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Making Meaning Of Multiplication: Integrating Virtual Manipulatives And Think-Alouds With Repeated Addition, Arrays, And Decomposing Numbers To Build Conceptual Understandings

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MAKING MEANING OF MULTIPLICATION: INTEGRATING VIRTUAL MANIPULATIVES AND THINK-ALOUDS WITH REPEATED ADDITION, ARRAYS, AND DECOMPOSING NUMBERS TO BUILD CONCEPTUAL UNDERSTANDINGS

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DEDICATION

To my husband, Scott, who encouraged me throughout this journey, and to my parents, Doug and Connie, who have always been my number one supporters and my biggest fans. My successes are your successes.

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ABSTRACT

The purpose of this action research was to evaluate the implementation of technology integration with multiplication concepts (i.e., repeated, arrays, and decomposing numbers) for struggling third grade mathematics students. This study incorporated the use of virtual manipulatives and student think-aloud recordings to measure students' conceptual understanding of basic multiplication. This study focused on two overarching research questions: (1) The first question explored how technology integration with multiplication concepts (i.e., repeated addition, arrays, and decomposing numbers) impacted student understanding; and (2) the second question explored how students select and explain strategies for solving multiplication problems. Data collection consisted of teacher-made pre- and posttests with virtual manipulatives and student think-aloud recordings. Data analysis incorporated an evaluative mixed-methods approach using objective assessment data with non-parametric tests and constant comparative method. After transcribing, reviewing, and coding data, overlapping themes emerged, including students' conceptual understandings, students' conceptual misunderstandings, and students' correct methodology with careless errors. (Careless errors in this study refers to simple errors in counting or adding. In several cases, the students used virtual manipulatives to build the problems correctly but made errors when counting or adding the manipulatives).

Findings revealed that virtual manipulatives significantly improved participants' conceptual understandings of all three given multiplication strategies. The impact of

virtual manipulatives is reflected in the increased percentages of students who demonstrated conceptual understanding of the three strategies from the end of week one to end of the innovation. Conceptual understanding for each of the strategies (i.e., repeated addition, arrays, and decomposing numbers) increased by 40 percent over the course of this innovation.

In addition, the impact of virtual manipulatives and think-aloud recordings is reflected in the increase of correct scores from the pretest to posttest. Four Wilcoxon Signed-ranks tests were conducted for (1) overall pretest-posttest scores and the three specific strategies (2) repeated addition pretest-posttest scores, (3) arrays pretest-posttest scores, and (4) decomposing numbers pretest-posttest scores. All four tests were statistically significant with posttest scores higher than pretest scores.

The student think-aloud self-recordings provided valuable insight into students' developing conceptual understandings, and consequently helped guide and direct remediation throughout this innovation. By listening to their own recordings, students were able to evaluate their work, identify mistakes, and correct careless errors before turning in their recordings. Consequently, the think-aloud recordings promoted student self-reflection and were essential in providing specific, individualized instruction for all participants.

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CHAPTER ONE

INTRODUCTION

National Context

Achievement in the area of mathematics is slowly improving over time for some United States students; however, it continues to be elusive for many others. Data results from *The Nation's Report Card,* which provides results of a nationally representative assessment administered by The National Assessment of Educational Progress, indicate that the 2015 mathematics scores for United States fourth and eighth graders have declined since 2013, but remain higher than scores for those same groups in 1990 (National Center for Educational Statistics [NCES], 2015). The same study shows that, with the exception of 2015 results, there has been a steady growth nationally since 1990 of fourth and eighth grade students' scores increasing from the *Below Basic/Basic* score ranges to the *Proficient/Advanced* score range (NCES). Despite this increase, the highest percentage of fourth graders achieving *Advanced* in mathematics since 1990 was only eight percent (in 2013), and the highest percentage of eighth graders achieving *Advanced* since 1990 was nine percent (in 2013) (NCES). This national trend reveals that while United States' students are generally improving in the area of mathematics each year, most students are lacking the conceptual understanding needed to perform at the highest levels.

When compared to students in other countries, it is clear that United States' students are lacking in the area of mathematics. In several cross-national tests, assessment results indicate that American mathematics students perform well below their international peers (Desilver, 2017). The most recent results from the 2015 Programme for International Student Assessment (PISA) assessment, which measures reading, mathematics, science, and other critical skills among students in dozens of developed and developing countries, ranked United States' students 38th out of 71 countries in the area of mathematics (Desilver). Most recent results from a similar cross-national assessment, Trends in International Mathematics and Science Study (TIMSS), indicate that 10 countries out of 48 total had statistically higher average fourth-grade mathematics scores than the United States, while seven out of 37 countries had statistically higher average eighth-grade mathematics scores than the United States (Desilver).

From these statistical findings (Desilver, 2017; NCES, 2015), it is quite evident that while American students seem to be improving overall in the area of mathematics, there exist gaps in many students' conceptual understandings. By better addressing these specific misunderstandings, especially of major overarching mathematics concepts such as the four basic operations: addition, subtraction, multiplication, and division, and building a richer number sense for students at the primary and elementary levels, educators will provide a strong foundation for all higher-level mathematics skills (Boaler, Williams, & Confer, 2015; Cumming & Elkins, 1999; Heege, 1985; Solomon & Mighton, 2017; Wells, 2012). An improved fundamental understanding of mathematical thinking and reasoning strategies will enable students to reason through *why* methods work mathematically and apply those methods to new types of problems (Boaler et al.; Zhang, Ding, Barrett, Xin, & Liu, 2014). By teaching students to think mathematically from an early age, students should be better able to make connections across all levels of mathematics (Westenskow, Moyer-Packenham, & Child, 2017). As a result, students will be better prepared to apply mathematical reasoning in the classroom, in real-world settings, and in the global economy.

Specifically, in the area of multiplication, it is crucial for students to utilize a strong sense of numbers when exploring, discovering, and reasoning through basic multiplicative relationships. By developing a conceptual understanding of basic multiplication facts in the primary and elementary grades, students will be much better prepared for higher-level mathematics skills and real-world concepts which involve multiplication, such as multi-digit multiplication, division, fractions, decimals, and proportions (Wong & Evans, 2007). To achieve fluency of multiplication facts, students must be able to flexibly and accurately use an appropriate strategy in order to efficiently arrive at an accurate answer (Common Core State Standards Initiative, 2010; Kling & Bay-Williams, 2015). Therefore, students must learn and incorporate an assortment of strategies in order to increase motivation and to improve conceptual understanding of multiplication (Heege, 1985; Solomon & Mighton, 2017). By integrating a variety of engaging educational technology-based programs, applications, and manipulatives to enhance the learning of multiplication strategies, students are better able to develop deeper conceptual understandings of basic multiplication (Shin et al., 2017).

Local Context

This action research takes place at Friendly Elementary School (a pseudonym; FES), which is a public elementary school and part of Lake County School District (a pseudonym). State and state data references have been removed to protect the identity of participants. FES is a low-income, Title One elementary school located in a diverse, rural district in the southeastern United States. FES employs 30 teachers and enrolls approximately 400 students (214 boys and 210 girls) in pre-kindergarten through fifth grade with 65% qualifying for free or reduced lunch status. According to the State Department of Education's 2017 *School Report Card*, FES consists of 71% Caucasian, 16% African-American, 7% two or more races, 3% Hispanic/Latino, 2% Asian, and 1% American Indian students. The school has a 68.6% poverty rate. FES has been the recipient of several awards, including the Palmetto Gold Award and Red Carpet School Award in multiple years, and FES earned AdvancEd accreditation in 2016. The school's most recent State *School Report Card* rating in 2014 was *Excellent*.

Lake County School District is a leader both in the county and state in the area of technology integration. The district implemented a one-computer to one-student (1:1) initiative during the 2011-12 academic year with the goal of placing 1:1 tablet technology in the hands of every student in grades 3-12 and shared technology in the hands of students in grades K-2. According to the *School Report Card*, the district maintains 2,900 digital devices for use by its 2,927 total student population. Lake County School District provides 51-60% of students in each elementary school with 1:1 digital technology.

In my third grade mathematics courses, I incorporate various digital applications, software programs, formative assessments, virtual manipulatives, educational gaming, and video clips through 1:1 technology integration with Google Chromebooks into my mathematics lessons in order to scaffold learning and facilitate student understanding. These engaging and educational activities are enjoyable and enable my students to learn without stress or embarrassment. These interactive educational tools greatly assist my students in building a strong foundational understanding of mathematics concepts and reasoning. As a result, my students have consistently outperformed their peers across the state on the yearly state assessments. According to FES' 2016-17 Annual School Improvement Council Report to the Parents, my students performed well above the state and district averages for third grade mathematics. Even so, with only 53.4% of my students meeting or exceeding grade-level expectations on the State Ready test during the previous school year, it was evident that I needed to provide additional supports to foster growth. Therefore, I provided remedial, small-group instruction to meet the needs of my striving learners. Thirty-one percent of my students scored *Approaching* grade level expectations on the 2017 State Ready test. The students in this group were of most concern to me. With intensive remediation using a variety of multiplication strategies and engaging technology to improve conceptual understanding, I believed that many students, who would otherwise fall into this *Approaching* category, could very possibly meet grade level expectations on the spring State Ready test. In an effort to reach this group, I provided small group instruction using differentiated instructional strategies to help correct thinking and develop students' mathematical understandings. By incorporating a variety of strategies and technology tools, I expected my struggling students to build on their prior knowledge, develop a stronger understanding, and eventually apply concepts to more complicated mathematics problems.

Statement of the Problem

A fundamental understanding of key mathematics concepts such as multiplication is essential for succeeding in school as well as in a global economy (Wong & Evans, 2007). Mathematics is a discipline in which new concepts are built upon prior knowledge (Fuchs, 2005; Westenskow et al., 2017). As a result, misconceptions from prior learning often limit mathematical growth (Geary, 1993; Goldman, Pellegrino, & Mertz, 1988; Westenskow et al.; Woodward, 2006). Students who do not develop strong foundational mathematics skills tend to get farther behind as they move to more involved levels of mathematics courses. For example, basic multiplication is essential for many complex mathematical skills such as multi-digit multiplication, division, fractions, decimals, and proportions (Wong & Evans, 2007). If students do not have a strong conceptual understanding of basic multiplication, then they are likely to have great difficulty when applying multiplication to higher-level tasks. To prevent such a gap in understanding, it is critical that elementary mathematics, in particular, focus on a variety of strategies for developing a conceptual understanding of multiplication (Heege, 1985; Solomon & Mighton, 2017). Currently, on both the national and local levels, students are lacking such conceptual understandings needed to perform at the highest levels (Desilver, 2017; NCES, 2015). This study aims to build conceptual understandings of multiplication for struggling third graders so that they have a strong mathematical foundation from which to build.

Purpose Statement

The purpose of this action research was to evaluate the implementation of technology integration with multiplication concepts (i.e., repeated addition, arrays, and decomposing numbers) for struggling third grade students at FES in Lake County School District.

Research Questions

This action research was guided by two grand-tour questions and three strategy-specific sub-questions:

1. How and in what ways does technology integration with multiplication concepts impact student understanding?
    a. How do virtual manipulatives and student think-aloud self-recordings impact students' understanding of repeated addition?
    b. How do virtual manipulatives and student think-aloud self-recordings impact students' understanding of arrays?
    c. How do virtual manipulatives and student think-aloud self-recordings impact students' understanding of decomposing numbers?
2. How do students select and explain strategies for solving multiplication problems?

Researcher Subjectivities & Positionality

I consider myself an upper middle-class Caucasian female who grew up with many of the same childhood experiences as the student participants in my research study. As a child, I lived in a low-income, single-parent home with three siblings. For most of my K-12 experience, I qualified for and received assistance from the government subsidized free/reduced lunch program. In fact, I teach at the very same school that I attended during my elementary years. I can easily relate to these students because I am a product of the very environment from which my students come. Many of the students I teach are the children of my friends and acquaintances with whom I grew up. While I no longer live directly in this community, some of my family members still live there which allows me social involvement opportunities in this community outside of school.

Throughout my educational career, I always excelled in the area of mathematics. It always came easily for me, and I especially enjoyed the logical reasoning of more complex problems. I never felt the frustration that many of my third grade students must feel when they do not understand a key mathematical concept until I began taking higher level mathematics courses in my undergraduate studies. While I was not an eight-year-old child in an elementary mathematics class, I am quite sure I felt some of that same anxiety as a college student as I struggled to grasp certain concepts that my professor taught. In this regard, while I never experienced difficulty in elementary school, I can relate to my students who sometimes struggle understanding mathematical concepts.

My experience with technology differs greatly than that of my students. When I was in fourth grade, my school opened a computer lab with approximately ten Apple computers for the entire student body to share. There were only a handful of games (on large floppy disks) available for student use. Throughout the remainder of my K-12 career, computer technology was never available for classroom use except in the programming classes I took in high school. In my undergraduate degree, I pursued a major in computer science and mathematics, which is when I became more proficient in computer-based skills. My students have an entirely different experience in regards to technology. Many of my students had access to smartphones, tablets, and other digital devices before they were old enough to attend school. Beginning in kindergarten, students at my school have shared access to tablets or Chromebooks. As a result, students come to third grade with a relative amount of technological proficiency and an eagerness to learn more. This affords my students a variety of new and exciting learning resources and opportunities that were unavailable when I was their age. As a result, my students are building strong foundations in technology, providing them with many avenues for engaged, student-centered learning.

By allowing numerous strategies for understanding multiplication, students are able to direct their own path for learning. Vygotsky's (1978b) sociocultural theory of learning notes that the teacher should facilitate learning as the student becomes more successful with increasingly complex tasks and gains competence. In this study, students were allowed to choose manipulatives and strategies that worked best for them. In this constructivist approach, the teacher acted as a guide or resource, rather than sole source, for a student's learning. Students actively constructed knowledge in environments where they were allowed to be self-regulated learners, rather than in environments where they passively received information (Brophy, 2010). This means that students used their pre-existing knowledge of addition as a tool to help them construct new meanings as it related to multiplication. Students used their prior knowledge and experiences to explore new problems, investigate possible solutions, develop their ideas, and create new thinking (Pitler, Hubbell & Kuhn, 2012). Jerome Bruner (1995) would describe this as discovery learning. My students actively engaged in unique, hands-on, learning experiences as they incorporated an assortment of hands-on and virtual tools. These interactive learning opportunities allowed students more motivation and control over their own learning, challenging them to think analytically, critically, and collaboratively in ways that perhaps they had not done so before (Pitler et al.).

Despite the high poverty rate, 68.6%, at FES, all students in each grade have equal access to technology, as well as to a high-quality education. In the classroom, I believe that educators must ensure that knowledge and learning opportunities are equally available to all students, regardless of gender, race, socioeconomic status, sexual orientation, religion, age, and other perceived differences. All students should be given equal access to ideas and knowledge so that they are enabled to be productive contributors to the classroom community. This positive interaction based on equality and justice will assist in laying the groundwork for students to become successful (and empowered) members of society.

By incorporating digital technologies into mathematics instruction, even my struggling students quickly became engaged and excited about learning. When students are actively engaged in their own learning and are made to understand that their differences offer positive and unique perspectives (rather than seeing their differences as a hindrance), students are empowered to have the self-confidence and motivation needed to be successful in the classroom and in society. When students truly feel that they are viewed as equals, only then will they begin to feel empowered. By providing a liberating education, teachers give students the power to remove boundaries and barriers that once limited them, providing hope and justice for all learners.

Several influences caused my students to feel a sense of connectedness with me from the onset. I related with my students well as we shared similar life experiences. I personally know the parents of many of my students and have also taught the older siblings of many of my students. My interactions with my students were very positive, supportive, and encouraging, and as a result, my students displayed their affection towards me in both words and actions. The relationships that I built with my students created a sense of trust and safety which allowed them to feel comfortable enough to tell me their troubles - both school-related and otherwise. These multifaceted relationships enabled me to have a unique insider perspective of the happenings within my classroom (Herr & Anderson, 2005). Such a perspective enabled me to see a more complete picture of my students' abilities and of their conceptual understandings of multiplication within this study. Because my students were comfortable with me, they were not shy or hesitant about working with me individually or in small groups for remedial instruction. This level of student ease and willingness to participate better enabled me to thoroughly understand students' misconceptions and provide individualized remediation, as needed.

Definition of Terms

Arrays: To build arrays, manipulatives are arranged in rows and columns to represent the multiplicands in the problem (Barmby, Harries, Higgins, & Suggate, 2009). The equal sized rows and columns enable students to visualize the two-dimensions represented in multiplication problems

Careless errors: Careless errors in this study refers to instances where the correct methodology was used but simple mistakes were made in counting, adding, or in merely stating the final answer.

Concrete manipulatives: Concrete manipulatives build deep conceptual understanding because they provide a physical representation of the problem which aids in reconstructing concepts and aids in concrete thinking (Loong, 2014; Sowell, 1989; Yuan, 2009).

Decomposing numbers: Decomposing numbers is a multiplication strategy that allows students to break apart more difficult problems into smaller, less challenging problems that are easier to solve.

Explaining strategies: Explaining strategies refers to the students' ability to discuss specific multiplication strategies (repeated addition, arrays, and decomposing numbers) with peers and the teacher in order to demonstrate conceptual understanding of multiplication (Parker, 2006; Piccolo, Harbaugh, Carter, Capraro, & Capraro, 2008; Wohlhuter, Breyfogle, & McDuffie, 2010).

First-order barriers: First-order barriers to technology integration include factors that are extrinsic to the teacher (Ertmer, 1999; Ertmer et al., 2012). These barriers typically include different types of resources (i.e., equipment, training, time, and support) that are missing or insufficient. Common first-order barriers include finances, software and connectivity, time, and teacher training.

Number sense: Number sense refers to the flexibility with which a student thinks about numbers. The core of mathematics is reasoning, and students must be able to reason through why methods work mathematically (Boaler et al., 2015; Zhang et al., 2014).

Number talks: Number talks is a method for students to share their mental math strategies for solving a given mathematics problem (Boaler et al., 2015; Wohlhuter et al. 2010).

Peer talks: Peer talks to enable students to express and share their thinking using mathematical language (Kotsopoulos, 2010; Yang, Chang, Cheng, & Chan, 2016).

Repeated addition: Repeated addition is defined as a method for solving multiplication problems where one multiplicand is added for as many times as the other multiplicand (Zhang, Xin, Harris, & Ding, 2014).

Second-order barriers: Second-order barriers to technology integration are those barriers which are internal to the teacher (Ertmer, 1999; Ertmer, Ottenbreit-Leftwich, Sadik, Sendurer, & Sendurer, 2012; Francom, 2016). These barriers include teacher attitudes and beliefs about the importance of technology integration (Ertmer).

Virtual Manipulatives: Virtual manipulatives are interactive, web-based representations of physical objects used for constructing mathematical understanding (Moyer, Bolyard, & Spikell, 2002).

CHAPTER TWO

LITERATURE REVIEW

Introduction

The purpose of this action research was to evaluate the implementation of technology integration with multiplication concepts (repeated addition, arrays, and decomposing numbers) for third grade students at FES in Lake County School District. The review of related literature focuses on the main research question, "How and in what ways does technology integration with multiplication concepts impact student understanding?"

Based on the research question, five main variables were used to guide the literature search: (1) multiplication strategies, (2) technology integration, (3) repeated addition, (4) arrays, and (5) decomposing numbers. The resources for this review were collected through a variety of methods. Electronic databases, such as *ERIC*, *Education Source*, and *JSTOR*, were used to search for published articles by using combinations of the following keywords: elementary, multiplication, strategies, mathematics, virtual manipulatives, repeated addition, arrays, decomposing numbers, properties, technology integration, think-alouds, number sense, explaining strategies, and number talks. I also accessed additional resources by utilizing the PsycInfo database and Google Scholar website. By using the bibliography pages of some articles, I was able to locate related materials that were useful to my study.

The review of this literature is organized into two major sections. The first section takes an in-depth look at multiplication as it pertains to elementary mathematics education. The second section examines multiple aspects of technology integration in elementary mathematics education. I explore both areas and discuss how technology integration in elementary mathematics education impacts students' conceptual understanding of multiplication.

Multiplication

The learning of multiplication facts involves a progression of higher-order thinking skills in order for a student to reach fluency. Fluency of these facts is described as happening in three successive phases (Baroody, 2006; Kling & Bay-Williams, 2015; Rave & Golightly, 2014). Phase one involves modeling or counting to determine the answer. Phase two involves deriving the answer using reasoning strategies and critical thinking, and phase three is automatic retrieval or mastery of the facts. In order to better understand this progression through multiplication, I will focus on four key areas: (a) students' conceptual understanding, (b) barriers to conceptual understanding, (c) recommendations for teaching basic multiplication, and (d) strategies for teaching multiplication concepts. Each of these areas is critical to the overall purpose and success of this study.

Student Understanding

There are many levels of student understanding ranging from rote memorization to a much deeper conceptual understanding where students are able to derive answers using strategies that show they truly comprehend the mathematical reasoning. Reasoning

v. memorization, developing number sense, and eventual fluency of facts are all key to improving student understanding of basic multiplication.

Reasoning v. memorization. In order to reach fluency of multiplication facts, students must first develop a conceptual understanding of multiplication because "Students make more rapid gains in fact mastery when emphasis is placed on strategic thinking" (Kling & Bay-Williams, 2015, p. 551). It is essential that students learn a variety of reasoning strategies for solving multiplication facts so they will be able to derive an answer if they have forgotten it. Reliance solely on rote memorization of basic multiplication facts leads to an inability to reason through problems to find the correct solution (Boaler et al., 2015; Kling & Bay-Williams; Woodward, 2006). Teachers who rely primarily on traditional timed drills for assessing fluency of multiplication facts are not accurately assessing a student's conceptual understanding (Woodward). Instead, teachers must incorporate a variety of learning strategies to motivate students and to improve their conceptual understanding of multiplication (Heege, 1985; Solomon & Mighton, 2017).

Early research by Brownell and Chazal (1935) initiated an ongoing debate over the best approach for learning multiplication facts. Their work calls into question the use of traditional, rote memorization of facts. Brownell and Chazal found there had been very little research or attention paid until then to memorization as the main instructional strategy for multiplication facts. Their research indicated that learning, not drill, is key for understanding multiplication. "Drill makes little, if any contribution to growth in quantitative thinking by supplying maturer ways of dealing with numbers" but is exceedingly valuable for improving and maintaining fluency (Brownell & Chazal, 1935,

p. 26). Therefore, while drill does help students maintain fluency, it must be preceded by instruction which builds conceptual understanding.

Heege (1985) also examined the ways in which students successfully learn basic multiplication facts in his work, *The Acquisition of Basic Multiplication Skills*. Heege worked closely with elementary students and determined that cognitive achievement depends largely on a student's ability to figure out answers to basic multiplication facts through informal thinking strategies. For instance, Heege explains that children do not start learning multiplication with a "blank slate" (Heege, 1985, p. 382); rather they generally are prepared for multiplication by having the supports of basic additions up to the number twenty. These addition facts provide a foundation for learning multiplication. Heege notes six informal strategies that are crucial in learning basic multiplication, including:

- using the commutative property,
- adding a zero after the other factor when multiplying by ten,
- doubling the number when multiplying by two,
- halving familiar multiplication problems,
- adding on to familiar multiplication problems, and
- decreasing familiar products (p. 383).

These strategies enable students to think flexibly about numbers rather than relying on rote memorization. Researchers (Cumming & Elkins, 1999; Heege, 1985; Woodward, 2006) have shown that the didactic approach of blindly memorizing facts limits student understanding as it does not provide opportunities to become familiar with the operation of multiplication by using appropriate thinking strategies. Therefore, it is essential that students think of numbers flexibly as they use strategies and supports to derive solutions to basic multiplication facts.

Developing number sense. Cognitive achievement in the area of multiplication depends largely on students' ability to think mathematically and derive answers rather than depending on rote memorization (Boaler et al., 2015; Kling & Bay-Williams, 2015; Woodward, 2006). A developing number sense is crucial in building foundational skills in the area of mathematics. The most effective way to develop fluency of multiplication facts is to develop a strong number sense by working with numbers in different ways rather than merely blindly memorizing the basic facts (Boaler et al.; Cumming & Elkins, 1999; Heege, 1985; Solomon & Mighton, 2017; Wells, 2012). Students must understand how to reason through problems to derive the answer.

Researchers (Chambers, 1996; Garnett, 1992; Heege, 2006; Miller, Strawser, & Mercer, 1996; Sherin & Fuson, 2005; Solomon & Mighton, 2017; Thornton, 1990; Van de Walle, 2003) suggest developing number sense by focusing on patterns in the multiplication table, which capitalizes on a student's natural inclination for recognizing patterns. This pattern method for improving number sense and conceptual understanding engages students without overwhelming them (Solomon & Mighton). Some researchers (Miller, Strawser, & Mercer; Sherin & Fuson) focus more on patterns using the zero property and identity property of multiplication. Other researchers (Chambers; Garnett; Heege, 2006; Sherin & Fuson; Thornton; Van de Walle), however, concentrate on the importance of patterns such as doubles, times five, times nine, and squares, which they claim are easier for students to learn. By guiding children to appreciate the patterns in the different multiplication tables, students will begin to make sense of numbers and develop a basis for conceptual understanding (Solomon & Mighton). As a result, students will enthusiastically work to discover new patterns and "demonstrate their new knowledge as they acquire it" (Solomon & Mighton, 2017, p. 32).

By developing a strong number sense, students will have numerous strategies available to help them reason through problems, which will enable them to accurately find solutions that make sense mathematically. My action research implemented several such multiplication strategies (i.e., repeated addition, arrays, decomposing numbers) that aid in developing students' conceptual understanding of multiplication. In effect, my students, theoretically, should cultivate a much stronger foundation of multiplication from which to build higher-level mathematical skills.

Eventual fluency of facts. One of the key foundational learning challenges in elementary mathematics is developing fluency of the basic 0-12 multiplication facts (Polya, 2002; Skarr et al., 2014; Wong & Evans, 2007). Basic multiplication is an integral aspect of many routine mathematical tasks, both in the classroom and in real-world settings (Wong & Evans). Basic 0-12 multiplication facts form the basis for learning a variety of other mathematical skills, including multi-digit multiplication, division, fractions, decimals, and proportions. Unless students are able to recall these basic facts from memory, their focus will be shifted to solving basic facts rather than on solving the task at hand (Wong & Evans).

When students build fluency of multiplication facts, this knowledge becomes "automatized and stored in long-term memory," which frees up working memory to "attend to deeper or more conceptual aspects of mathematics" (Solomon & Mighton, 2017, p. 31). Since the capacity for working memory is limited (especially in children), it is easy for the working memory to become overwhelmed (Cowan, Morey, AuBuchon, Zwilling, & Gilchrist, 2010; Miller, 1956; Sweller & Chandler, 1994). Long term memory, however, is vast. By building fluency and committing multiplication facts to long-term memory, the learner is able to focus on more advanced applications within the problem (Burns, Ysseldyke, Nelson, Kanive, 2015; Houchins, Shippen, & Flores, 2004; Wong & Evans, 2007; Woodward, 2006). While many researchers (Boaler et al., 2015; Heege, 1985; Solomon & Mighton; Wells, 2012) stress the importance of learning strategies to derive multiplication facts, other researchers (Burns et al.; Houchins et al.; Wong & Evans) have shown that without fluency of the facts, it would be very difficult to demonstrate and assess students' understanding of higher-level thinking skills that involve multiplication. Therefore, fluency in basic multiplication facts is needed in order for students to engage in more complex problem solving.

Strategies for Teaching Multiplication Concepts

It is essential to teach a wide range of multiplication strategies so that students can see multiple representations for solving problems. Each child can then select a strategy that appeals to him or her. Common multiplication strategies include repeated addition, making equal groups, number lines, arrays, and decomposing numbers.

Repeated addition. Repeated addition is defined as a method for solving multiplication problems where one multiplicand is added for as many times as the other multiplicand (Zhang et al., 2014). For example, 6 x 3 is the same as adding 6 three times (See Figure 2.1). While some researchers (Devlin, 2008; Jacobson, 2009; Larsson, Pettersson, & Andrews, 2017) argue that defining multiplication in this way overgeneralizes it and causes misinterpretations when students later multiply with decimal numbers, other researchers (Heege, 1985; Sherin & Fuson, 2005; Wells, 2012) agree that repeated addition is the simplest, most efficient method for teaching the concept of multiplication, and it is the strategy that young students are most likely able to understand. Students must understand the relationship between repeated addition and multiplication in order to fully comprehend what multiplication means (Heege; Sherin & Fuson; Wells). Lower ability students may be more likely to need such repetitive practice with basic number facts in order to understand this concept (Fisher, 2001). While research findings (Larsson et al.; Vosniadou & Verschaffel, 2004) indicate that other strategies for solving multiplication problems are essential in order to bring about conceptual change, "products of small integers can only be calculated by repeated addition - and the conceptual link between multiplication and repeated addition remains important" (Wells, 2012, p. 38). Therefore, repeated addition is an integral step in the conceptual understanding of multiplication.

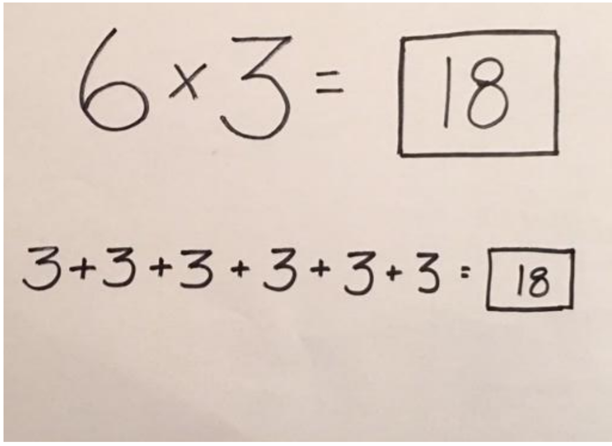

Figure 2.1. Repeated addition used to solve six times three.

Making equal groups. Making equal groups is one of the initial steps in making connections between multiplication and a student's prior knowledge of addition (Greer, 1992; Izsak, 2005) as it illustrates the link between the two operations (De Corte & Verschaffel, 1996). To make equal groups, students draw pictures or groups of tally marks to represent the multiplication problem. After drawing, the student counts the items or tallies to determine the product. For example, 7 x 3 could be drawn as seven equal sets of three tally marks (See Figure 2.2). The student would draw this representation and then count each circle to determine the product. This visual method for solving multiplication can be time-consuming as each student must count the number of groups, draw pictures, count the objects in each group, and then count the total (Barmby et al., 2009; Sherin & Fuson, 2005).

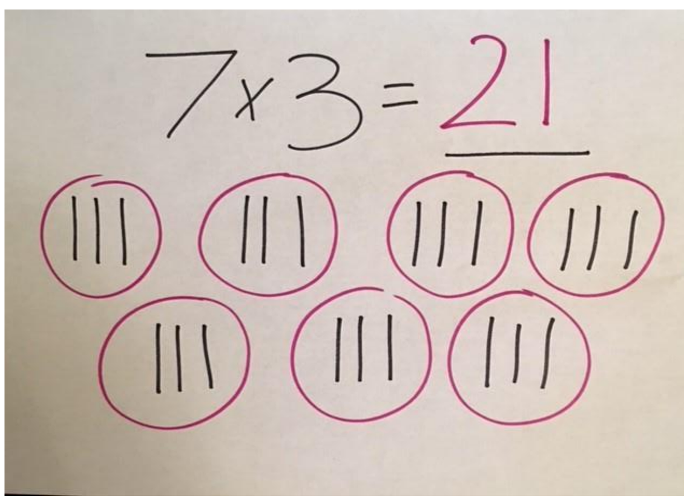

Figure 2.2. Seven equal groups of three tally marks.

While making equal groups is simple for many learners (Greer, 1992; Izsak, 2005), some researchers (Barmby et al., 2009; Larsson et al., 2017; Lo, Grant, & Flowers,

2008) warn that equal groups are asymmetrical and therefore do not reflect the commutative property accurately. For example, in four bags of six apples, where four (bags) is the multiplier and six (apples) is the multiplicand, it may not be evident that six bags of four apples would be the same amount (Barmby et al.; Greer, 1992). In effect, making equal groups serves as a simple transition from addition to multiplication, but students and teachers may opt for strategies that are less time-consuming and those which better reflect the multiplicative properties.

Number lines. Number lines can serve as a visual tool for representing multiplication as repeated addition problems. Number lines act as a visual-spatial representation (Gonsalves & Krawec, 2014; Kindle, 1976) that can be used as a counting-based (or sequence-based) problem-solving strategy (Yackel, 2001; Young-Loveridge, 2005). Teachers can use number lines to show how repeated addition is actually skip-counting on the number line (Grunke, 2016; Young-Loveridge). For example, 3 x 5 would be represented by starting at zero and jumping over five numbers at a time, for three jumps or iterations (see Figure 2.3). The teacher would explain that this is the same as adding 5 three times. Having a concrete, lifesize number line displayed in the classroom with which students can interact will enable them to better understand the abstract problems, attach meaning to solution strategies, and enable them to more easily solve multiplication problems (Bay, 2001; Gonsalves & Krawec). Number lines "facilitate the development of more sophisticated schematic diagrams to solve these more advanced problems, while simultaneously reinforcing students' conceptual understanding of operations and, more broadly, number sense" (Gonsalves & Krawec, 2014, p. 169). While the number line aids in calculating the product, researchers (Barmby et al., 2009) contend that (like the equal groups strategy) using the number line does not illustrate or make immediately clear why the commutative and distributive properties should apply. Therefore, number lines are helpful in learning multiplication as they provide visual-spatial representations which help build and reinforce conceptual understanding; however, they do not clearly illustrate the multiplicative properties.

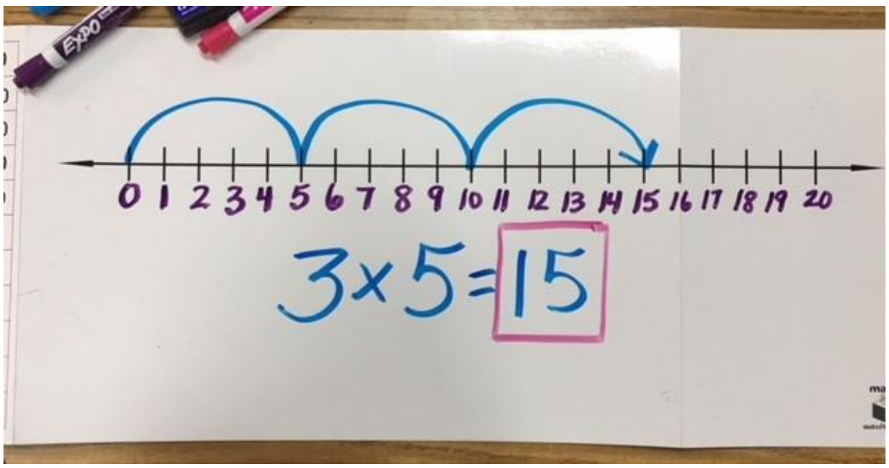

Figure 2.3. Number line model representing three groups of five.

Arrays. Arrays are helpful tools which enable many students to understand multiplication. To build arrays, manipulatives are arranged in rows and columns to represent the multiplicands in the problem (Barmby et al., 2009). The equal sized rows and columns enable students to visualize the two-dimensions represented in multiplication problems (Young-Loveridge, 2005). For instance, 5 x 2 can be represented with five rows of two or with two rows of five. The students see how the two numbers relate and then count the manipulatives to determine the solution.

Arrays also enable students to better represent and understand the commutative (Barmby et al., 2009; Charles & Duckett, 2008; Day & Hurrell, 2015; Hurst & Hurrell, 2017; Jacob & Mulligan, 2014; Kling & Bay-Williams, 2015) and distributive (Barmby et al.; Day & Hurrell; Hurst & Hurrell; Wall, Beatty, & Rogers, 2015) properties. For example, if a student builds a 3 x 4 array, so that it is three rows of four, then rotates the array so that it is four rows of three, the student can quickly see that the product does not change regardless of the position of the factors (see Figures 2.4 and 2.5). 3 x 4 is equal to 4 x 3, illustrating the commutative property.

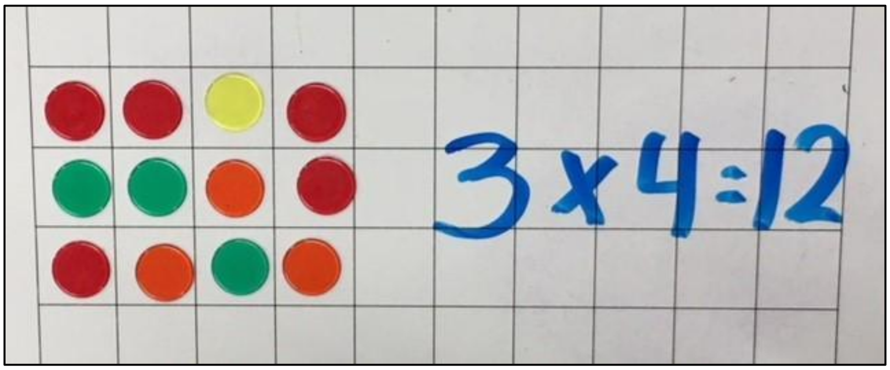

Figure 2.4. Array model representing 3 x 4 = 12.

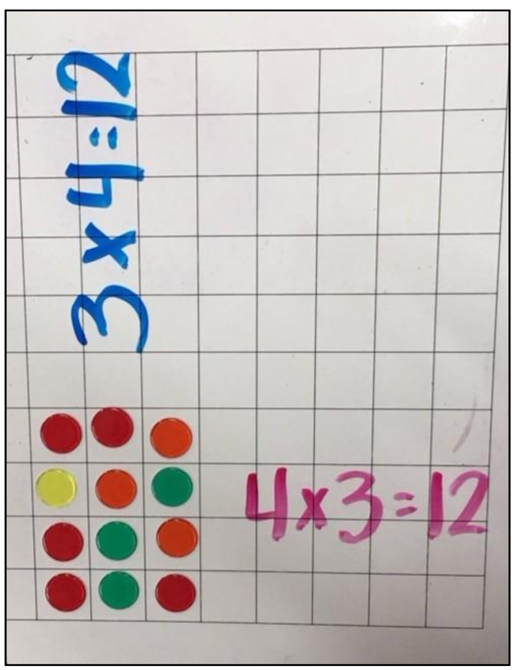

Figure 2.5. Array model rotated to represent the commutativity of 3 x 4 and 4 x 3.

Arrays can also serve as a visual representation of the distributive property (Barmby et al., 2009; Day & Hurrell, 2015; Hurst & Hurrell, 2017). For example, 12 x 3 can be represented as twelve rows of three. After making the array, students can separate the array so that there are ten rows of three and two rows of three. This will enable the student to separate the problem into two smaller problems using factors that are more familiar and easier to calculate.

Arrays shift student thinking from additive thinking (equal groups) to multiplicative thinking (factors and products) (Day & Hurrell, 2015; Jacob & Mulligan, 2014; Siemon et al., 2011). Through manipulation of arrays, students are provided with a strong understanding of factors, multiples, and products (Charles & Duckett, 2008; Day & Hurrell; Jacob & Mulligan). This visual, interactive representation of rows and columns enables students to develop a solid (collections-based) foundation of multiplication (Young-Loveridge, 2005).

In Barmby et al.'s (2009) study, researchers found that using arrays enabled students to successfully calculate products through simple counting strategies by using the distributive property to move groups within the array to make calculations easier. (The researchers noted that while students understood how to rearrange the arrays to aid in solving, many students did not relate their grouping strategies to the distributive property). Barmby et al. identified several possible difficulties with arrays, including the potential to over-use inefficient counting strategies, the unlikeliness of students to implement the distributive property, and some students' inability to represent the multiplication problem in a two-dimensions due to lack of understanding about the binary nature of arrays. The researchers concluded that having students create arrays was quite useful in gauging their conceptual understanding of multiplication.

In sum, using arrays to represent multiplication relationships provides a clear illustration of the multiplicative properties and relationship between factors and product. In addition, arrays are very useful tools in determining a student's conceptual understanding of multiplication.

Decomposing numbers. Decomposing numbers is a multiplication strategy that allows students to break apart more difficult problems into smaller, less challenging problems that are easier to solve. Students often find that it is easier to decompose larger numbers to enable them to solve multiplication problems (Zhang et al., 2014). They decompose one or both multiplicands and refer to known problems to find the answer, "such as derived fact (e.g., $6 \times 7 = 6 \times 6 + 6$), doubling (e.g., $8 \times 7 = 4 \times 7 \times 2$), doubling-again strategies (e.g., $8 \times 7 = 2 \times 7 \times 2 \times 2$)" (Zhang et al., 2014, p. 19). By decomposing more difficult numbers, students are able to use what they already know to help them solve more challenging problems.

A strong understanding of the distributive property of multiplication aids students in decomposing harder problems. The distributive property is a logical choice for students to use when decomposing numbers, as it allows them to think about numbers from multiple perspectives, consider numerical relationships, and develop the ability to estimate and make mental calculations (Baroody & Coslick, 1998; Benson, Wall, & Malm, 2013; Cumming & Elkins, 1999; Gerstan & Chard, 1999; Kilpatrick, Swafford, & Findell, 2001; Sowder, 1992). Teachers must explain this process of decomposing numbers in order to create smaller, less difficult multiplication problems as an extension of the distributive property so that students can make connections from one problem to the next (Benson et al.; French, 2005). By making these connections, students will better understand how and when to use the distributive property to decompose numbers. For example, students can decompose 7 x 3 by decomposing, or breaking apart, 7 into 5 + 2 (See Figure 2.6). The student would then distribute the 3 in order to solve: (5 x 3) + (2 x 3). This problem is much easier to solve in chunks with smaller numbers that students can much more easily manipulate.

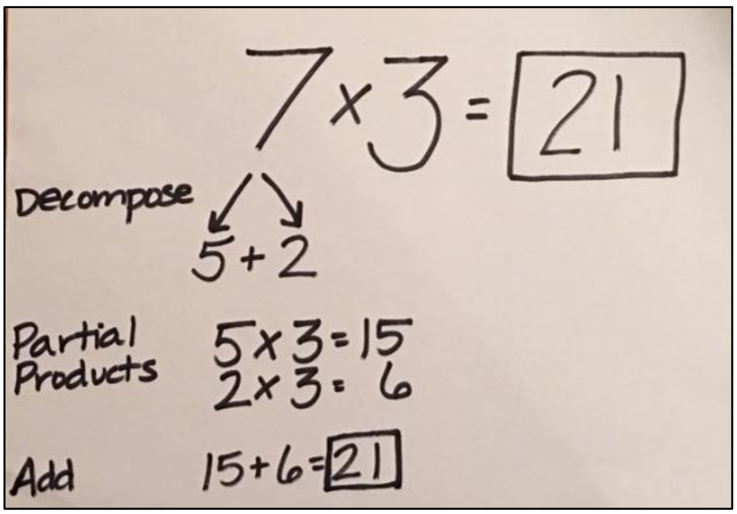

Figure 2.6. Decomposing numbers strategy.

As indicated above, many researchers rely on decomposition of numbers as a multiplication strategy that works with most students. It should be noted however, that some researchers (Cumming & Elkins, 1999; Geary, 1993; Goldman et al., 1988) argue that low achieving and learning-disabled students do not develop sophisticated facts strategies naturally and should be taught multiplication by integrating strategies such as decomposing numbers with timed practice drills. Cumming and Elkins (1999) explain that teaching strategies does increase a student's ability to use numbers flexibly, but that does not always lead to automaticity for this population. In this case, they argue that "frequent timed drill is essential" (Cumming & Elkins, 1999, p. 271). In sum, having a flexible understanding of numbers and strong conceptual understanding of multiplication allows students to decompose numbers so they can apply their knowledge to solve more complicated multiplication problems.

Barriers to Conceptual Understanding

For many students, there exist one or more barriers which impede students' conceptual understanding of multiplication. Such barriers include a lacking foundational number sense, prior knowledge that indicates gaps in learning, and a misunderstanding of the multiplicative properties. Each of these is discussed below.

Number sense. Number sense refers to the flexibility with which a student thinks about numbers. The core of mathematics is reasoning, and students must be able to reason through why methods work mathematically (Boaler et al., 2015; Zhang et al., 2014). A student's ability to understand how numbers relate to each other is key in solving mathematics at many levels. "When students demonstrate number sense, they are connecting ideas across characteristics of number[s] (e.g., magnitude, symbols, and representations) and the use of numbers (e.g., estimating, comparing, and operations)" (Westenskow et al., 2017, p. 1). Number sense provides foundational skills for all higher-level mathematics skills (Boaler et al., 2015).

In multiplication, for instance, students with a strong number sense may be able to figure out the answer to 8 x 9 even if they do not know the fact by memory. Students could easily determine the product of 8 x 10, and then subtract eight from their answer. Students who rely solely on memorization would be unable to derive answers that they have not memorized. “Low achievers are often low achievers not because they know less but because they don’t use numbers flexibly” (Boaler et al., 2015, p. 2). These students try to solve by memory instead of interacting with numbers flexibly, which can lead to learning a harder mathematics (Boaler et al.). Other researchers (Geary & Brown, 1991; Hanich, Jordan, Kaplan, & Dick, 2001; Hoard, Geary, & Hamson, 1999) indicate that low achieving students are more likely to rely on counting strategies than direct retrieval for solving basic multiplication facts. Without a strong number sense, however, these students are more prone than their peers to make retrieval and counting errors on basic addition and multiplication problems. Many mathematics educators and researchers (Boaler et al.; Geary, 1993; Goldman et al., 1988) agree that the best way to develop number sense is interventions which provide students the opportunity to work with numbers in many ways without relying on blind memorization of the facts. In effect, teachers must incorporate a variety of learning strategies to improve students’ conceptual understanding of multiplication (Heege, 1985; Solomon & Mighton, 2017). Therefore, building a strong and flexible sense of numbers is essential in helping all students achieve a more complex understanding of the relationships between numbers.

Prior knowledge. In mathematics, it is crucial for students to develop a conceptual understanding of basic skills before moving on to more difficult concepts. Mathematics is a discipline in which new skills and concepts are built on the foundation of previously learned concepts (Fuchs, 2005; Westenskow et al., 2017). Consequently, misconceptions from previous lessons or insufficient understanding of prior learning limits mathematical growth (Geary, 1993; Goldman et al., 1988; Westenskow et al.; Woodward, 2006). Many reasons for gaps in learning exist, including missed opportunities, inadequate teaching, absenteeism, second language learning, difficulties attending to instruction, or cognitive or physical disabilities (e.g., memory, visual perception, senses) (Dowker, 2005; Geary, 2010; Westenskow et al.). Before moving on to more challenging mathematics skills, students need remedial help to alleviate misconceptions or gaps in learning. For example, if students are having difficulty with basic addition, related skills such as multiplication will also prove unnecessarily difficult (Geary, 1993; Goldman et al., 1988). Early research by Brownell and Chazal (1935) indicates that rote memorization only reinforces students' poor methodology for solving basic facts. Other researchers (Anghileri, 1989; Baroody, 1997; Clark & Kamii, 1996; Isaacs & Carroll, 1999; Mulligan & Mitchelmore, 1997; Sherin & Fuson, 2005) agree that students will naturally develop strategies for correctly learning mathematics facts, specifically multiplication facts, if given the opportunity. In effect, teachers must work towards closing any gaps in student learning by identifying areas of students' conceptual weaknesses and providing ample opportunity for systematic practice (Geary, 1993; Goldman et al., 1988). Individualized instruction will allow teachers to differentiate lessons to provide effective remediation for each student.

Misunderstanding of the properties. Understanding the properties of multiplication can certainly improve students' abilities to reason through problems and derive answers that they do not know by memory (French, 2005; Kilpatrick et al., 2001; Kling & Bay-Williams, 2015; Sowder, 1992). The commutative property of multiplication states that the order of the factors has no effect on the resulting product (Denham, 2013). For example, 5 x 8 results in the same product as 8 x 5. When students learn basic facts through a multiplication table without being taught the multiplicative properties, students lack the conceptual understanding of multiplication (Denham; Kling & Bay-Williams). As a result, students do not realize the relationships created by the commutative property and are unable to understand why these multiplication problems have the same solution.

Similarly, students must understand fully how to apply the distributive property of multiplication in order to help them reason through a problem and find the correct solution (Benson et al., 2013; Day & Hurrell, 2015; Kinzer & Stanford, 2013; Wall et al., 2015). The distributive property states that multiplying two numbers is the same as multiplying the first factor by a sum of the parts of the second factor (Kling & Bay-Williams, 2015). For example, 3 x 12 is the same as multiplying 3 by the sum of the parts of 12 (i.e., 10 + 2). 3 x 12 equals (3 x 10) + (3 x 2). By learning how to use this property accurately, students may be able to derive answers that they do not yet know from memory, and it may allow them to reason through solving much larger multiplication problems (Benson et al.; Day & Hurrell; Kinzer & Stanford; Wall et al.).

Many teachers overlook the importance of teaching relationships among multiplication facts in order to improve fluency, as well (Hurst & Hurrell, 2017; Kling & Bay-Williams, 2015; Young-Loveridge, 2005). For instance, 6 x 4 is twice as large as 3 x 4. By teaching strategies, relationships, and properties, students can understand how to derive the answers to multiplication problems without needing to rely solely on rote memorization (Denham, 2013; Woodward, 2006). Therefore, it is essential for teachers to thoroughly represent multiplicative strategies, relationships, and properties in order to build a conceptual understanding of multiplication.

Recommendations for Teaching Multiplication

In order to remove these barriers and build conceptual understandings, teachers must accommodate the disparity in learning among students. This means that teachers must (a) incorporate concrete and virtual manipulatives and improve students' number sense by having students (b) explain strategies and participate in peer and number talks. Each of these recommendations is further discussed below.

Concrete and virtual manipulatives. Manipulatives are concrete (physical) or virtual tools which allow teachers and students to represent abstract thinking. By integrating manipulatives into mathematics instruction, students are better able to visualize the concepts being taught, scaffold their understanding, and simplify the abstract ideas (Burris, 2013; Loong, 2014; Sowell, 1989; Suh & Moyer, 2008). For example, students can interact with concrete or virtual manipulatives such as base-ten blocks, counters, tiles, connecting cubes, and number lines in order to construct quantities, aid in mathematical thinking, and solve problems. Without manipulatives, students have only an instrumental understanding and must rely on facts and procedures to help them solve mathematics problems without having the relational understanding needed to truly understand and explain the concepts (Loong; Skemp, 1976). To help students understand the foundations of multiplication, a variety of physical and virtual manipulatives should be incorporated in multiplication lessons (Loong; Moyer, Salkind, & Bolyard, 2008; Raphael & Wahlstrom, 1989; Terry, 1995). By scaffolding instruction using a combination of concrete and virtual manipulatives, teachers can help correct misconceptions and errors in students' thinking (Loong).

Concrete manipulatives build deep conceptual understanding because they provide a physical representation of the problem which aids in reconstructing concepts and aids in concrete thinking (Loong, 2014; Sowell, 1989; Yuan, 2009). Figure 2.7 illustrates how connecting cubes can be used to find a product by implementing the equal groups strategy.

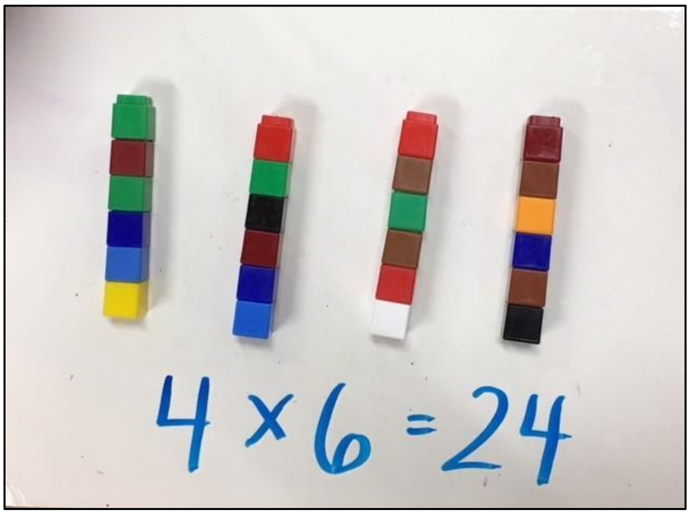

Figure 2.7. Equal groups model using connecting cubes to represent 4 x 6.

Similarly, *virtual manipulatives* are interactive, web-based representations of physical objects used for constructing mathematical understanding (Moyer et al., 2002). Virtual manipulatives can provide a different approach to teaching essential mathematical concepts. Virtual manipulatives provide metacognitive support by keeping record of the users actions and numeric notations (Moyer et al., 2008). Figure 2.8 illustrates such support provided by virtual manipulatives to solve the multiplication problem, 7 x 3. Also, virtual manipulatives provide immediate feedback, incorporate a larger range of problems, allow students to make connections with mathematical concepts, and can be utilized at home through personal computers (D'Andrew & Iliev, 2012). Some studies (Bolyard, 2006; Drickey, 2000; Kim, 1993; Smith, 2006; Steen, Brooks, & Lyon, 2006; Suh & Moyer, 2007; Takahashi, 2002; Terry, 1995), however, provide mixed conclusions for linking virtual mathematics manipulatives alone to student achievement.

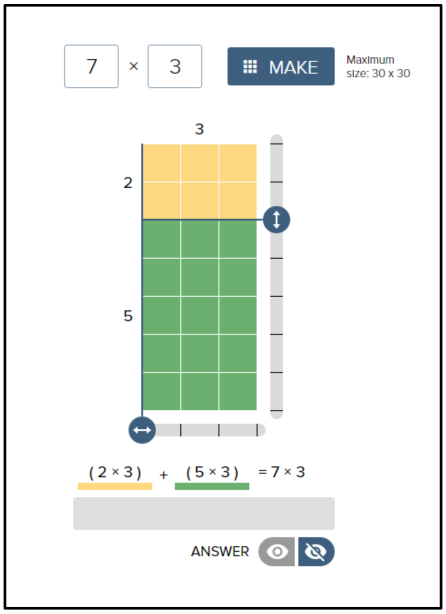

Figure 2.8. Virtual manipulative model which provides numerical and graphical support with immediate feedback. (From https://www.mathlearningcenter.org/web-apps/partial-product/)

Researchers have found that a combination of both concrete and virtual manipulatives help students make considerable gains as compared to students who used only virtual or only concrete manipulatives alone (Moyer et al., 2008; Terry, 1995). Burris (2013) found that students often used different strategies depending on whether they were working with concrete or virtual manipulatives. Both groups of students in this study (one group using concrete manipulatives and one group using virtual manipulatives) constructed quantities using standard and nonstandard representations with a variety of useful strategies. As concluded in Burris' study, both concrete and virtual manipulatives enable students to interact with quantities in nontraditional methods, enabling students to conceptualize the problem in a variety of meaningful ways. By using a combination of both concrete and virtual representations to connect procedures with conceptual understandings, students can better grasp abstract concepts (Burris; Martin, 2008). These manipulatives often enable students with poor relational understanding of mathematics concepts to clarify any misunderstandings and provide a necessary connection "using some form of concrete, kinesthetic, and/or visual experience so that an 'aha!' moment can occur" (Loong, 2014, p. 10). Therefore, it is recommended that teachers provide a variety of virtual and concrete manipulatives to remove barriers to conceptual understanding and to facilitate learning.

Explaining strategies, peer talks, number talks, student think-aloud protocols. Research shows that an effective method to "gaining insight into students' metacognition is asking them to verbalize their thoughts while working on a task" (Jacobse & Harskamp, 2012, p. 134). Qualitative data such as explaining strategies, peer talks, number talks, and student think-alouds provide meaningful insight into the thoughts and actions of the participants not otherwise available (Creswell, 2014). These strategies are explained below.

Explaining strategies. Explaining strategies refers to the students' ability to discuss specific multiplication strategies (repeated addition, arrays, and decomposing numbers) with peers and the teacher in order to demonstrate conceptual understanding of multiplication (Parker, 2006; Piccolo et al., 2008; Wohlhuter et al., 2010). When students explain their strategies for solving problems step-by-step, they "make invisible mental processes visible," (Silbey, 2002, p. 26) allowing the researcher to more completely view and understand the participants and problem. By incorporating think-aloud opportunities, teachers are better able to identify conceptual understandings and misconceptions that would possibly by difficult to identify otherwise (Basaraba, Zannou, Woods, & Ketterlin-Geller, 2013; Ericsson & Simon, 1993; Gorin, 2007). Talking aloud during mathematics enables students to "gain personal understanding, insight, and clarification" (Kotsopoulos, 2010, p. 1049). Students are better able to explain, clarify, and reinforce their own thinking when they talk through problems (Brennan, Rule, Walmsley, & Swanson, 2009; National Council for Teachers of Mathematics [NCTM], 2000). This type of mathematical conversation provides crucial information about misconceptions in students' understanding that may be overlooked otherwise (Secolsky et al., 2016). For example, as a student solves 6 x 4, he or she will reveal conceptual understandings while verbally explaining the strategy (i.e., repeated addition, number line, array, decomposing numbers, commutative property) that is used to solve the problem. Teachers should also probe students' thinking by asking them to explain *why* or *how* a strategy worked (Franke et al., 2009). By explaining their strategies, students reveal any misconceptions they may have (Basaraba et al.; Ericsson & Simon; Gorin). The teacher is then enabled to clarify and remediate, as needed. When students explain their thinking and reasoning processes, their conceptual understanding is made clear.

Peer talks. Research in mathematics education shows that a renewed emphasis has been placed on peer talks to enable students to express and share their thinking using mathematical language (Kotsopoulos, 2010; Yang et al., 2016). By learning to communicate mathematically, students are able to provide evidence of their mathematical ideas and understandings (Mooney, Hansen, Ferrie, Fox, & Wrathmell, 2012; Whitin & Whitin, 2000). When teachers promote student talk strategies (i.e., turn and talk, revoice, press for reasoning, debate the differences), they are providing students with the opportunity to explain their mathematical thinking with others (Chapin, O'Connor, & Anderson, 2009; Smith & Stein, 2011). By doing so, students can help each other bridge any gaps in understanding and solidify their own thinking. Such mathematical discourse affords students the opportunity to "develop strategic competence, adaptive reasoning, and productive dispositions" (Kastberg & Frye, 2013, p. 34). As a result, students become much more confident in their own mathematical abilities (Hufferd-Ackles, Fuson, & Sherin, 2004; NCTM, 2000; Walshaw & Anthony, 2008). By incorporating mathematical dialogues, such as peer talks, students are better able to clarify understandings, relate concepts, and formulate new knowledge.

Number talks. Number talks is a method for students to share their mental math strategies for solving a given mathematics problem (Boaler et al., 2015; Wohlhuter et al., 2010). This method, developed by Parker and Richardson (Parker, 2006), teaches number sense, mental math, and multiplication strategies at the same time. For example, the teacher may pose an abstract multiplication problem such as 18 x 5 and ask students to solve this problem mentally. After solving this problem, students will share how they derived the answer. For instance, one student may think of decomposing 18 as (9 x 2), so 18 x 5 would be like multiplying 9 x 2 x 5 which is the same as 9 x 1 a much easier problem (Boaler et al.). Other students will share their strategies so that every student understands a variety of different methods, applies the multiplication properties as needed, and develops a deeper conceptual understanding of the multiplication problem (Piccolo et al., 2008).

Student think-aloud protocols. Think-aloud protocols are the more structured method of making students' thinking visible. Think-alouds are the verbalization of one's step-by-step solution process (Silbey, 2002). To demonstrate how the think-aloud works, teachers may choose to model the type of thinking that builds conceptual understandings as well as appropriate ways of sharing their thinking (Trocki, Taylor, Starling, Sztajn, & Heck, 2015). Such a demonstration before high-level thinking assignments "promotes purposeful mathematical discourse for all students" and enables students to more readily share their mathematical thinking with others (Trocki et al., 2015, p. 278). By modeling think-alouds, teachers demonstrate the thinking process and how to reason through a problem in order to arrive at the correct solution. This process trains students how to think mathematically and how to engage in rich discussion. Mathematical communication focuses on the sharing of ideas which is necessary for students to express their own conceptual understanding and evaluate that of others (Yang et al., 2016). Students must reflect on their thinking process in order to clearly explain how they derived an answer. This careful reflection solidifies thinking and enables students to develop mathematical arguments (Yang et al.).

In the think-aloud protocol developed by Ericcson and Simon (1993), students were provided sample open-ended questions, shown how to share their thoughts, and told that the answers would be recorded for each problem (Secolsky et al., 2016). Students each solved five problems and were asked to continually report their thoughts aloud as their explanations were tape-recorded (Secolsky et al.). These think-alouds were later transcribed, sorted, and then students evaluated each one for correctness (Secolsky et al.). This enabled students and teachers to identify any misconceptions that existed. Teachers were then able to provide instructional interventions to directly address incorrect thinking (Secolsky et al.).

By using think-aloud protocols before instruction, teachers are able to identify misconceptions in advance of actual teaching (Secolsky et al., 2016). Therefore, preliminary think-alouds are very insightful as they directly inform and guide instruction while culminating think-alouds are extremely beneficial in determining the depth of a student's conceptual understanding after instruction has taken place.

Explaining strategies, peer talks, number talks, and student think-aloud protocols provide a unique and in-depth glimpse into participants' thoughts and actions (Creswell, 2014). Such qualitative data are essential in providing a comprehensive understanding of the participants and gaining valuable insight into their thought processes (Jacobse & Harskamp, 2012).

Technology Integration in Elementary Mathematics

As the availability of technology within the classroom increases, so do the opportunities for students to receive an individualized instructional plan through the digital curriculum and related resources. These opportunities provide technology-based learning which enable students to "employ higher-order critical thinking and reasoning skills - not just to arrive at the right answers, but to gain a deeper understanding of the concepts" (Smith, 2017, p. 24). In the next sections, I will discuss technology integration broadly, and will then discuss technology integration as it relates specifically to elementary mathematics.

Technology Integration Broadly

Digital technology such as tablets or iPads creates an abundance of new learning avenues for students of all ages and is recommended as a "viable instructional method" within the mathematics classroom (Ok & Bryant, 2016, p. 147). Since digital technology has become greatly immersed in the American culture and way of life, even the youngest students are eager and ready to learn how to use technology-based devices and programs. With increased technology exposure for students and ongoing professional training for teachers, the integration of 1:1 technology may facilitate higher levels of learning for students (Harris, Al-Bataineh, & Al-Bataineh, 2016). When applied effectively, technology increases student learning, understanding, and achievement (Liu, 2013; Pitler, Hubbell, & Kuhn, 2012; Ysseldyke & Bolt, 2007). Effective use of technology also facilitates conceptualization, encourages collaboration, and helps develop critical thinking and problem-solving skills (Pitler et al., 2012). Students are able to make connections between technology skills learned at school with those interrelated skills learned outside of school. As a result, students are able to realize the practicality of technology and understand its importance in their individual lives both in- and outside of school.

Theory and pedagogy. The integration of digital technology within the classroom provides a new means for allowing students to construct their own learning. According to constructivists such as Piaget, learning is an active process where students construct their own representations of the knowledge (Applefield, Huber, & Moallem, 2001). Students should be actively engaged in the learning process while the teacher guides learning. Vygotsky's (1978b) sociocultural theory of learning notes that the teacher should facilitate learning as the student becomes more successful with increasingly complex tasks and gains competence. According to Vygotsky's (1978a) zone of proximal development, the learner is much better able to build a conceptual understanding of multiplication when instruction is scaffolded (i.e., concrete and virtual manipulatives used in conjunction with multiplication strategies that build on one another) (D'Andrew & Iliev, 2012; Loong, 2014). In the constructivist approach, teachers act as a guide or resource, rather than sole source, for a student's learning. Students actively construct knowledge in environments where they are allowed to be self-regulated learners, rather than in environments where they passively receive information (Brophy, 2010). This means that students must use their pre-existing knowledge as a tool to help them construct new meanings and new knowledge. Students use their prior knowledge and experience to explore new problems, investigate possible solutions, develop their ideas, and create new thinking (D'Andrew & Iliev; Kling & Bay-Williams, 2015; Loong; Pitler et al., 2012). By integrating technology into the instruction, the classroom shifts from a teacher-centered to a student-centered learning environment (Pitler et al.). This type of constructivist classroom provides students with increased opportunities to work cooperatively, make choices, and play an active role in their own learning (Pitler et al.).

Technology-enhanced lessons enable students to become actively engaged, promoting effective differentiated and individualized learning. By allowing students the option to choose from a variety of teacher-selected web applications and sites, the students are given a sense of ownership in their own learning (Liu, 2013; Pitler et al., 2012; Ysseldyke & Bolt, 2007). Bruner (1995) would describe this as discovery learning.

Students are actively engaged in unique, hands-on, learning experiences as they incorporate a variety of online tools to create and enhance their own knowledge.

Technology-based instruction also prompts interaction from students who may not otherwise be as inclined for social learning. Collaboration is typically difficult for students diagnosed with cognitive disorders, autism, or other learning disorders (Cicconi, 2014). By using collaborative software, many of these students are enabled to interact with peers as they have never before (Cicconi; Pitler et al., 2012). Such software gives a voice to those students who had never been successful in traditional collaborative projects, allowing them to contribute and interact successfully with their peers.

Effective technology integration. Effective technology integration in the mathematics classroom must be engaging, improve students' conceptual understandings, and provide meaningful feedback.

Engaging. Technology-based learning provides the engaging, interactive, and effective instruction needed in the 21st century mathematics classroom (Lavin-Mera, Torrente, Moreno-Ger, Valleji-Pinto, & Fernandez-Manjon, 2009; Mansour & El-Said, 2009). Such games are objective-based which allows them to present educational content in a fun and engaging format (Hoffman, 2009). By relating examples to real-world problems, students are able to make connections as they interact with mathematics gaming technology (Allsopp, Kyger, & Lovin, 2007; Griffin, 2007).

In a 2010-2011 pilot study by Houghton Mifflin Harcourt and Amelia Earhart Elementary School in California, students in the experimental group were given both school- and home-access to iPads for the entire academic year. The results of this study indicated that students in the experimental group were "more motivated, more attentive in class, and more engaged" than students receiving traditional textbook instruction (Houghton Mifflin Harcourt, 2012, p. 3). In addition, achievement increased by 19% for students in the experimental group who scored proficient or advanced on the California Achievement Test.

Jackson, Brummel, Pollet, and Greer's (2013) examined the effects of interactive tabletops on math performance, attitudes, and gender differences using a sample of 53 elementary students over the course of one academic semester. Students were able to work in groups of four to work together to solve math problems as a group. This study enabled the elementary students to collaborate and solve mathematics problems as a team which enabled them to help each other with skills, as needed. Students reacted quite favorably to the program. The results of this study are similar to Liu's (2013) findings which suggest that the technology-based lessons increase math achievement and improve student attitudes towards mathematics. Jackson et al. (2013) also found that interactive tabletops, despite the cost, can "prove to be an effective instructional aide" (p. 327). Educational technology such as this which allows student collaboration, review, and feedback is extremely beneficial and promotes student growth and understanding (Carr, 2012; Lavin-Mera et al., 2009; Mansour & El-Said, 2009).

Improve conceptual understanding. Incorporating hand-held digital devices for student use in the classroom is a motivating factor for students as it provides them control over their own learning and enables students to more actively engage in the instructional process (Guha & Leonard, 2002; Ok & Bryant, 2016; Pitler et al., 2012; Rave & Golightly, 2014). In effect, the use of technology improves students' conceptual understandings as it challenges them to think analytically, critically, and collaboratively in ways that perhaps they have not done so before (Pitler et al.). Incorporating technology into classroom instruction creates an "open ended intellectual milieu" which allows a wide range of ideas to be developed and explored (Abramovich & Connell, 2014, p. 6). Students are much more attentive to technology-based mathematics lessons, are highly engaged during instruction, respond favorably to assigned tasks, and perform at higher levels (Bragg, 2006; Camp, 2016; Clark & Ernst, 2009; Huizenga, Admiral, Akkerman, & Dam, 2009; Liu, 2013).

In their five-week experimental study in a diverse Hong Kong primary school, Li and Pow (2011) found that 1:1 tablet technology immensely impacted student learning both formally at school and informally at home in less-structured learning environments. Li and Pow concluded that the integration of technology in primary classrooms enhances student motivation, develops cognitive skills, and improves learning strategies. In addition, the researchers found that students in the experimental group (using the 1:1 tablet technology) consistently outperformed students in the control group in the area of mathematical performance in their daily learning activities.

Educational websites and gaming applications that develop conceptual understanding of multiplication and provide meaningful feedback are seemingly quite effective tools in mathematics. While many free educational applications require only low-level thinking, Hoffman (2009) contends that effective gaming applications "require resolve, concentration, and the use of a variety of strategies, imagination, and creativity" (p. 122). For example, online multiplication games provide students a means of using various strategies to derive basic multiplication facts without the stress of timed tests (Kling & Bay-Williams, 2015). This meaningful and enjoyable practice allows students the opportunity to deepen their conceptual understandings of multiplication without even realizing that they are working.

Provide meaningful feedback. By allowing students to reason through multiplication problems and by providing effective and immediate feedback, many mathematics websites and applications help build a solid foundation in multiplication (Van de Walle, Karp, & Bay-Williams, 2010). Rather than focusing only on the final answer, many game-based mathematics applications focus on the strategies used to ensure conceptual understanding (Allsopp et al., 2007), which is equally as important as playing the mathematics games (Van de Walle et al.). Virtual games that review mathematics skills afford students the opportunity to think about and question misconceptions from prior learning. From a constructivist perspective, this "cognitive conflict" is a necessary step for overcoming mathematical misconceptions (Bragg, 2006, p. 7). By integrating a variety of technology-based strategies into curriculum, teachers can better engage and empower their learners to become conceptual thinkers.

Barriers to technology integration. Potential barriers to successfully integrating technology within the classroom include access to resources, technology training and support, time to plan and prepare, and teacher beliefs and attitudes about the usefulness of technology integration (Francom, 2016; Hew & Brush, 2007; Inan & Lowther, 2010; Kopcha, 2012; Reinhart, Thomas, & Toriskie, 2011; Ritzhaupt, Dawson, & Cavanaugh, 2012; Spotts & Bowman, 1993). These barriers, which can be categorized as either external (first-order) or internal (second-order) barriers, impede successful and effective technology integration in many schools and districts (Ertmer, 1999; Ertmer et al., 2012).

External (first-order) barriers. First-order barriers to technology integration include factors that are extrinsic to the teacher (Ertmer, 1999; Ertmer et al., 2012). These barriers typically include different types of resources (i.e., equipment, training, time, support) that are missing or insufficient. Common first-order barriers include finances, software and connectivity, time, and teacher training.

Finances. The significant expenditures for 1:1 computer- or tablet-based technology is not easily affordable in many districts (Harris et al., 2016; Hasselbring, 2014). It is also difficult for those districts that can afford this technology to keep their systems updated as often as needed, and therefore, those districts often operate on old technology (Hasselbring).

Software and connectivity. In many cases, schools may have the technology devices but are not provided with effective, instructionally-adequate, educational software, reliable connectivity, or sufficient bandwidth to accommodate a large number of devices at the same time (Hasselbring, 2014; Herron, 2010). Often, the software provided to the school district necessitates higher system requirements than what is available (Hasselbring). While access to technological resources and equipment has consistently increased (Inan & Lowther, 2010), studies show that limited technology integration continues to be a problem in many classrooms where computers and software are available (Hew & Brush, 2006; Lowther, Inan, Strahl, & Ross, 2008; Cuban, Kirkpatrick, & Peck, 2001) because it does not work properly (Clark, 2006; Lim &

Khine, 2006; Zhao, Pugh, Sheldon, & Byers, 2002) or because it is not useful (Norris, Sullivan, Poirot, & Soloway, 2003).

Teacher training. In addition to technology hardware and software issues, there also exists the concern that the majority of teachers have had little or no computer education training with up-to-date equipment and adequate resources and are therefore unable to select the most appropriate programs and applications to meet individual students' needs (Hasselbring, 2014; Shin et al., 2017; Snoeyink & Ertmer, 2001; Williams, Coles, Wilson, Richardson, & Tuson, 2000). Many school districts do not employ facilitators to aid in effectively using technology to promote higher-level thinking activities (Reinhart et al., 2001). Even in the districts which do employ technology facilitators, these specialists have limited training regarding accessibility of technology (Wisdom et al.). As a result, few schools are adequately prepared for highly effective technology integration.

Time. Research findings from many studies (Butzin, 2001; Cuban et al., 2001; Dawson, 2008; Kale & Goh, 2014; Karagiorgi, 2005; Lyons, 2007; O'Mahony, 2003) indicate that lack of time to plan and prepare is one of the most commonly reported barriers for technology integration. Teachers need hours to plan and prepare multimedia projects and those teachers who are willing to spend the extra time eventually become so overwhelmed by the lack of personal time that they eventually resign (Hew & Brush, 2007).

Internal (second-order barriers). Second-order barriers to technology integration are those barriers which are internal to the teacher (Ertmer, 1999; Ertmer et al., 2012;

Francom, 2016). These barriers include teacher attitudes and beliefs about the importance of technology integration (Ertmer, 1999).

Teacher attitudes and beliefs. Teachers' beliefs (suppositions) about technology integration is what determines their attitudes (specific feelings) (Bodur, Brinberg, & Coupey, 2000). These attitudes and beliefs about the usefulness and difficulty of technology can directly impact whether or not teachers choose to integrate technology within instruction (Inan & Lowther, 2010; Ottenbreit-Leftwich, Glazewski, Newby, & Ertmer, 2010; Vannatta & Fordham, 2004). Teachers who view technology as merely a means to keep students occupied do not see the relevance of technology in the instruction (Ertmer, Addison, Lane, Ross, & Woods, 1999). These teachers do not value technology integration and tend to place a priority on other subjects and skills - only using technology as a reward for finishing an assignment (Ertmer et al.).

Technology Integration in Elementary Mathematics

New and engaging educational technology programs and applications are continually being introduced to enhance learning within the elementary mathematics classroom. Technology-supported instruction, mathematics skill review games, and virtual manipulative applications promise considerable potential for teaching, interactive learning opportunities, collaboration, and creative expression (Johnson, Levine, Smith, & Haywood, 2010).

Technology-supported instruction. Technology supported instruction is a means of teacher-directed instruction which provides students with a variety of technology-centered supports. Such technologies enable learners to develop creativity through interactive learning opportunities, including presentation software, instructional platforms, and formative assessments.

Presentation software. To effectively integrate technology in the elementary mathematics instruction, students should be able to actively engage with technology to solve problems (Eskicioglu & Kopec, 2003; Goodwin, 2008; Liu, 2013; Wentworth & Monroe, 2011). Presentation software such as interactive whiteboards, PowerPoint presentations, ActivInspire software, and Prezi presentations should enhance instruction rather than be merely a tool used to create lessons (Wentworth & Monroe). Students who receive multimedia-based instruction are more engaged in the lesson and consistently outperform their counterparts (Malik, 2011; Milovanovic, Takaci, and Milajic, 2011). According to researchers (Ok & Bryant, 2016; Williams et al., 2000), the use of web-enhanced mathematics instruction has received widespread endorsement from agencies such as the NCTM as it provides more opportunities for student practice, feedback, and conceptual development. Technology-based mathematics instruction keeps students engaged in the learning process and fosters conceptual understanding (Williams et al.). As student engagement increases, the motivation for continued learning increases as well.

Connell and Abramovich (2016) offer pedagogical suggestions for incorporating technology in the elementary mathematics classroom. The researchers suggest that mathematical content should take precedence over technology methods, teachers should integrate technology effectively, and technology should be used to confirm thinking, not replace it (Connell & Abramovich). When these strategies are incorporated, students are able to explore the tools of technology to more effectively develop their ideas, and substantiate their own learning (Connell & Abramovich).

Despite the many benefits of presentation software, research findings by Eskicioglu and Kopec (2003) discuss several shortcomings of utilizing this technology for lesson delivery. Eskicioglu and Kopec explain that the students in their study were distracted by other websites, games, and attractions that did not pertain to the lesson. As a result, the students were disengaged in the lesson and the teachers felt that they did not have the full attention of their students. In addition, students and teachers experienced sporadic network connectivity and technical issues with both computers and printers, all of which took time away from instruction. Eskicioglu and Kopec also reported issues with visibility, screen size, and noise of the LCD projector.

Instructional platforms. Instructional platform sites, (also referred to as learning management systems and course management systems such as Edmodo.com and Google Classroom), allow teachers a safe and efficient means for disseminating assignments, sharing video tutorials, offering individualized review activities, and providing differentiated instruction. These blended learning platforms allow students to safely access a wide variety of sites where they can review lessons, practice skills, collaborate with peers, or investigate topics of interest. In the mathematics classroom, instructional platforms can be used to provide differentiated instruction by assessing student skill level and then providing instruction based on that level (Ysseldyke, Tardrew, Betts, Thill, & Hannigan, 2004). These instructional platforms also provide personalized goal setting, practice time, and immediate feedback (Ysseldyke et al.). Such instructional sites also allow students an opportunity receive valuable feedback from other students, teachers, parents, or outside experts by posting their work in a multimedia format for others to view and provide feedback (Pitler et al., 2012).

Communication and collaboration software platforms provide a way for students and teachers to interact in a timely manner. Research shows that collaboration software increases problem solving, critical thinking, written communication skills, in-depth writing and improves enjoyment, motivation, and learning (Gomez, Wu, & Passerini, 2010; Marjanovic, 1999; Prinsen, Volman, Terwel, & Vandeneeden, 2009). Classroom conversations are easy to maintain and provide a way for students to easily interact by sharing their mathematical understandings and explaining concepts. Teachers can effectively facilitate whole group or small group discussions where every student is an active participant (Pitler et al., 2012). Also, in the mathematics classroom, these instructional platforms offer formative and summative assessments that provide feedback so that the teacher can adjust instruction and provide additional supports and activities for students, as needed (Whetstone, Clark, & Flake, 2014). Instructional platforms allow students to safely receive assignments, blog their ideas, learn from others, share their work, and receive constructive feedback from both their teacher and their peers.

Formative assessments. Technology-based formative assessment tools such as online surveys and polling devices (i.e., Quizizz, Socrative, GoFormative, QuizletLive, ClassFlow, Kahoot!) are beneficial as they provide immediate feedback which promotes student learning (Zhang et al., 2014) during the course of instruction. This information can aid the teacher in assessing students' knowledge so that he or she can then adjust instruction to meet the specific needs of the students (Baroudi, 2007; Hodgen, 2007; Pitler et al., 2012). Such tools enable the teacher to engage and motivate the learner while assessing the needs of each student.

According to research findings by Klute, Apthorp, Harlacher, and Reale (2017), in a large-scale evaluation of education programs and practices, formative assessments were found to be very effective in elementary mathematics achievement. Formative assessment has the greatest positive effect on overall student achievement, especially in the area of mathematics (Goss, Hunter, Romanes, & Parsonage, 2015; Klute et al.; Polly et al., 2017; Wiliam, 2007). Klute et al. explain that students who participated in formative assessments scored higher on measures of academic achievement than those who did not. In addition, this study indicates that formative assessments in mathematics had more substantial effect than similar assessments in reading and writing (Klute et al.). Therefore, student-directed, teacher-directed, or computer-directed formative assessments are extremely beneficial in providing useful feedback and guidance to improve mathematical understanding (Whetstone et al., 2014; Ysseldyke et al., 2004).

Skill review. Multiplication skills review activities provide interactive opportunities for students to practice basic mathematics skills such as multiplication. Two of the most popular skill practice activities are technology-based multiplication drills and games.

Drills. Multiplication drills are often used in elementary classrooms to practice and assess fluency of basic facts. For example, Cumming and Elkins (1999) research findings indicate that multiplication strategy instruction alone does not build fluency. Instead, timed drills are an essential tool for multiplication instruction and must be taught in conjunction with the strategies (Cumming & Elkins; Woodward, 2006). According to Brownell and Chazal's (1935) research, however, multiplication drills do not have a place in the initial learning process, rather drills are "exceedingly valuable for increasing, fixing, maintaining and rehabilitating efficiency otherwise developed" (p. 26). Studies (Brownell & Chazal; Cumming & Elkins; Galfano, Rusconi & Umilta, 2003; Isaacs & Carroll, 1999; Witt, 2010) indicate that many elementary mathematics teachers tend to use drills for initial learning and then mistakenly assume students can maintain fluency without ongoing practice. However, researchers (Binder, 1996; Brownell & Chazal; Burns, 2005; Wong & Evans, 2007) found that teachers must continue to place an emphasis on the continued practice of basic multiplication facts after a student achieves fluency in order to maintain what they have learned. Consequently, repeated practice and drills after initial learning of basic multiplication facts are necessary components to maintaining fluency.

Games. Instructional interactives such as games that provide immediate feedback prove to be both educationally stimulating and entertaining. Many educational games encourage 21st century skills such as solving problems, collaborating with other players, and planning in a nonjudgmental environment (Pitler et al., 2012). For example, a struggling learner can repeat lessons and practice a skill as many times as necessary without fear that the instructor has grown frustrated (Pitler et al.). The endless options for educational applications in the iTunes store provide limitless options for online learning and educational gaming in the area of mathematics.

Much research on the effectiveness of tablet-based mathematics games has found that such games improve learning, student performance, and attitudes towards mathematics (Ching, Stampfer, Sandoval, & Koedinger, 2012; Ke & Grabowski, 2007; Shin, Sutherland, Norris, & Soloway, 2012). Few studies did not have similar findings (Carr, 2012; Ke, 2008a; Ke, 2008b). The mixed reviews are largely due to the quality of mathematics games used in the various research studies. To be effective, mathematics games should allow the learner to actively process the content, have an interesting context, clearly align learning goals with the game objectives, offer user-friendly challenges at appropriate difficulty levels, provide timely feedback, and scaffold instruction (Erhel & Jamet, 2013; Ke, 2008a; Shin et al., 2012; Young et al., 2012). In sum, mathematics games must be engaging, appropriately challenging, and standards-based in order to be an effective learning tool.

A study by Nusir, Alsmadi, Al-Kabi, and Sharadgah (2012) explored the impact of utilizing multimedia technologies (including educational games) on enhancing, or not, the effectiveness of teaching mathematics to primary students. One group of students was taught mathematics using traditional methods, while the experimental group was taught mathematics using programs with multimedia-enhanced methods. Results showed a positive impact on learning in that the technology-enhanced lessons were very effective in motivating students (Nusir et al.). The results also indicate that the experimental group significantly outperformed the traditional group as indicated by their (almost doubled) test scores (Nusir et al.). Clearly, the incorporation of educational games during mathematics instruction made a significant impact in this study.

Virtual manipulatives. Virtual manipulatives are visual models that the teacher can easily use to model mathematical thinking. Moyer et al. (2002) describes virtual manipulatives as interactive, web-based representations of physical objects used for constructing mathematical understanding. These instructional mathematics tools are easily accessible and can be used by students as they reason through mathematical problems (Shin et al., 2017). Studies show that students can use a variety of virtual representations (at the appropriate level) to represent their thinking to foster growth in conceptual understanding (Burris, 2013; Connell & Abramovich, 2016; Moyer-Packenham et al., 2013; Shin et al.). These virtual manipulatives can actually be used as an individualized learning accommodation for students with learning difficulties and enable all students to better understand abstract concepts (Shin et al.).

Clements and Sarama (2016) also note the importance of integrating the use of virtual manipulatives in mathematics instruction. The researchers (Clements & Sarama) noted that “a recent review of 66 studies found that the use of computer manipulatives raised a child from the 50th percentile to the 64th percentile” (2016, p. 89). Clements and Sarama attribute this positive effect on seven advantages of technology-based manipulatives: Virtual manipulatives bring mathematical ideas to conscious awareness, facilitate complete and precise explanations, support mental actions on objects, can change the nature of the shape by cutting apart virtual manipulatives (unlike concrete manipulatives), symbolize mathematical concepts, link concrete and abstract, and record and play students’ actions. As a result, the functionality of virtual manipulatives outweighs that of concrete manipulatives by far.

Connell and Abramovich’s (2016) research on virtual manipulatives in the elementary classroom clearly indicates that students must be developmentally ready to use abstract manipulatives to represent their thinking. Connell and Abramovich indicate that before using virtual manipulatives, sufficient time must be used with concrete, “real-world referents” so the learners have a strong understanding and can make the connection between the concrete and abstract representations (2016, p. 216). Consequently, teachers must be careful to select virtual manipulatives that match both the needs and experiences of their students in order to create meaningful learning experiences (Connell & Abramovich).

In Burris' (2013) study, he compared how third graders think mathematically when using virtual versus concrete base-ten blocks to learn place-value concepts. While students interacted with concrete and virtual manipulatives in much the same way, the researcher found that the "virtual models were advantageous to students as they generated nonstandard numbers more efficiently using technology" (2013, p. 235). Students were able to compose and decompose numbers more easily with the virtual base-ten blocks than with the concrete blocks. While both representations of base-ten blocks proved useful, the virtual blocks proved to have added benefits when constructing nonstandard representations of numbers (Burris). As a result of this study, Burris recommends a few considerations when deciding whether to incorporate concrete or virtual manipulatives: What is the purpose of the technology or virtual manipulative? How will students interact with this manipulative? How will students think mathematically with this manipulative? Therefore, virtual manipulatives should be used in mathematics instruction because they provide students an opportunity to interact with the numbers and foster opportunities to think mathematically.

Summary

This literature review regarding integrating technology in my elementary mathematics classroom will guide my action throughout this study. This action research study is similar to those presented in this literature review as it evaluates the implementation of technology integration with multiplication concepts (repeated addition, arrays, and decomposing numbers). This study is unique in that it uses virtual manipulatives and student think-aloud recordings to measure students' conceptual understanding of basic multiplication. The literature in this review is helpful in informing the current research and determining best practices of technology integration in the elementary mathematics classroom.

CHAPTER THREE

METHODS

Purpose Statement

The purpose of this action research was to evaluate the implementation of technology integration with multiplication concepts (i.e., repeated, arrays, and decomposing numbers) for struggling third grade students at FES in Lake County School District.

Research Questions

This action research was guided by two grand tour questions and three strategy-specific sub-question:

1. How and in what ways does technology integration with multiplication concepts impact student understanding?
    a. How do virtual manipulatives and student think-aloud self-recordings impact students' understanding of repeated addition?
    b. How do virtual manipulatives and student think-aloud self-recordings impact students' understanding of arrays?
    c. How do virtual manipulatives and student think-aloud self-recordings impact students' understanding of decomposing numbers?

3. How do students select and explain strategies for solving multiplication problems?

Research Design

In this study, I used action research to evaluate the implementation of technology integration with multiplication concepts (repeated addition, arrays, and decomposing numbers) using students in my own third grade classroom. This enabled me to determine multiplication strategies that most effectively enhance learning and conceptual understanding of my students. The results of this study will help guide my current and future teaching practices.

Action Research

Action research acts as a tool for teachers to study and understand their own students in order to improve the quality and effectiveness of their practice (Mertler, 2014). Mills (2014) defined action research as "any systemic inquiry conducted by teacher researchers, principals, school counselors, or other stakeholders in the teaching/learning environment to gather information about how their particular schools operate, how they teach, and how well their students learn" (p. 8).

Action research was essential to this study as it provided data that are persuasive and relevant, allowed immediate access to research findings, and challenged the intractability of educational reform (Mills, 2014). Unlike other research techniques, action research deals with problems and struggles in one's own classroom, making the findings both relevant and practical. It allowed me to identify specific problems within my classroom and then to conduct my own research in order to improve instruction. Action research is timely in that it allows the educator to start research as soon as he or she chooses and provides immediate results, enabling the educator to better understand and improve his or her practices (Mertler, 2014). This research design allows teachers the “opportunity to embrace a problem-solving philosophy and practice as an integral part of the culture of their schools” and “challenges the intractability of educational reform by making action research a part of the system rather than just another fad” (Mills, 2014, p. 16). As a result, teachers are able to examine their teaching practices through multiple lenses and are in effect, better able to identify and incorporate the best practices for their specific students.

To best gauge my own students’ learning and growth, I implemented an evaluation study with triangulation (Mertler, 2014) or convergent (Creswell, 2014) mixed method design using objective assessment data, non-parametric tests, and inductive thematic analysis. By using this approach, I was able to collect both qualitative and quantitative data to better understand the conceptual understanding of my students and compare different perspectives before, during, and after the learning takes place (Creswell). This enabled me to identify specific gaps in conceptual understanding as it pertains to the learning of basic multiplication facts for my third grade students. By better understanding their misconceptions, I was able to individualize my instruction to more accurately address each learning need.

Setting

This study focused specifically on students in third grade mathematics. I taught two classes of third grade mathematics with one class having seventeen students and the other eighteen students. My research was based on the students in these two particular classes.

In third grade mathematics, students work toward learning and developing fluency in zero through ten multiplication facts. In this study, I focused specifically on technology integration with multiplication concepts (i.e., repeated addition, arrays, and decomposing numbers).

The students in each class were grouped in tables of four. At each table, I assigned seating according to the following ability levels (as determined by MAP testing scores, summative assessments such as unit tests, and teacher observations): one high-performing student, two on-grade level students, and one student who performs below grade level in the area of multiplication. The seating is arranged in this way to enable students to effectively collaborate and help each other during group and partner work time (Vygotsky, 1978b). During twenty minutes of each ninety-minute mathematics block, I worked with the students who perform below-grade level (in multiplication) in a small group at the teacher table in my classroom.

At the beginning of each class, I spent approximately five minutes each reviewing homework and mathematics morning work (spiral review) before beginning the new lesson. I typically introduced the lesson to the whole class using either a five-minute BrainPOP Jr. video or other mathematics video clip, and I then used some type of real or virtual manipulative for approximately fifteen minutes to help students conceptualize their learning. Students had an opportunity to work collaboratively for about fifteen minutes to practice using manipulatives to solve multiplication problems in their workbooks. During the last fifteen minutes of class, I had students practice in leveled groups using tablet-based applications, real or virtual manipulatives, skill-based board games, flashcards, etc. to review the lesson of the day. During this time, I worked with the sample group to practice multiplication strategies (repeated addition, arrays, and decomposing numbers). My mathematics classes are very interactive and encourage collaboration. I strive to meet the individual needs of all learners by differentiating instruction in small groups and providing lessons that are compatible with many learner preferences.

Participants

I conducted this action research study in my third grade mathematics classroom. Two classes (one with seventeen students and the other with eighteen students) were involved in this study. These two intact classes were arranged by the principal based on past performance, gender, and ethnicity. As a result, the two classes are very similar in make-up, with similar numbers of high, average, and low-performing students. Four of these students also receive special education services.

Using initial Measures of Academic Progress (MAP) data and multiplication pre-test results, I identified ten struggling students in need of extra assistance to address gaps in learning and any misconceptions. These tests served as a measure of current academic performance in mathematics skills. Students who performed both in the 33rd percentile or lower on the Numbers and Operations section of the third grade mathematics MAP test and who also scored 33 % or lower (8 or less correct questions) on the teacher-made pre-test were selected for the sample.

The Northwest Evaluation Association (NWEA) provides the MAP test, which is a normative, computerized, and adaptive test where the difficulty of each question depends on how the student answered previous questions, for students in kindergarten through eleventh grade (NWEA, 2015). After completing the MAP test, students were assigned a Rasch Unit (RIT) scale score to reflect their performance level (NWEA, 2015). For grades 2-5, the possible RIT range for Numbers and Operations is “Below

161" to 230 (NWEA, 2015). The overall mean RIT score for the beginning of the year third grade mathematics MAP test is 190.4 (*SD* = 13.10), and by the end of the third grade, the mean mathematics RIT score is 203.4 (*SD* = 13.81). See Table 3.1 below for student status norms.

Table 3.1 *2015 MAP Mathematics Student Status Norms: Grade Three*

Begin-Year		**Mid-Year**		**End-Year**	
Mean	SD	Mean	SD	Mean	SD
190.4	13.10	198.2	13.29	203.4	13.81

Note. SD = standard deviation

Innovations

To address the needs of students in my sample group, I provided a variety of techniques (such as hands-on and virtual manipulatives, video clips, and games) and strategies (such as repeated addition, arrays, and decomposing numbers) to help students better understand the concept of basic multiplication. These techniques and strategies have been previously introduced in normal classroom instruction and were used to focus on intervention for students with low ability. By incorporating a variety of technology-based strategies in my instruction, students were empowered to construct their own learning and actively engage in the learning process (Guha & Leonard, 2002; Ok & Bryant, 2016; Pitler et al., 2012; Rave & Golightly, 2014). In mathematics, active engagement translates to the use of concrete and abstract tools to enable students a better conceptual understanding. Visual representations play an integral role in the way we develop mathematical concepts as learners, "moving from an operational or process view of a concept to a structural view (e.g. moving from multiplication as a process to multiplication as a static object, the properties of which can then be examined)" (Barmby et al., 2009, p. 223). By incorporating concrete and virtual manipulatives in mathematics instruction, students are better able to visualize the mathematical concepts, scaffold their understanding, and make sense of abstract concepts (Burris, 2013; Loong, 2014; Sowell, 1989; Suh & Moyer, 2008).

In this study, focus group students in both classes received the same treatment. Students used Chromebooks as a source for virtual manipulatives to aid in solving multiplication problems. Students had access to multiple websites and applications that provided user-friendly virtual manipulatives, such as base-ten blocks and number racks, for repeated addition. Students used virtual tiles, shapes, and counters for creating arrays, and students used partial product finders for decomposing numbers. These virtual manipulatives enabled students to use technology sources to help them reason through and successfully solve multiplication problems (Burris, 2013; Loong, 2014; Sowell, 1989; Suh & Moyer, 2008). Students also used their Chromebooks to access numerous applications and games where they practiced using repeated addition, arrays, and decomposing numbers. Lastly, students used Chromebooks to create student think-aloud recordings where they videoed themselves explaining their strategies while using virtual manipulatives for solving multiplication problems. The think-aloud recordings enabled me to identify and address any conceptual understandings and misconceptions that existed (Basaraba et al., 2013; Ericsson & Simon, 1993; Gorin, 2007).

It is essential for students to have access to multiple problem-solving strategies to enable them to solve more challenging problems. When students have certain key representations for a concept in their understanding and are able to reason between new and other representations that they already have, greater restructuring of student understanding will result (Barmby et al., 2009). Teaching a variety of multiplication strategies enables students to make connections between prior knowledge and new knowledge and develop a deeper conceptual understanding.

In the past, I have used a variety of such activities to guide my students to a better conceptual understanding of multiplication. I have used whole/small group instruction, partner work, real and virtual manipulatives, games, foldables, video clips, etc. to provide a wide range of learning activities to meet the needs of all types of learners. While I have used small-group instruction periodically in the past, I incorporated it on a regular basis with this group to provide much more frequent remediation.

In this research, I focused exclusively on students who are low-achieving in basic multiplication and worked extensively in small groups to improve their understanding of multiplication using three particular strategies (i.e., repeated addition, arrays, and decomposing numbers). I have selected these three strategies on which to focus to provide students with a variety of tools for solving basic multiplication facts. I specifically chose these three strategies for this study as they represent a progression of conceptual understandings. In addition, virtual manipulatives can be used with each strategy to help students visualize the concepts. After students learned each strategy, they were then able to select the strategy that works best for him or her and use that strategy(ies), as needed.

Repeated Addition

Third grade students must conceptualize multiplication as repeated addition by visually representing equal groups added together (Wall et al., 2015). This strategy is often the most effective strategy for teaching the concept of multiplication to younger students (Heege, 1985; Sherin & Fuson, 2005; Wells, 2012). I introduced this concept with concrete manipulatives to allow students a chance to build multiplication problems so that they can see what multiplication looks like. I used connecting cubes, tiles, and base ten blocks to help my students solve multiplication using repeated addition. Concrete manipulatives such as these build conceptual understanding because they help students represent abstract concepts (Loong, 2014). Students arranged the manipulatives according to the problem, and then counted to see how many manipulatives were used to solve the problem. After students seemed confident using the concrete manipulatives, I had them practice the same strategy with more abstract virtual manipulatives. Students also used online applications www.splashmath.com and www.sheppardsoftware.com to practice repeated addition. These strategies laid the groundwork for building a conceptual understanding of multiplication.

Arrays

Arrays are powerful tools for learning multiplication because they illustrate the multiplication fact family (Day & Hurrell, 2015). These visual representations also help students understand the multiplicative properties of commutativity (Barmby et al., 2009; Charles & Duckett, 2008; Day & Hurrell; Denham, 2013; Hurst & Hurrell, 2017; Jacob & Mulligan, 2014; Kling & Bay-Williams, 2015) and distributivity (Barmby et al.; Day & Hurrell; Hurst & Hurrell; Wall et al., 2015). My students created arrays using both concrete and virtual manipulatives. Using a combination of both physical and virtual manipulatives enables students to make much greater gains than using only one type of manipulative alone (Terry, 1995). I had my students create arrays by placing flat, round tiles on a gridded mat in rows and columns to match the given problem. Students then counted the tiles to determine the product. Students also used virtual manipulatives to create arrays online using their Chromebooks. After practicing with manipulatives, students also drew arrays on individual dry-erase boards and in their notebooks. They were able to both build and draw mathematical representations to help them find the product. Students were then able to review and practice this strategy using online games where they had to build arrays.

Decomposing Numbers

Decomposing numbers is an invaluable tool for multiplying more difficult numbers and will eventually lead students to an understanding of the distributive property (Baroody & Coslick, 1998; Benson et al., 2013; Cumming & Elkins, 1999; Gerstan & Chard, 1999; Kilpatrick et al., 2001; Kinzer & Stanford, 2014; Sowder, 1992). I first showed students how to decompose numbers using arrays. For instance, with an array for 7 x 2, I reminded students how the number seven can be broken apart into 5 + 2. As a result, I was able to break apart the group of seven tiles in my array into a group of five tiles and a group of two tiles. Then, I showed them how they can find the partial products, two sets of five and two sets of two, to solve the problem. We practiced using arrays to decompose numbers, and then practiced drawing it on our dry-erase boards and paper. The concrete manipulatives were essential in helping students understand such an abstract concept. By progressing from concrete to abstract, decomposing numbers became much easier for students to understand. Students also watched teacher-assigned YouTube videos and BrainPOP Jr videos for extra guidance in decomposing numbers.

Description of Data Sources

I used both qualitative and quantitative measures to collect data. The assessments were teacher-made and allowed students to use virtual manipulatives to solve multiplication problems using a given strategy. The assessments clearly indicate which strategy students should use for solving each problem. This enabled me to accurately identify how well each student understands the specific strategies. Each of the assessments addressed the following third-grade State College and Career Ready Standards for Mathematics:

- 3.ATO.1 Use concrete objects, drawings and symbols to represent multiplication facts of two single-digit whole numbers and explain the relationship between the factors (i.e., 0 – 10) and the product;
- 3.ATO.3 Solve real-world problems involving equal groups, area/array, and number line models using basic multiplication and related division facts. Represent the problem situation using an equation with a symbol for the unknown;
- 3.ATO.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers when the unknown is a missing factor, product, dividend, divisor, or quotient.

The instructional objectives for this research project included:

1. The learner will be able to use concrete objects, drawings, and symbols to represent multiplication facts of two single-digit whole numbers and explain the relationship between the factors (i.e., 0 – 10) and the product with 90 % accuracy.
2. The learner will be able to solve real-world problems involving equal groups, arrays, and decomposing numbers with 90% accuracy.
3. The learner will be able to determine the unknown whole number when the unknown is a missing factor or product with 90% accuracy.

Table 3.2 displays the alignment of research questions with the data sources.

Table 3.2 *Data Sources*

Research Questions	**Data Sources**
1. How and in what ways does technology integration with multiplication concepts impact student understanding?	
a.) How do virtual manipulatives and student think-aloud self-recordings impact students' understanding of repeated addition?	• Teacher-Made Pre-Post tests with virtual manipulatives • Think-aloud self-recordings • Think-aloud interview
b.) How do virtual manipulatives and student think-aloud self-recordings impact students' understanding of arrays?	• Teacher-Made Pre-Post tests with virtual manipulatives • Think-aloud self-recordings • Think-aloud interview
c.) How do virtual manipulatives and student think-aloud self-recordings impact students' understanding of decomposing numbers?	• Teacher-Made Pre-Post tests with virtual manipulatives • Think-aloud self-recordings • Think-aloud interview

Research Questions	Data Sources
2. How do students select and explain strategies for solving multiplication problems?	• Think-aloud self-recordings • Think-aloud interview

Multiplication Strategies Pretest-Posttest

The teacher-made pre-post tests consisted of 24 multiplication problems separated into three sections (see Appendix A and B). In Part A of both tests, students were asked to use repeated addition to solve each problem. In Part B, students were asked to solve by creating arrays, and in Part C, students were asked to solve by decomposing numbers. In each of Parts A, B, and C, students were given a variety of 0-12 basic multiplication problems to solve. In each section, the student solved for the product in four problems, solved for the multiplicand in three problems, and solved one multiplication word problem (McGraw-Hill, 2013). Table 3.3 displays the alignment of word problems to the corresponding research questions.

Table 3.3 *Pre-Post Test Word Problem Alignment*

Research Questions	Word Problems
1. How and in what ways does technology integration with multiplication concepts impact student understanding?	
a.) How do virtual manipulatives and student think-aloud self-recordings impact students' understanding of repeated addition?	• There are 5 spiders. Each spider has 8 legs. How many legs are there in all? (Use repeated addition and virtual manipulatives to solve).

Research Questions	Word Problems
b.) How do virtual manipulatives and student think-aloud self-recordings impact students' understanding of arrays?	• Lindsay made a poster to display her photos. She made 2 rows with 4 photos in each row. How many photos did Lindsay display? (Draw an array using virtual manipulatives to solve).
c.) How do virtual manipulatives and student think-aloud self-recordings impact students' understanding of decomposing numbers?	• Calvin puts his books on shelves in his room. How many books does Calvin have if he puts 10 books on each of 5 shelves? (Decompose numbers and use virtual manipulatives to solve).

The students also used virtual manipulatives to complete both the pretest and posttest. For instance, when students were assessed on their ability to use repeated addition, they had to represent the problem using virtual manipulatives to demonstrate their conceptual understanding (Loong, 2014). When I assessed students on their understanding of arrays, I had them select a virtual manipulative with which they built the arrays to solve the given problems. Students also had to use virtual manipulatives to demonstrate how to find the products by decomposing numbers. On the teacher-made pre- and posttests, each of the 24 questions counted one point. The maximum point value was 24.

I obtained quantitative data from objective assessments with virtual manipulatives. These tests included teacher-made pre-post multiplication tests given before and after the focus group remediation. The teacher-made test was reviewed by two other elementary math teachers and a local university professor of education before it was administered to students. I used a Wilcoxon Signed-rank test to determine the effectiveness of technology integration with multiplication concepts. Statistical significance was calculated with an alpha significance level of 0.05.

Student Think-Aloud Protocol

In this study, I used both formative and summative think-aloud self-recordings and culminating teacher-interview think-alouds as a type of summative assessment. Immediately before this study began, I modeled the think-aloud process for students and walked them through the self-recording process. Students practiced recording themselves during class time as they explained their thinking for several problems. This enabled students the opportunity to become comfortable with the recording and think-aloud process before the study began. During this study, students had to explain their thinking while demonstrating with virtual manipulatives how to solve given multiplication problems using specific strategies (repeated addition, arrays, and decomposing numbers). After students recorded their think-alouds, I transcribed students' responses in order to complete a thorough analysis of the data.

To gauge conceptual understanding of each strategy, I had students record their own explanations of how they use repeated addition, arrays, and decompose numbers to help them solve the multiplication problems. Eliciting self-explanations from students greatly improves their learning and their understanding (Barmby et al., 2009). This enabled me to better understand their thinking and allowed me to address misconceptions, as needed.

I used think-aloud recordings to obtain qualitative data for this study to provide a more in-depth snapshot of my students' conceptual understanding of basic multiplication. This enabled me to specifically address any misconceptions and correct thinking, as needed. Students self-recorded their thinking throughout the study as a means of formative assessment. For a summative assessment, I met with students individually to assess each child's learning by asking an icebreaker question as suggested by Creswell (2014), followed by conceptual understanding questions: 1) How can I solve 4 x 3 using repeated addition? (Show your thinking using virtual manipulatives). 2) How can I use an array to solve 6 x 4? (Show your thinking using virtual manipulatives). 3) How can I decompose 7 x 3 to help me solve the problem? (Show your thinking using virtual manipulatives). 4) Which multiplication strategy do you prefer and why? (Show your thinking using virtual manipulatives). In Table 3.4, the think-aloud questions are aligned to specific research questions. (See Appendix C for full think-aloud protocol and additional questions to build rapport). After recording the think-alouds, I transcribed the videos to obtain written data.

Table 3.4 *Think-Aloud Question Alignment*

Research Questions	**Think-Aloud Questions**
1. How and in what ways does technology integration with multiplication concepts impact student understanding?	
a.) How do virtual manipulatives and student think-aloud self-recordings impact students' understanding of repeated addition?	1) How can I solve 4 x 3 using repeated addition? (Show your thinking using virtual manipulatives).
b.) How do virtual manipulatives and student think-aloud self-recordings impact students' understanding of arrays?	2) How can I use an array to solve 6 x 4? (Show your thinking using virtual manipulatives).

Research Questions	Think-Aloud Questions
c.) How do virtual manipulatives and student think-aloud self-recordings impact students' understanding of decomposing numbers?	3) How can I decompose 7 x 3 to help me solve the problem? (Show your thinking using virtual manipulatives).
2. How do students select and explain strategies for solving multiplication problems?	4) Which multiplication strategy do you prefer and why?

Procedures & Timeline

The timeline for the procedures for this research is as follows: Phase 1: Participant Identification, Phase 2: Data Collection and Phase 3: Data Analysis. Each phase is described in detail below. Table 3.5 is included to detail the timeline of all the procedures.

Table 3.5 *Timeline of Participant Identification, Data Collection, & Data Analysis*

Phase	Expectation	Time Frame
Phase 1: Participant Identification	1. Mathematics MAP test 2. Teacher-Made Multiplication Pretest 3. Identify Participants 4. Contact Participants 5. Review Consent Form	2 weeks
Phase 2: Data Collection	1. Small-Group Multiplication Instruction Using Virtual Manipulatives 2. Multiplication Posttest 3. Student Think-Aloud Self-Recordings 4. Think-Aloud Interviews with Teacher	6 weeks (2 weeks per strategy)

Phase	Expectation	Time Frame
Phase 3: Data Analysis	1.Transcribe Student Think-Aloud Interviews 2. Wilcoxon Signed Rank Test (Repeated Addition) 3. Constant Comparative Method (Repeated Addition) 4. Wilcoxon Signed Rank Test (Arrays) 5. Constant Comparative Method (Arrays) 6. Wilcoxon Signed Rank Test (Decomposing Numbers) 7. Constant Comparative Method (Decomposing Numbers) 8. Constant Comparative Method (Think-Aloud Interviews)	5 weeks

Phase 1: Participant Identification

Participant identification for this study began in the spring of 2018 using the selection criterion identified earlier (mathematics MAP test and teacher-made multiplication pretest). Students who performed both in the 33rd percentile or lower on the Numbers and Operations portion of the third grade mathematics MAP test and who also scored 33 % or lower (8 or less correct questions) on the teacher-made multiplication pretest were invited to participate in this study. A total of ten students qualified and participated in this study.

Phase 2: Data Collection

I met with the focus group daily to provide remedial multiplication instruction using each of the following strategies: repeated addition, arrays, and decomposing numbers. In addition to the whole-group instruction and partner work that students receive daily during the ninety-minute-long mathematics class, I also met with the selected students for twenty minutes each day to provide intensive remediation with the indicated multiplication strategies. I taught students how to use virtual manipulatives to derive their answers and improve their conceptual understanding of basic multiplication (Burris, 2013; Loong, 2014). Students used virtual manipulatives and teacher-selected multiplication applications on their Chromebooks for guided and independent practice of each skill. Students self-recorded their thinking once each week to formatively assess conceptual understanding of multiplication. After six weeks of small-group instruction, students completed the teacher-made multiplication posttest. I then recorded student think-aloud (summative) interviews where students explained their reasoning for how they solved multiplication problems (Charters, 2003). These interviews provided key insight into students' conceptual understanding of multiplication. I also kept field notes to record my self-reflections and to document my observations regarding students' growth. As a result, I was better able to track the development in my students' understandings and address any specific misunderstandings that existed.

Phase 3: Data Analysis

After completing student think-aloud interviews, I transcribed each recording. I then analyzed each separate section (repeated addition, arrays, and decomposing numbers) of the teacher-made pre-post test data using a non-parametric Wilcoxon Signed-ranks test of related samples. I also determined the alpha levels for the tests. In addition, I used the constant comparative method (Creswell, 2014) to analyze each section of the teacher-made pre-post tests and the student think-aloud interviews.

Data Analysis Methods and Representation

This study necessitated an evaluative mixed-methods approach using objective assessment data with non-parametric test of related samples and constant comparative method (Creswell, 2014). Table 3.6 shows the alignment of research questions with the data sources and data analysis methods.

Table 3.6 *Research Questions, Data Sources, and Data Analysis Methods*

Research Questions	**Data Sources**	**Data Analysis Methods**
1. How and in what ways does technology integration with multiplication concepts impact student understanding?		
a.) How do virtual manipulatives and student think-aloud self-recordings impact students' understanding of repeated addition?	● Teacher-Made Pre-Post tests with virtual manipulatives ● Think-aloud/recording	● Wilcoxon Signed-ranks test ● Constant Comparative Method
b.) How do virtual manipulatives and student think-aloud self-recordings impact students' understanding of arrays?	● Teacher-Made Pre-Post tests with virtual manipulatives ● Think-aloud/recording	● Wilcoxon Signed-ranks test ● Constant Comparative Method
c.) How do virtual manipulatives and student think-aloud self-recordings impact students' understanding of decomposing numbers?	● Teacher-Made Pre-Post tests with virtual manipulatives ● Think-aloud/recording	● Wilcoxon Signed-ranks test ● Constant Comparative Method
2. How do students select and explain strategies for solving multiplication problems?	● Think-aloud/recording	● Constant Comparative Method

Qualitative Data

Students self-recorded their think-aloud assignments. Within the following four hours, I transcribed these videos to ensure accuracy of data (in the event of recording error). Student think-aloud transcriptions indicated each student's conceptual understanding of multiplication. I employed the constant comparative method (Creswell, 2014) to better understand students' performance. At the end of the innovation, I asked each student in the focus group the four think-aloud interview questions to discuss multiplication strategies (see Appendix C). I recorded each student's answers as he or she verbally responded. The responses to these questions also demonstrated whether each student understands how to solve multiplication problems using repeated addition, arrays, and decomposing numbers.

After transcribing and reviewing data, I coded the data into categories or "chunks" using *in vivo* terms (Creswell, 2014, p. 247). I separated the coded data into two sets: data collected early on in the innovation and data collected at the end of the innovation. For each set of data, I color-coded the categories and grouped like color-codes in a concept map (See Figures 3.1 and 3.2). This enabled me to identify themes and determine if overlapping themes exist (Creswell). These steps were ongoing in order to refine and understand emerging themes and how they interrelate (Creswell). I later merged both sets of data into one concept map (See Figure 3.3). The resulting themes include students' conceptual understandings, students' conceptual misunderstandings, and students' correct methodology with careless errors.

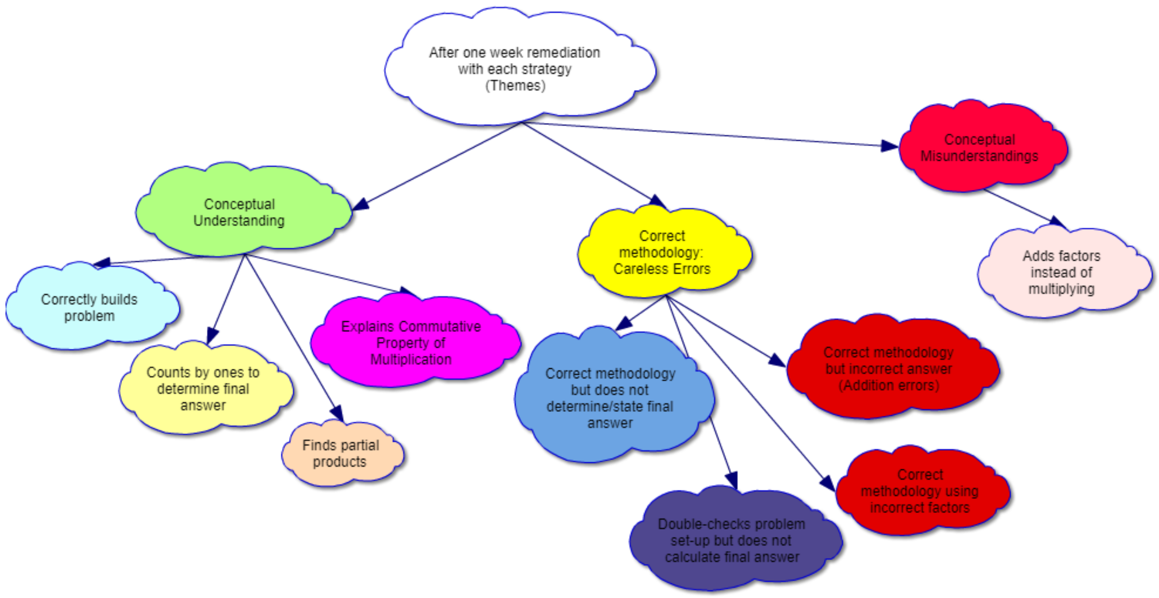

Figure 3.1. Emerging themes after one week of innovation.

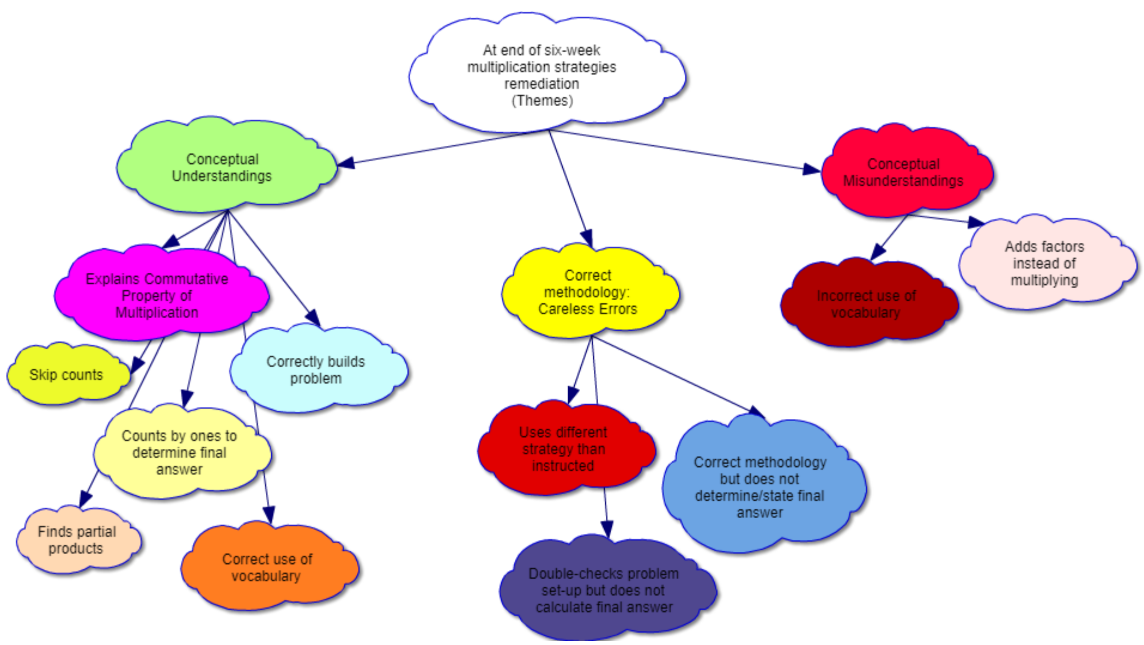

Figure 3.2. Emerging themes at the end of the innovation.

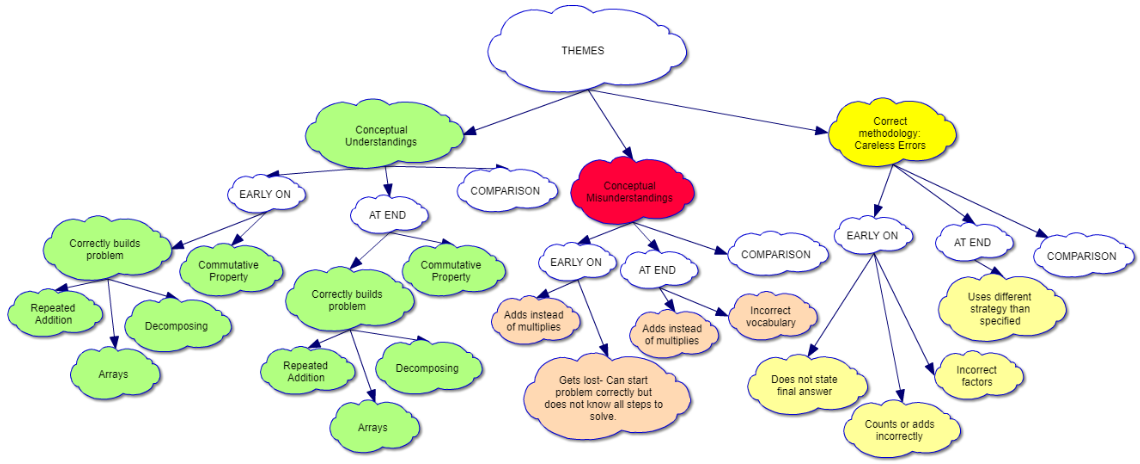

Figure 3.3. Overall themes that emerged from student think-aloud recordings

Quantitative Data

Teacher-made pre-post tests using virtual manipulatives provided data before and after the research study. I used a Wilcoxon Signed-rank test to determine the effectiveness of technology integration with multiplication concepts. Statistical significance was calculated with an alpha level of 0.05.

Representation

I represented my findings using narrative text through themes and thick, rich description (Merriam, 1998; Mertler, 2014). In this descriptive narrative, I included assertions and supporting evidence. In a table, I also displayed themes, theme-related components, and assertions collected from observations and student think-aloud interviews.

Rigor & Trustworthiness

Rigor and trustworthiness refer to how precisely and accurately the researcher has measured what he or she intended (Mertler, 2014). Validity and reliability are measures of rigor for trustworthiness in quantitative designs; however qualitative designs have other methods (Creswell, 2014; Mertler). The strategies for rigor and trustworthiness in this study include prolonged exposure; thick, rich description; triangulation; member checking; peer debriefing; and audit trail (Creswell; Mertler).

Prolonged Exposure

Prolonged exposure allows the researcher to become immersed in the study's setting, allowing the researcher to get to know participants and test any perceptions that may exist (Mills, 2014). By continuously observing and interacting with my students on a daily basis, I was able to identify patterns in their conceptual understandings of multiplication and was able to more thoroughly understand misconceptions that exist. By participating in this reflective action research study, I had the unique insider perspective of the happenings within my classroom (Herr & Anderson, 2005). I actively listened to and interacted with my students to gain valuable insight into their thinking processes. This enabled me to better understand the strengths and weaknesses in my students' conceptual understanding of basic multiplication so that I can more effectively bridge any gaps that may exist.

Thick, Rich Description

Merriam (1998) explains that thick, rich descriptions are vital to research because they allow the reader to determine how closely their own situations match the research and whether or not the results can be transferred (p. 211). In this study, students' think-aloud recordings and pre- and post-test data using virtual manipulatives were analyzed and described in detail to reveal students' conceptual understanding of multiplication strategies. The reader can then determine if the findings of this study are applicable to other classrooms.

Triangulation

To increase trustworthiness of a study, Creswell (2014) argues that the researcher should examine data from multiple sources and different perspectives in order to establish themes. In this study, I used methodological triangulation with mixed methods to determine if any themes exist. I triangulated data sources by interviewing students to obtain a rich understanding of students' ability levels (Shenton, 2004). From these interviews, I incorporated verbatim quotes and made specific observations to analyze and inform my research. I also examined and compared the qualitative and quantitative data to determine if any similar findings and correlations exist.

Member Checking

Member checking involves the sharing of data with participants in order to ensure accuracy (Mertler, 2014). The researcher should share any notes, interview transcripts, observer's comments, etc. with the participants to ensure that their thoughts and ideas are represented accurately (Mertler). I read my notes and student think-aloud interview transcripts to the students in my study in order to ensure accuracy. Also, I had another mathematics educator review the think-aloud recordings to assist in gaining insight to students' conceptual understanding.

Peer Debriefing

According to Mills (2014), peer debriefing is essential as it allows researchers to "test their growing insights" through interactions and collaborations with colleagues and other professionals. Peer debriefing allows the researcher to obtain multiple perspectives from expert sources which will act as an external auditor and help the researcher identify any holes or inconsistencies within the research. My dissertation chair and committee acted as my peer debriefing team and reviewed my decisions in order to provide insight throughout my research study.

Audit Trail

Lastly, Mills (2014) explains that an audit trail enables an external auditor to "examine the processes of data collection, analysis, and interpretation" (p. 116). By providing artifacts such as memos, researcher's journal, field notes, photographs, video recordings, etc., the researcher can enable the auditor to better understand decisions made about the research. I documented my decisions in a researcher's journal. The notes in my journal helped me to identify categories, codes, and themes within my data. This journal allowed me to organize my data. As a result, I was better able to justify changes in methods due to documentation of my observations.

Plan for Sharing & Communicating Findings

I plan to share and communicate my research findings with multiple audiences. I will share individual findings with the student participants at the end of the study. I will informally share my overall findings and implications for teaching with my principal and district mathematics coach. I will also discuss my research findings at a school-level professional development meeting for all teachers of mathematics and with a district-level administrator.

On a more formal level, I plan to present my findings in a poster session at an annual research conference held by a state organization such as State Educators for Practical Use of Research or the State Association for Educational Technology. I will also submit my study for possible publication in a relevant academic journal. When presenting my findings, I will protect students' identities by referring to participants using pseudonyms. I will not include any other identifying information that would compromise confidentiality.

CHAPTER FOUR

FINDINGS AND INTERPRETATIONS

The purpose of this action research was to evaluate the implementation of technology integration with multiplication concepts (i.e., repeated addition, arrays, and decomposing numbers) for struggling third grade students at FES in Lake County School District. It is expected that the findings of this study will provide insight regarding the impact of virtual manipulatives in the development of students' conceptual understanding of multiplication. This chapter presents findings obtained from both quantitative measures (i.e., teacher-made pre- and post-tests with virtual manipulatives) and qualitative measures (i.e., student think-aloud self-recordings and student think-aloud interviews). Data collection was guided by two grand-tour questions and three strategy-specific sub-questions:

1. How and in what ways does technology integration with multiplication concepts impact student understanding?
    a. How do virtual manipulatives and student think-aloud self-recordings impact students' understanding of repeated addition?
    b. How do virtual manipulatives and student think-aloud self-recordings impact students' understanding of arrays?
    c. How do virtual manipulatives and student think-aloud self-recordings impact students' understanding of decomposing numbers?

2. How do students select and explain strategies for solving multiplication problems?

Part One of this chapter reports the quantitative results and findings obtained from student pre- and post-tests. Part Two of this chapter identifies and explains three common themes that emerged from qualitative data sources.

Part One: Quantitative Data

Pretest-Posttest

The teacher-made pre- and post-tests allowed students to use virtual manipulatives to solve multiplication problems using a given strategy. These assessments provided quantitative data to clearly measure students' conceptual understanding for each of the three multiplication strategies: repeated addition, arrays, and decomposing numbers. The assessments indicated the specific strategy to use for each problem, which enabled me to accurately identify how well each student understands the specific strategies. Students used the website https://www.mathlearningcenter.org/resources/apps to access virtual manipulatives using the Numbers Racks, Pattern Shapes, and Partial Product Finder applications. (Students used the Number Racks application for solving repeated addition problems, Pattern Shapes for solving arrays, and Partial Product Finder for decomposing numbers).

Descriptive statistics. Descriptive statistics of the Multiplication Pre-Post Test scores are recorded in Table 4.1. The total number of questions in each section (i.e., repeated addition, arrays, and decomposing numbers) of the pre-posttests was 8.0. There was a grand total of 24 questions per test. Pretest means range from 0.4 to 2.0. Posttest means range from 6.6 to 7.0. The highest pretest mean was 2.0 (arrays), and the highest posttest mean was 7.0 (repeated addition).

Table 4.1 *Multiplication Pre-Post Test Scores (n=10)*

	Pretest		**Posttest**	
Multiplication Strategy	Mean (SD)	Median	Mean (SD)	Median
Repeated Addition	1.3 (2.83)	0	7.0 (0.82)	7
Arrays	2.0 (2.62)	0	6.9 (1.20)	7
Decomposing Numbers	0.4 (0.97)	0	6.6 (1.71)	7
Total	3.7 (4.55)	2	20.5 (2.84)	21

Non-parametric tests. Dependent *t*-tests were planned for comparing the pretest and posttest data. However, after visual inspection of the variances and subsequent tests of normality (i.e., Shapiro-Wilk), three of the four paired data sets were determined to be non-normal data. Therefore, non-parametric Wilcoxon Signed-ranks tests were conducted for each pair of pre-post data. I calculated a Bonferroni correction to guard against bias of repeated testing effects. Since I performed four tests on the same sets of data, I divided my desired alpha significance level, $\alpha = 0.05$, by four ($p = 0.05/4$ or $p = 0.0125$). *P*-values less than or equal to 0.0125 were considered significant. Each of the tests are reported below.

The first Wilcoxon Signed-ranks test compared overall pretest and posttest scores. The output indicated that posttest scores (*Mdn* = 21.00) were significantly higher than pretest scores (*Mdn* = 2.00), $Z = 2.814$, $p = 0.005$.

The next three tests examined the individual multiplication strategies. The second Wilcoxon Signed-ranks test compared repeated addition pretest and posttest scores. The output indicated that posttest scores (*Mdn* = 7.00) were significantly higher than pretest scores (*Mdn* = 0.00), $Z = 2.717$, $p = 0.007$. The third Wilcoxon Signed-ranks test compared array pretest and posttest scores. The output indicated that posttest scores (*Mdn* = 7.00) were significantly higher than pretest scores (*Mdn* = 0.00), $Z = 2.818$, $p = 0.005$. The final Wilcoxon Signed-ranks test compared decomposing numbers pretest and posttest scores. The output indicated that posttest scores (*Mdn* = 7.00) were significantly higher than pretest scores (*Mdn* = 0.00), $Z = 2.820$, $p = 0.005$.

All Wilcoxon Signed Ranks tests resulted in *p*-values below the adjusted significance level of $p = 0.0125$ and suggest all posttest scores improved with statistical significance after the innovations.

Part Two: Qualitative Data Themes

I used student think-aloud recordings and interviews to obtain qualitative data for this study to provide a more in-depth snapshot of my students' conceptual understanding of basic multiplication. Through student think-aloud self-recordings, participants used virtual manipulatives to demonstrate their understandings of each of the three multiplication strategies: repeated addition, arrays, and decomposing numbers. The student think-aloud recordings and interviews were transcribed verbatim in the students' own vocabulary to ensure authenticity. Three primary themes emerged from the analysis of the data (See Tables 4.2 and 4.3). Early on and at the end of the innovation, students' understanding of multiplication concepts using technology were reflected in their (a) conceptual understanding, (b) conceptual misunderstandings, and (c) correct methods with careless errors. Each of these themes is explained in detail below.

Table 4.2 *Primary Themes that Emerged from Qualitative Data – Early On*

Themes	Examples
1. Conceptual Understandings	• Correctly builds problems • Counts by ones to determine final answers • Finds partial products • Explains Commutative Property of Multiplication
2. Conceptual Misunderstandings	• Adds factors instead of multiplying
3. Correct Methods with Careless Errors	• Correct methodology but does not determine or state final answer • Correct methodology with incorrect factors • Correct methodology with counting or addition mistakes

Table 4.3 *Primary Themes that Emerged from Qualitative Data – At End*

Themes	Examples
1. Conceptual Understandings	• Correctly builds problem • Skip-counts • Correct use of vocabulary • Finds partial products • Explains Commutative Property of Multiplication
2. Conceptual Misunderstandings	• Adds factors instead of multiplying • Incorrect use of vocabulary
3. Correct Methods with Careless Errors	• Uses different strategy than instructed to use • Uses correct methodology but does not determine or state final answer

Conceptual Understanding

To achieve fluency of multiplication facts, students must be able to flexibly and accurately use an appropriate strategy in order to efficiently arrive at an accurate answer (Common Core State Standards Initiative, 2010; Kling & Bay-Williams, 2015). This means that a variety of learning strategies is needed to motivate students and to improve their developing understandings of what it means to multiply numbers (Heege, 1985; Solomon & Mighton, 2017). In this study, conceptual understanding of multiplication is defined as the ability to explain and apply each of the three specific strategies (i.e., repeated addition, arrays, and decomposing numbers) using virtual manipulatives. To demonstrate conceptual understanding of repeated addition, students were expected to determine the product by using virtual manipulatives to build equal groups (with the factors indicating the number of groups and amount within each group) then adding the sum of each group. To demonstrate conceptual understanding of arrays, students had to determine the product by utilizing virtual manipulatives to create arrays (with the factors indicating the size of the rows and columns). To demonstrate conceptual understanding of decomposing numbers, students were asked to use virtual manipulatives to decompose either factor, multiply to determine partial products, and then add partial products to determine the final answer. Students' conceptual understandings were assessed both (a) early on and (b) at the end of the innovation. The data were then (c) compared to show any growth or changes in conceptual understandings.

Early on. After one week of the innovation for each of the specific strategies (i.e., repeated addition, arrays, and decomposing numbers), students described their developing understandings of multiplication concepts. Students communicated their proficiency by accurately explaining how they solved the multiplication problems using the given strategy while demonstrating their thinking with virtual manipulatives. This early on assessment allowed me to diagnose specific conceptual misunderstandings and provide a more individualized remediation for each student in this focus group. In these early on self-recordings, students demonstrated their developing conceptual understandings by (a) correctly building the multiplication problems using virtual manipulatives and by (b) accurately explaining their understanding of the Commutative Property of Multiplication to illustrate conceptual awareness of the relationship between factors.

Correctly build problems. Approximately half of the students built the given multiplication problem and explained as they solved using each of the three given strategies. Using virtual manipulatives, five (out of ten total) students correctly built the problem with repeated addition, six students did so with arrays, and five students did so by decomposing numbers. To correctly build the problem with virtual manipulatives, students had to appropriately arrange equal groups of manipulatives to represent repeated addition, situate manipulatives in equal-sized rows and columns to represent an array, or decompose one factor to determine partial products. Students were to use these strategies to aid them in determining the product.

Repeated addition. Early on, only five students were able to use virtual manipulatives to correctly build equal groups that would then be used for repeated addition. Opal explained as she used virtual manipulatives to solve using repeated addition, “Hey guys! I’m doing seven times four. Seven times four is easy. [Makes four groups of seven circles]. So seven times two is 14, and 14 plus 14 equals 28. Seven times four equals 28." Opal broke the problem into two smaller addition problems before adding the partial products. Another student, Jim, similarly explained how he correctly built and solved the same problem:

> Today I will be walking you through seven times four. Seven times four is basically four groups of seven. I am going to go ahead and show you that. [Makes seven groups of four circles on his tablet]. Hang on. … Seven and seven is 14. And 14 plus 14 is 28. So that's your answer. 28.

Although Jim explained that he was building "four groups of seven" but actually built seven groups of four, he had the correct idea. Either way he solved the problem would have resulted in the correct answer. Since the Commutative Property of Multiplication states that the order of the factors does not matter when finding the product (Barmby et al., 2009; Charles & Duckett, 2008; Day & Hurrell, 2015; Hurst & Hurrell, 2017; Jacob & Mulligan, 2014; Kling & Bay-Williams, 2015), this study did not focus on whether students reordered the factors before solving. Rather, students were encouraged to apply the Commutative Property when they felt it would make the problem easier to solve. In both the above cases, the students built the problem with the correct number of groups and correct number of circles in each group. Each student then used repeated addition to determine partial products. They then added the partial products to determine the correct answer.

Since these early on self-recordings enabled me to see that only half of the students were able to correctly apply the repeated addition strategy to demonstrate conceptual understanding of multiplication, I was able to provide specific remediation in this area to assist students with conceptual misunderstandings as well as those with counting and addition mistakes. Students practiced by building equal sets of beads using the Number Racks virtual manipulatives and then counting the totals.

Arrays. Early on, six students correctly built and solved multiplication problems by using virtual manipulatives to draw arrays. Karla quickly built the array for four times nine. She stated, "Four times nine." She then made four rows of nine using square tiles and counts by ones as she built each row, "1, 2, 3, 4, 5, 6, 7, 8, 9." She then repeated this three more times. Finally, she counted by ones to determine her total: "36." Another student, Laura, explained in greater detail as she correctly built an array to solve nine times four:

> So I'm gonna show you how to solve nine times four. So you're gonna put one, two, three, … nine. [Makes one column of nine rhombuses]. Then you're going to put four going this way. [Makes the top row have four rhombuses]. You're gonna keep adding four going this way (See Figure 4.1). [Makes four in every row]. Keep going until you have all fours. Keep adding four. [Finishes building array with nine rows of four, then counts all shapes by ones as she writes the numbers inside the shapes]. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36. So the answer is 36 (See Figure 4.2). Bye!

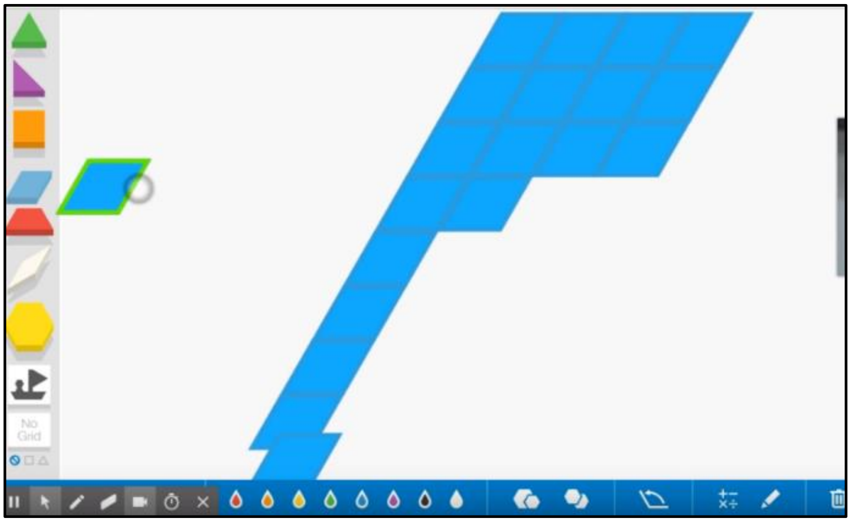

Figure 4.1. Laura used Pattern Shapes (virtual manipulatives) to begin building her array.

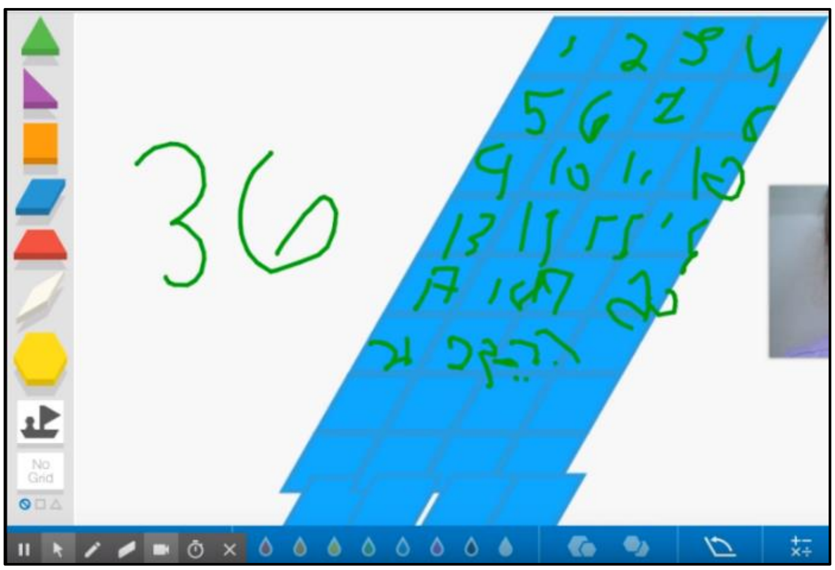

Figure 4.2. Laura completed her array and wrote in numbers as she counted to determine the total.

In both Karla's and Laura's array examples, each student chose different ways to build their arrays with virtual manipulatives to represent the problem. Karla was much more efficient in drawing her array than Laura; however, Laura was much more careful when counting to obtain the final answer. Both students were learning to demonstrate an accurate understanding of what it means to multiply. Each girl correctly built and displayed her array. Both students were also diligent in counting by ones to obtain the final answer.

The students' early-on self-recordings provided much-needed insight into students' developing conceptual understandings. The recordings enabled me to determine that six students demonstrated conceptual understanding of arrays and enabled me to diagnose and remediate misunderstandings (such as adding factors instead of multiplying, counting mistakes, and addition errors) of the other four students.

Decomposing numbers. After one week of innovation with each strategy, all ten students built the problem and decomposed factors correctly, although there was some difficulty in attaining the correct answer. (Some students were able to decompose a factor, but unsure of remaining algorithm. Others were able to decompose and establish partial products but did not seem to know what to do with the partial products). Johnny, however, correctly built and solved the given problem (See Figure 4.3).:

Okay. My name is Johnny and I'm gonna solve nine times six. This is easy because it's a fact that I know. Let's get it together. [Builds model onscreen using virtual manipulatives and decomposes the six into five and one]. Start with nine times five. Nine times five equals 45. Then nine times one equals nine. [Writes the partial products, "45 + 9", on screen]. Now let's write nine times six equals... Nine times six is 54. Yay!

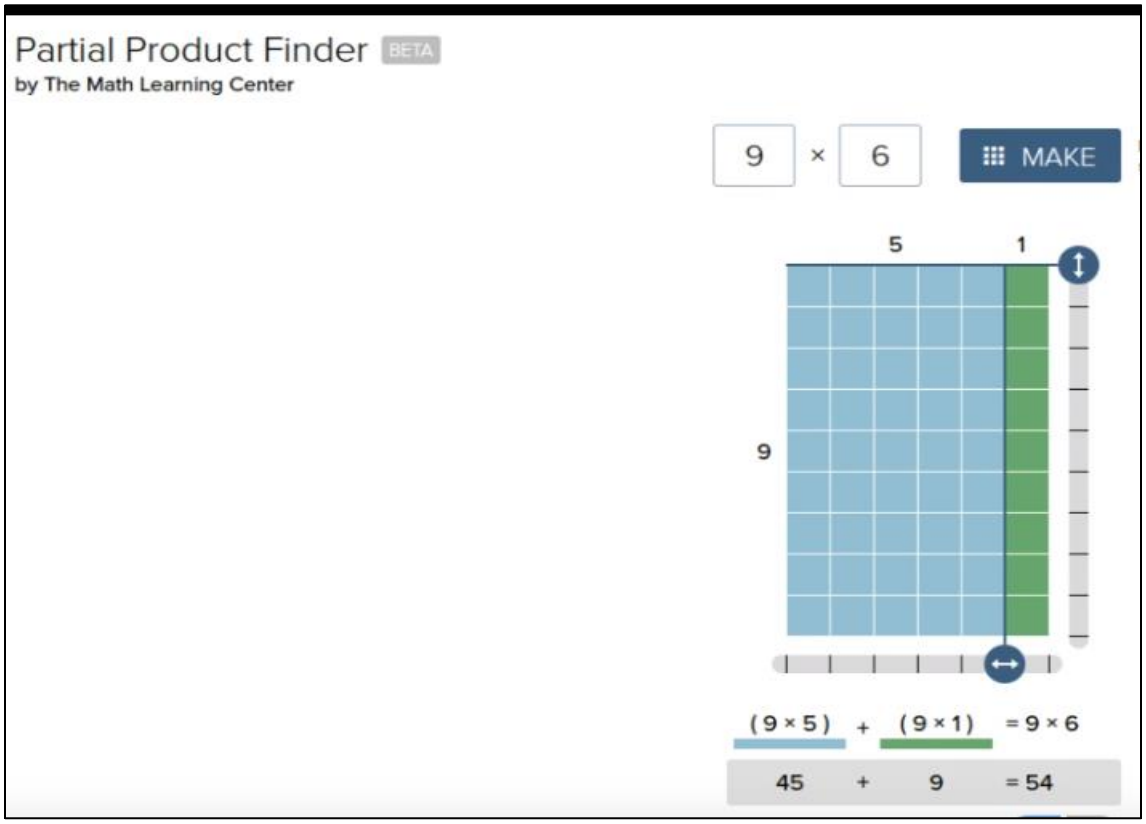

Figure 4.3. Johnny correctly decomposed the factor of six and found partial products to help him determine the final answer.

Grace used a different strategy to help her decompose this same problem. Instead of using the virtual manipulatives, she chose to work out the problem, 9 x 6, on paper. She first decomposed the six and made five plus one. She did not speak as she tried to solve the problem using the nine fingers trick strategy that she learned in class. She used this strategy to check the work she does on her paper. Finally, she explained, "Nine times five is 45. Nine times one equals nine. 45 plus nine is 54." Grace did not appear to be as confident with the virtual manipulatives as she is with paper and pencil, which is certainly a practical means of solving the problem using this strategy. Grace's recording allowed me to see that she needs continued practice using virtual manipulatives to improve her confidence level.

The students' early-on self-recordings for decomposing numbers certainly provided much-needed insight into students' developing conceptual understandings. While all of the students correctly decomposed one factor, Johnny and Grace were the only two students who were able to accurately determine and add the partial products. These formative recordings enabled me to diagnose and remediate misunderstandings in conceptual understanding (such as finding and adding partial products) for the other eight students.

Commutative Property of Multiplication. Without being asked, one student went a step further with demonstrating conceptual understanding and explained how the Commutative Property of Multiplication works. She represented her understanding of this property with virtual manipulatives as she built the assigned multiplication problems. Kate explained how to use the Commutative Property to solve nine times four using arrays:

> I'm going to show you how to do nine times four. [Builds four rows of nine using square tiles]. My eyes are keeping me so exhausted right now. So you can spend all your time making four nine times or you can make it easy and just do nine four times. You can do it the hard way or the easy way. So thank you. Bye! [Did not determine final answer].

Kate considered it easier to make fewer rows with the larger amount in each row. She explained that either way the array is arranged, it will result in the same product. This example is important as it reflects Kate's understanding of the Commutative Property of Multiplication. In her own words, Kate is explaining that, by reordering the factors, she can essentially make solving easier. This explanation represents Kate's thinking processes and is also indicative of her growing conceptual development of multiplication.

At the end. At the end of the six-week innovation, students' conceptual understandings of multiplication greatly improved as evidenced by their final student think-aloud recordings. In these summative self-recordings, students demonstrated their developing conceptual understandings by (a) correctly building the multiplication problems using virtual manipulatives and by (b) accurately explaining their understanding of the Commutative Property of Multiplication to illustrate conceptual awareness of the relationship between factors.

Correctly build problems. Almost all students in this study were able to correctly build the given multiplication problem and explain as they solved using each of the three given strategies. Using virtual manipulatives, nine out of 10 students each correctly built the given problem with repeated addition and decomposing numbers, while all ten students did so with arrays. As previously mentioned, to correctly build the problem with virtual manipulatives, students had to appropriately arrange equal groups of manipulatives to represent repeated addition, situate manipulatives in equal-sized rows and columns to represent an array, or decompose one factor to determine partial products. Students were to use these strategies to aid them in determining the product. Overall, students spoke with confidence as they used the virtual manipulatives to demonstrate their understandings of each of the three multiplication strategies.

Repeated addition. In the self-recordings at the end of the innovation, nine out of ten students used virtual manipulatives to correctly make equal groups which would then be used to assist with repeated addition. In the student think-aloud interviews, all ten students correctly explained how to use repeated addition as a multiplication strategy. Johnny explained that repeated addition is just like counting:

Okay. Welcome everybody. We are going to do four times three. [Makes four sets of three and then counts]. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. [Recounts]. Count. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. Count them, and you get 12. We can make sure by counting. And you count, and it equals 12.

Johnny correctly solved and then double-checked his work by counting a second time to ensure his answer is accurate. Similarly, Kate explained her understanding of repeated addition (See Figure 4.4).:

I'm gonna show you three times four. [Makes one set of three]. That's one set of three. [Makes another set of three]. That's two sets of three. [Makes another set]. That's three sets of three. [Makes one last set]. That's four sets of three. So this is three sets of four. Let's count 'em. [Counts by moving cursor to each shape, one at a time]. That's 12. It's easy! See?

Figure 4.4. To assist with repeated addition, Kate used virtual manipulatives to make four sets of three.

Kate's recording indicates that her conceptual understanding has grown over the course of this innovation. Although she stated, "So this is three sets of four" instead of "four sets of three", she did correctly set up the problem and accurately solved using repeated addition. (As previously mentioned, this study did not focus on whether students reordered the factors before solving. Rather, students were encouraged to apply the Commutative Property when they felt it would make the problem easier to solve). Kate was very careful to move her cursor to each shape as she counted in order to prevent any counting errors, as we had practiced during remediation.

Overall, Kate and eight of her peers were able to implement checks (i.e., moving cursor while counting, writing numbers while counting, etc.) to assist them in ensuring that they did not make careless counting or addition mistakes with repeated addition. As a result, at the end of this innovation, nine out of ten students were able to correct their own errors and accurately demonstrate a correct understanding of repeated addition. This extra step of careful counting and adding indicates a strong understanding of the concept of repeated addition as well as a newly created self-awareness of possible mistakes despite procedural overconfidence.

Arrays. In the self-recordings at the end of the innovation and in the student think-aloud interviews, all ten students used virtual manipulatives to correctly build an array to represent the assigned problem. Wesley quickly and correctly used virtual manipulatives to build an array in order to solve the given multiplication problem. He explained, "Okay, so three times four. [Counts as he builds three rows of four]. 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4. So there's your answer. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. So your answer is

12.” Jim similarly used virtual manipulatives to assist him in drawing an array to find the answer:

> I am showing how to decompose…no not decompose…draw an array for three times four. So I'm gonna do four groups of three. An array has to be the same shapes and you can't just make one row all the way across the thing because that wouldn't work. [Makes three rows of four squares]. So it's eight…no….4, 8, 12. Done.

Jim used the virtual manipulatives to help organize his thinking. He seemed a little nervous at the beginning and stated the wrong strategy, but he quickly corrected himself and kept working. He started to answer incorrectly, but then used the virtual manipulatives to help him skip-count by fours to arrive at the correct answer. Jim, as well as many other students at the end of this innovation, was able to quickly identify when he made a mistake in his thinking and then successfully self-corrected.

The ability of students to diagnose and address their own mistakes gives clear and meaningful insight into the students' growing conceptual understandings. At the end of this innovation, all 10 students were able to correctly solve multiplication problems using arrays. This indicates that students' conceptual understanding of multiplication using arrays improved, and according to the think-aloud interviews completed at the end of this study, most students (six out of 10) preferred working with arrays because they understood this strategy better than decomposing numbers. Students indicated that it also allowed them to provide their own visual which could then be counted. Students felt most confident with this strategy and therefore preferred using arrays over repeated addition and decomposing numbers.

Decomposing numbers. In the self-recordings at the end of the innovation, nine out of 10 students used virtual manipulatives to correctly decompose one factor and then find partial products. In the student think-aloud interviews, however, only four out of 10 students correctly explained how to decompose factors as a strategy for solving a given multiplication problem. For example, in decomposing to solve seven times three, Andrew used a different strategy (repeated addition) and explained that he would, "Add seven three times." He could not explain how to decompose but did accurately explain another strategy that would work to obtain the correct answer. Johnny explained his strategy for solving this same problem. "Decompose seven. Make five plus two. Add five plus two." In Johnny's explanation, he appears to understand how to decompose numbers but needs further remediation in order to understand how to find and add partial products to determine the final product. These types of errors (especially during the interviews) indicate a need for continued remediation to ensure that students understand all steps of this strategy, understand why it works mathematically, and have confidence when applying it. Jim explained his growing understanding of what it means to decompose as he solves three times four:

Hello. I'll be decomposing three times four. If you don't know, decomposing is like cutting it into a number what it equals. So for four, it would be two and two or one and three. For three it would be one and two. I'm gonna do [keep] three because it's the easiest. [Decomposes the four and makes two plus two. See Figure 4.5]. Two and two. Three times two is six. And what's six and six? 12. If it's not 12…it's right. My answer is right.

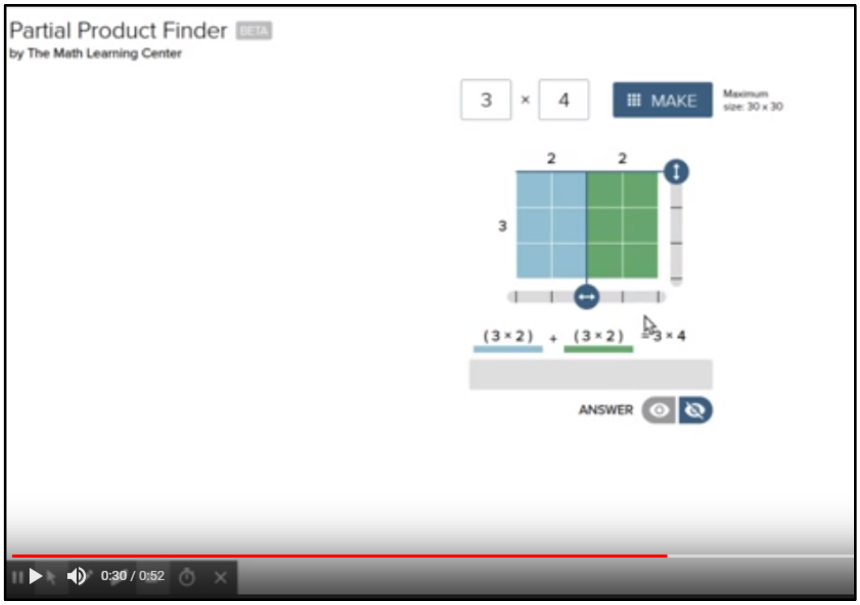

Figure 4.5. Jim used Partial Product Finder (virtual manipulative) to decompose and determine partial products.

Jim demonstrated a strong conceptual understanding as he explained that either factor can be decomposed to solve the problem, and then he proceeded to explain how to decompose both numbers. This lengthy explanation reveals a solid understanding of how to break apart larger numbers in order to find partial products. This is an essential step in achieving fluency of multiplication facts. As previously discussed in Chapter Two, cognitive achievement in the area of multiplication depends largely on students' ability to think mathematically and derive answers rather than depending on rote memorization (Boaler et al., 2015; Kling & Bay-Williams, 2015; Woodward, 2006). By decomposing numbers, students are thinking more flexibly about numbers, which reflects a strong number sense (Boaler et al.). As students become confident in decomposing, they become better able to use this as a mental math strategy which, in turn, will lead to automatization of multiplication facts.

Grace also demonstrated a strong understanding of decomposing numbers as she solved the same problem. Grace stated, "Three times four. [Decomposes four to make three plus one, then starts over and decomposes the three to make two plus one]. Two times four is eight. One times four is four. Eight plus four is 12." As she began working, Grace decided that the three would be easier than the four to decompose because she would be breaking apart the three into smaller numbers (i.e., one and two). That mental process of thinking through the problem and determining how to rearrange numbers in order to make decomposing and solving a simpler process is a clear indication of the student's conceptual understanding of this strategy and of multiplication (Baroody & Coslick, 1998; Benson et al., 2013; Cumming & Elkins, 1999; Gerstan & Chard, 1999; Kilpatrick et al., 2001; Sowder, 1992). The virtual manipulatives allowed Grace, as well as all the other students in this study, to visualize the abstract processes and, in effect, helped them to see and understand what it means to multiply.

Commutative Property of Multiplication. At the end of the innovation and without any prompts from the teacher, almost half the students explained the Commutative Property as they solved problems while self-recording. Students' initiative to explain the Commutative Property perhaps developed as a result of confidence in their growing conceptual understandings of multiplication strategies. Students were quite enthusiastic about recording themselves and were extremely proud of their academic growth over the course of this innovation. As a result, many exuded self-confidence and were excited to convey to me what all they had learned. In effect, these students wanted to ensure that they told me everything they learned about solving multiplication problems. These explanations of the Commutative Property do, in fact, provide insight into the students' conceptual understandings of multiplication. For example, Lisa explained how to solve three times four:

Hi everybody! [Builds an array with four rows of three squares. See Figure 4.6]. We have three times four today. And so we can do four times three or do [Sings] three rows, three rows, three rows like this. [Builds a second array with three rows of four squares. See Figure 4.6]. That's easy for me. Let's count 'em. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. Good job! Now let's count [first array]. Four times three is 12! It's not crazy. It's not hard. All you have to do is put four on three rows, three rows! You can do it and count 'em like this! It's still 12. It's not hard! You can get a piece of paper and you count them like I did. Up, down, and side to side.

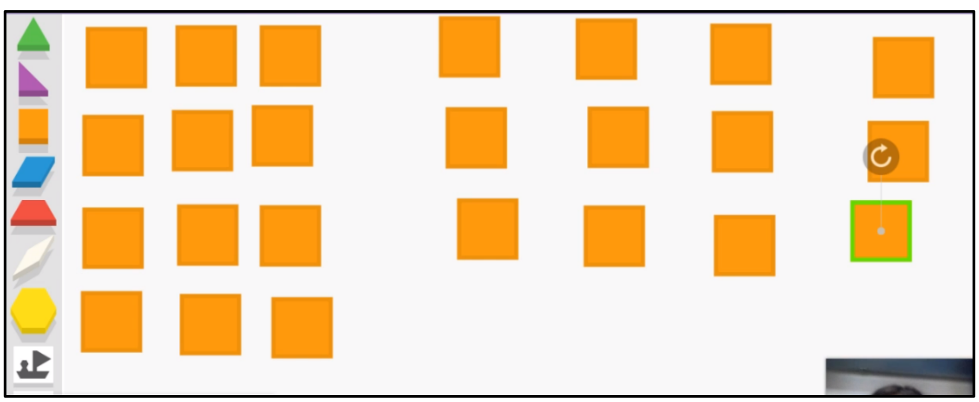

Figure 4.6. Lisa used Pattern Shapes (virtual manipulatives) to create two arrays to represent how the Commutative Property of Multiplication works.

Lisa has a definite understanding of how commutativity works in multiplication, which enables her to better conceptualize the meaning of multiplication.

Jim also explained his thinking regarding the commutativity of factors within a multiplication problem:

Okay. I am going to show you how to use repeated addition to solve three times four. So I'm gonna take three [makes one set of three]. Alright, gonna move that rack. [Makes four sets of three]. So you could add four three times or add three four times. I'm gonna add three 4 times. [Skip-counts by three to solve]. 3, 6, 9, 12. Four times three is 12.

Jim accurately explained how the Commutative Property of Multiplication applies in this problem. He described the two ways repeated addition could be used to solve this problem and then skip-counted to determine the final answer. Jim's detailed explanation of how the Commutative Property works is clearly indicative of his developing understanding of multiplication.

Comparing changes from early on to at the end. Students' conceptual understandings of multiplication greatly improved over the course of this six-week innovation. This increase in conceptual understanding is evidenced in the student think-aloud self-recordings. Students' ability to use virtual manipulatives to correctly build repeated addition problems increased from five out of 10 students early on in the study to nine out of 10 students at the end. Similarly, the number of students who used virtual manipulatives to correctly build and solve arrays increased from six to 10 (out of 10), with every student being able to demonstrate conceptual understanding of this multiplication strategy. The number of students who used virtual manipulatives to correctly decompose numbers increased from five to nine (out of 10) over the course of this innovation.

Early on, many students set up the problems for repeated addition, arrays, and decomposing numbers but failed to find the total. Also, as previously mentioned, several students made simple mistakes counting or adding when working early on but were much more careful about these type mistakes at the end of the innovation. At the end of the innovation however, there was only one careless mistake and one conceptual misunderstanding overall. All students showed great improvement in conceptual understandings of multiplication with the three specific strategies.

For instance, Opal's understanding of decomposing numbers improved significantly over the course of the innovation as evidenced by her think-aloud recordings. In her initial recording, she correctly decomposed and found partial products, but then mistakenly added the factors rather than adding the partial products:

> Hey guys! I'm gonna do six times nine. (Decomposes six into five and one). Nine times six equals…wait nine times six equals 45. And nine times one. That would equal nine. Then you would plus. Nine times five is 5, 10, 15, …55, wait, (recounts) 45. Nine times one equals nine. And then five plus nine equals 14. Then it equals 45. Bye guys!

This early on explanation indicates gaps in Opal's conceptual understanding of adding partial products, since she calculated partial products but then added the two factors. If Opal truly understood the concept of multiplication, she would have been able to assess the reasonable of her answer (Common Core State Standards Initiative, 2010). Since nine rounds to 10, the answer to six times nine would have to be close to six times ten, or 60. She should have realized that six sets of ten would not be close to 14, and then reworked to find her error.

At the end of the innovation however, Opal accurately explained her improved understanding of this strategy:

> Three times four. If I decompose three (Decomposes three to make two plus one). So two times four equals eight. And then one times four equals four. And then 8 and 4 equals 12.

In her final recording, Opal clearly and concisely explained exactly how to find the product by decomposing numbers. This indicates her growth in conceptual understanding of decomposing numbers.

By the end of the innovation, students' recordings reflected improved conceptual understandings. Overall, students were better able to explain how to solve with the given strategy rather than just moving the manipulatives and silently working the problems. Also, at the end of the innovation, students appeared much more confident with their work (especially with repeated addition and arrays) as they did not stumble upon words or make mistakes as they solved. These students were appropriately certain of their work as they accurately and straightforwardly explained their strategies. Students' confidence and certainty is reflected in the percentage of students who demonstrated conceptual understanding of the three strategies at the end of the innovation: 90 percent demonstrated conceptual understanding of repeated addition, 100 percent demonstrated conceptual understanding of arrays, and 90 percent demonstrated conceptual understanding of decomposing numbers. In addition, four of the 10 students (almost half) went an extra step without being prompted to demonstrate how the Commutative Property of Multiplication can be applied to the given problem to check the answer. These voluntary descriptions of the Commutative Property indicate a strong conceptual understanding of what multiplication means and a thorough understanding of the relationship between factors.

At the end of the innovation, all students were much more careful and intentional when solving the given multiplication problems. No students used incorrect factors at the end of this innovation. Students made thoughtful decisions as they worked. For example, Jim explained that how to decompose both factors in his problem, but then chose to decompose the four because “it’s the easiest.” Others double-checked to avoid careless mistakes. For example, Wesley explained how to create an array to solve three times four. He made three rows of four and counted as he made his array. “1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4. So there’s your answer.” He then went the extra step and double-checked by counting the total to ensure he had the correct answer. “1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. So you answer is 12.” Students also self-corrected when they found mistakes. For example, when solving three times four, Jim drew his array and then counted eight, but immediately realized he made a mistake. He then recounted and self-corrected, “4, 8, 12. Done.” Most students were very poised and proud to show what they had learned over the course of this six-week innovation. This was evident in their excitement, smiles, and eagerness to record their think-alouds. In addition, students enthusiastically watched their own videos, made self-corrections, and re-recorded if they felt necessary, before submitting their final recordings to me.

Conceptual Misunderstanding

As previously stated, by better addressing specific misunderstandings and building a richer number sense for students at the primary and elementary levels, educators provide a strong foundation for all higher-level mathematics skills (Boaler et al., 2015; Cumming & Elkins, 1999; Heege, 1985; Solomon & Mighton, 2017; Wells, 2012). An improved fundamental understanding of mathematical thinking and reasoning strategies will ensure that students are better able to reason through *why* methods work mathematically and apply those methods to new types of problems (Boaler et al., 2015; Zhang et al., 2014).

In this action research study, conceptual understanding of multiplication is defined as the inability to explain and apply each of the three specific strategies (i.e., repeated addition, arrays, and decomposing numbers) using virtual manipulatives. A conceptual misunderstanding of repeated addition would be evident if students were unable to determine the product by using virtual manipulatives to build equal groups (with the factors indicating the number of groups and amount within each group) then adding the sum of each group. A conceptual misunderstanding of arrays would be evident if students were unable to determine the product by utilizing virtual manipulatives to create arrays (with the factors indicating the size of the rows and columns). A conceptual misunderstanding of decomposing numbers would be evident if students were unable to use virtual manipulatives to decompose either factor, multiply to determine partial products, and then add partial products to determine the final answer.

In this study, students' conceptual misunderstandings of multiplication became evident in the application of three specific strategies (i.e., repeated addition, arrays, and decomposing numbers) using virtual manipulatives. Students' conceptual misunderstandings were assessed both (a) early on and (b) at the end of the innovation. The data were then (c) compared to show any growth or changes in conceptual understandings.

Early on. After one week of the innovation for each of the specific strategies (i.e., repeated addition, arrays, and decomposing numbers), students described their developing understandings of multiplication concepts, and while doing so, revealed some major gaps in learning. Several students understood that adding was a strategy for solving multiplication problems, but they did not understand the concept of repeated addition. In these cases, the students added, but then their algorithm became disjointed. For example, Wesley explained how to solve six times three:

Alright, so um, six times three. I have it set up like this. There are six of these and three more. And then when you add the three more, how I do it is go over here and so [starts writing tallies. See Figure 4.7]. That's one, two, three, four, five. Those are like fingers. Then one, two, three, four. So then you have got your answer.

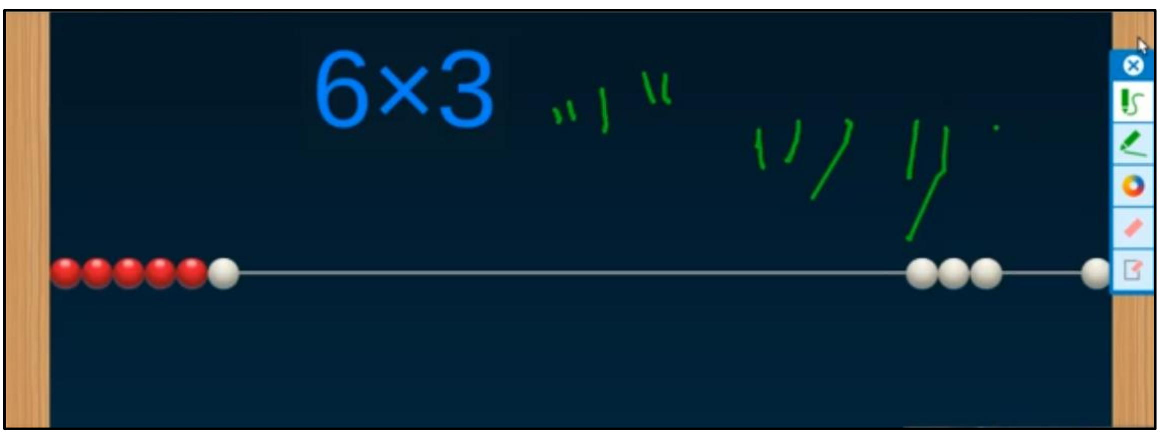

Figure 4.7. Wesley's use of Number Racks (virtual manipulatives) demonstrate his conceptual misunderstandings.

In this case, Wesley tried to add the two factors rather than multiply them. He understood that adding can be used to solve multiplication; however, he then became confused in his methodology and did not state a final answer.

Another student, Andrew, began decomposing correctly but then confused his work and arrived at an incorrect answer:

> Okay. I'm gonna be solving nine times six. [Decomposes the nine into eight and one]. And I start with six times six which I just already did. And they equal sixty-four. Now the answer is right there. That's how you solve it.

Andrew seemed to understand exactly how to decompose a factor, but his understanding of how to finish solving the problem was lacking. This indicated a very wide gap in understanding and enabled me to focus on specific skills for remediation, such as working individually with concrete and virtual manipulatives to demonstrate the quantity of each factor and how to correctly determine and add partial products.

Similarly, Opal seemed to understand how to decompose a factor, but she then confused her algorithm. She explained:

> Hey guys! I'm gonna do six times nine. [Decomposes six into five and one]. Nine times six equals…wait nine times six equals 45. And nine times one…that would equal nine. Then you would plus. Nine times five is 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, wait, [recounts] 45. Nine times one equals nine. And then five plus nine equals 14. Then it equals 45. Bye guys!

In this example, Opal is somewhat familiar with the steps in decomposing to solve a multiplication problem. However, she does not comprehend the concept of multiplication well enough to understand her mistakes. In other words, she is trying to remember all the pieces or steps without realizing the big picture. At the beginning, she knows she must decompose a factor, and at the end she understands that she must add two numbers. Her algorithm for completing the problem and finding partial products is where her misunderstandings occur. I used this information regarding her misunderstandings to help provide individualized instruction specific to decomposing numbers. I reworked this problem and several other problems with Opal independently to help her identify her own mistakes. By using the Partial Product Finder virtual manipulative and relating each problem to an array that she and I both drew on dry-erase boards, Opal began to understand that the product is the sum of all parts of the problem. She began to see the connection across all strategies: add all groups for repeated addition, add all items in each row for arrays, and add all partial products for decomposing numbers.

At the end. At the end of the six-week innovation, students' conceptual misunderstandings of multiplication greatly decreased as evidenced by their final student think-aloud recordings. In these self-recordings, very few students continued to demonstrate conceptual misunderstandings. Rather, most correctly built the problems using virtual manipulatives, explained their mathematical thinking, and accurately solved using the given multiplication strategy. For instance, only one student appeared to have a continued conceptual misunderstanding. Kate tried to explain how to decompose to solve three times four. She stated:

> I will show you three times four. You will make it up to two. [Decomposes the four to make three plus one. See Figure 4.8]. Three. Then that equals one. [Points to the three and one]. So three times one equals three. Put three plus one equals four. That's easy. Seven times three plus one equals four. Four. Four. Three times one equals three. So it's easy. Thank you.

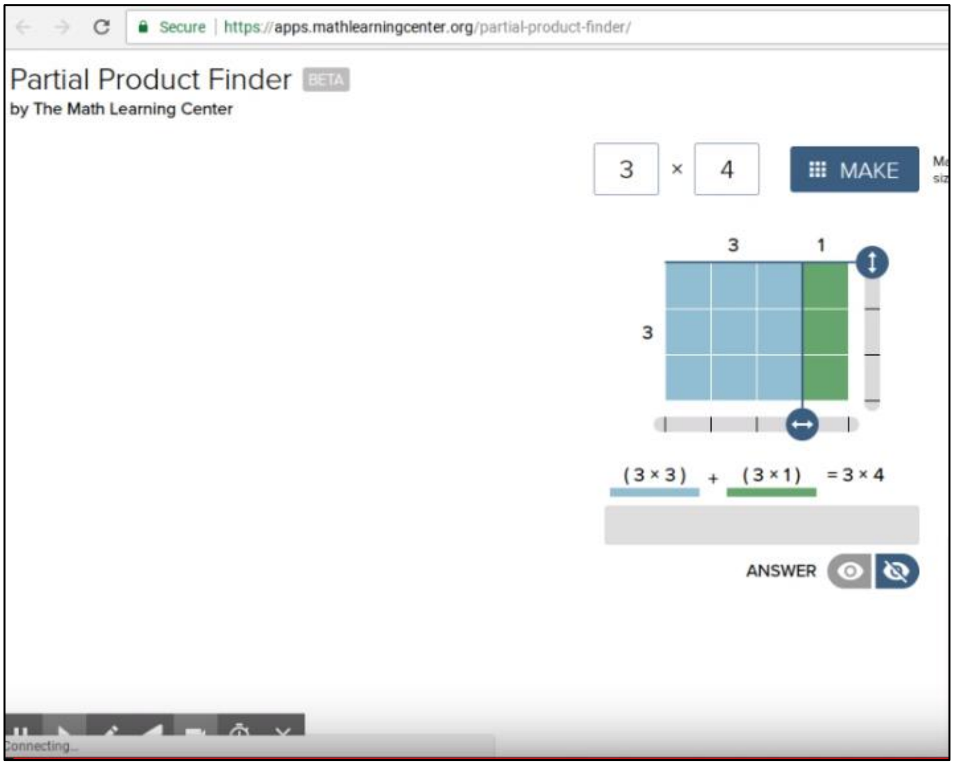

Figure 4.8. Kate correctly decomposed the factor of four, but did not understand how to find partial products and solve.

Kate correctly used the virtual manipulatives to decompose the number four, but then her methodology was quickly confused thereafter. She started adding and multiplying numbers without meaning. Clearly, this indicates that this student needs further remediation in finding partial products. Kate does understand how to decompose, but she is lacking the conceptual understanding of using the decomposed numbers to calculate and then add the partial products. At the end of the innovation, Kate was the only student who continued to demonstrate serious conceptual misunderstandings. This indicates that the innovation with virtual manipulatives successfully remediated all but one student.

The only other error at the end of the innovation included incorrect vocabulary by one student. Johnny explained how to decompose to solve three times four:

> Welcome back, guys. We gonna do partial products with this stuff right here. Three times four. [Decomposes the four to make two plus two]. We are doing three times four. We are going to find the numerator and denominator. We are practicing. Three times two is six and three times two is six. And we just trying to find three times four. And…we are just going back and forth. Three times four is twelve.

Johnny decomposed, found partial products, and solved correctly. However, he incorrectly stated that he was finding the "numerator and denominator." Including random, incorrect terminology into his explanation indicates that Johnny may benefit from remediation in the area of fractions. Several possible explanations exist regarding why Johnny would have used "numerator and denominator" when explaining multiplication strategies. One possible reason is that we recently completed the unit on fractions, and I still have fraction anchor charts hanging on the classroom walls. It is possible that Johnny had fractions in his recent memory and glanced at one of those charts, which made him think of numerators and denominators. A second possibility is that he confused 'decompose' with 'denominator' since they both have the same first two letters are similar in length. Lastly, it is possible that he does not have a strong conceptual understanding of fractions and may need remediation to ensure that he fully understands the meanings of numerators and denominators.

Comparing changes from early on to at the end. Eight students demonstrated conceptual misunderstandings early on. By the end of the innovation, only one student, Kate, demonstrated a major gap in conceptual understanding. One other student, Johnny, used incorrect terminology but otherwise used correct methodology to accurately solve the given multiplication problem. This reduction in misunderstanding indicates considerable student growth in conceptual understanding of the given multiplication strategies. This reduction also suggests that the ongoing practice with virtual manipulatives was successful in clarifying students' misconceptions of multiplication concepts.

Correct Methods with Careless Errors

Low achieving students are more likely to rely on counting strategies than direct retrieval for solving basic multiplication facts (Geary & Brown, 1991; Hanich et al., 2001; Hoard et al., 1999). Without a strong number sense, these students are more prone than their peers to make retrieval and counting errors on basic addition and multiplication problems (Geary & Brown; Hanich et al.; Hoard et al.). In this action research study, students' developing conceptual understandings of multiplication are reflected in the application of three specific strategies (i.e., repeated addition, arrays, and decomposing numbers) using virtual manipulatives. Some students' self-recordings reflected correct methodology with careless computational errors that indicated a somewhat weak sense of numbers. These students demonstrated a developing conceptual understanding of the multiplication strategies despite having counting errors that resulted in incorrect answers. Students' correct methodology with careless errors was assessed both (a) early on and (b) at the end of the innovation. The data were then (c) compared to show any growth or changes in conceptual understandings.

Early on. After one week of the innovation for each of the specific strategies (i.e., repeated addition, arrays, and decomposing numbers), students described their developing understandings of multiplication concepts, and while doing so, revealed an assortment of computational errors that resulted in incorrect answers despite using the correct methodology. Four students correctly built the repeated addition problems using virtual manipulatives but failed to count the final total. Similarly, four students used virtual manipulatives to correctly build arrays, but then miscounted (or failed to count) the total. Andrew explained:

> I am gonna be solving nine times four. [Makes two rows of nine, counting them as he builds each row. Erases everything. Starts over with smaller shapes that will fit on the screen and then builds four rows of nine]. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35. [Counts all triangles quickly and then recounts]. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35. [Pauses]. 35. [Types "4 + 9 = 35."]

In this case, the student clearly understood how to use the two factors to build an array. His conceptual understanding was accurate; however, he miscounted twice. Andrew even double-checked his work and recounted to ensure that his answer was correct. The problem in this case was a counting error that resulted in an incorrect answer, despite having the correct algorithm. Had he written the numbers on the shapes as he counted, he most likely would have realized his counting mistake.

Two students wrote the correct problem but confused the factors when solving. For example, instead of solving three times twelve, Johnny correctly solved three times ten using the Number Racks virtual manipulatives:

> Okay. Welcome to the number racks channel. I see you again. Now we are going to do another answer. Three times twelve. Everybody knows three times twelve.

> 1, 2, 3, 4, 5, 6, 7, 8 ,9, 10. And then mmmm. [Moves beads]. …Ok, let's count. [Skip counts by three for every virtual manipulative that he moved]. 3, 6, 9, 12, 15, 18, 21, 24, 27, 30. See?

While he used the correct methods to accurately solve three times ten, Johnny did not solve the assigned problem which was three times twelve. Clearly, Johnny understands how to solve using repeated addition; however, he needs to slow down and diligently check his work to ensure that he does not make careless mistakes.

Two students had difficulty with decomposing early on due to problems with addition. Both students correctly decomposed one factor and found partial products, but they did not add them together. For example, Lisa explained how to decompose and solve nine times six:

> Hi guys! We're gonna solve nine times six, okay? So it's good to decompose the six and make it five and one. Nine times one equals nine, plus nine times five equals 45, so nine times six equals 42. 42, guys, is not the right answer. Let's try again, guys. That's okay. I hope y'all come visit me soon.

Lisa accurately explained how to decompose the factor of six and find partial products. She also realized she made a mistake in adding but became frustrated and failed to rework the problem to determine the correct answer.

All of these computational errors indicate that students do have a conceptual understanding of the multiplication strategies; however, they need to become more conscientious regarding basic number sense skills such as counting and adding. When students do not diligently check their work for such mistakes, they are more likely to miss problems due to careless computations. As a result, it may appear that students are lacking conceptual understandings when in fact, they are just making absentminded mistakes.

At the end. In this action research study, students' developing conceptual understandings of multiplication are reflected in the application of three specific strategies (i.e., repeated addition, arrays, and decomposing numbers) using virtual manipulatives. Some students demonstrated an accurate understanding of each strategy, although they made minor adding or counting errors which resulted in incorrect answers. These computational errors indicated a weak number sense and a developing conceptual understanding of the multiplication strategies despite having minor counting errors that result in incorrect answers.

At the end of the innovation, only one student demonstrated a careless error in his think-aloud self-recording. Instead of solving the given problem by decomposing, as directed, Wesley used virtual manipulatives to build an array. He explained how to decompose but actually solved using an array for three times four:

> Three and four. [Decomposes the three to make two and one]. So there's three [points to the rows] and four in each group. So you'd go…1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. So you got your answer, now that's 12. So then really, all I did I got three and there's four in each group. So your answer is 12. Bye.

Wesley correctly solved the given problem, although he did not use the specified strategy as directed. This indicated that Wesley does in fact have a conceptual understanding of multiplication and realized that any of the strategies discussed would result in the correct product. While he may not have solved using the assigned strategy, he accurately applied a strategy (arrays) that he felt confident in using. While Wesley did not implement the suggested strategy (decomposing numbers), he thoughtfully chose the strategy that worked best for him, which is the goal of learning multiple strategies. No other careless mistakes were noted at the end of the innovation.

Comparing changes from early on to at the end. Early on in this innovation, there was a total of 12 instances where students made careless errors while using correct methodology to solve the given problems. At the end of this innovation, however, only one student made a careless error while solving. This indicates a remarkable increase in conceptual understanding. In this study, virtual manipulatives significantly improved students' conceptual understandings of three multiplication strategies: repeated addition, arrays, and decomposing numbers. These results are supported by prior research that suggests students are much more attentive to technology-based mathematics lessons, are highly engaged during instruction, respond favorably to assigned tasks, and perform at higher levels (Bragg, 2006; Camp, 2016; Clark & Ernst, 2009; Huizenga et al., 2009; Liu, 2013). This was certainly true of all students in this study. As students became more confident in their understandings (as evidenced by their accurate explanations, precise understandings, direct answers, and eagerness to show what they had learned), they were able to identify errors, such as counting and addition mistakes, were able to ensure that their methods were accurate, such as decomposing and finding partial products, and their answers were reasonable, such as close to an estimate.

CHAPTER FIVE

DISCUSSION, IMPLICATIONS, AND LIMITATIONS

This chapter positions the findings with the literature on technology integration involving multiplication concepts. The purpose of this action research was to evaluate the implementation of technology integration with multiplication concepts (i.e., repeated addition, arrays, and decomposing numbers) for struggling third grade students at FES in a rural southeastern state. Three primary themes emerged from the data analysis (see Figure 3.3). Early on and at the end of the innovation, students' understanding of multiplication concepts using technology were reflected in their conceptual understanding, conceptual misunderstandings, and correct methods with careless errors. Both quantitative (i.e., multiplication pretest-posttests) and qualitative methods (i.e., student think-aloud self-recordings and think-aloud interviews) were utilized for data collection and analysis. The (a) discussion, (b) implications, and (c) limitations of this research are examined below.

Discussion

It is important to situate these results within the larger context of research for technology integration with multiplication concepts. To specifically answer the research questions, the data were combined and considered through a lens of conceptual understanding of multiplication strategies and with research-based literature. The literature on technology integration also assists in explaining the significant changes in conceptual understanding when virtual manipulatives are utilized. The discussion is organized by the two grand tour research questions.

Research Question 1: How and in what ways does technology integration with multiplication concepts impact student understanding?

Fluency of multiplication facts involves a progression of higher-order thinking skills and is described as happening in three successive phases (Baroody, 2006; Kling & Bay-Williams, 2015; Rave & Golightly, 2014). Phase one involves modeling or counting (i.e., repeated addition and arrays) to determine the answer. Phase two involves deriving the answer using reasoning strategies and critical thinking (i.e., decomposing numbers), and phase three is automatic retrieval or mastery of the facts. By using virtual manipulatives with given multiplication strategies (i.e., repeated addition, arrays, and decomposing numbers) during this innovation, students were able to progress through phases one and two as they were working towards building fluency (phase three).

It should be noted that the participants in this study had received differentiated small-group mathematics instruction daily over the course of the school year, from August until this study began in late March. In addition, the students had already completed the unit on basic multiplication concepts earlier in the school year. Despite ongoing remediation incorporating individualized instruction with various multiplication strategies, the participants had not responded to previous instructional interventions. By changing the structure of my instruction, participants were provided the framework to better promote independent thinking and learning. This intervention was an upfront investment with sustainable results. Although these participants faced numerous obstacles (i.e., gaps in foundational mathematics skills, low instructional level, intervention at end of school year, ongoing frustrations with mathematics, placement within special education), this intervention enabled them to make great improvements in their conceptual understandings of multiplication.

The research findings suggest that students' conceptual understanding of multiplication strategies (i.e., repeated addition, arrays, and decomposing numbers) was positively impacted by the use of (a) virtual manipulatives and (b) student think-aloud self-recordings.

Virtual manipulatives. Vygotsky's (1978a) zone of proximal development theory proposed that the learner is much better able to build a conceptual understanding when instruction is scaffolded (D'Andrew & Iliev, 2012; Loong, 2014). By using concrete and virtual manipulatives in conjunction with multiplication strategies that build on one another, students in this study utilized their prior knowledge of addition and multiplication to explore more difficult multiplication problems, investigate possible solutions, develop their ideas, and create new thinking (D'Andrew & Iliev; Kling & Bay-Williams, 2015; Loong; Pitler et al., 2012). The resulting student-centered, constructivist learning environment provided students with increased opportunities to play an active role in their own learning (Pitler et al., 2012).

This study confirms Loong's (2014) research which contends that scaffolding instruction using virtual manipulatives can help correct misconceptions and errors in students' thinking. In this study, virtual manipulatives significantly improved participants' conceptual understandings of all three given multiplication strategies: repeated addition, arrays, and decomposing numbers. The impact of virtual manipulatives is reflected in the increased percentages of students who demonstrated conceptual understanding of the three strategies from the end of week one to end of the innovation: conceptual understanding of repeated addition increased from 50 percent to 90 percent, conceptual understanding of arrays increased from 60 percent to 100 percent, and conceptual understanding of decomposing numbers increased from 50 percent to 90 percent.

In addition, the impact of virtual manipulatives is reflected in the increase of correct answers from the pretest to posttest. The median number of correct problems increased from two on the pretest to 21 on the posttest. Similarly, there was a significant increase in the median number of correct problems for each of the given strategies (i.e., repeated addition, arrays, and decomposing numbers). The pre-posttests included eight problems for each of the three given strategies. From pretest to posttest, the median number of correct problems for repeated addition increased from zero to seven, the median number of correct problems for arrays increased from zero to seven, and the median number of correct problems for decomposing numbers increased from zero to seven.

These results are supported by prior research that suggests technology-based mathematics lessons provide a much more engaging and interactive learning environment that highly motivates students and enables them to perform at higher levels (Bragg, 2006; Camp, 2016; Clark & Ernst, 2009; Huizenga et al., 2009; Liu, 2013). These findings also corroborate those of previous studies which reported that by integrating manipulatives into mathematics instruction, students are better able to visualize the concepts being taught, scaffold their understanding, and simplify the abstract ideas (Burris, 2013; Loong, 2014; Sowell, 1989; Suh & Moyer, 2008).

Student think-aloud self-recordings. The student think-aloud self-recordings allowed students to make their thinking visible (Silbey, 2002), which was an essential step in the process of assessing conceptual understanding. For instance, Jim thoroughly explained his conceptual understanding of the decomposing numbers multiplication strategy:

> If you don't know, decomposing is like cutting it into a number what it equals. So for four, it would be two and two or one and three. For three it would be one and two. I'm gonna do [keep] three because it's the easiest. [Decomposes the four and makes two plus two. See Figure 4.5]. Two and two. Three times two is six. And what's six and six? 12.

The ability to explain his thought processes while accurately demonstrating how he has decomposed a factor, found partial products, and correctly solved the problem reflects Jim's conceptual understanding and progression through phase two of multiplication fact fluency (Baroody, 2006; Kling & Bay-Williams, 2015; Rave & Golightly, 2014). Researchers (Chapin et al., 2009; Smith & Stein, 2011; Walshaw & Anthony, 2008) submit that having students make their thinking visible by explaining reasoning strategies has been proven to positively impact students' conceptual understandings. Likewise, in this study, the summative think-aloud allowed me to see that this student, Jim, thoroughly understands the concept of multiplication.

Before submitting self-recordings, students were asked to listen to their own recordings to ensure accuracy. Researchers (Chi, 2000; Hatano, 1993; Ing et al., 2015;

Roscoe & Chi, 2008) suggest that by making thinking visible and then reflecting on their own work, students are provided with an opportunity to monitor and revise their thinking while identifying any misconceptions, errors, or incomplete understandings. In several recent studies, researchers have found that elementary students' explanations of strategies and high-level discussions of concepts significantly predict student achievement (Webb et al., 2014) and that student explaining and re-explaining ideas were associated with the growth of students' mathematical understandings (Ing et al.; Warner, 2008). Therefore, listening to their own think-aloud recordings was an essential step in providing my students an opportunity to evaluate their own work, identify mistakes, and correct careless errors before turning in their recordings. As students in this study became more confident and certain of their understandings, they were able to identify their own errors in their self-recordings, make corrections, and even rerecord when needed.

In addition, the students were able to ensure that their methods were accurate and that their answers were reasonable by solving the same problem using multiple strategies or by checking the problem using the Commutative Property of Multiplication. According to researchers (Heege, 1985; Hurst & Hurrell, 2017; Kling & Bay-Williams, 2015; Van de Walle et al., 2010; Young-Loveridge, 2005), students must be provided with a progression of developmentally-appropriate reasoning strategies to solve multiplication problems and build fluency. In this study, students incorporated a variety of progressive strategies. By the end of the innovation, students appeared much more confident with the multiplication strategies as they accurately and straightforwardly explained their strategies. In addition, the total number of instances where students made careless errors while using correct methodology to solve the given problems decreased from 12 to one from week one to the end of this innovation.

Research Question 2: How do students select and explain strategies for solving multiplication problems?

The think-aloud protocol developed by Ericcson and Simon (1993) was used early on to identify misconceptions soon after the beginning of intense remediation. These think-alouds provided meaningful and consequential understanding of the participants' thoughts and actions (Creswell, 2014). The preliminary think-alouds were very insightful as they directly informed and guided instruction. Culminating think-alouds were also used at the end of the study to determine the depth of a student's conceptual understanding after remedial instruction had taken place. The think-aloud protocols allowed me to understand how students (a) select and (b) explain strategies for solving multiplication problems.

Select strategies. The teacher-interview think-alouds provided insight into students' selection and understanding of multiplication strategies (i.e., repeated addition, arrays, and decomposing numbers). Students discussed which strategy they preferred and would select when needed. Six out of the 10 students stated that they preferred using arrays for reasons such as, "I like to draw," "Arrays help me get the answer. You just count the dots," and "When you make them, you can count them, and that will give you the answer." Three students reported preferring repeated addition and cited reasons such as, "I just keep adding," and "It's just counting over and over." Only one student preferred decomposing numbers as a strategy of choice. That student explained, "Decomposing numbers is easiest when the problem has big numbers. I can just make the problem into two easier problems, and then add." While these students are in the process of attaining fluency of multiplication facts, the strategy that they prefer and would select reveals much about their progression of conceptual understanding. Students who felt most comfortable with using repeated addition or arrays indicated that they remain in phase one of multiplication fluency because they are using modeling or counting to determine the answer (Baroody, 2006; Kling & Bay-Williams, 2015; Rave & Golightly, 2014). Students who preferred decomposing numbers (and accurately used this strategy to solve) indicated that they have progressed to phase two of the continuum because they are deriving answers using reasoning strategies and critical thinking skills (Baroody; Kling & Bay-Williams; Rave & Golightly). Students in both phases one and two should continue intensive remediation until they have progressed through the continuum and reached phase three, which is multiplication fluency or automatic retrieval of facts (Baroody; Kling & Bay-Williams; Rave & Golightly).

Explain strategies. The student think-aloud self-recordings allowed students to explain their conceptual understanding of repeated addition, arrays and decomposing numbers as multiplication strategies. The previous section discussed how students selected their preferred strategies and explained why they chose one strategy over the others. This section discusses how students explained the steps of solving a multiplication problem with each of the three strategies discussed in this study (i.e., repeated addition, arrays, and decomposing numbers).

After students selected their preferred strategies, they were required to explain how to use all three strategies. These think-aloud recordings enabled me to identify misconceptions that existed in students' understanding and provide instructional interventions to directly address incorrect thinking. As suggested by researchers (Creswell, 2014; Jacobse & Harskamp, 2012; Secolsky et al., 2016), early on think-aloud self-recordings were very insightful as they directly informed me and guided my instruction. For instance, in Wesley's early on think-aloud, he said, "There are six of these and three more." This indicated that he was adding rather than using repeated addition to solve the multiplication problem. Specifically targeting areas of need is an essential step in addressing mathematical content misconceptions, lack of flexible number sense, and/or a negative mindset (Dettori & Ott, 2006; Dowker, 2005; Ma, 1999; Westenskow & Moyer-Packenham, 2017). As a result of targeted intervention, I was able to provide specific remediation using virtual manipulatives to address the difference between addition and multiplication.

I met daily with the participants in a small-group setting to target multiplication strategies. We began by using concrete manipulatives and dry-erase boards to visualize concepts, and then quickly moved to virtual manipulatives on ChromeBooks to demonstrate thinking. Students periodically conducted think-aloud recordings so that they and I could gauge their conceptual understandings. For example, this enabled Wesley to visualize the concepts of addition and multiplication and differentiate between the two. The early on think-alouds also revealed that two other students, Andrew and Opal, were able to decompose a factor but were unable to find partial products. The think-alouds allowed me to specifically address each student's area of weakness and provide targeted remediation which was essential in improving conceptual understanding.

Similarly, student think-aloud self-recordings at the end of this innovation were beneficial in determining the depth of each student's conceptual understanding (Creswell, 2014; Jacobse & Harskamp, 2012; Secolsky et al., 2016). The final self-recordings revealed students' ability to explain each of the multiplication strategies (i.e., repeated addition, arrays, and decomposing numbers) while solving using virtual manipulatives. For example, the think-aloud self-recordings allowed Jim to make his thinking visible, which served the dual purpose of helping him clarify his thinking and enabling me to understand his thought processes. Jim self-corrected his strategy and counting error as he explained his understanding of the given strategy:

> I am showing how to decompose…no not decompose…draw an array for three times four. So I'm gonna do four groups of three. An array has to be the same shapes and you can't just make one row all the way across the thing because that wouldn't work. [Makes three rows of four squares]. So it's eight…no….4, 8, 12. Done.

Using the think-aloud enabled Jim to hear his own explanation and address missteps as they happened. As suggested by researchers (Chi, 2000; Hatano, 1993; Ing et al., 2015; Roscoe & Chi, 2008), explaining one's strategies is an essential step in monitoring, revising, and evaluating one's own thinking. The think-aloud self-recordings with virtual manipulatives proved to be an effective method for building and improving conceptual understanding of multiplication strategies.

Implications

This research has implications for me, practitioners, as well as scholarly practitioners and researchers. Four types of implications are considered: (a) personal implications, (b) implications for teaching multiplication strategies, (c) implications for technology integration in mathematics, and (d) implications for future research.

Personal Implications

As a result of this study, I have learned several personal lessons that I will use as a teacher in practice. These include (a) considering both quantitative and qualitative data analysis methods, (b) a comprehensive literature review, (c) sharing and communicating my findings.

Considering both quantitative and qualitative data analysis methods. To better evaluate my own students' learning and growth, I implemented an evaluative study with triangulation (Mertler, 2014) or convergent (Creswell, 2014) mixed method design using objective assessment data, Wilcoxon Signed Rank Testing, and inductive thematic analysis. This approach enabled me to collect both qualitative and quantitative data and as a result, allowed me to better understand the conceptual understanding of my students. Too often, educators rely heavily on quantitative data (i.e., test scores) to determine a student's academic ability (Mills, 2014); however, test scores alone do not provide a complete representation of conceptual (mis)understandings (Mertler; Mills). By including data obtained from student think-aloud self-recordings and interviews, I was able to better assess conceptual understandings and identify specific gaps in understanding as it pertained to the learning of basic multiplication facts for my third-grade students. The think-aloud recordings reflected a progression of students' understanding which aligns with Vygotsky's (1978a) zone of proximal development theory and were essential in providing valuable, comprehensive insight into students' preliminary and culminating understandings (Creswell; Jacobse & Harskamp, 2012). Such insight helped guide and direct remediation throughout this innovation. Through this triangulated mixed methods approach, I learned that incorporating both quantitative and qualitative methods better enabled me to understand specific misconceptions and individualize my instruction to more accurately address each learning need. Unlike the explanatory and exploratory mixed methods designs, the triangulated mixed-methods design allowed me to collect qualitative and quantitative data simultaneously (Mertler). By collecting both forms of data simultaneously and giving them equal emphasis, I was able to combine the strengths of each data set and merge the results so that the data analyses could be used concurrently to better understand the research problem (Mertler).

Comprehensive literature review. The literature review presented an abundance of related research which provided a solid foundation for this innovation and allowed me to learn about strategies that have been successful or unsuccessful in the past. The research and theories presented in the literature review were essential in presenting a complete understanding and framing of the current study. Before beginning this action research study, I did not realize the significance of considering past research and theories as they related to my own study. While conducting the literature review, I found much information regarding technology integration in elementary mathematics, virtual manipulatives, and multiplication strategies. I learned that past research must inform current studies because it provides the understanding and insight needed to place the research topic in an appropriate framework (Mills, 2014). In addition, administrators and practitioners should use past research to inform their practices of strategies and data collection methods that have or have not been effective to help them avoid repeating the mistakes of others (Mills). Past research provided essential background knowledge that helped to guide my research and direct my thinking. Consequently, I considered my research through the lens of multiple theories and decided that constructivism provided a solid, descriptive framework for this innovation. The constructivist approach allowed for a more engaging learning opportunity where students used prior knowledge, virtual manipulatives, and self-recordings to construct meaning. The different facets of this learning construct helped me to identify strategies on which to focus and allowed me to collect and analyze both qualitative and quantitative data in order to better understand the overall effectiveness of my teaching methods. I learned that this theoretical framework essentially provided the structure on which I built my research. Therefore, the literature review proved to be a very crucial piece of this study, as it should.

Sharing and communicating my findings. At the end of the study, I shared posttest data with the participants. They were very excited to hear that the scores had significantly improved, and several students told me that they “knew” their scores on the posttest were much better because they “finally understood” how to multiply. The virtual manipulatives had helped them to represent the problems in various ways and helped them to “see what was going on” in each problem.

I also shared my successes with my principal, math coach, and district administrator. They were all intrigued at the success of each of these students and realized the importance of the think-aloud recordings and virtual manipulatives for formative assessments. The district administrator suggested that other teachers across grade levels and content areas begin incorporating think-aloud self-recordings so that all teachers had access to such valuable learning data. The math coach agreed and felt that the virtual manipulatives were also definitely worth implementing in other mathematics classes. She agreed to help other mathematics teachers with this type of technology integration immediately.

Implications for Teaching Multiplication Strategies

This study suggests two major implications for educators who teach basic multiplication strategies in whole-group instruction and/or in small-group remediation. Multiplication should be taught (a) in three phases and (b) in conjunction with number sense.

Three phases. Past research supports the teaching of multiplication in three progressive phases: (1) modeling or counting to determine the answer (i.e., equal groups, repeated addition, and arrays); (2) derive the answer using reasoning strategies and critical thinking (i.e., decomposing numbers); and (3) automatic retrieval or mastery of the facts (i.e., drills) (Baroody, 2006; Kling & Bay-Williams, 2015; Rave & Golightly, 2014). This progression of skills allows students to use their prior knowledge to build a conceptual understanding of multiplication, and students must not move from one phase to the next until they demonstrate a solid understanding of the strategies within each given phase. I used this progression during the innovation by having students begin with repeated addition, then arrays, and finally decomposing numbers. Each strategy helped my students better understand the next, more complex strategy. By teaching multiplication in the three progressive phases, students' learning remained strictly within their zone of proximal development (Vygotsky, 1978a). This progression allowed each student to learn at their pace and provided specific individualized remediation, as needed. In addition, I learned that as students became more comfortable with phase two (i.e., decomposing numbers), they were much better able to reason through problems and accurately derive answers.

This research should inform both preservice and inservice teacher education programs as it provides best practices for the teaching of basic multiplication. The teaching of multiplication in three progressive phases is essential to building a solid, conceptual understanding of multiplication (Baroody, 2006; Kling & Bay-Williams, 2015; Rave & Golightly, 2014). Rather than merely focusing on memorization of facts, which is all too common in many classrooms, the teaching of multiplication must focus on a strong and flexible sense of numbers (Boaler et al., 2015). Therefore, it is essential that multiplication is taught in this progression to develop conceptual meanings which is ultimately a basis for true fluency of the facts.

Number sense. This study taught me that students must attain a flexible sense of numbers in order to achieve fluency of multiplication (Boaler et al., 2015; Cumming & Elkins, 1999; Heege, 1985; Solomon & Mighton, 2017; Wells, 2012). While many teachers (Hurst & Hurrell, 2017; Kling & Bay-Williams, 2015; Young-Loveridge, 2005) overlook the importance of teaching relationships among multiplication facts in order to improve fluency, it is an essential step in deriving answers. For instance, 2 x 12 is twice as large as 2 x 6. I learned that by teaching strategies and multiplicative properties, students can begin to understand how numbers relate, which will enable them to derive the answers to problems without needing to rely solely on rote memorization (Denham, 2013; Woodward, 2006). This flexible sense of numbers was integral for students as they learned how to decompose numbers during this innovation. Students used this flexibility as they discovered how to break apart larger factors into smaller, easier numbers with which they were more familiar. As a result, my students were able to successfully derive answers to more difficult multiplication problems.

Teaching relationships among numbers is often the missing piece in building conceptual understandings of multiplication (Hurst & Hurrell, 2017; Kling & Bay-Williams, 2015; Young-Loveridge, 2005). Therefore, it is essential that both preservice and inservice teachers receive training to underscore the importance of this critical step in teaching multiplication.

Implications for Technology Integration in Mathematics

While numerous technologies are available for integration in mathematics, this study focused specifically on two. Implications for both preservice and inservice elementary mathematics teachers using (a) virtual manipulatives and (b) think-aloud recordings are explained below.

Virtual manipulatives. Virtual manipulatives are essential web-based representations of physical objects used for constructing mathematical understanding (Moyer et al., 2002). I learned that by integrating technology through virtual manipulatives into mathematics instruction, students are better able to visualize the concepts being taught, scaffold their understanding, and simplify the abstract ideas (Burris, 2013; Loong, 2014; Sowell, 1989; Suh & Moyer, 2008).

Integrating technology through the use of virtual manipulatives is considered more advantageous than using only concrete manipulatives for a variety of reasons. First, they are more easily accessible than concrete manipulatives and can easily be used by students as they reason through mathematical problems (Shin et al., 2017). Second, studies show that virtual manipulatives provide a variety of representations (at the appropriate level) to represent students' thinking to foster growth in conceptual understanding (Burris, 2013; Connell & Abramovich, 2016; Moyer-Packenham et al., 2013; Shin et al.). Third, virtual manipulatives are essential for teaching mathematics because they can be used as an individualized learning accommodation for students with learning difficulties, and they enable all students to better understand abstract concepts (Shin et al.). For example, virtual base ten blocks are more beneficial than concrete because the virtual tools allow students to more easily compose and decompose nonstandard numbers (Burris). For example, the standard form of 125 would be one hundred, two tens, and five ones cubes. Nonstandard representations would include other ways of building 125 with base-ten blocks, such as twelve tens and five ones. Other advantages of virtual manipulatives include that they bring mathematical ideas to conscious awareness, facilitate complete and precise explanations, support mental actions on objects, can change the nature of the shape by cutting apart virtual manipulatives (unlike concrete manipulatives), symbolize mathematical concepts, link concrete and abstract, and record and play students' actions (Clements & Sarama, 2016). Therefore, the functionality of technology-integrated virtual manipulatives significantly outweighs that of concrete manipulatives (Clements & Sarama).

Students must be taught how to use a variety of virtual manipulatives and then allowed to use whichever matches their needs and experiences when solving (Connell & Abramovich, 2016). Students must be developmentally ready to use abstract manipulatives to represent their thinking and have a strong understanding so they can make the connection between the concrete and abstract representations (Connell & Abramovich).

Virtual manipulatives should be provided for all students in order to scaffold understanding and represent abstract mathematical operations (Clements & Sarama, 2016; Shin et al., 2017). Consequently, due to limited technology training, preservice and inservice mathematics teachers must be made aware that such tools exist and then trained how to use virtual manipulatives so that they can effectively implement them in the classroom. Students in this study learned how to use three specific, highly-engaging manipulatives (i.e., Number Racks, Pattern Shapes, and Partial Product Finder) and enjoyed using them to solve multiplication problems. All students in this study were significantly more successful in solving multiplication problems when using the given virtual manipulatives.

Think-aloud recordings. Think-aloud recordings are the verbalization of one's step-by-step solution process (Silbey, 2002). Think-alouds are a type of technology integration that is highly-engaging for students and provides beneficial formative assessments for teachers. In this study, I learned that students must reflect on their thinking process in order to clearly explain how they derived an answer, which solidifies their thinking and enables students to develop mathematical arguments (Yang et al., 2016). This step in the thinking process is crucial for refining and evaluating one's own thinking and building mathematical understandings and reasoning (Chapin et al., 2009; Smith & Stein, 2011; Walshaw & Anthony, 2008).

Technology integration through student think-aloud self-recordings and interviews provide valuable insight into students' conceptual understanding of multiplication strategies. In this study, I learned that think-alouds were essential in providing a comprehensive look at students' conceptual understandings. Consequently, preservice and inservice teachers should be made aware of these technology integration strategies and trained to use both forms of think-alouds (i.e., student self-recordings and teacher interviews) to enable students to explain their thinking, identify their own mistakes, and demonstrate their levels of conceptual understandings. In addition, think-alouds should be used to allow the teacher to identify specific misconceptions and errors in thinking so that he/she can effectively remediate.

Implications for Future Research

The findings of this study offer implications for scholarly practitioners carrying out systematic inquiry within their own contexts and researchers who may be interested in studying technology integration with multiplication concepts in an elementary school classroom. Recommendations for future research include:

- Replicating this study in other third grade classrooms at the same school and/or at other schools. Research could include a broader selection of multiplication strategies such as making equal groups (De Corte & Verschaffel, 1996; Greer, 1992; Izsak, 2005) or using number lines as a spatial model (Gonsalves & Krawec, 2014; Woods, Geller, & Basaraba, 2018). Additional technologies such as online multiplication games (Denham, 2013; Zhang, 2015), tutorials, virtual flash cards, etc., and/or a broader selection of virtual manipulatives could also be incorporated. A wider variety of strategies or technologies would better allow the researcher to determine the most effective approach to teaching multiplication.
- Expanding this study to include students in higher grades who demonstrate a need for remediation in multiplication concepts. This will provide targeted and individualized instruction for students who do not demonstrate a strong sense of numbers and who have not achieved fluency of basic multiplication facts (Boaler et al., 2015; Zhang et al., 2014); and

- Replicating this study over a longer time frame and include the progression into phase three (i.e., automatic retrieval or mastery of the facts) of multiplication fluency (Baroody, 2006; Kling & Bay-Williams, 2015; Rave & Golightly, 2014).

If I chose to repeat this same action research study for multiple cycles, I would begin this study earlier in the school year to allow more time for students to progress through each phase of the multiplication continuum (Baroody, 2006; Kling & Bay-Williams, 2015; Rave & Golightly, 2014) to ensure that students had the opportunity to build a flexible sense of numbers and strong fluency of facts. The goal of this study would be for students to attain fluency by building a solid conceptual understanding of multiplication, as multiplication is the foundation for so many higher-level mathematical skills. By allowing more time for students to progress into phase three (Baroody; Kling & Bay-Williams; Rave & Golightly), I would expect students to build even greater fluency and improve automatic retrieval of facts.

Limitations

As with any research study, there are limitations that should be noted. These include limited resources, limited grade-levels, and the novelty effect of technology. The most significant of limitations was the number of resources utilized in this innovation. In order to hone in on specific skills and strategies, I purposefully limited the number of resources. For example, while there are numerous multiplication strategies, this study focused on three essential strategies (i.e., repeated addition, arrays, and decomposing numbers). I selected these three strategies for this study as they represent a progression of conceptual understandings. Likewise, while there exists a vast amount of technology integration possibilities for teaching multiplication, only two tools (i.e., virtual manipulatives and think-aloud recordings) were implemented in this study because they could easily be implemented with each multiplication strategy. Similarly, while there are many free virtual manipulatives available for elementary student use, this study focused only on three (i.e., Number Racks, Pattern Shapes, and Partial Product Finder). By controlling the number of resources, I was better able to target specific skills using technologies with which the students were familiar; however, this limitation prevented students from being able to use a large variety of multiplication strategies, technologies, and virtual manipulatives.

Implementation of this innovation was limited to ten participants in one third grade classroom. Since all students were not involved in this study, the findings may not be representative of the entire class or grade level. As is typical of classroom action research (Mertler, 2014), this study does not attempt to generalize findings beyond my own context. So, the applicability of these findings into other contexts remains with the reader's interpretations. In my own school context, additional students in other third-grade classrooms and other upper elementary grade-levels, such as fourth and fifth grades, might also benefit from this innovation.

The reliability of the pre- and posttests may also be a limitation of this study since they were teacher-made. While the tests did include some word (i.e., story) problems from a published textbook (McGraw-Hill Education, 2013), the remainder of the test questions were teacher-created.

One final limitation is the novelty effect of technology integration. Researchers (Montrieux, Vanderlinde, Schellens, & De Marez, 2015) suggest that incorporating new technology often results in an initial positive impact; however, as the technology becomes more commonplace, the learners lose interest and the technology loses its effectiveness. Therefore, this innovation may not have such a positive impact over an extended period of time.

Closing Thoughts

While American students seem to be improving overall in the area of mathematics (Desilver, 2017; NCES, 2015), there exist gaps in many students' conceptual understandings of major overarching concepts, such as the four basic operations (i.e., addition, subtraction, multiplication, and division). For many upper elementary students (both nationally and locally), a major area of conceptual misunderstanding is multiplication (Desilver, 2017; NCES, 2015). To achieve fluency of multiplication facts, students must be able to flexibly and accurately use an appropriate strategy in order to efficiently arrive at an accurate answer (Common Core State Standards Initiative, 2010; Kling & Bay-Williams, 2015). This means that teachers must provide students with a strong number sense and a variety of multiplication strategies to improve mathematical reasoning. By integrating technologies such as virtual manipulatives and student-think aloud recordings, students can make their thinking about abstract concepts visible, which will provide comprehensive insight into their conceptual understandings of multiplication strategies. As a result, teachers can then provide specific individualized remediation for each student. Together, these strategies will enable students to think more flexibly about numbers, and consequently, students will be better prepared to apply mathematical reasoning in the classroom, in real-world settings, and in the global economy.

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APPENDIX A

MULTIPLICATION STRATEGIES PRETEST

PRETEST

Name________________________

Multiplication Strategies Test

Read each question carefully, then solve using the virtual manipulatives on the https://www.mathlearningcenter.org/resources/apps website.

Part A. For questions 1-8, use repeated addition to solve. (Be sure to use the virtual math tools at the website above to help you solve each problem.)

1) 3 x 5 = _____

2) 4 x 6 = _____

3) 11 x 2 = _____

4) 8 x 3 = ____

5) 5 x _____ = 35

6) 4 x _____ = 12

7) _____ x 2 = 16

8) There are 5 spiders. Each spider has 8 legs. How many legs are there in all?

_____

Part B. For questions 9-16, draw an array to solve. (Be sure to use the virtual math tools at the website above to help you solve each problem.)

9) 4 x 2 = _____

10) 8 x 4 = _____

11) 9 x 3 = _____

12) 2 x 12 = ____

13) 5 x _____ = 50

14) 3 x _____ = 21

15) _____ x 6 = 30

16) Lindsay made a poster to display her photos. She made 2 rows with 4 photos in each row. How many photos did Lindsay display? _____

Part C. For questions 17-24, decompose numbers to solve. (Be sure to use the virtual math tools at the website above to help you solve each problem.)

17) 9 x 4 = _____

18) 7 x 3 = _____

19) 4 x 3 = _____

20) 6 x 8 = ____

21) 12 x ______ = 36

22) 11 x ______ = 55

23) ______ x 6 = 12

24) Calvin puts his books on shelves in his room. How many books does Calvin have if he puts 10 books on each of 5 shelves? ______

APPENDIX B

MULTIPLICATION STRATEGIES POSTTEST

POSTTEST **Name ______________________**

Multiplication Strategies Test

Read each question carefully, then solve using the virtual manipulatives on the https://www.mathlearningcenter.org/resources/apps website.

Part A. For questions 1-8, use repeated addition to solve. (Be sure to use the virtual math tools at the website above to help you solve each problem.)

1) 3 x 5 = _____

2) 4 x 6 = _____

3) 11 x 2 = _____

4) 8 x 3 = ____

5) 5 x _____ = 35

6) 4 x _____ = 12

7) _____ x 2 = 16

8) There are five spiders. Each spider has eight legs. How many legs are there in all?

_____

Part B. For questions 9-16, draw an array to solve. (Be sure to use the virtual math tools at the website above to help you solve each problem).

9) 4 x 2 = _____

10) 8 x 4 = _____

11) 9 x 3 = _____

12) 2 x 12 = ____

13) 5 x _____ = 50

14) 3 x _____ = 21

15) _____ x 6 = 30

16) Lindsay made a poster to display her photos. She made 2 rows with 4 photos in each row. How many photos did Lindsay display? _____

Part C. For questions 17-24, decompose numbers to solve. (Be sure to use the virtual math tools at the website above to help you solve each problem).

17) 9 x 4 = _____

18) 7 x 3 = _____

19) 4 x 3 = _____

20) 6 x 8 = ____

21) 12 x _____ = 36

22) 11 x _____ = 55

23) _____ x 6 = 12

24) Calvin puts his books on shelves in his room. How many books does Calvin have if he puts 10 books on each of 5 shelves? _____

APPENDIX C

THINK-ALOUD QUESTIONS/ INTERVIEW PROTOCOL

Date: __________________ Interviewee: ___________________________

Location: _______________ Interviewer: ___________________________

Instructions: Meet with each student in the focus group individually to discuss the following questions. Each student should have access to his or her Chromebook and the virtual manipulatives on the https://www.mathlearningcenter.org/resources/apps or on http://www.abcya.com/third_grade_computers.htm#numbers-cat website. The interviewer should video record the student responses and also take notes as the student responds (in case of video error). Be sure to thank the student upon completion.

1) How do you feel about your understanding of multiplication? Has your understanding of multiplication improved since we started working together in the focus group?

2) How can I solve 4 x 3 using repeated addition? (Show your thinking using virtual manipulatives).

3) How can I use an array to solve 6 x 4? (Show your thinking using virtual manipulatives).

4) How can I decompose 7 x 3 to help me solve the problem? (Show your thinking using virtual manipulatives.)

5) Which multiplication strategy do you prefer and why?

Upon completion of interview, state to student: "Thank you so much for meeting with me today and discussing what you know about multiplication strategies."