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Global Problem-Based Learning In Math

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GLOBAL PROBLEM-BASED LEARNING IN MATH

by

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DEDICATION

For my loving husband who has been by my side since a day long ago when I made the decision to become a teacher. You have been my partner, my helper, my cheerleader, my thesaurus, my encourager, my sounding board, and my comforter. Thank you, David, for helping me think.

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ABSTRACT

In the high school of focus in this study, mathematics students struggle to transfer and apply skills to unfamiliar contexts. Also, minority students are underrepresented in advanced mathematics course enrollments. This two-part study consists of a longitudinal analysis of enrollment trends in mathematics courses as well as an action research study of the impact of Global Problem-Based Learning on achievement and attitudes in advanced mathematics classes. The study was conducted at a southern U.S. International Baccalaureate (IB) high school with an annual enrollment of approximately 1300 students. The longitudinal study examined three cohorts of high school graduates, 797 students, beginning with their enrollment in advanced Algebra 1. The sample used in the action research study was comprised of 25 IB Mathematical Studies students. A mixed-method structure including summative assessment scores, observations, questionnaires, discussion boards, and focus group interviews generated both quantitative and qualitative data. Enrollment data revealed a disproportional number of minority students do not sustain an advanced level of mathematics courses. The action research results indicate the use of Global Problem-Based Learning had positive impacts on both student achievement and attitudes. These findings suggest that incorporating Global Problem-Based Learning could improve access to advanced mathematics for all students as well as motivate mathematics students to persevere.

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CHAPTER 1

INTRODUCTION

Mathematics is often considered to be a challenge, not because of students' lack of ability, but because the study of mathematics is often deemed boring or irrelevant, which lends to insufficient focus and limited engagement. One reason studying mathematics is perceived as such by students is that the language of mathematics takes years to develop, and even then, mathematics is difficult to discuss as a narrative, unlike nearly all other academic subjects (Medoff, 2013). Students (and adults) often quickly develop a self-assigned weakness in mathematical ability, and this attitude is socially acceptable. It is not uncommon to hear students and adults admit their self-perceived weak abilities in math, even that they hate mathematics (Soni & Kumari, 2017). Furthermore, there is little motivation to excel in mathematics; the social respect given a mathematician is not comparable to that given a doctor or lawyer, even though the scholarly pursuits are comparable (Larkin & Jorgensen, 2016). The result is an education system that focuses on individual skills delineated by state and national standards, which lends to a lack of access to deep mathematical understanding. Another significant problem in mathematics classrooms is the ethnically disproportionate enrollment in advanced mathematics classes. With the rapidly changing demographics of communities and schools within the United States, students need strategies to know, appreciate, understand, and work with people from backgrounds and cultures different than their own. "When students experience the strength of diversity they begin to value others in new ways" (Hutchings & Standley, 2000, p. 113).

Theorist John Dewey (1938) detailed how children learn by working through real-world problems. Problem-Based Learning (PBL) is a methodology anchored in progressive theory that facilitates access to the relevance and application of mathematics; Global Problem-Based Learning (GPBL) extends that access to all students and simultaneously engages them to become global citizens (Mansilla, 2011; Moses & Cobb, 2001; Rosa, 2011; Strayhor, 2010). GPBL opens the contexts of mathematics to a worldwide view while building upon established, research-based strategies. It leads students of all cultures to engage with contemporary global issues, thereby reinforcing the relevance of mathematics. This research will seek to better understand enrollment trends in mathematics classes in terms of gender and race. This research will also explore the utilization of GPBL to stimulate all mathematics students to make connections and transfer skills among topics, global contexts, and applications.

Problem in Mathematics Classes

In the high school of interest to this study, students do not appear able to transfer and apply their understanding of mathematics skills they are learning to other mathematics skills, topics, and application contexts. This inability is evidenced by the daily requirement to review and reteach, student resistance to persevere with application problems, and by inconsistent achievement. The school of interest in this study is an International Baccalaureate® (IB) "world school" that is committed to teaching with specific emphasis on global issues. This commitment is affirmed in the IB Mission Statement, "The International Baccalaureate® aims to develop inquiring, knowledgeable and caring young people who help to create a better and more peaceful world through intercultural understanding and respect" (IBO Mission, 2016). However, integration of global issues in mathematics classrooms is at best inconsistent. Furthermore, enrollment in IB mathematics classes, the most rigorous courses available, does not reflect the demographic identity of the school. A review of enrollment for the last two years revealed 80% Caucasian, 5% African-American, 10% Hispanic and 5% represented by students of other ethnicities. Meanwhile the overall school demographics for this same time period was 53%, 10%, 33%, and 4% respectively. Because students choose their mathematics courses, it is not clear if the discrepancy is due to student choice or student preparation for rigorous mathematics courses. Regardless of the reason, this discrepancy is inconsistent with the IB philosophy (IBO Mission, 2016). The superficial depth of mastery, a persistent lack of student diversity, and artificial contexts within advanced mathematics classes demand a need for globally linked instructional practices facilitating a deeper understanding of and connections among mathematical topics for all students.

Thinking is learning, and teachers have a unique opportunity to influence long-term effects on the future. Experiential learning, such as that at the core of progressivism, is a means for this teacher-researcher to compete for the attention of immediate-gratification seeking, technology-habituated teenagers. The current emphasis on assessment results and grade point averages and even community service (logs), too often results in students (and parents) focusing on resume building, rather than building thinkers. John Dewey (1938) closes his book *Experience and Education* with his thoughts on education itself by challenging the reader to consider what is worthy of the name education.

Objectives

GPBL is a method by which to challenge teachers to focus content in a way that empowers students to think deeply and considerately, to recognize the strengths and contributions of all peoples, and to persevere with a purpose. Not only will these students be best prepared for further education and the global job market, but they will also be best prepared to be global citizens (Mansilla & Jackson, 2011; Martin, 2010). The following research questions will be used to guide the exploration of the stated problem of practice. *1) What are historical enrollment trends in mathematics classes at the high school of focus? And 2) What impact does focusing mathematics lessons on specific, authentic global problems have on achievement and attitudes in advanced mathematics classes?*

This study has four goals: (a) to reconsider the obsolete instructional model typical in mathematics classrooms of fact – example – practice; (b) to examine the necessity of teaching mathematics with relevance, connections, and applicability; (c) to understand and act on current realities regarding limited access to advanced mathematics for minorities; and (d) to reframe the study of mathematics as an accessible structure for understanding our global society and its challenges.

Glossary of Key Terms

In conducting this study, there is terminology specific to the context of this study. *Global Competency* refers to the development of an understanding of international issues and the ability to work with people from various cultural backgrounds (Mansilla & Jackson, 2011).

Global Citizenship refers to a person's recognition of their relationship to the world equal to national or local relationships (Martin, 2010).

Problem-Based Learning refers to a student-centered approach to education in which students learn a subject through solving open-ended problems (Scott, 2014).

Social Justice refers to an expectation for all people to have equal access to the distribution of economic, political, and social rights (Garii & Rule, 2009).

Cultural Relevance refers to acknowledgement and integration of the home community culture of students, including cultural experiences, values, and understandings (Brown-Jeffy & Cooper, 2011).

Purpose of this Study

Experts widely accept the benefits of PBL in the field of education; the research supporting PBL is extensive (Barrows, 1985; Delisle, 1997; Scott, 2014). In her book Creating Standards-Based Integrated Curriculum, Susan Drake (2012) states "PBL is superior to traditional education for long-term retention of knowledge, skills development, and the satisfaction of students" (p. 23). In Chapter 2 the findings related to the strengths of PBL by multiple researchers are discussed, as are the challenges of implementing PBL. "Today's employers are looking for candidates who know how to work as a team, adapt to change, access and analyze information, and think creatively to solve problems" (Boss & Krauss, 2014, p. 16) "PBL is based on the constructivist finding that students gain a deeper understanding of material when they actively construct their understanding by working with and using ideas" (Krajcik & Blumenfeld, 2006, p. 317).

The quickly changing demographics of the global job market as well as within schools in the United States require a shift to a more culturally diverse workforce.

> Multicultural education refers to establishing and maintaining a classroom climate or culture in which students appreciate diversity and allow themselves to be enriched by the opportunity to work with students from many cultural backgrounds, including differences in ethnicity, political affiliation, socioeconomic status, ability level, and religion. (Henson, 2015, p. xvi)

The U.S. Department of Education published A Blueprint for R.E.S.P.E.C.T. (Recognizing Educational Success, Professional Excellence, and Collaborative Teaching) in April 2013. This report states: "Every child in America deserves a high-quality education that prepares her or him for college, a career, and responsibilities of citizenship" (p. 1). Moreover, "…students are confronting unprecedented challenges and heightened competition in an increasingly knowledge-based, global job market" (p. 1). The urgent message of this report specifies that "…to productively engage in our democracy and compete in our global economy, students will need strong, well-rounded academic foundations; cultural and global competencies; the ability to collaborate, communicate, and solve problems; and strong digital literacy skills" (p. 19).

The purpose of the present action research study is to explore the impact of GPBL in mathematics classes. In addition to focusing on relevant contexts to address the observed struggles of mathematics students to make connections and transfer skills, this study also addresses the inconsistent integration of global issues that contribute to a limited perception of mathematics as a tool to find the *correct* answer. GPBL offers students a method of modeling "through replicable and empirical demonstrations, the nature and intersections of global and local power systems in a way that students can comprehend" (Bond & Chernoff, 2015, p. 29). Also, with a goal to improve the accessibility of advanced mathematics courses for all students through better understanding the current environment, this study includes exploring and describing the enrollment trends of minority students in advanced mathematics classes.

Overview of Related Literature

PBL is a constructivist-based instructional strategy intended to promote engaged learning (Larrier, Y. I., Hall, K., Linton, J. M., Bakerson, M., Larrier, I. M., & Shirley, T. S., 2016). This curricular approach develops problem-solving as well as interdisciplinary knowledge, and inquiry skills (Aufdenspring, 2003). There are several benefits of PBL, including increases in motivation for learning, retention of information, and the ability to apply knowledge across academic disciplines and life situations (Larrier, et al., 2016). Students learn to "integrate multiple perspectives and discover that in the real world, 'right answers' are rare to nonexistent" (Aufdenspring, 2003, p. 19). PBL helps to connect students to the learning. Aufdenspring concludes that students develop respect for various points of view while developing the ability to defend their points of view.

Widyatiningtyas, Kusumah, Sumarmo, and Sabandar (2015) studied the impact of PBL on the critical thinking ability of high school mathematics students. They found that within the parameters of their study, students taught using a PBL approach had better mathematical critical thinking ability than students who learned in a conventional approach. Comparably, in a presentation at the 2008 International Conference on Learning, German Pliego presented results of a similar study. Pliego concludes that the problem-based approach is well-suited for studying introductory statistics. "[PBL] provides the infrastructure to develop and implement a basic statistics course to put students in the driver seat to learn statistics by attempting to solve problems" (para. 2). A detailed discussion of research related to PBL is included in Chapter 2.

Regarding the integration of a more global perspective into the mathematics classroom, research by Fox, Finer, Khourey-Bowers, and Heaphy (2016) emphasizes the importance of courses that "can add to the students' understanding diverse worldviews, even when those students are place-bound" (p. 54). The expectations for students to be knowledgeable about issues beyond their community is being articulated by employers, politicians, and educators (Fox, et al., 2016). This topic is reviewed further in Chapter 2.

Social justice is fairness regarding the distribution of wealth, opportunities, and privileges within a society (Oxford Dictionaries, 2016). The civil rights activist and mathematics teacher Bob Moses (Moses B. , 1995), led the country to consider mathematics literacy and economic access as essential to "give hope to the young generation" (p. 56). Moses' focus was to build a curriculum based on concrete yet complex events as a pathway for African American students to master algebra, and through algebra, college preparatory mathematics. "Education [of African American students] has to produce graduates who can think in a critical way with quantitative data" (p. 58). Bond and Chernoff (2015), writing on the topic of social justice integrated into content classrooms, name social justice as a unique challenge for all teachers because of the dynamic and on-going process it represents. "It requires the maintenance of extensive and constantly evolving understandings of every student in a given classroom" (p. 29). There is an urgency to face the truths of diversity and work toward social justice, "because the classroom itself is one of the primary sources of the socialization that shapes social inequality, an uncritical pedagogy serves only to enforce existing systems of dominance and inequity" (p. 27).

At the school of interest to this teacher-researcher, the enrollment in advanced (IB) mathematics classes does not reflect the demographic identity of the school. For minority students, this results in a restriction of access to the most rigorous program of study available at this school. Such restriction of access is a foundational element of oppression (Adams, et al., 2013).

Limitations and Significance

Although the goals of this study are broad, the nature of the study presents some limitations; action research does not produce results that are generalizable. The student population at the high school under study has unique characteristics including a somewhat transient population, and the school is a school of choice within the district of six high schools. Although a student's home school zone is not a variable of this study, this could have an influence on enrollment in IB courses. The teacher-researcher is assuming that all students enrolled in the course understand and accept the expectation of independent work more rigorous than that of previous mathematics courses. Also, it is notable that the mathematics course progression of minority students, particularly those who did not attend elementary and middle schools in the United States, may be considerably different among themselves and from their American-educated peers. However, these potential limitations and assumptions do not diminish the potential insights of this study. The field of mathematics education is experiencing a continuing deficit of applicants to the number of teaching positions. In-service and pre-service mathematics teachers face the dual challenge of engaging students mathematically and teacher shortages that often result in large class sizes. An updated teaching model is desperately needed to improve the buy-in of all students to the necessity and empowering nature of mathematics.

Summary

The demographic profile of students enrolled in advanced mathematics classes skews toward the white male majority (Atuahene & Russell, 2016; Useem, 1991). However, even for those students who take advanced mathematics in high school, the level of proficiency is low (The College Board, 2016). Topics and contexts typical in mathematics classes are not engaging. GPBL is an instructional model that frames the study of mathematics within real-world global issues. This increased relevancy may lead to greater access for all students.

The first chapter of this dissertation in practice introduced the reader to the identified key elements framing this action research study. These elements include the problem of practice with an overview of the context of the study, the research questions that will guide the study, a purpose statement that justifies the study, a brief overview of related literature as well as the action research design, and a discussion of ethical considerations related to this study. The second chapter will examine and review the related literature relevant to GPBL and this study. The discussion will include research on assessment, gaps in achievement, the importance of global competencies and social justice, and PBL, as well as the theory undergirding this strategy. The third chapter will detail the specific methodology used to collect and analyze the data gathered as part of this study. The fourth chapter will consist of reporting the findings, a discussion of these findings, and an interpretation of the results of the study. The fifth chapter will report the conclusions and implications of the study. The discussion will include a summary of major points, conclusions based on the findings and implications for both teaching and further research.

CHAPTER 2

LITERATURE REVIEW

The proposed adaptation of PBL into a new GPBL is intended to focus content in a way that empowers students to persevere and think deeply, and to embrace the benefits of collaboration among a diverse group of students. It is the expectation that such a focus will result in students who are not only prepared for further education and the global job market but also prepared global citizens. The braiding of Problem Based Learning along with Cultural Relevance Pedagogy (CRP) and Social Justice Pedagogy (SJP) creates a structure for learning mathematics that is known as GPBL. The combining of these instructional frameworks and strategies is necessary because none alone "fully address major shortcomings in the mathematics classroom" (Leonard, Brooks, Barnes-Johnson, & Berry, 2010).

A review of current literature related to mathematical achievement, the disproportional demographics represented by students who pursue advanced mathematics in high school, and the PBL approach resulted in the emergence of four inter-related themes. This chapter is therefore organized into four parts, based on these major themes. To document the need for this study, the literature review's first two parts are comprised of research on the current reality of mathematics classrooms and assessment expectations. Next, it includes a summary of findings from research on the depth of understanding demonstrated on mathematics assessments, including identified gaps in achievement. This section also includes findings that support a needed change in scope of instruction in the form of summaries of current research on the importance of global competencies, internationalism, global citizenship, and social justice. Finally, to articulate and support the need for a change in instructional strategies, PBL research includes foundations and strengths, as well as the challenges of implementing PBL; a necessary basis for the proposed adaptation to GPBL. Current peer-reviewed research represents the majority of the sources for this literature review, also included are foundational and seminal works, these being of essential value for theoretical and historical perspectives.

Most of the literature review is built upon primary sources in the form of current, peer-reviewed research in U.S. and international academic journals. Other sources used in this chapter include textbooks on curriculum theory and studies. These secondary sources are current textbooks authored by practitioners established in the field and reflect the current understanding of topics relevant to this study. In addition to the textbooks, the literature is composed of government documents that detail educational policy, a key component of both content and assessments. International Baccalaureate documents are also used, relevant as the school in this study's setting is an IB World School and therefore guided by the International Baccalaureate Organization. These sources represent the previous and current literature on the problem of practice addressed in the current study. Although there exists a growing body of research on teaching mathematics using social justice, this study serves to add the perspective of integrating CRP, SJP, and PBL into GPBL. The research is within the most recent five years, except in the case where there was no research on a specific topic found within that time frame. The university library was invaluable for conducting this research, drawing on ERIC (EBSCO) and Education Source databases for most of the academic journals.

Current Reality of Mathematics Classrooms

A thorough understanding of the root causes of the problem of practice requires an honest look at the current reality of today's mathematics classrooms. This reflection bears out a need for research in areas including the focus of skills, quantitative literacy, the perception of mathematics, and textbooks. The following summarizes research for each of these features of the mathematics classroom in more detail.

Skills Focus

In 1983 *A Nation at Risk* was published by the National Commission on Excellence in Education. This set into motion the standards movement, which remains at the core of American education. This came at a time when the Learner-Centered ideology, with a goal of self-actualized growth, teachers as facilitators, and authentic assessment, as well as the Social Reconstruction ideology with its goal to attend to our society's problems and reveal the influences of the hidden curriculum were both actively influencing American education (Schiro, 2013). However, for decades following the turn to a standards-based curriculum these progressive ideologies have lost momentum. In today's classrooms, the emphasis is on accountability and testing. Teaching strategies follow a transmittal pedagogy, even to the extent of teaching to the test. The transmittal pedagogy is nowhere more evident than in the mathematics classroom where students "associate mathematics with memorizing techniques out of context, completing long lists of mindless exercises, and being judged on the number of right answers" (Allen, 2011, p. 4). Although there has been a push for an increase in reasoning to complement the computation, the progress toward this goal has been slow.

One of the boldest movements towards the goal of changing the measurement of mathematics competency has been the Common Core State Standard Initiative, a state-led effort to design a new framework for educators. Led by the Council of Chief State School Officers (CCSO) and the National Governors Association (NGA) Center, common core state standards were “based on rigorous content and application of knowledge through higher-order thinking skills” (About the standards, 2017, p. 1). But although many states initially garnered support for these new standards, several did not see it through. As with most features of public education, political factors have a strong influence (Supovitz, Daly, & del Fresno, 2015).

Lack of Quantitative Literacy

In 1989, the Mathematics Association of America formed the Quantitative Literacy Subcommittee of the Committee on the Undergraduate Program in Mathematics (MAA, 1994). This subcommittee issued guidelines for quantitative literacy programs that brought into clear focus the desired outcome of a mathematical education; an ability for mathematical skills to be applied to real world problems. What followed was a surge in the constructivist-based curriculum. The constructivist approach is anchored in the works of theorist John Dewey (1910) who said “The origin of thinking is some perplexity, confusion, or doubt. Thinking is not a case of spontaneous combustion; it does not occur on ‘general principals.’ There is something specific which occasions and evokes it” (p. 12). There is a clear and supportive connection between constructivism and the *Principles and Standards for School Mathematics* published by the National Council of Teachers of Mathematics in 2000 (Stemhagen, 2011). However, as Allen (2011) points out, the implementation of any practice is varied, influenced by the individuality of the teacher greatly. “If the teacher does not make use of a variety of strategies to meet the range of students’ needs, constructivist practices can perpetuate bias and exacerbate problems of inequity rather than reduce them” (p. 4).

To explore this idea of the impact that teachers bring to implementation of teaching strategies, Allen (2011) composed the following list based on the previous studies led by Benilower and by St. John, of common challenges teachers encounter:

- They did not learn mathematics themselves using these strategies.
- They have not seen strong models for using these strategies in real classrooms.
- They feel a lack of confidence in and control over the lesson.
- They fear the questions that students may ask if the mathematics becomes too open-ended.
- They have trouble tolerating the noise associated with constructivist instructional practice.
- They question the effectiveness of small groups of engaging all learners. (p. 4)

Quantitative literacy, as detailed by Boersma, Diefenderfer, Dingman, and Madison (2011) is based on a set of core competencies that include interpretation, representation, calculation, analysis/synthesis, assumptions, and communication. Writing Across the Curriculum, one of the longest-running educational reform movements holds as the driving tenant that writing should be an integral part of the learning process in all subjects. Furthermore, although students arrive with varied experiences and at varied literacy levels “all students can learn to become more proficient writers” (Cox, et al., 2014, p. 1). Similar to Writing Across the Curriculum, there is a movement to incorporate quantitative literacy or numeracy across the curriculum. A comparison of numeracy education, also known as quantitative literacy, and the Writing Across the Curriculum movement was conducted by Hillyard (2012) who found that there are many similarities in the challenges of both endeavors. She draws upon the successes of Writing Across the Curriculum for suggestions to offer to schools and teachers. First, it is essential to document that numeracy education increases students' critical thinking and decision-making skills. Also, resources are needed for teachers to integrate numeracy education into their disciplines, an electronic compilation is a good starting point.

Mathematics is Perceived as Too Hard

Seemingly unaware that innumeracy, the inability to do basic mathematics – the equivalent to mathematical illiteracy (Innumerate, 2017), is a limiting factor in the pursuit of life goals, many people, adults as well as children, readily admit to their poor mathematics skills (Allen, 2011). It has become socially acceptable to laugh at one's inabilities in math. According to the findings of Beilock, Gunderson, Ramirez, and Levine (2010) teachers, especially female teachers, frequently experience mathematics anxiety at such a level that the anxiety transfers to their students. Nearly all elementary school teachers are female, more than 90% of them. Beilock et al. conducted a study with elementary teachers and their students who were pre- and post-tested to measure achievement. The students were also given the task at the beginning of the year and again at the end, to draw characters described in gender-neutral stories, these drawings were used to measure gender stereotypes. The study revealed that although at the beginning of the school year there was no relation between a teacher's mathematics anxiety and her students' mathematics achievement, that at the end of the year, the more anxious teachers were about math, the lower the girls' mathematics achievement. Furthermore, those girls endorsed the commonly held stereotype that “boys are good at math, and girls are good at reading” (Beilock et al., 2010, p. 1860). Elementary teachers are certainly not the cause of all low mathematics achievement among female students, but these findings are significant and contribute the overall understanding of the lack of motivation.

Motivation, which can be related to perceptions of the difficulty of math, has also been documented in other studies relevant to low performance in mathematics. Hardre (2012) conducted studies on motivating and retaining rural high school students over a 15-year period then synthesized her findings into what she refers to as strategic principles to help rural teachers. She found “their [rural students’] lack of motivation leads to disengagement and dropout from school and educational pursuits” (p. 11). Her 15 years of research on motivating rural secondary students revealed that motivation in mathematics was among the lowest. Mark Russo (2015), to combat poor motivation among his students, conducted action research to explore the impact of co-construction on students’ quantitative literacy and attitudes towards math. A constant comparative method was used to analyze data on assessment and teacher-researcher class observations and reflections. He documented a lack of interest and lack of relevance common among his mathematics students. Even with the opportunity to plan what mathematics topics and contexts to cover in the course, his students were not familiar with mathematics topics relevant to their lives and therefore could not provide much input about what they wanted to study (Russo, 2015).

Problems with Textbooks

Textbook publishing is a fourteen-billion-dollar industry divided into two approximately equal sectors: K-12 and Higher Education; an industry that raised the price of textbooks over the last 30 years more than that of medical expenses, new homes or the Consumer Price Index, a whopping 800% (Band, 2013). Before 2008 publishers released to the National Association of College Stores a breakdown of the costs that revealed that approximately 77.4 cents of every dollar went to the publishers. This money was distributed among marketing, authors, materials, and of course, profit (Kurtzleben, 2012). With the K-12 market dominated by three publishers (Pearson, McGraw-Hill, and Houghton Mifflin Harcourt) and mathematics being a core subject area in each of the US states, it follows to reason that the textbooks available would be contextually stimulating and culturally neutral, but that is not the case as described in the following research reviews.

In 2016 Booth, Begolli, and McCann conducted a lengthy study to test the effects of changing textbooks by adding a variety of worked examples in algebra textbooks. They tested conceptual understanding, procedural skills, and error anticipation. The original form of a textbook was used to teach two classes, and another two classes with the same textbook augmented with "approximately 26% of the problems replaced with correct, incorrect, or partial solutions to that problem" (Booth, Begolli, & McCann, 2016). Pre- and post-tests were used, and the findings revealed that with the use of the augmented textbook the procedural problem-solving skills increased as did the error anticipation. This study suggests that textbooks are lacking in powerful error analysis examples, which have been shown effective.

In 2011 the National Council for Teachers of Mathematics held its first ever Interactive Institute: *Infusing Classrooms with Reasoning and Sense-Making* (Shaughnessy, 2011). This push for increasing reasoning and sense making took the form of incorporating tasks into the curriculum. Tasks were meant to be an extended, context-rich scenario requiring modeling and multiple problem-solving skills as well as contextual communication of the possible solutions. Following this, some mathematics textbooks began to include tasks.

Xiong Wang (2016) conducted a study of the predominant textbook, analyzing the textbook tasks, the practice and homework sets, and the teacher resource guide. She used coding to establish the knowledge level of the tasks, the cognitive demand of the practice and the appropriateness and usefulness of the teacher guide. Her results revealed that the tasks were at sub-medium knowledge level, the cognitive demands were at a medium-level while communication was higher-level and that a major factor in the supportive strategies provided for teachers was scaffolding students' thinking and reasoning. Overall Wang found that problem solving and understanding were lacking. "For instance, 'Investigate' in each lesson is designed for doing mathematics. However, the tasks involved lower levels of knowledge, and their cognitive demands failed to achieve the curriculum goal" (Wang, 2016, p. 18).

With the current availability of digital resources including the Internet and wide variety of devices, it follows that textbooks could be enhanced digitally to meet the needs of students better. There is a growing trend of using digital or etextbooks, supplemented with auxiliary materials provided by the publishers intended to contextualize further and otherwise enhance the content. To measure the effectiveness of using such resources Nagra, Eng, and Karrass (2013) conducted a study of the effectiveness of etextbooks, traditional textbooks and a bundled approach of both traditional plus online resources. The two variables tested were student interaction with the text and their interest in mathematics as a result of the form of resource. Three sources of data were used to triangulate their findings: pre- and post-surveys of the students, a follow-up focus group, and an interview of the mathematics teacher upon completion of the course. Their findings revealed that the strongest results were for the bundled traditional text with online resources, followed by the etextbook and then the traditional textbook as a singular resource. In all three forms, there was an increase of time spent in solving mathematics problems due to the increase in interest in learning mathematics, though the results were proportional to the overall improvement by method (Nagra, Eng, & Karrass, 2013). The last place finish of the traditional textbooks further emphasizes that the resources available are lacking. Add to that the common practice of states adopting textbooks on a five to ten-year cycle and the result is a stuck curriculum, confined by budgetary limitations. Add to that the still pervasive occurrence of teachers who rely heavily on the textbook as the curriculum and the result is an often decade-long slow march hobbled by limited resources for mathematics classrooms (Whitman, 2004).

But the mathematical content and rigor of the examples, tasks and support resources is the not the total picture of what is wrong with textbooks. The very contexts used in the textbooks is a measurable encumbrance to progress toward social justice. A recent study by a university professor of Mathematical Methods for Elementary Teachers harnessed the force of 58 pre-service teachers to examine the content of mathematics books critically. Anita Bright (2016) used two methods: collaborative analysis of the text and individual identification of “troubling” mathematics problems. With each method, the participants looked to identify two features: what is valued in the problems and mathematics for whom? The findings were powerful, suggesting that current textbooks frequently exploit peoples and disregard the environment. However, it is important to note that a few of the 58 pre-service teachers involved in this study did not describe their experience as eye-opening. Instead, they felt the experience one of hyper-sensitivity equivalent to looking for insult in every problem. That mathematics is just math. Perhaps, but in a subject such as mathematics where the end goal and in fact the students' grade is directly related to being "right", there is due consideration that contexts can and likely are interpreted by students as being "the 'right' way of being, the 'right' kind of home, the 'right' kind of vacation, the 'right' kind of leisure activity" (Bright, 2016, p. 18).

Assessment Structure

Since 1900, when the College Entrance Examination Board was established in response to the call for a more uniform method of assessing college applicants, standardized testing has increased steadily (U.S. Congress, 1992). During World War I the US Army used standardized tests in the form of aptitude quizzes for servicemen to be assigned to tasks. Shortly following the war, the first SAT was administered. But the momentum was boosted even more by the 1965 landmark legislation known as the Elementary and Secondary Education Act (ESEA) that set in motion new and increased use of standardized tests for the purpose of the evaluation of schools and educational programs (U.S. Congress, 1992). In 2001 the ESEA was renewed and given a new call sign: No Child Left Behind, and with this renewal came heightened accountability and sanctions for states based on predetermined achievement levels known as Adequate Yearly Progress (AYP). Since No Child Left Behind was enacted, the emphasis on tests and standards increased, at the expense of thinking and competence (Smith & Szymanski, 2013).

American education has been greatly influenced by four curriculum philosophies known as the Scholar Academic, Social Efficiency, Learner-Centered, and Social Reconstruction ideologies. These ideologies advocated different purposes and methods of achieving those respective purposes for schooling (Schiro, 2013). In brief, the Scholar Academic ideology views school-based education as a process to prepare students to become good citizens. The Social Efficiency ideology propounds that education should prepare students for life by preparing them for the performance of specific tasks through carefully sequenced learning experiences representing the behavior to be learned. The Learner-Centered ideology, as the name suggests, focuses on the learner with schools organized around the needs and interests of the students. Finally, the Social Reconstruction ideology proposes that education is to facilitate the necessary changes for a more just society, one that offers maximum opportunities for all of its members (Schiro, 2013).

Similarly, assessment and evaluation have been shaped through these four distinct lenses. As stated by Green and Johnson (2010), assessment is “the variety of methods used to determine what students know and can do before, during, and after instruction” (p. 14). The Scholar Academic ideology uses norm-referenced assessments for the purpose of ranking the students, whereas the Social Efficiency ideology employs criterion-referenced assessments to certify for groups of stakeholders including parents, administration, and society, that students have specific skills. These two views are known as an assessment *of* the learning, the purpose is assigned to the outcome or result, after instruction (Stiggins, 2002). The Learner-Centered and Social Reconstruction have somewhat similar views on the purpose and method of assessments, to diagnose students’ abilities to facilitate growth and to measure progress on those abilities, respectively. Both use informal subjective diagnosis (Schiro, 2013), this being an example of assessment *for* learning, meant to inform instruction in progress (Stiggins, 2002).

Assessment of learning, typically used to evaluate schools, attempts to measure achievement. Academic achievement represents performance outcomes that indicate the extent to which students accomplish specific goals that were the focus of instruction and should be considered a "multifaceted construct that comprises different domains of learning" (Steinmayr, MeiBner, Weidinger, & Wirthwein, 2014, p. 1). There exists, however, a concern that achievement is not equal to fulfilling potential; there is a gap between potential and actual achievement. Hardre (2012) argues that "even high achievers may be doing well but not achieving at full ability" (p. 12).

Achievement in mathematics has garnered the attention of academic leaders, politicians, and citizens who refer to assessments to compare the performance of American schools both locally and internationally. The Programme for International Student Assessment (PISA) results were analyzed and summarized by Hanushek (2010) who stated "The percentage of students scoring at the advanced level varies considerably among the 50 states, but none does well in international comparison…shockingly below those of many of the world's leading industrialized nations" (p. 4). Even though Hanushek stipulates that following the passage of NCLB legislation the percentage of students performing at a high level in mathematics steadily climbed, he claims the "incapacity of American schools to bring students up to the highest level of accomplishment in mathematics" (p. 5) is more significant than challenges associated with NCLB requirements. In rebuttal of Hanushek's claims, Jeremy Kilpatrick (2011) points out three significant limitations to the interpretation of the PISA results. The international sample was composed by age, not by grade and there was great variability in the proportion of students in the study who were included in the comparison. Also, the method of reporting masks clustering of scores and exaggerates small differences in scores. Finally, different tests were used internationally, and these tests tested different domains of mathematics. Furthermore, internationally tests contained relatively few test items (Kilpatrick, 2011).

Depth of Understanding

Daniel Willingham (2010) is a leader in the field of cognitive science, an interdisciplinary field of researchers who seek to understand the mind. Based on his 20 years of research in this field he professes an innate connection between numbers and space (a) multiple cultures use a spatial representation of numbers; (b) numbers and space are represented in overlapping areas of the brain, verified through documentation of damage to this area (intraparietal sulcus) resulting in an inability to process numbers and difficulties with spatial relations; and (c) there is documented success predicting mathematics operations based on brain data related to eye movement. He draws upon his expertise to offer “While it is true that some people are better at math than others – just like some are better than others at writing or building cabinets or anything else – it is also true that the vast majority of people are fully capable of learning K-12 mathematics” (p. 14).

Gaps Among Student Groups

As measured by standardized tests, there exist achievement gaps among multiple groups of students (Domina & Saldana, 2012). The following is a summary of research relevant to groups including English language learners, Black/African American, gender, rural location, and socio-economic level.

English language learner (ELL). The NCLB Act requires that all demographic groups of students perform at a proficient level (Domina & Saldana, 2012). However, ELL students score lower than other groups on standardized tests of mathematics, propelling school leaders to seek a different approach to mathematics instruction (Rosa, 2011). Rosa (2011) found that for nearly 85% of school leaders, the gaps in performance on standardized mathematics assessments of ELL students in his study attributable to language, they attributed the heavy reliance on English language skills in most mathematics classes as the base for instruction as a barrier for these hard working students. One of the purposes of this study was to determine how the perceptions of principals and vice-principals concerning the challenges faced by ELL students can be influenced by an understanding of the influences of the diverse linguistic and cultural backgrounds of ELL students on their academic performance in mathematics.

Improvements in achievement for these students is likely going to be based on changes in understanding and strategies used by content teachers. Recently, a programmatic restructuring that implemented course work specifically designed to prepare pre-service and in-service mathematics, science, and English speakers of other languages (ESL) teachers to work with ELLs in their content areas was studied and found to have promising results. The study made use of a mixed-methods approach that included both quantitative and qualitative data collection and analysis. Data came from several different sources. Proficiencies related to the effective instruction of ELLs in the mainstream classroom were tested using pre- and post-tests. Quantitative data were obtained from a pre- and post-survey. Qualitative data were obtained from reflective writing, position papers, journals and assignments (DelliCarpini & Alonso, 2014). Results indicated that content teachers showed positive growth in skills related to working with English language acquisition. All teacher candidates developed a deeper understanding of collaborative partnerships between ESL and content teachers.

Black/African-American. In a 2010 study of the role of schools, families, and psychological variables on mathematics achievement of black high school students, Terrell Strayhorn referenced data from the U.S. Department of Education. The data indicate that achievement disparities between white and black students have resisted the decades of school reform efforts and federal influence aimed at reducing the black-white achievement gap. In fact, the black-white achievement gaps in mathematics were reported as 23 percentage points for age nine students, 26 percentage points for age 13 students, and 31 percentage points for all public school students (U.S. Department of Education, 2015). Strayhorn's study employed hierarchical linear regression analyses, with a nested design and sought to estimate the influence of affective variables including parent involvement, teacher perceptions, and school environments, on black students' mathematics achievement in grade ten. He found that all three systems affect black students' mathematics achievement, accounting for approximately 20% of the variance in scores (Strayhorn, 2010).

Gender. There has been a lingering presumption that boys are better at mathematics than are girls. However, the data does not bear that out. In 2010, Scafidi and Bui attempted to replicate the findings of Hyde, Lindberg, Linn, Ellis, and Williams' 2008 study, based on data from ten states, that found gender similarities in performance on standardized mathematics tests. Scafidi and Bui (2010) used national data and extended the previous study by examining whether gender similarities in mathematics performance are moderated by race, socioeconomic status, or mathematics ability in grades eight, 10, and 12. They used data from the National Education Longitudinal Study that began in 1988 and conducted follow-up assessments of the original participants in 1990, 1992, 1994, and 2000. Using a significance level at 0.001, the researchers conducted a multivariate analysis of variance to examine the overall effect of gender on the mathematics test scores. They found that although the average score for girls was slightly lower than that of boys, the difference was not significant. The results showed that gender did not have an overall effect on students' test scores. Follow-through analyses showed that race or mathematics ability did not moderate gender similarities in mathematics performance. A 2008 study by Frank, Muller, Schiller, Riegle-Crumb, Mueller, Crosnoe, and Person, focused on the social dynamics of mathematics course taking in high school. The researchers found that girls are highly responsive to the social norms in their local positions, which contributes to homogeneity within and heterogeneity between local positions. Furthermore, they found that girls had slightly higher college aspirations than boys' and girls' parents and boys' parents expressed nearly equal disappointment over their children's failure to attend college. Interestingly, they found that girls' friends had slightly higher levels of mathematics course taking than did boys' friends. These studies reveal a similarity in gender mathematics achievement but a possible influence on mathematics course taking by members of students' social contexts.

Rural location. The achievement gap between rural schools, when compared to their more urban counterparts, is not as widely researched as the ethnic and gender gaps.

In 2012, Edward Reeves conducted a study of the effects of opportunity to learn, family socioeconomic status, and friends on the rural mathematics achievement gap in high school. Using the database of the Educational Longitudinal Study of 2002-2004, he explored the achievement gap present during the last two years of high school. His approach focused on the geographic disparities in the opportunity to learn advanced mathematics. Reeves used a survey regression in three phases. Phase one was multi-variable: frequency of enrollment in advanced mathematics courses, a family legacy of enrollment in advanced mathematics courses, and quality of instruction as measured by the percentage of advanced mathematics credits earned. Phase two focused on the efficiency of models used to predict student success. And phase three on the assignment of specific model based on the predictor variables. The findings showed that geographic variation in high school resources and practices, (operationalized as the availability of advanced mathematics courses, evidence for track assignment by family background, and the quality of instruction) do not account for the rural mathematics achievement gap. However, geographic variations in the opportunity to learn that result from differences in family socioeconomic status and the influence of friends' academic commitments and aspirations do help to explain why rural high school students learn less mathematics than their non-rural counterparts (Reeves, 2012).

In a more recent study, Harmon and Wilborn (2016) explored the need for preparing STEM technicians for the new rural economy. They used seven high schools and seven middle schools, deemed representative of typical rural areas in the U. S. that seek to grow a new economy consistent with changing realities of global competition and regional economic development opportunities. The researchers set out to identify mathematics competencies used by technicians in the workplace compared to standards of learning required in the public school curriculum. The project mathematics specialist and the team of teachers identified four types of learning gaps: (a) mathematics competencies not included in state standards; (b) mathematics competencies included in state standards taught prior to Algebra I, Algebra II, Geometry, and Algebra Functions and Data Analysis courses; (c) mathematics competencies included in high school state standards that students struggle to learn; and (d) mathematics competencies community college students struggle to learn (Harmon & Wilborn, 2016). The consequences of the mathematics achievement gap among rural students are evident in their communities where technology has replaced the manual work often dominant in these mostly agricultural and mining regions of the country.

Socio-economic level. Students from low socio-economic environments score significantly lower than their peers (U.S. Department of Education, 2015). This deficiency in mathematics skills serves as a hindrance to competitiveness in today's technological world. According to Galindo and Sonnenschein (2015) who used data from the Early Childhood Longitudinal Study-Kindergarten Class of 1998–99 that included a nationally representative sample of about 21,000 kindergarteners in over 1,000 schools. The researchers tested mediation using the KHB-method to examine the associations of multiple mediators simultaneously when control variables are also included in the model. The results of the study were clear and the implications strong, "Starting kindergarten proficient in math and experiencing a supportive home learning environment significantly decreased SES achievement differences. Proficiency in math at the start of kindergarten accounted for the greatest decrease in the SES-math achievement gap" (Galindo & Sonnenschein, 2015, p. 25).

Change of Scope Needed

The modern movement to change access and scope of mathematics education is anchored in the work of Robert Moses and The Algebra Project (Silva, Moses, Rivers, & Johnson, 1990). The goal of The Algebra Project was to make college preparatory mathematics accessible to minority students. Moses, a civil rights activist and mathematics educator, acknowledged the emerging need for "mathematical literacy" (p. 376), and the necessity for all students to achieve such literacy in the form of reasoning, inference, and problem solving in order to find "a meaningful place in the changing political and economic institutions" (p. 388). Similarly, the construct of Culturally Relevant Pedagogy acknowledges the power of harnessing cultural norms in the mathematics classroom as a means to develop academic success, cultural competence, and the development of critical consciousness (Leonard, Brooks, Barnes-Johnson, & Berry, 2010).

The Social Justice Pedagogy builds on the theoretical constructs of Paulo Freire, (Apple, 2011) who avows we need to "see the world through the eyes of the dispossessed and act against the ideological and institutional processes and forms that reproduce oppressive conditions" (p. 229). The driving force for teaching mathematics for social justice must be informed citizenship (Moses & Cobb, 2001) and self-empowerment (Leonard, et al., 2010), not only increasing participation in advanced mathematics courses and access to higher education. The problem with the depth of understanding in mathematics does not stop only among the demographic groups of students and their specific challenges to achieve. The problem can also be seen in the limited perspective of applications and contexts, focused on a time before the Internet connected the world, creating the need for a new definition of neighbor.

Global competencies and citizenship. An understanding of what is meant by global competencies is necessary; according to Veronica Mansilla (2011), "Globally competent students understand the Earth as a system, they understand multiple local contexts well, and they are familiar with the pressing issues defining our times" (p. 4). Martin (2010) expands on this, "Technological advances in communications, transportation, and information processing have deepened and broadened connections on multiple levels, local through global, thickening the webs of interactivity that bind us to each other economically, politically, militarily, socially, culturally, environmentally, and ethically" (p. 70). Mansilla focuses on the substantive foundation of global competence and posits that students must develop this competence while they are gaining fundamental disciplinary knowledge and skills, not afterward. She goes on to describe the core of educating for global competence as creating a school culture where inquiring about the world through consideration of cultural, religious, class, and regional perspectives. As stated by Martin (2010), "Many problems in our world – extensive poverty, pandemics, economic or political instability, environmental degradation, fanatical ideologies, or terrorism, to name but a few – easily spill over borders and generate transnational challenges that cannot be ignored nor effectively redressed unilaterally" (p. 70). Mansilla describes four distinct global competence capacities: investigate the world, recognize perspective, communicate ideas, and take action.

Martin (2010) justifies the importance and urgency of global competencies and global citizenship.

> In a global context this means preparing students for the myriad opportunities and challenges that they will face in their adult lives and help them to grasp the ongoing changes in our world; teaching them to be flexible and adaptive, reflective and imaginative, forward-looking and growth-oriented; and instilling in them a healthy respect for themselves and others, given how often they will need to work with others to solve common problems or realize shared objectives. (p. 70)

However, teachers and schools struggle to embrace this need for global competence while governed by prescribed standards and high-stakes testing. Klein (2013) presents a research-based comparison and synthesis of strategies to “argue that for education to be meaningful and transformative, and for global citizenship to flourish deeply and authentically, we need to integrate content knowledge with student-driven inquiry” (p. 482). She goes on to state that the skills garnered from global experiences have a positive impact on student understanding in the form of increasing flexibility, resiliency, creativity, critical thinking, and collaboration. Furthermore, she calls for “strong assessments that tie global competencies to core knowledge and 21st-century skills in legitimate, quantifiable terms” (p. 488). The ethical and educational implications of our continuously re-connecting world could hardly be more important (Martin, 2010).

Research suggests schools transform the curriculum and approach to teaching to answer the call for global competencies and the potential for revised high-stakes testing. Hassan, Gatto, and Walraven (2012) explore the importance of integrating global issues and perspectives into teaching and learning through their guide *A Model for Taking Classrooms Global* available through the online charitable organization Taking IT Global. They offer a variety of resources for teachers to enhance their understanding of global awareness and citizenship, and the connections to teaching and learning for the 21st century (Hassan, Gatto, & Walraven, 2012). In 2011, Morais and Ogden reported on the initial development of a theoretically grounded and empirically validated scale to measure global citizenship. They employed a multi-faceted methodology that included two expert face validity trials, extensive exploratory and confirmatory factor analyses with multiple data sets, and a series of three small-group interviews utilizing nominal group technique to verify the scope of the global citizenship construct (Morais & Ogden, 2011). The results of their study provide support for a three-dimensional Global Citizenship Scale that encompasses social responsibility, global competence, and global civic engagement. Each with three reliable sub-dimensions that add to refine the construct further. Global competence and global civic engagement are both strong dimensions of global citizenship. Social responsibility was less clearly defined, leading Morais and Ogden to conclude that additional work is needed to operationalize better this dimension.

In their 2016 study on fostering global citizenship across the content areas, Tichnor-Wagner, Parkhouse, Glazier, and Cain explored the signature pedagogies of ten North Carolina K-12 teachers who teach for global competence in math, music, science, English, social studies and language. This qualitative multiple case study purposefully chose cases to represent varying contexts in which the phenomenon of globally competent teaching occurs in regards to location, school level, and content areas. The results of the Tichnor-Wagner et al. study revealed globally competent teaching practices in the form of three signature pedagogies:

- intentional integration of global topics and multiple perspectives into and across the standard curriculum;
- ongoing authentic engagement with global issues; and
- connecting teachers' global experiences, students' global experiences, and the curriculum. (p. 2)

These findings suggest pedagogies for global competencies should be open and flexible, allowing for adjustments by teachers in response to the needs and experiences of their students.

In addition to incorporating global competencies into the K-12 curriculum, it is also crucial, according to Appe, Rubaii, and Stamp (2016), to promote and facilitate international service learning. These researchers collected quantitative data through a survey and followed up with interviews that produced qualitative data. They thematically coded responses to reflect the "richness and range of responses" (p. 73). A measure of interrater reliability was provided by all three authors' independent evaluations. The findings show a lack of clear understanding of international service learning and that the current international service learning programs lack a systematic assessment, specifically lacking is a method for evaluating their contributions to global cultural competencies. Whether it is in the K-12 classroom or international service learning, there is strong evidence in support of global competencies and citizenship.

Social justice. Adolescent and young adult students are at a transitional age between childhood and adulthood; they are making decisions that will affect the way they live the rest of their lives and this is a prime time to introduce to them important concepts such as the need to seek social justice (Gerstung, 2013). Working with middle school students, Gerstung employed lessons focusing on social justice issues, in conjunction with regular lessons. These lessons were taught over a 7-week period; a brief pre- and post-survey was used to measure the effectiveness of instruction as well as student perceptions. There was not a significant change in most responses judging purely on the pre-survey and post-survey responses. The most significant increase was found in students' confidence in their ability to help those in poverty and otherwise suffering from social justice ills (Gerstung, 2013).

A 2009 study by Garii and Rule consisted of a qualitative analysis of the mathematics and science lessons taught by pre-service teachers. The results were mixed, although four of the ten elementary lessons and 14 of the 16 secondary lessons approached the academic content through an appropriate social justice lens, the integration of social justice and academic content was incomplete. Ultimately, the lessons themselves focused on either the issue of social justice concern or academic content but not both (Garii & Rule, 2009). Bond and Chernoff (2015) propose evidence to suggest that mathematics and social justice form a symbiotic pedagogy, their conclusions are three-fold.

- Mathematics is a uniquely well-suited partner with social justice because it can model, through replicable and empirical demonstrations, the nature and intersections of global and local power systems in a way that students can comprehend.

- Social justice provides engaging, empowering, and authentic contexts for projects in which mathematics skill sets can come alive and transcend the traditional limited and abstract operations that have isolated and discouraged too many students for too long.
- Mathematics can be employed to argue social justice issues without the succumbing to the pitfalls of emotional backlash. Social justice can elicit intrinsic motivation in students, that inspires growth far beyond the basic traditionally requisite mathematical skill set.

Creating a classroom environment in which the learning of mathematics is accessible to all students is a necessary first step toward making the science of mathematics available to every student (Bond & Chernoff, 2015). Leonard, et al., (2010) argue that “culturally relevant and social justice instruction can offer opportunities for students to learn mathematics in ways that are deeply meaningful and influential to the development of a positive mathematics identity” (p. 261). The classroom setting requires a significant overhaul to make possible an equitable access for all students.

Need for a New Instructional Model

A new model of mathematics instruction is needed to meet the challenge of offering a change in scope. The basis for this new model is found in the strategy of Problem Based Learning, a method that challenges much of what we consider a traditional mathematics classroom. There is considerable research describing and supporting the use of Problem-Based Learning in a multitude of settings.

Problem Based Learning

Problem Based Learning (PBL) was developed by Dr. Howard Barrows while a professor at McMasters University Medical School. Dr. Barrows sought to develop an instructional method that would meet three distinct yet interdependent elements

> (a) an essential body of knowledge, (b) the ability to use knowledge effectively in the evaluation and care of patients' health problems, and (c) the ability to extend or improve that knowledge and to provide appropriate care for future problems which they must face (Barrows, 1985, p. 3).

Since then, PBL has been employed as a strategy for increasing student achievement in a growing number of K-12 schools Teachers use PBL as a structured method to build thinking and problem-solving skills while students master essential subject knowledge (Delisle, 1997). Teachers search for ways to engage students in problem-solving rather than focusing teaching on "functional-specific factual knowledge" (Scott, 2014, p. 2). John Dewey (1938) asserted that "education is conceived in terms of experience" (p. 73); PBL provides opportunities for students to experience learning in context. Scott (2014) described five features that differentiated PBL from other experiential learning techniques (a) start with a problem, (b) require student-directed learning throughout the process (c) reflect on problem-solving and learning, (d) small group collaboration, and (e) facilitation to guide learning. Problem-solving is a key element of the PBL model and has been a fundamental goal of the study of mathematics throughout history.

Marra, Jonassen, Palmer, and Luft (2014) described PBL as "an instructional method that drives all learning via solving an authentic problem" (p. 221). They elaborated that "problems are a critical aspect of PBL…[and] the two most important criteria …[are that] the problems should stimulate thinking or reasoning and lead to self-directed learning in students" (p. 231).

History of problem-solving. Successful PBL requires successful problem-solving however, problem-solving has a rich and somewhat tumultuous history in the mathematics curriculum. Joseph Ray, as quoted by D'Ambrosio (2003) said "the mind of the pupil may be disciplined and strengthened so as to prepare him, either for pursuing the study of Mathematics intelligently or more successfully attending to any pursuit in life" (p. 39). Ray was one of the most prolific textbook authors in the United States during the 19th century. This attitude toward the inherent power and benefits of learning mathematics came to be known as the Mental Discipline Theory and promoted the study of mathematics as the "primary vehicle for developing reasoning" (D'Ambrosio, 2003, p. 40). However, a bit of a controversy developed regarding the place of problems and problem-solving in the school mathematics curriculum in the early 20th century. One side of this controversy, represented by scholars such as Smith and Young, promoted the platform that mathematics develops the brain and is thereby good for everyone. Their opponents, represented by Felix, Klein, Perry, and Moore, professed that pure mathematics was not the answer, the application of mathematics through problem-solving was what benefitted students most (D'Ambrosio, 2003). Around this same time, Edward Thorndike began to gain credibility for his claims that discredited the Mental Discipline Theory and so-called transfer of training.

In 1988 George Stanic and Jeremy Kilpatrick authored a seminal article tracing the history of problem-solving in the mathematics curriculum. Three general themes on problem-solving became the focus of their writings: problem-solving as content, skill, and as art (D'Ambrosio, 2003). D’Ambrosio (2003) includes the Stanic and Kilpatrick discussion of these problem-solving themes in its entirety. In addition to describing in detail the comparison of problem-solving as content versus problem-solving as skill, they refer to George Polya and his theories on problem-solving as art. Stanic and Kilpatrick include, in response to the ongoing debate over whether mathematics should be taught to students or discovered by students, Polya’s posit “Finished mathematics requires demonstrative reasoning, whereas mathematics in the making requires plausible reasoning. Won’t students understand mathematics better if they see how it was created in the first place and if they can get some taste of mathematical discovery themselves?” (as cited in D’Ambrosio, 2003, p. 43) Stanic and Kilpatrick also point out the inequity of the hierarchical method of teaching problem-solving in the majority of the schools of their time. A hierarchical system that resulted in only those most accomplished students being challenged with non-routine problems; because non-routine problems were only taught after routine problems had been mastered.

Theories of problem-solving. Another expert of education of the time was John Dewey, while he did not specifically write about problem-solving, Dewey did write extensively on reflective thinking. Stanic and Kilpatrick wrote, “We believe that Dewey’s ideas about problem-solving complement those of Polya” (as cited by D’Ambrosio, 2003, p. 44). Dewey is most well recognized for his constructivist theory, incorporating interactions with the learning environment and the perception of sensory information as a result of the interactions. Shiro (2013) describes Jean Piaget’s constructivist theories concerning “how people construct knowledge through the processes of assimilation and accommodation” (p. 131). Schiro also relates the evolution of the Learner-Centered ideology that examines the continuity of belief fusing “the work of Francis Parker in the 19th century, the progressive education movement in the first half of the 20th century, and the open education, developmentally appropriate practices, and constructivist movements of the last 50 years” (p. 7).

David Jonassen has written extensively on the topic of both problem-solving and problem-based learning. In his 2011 book *Learning to Solve Problems*, Jonassen describes problem-solving as a schema-based activity. He states that “learners must engage with problem-solving and attempt to construct schemas of problems, learn about their complexity, and mentally wrestle with alternative solutions” (p. xxi). He goes on to state that “until the problem solver constructs a model of the problem in its context, a viable solution is only probable, while understanding and transfer are improbable” (p. 4).

Research on PBL. Problem-based learning, a combination of cognitive and social constructivist theories, has been the focus of extensive research in the educational community. More than ten studies, including a meta-analysis, were identified that explored the impact of PBL on learning and attitudes. In Demirel and Dagyar’s (2016) meta-analysis of 47 studies, they attempted to determine the effects of PBL on students’ attitudes as compared to traditional teaching; they found PBL more effective at improving students’ attitudes. Upon completing his quasi-experiment on achievement level and methodology, Mudrikah (2016) found that “PBL is appropriate to be used to improve students’ high order mathematical thinking ability” (p. 133).

Students benefit from PBL by reformulating hypotheses, choosing how to solve problems based on multiple sources, multiple points of view, and by defending their point of view (Aufdenspring, 2003). Schmidt, Rotgans, and Yew (2011) summarize recent studies in which a new “micro-analytical” methodology was used to trace the process of PBL in the natural classroom setting. They report evidence that “problems arouse situational interest that drives learning” (p. 792). More recently, Uce and Ates compared PBL with traditional instruction in a high school chemistry class. Using a pre- and post-test with a scientific achievement scale and t-test, they found significant differences between the PBL group and the traditional group (2016). PBL has been documented to increase motivation, retention of information and the ability to apply knowledge across disciplines and life situations (Larrier, et al., 2016). In a qualitative study in the primary grades, results showed that students, including those from under-served populations, showed mathematical promise after being taught with the PBL methodology (Trinter, Moon, & Brighton, 2015).

Rattanatumma and Puncreobutr (2016), using mean, standard deviation, and ANCOVA a covariance analysis concluded that although both PBL and Cooperative Learning resulted in greater mathematics learning achievement than did traditional methods, Cooperative Learning outperformed PBL. Similarly, Scott (2014) examined how best to design, implement, and evaluate PBL; she cautioned that her findings “affirm the importance of problem design characteristics and effective team facilitation” (p. 2). The interpretation of research can vary, often as a result of how best to evaluate the use of PBL. In response to such inconsistencies in evaluating the effectiveness of PBL, Sipes (2016) created a matrix, a tool to be applied in the examination of PBL to create consistency in “methodology, definitions, and language among scholars” (p. 2). Future research can now employ this matrix thereby providing a more consistent basis for comparison of results.

Summary

The review of literature serves to document the need for this study through research on the current reality of mathematics classrooms and assessment expectations. A summary of findings from research on the depth of understanding demonstrated on mathematics assessments, identified gaps in achievement, and findings that support a needed change in scope of instruction. Research on the importance of global competencies, global citizenship, and social justice support this proposed change in scope. Finally, to articulate and support the need for a change in instructional strategies, PBL research includes the history and the theoretical basis for problem-solving as well as the strengths, and the challenges of PBL. “PBL transfers the active role in the classroom to students through problems that connect to their lives and procedures that require them to find needed information, think through a situation, solve the problem, and develop a final presentation” (Delisle, 1997, p. 119). This action research study proposes to adapt PBL into GPBL by harnessing the strengths of PBL and infusing opportunities to develop global competencies with CRP and SJP, and thus provide high-quality mathematical education to all students.

CHAPTER 3

METHODOLOGY

The purpose of this study is to explore historical enrollment trends in mathematics courses and determine the impact of GPBL-based instruction in mathematics classes on achievement and perceptions at a small, southeastern high school. Specifically, the tasks are to understand enrollment trends and to document the outcomes of focusing on relevant global contexts to address both the struggles of mathematics students to make connections and to transfer skills among mathematical topics. It is anticipated that this information will inform efforts to encourage higher levels of enrollment of minority students and increase performance in advanced mathematics courses. The study was framed using two research questions:

1) What are historical enrollment trends in mathematics classes across the curriculum at the high school of study? 2) What impact does focusing mathematics lessons on specific, real-world global problems have on achievement and attitudes in advanced mathematics classes? This study examines the inconsistent integration of global issues that may contribute to the diminished value of mathematical skills and students' lack of enrollment and perseverance.

Action Research and the Researcher Role

The American educational philosopher John Dewey (1938) believed that children learn by working through real-world problems and exposure to such problems are an integral element of their education. The data collection period consisted of two units, each framed by a global issue (see Appendix A for a list of units and global issues). The functions and modeling unit was framed by the global issue of poverty while the geometry and trigonometry unit was framed by the global issue of cultural diversity. Each unit consisted of an introduction to the global issue, problem sets related to the issue and an in-depth reasoning and sense-making task drawing on the mathematics of the unit as a means to solve or improve the global issue. The problem sets for poverty included conversions, cost functions, population growth, virus spreading, and depreciation. The task focused on ethical compensation – a fair wage. For the global issue of cultural diversity, the problem sets addressed triangulation, map-making, bearings, and architecture and design. The task involved map-type comparisons – Mercator versus Peters – and the implication of perspective and proportions (see Appendices B and C for unit plans used in the study). The intent was to connect the set of skills in the unit as requisite to understanding and, to varying degrees, pose plausible resolutions for engaging global issue.

Both qualitative and quantitative data were collected and analyzed. Because of the nature of the teacher as a participant and a researcher, there existed unplanned adjustments to the activities in response to student actions. For example, during the ethical compensation task, students were much more familiar with the workplace than anticipated. In response to this background knowledge, the teacher-researcher modified the order of the activity steps and omitted steps aimed at orienting the students to the workplace. This change may explain the less than enthusiastic response of the students, as well as their demonstrated difficulty considering the social topic from alternative perspectives. Upon reflecting on this possible influence, the teacher-researcher was more cautious during the map type comparison task in the next unit. For example, when students struggled to agree on a strategy within each group, the teacher-researcher resisted the urge to step in and define a process. This resulted in student groups approaching the challenge differently, sometimes with much more detail than necessary. However, when groups began to calculate area conversions incorrectly I did step in. This intervention however was deemed necessary to avoid completing a considerable amount of mathematical calculations incorrectly and thereby risking setting a pattern that is in error.

In addition to these reaction-based influences, some assumptions may limit the study as well. The student participants were pre-assigned to classes, so there is an assumption that all are proficient in prerequisite skills for the coursework. Similarly, attendance may have limited the students' experience with the strategy. However, this study could help mathematics teachers not only facilitate deeper mathematical understanding but also improve the accessibility of the advanced mathematics courses for all students by featuring international contexts.

"The goal of action research is to improve one's teaching practice or to enhance the functioning of a school" (Johnson, 2008, p. 34). Action research is an appropriate methodology for this study: The purpose is to explore the impact of GPBL in mathematics classes on student achievement and attitudes, that may serve to increase enrollment in advanced mathematics courses among underrepresented student groups. The teacher-researcher endeavored to remain an impartial observer, carefully working through the study process while remaining cognizant of her influences and interactions with the subjects. This action research study served as a method to understand better the current reality of the classroom and the impact of implementing specific strategies to improve the learning experience.

Site and Participants

As described in Chapter 1, this is an IB high school. More specifically, a small public high school in a resort town of 39,000 residents. The community represents a wide range of socio-economic diversity among the residents and hence the students. At the time of this study, there were 1354 students in this school, 34% qualify for free or reduced lunch, 53% are white, 33% Hispanic, 10% African-American, and 4% represent other ethnicities. The classes under study meet for 90 minutes, every day for one 90-day semester. The students in these classes are in 11th or 12th grade and have demonstrated at least minimal proficiency in their previous mathematics courses.

The teacher-researcher used historical data for the longitudinal study of three consecutive cohorts of graduated seniors, a total of 795 students. For the action-research component of this study, the course of focus for this study is taken by students who are college bound but do not intend to pursue a university degree in science, technology, engineering, or mathematics. The teacher-researcher used both sections of this course, which consisted of 25 students total; although this is a rather small sample size, this is a sufficient number of subjects to identify potential themes rather than idiosyncrasies. Because the nature of high school course scheduling determines the group of participants, the sample size was not negotiable. Of the 25 students, 23 were in 12^{th}-grade and two were in 11^{th}-grade; 18 were white, five Hispanic, and two of other ethnicities. The demographic distribution in terms of gender, ethnicity, and experiences in previous mathematics classes are detailed in Table 3.1.

Table 3.1 Sample Demographic Information

	Female			Male			
Previous math classes	White	Hispanic	Other	White	Hispanic	Other	Total
Algebra 2	0	2	0	0	0	0	2
Algebra 2 honors	8	0	1	2	1	0	12
Pre-Calculus honors	4	1	1	4	1	0	11
Total	12	3	2	6	2	0	25

Three of the students in the sample received support as English speakers of other languages (ESOL), and none of the students received services through a 504 plan or an individual education plan (IEP). Among the 25 students in the sample, six qualified for free or reduced lunch.

Ethical Considerations

This study does not pose any risk to the students. Although the entire class was taught using the strategy, no student was compelled in any way to agree to the use of data directly related to them. The strategy and an overview of the action research study was discussed with the students. Research intentions were also shared with parents. Student, school, and town names were not included in the results. As required, permission and authorization to conduct this research was obtained from the school district. Students were asked to give their assent to be included in the results. Because these students are young adults, it was particularly important to respect their increasing responsibility as decision makers. Respecting the students as decision makers is a timely issue because many of these high school students had the opportunity to vote in public elections for the first time during this study. The students will maintain the power to withdraw, without penalty, from inclusion in the research results, should they so choose.

Data Collection Methods

A teacher-researcher's goal when conducting action research is to understand what is happening in the classroom and determine what might improve learning in that context. With this goal in mind, it was essential to determine what data would contribute to the understanding and resolution of the problem in practice: enrollment in advanced mathematics classes does not reflect the school demographics and students do not appear sufficiently proficient in their understanding of mathematics skills for these skills to transfer to other mathematics skills, topics, and application contexts. Mills (2011) states "…researchers should not rely on any single source of data, interview, observation, or instrument" (p. 96). For this action research study, the teacher-researcher used multiple methods of data collection that produced both quantitative and qualitative data.

Historical Enrollment Trends

The teacher-researcher explored historical enrollment trends in mathematics courses at the high school of focus. Data was gathered from the district database, for an eight-year period, focusing on the enrollment in mathematics courses from grade seven through 12 for three cohorts of high school seniors.

Student Achievement and Attitudes

The teacher-researcher used summative assessments following each of two units of study to explore skill proficiency and problem solving. In addition, the teacher-researcher used observations, student questionnaires, discussion boards, and focus groups to explore student attitudes.

Assessment Process

To measure the impact on students' ability to apply their mathematical skills to unfamiliar contexts, a summative assessment was used for each unit. The teacher-created assessments each consisted of short-response questions that vary in complexity. The questions were selected by the teacher from the test bank provided by the textbook publisher (Blythe, Fensom, Forrest, & Waldman de Tokman, 2012) and a course-specific bank of previous IB examination questions (Questionbank dp mathematical studies, 2017). The textbook resources include samples of problems requiring the application of multiple content skills with the flexibility to construct a solution in various ways. These problems are consistent with the approach to problem solving essential to PBL. The question bank includes problems from previous IB exams frequently framed by global contexts. Student responses were scored for accuracy and completeness using a detailed mark scheme. The teacher-research recorded the accuracy of problems completed by each student. A comparison of the proportion of problems accurately completed was analyzed. This student work data was useful to better understand the impact of global problem-based lessons on mathematics students' deeper understanding and ability to apply the concepts.

Observations

Teacher-researcher observations are frequently used as a rich source of such data. Because of the multitude of activities in a typical class period, it can be difficult to conduct structured observations. Therefore semi-structured observations were made and recorded in the form of a teacher-researcher journal. Frequent journal entries of objective notes regarding what was occurring in the classroom, as well as reflections throughout the data collection period, were useful to understand the impact on students' attitudes toward attempting problems. Schwalbach (2003) refers to the teacher journal as the "most important way that you will have to keep track of the details of your study" (p. 59). Reflection is a key component of action research. Zeichner and Liston (2010) point out that John Dewey was "one of the first educational theorists in the United States to view teachers as reflective practitioners, as professionals who could play very active roles in curriculum development and educational reform" (p. 8).

Surveys, Focus Groups, and Discussion Boards

To gain insight into the students' perceptions of their engagement during each unit, the students were asked to rank characteristics of their experience using a Likert scale questionnaire (see Appendix D). This instrument was appropriate because it utilizes levels of agreement as a means of measuring attitude. The brief questionnaire targeted the students' self-assessment of their level of engagement in mathematics class. The questions were adapted from a school-level student engagement survey (Student Engagement Survey, 2012).

At the beginning and end of the data collection period, the teacher-researcher conducted a focus group interview with each class of participants. Questions for the focus group were opened-ended prompts to discuss the importance and relevance of mathematics (see Appendix E). The teacher-researcher recorded the discussion and remained a facilitator with limited participation to encourage genuine input from the student participants. Following the focus group interviews, the teacher-researcher transcribed the recordings for the process of coding and analysis. Also, a discussion board involving all participants was used to gather student perspectives on the use of global issues as a frame for learning mathematics (see Appendix F). It is often the case that students who are willing to speak up in a class discussion are hesitant to express their ideas in writing and vice versa. The use of both the focus group and the discussion board provided a means for the teacher-researcher to gain feedback from all student-participants. Because even closed questions such as those on a Likert scale questionnaire are subject to misinterpretation (Schwalbach, 2003), the teacher-researcher worded the questions and prompts in a way that minimized potential ambiguity.

Assurances and Timeline

There are drawbacks and potential limitations to some of these data collection techniques. Observations can be undermined by the very act of observing; students tend to behave differently when they know they are being watched. Also, field notes can be insufficient in fully describing the details of observations. With quantitative data, there is a risk of using instruments that stray from the problem being researched (Mertler, 2014).

Triangulation is a needed strategy to confirm the credibility of conclusions drawn. Findings are supported when independent measures agree with each other or at least do not directly contradict each other (Mertler, 2014). Furthermore, there is an enhanced opportunity for learning when discrepancies arise (Dana & Yendol-Hoppey, 2014). The five data collection techniques, summarized in Figure 1, provided information useful to address the research question: What impact does focusing mathematics lessons on specific, real-world global problems have on achievement and enrollment in advanced mathematics classes? A timeline of these five data collection methods is provided in

Figure 3.1.

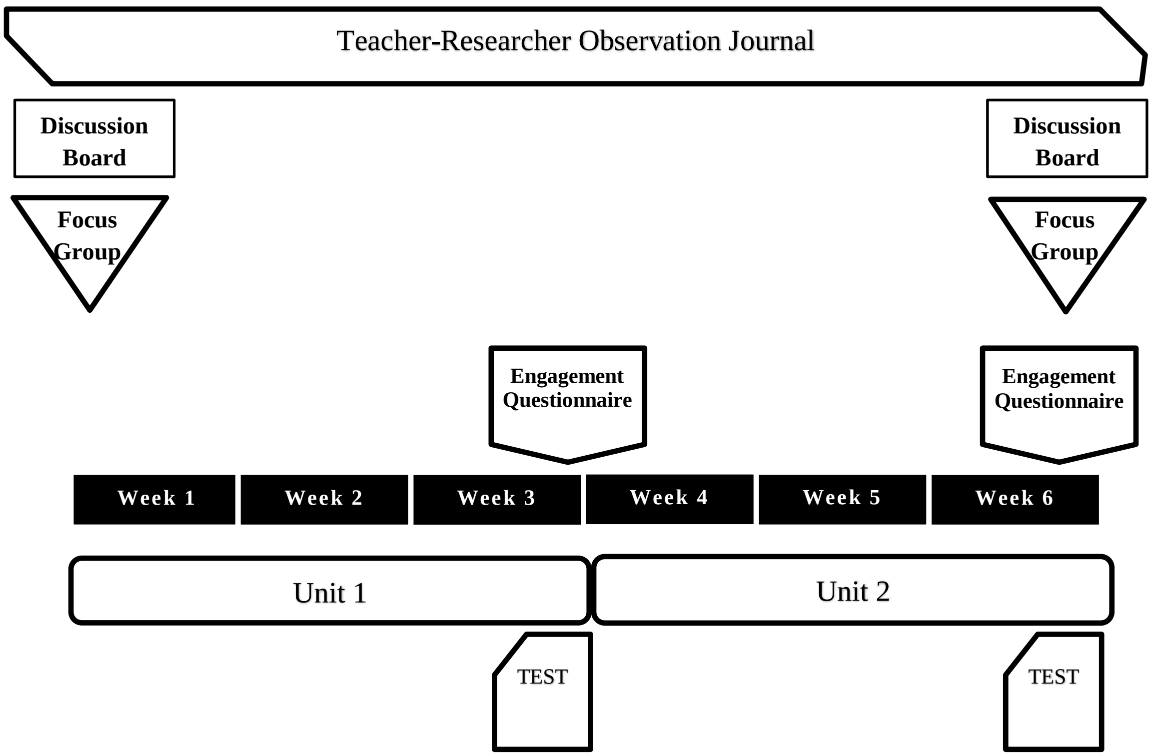

Figure 3.1. Timeline for data collection. Including one quantitative method, and four qualitative methods. The unit tests, attitude questionnaires and focus group discussions were conducted at a class (period) level, the discussion board was at the multi-class level, and the teacher journal included both class level and overall multi-class level observations.

Data Analysis Methods

Data analysis consists of a process for reviewing, examining, reorganizing, and modeling data with the goal of discovering useful information and informing conclusions. To this end, each type of data will be analyzed individually and as a component of the entire body of data.

Historical Enrollment Trends

The teacher-researcher analyzed trends in enrollment including the representativeness of enrollment based on the school population in mathematics classes. At the high school of focus, students typically follow a sequence of mathematics courses from ninth to 12th grade, with advanced students beginning that sequence as early as seventh grade. To earn the state diploma a minimum of four mathematics courses are required. All students must take two algebra courses and one geometry course. With these three foundational courses complete, students can choose from five further courses. Because the school of focus operates on a semester system, students can take two mathematics courses each year. To reach the most rigorous mathematics course offered (AP Calculus BC), a student would *typically* take a total of seven mathematics courses. The teacher-researcher looked at points where enrollment patterns change substantially within the sequence of courses. The teacher-researcher compared the enrollment in advanced mathematics courses based on ethnicity and gender.

Impact of Global Problem-Based Instruction

Because all forms of PBL involve students working collaboratively on a carefully structured open-ended problem (Delisle, 1997), the impacts can be obvious or masked, significant or minor. To better understand the impact of GPBL, several forms of data collection were used.

Summative Assessment The student work data is quantitative and represents the percentage of marks earned out of the total possible for the set of questions making up the test. Each item varied in point value and was scored according to a set mark scheme. This quantitative data was displayed using a histogram that gave a visual representation of the dataset as a whole and allowed for a determination of the shape. The shape is useful to understanding the set of data as a whole because it reveals potential features such as extreme values, outliers, gaps, and clusters. The analysis continued with the use of descriptive statistical processes including the mean and median to determine the typical value of the variable and the standard deviation and interquartile range (IQR) to describe the dispersion of the data with regards to the center. The mean and median were also interpreted in regard to letter grade bands.

Observations, Questionnaires, Surveys, and Focus Groups. The journal of frequent notes and reflections by the teacher-researcher, the engagement questionnaire, the focus group comments, and the discussion board content were analyzed using an open and axial coding process. Open coding is defined by Strauss and Corbin (1998) as an analytic process of identifying concepts, their properties, and dimensions. The teacher-researcher read and reread to conceptualize the data by identifying central ideas. The concepts were then grouped based on properties or characteristics. Although there are various ways of doing open coding, the teacher-researcher used sentence and paragraph analysis. Grouping of initial concepts related to higher order concepts resulted in categories. The goal of axial coding is to systematically relate categories "along the lines of their properties and dimensions" (p. 124). Schwalbach (2003) states "the process [of coding] is often recursive and complex" (p. 78). Coding was a process completed over multiple days to assure the teacher-researcher's perspective. Caution was taken to ensure the maintaining of objectivity throughout the coding process (Schwalbach, 2003).

The data resulting from the questionnaire required cautious analysis as well. The analysis began with a table to show the percentage of each response per question as parts of all the response choices to the question. Analysis resulted in the identification of patterns among the results.

Findings and Results

The teacher-researcher thoughtfully examined the analysis results and conclusions were drawn based on each source of data. Following the individual data source conclusions, the five sets of analysis results and conclusions were consolidated and overall conclusions synthesized. Specific evidence from the analysis is cited to support each claim made. Consideration was given to both consistencies among the related conclusions as well as inconsistencies. By the evidence, the teacher-researcher provided a plausible explanation for the findings and answered the research question. She remained objective and resisted interpreting or expressing any value judgments. Because this is action research, there was not an attempt to generalize to all students the results.

Summary

GPBL is a strategy to help make advanced mathematics in high school more accessible and engaging for all students. This study was designed to gather information on enrollment trends and on the impact of using GPBL in a high school IB mathematics class. Action research is a powerful vehicle for studying this topic. Data was collected from multiple sources and analyzed to identify emerging themes. It is anticipated that these themes will lead to better understanding why students struggle to apply their mathematical understanding and why few minority students enroll in advanced high school mathematics courses.

CHAPTER 4

DATA ANALYSIS AND FINDINGS

In the high school of interest to this study, students do not appear able to transfer and apply their understanding of mathematics skills to other mathematics skills, topics, and application contexts. This inability is evidenced by the daily requirement to review and reteach, student resistance to persevere with application problems, and by inconsistent achievement. Although the high school of interest in this study is an IB World School, integration of global issues in mathematics classrooms is at best inconsistent. Furthermore, enrollment in IB mathematics classes, the most rigorous courses available, does not reflect the demographic identity of the school.

The purpose of this study was to explore historical enrollment trends in mathematics courses and determine the impact of GPBL-based instruction in mathematics classes on achievement and perceptions. It is anticipated that this information will inform efforts to encourage higher levels of minority student enrollment and increased performance in advanced mathematics courses. The study will be framed using two research questions:

1) What are historical enrollment trends in mathematics classes across the curriculum at the high school of study? 2) What impact does focusing mathematics lessons on specific, real-world global problems have on achievement and attitudes in advanced mathematics classes? This study examines the inconsistent integration of global issues that may contribute to the diminished value of mathematical skills and students' lack of enrollment and perseverance.

Overview of Study

This study consists of two components, a longitudinal study of enrollment patterns in mathematics classes and an action-research study of the impact of GPBL in mathematics classes on achievement and perceptions. For the longitudinal component of the study, data was gathered from an eight-year period, focusing on the enrollment in mathematics courses from grade seven through 12 for three cohorts of high school seniors. For the action-research component, the teacher-researcher used summative assessments within two units of study to explore skill proficiency and problem solving. In addition, the teacher-researcher used observations, student surveys, discussion boards, and focus groups to explore student attitudes.

The small southeastern high school studied has an enrollment of approximately 1300 students, of whom 53% are white, 33% Hispanic, 10% African American, and 4% other ethnicities. The longitudinal study of historical enrollment trends consists of data from 797 students over an eight-year period. This component examined enrollment in high school mathematics courses from Algebra 1 through 12th-grade for three cohorts of high school graduates. The sample used in the action-research component consisted of 25 students, in the 11th and 12th-grades, of whom 72% are white, 20% Hispanic, and 8% other ethnicities. Before enrolling in the IB mathematics course at the focus of this study, 12 of the students were enrolled in Algebra 2 Honors and 11 were enrolled in Pre-Calculus Honors. However, two students did not matriculate through the honors program, these two students enrolled in the IB mathematics course after taking Algebra 2. All subjects had successfully completed the first of this two-course sequence of IB Mathematical Studies.

Implementation of GPBL

One of the tasks of this action-research study was to document the outcomes of focusing on relevant global contexts to address the struggles of mathematics students to make connections and to transfer skills among mathematical topics. The teacher-research focused on two of the six units that occured during the semester-based course, Functions and Modeling, and Geometry and Trigonometry. Each unit was framed by a global issue (see Appendix A for a list of units and global issues). The global issue was threaded throughout the topical mathematics lessons. In each unit, a reasoning and sense-making tasks required small groups to apply the skills and concepts of the unit to address the global problem (see Appendices B and C for unit plans used in the study). Although reasoning and sense-making tasks are often a culminating activity, that was not the case for the two units of this study. For each unit, the task was used during the first half of the unit. This was due to the mathematical topics needed to complete the tasks.

Results

The findings from this study are presented in response to the research questions and based on the data sources. The longitudinal study of historical enrollment trends is presented first. This data analysis includes descriptive statistics and graphic representation of the data. Findings from the implementation of GPBL begin with the presentation of the summative assessment data, also in the form of descriptive statistics and graphic representation. This is followed by the three sources of qualitative data, presented using tables to show student responses as well as discussion of themes resulting from the open and axial coding process.

Historical Enrollment Trends

The teacher-researcher collected the level of mathematics classes taken from Algebra 1 and beyond for three cohorts of graduates from the high school of interest in this study. Levels were defined as either College Prep or Advanced. The graduating classes of 2018, 2017, and 2016 were included in this longitudinal study. In 2010 the Common Core State Standards for Mathematics (CCSSM) brought about an increase in rigor across mathematics courses throughout the country (About the standards, 2017). Even in states that did not adopt the CCSSM, the movement resulted in revised and restructured state standards. In South Carolina, advanced students often take Algebra 1 in the seventh-grade. For these reasons, the cohorts were selected to include students who took Algebra 1 in any grade, beginning in the 2010-2011 school year. Furthermore, South Carolina requires at least four courses of mathematics to earn a high school diploma. For this reason, the sample was further delimited to include only those students who had earned at least four mathematics credits. The data obtained from the school database consisted of 7,449 records of all students who had taken any mathematics course between fall 2010 and spring 2018. From those records, the teacher-researcher identified 1,062 records for students who were in the cohorts to graduate between 2016 and 2018. Those records were further filtered to eliminate students who did not complete at least four years of math. The final data set consisted of complete records for 797 students. Each of these records were examined and coded using the coding scheme that grouped students in one of seven categories as described in Table 4.1.

Table 4.1 Coding Scheme

Code	Meaning
A	All mathematics courses at advanced level
AC	Mathematics courses began at advanced level then moved to college prep level
ACA	Mathematics courses began at advanced level, moved to college prep level, then returned to advanced level
ACAC	Mathematics courses began at advanced level, moved to college prep level, returned to advanced level, then returned to college prep level
C	All mathematics courses at college prep level
CA	Mathematics courses began at college prep level then moved to advanced level
CAC	Mathematics courses began at college prep level, moved to advanced level, then returned to college prep level

The number of students with each pattern of levels for each of the three years included in this study is displayed in Table 4.2.

Table 4.2 Frequency of Levels of Mathematics Classes

Code	Class of 2016	Class of 2017	Class of 2018	Total
A	85	83	75	243
AC	74	102	90	266
ACA	5	4	3	12
ACAC	0	0	1	1
C	64	64	84	212
CA	1	3	3	7
CAC	16	21	19	56
Total	245	277	273	797

Review of the data reveals that three sequences are most common: Sustained Advanced (A), Advanced then College Prep (AC), or Sustained College Prep(C). These three sequences constitute 91.02% (2016), 89.89% (2017), and 91.21% (2018). The sample was further delimited to include only students in these three sequences. These sequences and their corresponding percentages are seen in Table 4.3.

Table 4.3 Level Sequences Percentages

Sequence	2016	2017	2018	Total
Sustained Advanced	38.12	33.33	30.36	33.70
Advanced then College Prep	33.18	40.97	36.44	36.90
Sustained College Prep	28.70	25.70	33.20	29.40

Review of this data revealed that slightly more than half of the students who begin taking mathematics courses at the advanced level do not sustain that level. This finding led to further analysis of ethnicity and gender of the students who begin taking courses at the advanced level of study (A and AC). The ethnicity distribution of this refined sample is shown in Table 4.4. The frequency of course levels over ethnicity and gender is shown in Table 4.5.

Table 4.4 Ethnicity Distribution of Sample

Ethnicity	2016	2017	2018	Total
Black (B)	30	8	9	47
Hispanic (H)	35	35	33	103
White (W)	102	137	116	355
Other Ethnicities (O)	9	5	7	21
Total	159	185	165	509

Table 4.5 Ethnicity and Gender for Level Sequences

Year	Sequence	B	H	W	O	F	M	Total
2016	Sustained advanced	2	16	59	8	46	39	85
	Advanced then college prep	11	19	43	1	31	43	74
	Total	13	35	102	9	77	82	159
2017	Sustained advanced	2	13	66	2	50	33	83
	Advanced then college prep	6	22	71	3	53	49	102
	Total	8	35	137	5	103	82	185
2018	Sustained advanced	3	11	57	4	42	33	75
	Advanced then college prep	6	22	59	3	48	42	90
	Total	9	33	116	7	90	75	165
Overall	Sustained advanced	7	40	182	14	138	105	243
	Advanced then college prep	23	63	173	7	132	134	266
	Total	30	103	355	21	270	239	509

Note. F = female; M = male.

To further explore, the percentage of students who began at an advanced level were calculated for those who sustained the advanced level and those who did not. This data is shown in Table 4.6.

Table 4.6 Percentages of Level Sequences by Ethnicity and Gender

Year	Sequence	B	H	W	O	F	M	Total
2016	Sustained Advanced	15.38	45.71	57.84	88.89	59.74	47.56	53.46
	Advanced then College Prep	84.62	54.29	42.16	11.11	40.26	52.44	46.54
2017	Sustained Advanced	25	37.14	48.18	40	48.54	40.24	44.86
	Advanced then College Prep	75	62.86	51.82	60	51.46	59.76	55.14
2018	Sustained Advanced	33.33	33.33	49.14	57.14	46.67	44	45.45
	Advanced then College Prep	66.67	66.67	50.86	42.86	53.33	56	54.55
Overall	Sustained Advanced	23.33	38.83	51.27	66.67	56.79	43.21	47.74
	Advanced then College Prep	76.67	61.17	48.73	33.33	49.62	50.38	52.26

Note. F = female; M = male.

Overall approximately one-half of all students who began taking mathematics courses at an advanced level sustained that level based on data for three cohorts of high school graduates. Furthermore, the percentage of those dropping rigor level is much higher for Black and Hispanic students. In terms of gender, the percentage of students maintaining the advanced rigor level is similar for females and males, with slightly more males dropping rigor level. These findings are useful to answer the first research question of this study. At the high school of focus, enrollment trends suggest less than one-half (47.74%) of all students who take Algebra 1 at an advanced level, sustain an advanced level throughout all mathematics courses. However, this proportion does not hold across race or gender. In terms of race, 51.26% of white students compared to 39.61% of minority students sustained an advanced level of study. Similarly, 56.79% of female students compared to 43.21% of male students sustained an advanced level of mathematics courses.

Student Achievement and Attitudes

Enrollment trends are important to understand as they impact matriculation into advanced level mathematics classes. In addition to enrollment, this study explored the implementation of GPBL to understand how students perceive the study of mathematics. Three methods were used to explore the impact of the intervention on student achievement and attitudes.

Summative assessments. A mark scheme was used to score the summative assessments for each unit. After scoring, a percentage score was calculated. Letter grades are based on a ten-point scale with 90-100 representing a letter grade of A. The distribution of the assessment scores can be seen in Figure 4.3. The histograms suggest that the performance on the two units is very similar.

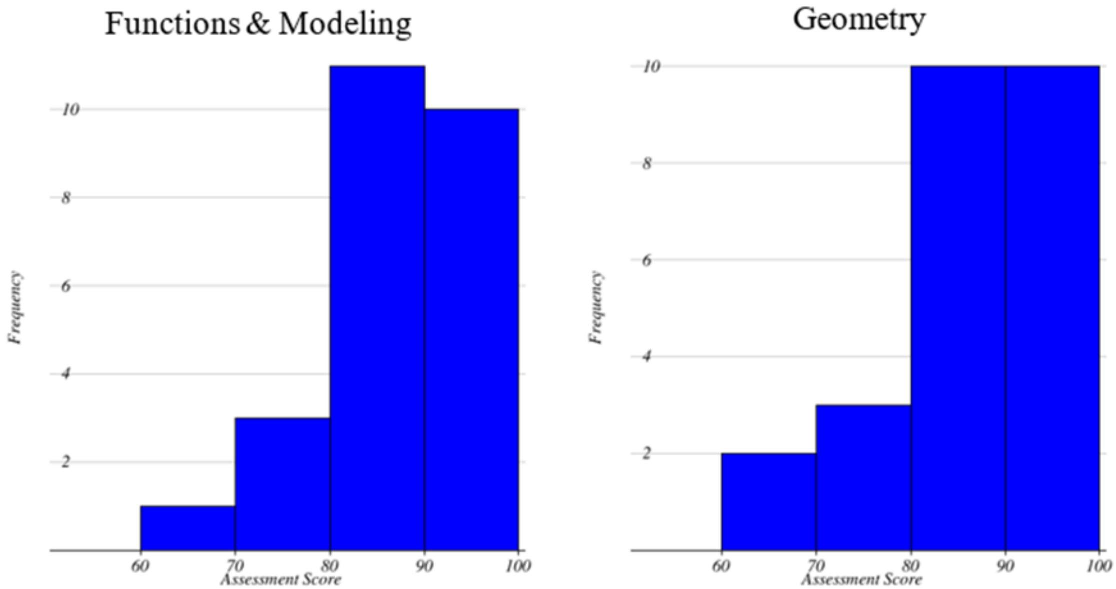

Figure 4.1 Comparison of Summative Assessment Scores

Calculation of descriptive statistics (Table 4.7) confirms that the student performance on the summative assessment for each of the two units of interest were similar, with the Geometry unit assessment having slightly greater variation than the Functions & Modeling unit assessment.

Table 4.7 Descriptive Statistics for Summative Assessments

Statistic	Functions & modeling assessment	Geometry assessment
Mean	86.48	85.44
Standard Deviation	9.11	10.24
Median	88	87
Interquartile Range (IQR)	14	13

For each unit, the mean grade was around the mid-point of the B grading scale range. By further disaggregating this data, comparisons among ethnicities and between genders can be explored.

Table 4.8 Summative Assessment Frequencies by Ethnicity and Gender

Letter Grade	Numeric Grade Range	H	O	W	F	M	Total
A	90-100	2	2	16	11	9	20
B	80-89	7	2	12	16	5	21
C	70-79	1	0	5	4	2	6
D	60-69	0	0	3	3	0	3
Totals		10	4	36	34	16	50

Note. F = Female; M = Male

In an advanced mathematics class, proficiency is considered as letter grade level of at least B. Work scored below the letter grade level of B are considered not proficient. Table 4.9 shows the distribution of proficiency over ethnicity and gender.

Table 4.9 Mastery Percentages by Ethnicity and Gender

Performance	H *(n = 10)*	O *(n = 4)*	W *(n = 36)*	F *(n = 34)*	M *(n = 16)*	Total *(N = 50)*
Proficient	90	100	77.78	79.41	87.50	82
Not Proficient	10	0	22.22	20.59	12.50	18

Note. Proficient = letter grade A and B; Not Proficient = letter grade C and D;
F = Female; M = Male

Mastery was highest for students in the subgroups of Hispanic, other ethnicities, and male. Each of these subgroups exceeded the overall percentage of those achieving mastery. The weakest performance was the three scores in the D grade range, each of which correspond to students in the subgroups of white and female.

Questionnaire: Engagement during GPBL units. All participants completed the five-item questionnaire following each of the two units of the study. Two items were identical, the other three items were parallel but specific to the GPBL task of the unit. The results are shown in Table 4.10.

Table 4.10 Engagement Questionnaire Response Percentages

Questionnaire Item	Strongly agree	Agree	Neither agree nor disagree	Disagree	Strongly disagree
Functions and Modeling Unit					
My teacher believes that I can do well in math class.	64	28	4	0	4
I feel that the math of the Ethical Compensation Task is important.	48	44	4	4	0
I am interested in what I am learning in the Ethical Compensation Task.	44	52	0	0	4
During the Ethical Compensation Task, I tried to do my best in math class every day.	32	60	8	0	0
I am praised for doing good work in math class.	32	60	4	4	0
Geometry Unit					
My teacher believes that I can do well in math class.	72	24	4	0	0
I feel that the math of the Map Type Comparison Task is important.	44	36	16	4	0
I am interested in what I am learning in the Map Type Comparison Task	40	48	12	0	0
During the Map Type Comparison Task, I tried to do my best in math class every day	28	68	4	0	0
I am praised for doing good work in math class.	36	52	8	4	0

Note. Because permission to use copyrighted material was not granted, the name of the activity was changed to a descriptive task title for this paper.

There appears to be more responses of disagreement following the functions & modeling activity focused on ethical compensations than there were following the geometry map type comparison task which focused on maps of differing geographical perspectives and proportions. The clearest difference is found in the responses of agreement related to the importance of the mathematics used and the student's interest. Although results were similar, it appears that the ethical compensation task was considered more important than the map type comparison task (92% versus 80% agreement), and of more interest to the students (96% versus 88% agreement). For both activities, the students responded frequently that they had tried their best during the unit. The responses to the two items related to the teacher were similar, however, responses for being praised for good work were agreed upon with slightly less frequency.

Discussion board: Usefulness of mathematics to solve global issues. Students responded to two prompts related to the usefulness of mathematics to solve global issues both prior to beginning the two units for which GPBL was implemented and again after the two units. The prompts prior to and following the students' GPBL experience were identical. Table 4.11 presents the topics discussed by the participants including global issues that require mathematics to resolve and those that do not require mathematics to resolve. Because of the nature of a discussion board, participants had the option to create a new thread or to respond to another student's comments, or both. Because of this structure, not all students gave unique examples. Student responses were conceptualized by identifying central ideas and grouping responses by common theme.

Table 4.11 Discussion Board: Global Issues and Mathematics

Prompt	Prior to the GPBL Experience	Following the GPBL Experience
Give an example of a global issue that is likely to require mathematics to improve or resolve. Why does it require math? What type(s) of math? Who do you think is required to have the necessary math skills? Why?	• Pollution due to fossil fuels • Infectious diseases • Natural disasters • Poverty • Erosion • Refugee crisis • Clean water • World hunger • Global warming	• All • Energy crisis • Preservation of biodiversity • Predictions of natural disasters • Financial education • Poverty • Erosion • Climate change • Pollution • Population growth • Global warming • Traffic accidents in cities • Internet safety • Global banking • Renewable energy
Give an example of a global issue that is not likely to require mathematics to improve or resolve. Why does it not require math? Do you think the issue will be improved or resolved in your lifetime? Why?	• Racism • Gender equality • Terrorism • Assisted suicide • Religious issues • None	• Everything requires math • Bigotry • Stereotyping • Disaster reconstruction • Assisted suicide • Equality • Religious issues • World peace • Racism • Humanitarian assistance

The most obvious difference between the pre- and post-discussions is the increase in the number of examples generated by the students of global issues for which mathematics may be useful. Also noticeable is the multiple responses of “all global issue require math” in the discussion following the GPBL experience.

The majority of examples of the global issues that require mathematics to resolve given prior to the GPBL experience can be classified as science or logistics-based. However, following the GPBL experience, in addition to the science and logistics-based examples, students also gave examples related to finances and one example related to social justice. This evidence suggests a possible increase among the students in recognizing the relevance of mathematics and its usefulness. However, the examples that do not require mathematics to resolve are somewhat contradictory to this conclusion. Examples of topics for which mathematics is not useful can be classified as social justice-based and ethics-based were given both prior to and following the GPBL experience. However, two examples of topics for which mathematics is not useful can be classified as logistics-based were also given following the GPBL units. One student followed through on her example of racism as a topic for which mathematics is not useful by stating “It takes a bit more than statistics for someone to have a complete change of heart.”

Focus groups: Studying mathematics. During the week prior to beginning the two units for which GPBL was implemented, the students participated in a focus group interview. The same questions were used in a focus group interview following the two GPBL units. Students voluntarily responded, which resulted in some questions receiving few responses and others receiving many. Student responses were conceptualized and grouped by common themes. Representative responses by the students to the six questions asked both before the GPBL experience and after, are shown in Table 4.12.

Table 4.12 Focus group: Studying mathematics

Question	Prior to the GPBL experience	Following the GPBL experience
Why do we study mathematics?	To understand the world We always use basic math If you want to be a doctor	To understand the world Required in school To organize and measure
What does it take to be good at math?	Good thinker Good memory Logical thinker	Concentration Motivation A lot of practice Effort Open-mindedness
What prevents you from being good at math?	Self-doubt Closed-minded It is confusing Requires logic Can't memorize it	Myself Lack of motivation Lack of basic skills Poor work ethic Failure to practice
What is wrong with studying math?	Takes a long time Requires lots of practice There's only one right answer It's forced on you	Too complicated Isolated from other subjects Not taught applications Integration is the key
Can everyone be good at math?	10 "yes" responses if: dedicated, good work ethic, determined, want to, believe you can nine "no" response: Requires aptitude, depends on your brain, mental predisposition	To some level Requires talent and desire Everyone can be sufficient If they care enough If determined
Is there such a thing as math-mindedness?	Every response was "yes"	There is natural talent Math-minded IS a thing! No, everyone can learn with enough practice

Student responses to the question regarding why we study mathematics were vague and may suggest a lack of personal engagement with mathematics. When asked what it takes to be good at mathematics the responses both before and after the activities were overwhelmingly related to effort on the students' part. Student responses to what prevents each of them from being good at mathematics revealed a slight shift from difficulty level, lack of motivation, and lack of aptitude to a resounding lack of motivation. This may suggest recognition that *if* motivated, improved mathematics achievement could result. This need for motivation is reinforced with the shift of responses to the question asking what is wrong with studying mathematics from difficulty level, lack of motivation, and lack of relevance to consistent responses related to lack of relevance. One student stated, “it’s really important to teach students from the beginning the real uses of math and how that can benefit them in the future instead of just giving them equations and multiplication tables to memorize.” The most pronounced change is seen in responses to whether everyone can be good at math. Prior to the GPBL units, most of the students responded and the responses were nearly equally split between “yes” and “no.” However, following the GPBL units, responses revealed a growing consensus that although talent exists, proficiency is attainable by all. This is further confirmed by the shift from a unified response affirming math mindedness to “there is mental predisposition that makes you better at it but everybody has the capability to understand math if they really want to.”

The second research question framing this study considered the impact on achievement and attitudes of focusing mathematics lessons on specific, authentic global problems. Achievement was positively impacted as evidenced by the summative assessment following each of the two units that showed similar results for each unit and that at least 90% of the students demonstrated mastery of the content. Attitudes were also positively impacted, albeit in some areas more than others. The engagement questionnaire revealed a large majority of positive responses for the perceived importance of the mathematics needed, interest in the topic, and effort given for both units. The discussion board was used to measure the students’ perceived usefulness of mathematics as a tool to lesson or resolve global issues. The responses revealed a perceived limited usefulness for topics related to social justice. Amy (pseudonym) stated “Math can be used to show the impact of the situation but doesn’t necessarily change a person’s mindset or beliefs.” Finally, the focus groups discussed the students’ role in the study of mathematics. The results suggest a developing understanding, demonstrated by the increased detail and specificity of responses to the prompts. For example, when asked what it takes to be good at math, the students initially responded in general terms such as “good thinker”, “logical thinker”, and “good memory.” However, following the GPBL experience, the responses revealed increased onus by using terms including “concentration”, “motivation”, “a lot of practice”, “effort,” and “open mindedness.”

Summary

This study was guided by two research questions that examined the enrollment trends for advanced mathematics classes as well as the impact of implementing GPBL in advanced mathematics classes. Data was collected from historical enrollment records and was analyzed to identify enrollment trends related to the level of mathematics courses. Specifically, records for students who begin their high school mathematics sequence of courses at an advanced level in Algebra 1 were examined. Of this group, approximately one-half sustained an advanced level throughout their mathematics course progression. However, of those who sustain the advanced level, there is inconsistency in terms of race and gender, with minorities and males underrepresented. Additional data was collected via an action-research model. Summative assessments showed mastery and an engagement questionnaire revealed strong engagement with the specific GPBL activities. Discussions in the form of focus groups and discussion boards revealed an increase in the students’ perceived usefulness of mathematics to solve or improve global issues as well the students’ self-assessed potential ability in studying mathematics.

CHAPTER 5

DISCUSSION, REFLECTIONS, AND IMPLICATIONS

In the U.S., the demographic profile of students enrolled in advanced mathematics classes skews toward the white male majority (Atuahene & Russell, 2016; Useem, 1991). Furthermore, the topics and contexts often found in textbooks and mathematics lessons fail to engage students. GPBL is an instructional model that frames the study of mathematics within real-world global issues. This increased relevancy may lead to increased achievement, improved attitudes, and greater access for all students.

Synopsis of this Study

In the high school of interest to this study, students do not appear able to transfer and apply their understanding of mathematics skills to other mathematics skills, topics, and application contexts. Although the high school of interest in this study is an IB World School, integration of global issues in mathematics classrooms is at best inconsistent. Furthermore, enrollment in IB mathematics classes, the most rigorous courses available, does not reflect the demographic identity of the school. GPBL has its foundation in a combination of cognitive and social constructivist theories and is the product of braiding the strengths of PBL, CRP, and SJP. The combining of these instructional frameworks and strategies is necessary because none alone "fully address major shortcomings in the mathematics classroom" (Leonard, Brooks, Barnes-Johnson, & Berry, 2010). An updated teaching model – where math is taught within context and with a clear purpose - is needed to improve the buy-in of all students to the necessity and empowering nature of mathematics. If students can see that mathematics can be useful to solve problems within one context, they may more readily recognize the usefulness of mathematics in various contexts.

The southeastern high school studied has a persistently increasing enrollment of approximately 1,300 students of whom 53% are white, 33% Hispanic, 10% African American, and 4% other ethnicities. This study consists of two components, a longitudinal study of enrollment patterns in mathematics classes and an action-research study of the impact of GPBL in mathematics classes on achievement and attitudes.

The longitudinal study of historical enrollment trends consists of data from 797 students over an eight-year period. The data gathered focused on the enrollment in mathematics courses from Algebra 1 through grade 12 for three cohorts of high school graduates. Of this group, approximately one-half sustained an advanced level throughout their mathematics course progression. However, of those who sustain the advanced level, there is inconsistency in terms of race and gender, with minorities and males underrepresented.

The sample used in the action-research component of this study consisted of 25 students, in 11th and 12th-grade, of whom 72% are white, 20% Hispanic, 0% African American, and 8% other ethnicities. For the action-research component, the teacher-researcher used summative assessments following each of two units of study, the results of which showed strong mastery. These results suggest that the use of GPBL positively impacted achievement. Observations, student surveys, discussion boards, and focus groups were used to explore student attitudes. Results revealed strong engagement with the specific GPBL tasks and an increase in the students' perceived usefulness of mathematics to solve or improve global issues, as well as a more open-minded perspective on potential ability in mathematics. These results suggest that attitudes were also positively impacted, although in some areas more than others.

Findings Related to Existing Literature

The longitudinal study of enrollment trends at this high school revealed that most students (90.46%) either sustain a college prep level of study, sustain an advanced level, or begin at advanced level and drop to college prep level. Although approximately two-thirds (65.50%) begin at the advanced level, essential half of those (52.26%) drop to the college prep level. Furthermore, although the percentage of male students who dropped rigor level (50.38%) is slightly greater than that of females (49.62%), the difference in terms of ethnicity was clear. Compared to the percent of white students who drop rigor level (48.73%), the percent of students of Hispanic and black ethnicity who dropped rigor level was much greater, 61.17% and 76.67% respectively. It is interesting to note that the difference of 27.94% between white and black students is consistent with Strayhorn's 2015 findings based on U.S. Department of Education data that showed a 31% black-white achievement gap for all public-school students. Although enrollment and achievement are not directly comparable, the 27.94% of black students who dropped rigor level began their high school mathematics sequence at the advanced level. This option is only available to students who have demonstrated mathematical proficiency and academic scholarship. It follows logically that these students likely did not reach their mathematical potential.

Analysis of the summative assessment data following the two instructional units of this study, revealed similar results on the two unit assessments, each with an average score in the mid B grade range. Further analysis revealed nine out of the 50 scores failed to demonstrate mastery of concepts resulting in an 82% mastery rate overall. Similar to the results found by Scafidi and Bui in their 2010 analysis of national mathematics standardized test data, this study confirms gender similarities in performance.

Qualitative data collected purposed to measure students' attitudes about the study of mathematics, their own engagement with mathematics lessons, and the usefulness of mathematics to address global issues. Students self-assessed having strong engagement with the specific GPBL activities as well as an increased perception of the usefulness of mathematics to solve or improve global problems. Interesting themes were revealed among responses related to students' potential ability in mathematics.

The comments made during the focus group interview both prior to beginning the GPBL units and following the conclusion of the second unit were coded and analyzed. The most pronounced change is seen in the positive responses to whether everyone can be good at math. This indicates an improved self-assessed potential for mathematical proficiency. Also interesting are the responses to the question asking what keeps you from being good at math, the post-interview response was a chorused "motivation." This insight of the importance of motivation is encouraging and consistent with previous studies. Larrier, et al (2016) names motivation as one of the benefits of PBL. Hardre (2012) states that motivation can be related to perceptions of the difficulty of mathematics. Also, Bond and Chernoff (2015) describe the symbiotic relationship between mathematics and social justice stating that social justice can elicit intrinsic motivation in students.

Implications for Teaching and Plan for Action

The research questions framing this study presented two tasks (a) to better understand enrollment trends, and (b) to teach mathematics in a way that engaged students in substantial mathematical problems framed by culturally relevant global contexts. The longitudinal study of historical enrollment trends revealed that nearly two-thirds of students who graduate from the high school of focus take Algebra 1 at an advanced level and of those students who begin mathematics at an advanced level, approximately half do not sustain that rigor level. This reveals a need for classroom teachers to create an environment in which students can reflect and communicate; provide opportunities to dialog about frustrations and struggles. Through this, teachers might identify and support students who may otherwise abandon the advance courses and thus fail to reach their full mathematical potential. The action research findings indicate that students are motivated by engaging contexts.

Although findings of action research are not generalizable, they may be transferable to other teaching settings. The GPBL structure provides a strategy to engage students in constructivist mathematics, while modeling cultural relevance, and social justice. This structure could be adapted for various high school classes. If implemented in mathematics courses taken earlier in a student's matriculation, GPBL could promote higher enrollment of under-represented student groups as well as improved perseverance in advanced mathematics classes. The frequency of using GPBL could be customized according to subject, grade, and class size. Sharing experiences of implementing GPBL through teams, departments, and other professional learning communities would be an effective way to continuously refine its incorporation into the curriculum.

Reflection on Limitations

As an action research study, the findings are not generalizable. Furthermore, there are a few other limitations and delimitations related to this study. Some of these were decision made throughout the data analysis process, others were discovered during the interpretations of the findings. Upon reflection, I recognize that because this is an IB World School - and therefore students are familiar with considering a broader, more global perspective, the implementation of GPBL may have met less resistance than could be true for high schools overall.

One limitation was the decision to include only students who began to take Algebra 1 during the 2010-2011 school year and after. This decision was necessary because of the impact of the Common Core State Standards for Mathematics initiative. A comparison of students taught advanced Algebra 1 prior to 2010 would not be appropriate because the redistribution of standards across grade levels changed considerably. Another delimitation was the inclusion of only students who had completed the requisite four courses of high school math. This decision was necessary to include only students who had the opportunity to take an IB mathematics course. Because IB mathematics is a fourth or fifth mathematics class in the sequence, it was necessary to include only students who had taken at least four courses.

Two unexpected limitations were discovered during the interpretation of results. Although the teacher-researcher acknowledged the deficit of applicants to teach mathematics as a rationale to rethink how mathematics instruction is delivered, the study results did not address how GPBL may affect teacher recruitment and retention. Also, although the enrollment trends and achievement were analyzed in terms of ethnicity and gender, this study did not include such demographics for students at the beginning of the mathematics sequence, when they enrolled in advanced Algebra 1. Therefore, although comparisons between and among subgroups can be made, a change from the beginning cannot be quantified.

Implications for Further Research and Plan for Action

The longitudinal study of historical enrollment trends was structured from the graduating class backwards to the enrollment in advanced Algebra 1. However, this structure does not allow for comparison of enrollment trends based on the grade in which the student first enrolls in advanced Algebra 1. It would be useful to compare students who begin in seventh grade to those who begin in eighth grade and ninth grade. There has been a trend of starting students in Algebra at earlier grades, it would be interesting to know if they are sustaining the advanced level of study similar to the group at large. Further research is needed regarding the impact of taking advanced Algebra 1 in earlier grades on students' mathematical course trajectory. Algebra 1 is a foundational course for all mathematics courses. Further research may lead to better understanding the potential of a strong instructional foundation leading to sustained advanced work versus the potential of struggling with foundation concepts leading to limited motivation and curtailed potential. I intend to further analyze the collected data to compare the sustaining rates of grades 7, 8, and 9 to the overall sustaining rate. In addition to the findings of this study, I will share these grade-specific findings at the school and district level.

The structure of this study also did not consider during *which* courses the drop in rigor level is most frequent. Better understanding when the students drop rigor level may lead to better course sequencing and planning. I intend to further analyze the collected data to identify the frequency with which courses are taken at a decreased rigor level. I will share these findings at the school and district level.

Also, the analysis of enrollment trends showed mixed results in terms of gender and clear results in terms of ethnicity. However, the study did not consider the gender distribution for each ethnic subgroup. Further research may lead to better understanding potential differences in terms of gender over ethnicity.

One of the challenges to the implementation of GPBL in this high school mathematics class was the availability of resources. The teacher-researcher ultimately had to revise and refine materials only somewhat appropriate for the target audience of students. This was a time-consuming endeavor. Further research may be needed in the preparation of in-service and pre-service teachers to facilitate GPBL with fidelity. I will endeavor to refine the units used during this study and to create additional units for other mathematics courses. I will explore methods by which to make these resources available to other teachers.

Summary

Understanding enrollment trends is a necessary step toward avoiding the creation of an education system that limits access to advanced mathematics courses. This study revealed that although nearly two-thirds of students begin high school mathematics at an advanced level, approximately half of those students do not sustain that level. Furthermore, the demographics of students who drop rigor level is strongly skewed toward minorities. GPBL is a strategy through which advanced mathematics in high school is more accessible and more engaging for all students. This action research study revealed positive results following the implementation of GPBL for two units of study. Students mastered the content and reported strong engagement with the GPBL units, an increased perception of the usefulness of mathematics to solve or improve global issues, and an improved perception of their own mathematical abilities. GPBL is a method by which to challenge teachers to focus content in a way that empowers students to think deeply and considerately, to recognize the strengths and contributions of all peoples, and to persevere with a purpose.

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APPENDIX A─GLOBAL ISSUES BY UNIT

Unit	Global Issue
Sets and Logic	Sexism
Number and Algebra	World Hunger
Functions and Modeling*	Poverty
Geometry and Trigonometry*	Cultural Diversity
Calculus	Environmental Protection
International Finance	Racism

* Included in this study

APPENDIX B─GEOMETRY AND TRIGONOMETRY UNIT PLAN

<table>
<tr><td>School</td><td>XXXX</td><td>Course</td><td>IB Math Studies</td><td>Semester, Year</td><td>Spring 2018</td></tr>
<tr><td>Unit</td><td colspan="5">Geometry and Trigonometry</td></tr>
<tr><td colspan="5">Essential Questions</td><td>Global Connection</td></tr>
<tr><td colspan="5">How can I define essential concepts (e.g. angle, circle, segment) in terms of undefined concepts?
How can triangles be used to find indirect measurements?
How are two dimensional relationships connected to the properties of two dimensional objects?</td><td>Cultural Diversity</td></tr>
<tr><td colspan="6">Objectives</td></tr>
<tr><td colspan="6">
Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles (angles of elevation and depression)
Use Law of Sines and Law of Cosines to solve non-right triangles.
Use trig to find area of a non-right triangle
Use Geometry of three-dimensional solids: cuboid; right prism; right pyramid; right cone; cylinder; sphere; hemisphere; and combinations of these solids.

The distance between two points; eg between two vertices or vertices with midpoints or midpoints with midpoints.
The size of an angle between two lines or between a line and a plane or between two planes

Volume and surface area of tree-dimensional solids
</td></tr>
<tr><td colspan="6">Tasks</td></tr>
<tr><td colspan="6">Triangulation problem set
Map-making problem set
Bearings problem set
Architecture and design problem set
Map type comparison task (Mercator vs. Peters)</td></tr>
<tr><td colspan="6">Materials</td></tr>
<tr><td colspan="6">Teacher-made presentations
Problem set handouts
Task handout</td></tr>
<tr><td colspan="6">References</td></tr>
<tr><td colspan="6">IBO problem bank, IBID textbook, Mathelicious® free webpage</td></tr>
</table>

APPENDIX C─FUNCTIONS AND MODELING UNIT PLAN

<table>
<tr><td>School</td><td>XXXX</td><td>Course</td><td>IB Math Studies</td><td>Semester, Year</td><td>Spring 2018</td></tr>
<tr><td>Unit</td><td colspan="5">Functions and Modeling</td></tr>
<tr><td colspan="5">Essential Question</td><td>Global Connection</td></tr>
<tr><td colspan="5">How can we use math to describe the larger-world interactions and make predictions?
Does a graph without labels or indication of scale have meaning?</td><td>Poverty</td></tr>
<tr><td colspan="6">Objectives</td></tr>
<tr><td colspan="6">• Concept of a function, domain, range and graph.
• Function notation ($f(x)$, $v(t)$, $C(n)$)
• Concept of a function as a mathematical model (equations, tables, graphs).
• Model linear functions (parallel, perpendicular)
• Model quadratic functions (symmetry, vertex, intercepts, equation of the axis of symmetry)
• Model exponential functions (asymptotes)
• Combinations of functions</td></tr>
<tr><td colspan="6">Applications and Tasks</td></tr>
<tr><td colspan="6">Conversion (temperature, currency) problem set
Cost functions problem set
Projectile motion problem set
Area functions problem set
Population growth problem set
Virus spreading problem set
Depreciation problem set
Ethical Compensation Task</td></tr>
<tr><td colspan="6">Materials</td></tr>
<tr><td colspan="6">Problem set handouts
Task handout</td></tr>
<tr><td colspan="6">References</td></tr>
<tr><td colspan="6">IBO problem bank, IBID textbook, Mathelicious® free webpage</td></tr>
</table>

APPENDIX D─ENGAGEMENT QUESTIONNAIRE

During this unit, I tried to do my best in math class every day

Strongly Disagree	Disagree	Neither Agree Nor Disagree	Agree	Strongly Agree
1	2	3	4	5

My teacher believes that I can do well in math class.

Strongly Disagree	Disagree	Neither Agree Nor Disagree	Agree	Strongly Agree
1	2	3	4	5

I feel that the math of this unit is important

Strongly Disagree	Disagree	Neither Agree Nor Disagree	Agree	Strongly Agree
1	2	3	4	5

I am interested in what I am learning in this math unit.

Strongly Disagree	Disagree	Neither Agree Nor Disagree	Agree	Strongly Agree
1	2	3	4	5

I am praised for doing good work in math class.

Strongly Disagree	Disagree	Neither Agree Nor Disagree	Agree	Strongly Agree
1	2	3	4	5

APPENDIX E—FOCUS GROUP QUESTIONS

1. Why do we study mathematics?
2. What does it take to be “good” at math?
3. What prevents you from being good at math?
4. What is wrong with studying math?
5. Can everyone be good at math?

APPENDIX F─DISCUSSION BOARD PROMPTS

Give an example of a global issue that is likely to require mathematics to improve or resolve. Why does it require math? What type(s) of math? Who do you think is required to have the necessary math skills? Why?

Give an example of a global issue that is not likely to require mathematics to improve or resolve. Why does it not require math? Do you think the issue will be improved or resolved in your lifetime? Why?

How do you know when to use math in your daily life?