement in the process. The teacher-researcher used open coding to identify concepts in the observations. These were placed into categories as well.

Strauss and Corbin (1998), define axial coding as, "the process of relating categories to their subcategories, termed "axial" because coding occurs around the axis of a category, linking categories at the level of properties and dimensions" (p. 123). The teacher-researcher utilized axial coding to collate the data from the interviews and observations to analyze similarities and differences.

Ethical Considerations

Mertler (2014) states, "Making sure that action research adheres to ethical standards is a primary responsibility of the educator-researcher" (p. 106). The high school in this study has a process in place to ensure ethical research and evaluation. A request must be submitted to the district office and approved before any research can proceed. Students, parents and administrators were assured of confidentiality. No identifiable information will be released about a student. All students were made aware of the fact that they were participating in a research study. The participants will remain anonymous. Fictional names are used for all participants. Students were told ahead of time that they did not have to participate in the case study and that there were no right or wrong answers to the interview questions. This was to encourage thoughtful responses. The students were made aware of how this work could help other students and teachers in the problem-solving process.

The purpose of the research is to help students. “It is significant to take into consideration the fact that ethics is not merely something to consider at the commencement of a research project or fieldwork but should be rather borne in mind throughout the entire research process as concerns about the ethics ought to be the leading consideration of any research study” (Abed, 2014, p. 1).

Summary

A mixed methods action research design was used to study the research question: “*What impact will the application of the first two steps of Polya’s four problem-solving steps have on an advanced mathematics classroom*?” The intervention of applying problem-solving strategies was used on the entire class of twenty students with special emphasis on three students from underrepresented groups. The student participants were juniors and seniors in high school taking an advanced math class.

The students were given a pretest at the beginning of the study. Problem-solving strategies were introduced and modelled, then the students applied those strategies to a problem each week. Data was collected in the form of assessments, interviews and observations. The scoring data was compared to the national mean and standard deviation. The pretest scores were compared to the posttest scores. The number of problem-solving strategies used by the students was counted to see if there were more strategies used as the study progressed. The interviews were transcribed, coded, and sorted into overarching themes.

The study was approved by the school district. Parental consent forms were signed and collected from all twenty families and students assent forms were signed and collected from all twenty students. The names of the students were changed to protect their identity.

CHAPTER 4

PRESENTATION AND ANALYSIS OF DATA

This chapter presents the analysis of the data collected by the teacher-researcher and findings to answer the research question: *what impact will the application of Polya's first two steps to problem-solving have on an advanced high school mathematics classroom?* More specifically: *what is the impact on underrepresented groups?* The teacher-researcher noticed a trend that students do not complete long answer word problems on assignments or assessments and often do not even attempt the problems. The purpose of this study was to determine if problem-solving strategies make it easier for students to find an entry point to start solving the problem or give them more avenues from which to attack the problem, so they see it through till the end.

Overview of Study

The study was conducted in a small suburban high school in the southeastern United States. The latest demographic figures are about 54% white, 31% Hispanic, 11% African American, and 4% other. About 40% of the students qualify for free or reduced lunch. Of the approximately 40% who qualify for free or reduced lunch, the racial breakdown does not follow the school-wide distribution. Those who qualify for free and reduced lunch are represented by 56% Hispanic, 33% African-American and 11% white. The classes being used for this study are predominately white. The classes do not match the racial make-up of the school population. Over an eight-week period the teacher-researcher modeled the use of problem-solving strategies, posted the strategies in the

Google Classroom, and wrote her observations in a journal. A pretest was administered, then a practice problem was administered once-a-week, followed by the administration of a posttest. Interviews were conducted with the three students from underrepresented groups.

Intervention

The goal of this action research study was to document the outcomes of using problem-solving strategies in an advanced math class, especially on underrepresented groups. Problem-solving strategies were modeled and practiced in the class. Results were recorded on practice problems, through interviews, and observation field notes.

Results

Data collected from the practice problems was analyzed for the number of strategies applied and for the score obtained using the College Board rubric. Observations were recorded in a journal to be coded for themes and patterns. Interviews were transcribed and coded for themes and pattern also. Following are the results.

Practice Problem Results

Each week, students completed one practice problem. The teacher-researcher scored the weekly practice problems according to the College Board AP Calculus AB Exam rubric. Each problem was scored on a scale of zero to nine. The mean score for the class on each problem (Table 4.1 and Figure 4.1) was above the national mean score calculated by the College Board. The pretest mean score nationally was 2.68 and the class mean score was 3.35. Eight weeks later, on the posttest, the mean score nationally was 2.74 and the class mean score was 4.70. The standard deviation shows a similar spread of scores. The standard deviation for the eight problems was 2.30 while the class average standard deviation was 2.24.

Table 4.1 Results from Weekly Practice Problems

Weekly Practice Problem	Year	Question Number of Weekly Practice Problem	National Mean for the Question	National Standard Deviation for the Question	Class Mean for the Question	Class Standard Deviation for the Question
Pretest	2003	4	2.68	2.04	3.35	2.10
Practice #1	2004	5	2.63	2.44	3.85	2.43
Practice #2	2009	6	2.07	1.89	3.40	2.24
Practice #3	2010	5	1.75	1.99	3.25	2.07
Practice #4	2011	4	2.44	2.26	2.90	2.14
Practice #5	2012	3	2.67	2.58	4.15	2.20
Practice #6	2013	4	2.62	2.65	3.00	2.41
Posttest	2016	3	2.74	2.53	4.70	2.30

Note. Each practice problem scored on a zero to nine scale.

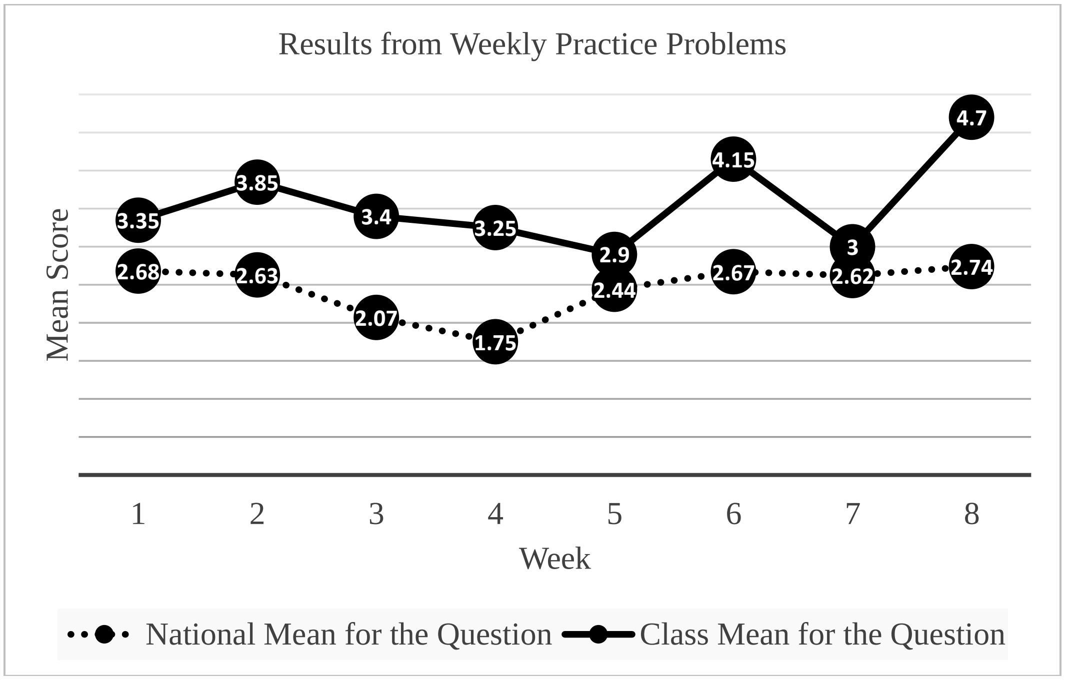

Figure 4.1 Results from Weekly Practice Problems

The mean scores for the class do not correlate strongly with the national mean scores for the exam. The lowest national mean score for the exam is not the lowest mean score for the class but the highest national mean score for the exam is the highest mean score for the class. The class scores also grouped more closely together than the national scores.

Problem-solving strategies (Appendix A) were documented in the teacher-researcher's journal after the pretest, each practice problem, and the posttest. The number of students using the easily identifiable strategies are recorded in Table 4.2. Multiple strategies were used by some students whereas other may have used one or none. It was difficult to determine which students used a guess and check strategy. When the strategy was unclear, the teacher-researcher questioned the student to determine if a guess and check strategy was used. Table 4.2 displays the strategies used by the students for each practice test as well as the pretest and posttest.

Table 4.2 Strategy Count from Weekly Practice Problems

Weekly Practice Problem	Circle or Underline Key Information	Draw on a diagram	Use a model	Solve a simpler problem	Make a list	Guess and Check
Pretest	6	5	1	0	1	4
Practice #1	12	13	1	7	1	3
Practice #2	14	11	1	2	1	4
Practice #3	14	15	2	3	6	4
Practice #4	11	18	12	3	1	2
Practice #5	8	18	8	3	1	1
Practice #6	10	14	1	2	0	4
Posttest	14	15	6	2	3	3

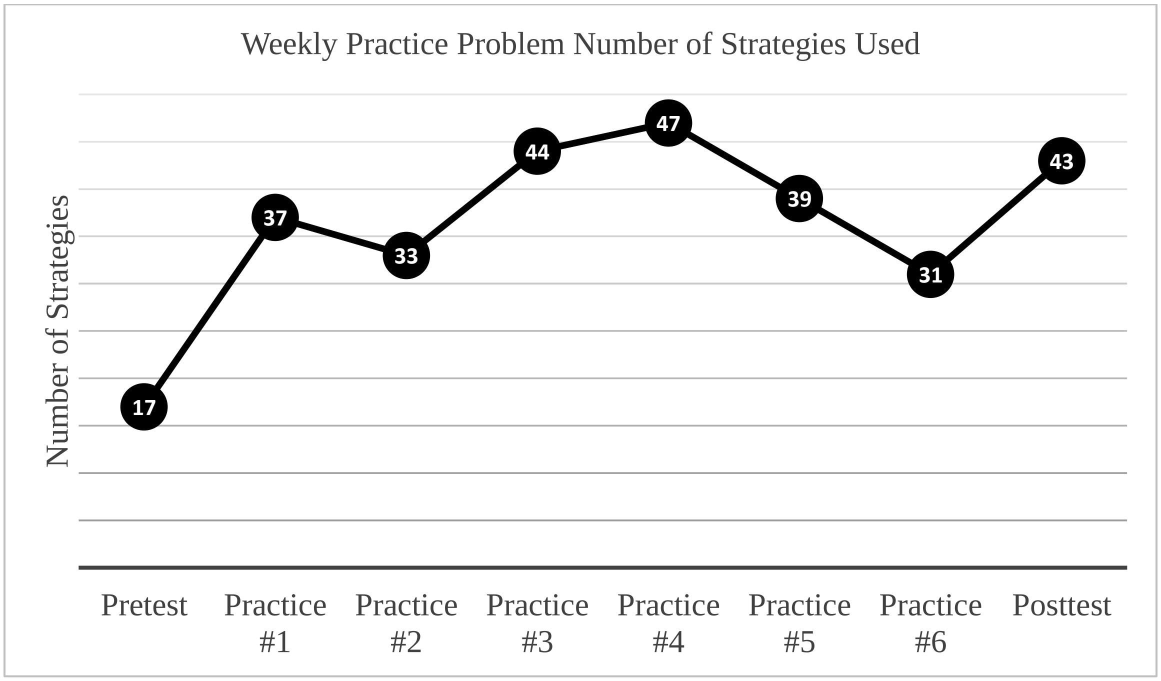

Figure 4.2 Strategy Count from Weekly Practice Problems

The first line of data contains the pretest usage of strategies. As expected, there were many fewer strategies used on the pretest. The number of strategies used over the next six weeks doubled or tripled at times. It must be noted that one student may have used multiple strategies whereas another student may not have used any strategies.

Observations

When it was announced that the class was going to participate in the teacher-researcher's study there was a general feeling of excitement and trepidation from the students. What's the study about? Is it going to be more work? Is it going to be graded? Can we celebrate when you are done? The students supported the study and expressed their desire to improve their problem-solving skills.

Prior to the study, the teacher-researcher noticed that students often did not attempt or successfully complete word problems. Journaling was used to record observations about how the students were trying or not trying to incorporate problem-solving techniques. During the eight-week period of the study, the teacher-researcher wrote daily comments in the journal about problem-solving conversations between the students and between the teacher-researcher and the students. At first, the students were just telling each other what to do, if they could figure it out. After two weeks of the teacher-researcher modeling the problem-solving strategies and prompting the students to ask themselves probing questions, the students started using the same questioning techniques with each other:

- "Did you circle the initial point? You know we're going to have to use that later" (Student A, personal communication, March 29, 2018).
- "How did you know to go backwards to find the area under that curve?" (Student B, personal communication, April 4, 2018).
- "I drew triangles, rectangles and trapezoids under my curve, what did you draw?" (Student C, personal communication, March 21, 2018).
- "Will a sign chart help me figure out where it is positive or negative?" (Student D, personal communication, April 17, 2018).
- "I circled that $f(x)$ was the first derivative of $g(x)$ because I know it's important, but what does it mean? How do I use it?" (Student E, personal communication, March 23, 2018).

The comments and questions from the students demonstrated their perseverance in trying to gain access to these word problems. When the teacher-researcher displayed the techniques used in a model problem, the students could show the use of similar techniques and show and explain their own techniques if they found a different way.

The pretest, practice problems, and posttest were reviewed for evidence of circling or underlining key information, drawing a diagram or drawing on a diagram, using a model, solving a simpler problem, making a list, and guessing and checking. The techniques did not follow a linear pattern of growth over the eight-week period. Every problem-solving strategy was not applicable to every practice problem. Table 4.2 shows circling or underlining key information and drawing on a diagram were the most used problem-solving strategies. These two strategies allowed the students to gain access to the problem where they otherwise might have left the problem blank without even trying to solve the problem. The strategy of using a model was applied by more than half of the students only one time. None of the other strategies, solving a simpler problem, making a list, or guess and check were applied by more than seven out of twenty students. The students did not find those strategies to be as helpful on these problems.

After the AP Calculus AB Exam, the teacher-researcher surveyed the students and nineteen of twenty students claimed to have attempted every problem and not leave a single problem blank. After the IB Math SL Exam, seventeen of twenty students made the same claim of not leaving any problems blank. All twenty students claimed to use one or more of the strategies on word problems during each exam. When the teacher-researcher surveyed the students after the exam, fourteen of the students felt confident in their ability to make some progress on the word problems. Only two students struggled to use the strategies on the exam claiming a lack of confidence in their problem-solving abilities. Overall, the students had a better attitude towards word problems, felt more confident using problem-solving strategies, and attempted to use a few strategies after the conclusion of the study on the AP Calculus AB Exam and the IB Math SL Exam.

The teacher-research identified three students to gain more in-depth information related to the use of the problem-solving strategies. Table 4.3 shows the comparison of Fernanda's, Jenna's, and Arjun's scores with the class mean scores and the national mean scores. Fernanda's scores are at or above the mean score for the class and the nation. She did well on all the problems except one and this was concurrent with her overall class performance. Jenna struggled with the academic language, with the time constraint of the problems, and with the calculus concepts. Her first and last scores were her best and they were above the mean score of the class and the nation, but those were the only two times that happened. Arjun was below the mean score for the class and the nation on every problem except the third and the last where he exceeded the national mean but not the class mean. Arjun's use of problem-solving strategies was very limited as was his understanding of the calculus concepts.

Table 4.3 Comparison of Case Scores to Class Mean and National Mean

Year	Question Number of Weekly Practice Problem	Fernanda's Score for the Question	Jenna's Score for the Question	Arjun's Score for the Question	Class Mean for the Question	National Mean for the Question
2003	4	6	5	0	3.35	2.68
2004	5	8	1	1	3.85	2.63
2009	6	7	2	3	3.40	2.07
2010	5	5	3	0	3.25	1.75
2011	4	5	2	2	2.90	2.44
2012	3	7	1	0	4.15	2.67
2013	4	3	1	2	3.00	2.62
2016	3	7	5	3	4.70	2.74

Case 1: Fernanda

Fernanda is a Hispanic female. She is a junior in high school, a leader of the junior class and well-respected by her peers and the faculty. Her mother and father were born in Mexico. Her father did not finish middle school because his family was very poor. He came to the United States when he was 18 years old. Her mom went to high school but was not able to finish. Her father went back to Mexico married her mother at age 22 and returned to the United States. Fernanda and her three siblings were born in the United States, lived in Mexico for a few years, lived in another state for a few months and settled in their current location.

During an interview, Fernanda shared that her parents impressed upon her and her siblings how important it is to get an education:

> My dad works in landscaping and my mom in cleaning. My sister and I have gone with her to help since we were younger. We live a happy life. With church, that's like what we need, we need love and being together. But they say they don't want to see us struggle as much as they did. They don't want to see us come home after landscaping for the whole day in the hot sun. So, they are like even if you get a small little job, like being a nurse's assistant, or if you do cut hair, become a barber, that's ok, get a little education so you don't have to do what they do. (Fernanda, personal interview, March 5, 2018)

Fernanda understands the importance of a solid education and how she will have better opportunities if she does well in school. Math is Fernanda's favorite subject. It always came easy to her. She studied, memorized formulas and received 100 percent on her final exams. Then she started struggling in IB Math SL and AP Calculus AB. She described it as an unusual and disturbing feeling. She felt defeated when she was not getting those good grades anymore but she kept working on it. We had many conversations throughout the year about how to study, when to come in for tutoring, and what other resources were available outside of school. During the interview, she tried to explain how she felt.

> Honestly, in the beginning, I didn't understand your teaching style, I had never had a teacher like that. Because you actually went and taught us why it is that we do things, so it was more to learn, versus my other math classes they just teach us do this, do this. But you teach us why it is, so then it is more to think about. But I think in the end it has really helped me. Cause now I am able to understand why it happens. (Fernanda, personal interview, May 2, 2018)

When Fernanda was asked to think about all her high school math classes and how often she saw word problems she replied that she very seldom saw word problems except in geometry where you had to do a lot of proofs. When asked if she liked proofs, she responded, "It made me happy when things at the very end line up and I was able to prove the answer to my work". Her main strategy for solving word problems was to memorize formulas.

> Like I would write them on sticky notes and have them with me all throughout that chapter and then review them at the very end. I think that was it. And also, I never took shortcuts, I always did it the long way and I never took shortcuts just to be sure. Like in class sometimes, you know you like skip steps because it should be common sense but I really like to write down because I need that to be able to keep the logic. I think that was basically it. (Fernanda, personal interview, March 5, 2018)

Being a conscientious and bright student, Fernanda thought IB Math SL and AP Calculus BC were going to be as easy as her previous math classes. Fernanda went on to say:

> I think it was harder. Honestly, in the beginning, I didn't understand your teaching style, I had never had a teacher like that. Because you actually went and taught us why it is that we do things, so it was more to learn, versus my other math classes they just teach us do this, do this. But you teach us why it is, so then it is more to think about. But I think in the end it has really helped me. Cause now I am able to understand why it happens. (Fernanda, personal interview, May 2, 2018)

Fernanda was a confident math student to this point in her high school career. Her strategies included memorizing formulas, taking very good notes and practicing as much as she could. She wanted to remain successful, so she incorporated the new strategies we worked on in class; circling or underlining key information, drawing diagrams or drawing on diagrams, using a model, making a list, or guessing and checking. During the six weeks of practice problems, Fernanda commented several times about using circling or underlining key information and how she would reflect on that information while working on the problem to see where she could apply it. She stated, "I think I have gotten better at them, especially particle motion. I now know what acceleration will give me and what velocity will give me". She confirmed during the interview she starts circling numbers, underlining formulas and key words, and deciphering the question. When asked if she was comfortable drawing on a given shape she said, "Most definitely, I think I am a more visual person when it comes to math, so the diagrams are a big help, even when I have to draw one myself".

Fernanda regarded herself as much more confident in solving word problems after practicing strategies. She commented that these are some of the same strategies that she uses in her English Literature class. Marking comments in her English Literature class is transferrable to her IB Math SL class. Polya (1945) believes that most problem-solving skills are transferrable among academic disciplines. In the final interview, at the end of the study, Fernanda thought she definitely had the skills to parse a word problem and she already was confident memorizing formulas and applying them in the context of the problem. She felt she would never leave a word problem blank because she would always be able to choose on strategy or another to get the problem started.

Fernanda passed her AP Calculus AB Exam and her IB Math SL Exam. She was nervous about her scores after taking the exam, but confided in the teacher-researcher that there was not a single problem she left blank, she tried several strategies on the difficult problems, and she did not think she did as well as she expected to but thought she would pass. She felt especially comfortable with the calculus question that most resembled the practice questions used during the study.

An analysis of Fernanda's practice questions revealed a steady increase in the use of problem-solving strategies. The pre-assessment showed two instances of circling key information and one instance of creating a diagram, but no other strategies. The posttest showed six instances of applying a problem-solving strategy. Not every problem allowed for such a variety of strategies, but Fernanda was a very thorough student. Her scores were also very consistent on the nine-point scale with only one problem scoring a three which was much lower than the other scores. The teacher-researcher predicted a four or higher for Fernanda on her AP Calculus AB Exam, she earned a three.

Case 2: Jenna

Jenna is a Hispanic female. She is a junior in high school and very engaged in the community inside and outside of school. Her parents are from Mexico. Her mother brought her to the United States when she was five years old. Her father tried to get a degree in education and her mother went to a community college. He couldn't make money as a teacher and support a growing family, so he got a visa to come to the United States to work in the fields harvesting tobacco, onions, and watermelon. Jenna and her mother came next and since their arrival it's been nothing but "working, working, working, day and night, day and night". When Jenna was asked about how her parents supported her education she said:

> To be honest, no they never, um, … like you know, usual parents are always on your back and emailing, but they are not like that. They never um like I don't want to say care, because they do care. They like support me but it is more like a you're on your own thing. (Jenna, personal interview, March 6, 2018)

Jenna discussed how her parents could not help her with homework. She had a to sit with a dictionary and try to figure it out herself. She is very independent, yet not afraid to ask for help.

As a junior in high school, Jenna is bilingual and she has passed her ELDA testing which exempts her from services of English for Speakers of Other Languages. But she feels as though she still struggles with English just a little. So, when it comes to word problems she says she must read them repeatedly to understand the problem. Sometimes that involves a dictionary for translation. (Jenna, personal interview, March 5, 2018)

She is also very disheartened when she sees Hispanic students being treated unfairly in a very stereotypical manner.

> I would see how like my white classmates here just like popular kids, they make fun of things and you try to like rub it off and not really pay attention, but, it's always like little comments little remarks, like Hispanic students and that. That just made me realize like why be like one of them, you can't stand and show them not everyone is like that. We are not all lazy and stuff. I get really sad that um, a lot of my Hispanic friends, they have everything you know, papers and they're not going to college. (Jenna, personal interview, March 5, 2018)

Jenna recognized that some of her Hispanic classmates were not taking full advantage of the opportunities available to them. "For me it's like ON ME, you know. I choose what I want to do. They (my parents) trust me. So, I have chosen to do good."

When asked if she always felt like she was good at math, Jenna had a lot to say about her math class history:

> Not like necessarily good, more like I try my best. And I like it, like I really like it. I love math like ever since geometry with Mr. Barker. I had to retake it with Mrs. Trenary, since you weren't supposed to get a C and take the credit for the class. But I enjoyed it. It's like the only class I truly enjoyed at school other than art and stuff. And then I got to Mr. Larkin and he is like a very enthusiastic teacher, really good at visuals. He is more of a visual person and I am also a visual person. So, I was able to like really catchup and got a good grade in his class. And here it is like, I love my friends you know and they're like good at math and there is a math connection and it just makes me happy. Every time I

come into your room most of the time I know what I am doing, but I am a really slow learner, so it like takes time for me to understand stuff, but I really like it, I enjoy it. And it is hard, like I know this class is hard, but I've tried my best. I could try harder, but you know sometimes procrastination sets in. (Jenna, personal interview, May 3, 2018)

Jenna has a strong sense of self. She recognizes her strengths and weaknesses. But she did find a point in her math career where she did not fully understand topics and did not take the time to master those topics. She felt she would be able to catch-up somewhere along the way and it never happened. Jenna identified deficits in her content knowledge, because she got lazy at times. When asked about problem-solving strategies that she uses Jenna said she likes to draw and visualize, "but it takes time to do that when other people can solve it really fast in their heads and stuff". Jenna is using one of the problem-solving strategies as a coping mechanism for not having the content vocabulary for advanced math, which is to read the problem for understanding.

When the teacher-researcher questioned Jenna about her natural inclination when encountering a word problem, Jenna replied that she feels anxious, because she is a native Spanish speaker and she has a weak academic vocabulary. She tries to compensate for this weakness by surrounding herself with English speaking students who have a strong academic vocabulary, then Jenna reads, rereads, and reads again. Sometimes she must go word by word trying to decipher the meaning of the word problem. Jenna said the problem-solving strategies helped a little when she stated:

Yeah, like you told me to circle when they give you an initial condition. To me it's more of a memory thing, like when I see a word and circle it and I

> know I am going to have to use it later, so I remember to look for it. For example, like initial condition is something you would have to circle and that's always stuck with me. We're going to use it again in part a, b, orc and there will be a problem where we are going to do that. Then I understood like having a positive point and going forward or backward meant the area would have a positive or negative sign. So, when I divided the area with vertical lines, I could find the area and whether it was positive or negative. (Jenna, personal interview, May 3, 2018)

Jenna could apply two of the problem-solving strategies consistently, which helped her to gain access to difficult word problems. She is convinced these strategies helped her pass the AP Calculus AB Exam. She wants to share her knowledge with future classes by speaking to them about the problem-solving strategies helped her.

An analysis of Jenna's practice questions revealed a steady increase in the use of problem-solving strategies. On the pretest, Jenna circled two key pieces of information and did some appropriate drawing on the diagram. On the posttest, Jenna calculated appropriate areas on the diagram by doing it in sections, circled key information, created a sign table, and used a model to find a value. She achieved her highest score on the posttest. The teacher-researcher predicted a score of three on the AP Calculus AB Exam due to limited language acquisition and time constraints. Jenna earned a score of three.

Case 3: Arjun

Arjun is a junior in high school. He loves science, especially biology. His parents are from India and both went to graduate school. His father went to medical school in India and then again in the United States. His mother got her law degree in the United States. Both of Arjun's parents stress the importance of getting a good education. Arjun feels the pressure to perform well in school. Arjun felt like math was easy for him through middle school and ninth grade. Then math started getting a little harder for him each year. Precalculus was tough, IB Math SL and AP Calculus AB are even harder for him.

Arjun was the only student interviewed who said he likes word problems, if he can figure them out. When questioned about what he means by "if" he replied that it was easier if he had seen that type of problem before. He may not recognize that he is using one of the problem-solving strategies which is to use previous knowledge to start a new problem. Arjun admitted that when the study first began, he did not underline or circle key terms nor did he draw on diagrams. He claimed, during the interview, that he did try to use some of the problem-solving strategies including circling key terms, drawing diagrams, and splitting the problem into smaller parts.

Arjun spent a lot of class time trying to solicit information from other students he thought might be able to help with solving problems. He believed seeing someone else work through a problem was going to be all he needed to help him solve similar problems. It was an infrequent occurrence to observe Arjun taking notes during class, completing assignments, or attempting to solve problems on his own. Arjun berated himself in front of his classmates calling himself dumb or stupid, which the teacher-researcher had to address on each occasion trying to reassure him that he did not get into an advanced math class by accident.

After the interview, Arjun said he would prefer to focus on biology and would not register to take the next level of calculus. Instead he opted to take probability and statistics. When the teacher-researcher explained the content of the course, which includes many word problems, Arjun felt confident he would be equipped to handle the course. He said the problem-solving strategies made the problems easier.

An analysis of Arjun's practice questions revealed a slow progression in the use of problem-solving strategies. On the pretest, Arjun drew on the diagram appropriately and created his own sign table. On the posttest, Arjun created a sign table and did not use any other strategy. He achieved his highest score on the posttest which was a score of 3 out of nine. He consistently scored three or less and received zero points for three of the practice problems. There was no evidence in Arjun's work that he was attempting the problem-solving strategies. During the interviews, he claimed he was trying to use them and that the strategies were helpful and made him a more confident problem-solver. The teacher-researcher predicted a score of two on the AP Calculus AB Exam since there was little evidence of progress in the course. Arjun earned a score of one.

Themes Through the Cases

After transcribing the interviews, coding the interviews and observation notes, and reviewing the strategies used in the practice problems, three themes emerged related to the first two steps Polya's problem-solving strategies; the lack of problem-solving strategies introduced in previous math classes, the independence of the student, and the expectations of the student and/or their parents.

Fernanda, Jenna, and Arjun each mentioned in their interviews that their previous math classes did little to prepare them for difficult word problems. All three believed that their previous math classes were very easy and not nearly as challenging as the AP Calculus AB and IB Math SL class they were currently taking. Arjun is the only student who said he enjoyed problem-solving if he knew how to get started. Fernanda and Jenna struggled with word problems and were pleased to have some strategies to help access these difficult problems.

The student's ability to work independently and make decisions regarding his or her education is a key element to be a problem-solver (Polya, 1945). Fernanda was very thorough in her note-taking during class, she completed her assignments knowing that it was good practice and sought tutoring before school, during ILT and after school when necessary. Fernanda was very self-sufficient. She knew when to seek help and when to do the work herself. Jenna was also a very good note-taker during class and she tried to complete all her assignments. Jenna's language acquisition left her struggling with academic vocabulary, but she could use a dictionary and her friends to overcome difficulties. Jenna also took advantage of tutoring before school, during ILT, and after school as necessary. Jenna would often go over a problem with the teacher-researcher multiple times before she understood the concept. Arjun only came for tutoring when forced to by his parents' disappointment in his grades. He was not an independent learner and only completed assignments occasionally. Arjun's goal is to be a medical doctor and there may be a disconnect between what he wants to do and what he needs to do to be successful.

The third theme involved the expectations placed on the student by the themselves or the student's parents. Fernanda had the support of her parents to do well in school so she would have a better life than they did. The encouragement from her parents helped motivate Fernanda to do her best in school. When a difficult problem gave Fernanda trouble she knew she had to seek help before she fell behind in her studies. The problem-solving strategies she started using in math class helped her access word problems that were difficult in the past. She had to practice the strategies many times to become comfortable with them and she believed the strategies were helpful and made her a more confident problem solver. Jenna's parents did not push her to do well, but Jenna pushed herself to do well. Her expectations for herself were very high and she knew she had to work hard to achieve success and be better off than her parents. She wanted to take advantage of every opportunity to get a good education. This was Jenna's motivation. Arjun's parents were both very successful and expected the same for Arjun. Arjun carried their big dreams for him on his shoulders, but seemed weighed down by the expectations instead of motivated. His desire to be a medical doctor does not match his current work ethic or academic performance. He can add to a classroom conversation, but does not perform as well on individual assessments.

Summary

The combined AP Calculus AB and IB Math SL class consisted of twenty students who participated in the study. Each student was given a list of Polya's problem-solving strategies and those strategies were modeled by the teacher-researcher throughout the eight-week study. The practice problem results showed the class mean for each practice question was greater than the national mean for each question. The class standard deviation was less than the national standard deviation on five of the eight problems. The class was outperforming the national averages.

The pretest showed minimal usage of problem-solving strategies by the students. The number of strategies increased through the study, but not in a linear relationship. The posttest showed more strategies used than the pretest. The posttest also showed appropriate attempts in student work for each section of the word problem. No subparts of the posttest problem were left blank. The students were accessing the word problems through the problem-solving strategies.

The three cases from underrepresented groups, Fernanda, Jenna, and Arjun, were high-performing, average-performing, and low-performing respectively. The performance on the practice problems followed the same pattern as their work throughout the year. Fernanda scored the highest on average, Jenna performed in the middle, and Arjun performed the lowest. All three students claimed to have more confidence in their problem-solving skills after the study. Fernanda believed that the problem-solving strategies were helpful and that she would be able to continue to improve her own problem-solving ability with more practice. Jenna was afraid of word problems in the beginning of the study and much of that was due to her lack of an academic vocabulary. She persisted with using the problem-solving strategies and became much more confident in her problem-solving ability. Arjun claimed during the interview that he used the problem-solving strategies and that he was a better problem solver by the end of the study. Unfortunately, there was little evidence in his work or in the teacher-researcher's observation notes to back up that claim.

Fernanda's note-taking ability, her study habits, and her independence, allowed her to overcome her lack of experience with word problems. She practiced using the problem-solving strategies and found the strategies helpful. Fernanda had high expectations for herself and she wanted to do well for her parents. Jenna also had good note-taking ability, study habits, and was very independent. She used the problem-solving strategies to improve her problem-solving ability, build her confidence in problem-solving, and learn more academic vocabulary. Arjun agreed with Fernanda and Jenna about not having much problem-solving experience from prior math classes, but he still felt confident in his ability to solve problems. He had very high expectations for himself and he also felt pressure from his parents to perform at a very high level. He tried to use some of the strategies, but did not consistently apply the strategies and therefore the strategies were not helpful for Arjun.

The use of problem-solving strategies improved during the study for eighteen out of twenty students. There was evidence of appropriate use of problem-solving strategies in the practice problems scored by the teacher-researcher. Two out of three of the cases showed improvement in the use of problem-solving strategies. The interview transcripts for Fernanda, Jenna, and Arjun show a positive shift in attitude towards problem-solving as well as a positive shift in their belief of their ability to solve difficult problems.

CHAPTER 5

DISCUSSION, CONCLUSIONS, AND RECOMMENDATIONS

This chapter presents the discussion, conclusions, and recommendations of the teacher-researcher after completing the study on the research question: *what impact will the application of Polya's first two steps to problem-solving have on an advanced high school mathematics classroom?* More specifically: *what is the impact on underrepresented groups?* The teacher-researcher noticed a trend that students do not complete long answer word problems on assignments or assessments and often do not even attempt the problems. The purpose of this study was to determine if problem-solving strategies help students to find an entry point to start solving the problem or give them more avenues from which to attack the problem, so they see it through till the end.

Overview of Study

Twenty students in an IB Math SL and AP Calculus AB class were given questioning strategies and the teacher-researcher modelled those strategies over an eight-week period. The strategies were posted in the Google Classroom. The students worked on a practice problem every week showing evidence of applying the problem strategies. The teacher-researcher assessed each practice problem for the number of problem-solving strategies used and for how many points the students scored based on an AP Calculus AB rubric. The students increased the class mean score from 0.67 above the national average on the pretest to 1.96 above the national average on the posttest. This is an increase of 1.29 points on a scale of zero to nine.

The teacher-researcher wrote her observations in a journal while the students worked on problem-solving strategies. The teacher-researcher coded the journal observations for themes and patterns. Three students from underrepresented groups were selected for in-depth interviews about how they felt using problem-solving strategies. All three students felt as though the questioning techniques were helpful in gaining access to each problem and with practice they each felt more comfortable applying the techniques while solving problems. The transcripts from the three interviews were also coded for themes and patterns. The three themes that emerged from the notes and interviews were; the lack of problem-solving strategies introduced in previous math classes, the independence of the students (some need more guidance than others), and the expectations of the student and/or their parents.

A pretest was administered, then a practice problem was administered once per week, followed by the administration of a posttest. Data was collected from the pretest, posttest and practice problem administrations. The mean score and standard deviation were calculated for each problem and compared to the national mean score and standard deviation for the entire class. The number of problem-solving strategies applied by each student were recorded. The pretest showed the students applying seventeen strategies overall. On the posttest, forty-three different problem-solving strategies were applied by the students. That averages to about two unique application of a strategy per student.

Overall, the mean score of the class improved and increased more than the national mean score. The number of strategies applied by the students increased over the eight-week period of the study. The teacher-researcher observed fewer questions being left blank on the practice problems and on the posttest no problems or portions of problems were left blank. The students who were interviewed felt more confident in their problem-solving ability and were thankful for the problem-solving strategies. Results of the study reveal a positive impact on problem-solving when applying the first two steps of Polya's problem-solving strategies.

Connection to Existing Literature

The approach to classroom instruction differs when observing a traditional approach to teaching concepts versus a problem-solving approach. The teacher-researcher modelled questioning techniques for students to help them persevere in problem-solving. This is consistent with Polya's (1981) statement, "solving problems is a practical art… you can learn it only by imitation and practice" (p.xi). The role of the teacher-researcher was to select the appropriate problems and model the application of appropriate techniques. With proper guidance, the students could solve complicated problems independently which mirrors the results of Sakshaug and Wohlhuter's 2010 study helping teachers incorporate problem-solving in their math lessons.

Students worked in groups one day when the teacher-researcher was observing their group dynamic. Notes from the teacher-researcher journal indicate deeper more meaningful questions being asked and answered by all the group's members. This was at week six of the study and the students were very familiar with the questioning techniques at this point. The teacher-researcher noted the quality and quantity of techniques being used. Asking deeper more meaningful questions concurs with the results of the Di Teodoro et al., (2011) study following four elementary school teachers while they tried to get their students to recognize questioning may help lead to better understanding. This eight-week study began eight weeks prior to the administration of IB Math SL and AP

Calculus AB exams. Four of those eight weeks were already scheduled as exam review time in class. So, the teacher-researcher incorporated problem-solving strategies in a way that did not interrupt teaching standards that were to be assessed on the IB and AP exams. The shift from a traditional approach teaching math to a problem-solving approach changes the way assessments are created and utilized. If problem-solving were added to standardized tests, it would take much longer to grade and the results would not get back to the teachers in a timely manner. The teacher-researcher navigated through learning how to assess students working in groups, by listening and recording observations in a journal. The teacher-researcher also learned how to use more open-ended assessments, so the students had to apply their questioning techniques and reflect on their work throughout the problem-solving process. This suggestion was incorporated to align with Schoenfeld's (2004) treatise, *The Math Wars*. The students in the study were able to apply the problem-solving techniques and the teacher-researcher believes this encouraged them to think mathematically, beyond the borders of specific math content.

The three students from underrepresented groups expressed an attitude of achievement at the end of the study. They felt as though they had become better problem-solvers even when assessment results did not show the same level of success. Each of the three students interviewed expressed a desire to go to college. Fernanda also believed that the problem-solving strategies would easily transfer to other classes, which demonstrates gaining independence and the ability to transfer skills based on her developing problem-solving skills. The skill of solving problems will open avenues for Jenna which will allow her to continue being independent. Since her parents give her freedom and trust her to do what she is supposed to do, her ability to think her way through problems will further her commitment to social justice. Arjun continues to aspire to be a doctor and he still must contend with the high expectations of his parents. His belief that the problem-solving strategies were effective may carry over to his other studies and benefit him in other areas.

Recommendations for Teaching

This study's primary focus was the impact of applying Polya's first two steps of problem-solving on and advanced math class. The secondary focus was the impact on underrepresented groups. Analysis of the data collected reveals an increase in the mean score. The teacher-researcher recorded a class mean of 4.70 versus a national mean of 2.74 out of nine points on the posttest. The posttest demonstrated the largest difference in mean scores and was larger than the difference of mean scores on the pretest and all the practice problems. The results suggest the first two steps of problem-solving had a positive impact on the students in the advanced math class. This reveals a need for teachers to apply problem-solving techniques giving students the opportunity to access difficult problems.

An analysis of the number of strategies used shows an average increase of over two strategies per person per problem. Some strategies were used more often than others, yet several different strategies were used overall. This result would suggest teachers take the time to model different strategies during the problem-solving process. Students, working in groups, need to share their ideas and reflect on which strategies were successful and why. The data collected from the three students of underrepresented groups showed Fernanda using the largest number of strategies, Jenna used less strategies but used them well and Arjun had a difficult time applying strategies successfully. This may imply more individual attention is necessary with some students from underrepresented groups.

The transcripts of the interviews from the three students show each one felt a sense of accomplishment from using the problem-solving strategies. Their attitudes about problem-solving were positive and they were hopeful this would be useful not only in math class but possibly in other classes as well as other aspects of life. Since Arjun's positive attitude did not quite match his performance on the practice problems or posttest, teachers may need evaluate the process more frequently to keep the students focused on the strategies.

Limitations and Suggestions

There are some limitations and delimitations related to this study. As an action research study, the findings may not be generalizable. At the beginning of the study a decision was made to use one combined class of IB Math SL and AP Calculus AB taught by the teacher-researcher versus using two combined classes with one taught by the teacher-researcher and the other class taught by a different teacher. This would have included more subjects, but also would have involved more variability in the modelling and application of the questioning techniques and problem-solving strategies.

The research question is: *what impact will the application of Polya's first two steps of problem-solving have on an advanced mathematics classroom?* Specifically, *what impact will the application of the problem-solving techniques have on underrepresented groups in the classroom?* Polya's problem-solving techniques involve a set of four principles: understand the problem, devise a plan, carry out the plan, and look back. This study focused on the first two steps only to give students access to solving difficult problems. Since this study showed a positive increase in the use of problem-solving strategies and showed an increase in the mean score from the pretest to posttest, maybe using all four steps of Polya's problem-solving would have had a larger positive impact.

The types of problems used in the study focused on a very narrow calculus concept that was about to be tested on the IB and AP exams. The students became more familiar with the topic as the weeks went on and therefore may have been able to produce better scores just because of their familiarity with the concept. The results may have been different if a wider variety of concepts were included in the practice problems. Even though a wide variety of examples were modelled by the teacher-researcher, the students were focused on one concept, which was typically the most difficult concept for students to score well on when they took the IB and AP exams.

The interviewing process may be another limitation for this action research study. Five students from underrepresented groups were enrolled in the IB and AP class. Only three of those students were selected to be interviewed. Those three consented to being interviewed. Five voices may have been better than three. It is also possible that the three students answered interview questions in a way they believed would be pleasing to the teacher-researcher doing the interview. Fernanda and Jenna appeared authentic in their interview responses and showed great feeling in their choice of words. They were willing to share their experiences. Arjun displayed less comfort during the interview process, giving short one-word answers to several questions. He still displayed a positive attitude toward problem-solving and his ability to solve difficult problems using the strategies.

The data did not back up his claims. So, he may have been telling the teacher-researcher what she wanted to hear instead of what he truly felt.

Recommendations for Future Research

To further enhance the ability of students to solve difficult problems, future research may include more students than just a single class of twenty students. This would allow for the collection of more data to see if there is still a positive impact. From the students' interviews, there was a sense that these problem-solving techniques need to be introduced at a much younger age. A study factoring in when would be an appropriate age to start teaching problem-solving techniques may help more students in the long run. This study would warrant another study with more attention paid to working independently versus working in groups. Those students who work independently have a more difficult time sharing information in a group setting. Would that hinder their ability to solve difficult problems or enhance it? Would those students who prefer to work in groups fair as well under the stress of a standardized test?

This study focused on only the first two steps of Polya's problem-solving techniques. Future research may include using all four steps. This would include assessment of completing the problem and a reflection on the process as well as the reasonableness of the solution. Students would be given the opportunity to show their thought process through the entire journey of problem-solving, lending credence to the questioning techniques.

Summary

Giving students the ability to access difficult problems and thereby the opportunity to solve difficult problems is the hallmark of producing productive citizens for our democratic society. This action research study, which implemented two of Polya's problem-solving steps in an advanced math class, revealed a positive impact in three areas. The twenty students in the class showed an increase in their mean scores from pretest to posttest. The same twenty students showed an increase in the number of strategies used on each problem from pretest to posttest. The three individuals who were interviewed show a positive attitude towards using the problem-solving strategies during the study. These problem-solving strategies may help this group of students be more critical thinkers and problem-solvers in their future endeavors.

The teacher-researcher will share these results with other math teachers in the school, and in the school district. Professional development sessions may be created to allow those teachers who are interested in implementing the strategies in their own classroom a chance to work with the teacher-researcher on how to make that happen. The teacher-researcher would also like to complete the four-step process laid out by Polya and see if the impact on students shows an even larger increase in average scores or an even larger increase in the use of strategies so students may perform better on difficult problems.

REFERENCES

Abed, M. G. (2014). A consideration to two main ethical issues in educational research, and how may these be addressed. *Journal on Educational Psychology, 8*(3), 1-14.

Adams, M., Blumenfeld, W. J., Castaneda, Carmelita, Hackman, H. W., Peters, M. L., & Zuniga, X. (Eds.). (2013). *Readings for diversity and social justice* (3rd ed.). New York, NY: Rutledge.

Allen, K. (2011). Mathematics as thinking: A response to "Democracy and School Math". *Democracy and Education, 19*(2), 1-7.

Aufdenspring, D. (2003). Project- and problem-based learning. In S. O'Hara, *National educational technology standards for students curriculum series: Social studies units for grades 9-12* (pp. 15-20). Eugene, OR: ISTE.

Boaler, J. (2008). *What's math got to do with it? Helping children learn to love their least favorite subject - and why it is important for America.* New York: Viking.

Bond, G., & Chernoff, E. J. (2015, July). Mathematics and social justice: A symbiotic pedagogy. *Journal of Urban Mathematics Education, 8*(1), 24-30. Retrieved from http://education.gsu.edu/JUME

Boss, S., & Krauss, J. (2014). *Reinventing project-based learning: Your field guide to real-world projects in the digital age.* International Society for Technology in Education.

Buckley, J. (2012). *NCES Statement on PIRLS 2011 and TIMSS 2011.* Washington, D.C.: National Center for Education Statistics.

College Board. (2017, 02 19). *AP Course and Exam Description.* Retrieved from College Board: https://secure-media.collegeboard.org/digitalServices/pdf/ap/ap-calculus-ab-and-bc-course-and-exam-description.pdf

Conner, A., Edenfield, K. W., Gleason, B. W., & Ersoz, F. A. (2011). Impact of a content and methods course sequence on prospective secondary mathematics teachers' beliefs. *Journal of Mathematics Teacher Education, 14*, 483-504.

D'Ambrosio, B. S. (2003). Teaching Mathematics through Problem-solving: A Historical Perspective. In H. L. Schoen, *Teaching Mathematics through Problem-solving.* (pp. 39-52). Reston, VA: NCTM.

Dewey, J. (2015). *Experience and education.* New York, NY: Free Press.

Di Teodoro, S., Donders, S., Kemp-Davidson, J., Robertson, P., & Schuyler, L. (2011). Asking good questions: Promoting greater understanding of mathematocs through purposeful teacher and student questioning. *Canadian Journal of Action Research*, *12*(2), 18-29.

Education, US Department of. (2013, April). *A blueprint for R.E.S.P.E.C.T.* Retrieved February 21, 2016, from US Department of Education: http://www2.ed.gov/documents/respect/blueprint-for-respect.pdf

Fox, L., Finer, K., Khourey-Bowers, C., & Heaphy, L. (2016, February). The genesis and evolution of two interdisciplinary courses focused on global learning. *Currents in Teaching and Learning*, *7*(2), 46-57.

Green, S. K., & Johnson, R. L. (2010). *Assessment is Essential.* New York, NY: McGraw-Hill.

Grouws, D. A. (2003). The teacher's role in Teaching Mathematics through problem-solving. In B. S. D'Ambrosio, *Teaching Mathematics Through Problem-solving: Grades 6 - 12* (pp. 129-141). Reston, VA: The National Council of Teachers of Mathematics, Inc.

Henson, K. T. (2015). *Curriculum planning: Integrating multiculturalism, constructivism, and education reform* (5th ed.). Waveland Press.

Hutchings, K., & Standley, M. (2000). *Global project-based learning with technology.* Eugene, OR: Visions Technology in Education.

IBO Mission. (2016). Retrieved July 10, 2016, from About the IB: http://www.ibo.org/about-the-ib/mission

Inglis, L., & Miller, N. (2011). Problem based instruction: Getting at the big ideas and developing learners. *Canadian Journal of Action Research, 12*(3), 6-12.

International Baccalaureate Organization. (2017, January 28). *IB Programmes*. Retrieved from International Education - International Baccalaureate : www.ibo.org

Krajcik, J. S., & Blumenfeld, P. C. (2006). Project-Based learning. In R. Sawyer (Ed.), *The Cambridge handbook of the learning sciences* (pp. 317-334). Cambridge University Press.

Labaree, D. F. (2011). Consuming the public school. *Educational Theory, 61*(4), 381-394.

Larrier, Y. I., Hall, K., Linton, J. M., Bakerson, M., Larrier, I. M., & Shirley, T. S. (2016). Problem based learning: A viable school counseling intervention to promote student engagement with at-risk high school students. *National Teacher Education Journal, 9*(2), 11-20.

Lee, C. Y., & Chen, M. J. (2015). Effects of Polya questioning instruction for geometry reasoning in junior high school. *Eurasia Journal of Mathematics, Science & Technology Education, 11*(6), 1546-1561.

Lewis, C. (2016). How does lesson study improve mathematics instruction. *ZDM Mathematics Education, 48*, 571-580.

Mann, H. (1848). *Twelfth Annual Report.* Boston: Massachusetts State Board of Education.

McDougal, T., & Takahashi, A. (2014, Fall). *National Association of Independent Schools*. Retrieved from Independent Teacher: http://www.nais.org/Magazines-Newsletters/ITMagazine/Pages/Teaching-Mathematics-Through-Problem-Solving.aspx

Mehta, J. (2015). Escaping the shadow: "A Nation at RIsk" and its far-reaching influence. *American Educator, 39*(2), 20-26.

Mertler, C. A. (2014). *Action reasearch: Improving schools and empowering educators* (4th ed.). Thousand Oaks, CA: Sage Publications.

Mertler, C. A. (2014). *Action research: Improving schools and empowering educators.* (4th ed.). Thousand Oaks, California: SAGE Publications, Inc.

Mills, G. E. (2007). *Action research: A guide for the teacher-researcher* (3rd ed.). Upper Saddle River, NJ: Pearson Merrill Prentice Hall.

National Commission. (1983). *A nation at risk: The imperative for educational reform.* Washington, DC: Author.

National Council of Teachers of Mathematics. (2010, October 20). *Principles to actions: Executive summary*. Reston, VA: NCTM. Retrieved June 26, 2016, from NCTM, National Council Of Teachers of Mathematics: http://www.nctm.org/PtA/

National Council of Teachers of Mathematics. (2010). *Why is teaching with problem-solving important to student learning?* Reston, VA: National Council of Teachers of Mathematics.

National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics.* Washington, DC: National Governors Association Center for Best Practices, Council of Chief State School Officers. Retrieved from www.corestanddards.org/math/

O'Donnell, B. D. (2006). On becoming a better problem-solving teacher. *Teaching Children Mathematics, 12*(7), 346-351. Retrieved from http://www.jstor.org/stable/41198763

Oxford Dictionaries. (2016). Retrieved March 18, 2016, from http://oxforddictionaries.com

Perveen, K. (2010). Effect of the problem-solving approach on academic achievement of students in mathematics at the secondary level. *Contemporary Issues in Education Research, 3*(3), 9-13.

Pinkerton, M., & Shafer, K. G. (2013). Punch up Algebra with POWs. *The Mathematics Teacher, 106*(6), 424-431. doi:Retreived from http://jstor.org.pallas2.tcl.sc.edu/stable/10.5951/mathteacher.106.6.0424

Pliego, G. J. (2008). Problem-Based learning and teaching introductory statistics. *The International Conference on Learning* (p. n.p.). Chicago, Ill: Common Ground.

Polya, G. (1945). *How to solve it: A new aspect of mathematical method.* Princeton and Oxford: Princeton University Press.

Polya, G. (1981). *Mathematical discovery: On understanding, learning, and teaching problem-solving.* New York, NY: John Wiley & Sons.

Polya, G. (1985). *How to solve it: A new aspect of mathematical method.* Princeton and Oxford: Princeton University Press.

Polya, G. (2002). The goals of mathematical education. (T. C. O'Brien, Interviewer)

Sakshaug, L. E., & Wohlhuter, K. A. (2010). Journey toward teaching mathematics through problem-solving. *School Science and Mathematics, 110*(8), 397-409.

Schiro, M. S. (2013). *Curriculum theory: Conflicting visions and enduring concerns.* Los Angeles: Sage Publications, Inc.

Schoenfeld, A. H. (2004). The math wars. *Educational Policy, 18*(1), 253-286.

Silva, C., Moses, R., Rivers, J., & Johnson, P. (1990). The Algebra Project: Making Middle School Mathematics Count. *The Journal of Negro Education, 59*(3), 375-391. doi:10.2307/2295571

Stanic, G. (2003). *A History of School Mathematics, Volume 1*. (G. Stanic, & J. Kilpatrick, Eds.) Ann Arbor, MI: National Council of Teachers of Mathematics.

Stanic, G. M., & Kilpatrick, J. (1989). Historical perspectives on problem-solving in the mathematics curriculum. *The teaching and assessing of mathematical problem-solving, 3,* 1-22. (R. I. Charles, & E. A. Silver, Eds.) Hillsdale, N.J.: Lawrence Erlbaum Associates, and Reston, VA: NCTM.

Stemhagen, K. (2016). Deweyan democratic agency and school math: Beyond constructivism and critique. *Educational Theory, 66*(1-2), 95-109.

Sury, B. (2014). George Polya - educator extraordinaire. *Journal of Science Education,* pp. 303-306.

United States. National Commission on Excellence in Education. (1983). *A nation at risk: The imperative for educational reform. A repot to the Nation and the Secretary of Education, United States Department of Education.* Washington, DC: The Commission.

US Congress, O. o. (1992). *Testing in American schools: Asking the right questions, OTA-SET-519.* Washignton DC: US Government Printing Office.

Voskoglou, M. G. (2011). Problem-solving as a component of the constructivist view of learning. *Journal of Education Research, 4*(3), 187-204.

Widyatiningtyas, R., Kusumah, Y. S., Sumarmo, U., & Sabandar, J. (2015, July). The impact of problem-based learning approach to senior high school students' mathematics critical thinking ability. *IndoMS-JME, 6*(2), 30-38.

Zeichner, K. M., & Liston, D. P. (1996). *Reflective teaching an introduction.* Mahwah: Lawrence Erlbaum Associates, Inc.

Ziebarth, S. W. (2003). Classroom assessment issues related to teaching mathematics through problem-solving. In B. S. D'Ambrosio, *Teaching Mathematics Through Problem-solving: Grades 6 - 12* (pp. 177-189). Reston, VA: The National Council of Teachers of Mathematics, Inc

APPENDIX A

PROBLEM-SOLVING GUIDE FOR STUDENTS

First Principle: *Understand the problem*

- Do you understand all the words used in stating the problem?
- What are you asked to find or show?
- Can you restate the problem in your own words?
- Can you think of a picture or diagram that might help you understand the problem?
- Is there enough information to enable you to find a solution?

Second Principle: *Devise a plan*

- Guess and check
- Look for a pattern
- Make an orderly list
- Draw a diagram or draw on a diagram
- Solve a simpler problem use symmetry
- Use a model
- Consider special cases work backwards
- Use direct reasoning

APPENDIX B

INTERVIEW QUESTIONS

First Interview

- Please discuss your level of comfort with the subject of mathematics.
- What is the first thought that comes to your mind when you see a word problem on a math test?
- How comfortable are you with multi-part problem? Explain.
- When you get stuck on a difficult problem, what do you try?
- Have you found success with any of your techniques?
- Do you feel that problem-solving will be an integral part of your future profession?
- What is your parent's educational background. How do they support you?

Second Interview

- How would you describe your comfort level with problem-solving after these eight weeks?
- Describe any success you had using Polya's problem-solving techniques.
- Were there any techniques, in particular, that resonated with you? That you would continue to use in the future?
- Did this process help you to become a better problem-solver?