Modeling Of Tow Wrinkling In Automated Fiber Placement Based On Geometrical Considerations

Roudy Wehbe
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MODELING OF TOW WRINKLING IN AUTOMATED FIBER PLACEMENT BASED ON GEOMETRICAL CONSIDERATIONS

by

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The Lebanese American University, 2015

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College of Engineering and Computing

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ABSTRACT

Automated manufacturing of fiber reinforced composite structures via numerically controlled hardware yields parts with increased accuracy and repeatability as compared to hand-layup parts. Automated fiber placement (AFP) is one such process in which structures or parts are built by adding bands of prescribed number of tows or slit-tape with prescribed width using robotic machine heads over 3D surfaces following prescribed paths. Despite the improved accuracy, different types of defects or manufacturing features arise during fabrication. These defects can be due to geometrical features, materials, and process planning parameters and are detected in the form of wrinkling, tow twist, tow folding, overlap, gaps and several others.

This thesis presents a thorough investigation of wrinkling within a path on a general surface for a composite tow constructed using the AFP process. Governing equations and assumptions for the presented model are derived based on geometric considerations only, neglecting the elastic properties of the material, and formulated for an arbitrary curve on a general three-dimensional surface. A simple form of the wrinkled shape is assumed and applied to the inner edge of the tow path. A numerical solution is implemented within Mathematica to visualize the curved paths and to indicate potential regions for wrinkling on the surface. Several examples are presented to demonstrate the model, including constant angle paths on a double-curved surface and curved paths on a flat surface. The obtained deformed patterns are compared with actual data from digital image correlation (DIC) of several towpaths.
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</tr>
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<tbody>
<tr>
<td>(a)</td>
<td>Amplitude of the wrinkle</td>
</tr>
<tr>
<td>(B)</td>
<td>Binormal vector</td>
</tr>
<tr>
<td>(C)</td>
<td>Curve in parametric form</td>
</tr>
<tr>
<td>(C_p)</td>
<td>Curve in parametric form parallel to (C)</td>
</tr>
<tr>
<td>(d)</td>
<td>Distance</td>
</tr>
<tr>
<td>(e)</td>
<td>Coefficient of the second fundamental form</td>
</tr>
<tr>
<td>(E)</td>
<td>Coefficient of the first fundamental form</td>
</tr>
<tr>
<td>(f)</td>
<td>Coefficient of the second fundamental form</td>
</tr>
<tr>
<td>(F)</td>
<td>Coefficient of the first fundamental form</td>
</tr>
<tr>
<td>(g)</td>
<td>Coefficient of the second fundamental form</td>
</tr>
<tr>
<td>(G)</td>
<td>Coefficient of the first fundamental form</td>
</tr>
<tr>
<td>(G)</td>
<td>Geodesic curve</td>
</tr>
<tr>
<td>(k)</td>
<td>Amplitude of the wrinkling rotation angle</td>
</tr>
<tr>
<td>(k_g)</td>
<td>Geodesic curvature</td>
</tr>
<tr>
<td>(K)</td>
<td>Stiffness of the elastic substrate</td>
</tr>
<tr>
<td>(K)</td>
<td>Gaussian curvature</td>
</tr>
<tr>
<td>(K_V)</td>
<td>Knot vector for a 2D curve</td>
</tr>
<tr>
<td>(l)</td>
<td>Wrinkle length</td>
</tr>
<tr>
<td>(L)</td>
<td>Length of a tow</td>
</tr>
</tbody>
</table>
\( S \)  Parametric surface  
\( n \)  Parameter in the normal direction  
\( N \)  Normal vector  
\( N \)  Number of wrinkles  
\( N_{i,p} \)  B-spline basis function of degree \( p \)  
\( p \)  Degree of a spline curve or degree of a spline surface in the \( u \)-direction  
\( P \)  Point in three dimensional coordinates  
\( q \)  Degree of a spline surface in the \( v \)-direction  
\( \mathcal{R} \)  Rodrigues’ rotation matrix  
\( t \)  Parameter in the tangential direction  
\( T \)  Tangent vector  
\( u \)  Surface parameter in the first direction  
\( U \)  Knot vector of a spline surface in the \( u \)-direction  
\( v \)  Surface parameter in the second direction  
\( V \)  Knot vector of a spline surface in the \( v \)-direction  
\( w \)  Width of a tow  
\( w_i \)  Weight of a control point \( i \)  
\( \beta \)  Wrinkle rotation angle  
\( \Gamma^i_{jk} \)  Christoffel symbols  
\( \varepsilon_i \)  Strain in the longitudinal direction  
\( \lambda \)  Amplitude of wrinkles on elastic substrate  
\( \hat{\ } \)  Unit Vector  
\( \mathbf{B} \)  Vector represented in Bold style
LIST OF ABBREVIATIONS

AFP ................................................................. Automated Fiber Placement
ATL ................................................................. Automated Tape Laying
CAD ................................................................. Computer Aided Design
CTS ................................................................. Continuous Tow Shearing
CFRP ............................................................... Carbon Fiber Reinforced Plastics
DIC ................................................................. Digital Image Correlation
FEA ................................................................. Finite Element Analysis
FMM ................................................................. Fast Marching Method
FRP ................................................................. Fiber Reinforced Plastics
GFRP ............................................................... Glass Fiber Reinforced Plastics
LED ................................................................. Light Emitting Diode
NURBS ............................................................. Non-Uniform Rational B-Spline
ODE ................................................................. Ordinary Differential Equation
RTM ................................................................. Resin Transfer Molding
STL ................................................................. Stereolithography
UD ................................................................. Unidirectional
VARTM .......................................................... Vacuum Assisted Resin Transfer Molding
CHAPTER 1

INTRODUCTION

1.1 PREAMBLE

Composite materials are known to man since thousands of years; the usage of wood and natural glue, straw reinforced clay bricks to build structures are few of these examples. Recent development in the constituent materials, mainly continuous fibers and polymer matrices, made the newly engineered composite materials more competitive and desirable. This advancement attracted several industries such as the aerospace, automotive, and renewable energy, due to the possibility of simultaneously increasing the performance and reducing the weight of structures.

Specifically for the aerospace industries, fiber reinforced plastics (FRP) are attractive compared to their metallic counterparts due their improved mechanical properties, possibility of weight reduction, better corrosion resistance, and enhanced maintenance. However, these advantages can be undermined due to defects that occur during the manufacturing process, resulting in weaker parts than designed.

Manufacturing high performance composites part for aerospace applications is historically done using glass-fiber reinforced plastics (GFRP), or carbon-fiber reinforced plastics (CFRP) in a manual layup process. Hand layup consists of cutting and laying the fibers in a mat form on an open mold successively to achieve the required part thickness in a prescribed set of orientations referred to as the stacking sequence. Hand layup can use
either dry or wet fibers, and the fibers can be unidirectional (UD) or woven. For the dry process, dry fibers are placed on the surface of the mold, and resin is either poured or infused to the part under vacuum, in resin transfer processes such as resin transfer molding (RTM) or vacuum assisted RTM (VARTM). Consequent oven curing might be necessary to achieve the desired properties. To avoid the risk of partially wetting the fibers, getting high void contents, and/or low fiber volume fraction, high performance composites usually use pre-impregnated fibers or also called prepreg. The process of hand laying prepreg is similar to the dry fibers, with additional option to use debulking process between the consecutive layers to reduce the amount of air entrapped between layers. The curing process is usually done at high pressure and temperature in an autoclave to further reduce the amount of voids. For some complex parts, hand layup can be done on a flat tool and later forming process is used to obtain the final shape. Quality control during the manufacturing process of the hand layup is usually done by visual inspection, with recent notable efforts to have automated inspection systems.

To meet the increasing demand of the aerospace industry, the productivity of the composite manufacturing has to be augmented. Several programs such as the Boeing 787 and the Airbus A350 have approximately 50% by weight of their structural components made from composites. Automation is a solution to achieve this high volume of production. First, Automated Tape Laying (ATL) has been introduced in the 1970s, then Automated Fiber Placement (AFP) was commercially available in the 1980s [1] to answer both productivity and consistency of the manufacturing process. ATL consists of delivering a wide tape of prepreg, generally on flat or slightly curved surfaces, while removing automatically the backing material from the prepreg. Temperature, speed and tape tension
can be varied during the process to achieve a good layup. AFP is similar to the ATL process, however the wide tape is replaced by several narrow prepreg slices called tows which can be individually controlled. The advantage of doing that is two folds: (1) to reduce waste, (2) to increase process flexibility enabling advanced capabilities such as manufacturing complex surfaces.

![Figure 1.1 Gantry type Automated Fiber Placement Machine at the McNair Center for aerospace innovation and research](image)

1.2 AFP PROCESS DESCRIPTION

Automated fiber placement consists of using numerically controlled robot or gantry type machine (Figure 1.1) to deliver the material with required position and orientation. Typical material that is laid down using AFP machines have a width of \(\frac{1}{8}\) in, \(\frac{1}{4}\) in, or \(\frac{1}{2}\) in. A single strip of material is named tow. Normally AFP delivers multiple tows in a single
sequence to form a course, while a sequence of courses is termed ply. Industry has a tendency to optimize production rates and thus higher number of tows and bigger tow width are preferred. The choice of tow width heavily depends on the complexity of the manufactured part to avoid defects such as wrinkling which will be demonstrated in this work. Smaller width tows are capable of manufacturing more complex shapes, while wider ones are used for flatter parts. Hence, reducing the amount of defects by using narrower tows comes to the cost of lower machine productivity.

The main material used for AFP is impregnated tows or slit tape. Other material such as dry fiber or thermoplastic are used to a lesser extent with a tendency to increase in the future. Individual tows are wound on a bobbin and stored in the creel for the gantry type machine or directly mounted on the machine head for the robotic type. Additional backing material is supplied between the tows to reduce the shredding defect while unspooling. The tows are fed through the machine head to the tow tensioner then through the compaction roller where additional heat and pressure is applied for the material to adhere to the surface. The roller material is usually flexible to increase the contact area and to reduce the void between the layers. Typical external heating sources used in AFP are torches, laser, and infrared radiation. Another advantage of the AFP process is the ability to control the tows individually and to cut and restart the tows as necessary. The cut and restart capability allows for further reduction in waste material compared to the ATL and hand layup, while controlling the speed of the tows individually makes it possible to lay material over complex surfaces and some in-plane steering. The limitation of the cutting and restarting mechanism is dictated by the minimal course length which corresponds to
the distance between the nip point and the cutting mechanism and dictates the smallest features that the AFP can manufacture.

1.3 DEFECTS IN THE AFP PROCESS

Due to the complexity of the AFP mechanism, the material being used, or the part to be manufactured, several defects may arise during the manufacturing process and can be detrimental to the produced part. Some of these defects can be related to the material variability, some to the machine or process parameters and some to the design or geometrical model by itself.

For instance, defects such as gaps and overlaps (see schematic in Figure 1.2) occurring between adjacent tows or courses can be related to some machine tolerances, calibration error or material variability, and sometimes cannot be detected during the design phase. However, these features are necessary and present in the design especially for the case of complex surfaces and variable stiffness panels to avoid excessive steering that might lead to other defects such as wrinkling. Other similar position errors can occur at the edge of the part or at the start or end of a tow, whereas a twist and a fold can sometimes be related to the machine rotational movements or material tack. A spliced tow is also present in the material provided by the manufacturer and corresponds to a small overlapping region necessary to join two tows in the spool. A schematic of these defect is presented in Figure 1.2. The presence of these defects during the manufacturing is usually undesirable, and in most cases, the process is interrupted for manual repairs, which leads to a decrease in the productivity. An experimental study [2] was conducted on the coupon level isolating 4 types of these defects (gap, overlap, half gap/overlap, and twist) and trying to understand their effect on the ultimate strength. However, the influence on the overall
structure is not well understood, and further research is needed to decide whether or not a repair is necessary during manufacturing.

![Typical Defects during AFP Process](image)

Figure 1.2 Schematic of typical defects during AFP process

Wrinkling or sometimes referred to as puckering, or tow buckling is the out-of-plane deformation of the tow edges that resembles buckling patterns. Tow wrinkling during AFP process is mainly observed on flat surfaces while steering the fiber-tow to follow non straight (curved/non-geodesic) paths. It also occurs on complex curved surfaces even if the center of the tow follows a geodesic path due to the finite width of the tow and the surface curvature. The primary reason for wrinkling occurrence is the mismatch in length between the prescribed path on the surface and the actual length of the tow delivered from the machine head (Figure 1.3). If the length of the tow fed from the machine head has a length \( L \), and it has to be placed on a curved path whose centerline has a length \( L \), then the
portion of the tow laying inside the vicinity of the curve has to absorb an additional length of $\Delta L$, and the portion laying on the outer side of the vicinity has to extend an additional length of $\Delta L$.

![Figure 1.3 Length mismatch between a tow and a curved tow-path](image)

To absorb these differential lengths caused by the mismatch between the tow and the prescribed path, six deformation mechanisms are suggested and presented in Figure 1.4. These mechanisms can be classified under three categories: (I) elastic deformations, (II) in-plane deformations, and (III) out-of-plane deformations. For the case (I) of elastic deformations, the excess of length on the shorter/inner side of the tow-path is absorbed by compressive strains in the tow, and the shortage in length on the longer/outer side of the tow-path is absorbed by tensile strains in the tow, in a way similar to the beam bending theory. Also, shear deformations between individual fibers within the tow especially near the start and the end of the tow-path can absorb some of the differential length. As for the case (II) of in-plane deformations, tow bunching occurs on the tensile side of the tow and is characterized by a local increase in the thickness of the tow and fiber misalignment. In this case, the fibers laying on the outer side of the tow-path are actually shorter than the path itself and tend to move towards the centerline where the length is somehow similar. The presence of good tackiness or adhesion between the tow and the surface beneath (other tows, or the mold surface) is essential for this mechanism, otherwise the tow will tend to lift from the surface and fold over itself as presented in the out-of-plane mechanism (III).
As for the compressive side, the in-plane waviness can occur in the presence of a good surface adhesion, otherwise, out-of-plane wrinkling can occur.

![Diagram of deformation mechanisms](image)

Figure 1.4 Deformation mechanisms for differential length absorption

Modeling tow wrinkling is a complex task that involves process parameters (temperature, pressure and speed), material properties (viscoelastic behavior and tackiness), and geometry. Literature exists that applies the methods of stability analysis of rectangular plate-like composites with appropriate boundary conditions under loads and deformations. For example, modeling of buckling has been studied in [3], as applied to slit-tape during automated fiber placement manufacturing to establish the critical radius at which tow wrinkling occurs. Similarly, [4] used the same physical foundation to find a closed form solution for the critical steering radius in automated dry fiber placement by assuming a cosine shape function. Experimental validation for these models is difficult,
since the boundary conditions are difficult to establish, in particular based on the lack of information on the tackiness of the foundation which is a complex parameter to identify.

In this thesis, the stability analysis based on finite size plate-like element assumption is abandoned in favor of a functional form of the tow-path with finite width. In addition, the effects of the process parameters, the viscoelasticity of the material, the tackiness of the tow with respect to its underlying surface, and the elastic properties of the tows are not investigated. In some sense, the slit-tape or fiber tow is approximated by a finite-width ribbon with infinitely large in-plane stiffness and negligible bending stiffness. Hence, this work will focus on the following points concerning the geometrical aspect of wrinkling formation:

- Numerically investigate the effect of the tow width and curvature on the amplitude of the wrinkles.
- Examine wrinkling formation on geodesic and non-geodesic paths placed on general surfaces.
- Generate a color coded map for layups on general surfaces indicating possible regions of wrinkling
- Validate the model by experimental procedures, using out-of-plane displacement measuring techniques such as digital image correlation applied to tows placed on curved paths

1.4 THESIS OUTLINE

To answer these points, and to fully understand the wrinkling formation from a geometrical perspective, this thesis is organized as follows:
Chapter 2 presents a literature review of the wrinkling phenomenon occurring in several aspects and fields, and the effort made to model it. Then, the wrinkling term is investigated in several manufacturing processes of composites laminates, especially automated fiber placement, and the main reason why this thesis only focuses on the geometrical aspect of wrinkling. Finally, several layup strategies are demonstrated as potential candidates for wrinkling analysis.

Chapter 3 develops the governing equations of tow wrinkling during automated fiber placement of tows on general surfaces. A simplified version of the governing equations is presented for tows placed on a flat surface.

Chapter 4 implements the developed equations in several examples of curved tows placed on flat plates to investigate the effect of tow width and path curvature. Examples of wrinkled tows placed on geodesic and non-geodesic paths on general surfaces are also presented. Wrinkling analysis of several layups on a general surface is conducted, and the results are presented in a color map indicating possible regions of wrinkling.

Chapter 5 validates the developed equations by comparing the model to the shape of actual tows placed on several curved paths where measurements are made using digital image correlation.

Chapter 6 concludes the presented work with recommendations and future research opportunities.
CHAPTER 2

LITERATURE REVIEW

This literature review chapter starts by discussing the wrinkling phenomenon in nature, and the effort made to model it as a thin film on a compliant substrate. Then, an overview of wrinkling during composite manufacturing is presented such as wrinkling during forming process, and during consolidation over an internal or an external radius. Afterwards, wrinkling during the AFP process is described, possible reasons behind are discussed based on the current literature, and the attempts to model it are also presented. Finally, the geometric aspect behind the layup strategies for the AFP process is presented, from the creation of the reference curves to obtaining total surface coverage and their relationship to tow wrinkling.

2.1 DIFFERENT ASPECTS OF WRINKLING PHENOMENON

A typical English dictionary definition of a wrinkle, is the word referring to the creation of lines, ridges, or creases on a surface due to a contraction, folding, crushing, or the like, and it is a common phenomenon in life, mainly observed in elderly people skin, fabric, or clothing items. However, this phenomenon stretches to several other aspects and can be observed in the nature in flowers and leaves [5] due to the natural excessive growth of cells making some wavy patterns (Figure 2.1). In addition, the scale of this phenomenon extends from elastomers at the nanoscale to mountains over several kilometers as presented in [6] and reproduced here in Table 2.1. Other case studies on wrinkling were discussed in
which were based on the buckling model of a thin film laying on soft foundation (Figure 2.2), such as wrinkling in human skin, wrinkling in rigid sheets on elastic and viscoelastic foundations. It was concluded that wrinkling is not always a bad phenomenon as depicted in the wrinkling of human skin, it can also be used as a guide in materials assembly, fabrication of several functional devices and assist in measuring material properties.

![Figure 2.1 Wavy pattern in an originally flat eggplant leaf after 12 days of injection of growth hormone [5]](image1)

![Figure 2.2 Basic wrinkling/buckling model of a thin film laying on soft foundation [6]](image2)

Table 2.1 Examples of wrinkling of skins on softer elastic foundations reproduced from [6]

<table>
<thead>
<tr>
<th>Context</th>
<th>Wavelength (m)</th>
<th>Skin</th>
<th>Foundation</th>
<th>Compressive force</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mountains</td>
<td>$10^3$</td>
<td>Earth crust</td>
<td>Earth mantle</td>
<td>Tectonics</td>
</tr>
<tr>
<td>“Mosquito” wing failure</td>
<td>$10^{-1}$</td>
<td>Plywood</td>
<td>Balsa</td>
<td>Bending</td>
</tr>
<tr>
<td>Modern sandwich failure</td>
<td>$10^{-2}$</td>
<td>Glass fiber-reinforced epoxy</td>
<td>Polyurethane foam</td>
<td>Bending</td>
</tr>
<tr>
<td>Skin wrinkling</td>
<td>$10^{-3}$</td>
<td>Epidermis</td>
<td>Dermis</td>
<td>Skin stretching/compression</td>
</tr>
<tr>
<td>Fruits</td>
<td>$10^{-3}$</td>
<td>Skin</td>
<td>Flesh</td>
<td>Drying</td>
</tr>
<tr>
<td>Physically treated elastomers</td>
<td>$10^{-8}$-$10^{-3}$</td>
<td>Metal or oxide film</td>
<td>Elastomer</td>
<td>Pre-stretching, stretching or thermal expansion</td>
</tr>
</tbody>
</table>
A general theory for wrinkling is presented in [7] based on the balance between the bending energy and the stretching energy of the thin film on a substrate, and constraining the problem by imposing the geometry with a Lagrange multiplier, and minimizing the total energy of the system. This theory can be extended to cover a large amount of physical problems using a scaling law, in which the wavelength of the wrinkles $\lambda$ is proportional to $K^{-1/4}$ with $K$ being the stiffness of the elastic substrate, and the amplitude of the wrinkles is proportional to the wavelength $\lambda$. In addition, this work identifies two main wrinkling behaviors: the compression wrinkles which arise in one dimensional stress field and the tensile wrinkles which arise in two-dimensional stress field (Figure 2.3). The general theory presented in [7] was inspired from wrinkling problem that appears in thin stretched elastic sheets as shown in Figure 2.4. These wrinkles appear due to compressive stresses induced by Poisson’s effect in the central region of the sheet [8]. However, it has been recently shown that Poisson’s effect is not only the main reason, but also shear warping deformations triggering the wrinkles in stretched sheet [9].

Figure 2.3 Wrinkle patterns in a film subjected to non-uniform strain field [10]

Figure 2.4 Wrinkles in a polyethylene sheet under uniaxial tensile strain [7]
Although the wrinkling phenomenon have been heavily studied for the past decades, three main issues concerning the wrinkling/buckling theories presented above were highlighted in [6], and are necessary to fully understand the wrinkling problem:

1. The morphology of buckles and wrinkles depends on the direction of the force acting on the foundation, and for the case of multidirectional stresses, wrinkling can be hard to predict.

2. Simple theories which were presented in [6], assumes the skin to be much stiffer than the foundation, however in other instances the skin is not much stiffer than the foundation.

3. The system is assumed to be bilayer neglecting the effect of the interface, and laying on completely flat surface.

Relating tow wrinkling during AFP to the wrinkling phenomenon presented in the literature above, these main issues hold true. First, the stresses applied on the tow are not uniform even for the simple case of 2D steering since the tow undergoes in-plane bending to follow its path which results in a combination of both tension and compression depending on the side of the tow. These stresses are even more complex especially for the case of doubly curved surfaces where out-of-plane bending and twisting are involved and necessary to adhere to the surface. Second, it is true that the tow in the fiber direction is much stiffer than the viscoelastic foundation, however in the transverse direction, the stiffness of the tow and the stiffness of the foundation are fairly close. Third, the tow is not always laying on a completely flat surface even for the case of the 2D steering, where the surface can be flat if directly placed on the mold, or it can be non-uniform depending on the existence of previous layers, their direction, and the presence of possible
features/defects as discussed earlier and presented in Figure 1.2. For these highlighted issues, tow wrinkling during AFP process can be hard to predict and a complex task to model.

2.2 WRINKLING IN COMPOSITE LAMINATES

Wrinkling during manufacturing of composites parts appears in several processes. For instance, during the process of diaphragm forming shown in Figure 2.5, laminate wrinkling can appear while trying to shape a flat stack of preform on a curved mold. A laminate wrinkling scaling law is derived in [11] based on ideal kinematics assumptions. The shear strain between the fibers is assumed to absorb the differential length, and a general formula for the complex shapes is derived based on ideal kinematics law. Differential geometry examples are discussed with two examples: hemisphere and curved C-channel. The difference between the proposed ideal prediction and actual results is explained by the deviation of the actual material from the ideal kinematics laws, and an empirical scaling law is proposed.

Figure 2.5 Schematic of the diaphragm forming process [11]
Another example of wrinkling in composites manufacturing occurs during consolidation of plies over an external radius (Figure 2.6). As the outer layer is forced to a tighter geometry and if the layers cannot shear between each other, these layers can form wrinkles (Figure 2.7). A 1D model is presented in [12] and consists of solving the buckling problem by minimizing the total potential energy of the one dimensional system assuming that the laminate is laying on an elastic foundation and constrained at both ends with point forces. As a result, the critical conditions of wrinkling appearance can be established. Results from this model show a good agreement with wrinkles observed on a spar demonstrator.

![Figure 2.6 Consolidation over external radius process [12]](image1)

![Figure 2.7 CT image of a corner wrinkle on a spar [12]](image2)

A similar wrinkling behavior can appear while laying prepreg material over internal radius (Figure 2.8). Two layup procedures were investigated in [13] for this case: the conventional layup consisting of laying the prepreg mats consecutively directly on the mold, and an alternate method consisting of laying the material on a flat surface and then bending the whole stack to conform to the inner radius. It was shown that in the later method the amount of wrinkling on the inner radius is increased, however, the spring-in values were decreased. These values were confirmed using Finite Element Analysis (FEA) simulation. A recent study on internal wrinkling instabilities in layered media is presented.
in [14] and uses large deformation Cosserat continuum model to capture these instabilities at a fraction of computational cost compared to conventional modeling techniques. The same wrinkling problem occurring during consolidation over external radius discussed in [12] is modeled in [14] showing good agreement in the results.

![Figure 2.8 Wrinkles in layup over internal radius (Left), wrinkles converting to in-plane fiber waviness after cure (Right) [13]](image)

Another mechanism for wrinkling formation is presented in [15] and it is due to shear between plies (Figure 2.9). For this case, the shear forces are generated as a mismatch between the thermal expansion of the tool and the composite, as well as ply slippage during consolidation into an inner radius. It was found that this type of wrinkling as well as fiber misalignment in the plies can be eliminated by increasing the frictional shear stress between the tool and the part by removing the release film slip layer.

![Figure 2.9 Wrinkle formation due to shear slip between the composite and the mold [15]](image)
In addition to ply wrinkling, fiber wrinkling, waviness and misalignment can occur during manufacturing, and are considered as defects. A general overview of these defects, their origin and variability in composite manufacturing is presented in [16]. The main focus of this work is on defects related to autoclave and resin transfer molding, however some of the listed defects are also common to the automated fiber placement process. Specifically, fiber waviness or wrinkling that occurs when draping unidirectional prepreg over a single curved (Figure 2.10) or doubly curved surface. This kind of defect is not optional and does not occur due to poor manufacturing, instead it is a function of the geometry [16]. This defect will lead to fiber misalignment and thus degradation in the overall properties of the produced part. This topic was further developed in [17], by discussing the effect of variability, misalignment and fiber waviness defects on the properties of the composite laminate or structure for unidirectional and woven fibers. It differentiates between design induced defects which can only be eliminated at the design stage, and process induced defects which can be eliminated by the correct choice of manufacturing process. Furthermore, examples of defects caused by the excess of length on one side of the unidirectional prepreg are shown in [17]. This excess of length can cause waviness for the unidirectional prepreg when wound on a drum for transportation or storing purposes, or it can cause wrinkling, or out-of-plane buckling for tows following curved trajectory during the AFP process, or it can lead to wrinkling and waviness of lay-up around sharp corners of mold sections.

Concerning the AFP process, an extensive overview of the automated prepreg layup is presented in [1] by discussing the history and past issues of both ATL and AFP, and for both thermoset and thermoplastic materials. Future research opportunities for the AFP were
also discussed, such as productivity, steering and control and others. Specifically, [1] classifies the defects that arise from fiber steering in three categories as illustrated in Figure 2.11: tow buckling occurring on the inside radius of the tow due to compressive forces, tow pull-up occurring on the outside of a tow due to tensile forces, and tow misalignment occurring as a result of variability in the layup system, layup control of prepreg material.

Figure 2.10 Level of misalignment generated by draping narrow and wide UD prepreg strips across a 100mm diameter hemisphere [16]

Figure 2.11 Overview of the most common steering defect [1]

A discussion of the effect of the tape width on the out-of-plane buckling of the tows when placed on a curved surface is presented in [17]. This defect is increased by increasing
the width of the tape, or if the path is non-geodesic on the surface. Experimental trials on the effect of tow width on the minimum steering radius to avoid wrinkling during AFP were carried out in [18]. A minimum radius of 1250 mm was found for ¼ in tows for a defect free layup, whereas 500 mm was the minimum radius if using ⅛ in wide tows. This vast difference between the two widths makes the ⅛ in tow more favorable for steering and for the fabrication of geometrically complex parts.

An analytical approach to model wrinkling of tows during the AFP process is presented in [3]. The model assumes the tow to be a thin plate laying on an elastic foundation, and it solves parametrically for the critical steering radius. Parametric results show that the steering radius can be decreased by decreasing the width of the tape similarly as discussed earlier in [17] and [18]. Also, increasing the tackiness or stiffness of the foundation reduces the limit of the steering radius. However, the theoretical solution was not backed up by experimental results, since other process parameters were not included in the model such as the speed of the layup, the applied temperature and pressure, and the viscoelastic behavior of the resin.

A similar approach is used in [4] where a closed form solution is developed based on the theory of buckling of orthotropic plate under non-uniform loading. A closed form solution was obtained by assuming a cosine shape function to the buckled edge of the tow. Results show that tack stiffness has the most influence on wrinkling formation. In addition, parametric studies show that increasing the temperature of the layup within the admissible limits can increase the quality of the layup. However, this work is limited to the study of dry fiber and does not include any viscoelastic effect of the resin, if thermoset prepreg were to be used. In addition, very limited amount of experimental results with undefined
parameters were conducted to back up the results. The stiffness parameter used in this work is also hard to measure and quantify even for the dry fibers, hence assumed values were used in this work.

To sum up, wrinkling during manufacturing of composite structure is a common phenomenon and it was discussed and modeled in several cases such as diaphragm forming, consolidation of preregs over an external or an internal radius, or it can happen due to shear between the layers, or due to a layup over complex surfaces. Concerning the AFP process, wrinkling was mainly investigated during steering on a flat mold, where the process parameters and tow width were changed to achieve tighter radii of curvature. Some attempts were made to model wrinkling during the AFP process, however these attempts fell short on the experimental validation of the models, and in the complexity of measuring and determining the stiffness of the foundation parameter which was determined to be one the most important aspects of wrinkling.

2.3 LAYUP STRATEGIES FOR AFP

This section describes the geometric aspect behind the wrinkling phenomenon during the AFP process. Starting from a given mold shape, a coverage strategy is necessary to obtain the path that the machine head has to follow to lay down the fiber tows and fabricate the part. Typical strategies start from a given reference curve depending on the load requirement or fiber direction needed to achieve this load. Then, this curve can be propagated in several ways, or other curve-path can be recreated to achieve full coverage. This summary of the literature contains two major section: the first one discusses several ways to create reference or initial curves on the mold surface such as fixed-angle curves,
geodesic curves, and others… The second section discusses strategies to propagate these curves to cover the mold surface such as the parallel method or shifted method.

2.3.1 Reference Curves

Before covering the surface, an initial (or reference) curve is needed. In this section, the different strategies to find the reference curve will be detailed. Both, a parametrical approach, and the use of a mesh, can be found in the literature. Each method has its advantages and drawbacks which have been summed up by [19] for automated spray painting: a mesh provides useful information, such as the different areas of the facets and their normal, which are important to generate the toolpath along the course, however the mesh is an approximation of the geometry and more precision comes at the computational cost. Most references found in the literature use a parametrical approach, since the surface would be precisely known. In this review, reference curves are classified under three categories: fixed-angle reference curves, geodesic curves, and the remaining types of reference curves are grouped under variable-angle guide curves.

2.3.1.1 Fixed angle reference curves

Using a fix angle strategy, the fiber angle in the reference curve is constant all along the surface. This layup strategy is very used as it allows a complete control of the fiber angles along the surface and resembles the conventional layup of quasi-isotropic laminates where the angles 90°, 0° and ±45° are the main used ones. Starting from a meshed surface, the method presented in [20] uses the mesh information contained in an STL (Stereolithography) file to generate the topology of the surface including the vectors/edges, the nodes, the facets, and the normal vectors of the facets. From these features, a slicing algorithm is used to find the reference curve which is the intersection between a plane at a
fixed direction and each triangular element on the meshed surface. An advantage of this method is that finding another fix angle path is relatively simple: the meshed plane can be rotated by the required angle from the previous path to obtain the new one.

A similar approach to find a fixed angle curve on a parametric surface is found in the literature ([21], [22] [23]) by intersecting the parametric surface $S(u, v)$:

$$S(u, v) = X(u, v)i + Y(u, v)j + Z(u, v)k,$$

with the plane $P(x, y, z)$ defined by the projection of the major axis into the surface (Figure 2.12):

$$P(x, y, z) = ax + by + cz + d = 0,$$

resulting the following equation:

$$f(u, v) = aX(u, v) + bY(u, v) + cZ(u, v) + d = 0.$$

A numerical solution for equation (2.3) is usually required to find the reference curve if the parametrization of the surface is complex, and geometrical algorithms such the ones presented in [24] have to be used. For the case of simple surface parametrization such as the case of conical shells presented in [25], a closed form solution for the reference curve at a fixed angle can be obtained, hence resulting in fast and efficient substitution.

Another approach to generate a fixed angle path is presented in [26] and [27]: starting from a given point $P_i$, a vector $d$ tangent to the surface is projected following the major direction (Figure 2.13). Another vector $t$ is found by rotating $d$ by an angle $\psi$ around the normal to the surface $n$. Another point $P_f$ is defined on the vector $t$ at a small distance from $P_i$. Finally, $P_f$ is projected on the surface to find the next point $P_{i+1}$. This process iterates until the path reaches the boundary of the surface.
The main disadvantage of using this method as a reference curve is it might violate the amount of curvature or the minimal steering radius at which the fiber tows start to buckle, hence resulting in a difficult manufacturing process if not impossible in some severe cases.

2.3.1.2 Geodesic guide curves

A layup strategy to avoid steering is to compute a geodesic guide curve. The geodesic path can also be known as the natural path, it is the shortest path between two points along a three-dimensional surface in Cartesian space. For the case of a flat plate, a geodesic path is only a straight line connecting two points. Also, a geodesic path can be obtained by specifying a starting point and a direction of travel. For a general parametric surface, a geodesic path has to satisfy a system of differential equations (discussed later in section 3.3 Geodesic Path Definition and Geodesic curvature, equation (3.16)) for which a numerical solution is needed. For the case of a cone, a closed form solution for a geodesic path can be obtained [25].
For more complex shapes, such as the Y shape investigated in [27], geodesic paths cannot be easily generated. Starting from one branch of the Y surface, and given an initial fiber angle, a geodesic path can defined. However, once at the junction of the Y, the geodesic path might change or won't be able to propagate on the remaining part of the surface. Several solutions to continue the path were presented: proceed in the direction of the minimum curvature, try to reach a geodesic path on the other branch of the Y (Figure 2.14) or create a straight path on the other branch respecting the steering conditions for the courses.

![Figure 2.14 Geodesic reference curve on a Y shape tube [27]](image)

### 2.3.1.3 Variable angle reference curves

It has been shown in the literature that fiber-steered composite laminates have higher mechanical performance than conventional straight-fiber laminates, such as improvements in the buckling load [28], and many others. The linear angle variation strategy is the most used method to generate variable stiffness plates.

This strategy consists of linearly varying the fiber angle between two points, each one having a different fiber angle $T_0$ and $T_1$ separated by a distance $d$ (Figure 2.15). $T_0$
defines the angle at the starting point of this path which is usually placed at the center of the rectangular panel. The axis system of fiber orientation is defined by rotating the rosette by an angle $\phi$. The fiber path is then defined by $\phi < T_0 | T_1 >$ and varies linearly along the radial distance $r$ from $T_0$ to $T_1$. Hence, the representation of such path in polar coordinates can be:

$$\theta(r) = \begin{cases} 
\phi + (T_0 - T_1) \cdot \frac{r}{d} + T_0, & -d \leq r \leq 0 \\
\phi + (T_1 - T_0) \cdot \frac{r}{d} + T_0, & 0 \leq r \leq d
\end{cases} \quad (2.4)$$

The reference curve repeats indefinitely with a $2d$ period until it reaches a boundary.

![Figure 2.15](image.png)

Figure 2.15 Linear angle variation reference curve [29]

Another approach to obtain variable angle guide curves is to use non-linear angle variation. Non-linear angle variations have been employed to obtain higher structural performance [30]. The method described in [30] uses a streamline analogy to predict the thickness build up as a function of the fiber orientation. Piecewise quadratic Bezier curves were used in [31], whereas spline functions were used in [32] to describe variable angle curves. Another mathematical representation of non-linear variable fiber angle is also presented in [33] using Lagrange polynomials.
2.3.2 Coverage strategies

To assure that the surface is totally covered by the material, three different techniques are highlighted here. The first one consists of recreating other independent guide curves using one of the techniques presented earlier until full surface coverage is obtained. The second method consists of shifting the guide curve to cover the surface. This technique is usually used in variable stiffness plates [28]. And the third method consists of creating curves parallel to the reference guide curve. The last two methods were investigated in [34], where the parallel method had more restriction on the radius of curvature, thus resulting in a reduced design space compared to the shifted technique. However, gaps and overlaps are inevitable if the shifted methods has to be used, resulting in possible degradation of the properties.

In this section, the different techniques used in the literature to compute parallel curves are only presented here, since shifted curves can be easily obtained by applying a simple translation. In addition, parallel curves are necessary to understand in the scope of this thesis, since the edges of the tow are computed from a given centerline using this technique, as well as other parallel paths within a course using the AFP process.

For the case of a planar curve derived in [35], a parallel curve to a regular plane curve $\alpha$ at a small distance $s$ is the plane curve given by:

$$\text{parcurve}[\alpha][s](t) = \alpha(t) + s \frac{J\alpha'(t)}{\|\alpha'(t)\|},$$

(2.5)

where the operator $J$ is given by $J[[p1,p2]] = \{–p2,p1\}$, and $s$ can be either positive or negative. If $s$ is large, the parallel curve can self-intersect [35].
For the case of a general surface, a closed form solution for the parallel curves does not exist in most cases. Hence several algorithms ([21], [22], [23], and [36]) have been developed to compute parallel curves or also referred to as offset curves numerically.

For instance, a similar approach for the planar case is used in [21] to find parallel curves by following the vector normal to the reference curve. This vector named \( \mathbf{O} \) (Figure 2.16) can be found by taking the cross product between the tangent vector to the curve and the normal vector to the surface. Then at a distance \( d \) along the vector \( \mathbf{O} \), a point \( P' \) is projected to the surface following the normal vector using a Global Closest Technique. This process is repeated at every point-step along the curve to obtain the new parallel curve.

The resulting error from using this technique (Figure 2.17) is reported to be [21]:

\[
Error = d \left( 1 - \frac{\psi}{\tan \psi} \right)
\] (2.6)

Therefore, the error increases by taking a further offset curve (in the case of wider roller), and in the case of highly curved surface.
A more accurate method is presented in [22] and [23] by taking the intersection between the plane perpendicular to the curve and the mold surface (Figure 2.18). To do so, a numerical approach presented in [24] is used to determine the resulting curve. Then, the offset point can be found by taking the required distance along the perpendicular arc. A last step is needed to obtain a complete offset path in the case where the reference path is shorter than the offset one that does not reach a boundary. In this case, the offset curve is completed by interpolating the last point from the calculated ones until it reaches the boundary.

![Figure 2.18](image)

Figure 2.18 (a) Initial path generation, (b) Curve offset by taking perpendicular arcs, (c) Path extension [22]

Three other methods are presented in [36] to compute parallel curves on a parametric surface. The first method named section curves is similar to the ones presented in [22] and [23]. The other two consist of generating orthogonal curves to the reference by either taking vector-field curves or geodesic curves. Once the orthogonal curve is defined in either of these methods, the offset points can be calculated at the required distance from the reference curve, and finally the new parallel curve is obtained by interpolating these points.
For the case of a meshed surface, creating parallel curves on the surface has been introduced by [37] in a technique named Fast Marching Method (FMM) which is based on the Eikonal equation. This equation is mostly used in optic to calculate the propagation of a wave with a particular speed. Hence, from a wave, one can calculate the different position of this wave at every time once it starts propagating.

![Diagram of the Fast Marching Method](image)

**Figure 2.19 Different steps of the Fast Marching Method to offset a reference curve [37]**

This method starts from a random discretized reference curve on the surface. For initialization, all the points have a time value of 0. Then, the reference curve is propagated at a defined speed so every node of the mesh will hit the propagated curve at a certain time. Knowing the time value of two nodes of one triangular mesh, one can calculate the time value of the last node based on the geometry of the triangle. Using the FMM method, this principle is propagated through all the mesh. Finally, the offset curve is obtained by
connecting the points having the same propagation time, indicating that they are equidistant from the reference curve.

To conclude, several mathematical algorithm exist to create reference curves on a surface and to propagate them based on the mechanical load requirements or geometrical and manufacturability constraints. Investigation of each of these method concerning the wrinkling aspect is important during the design phase to detect possible defects and avoid them during the manufacturing phase. Some of these methods are more prone to wrinkling such as the parallel offset coverage strategy compared to the shifted one, or the fixed-angle reference curve compared to the geodesic one, hence a tradeoff analysis has to be done to determine the optimal strategy.

2.4 SUMMARY

After this extensive state-of-art literature review, the main points concerning wrinkling can be summarized as follow:

- Wrinkling is a very broad term and can occur in several physical aspect such as human skin, fabric or clothing items, flowers and leaves due to excessive growth [5], and it can occur from large scale as mountains folds to smaller sandwich structures failure and even to the nanoscale of physically treated elastomers (Table 2.1, [6]), and even in stretched sheets [8]. The model used in the literature to analyze this phenomenon is based on the wrinkling/buckling model of a thin stiff film laying on soft foundation [7]. However, this model does not fully translate to the AFP process due to three main concerns: multidirectional stress applied to the tow during the layup, the highly anisotropy of the tow making it much stiffer in the longitudinal
direction compared to the transverse one, and lastly the irregularities of the underlying surface leading to the necessity of multi-layer analysis.

- Wrinkling during manufacturing of composite structure has been discussed in the literature for several processes, such as diaphragm forming [10], draping (hand layup) [16], consolidation of prepreg over an external [12] and internal [13] radius, or due to shear between the prepreg and the mold [15]. The common reason behind such defect is the mismatch in dimension between the material and the mold surface.

- Wrinkling during the AFP process is also referred to as out-of-plane buckling or tow buckling ([1], [3], [4]), it happens on the compressive edge of the tow due to mismatch in length between the tow and the path. It is investigated during in-plane steering [18] where the process parameters where changed to obtain a defect free layup.

- Physics based model to describe wrinkling ([3] and [4]) use the buckling model of a plate laying on elastic foundation. These models have limited experimental backup, since the material properties of the prepreg and the stiffness of the foundation are hard to measure.

- Several algorithm to describe various types of layup strategies have been developed in the literature, including but not limited to: fixed-angle reference path ([20]-[27]), geodesic path ([25], [27], [35]), and other linear and non-linear angle variation reference curves ([28]-[33]), as well as the shifted coverage method ([28], [34]) or the parallel offset one ([21]-[23], and [35]-[37]). The choice of the
reference curve, and the propagation technique to achieve full coverage of the surface heavily affect the wrinkling creation during manufacturing.
CHAPTER 3

TOW-PATH MODELING ON GENERAL SURFACES

This chapter presents the relevant geometrical parameters for wrinkling analysis on a general surface. Those parameters are the mold surface and its corresponding tangent and normal vectors, the relevant path on the surface, its tangent, normal and bi-normal vectors, its geodesic curvatures, computing parallel curves on the surface, the tow surface and the resulting strain, and finally the assumed shape function of the wrinkles and the concerning assumptions. The wrinkling model applied to the general surface is further reduced for the simple case of a flat surface where relevant simplified equations are shown.

3.1 SURFACE MODELING

A general three-dimensional surface is represented through surface parameters \((u, v)\) such that:

$$S(u, v) = X(u, v)i + Y(u, v)j + Z(u, v)k$$  \hspace{1cm} (3.1)

where the coefficient functions \((X, Y, Z)\) are defined for each unit vector in three-dimensional Cartesian coordinates. Once the surface is defined, an orthonormal frame on that surface can be defined using the two tangent vectors and the normal vector in the \(u\) and \(v\) directions. The tangent vectors along the surface parameters \(u\) and \(v\) are denoted by \(S_u\) and \(S_v\), and are given by:
\[ S_u(u,v) = \frac{\partial S(u,v)}{\partial u}, \quad S_v(u,v) = \frac{\partial S(u,v)}{\partial v}, \] (3.2)

where the subscripts represent differentiation with respect to those variables. These vectors are often normalized to produce unit vectors in the surface directions:

\[ \mathbf{\hat{S}}_u(u,v) = \frac{\partial S(u,v)/\partial u}{\|\partial S(u,v)/\partial u\|}, \quad \mathbf{\hat{S}}_v(u,v) = \frac{\partial S(u,v)/\partial v}{\|\partial S(u,v)/\partial v\|}, \] (3.3)

where the operator \(\|\cdot\|\) denotes the Euclidean norm and the hat symbol \(\mathbf{\hat{\cdot}}\) signifies a unit vector.

The unit normal vector to the surface is defined as the cross product between the surface tangents and represents the third vector of the orthonormal frame for a surface:

\[ \mathbf{\hat{N}}(u,v) = \frac{S_u \times S_v}{\|S_u \times S_v\|} \quad \text{or} \quad \mathbf{\hat{N}}(u,v) = \mathbf{\hat{S}}_u \times \mathbf{\hat{S}}_v. \] (3.4)

For distance calculations on the surface, the infinitesimal distance between two points \(P(u,v)\) and \(P(u + \Delta u, v + \Delta v)\) on the surface is given by:

\[ ds^2 = E \, du^2 + F \, du \, dv + G \, dv^2, \] (3.5)

where the scalar quantities \(E, F\) and \(G\) are the coefficients of the first fundamental form relative to the surface \(S(u,v)\) and given by:

\[ E(u,v) = S_u \cdot S_u, \] (3.6)
\[ F(u,v) = S_u \cdot S_v, \] (3.7)
\[ G(u,v) = S_v \cdot S_v, \] (3.8)

and the symbol "\(\cdot\)" denotes the dot product.
Many surface curvatures exist, however, one of the most important ones concerning the discussed problem is the Gaussian curvature $K$ and is given by:

$$K = \frac{e g - f^2}{E G - F^2}, \quad (3.9)$$

where $e(u,v)$, $g(u,v)$, and $f(u,v)$ are the coefficients of the second fundamental form relative to the surface $S(u,v)$ and given by:

$$e(u,v) = N(u,v) \cdot S_{uu}(u,v), \quad (3.10)$$
$$f(u,v) = N(u,v) \cdot S_{uv}(u,v), \quad (3.11)$$
$$g(u,v) = N(u,v) \cdot S_{vv}(u,v). \quad (3.12)$$

The Gaussian curvature can be positive on a hill, negative on a saddle point or zero as in developable surfaces. The importance of the Gaussian curvature from a manufacturing point of view is that for instance, in a milling operation where the tool has a spherical head, the radius of the tool should be less than the smallest radius of curvature of the surface. From an AFP consideration, Gaussian curvature sets a limit on the manufacturability of a given surface especially for the case of a negative curvature where collision between the machine head and the surface might occur due to high curvature (in absolute value). In addition, a surface with zero Gaussian curvature called developable surface, can be laid down on a flat surface without stretching or distorting it, hence the possibility of obtaining wrinkle free layup for a given path direction.

3.2 PATH DEFINITION

The previous section defined an arbitrary surface on which the fiber tows have to be laid. An infinite number of possibilities exist for the definition of a fiber path over a general 3D surface, but the most common paths used in the literature are surface-plane
intersection curves ([21], [22], [23], [24]), geodesics ([25], [27], [35]), constant angle paths ([25]), and constant curvature paths ([25], [35]) as discussed previously in Chapter 2, Reference Curves section. In the scope of this Chapter only the derivation of a geodesic path will be presented since it has direct relevance to the solution process, though the formulation allows for any arbitrary definition of a path. For the definitions of the surface and path Figure 3.1 is used as a reference.

Figure 3.1 Surface and Path definition

An arbitrary path on the surface is defined by \( \mathbf{C}(t) = \mathbf{S}(u_c(t), v_c(t)) \) where \( u_c(t), v_c(t) \in \Omega \), the domain of definition of the surface. The unit tangent vector to the path can be defined by:

\[
\hat{T}(t) = \frac{\mathbf{dC}(t)/dt}{\|\mathbf{dC}(t)/dt\|} = \frac{\mathbf{S}_u d u_c/dt + \mathbf{S}_v d v_c/dt}{\|\mathbf{S}_u d u_c/dt + \mathbf{S}_v d v_c/dt\|}.
\]

(3.13)

In the analysis of a path along the surface, an additional useful orthonormal frame is defined using the following three vectors: \( \hat{T} \) the unit tangent to the path, \( \hat{N}(t) = \hat{N}(u_c(t), v_c(t)) \)
the unit normal to the surface along the path, and \( \hat{\mathbf{B}} \) the unit in-plane normal. This last vector \( \hat{\mathbf{B}} \) (often referred to as the bi-normal or bi-tangent in the literature) is tangent to the surface at a given point but orthogonal to the tangent vector along the curve. It is represented through a cross-product:

\[
\hat{\mathbf{B}}(t) = \hat{\mathbf{N}}(t) \times \hat{\mathbf{T}}(t) .
\] (3.14)

Another important feature in analyzing the path is the distance (arc length) along the path. The distance between two points on the surface along the curve \( \mathbf{C}(t) = \mathbf{S}(u_c(t), v_c(t)) \) can be found through integration along the path of the distance between two points on a surface from equation (3.5):

\[
d = \int_{t_a}^{t_b} \sqrt{E \left( \frac{du_c}{dt} \right)^2 + F \frac{du_c}{dt} \frac{dv_c}{dt} + G \left( \frac{dv_c}{dt} \right)^2} \, dt .
\] (3.15)

### 3.3 Geodesic Path Definition and Geodesic Curvature

A geodesic path on a surface is the shortest path connecting two points on that surface. A geodesic path can also be defined starting at a point with a given direction of travel. The equations governing the path of a geodesic arise from minimization of the integral in (3.15) with appropriate boundary conditions, and for a general surface is calculated numerically. A geodesic path has to satisfy the following system of differential equations [35]:

\[
\begin{align*}
&u'' + \Gamma^1_{11} u'^2 + 2 \Gamma^1_{12} u'v' + \Gamma^1_{22} v'^2 = 0 \\
v'' + \Gamma^2_{11} u'^2 + 2 \Gamma^2_{12} u'v' + \Gamma^2_{22} v'^2 = 0
\end{align*}
\] (3.16)

where primes (\( ^{'} \)) represent differentiation with respect to the parameter \( t \) and \( \Gamma^i_{jk} \) are the Christoffel symbols of the surface \( \mathbf{S} \) given by:
In order to solve this system of second order differential equations, four initial conditions have to be prescribed: 

\[ u(t_0) = u_0, \ v(t_0) = v_0, \ u(t_1) = u_1, \ v(t_1) = v_1 \]

for the geodesic path between two points \( P_0 = S(u_0, v_0) \) and \( P_1 = S(u_1, v_1) \); or 

\[ u(0) = u_0, \ v(0) = v_0, \ u'(0) = u_0', \ v'(0) = v_0' \]

for the geodesic path starting at \( P_0 = S(u_0, v_0) \) with a direction \( (u_0', v_0') \).

Lastly, several specific measures of curvature can be defined for the geometric formulation that has been discussed. Gaussian curvature \( K \) applies to a surface and represents the intrinsic curvature of the surface at a given point as shown earlier in equation (3.9). For a space curve (one not laying on a specified surface), a curvature parameter can be derived from the path definition \( C(t) \) and its derivatives, where the curve tangent, normal, and binormal vector fields are related to the curvature and torsion by Frenet formulas. However, the most important estimate of curvature for the problem under consideration is referred to here as the geodesic curvature \( k_g \), which measures the curvature within the tangent plane for a curve on a surface. This parameter depends on both the

\[
\Gamma_{11}^1 = \frac{G E_u - 2 F F_u + F E_v}{2(E G - F^2)} \quad (3.17)
\]
\[
\Gamma_{11}^2 = \frac{2 E F_u - E E_v - F E_u}{2(E G - F^2)} \quad (3.18)
\]
\[
\Gamma_{12}^1 = \frac{G E_v - F G_u}{2(E G - F^2)} \quad (3.19)
\]
\[
\Gamma_{12}^2 = \frac{E G_u - F E_v}{2(E G - F^2)} \quad (3.20)
\]
\[
\Gamma_{22}^1 = \frac{2 G F_v - G G_u - F G_v}{2(E G - F^2)} \quad (3.21)
\]
\[
\Gamma_{22}^2 = \frac{E G_v - 2 F F_v + G G_u}{2(E G - F^2)} \quad (3.22)
\]
surface parameters and the path definition and provides a direct indication on where wrinkles might form for tow path. It is calculated as:

\[
k_g = \left[ (u_c'' + \Gamma_{11}^1 u_c'^2 + 2 \Gamma_{12}^1 u_c' v_c' + \Gamma_{12}^2 v_c'^2) v_c' - (v_c'' + \Gamma_{11}^2 u_c'^2 + 2 \Gamma_{12}^2 u_c' v_c' + \Gamma_{22}^2 v_c'^2) u_c' \right] \\
\times \frac{\sqrt{EG - F^2}}{(E u_c'^2 + 2F u_c' v_c' + G v_c'^2)^{3/2}}.
\] (3.23)

Note that for a geodesic curve that satisfies equation (3.23) the geodesic curvature is identically zero, which affirms that the geodesic is the “straightest” path on a surface.

3.4 PARALLEL CURVES ON THE SURFACE AND TOW SURFACE

For a given base curve (reference curve) on the surface, parallel paths (see Figure 3.2) on one or both sides of the base curve have to be defined in order to show the area covered by the carbon fiber tow. In addition, parallel curves can be used to generate the centerlines and the edges of adjacent tows, in the case if more than one tow is fed during a single course. Those parallel curves are computed by taking the same distance along the geodesics orthogonal to the base curve.

The algorithm to compute those curves is similar to the one presented in [36], and is summarized here as follows:

- Take \( n \) points \( P_i \) along the base curve: \( P_i = \{P_1, P_2, ..., P_n\} \), \( i = 1 \ldots n \)
- Find each geodesic \( G_i \) starting at \( P_i \) in the direction orthogonal to the base curve
- Find the points \( Q_i \) on the geodesics at a distance \( d_i \) equal to the tow width or half-tow width, depending on the application
- Generate the parallel path \( C_p(t) \) by interpolating the points \( Q_i \)
From these parallel path curve calculations, their lengths on the surface and geodesic curvatures can be computed from equations (3.15) and (3.23), respectively. Another important parameter to obtain from the geometry once the parallel path is computed, is the longitudinal strain $\varepsilon_l$ or the strain along the length of the tow. $\varepsilon_l$ is given by:

$$
\varepsilon_l|_{t_a \rightarrow t_b'} = \frac{d_c|_{t_a' \rightarrow t_b'} - d_c|_{t_a \rightarrow t_b}}{d_c|_{t_a \rightarrow t_b}},
$$

(3.24)

where $d_c|_{t_a \rightarrow t_b}$ is the distance between two points $A$ and $B$ along the reference curve $C(t)$, $d_c|_{t_a' \rightarrow t_b'}$ is the distance between $A'$ and $B'$ the corresponding points on the parallel curve $C_p(t)$, hence $\varepsilon_l|_{t_a' \rightarrow t_b'}$ is the average strain between the two points $A'$ and $B'$ on the parallel
curve. It is very important to first calculate the geodesic curvature $k_g$ and to detect the location where there is a change of sign, its corresponding parameter $t$, and the boundaries associated with this change. If there is no change in the sign of the geodesic curvature, the boundaries $A$ and $B$ can be considered the start and the end of the tow. Then the strain calculation can be carried further to determine the severity of the anticipated defect. For the case where the computed strain is negative, defects such as wrinkling or in-plane waviness can be expected. For the case where the strain is positive, defects such as bunching or folding can be expected. Lastly, for the case of zero strain, a defect free layup is anticipated such as the case of a straight path on a flat surface. Note that, this strain analysis is based on the length of the reference curve, or in other words, it is assumed that the machine head will deliver the exact amount of material as the length of the reference path.

The surface that should be covered by the tow is bounded by the initial path $C(t)$ and the interpolated parallel curve $C_p(t)$. The equation of the tow surface can be generated by first creating a ruled surface $S_{tow,(u,v)}(t, n)$ in the $(u, v)$ domain, where $n = 0$ corresponds to the initial reference path $\{ u_c(t), v_c(t) \}$ and $n = 1$ corresponds to the parallel path $\{ u_p(t), v_p(t) \}$:

$$
S_{tow,(u,v)}(t, n) = \begin{cases} u_{tow}(t, n) \\ v_{tow}(t, n) \end{cases} = (1 - n) \begin{cases} u_c(t) \\ v_c(t) \end{cases} + n \begin{cases} u_p(t) \\ v_p(t) \end{cases}.
$$

(3.25)

Then, the actual surface of the tow on the mold is generated by mapping $S_{tow,(u,v)}(t, n)$ from $(u, v)$ domain to the physical domain by:

$$
S_{tow}(t, n) = S(u_{tow}(t, n), v_{tow}(t, n)).
$$

(3.26)
3.5 MODELING ASSUMPTIONS FOR WRINKLED TOW

The previous sections provided the necessary equations to calculate lengths and curvatures of prescribed paths and their parallel curves on a general surface, as well as the induced strain in the process. For paths with non-zero geodesic curvatures, the relative lengths of the inner and outer edges of a tow with a finite width will differ on the surface of a part. The AFP hardware on the other hand dispenses tows that have equal length on both edges. The differential length on the part surface between the two edges of the tow has to be somehow absorbed through a deformation mechanism (Figure 1.4) within the tow to take the excess length on one edge of the tow that has to fit onto the part surface. Here it is assumed that the edge of the tow separates from the shorter edge on the surface while the tow’s edge corresponding to the longer edge on the surface remains on its prescribed path. This will generate out-of-plane wrinkles along the length of the tow (see schematics in Figure 1.4 (e) and Figure 3.3). To summarize, the following assumptions are therefore considered in the modeling of the wrinkles:

1. The wrinkle will appear on the compressive side of the tow and the remaining regions of the tow are not modeled since other length absorption mechanisms (Figure 1.4) might appear which are outside the scope of this work.

2. The edge of the tow coinciding with the reference curve \( C(t) \) will remain on the surface of the mold after placement, since the tow fed from the machine head has the same length as the reference curve \( C(t) \).

3. The other edge of the tow coinciding with the parallel edge \( C_p(t) \) will totally lift from the surface to form the wrinkle, since the length of the tow is longer than the parallel path \( C_p(t) \).
4. The wrinkling formation is approximated by the rotation of the parallel edge around the reference curve, while the boundaries remain on the surface.

5. The rotation angle is assumed to be a cosine function so that the shape of the obtained wrinkled is similar to the buckling of a beam with clamped-end conditions.

6. The shape of the buckled tow in the transverse direction will remain the same as the tow surface \( S_{\text{tow}}(t, n) \) before buckling (linear for the case of a flat plate, higher order for general surfaces).

![Figure 3.3 Schematic of a wrinkle of a general surface](image)

Based on these assumptions, the wrinkled surface of the tow \( S_w(t, n) \) (Figure 3.3) is derived from the tow surface \( S_{\text{tow}}(t, n) \) in the following equation:

\[
S_w(t, n) = S_{\text{tow}}(t, 0) + \left( S_{\text{tow}}(t, n) - S_{\text{tow}}(t, 0) \right) \cdot R \left( \hat{T}(t), \beta(t) \right),
\]  

(3.27)
where $\mathcal{R}\left(\vec{T}(t), \beta(t)\right)$ is Rodrigues’ rotation matrix corresponding to the rotation by an angle $\beta(t)$ about the axis $\vec{T}(t)$, the unit tangent to the original path (assumption # 4). The first term in equation (3.27) $S_{tow}(t, 0)$ corresponds to the reference path (as discussed in the previous section), and represents the 2nd assumption concerning the reference edge remaining on the surface. The second term in equation (3.27) represents the shape of the wrinkled tow in the transverse direction $n$ at each location $t$. This shape remains unchanged as stated in assumption # 6. Rodrigues’ rotation matrix $\mathcal{R}(\hat{\omega}, \theta)$ computes the rotation by an angle $\theta$ about the unit axis $\hat{\omega} = \{\omega_x, \omega_y, \omega_z\}$ and is given by

$$\mathcal{R}(\hat{\omega}, \theta) = e^{\hat{\omega}\theta} = I + \hat{\omega} \sin \theta + \hat{\omega}^2 (1 - \cos \theta)$$

where $I$ is the $3 \times 3$ identity matrix, and $\hat{\omega}$ is the cross product matrix of the vector $\hat{\omega}$ given by:

$$\hat{\omega} = \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix} \tag{3.29}$$

A representation of Rodrigues’ rotation in matrix form is as follows:

$$\mathcal{R}(\hat{\omega}, \theta) = \begin{pmatrix} \cos \theta + \omega_x^2 (1 - \cos \theta) & -\omega_y \omega_x (1 - \cos \theta) & \omega_z \sin \theta + \omega_x \omega_z (1 - \cos \theta) \\ \omega_y \sin \theta + \omega_x \omega_y (1 - \cos \theta) & \cos \theta + \omega_y^2 (1 - \cos \theta) & -\omega_z \sin \theta + \omega_y \omega_z (1 - \cos \theta) \\ -\omega_y \sin \theta + \omega_x \omega_y (1 - \cos \theta) & \omega_z \sin \theta + \omega_y \omega_z (1 - \cos \theta) & \cos \theta + \omega_z^2 (1 - \cos \theta) \end{pmatrix} \tag{3.30}$$

One of Rodrigues’ rotation’s conditions is that the rotation axis has to pass through the origin of the reference system. Hence the third term in equation (3.27) ensures that conditions by applying the necessary translation of “$-S_{tow}(t, 0)$”. Also, by choosing the rotation axis to be the unit tangent vector $\vec{T}(t)$, assumptions 2, 3, and 4 are respected. Lastly, the remaining term in equation (3.27) is the rotation angle $\beta(t)$. This angle is assumed to have a cosine shape function as stated in assumption # 5, and is given by:
Here $t_i$ and $t_{i-1}$ are the parameters corresponding to the starting and ending points of the tow section assuming rotation-free boundary conditions at the two ends along the length, and the amplitude $k$ is computed so that the length of the shorter edge is set equal to the length of the longer one. The integer $N$ in (3.31) represents the number of wrinkling waves along the tow or also referred here as the mode shape. To solve for the transition points between each wave (the $t_i$ locations), the length $L = d_C|_{t_a=t_b}$ of the initial curve between the boundary points $A$ and $B$ is divided into equal $N$ segments and the transition points are found from equation (3.15) based on the initial curve, such that:

$$
\int_{t_{i-1}}^{t_i} \sqrt{E \left( \frac{du_c}{dt} \right)^2 + F \frac{du_c}{dt} \frac{dv_c}{dt} + G \left( \frac{dv_c}{dt} \right)^2} \, dt = \frac{L}{N} \quad , \quad \text{for } i = 1, \ldots, N . \tag{3.32}
$$

Note that for $N = 1$ ($1^{\text{st}}$ mode shape, or only one wrinkle is assumed) the end points are equal to the start and end of the curve $C(t)$. To calculate the amplitude $k$ of the cosine shape function in equation (3.31), the equation for the parallel curve at a distance $n = 1$ in equation (3.27) is computed and inserted into the length calculation of equation (3.32) with the corresponding start and end points $t_i$ and $t_{i-1}$, then an iterative method (such as Newton’s method) is used to converge for a final value of $k$ such that the total length of the shorter parallel curve is equal to the length of the longer initial path.
3.6 SIMPLIFICATION FOR FLAT SURFACE

To provide a realistic example and verification with actual experiments for tow wrinkling, the previous equations are simplified for a flat surface. The equation for the surface (3.1) can be represented in three-dimensional space as:

\[ S(u, v) = u \hat{i} + v \hat{j}. \] (3.33)

Computations of the ensuing equations leads to the following relevant equations for the surface:

\[ \vec{S}_u(u, v) = \hat{i}, \quad \vec{S}_v(u, v) = \hat{j}, \]
\[ E = 1, \quad F = 0, \quad G = 1, \quad \text{All } \Gamma_{jk}^i = 0. \] (3.34)

The coefficients of the second fundamental form of the flat surface are all zero, leading to a zero Gaussian curvature as expected for the special case of a ruled surface:

\[ e = f = g = 0, \text{ and } K = 0 \] (3.35)

The normal, tangent, and in-plane normal vectors (Figure 3.4) to the reference curve \( C(t) = S(u_c(t), v_c(t)) = u_c(t) \hat{i} + v_c(t) \hat{j} \) become:

\[ \vec{N}(u, v) = \hat{k} \] (3.36)

\[ \vec{T}(t) = \frac{u_c'(t) \hat{i} + v_c'(t) \hat{j}}{\left( u_c'^2(t) + v_c'^2(t) \right)^{1/2}} = \hat{\omega}_c(t) \hat{i} + \hat{\nu}_c(t) \hat{j} \] (3.37)

\[ \vec{B}(t) = \frac{-v_c'(t) \hat{i} + u_c'(t) \hat{j}}{\left( u_c'^2(t) + v_c'^2(t) \right)^{1/2}} = -\hat{\nu}_c(t) \hat{i} + \hat{\omega}_c(t) \hat{j} \] (3.38)

with \( \hat{\omega}_c(t) = \frac{u_c'(t)}{\sqrt{u_c'^2(t) + v_c'^2(t)}} \), and \( \hat{\nu}_c(t) = \frac{v_c'(t)}{\sqrt{u_c'^2(t) + v_c'^2(t)}} \).
The equations for a geodesic path (3.16) reduce to a simple system of separable ODE’s (Ordinary Differential Equation), reaffirming that the shortest path on a flat surface is a straight line:

\[
\begin{align*}
    u'' &= 0 \\ 
    v'' &= 0
\end{align*}
\]

\[
\Rightarrow \quad \begin{align*}
    u(t) &= c_0 t + c_1 \\ 
    v(t) &= c_2 t + c_3
\end{align*}
\]

(3.39)

The parallel edge curves can be stated more simply as:

\[
\begin{align*}
    \mathcal{C}(t) : \begin{cases} x(t) = u_c(t) \\ y(t) = v_c(t) \end{cases} , \\
    \mathcal{C}_p(t) : \begin{cases} x_p(t) = u_c(t) - d \hat{v}_c(t) \\ y_p(t) = v_c(t) + d \hat{u}_c(t) \end{cases}
\end{align*}
\]

(3.40)

similarly to the parametrization of a parallel curve presented in [35] and shown previously in equation (2.5). The in-plane curvature (simplified version of the geodesic curvature) is calculated from:

\[
k_g(t) = \frac{u'_c v''_c - u''_c v'_c}{(u'^2_c + v'^2_c)^{3/2}}.
\]

(3.41)

Figure 3.4 Schematic of a wrinkle for the simplification of flat surface
The assumed shape of the wrinkled tow surface $S_w(t,n)$ shown in Figure 3.4 is represented as:

$$S_w(t,n) = C(t) + n \, d \left[ \cos(\beta(t)) \, \hat{B}(t) + \sin(\beta(t)) \, \hat{N}(t) \right], \quad (3.42)$$

or explicitly in terms of the Cartesian three dimensional space:

$$S_w(t,n) = \begin{cases} 
    x_w(t,n) = u_c(t) - n \, d \, \hat{v}_c'(t) \cos(\beta(t)) \\
    y_w(t,n) = v_c(t) + n \, d \, \hat{u}_c'(t) \cos(\beta(t)) \\
    z_w(t,n) = n \, d \, \sin(\beta(t)) 
\end{cases} \quad (3.43)$$

where $\beta(t)$ is the same cosine shape function for the wrinkle rotation angle as shown earlier in equation (3.31).

Lastly, the length calculations for the edges are found from:

$$\int_{t_{i-1}}^{t_i} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt = \int_{t_{i-1}}^{t_i} \sqrt{\left(\frac{dx_w}{dt}\right)^2 + \left(\frac{dy_w}{dt}\right)^2 + \left(\frac{dz_w}{dt}\right)^2} \, dt = \frac{L}{N} \quad (3.44)$$

for $i = 1, \ldots, N$.

3.7 SUMMARY

Governing equations for tow wrinkling along an arbitrary path on a general surface are derived in this chapter based on geometrical considerations only. The tow is assumed to wrinkle on its compressive edge where the length of the tow-path is smaller than the length of the actual tow. For this case, the outer edge will lift totally from the surface to form a wrinkle whose shape is assumed to be a cosine function similar to the buckling of a beam with clamped-ends conditions. In addition, the shape of the tow in the transverse direction is assumed to remain the same as the placement surface. Based on these assumptions, relevant geometrical parameters to tow wrinkling are derived and presented.
for arbitrary paths on general surfaces. Furthermore, the relevant equations are simplified for the special of curved paths placed on a flat mold surface.
CHAPTER 4
MODELING RESULTS

This chapter investigates the derived equations in the previous chapter, and implements the assumed shape of the wrinkled tow to examples of curved paths on a flat plate. The effect of the tow width and the curvature on the amplitude of the wrinkles are numerically investigated. Furthermore, the implementation of the model is shown on a quasi-random doubly curved NURBS surface, where a color map of different layup strategies is also shown indicating potential wrinkling regions. The study is carried further on an industrial scale doubly curved mold where several paths were investigated for wrinkling.

4.1 PATHS ON FLAT SURFACE

This section focuses on the simplified form of the developed equations for wrinkling of tows placed on a flat surface. The first example considers a simple circular path where the effect of the wrinkles number and tow width are investigated while keeping the curvature constant. The second example shows the effect of the curvature on the obtained shape and the amplitude of the wrinkles by analyzing a NURBS curve. The third example investigates the possible issue with curves having inflection points and the way to handle that. Finally, the last example investigate wrinkling for a variable angle layup of a plate with a hole.
4.1.1 Circular Path Analysis

To investigate the presented algorithm, a simple constant curvature path is first considered. This path is only a circular arc, with a possible parametrization defined as:

$$\mathcal{C}(t) = \left\{ \cos\left(\frac{\pi}{2} t\right), \sin\left(\frac{\pi}{2} t\right) \right\}, \quad 0 \leq t \leq 1$$

(4.1)

where the radius of curvature is constant with a value of 1, hence the curvature of the path is $\kappa(t) = \frac{1}{\rho} = 1$. The corresponding tangent and in-plane normal vectors are calculated from equations (3.37) and (3.38) respectively:

$$\mathbf{T}(t) = \left\{ -\sin\left(\frac{\pi}{2} t\right), \cos\left(\frac{\pi}{2} t\right) \right\}, \quad 0 \leq t \leq 1$$

(4.2)

$$\mathbf{B}(t) = \left\{ -\cos\left(\frac{\pi}{2} t\right), -\sin\left(\frac{\pi}{2} t\right) \right\}, \quad 0 \leq t \leq 1$$

(4.3)

A tow width of $w = 0.2$ (non-dimensional units) is first considered, and the parallel path is computed from (3.40), then the wrinkled shaped of the tow (3.42) for the first 4 mode shapes ($N = 1, ..., 4$) is calculated and visualized in Figure 4.1.

In Figure 4.1, the reference curve $\mathcal{C}(t)$ shown in blue remains on the surface (assumption #4) while the parallel edge shown in orange will totally lift from the surface and form the wrinkle (assumption #3). From visual inspection of Figure 4.1, it can be directly observed that the amplitude of the wrinkles is affected by the mode shape. With higher mode shape the amplitude of the wrinkles decreases significantly. In addition, within a mode shape the amplitude of the wrinkles for this case is the same; i.e. the three wrinkles in the 3rd mode shape have the same amplitude, similarly for the four wrinkles in the 4th mode shape. This is due to the fact that this circular path has a constant curvature of $k_g = 1$ and constant width $w = 0.2$ leading to a constant differential length.
To numerically investigate the effect of the mode shape on the amplitude, the height of the wrinkles in the $z$-direction is considered. From (3.43), the height is given by:

$$z_w(t, n) = n \cdot d \cdot \sin(\beta(t)).$$  \hspace{1cm} (4.4)

The maximum value of $z_w(t, n)$ occurs at $n = 1$ and $d = w = 0.2$ at the wrinkled edge. In addition, the value of $\beta(t)$ has to be maximum at a given mode shape $N$. From the assumed cosine shape of $\beta(t)$ defined in (3.31), the maximum value is:

$$\beta_{\text{max}} = 2 \cdot k,$$  \hspace{1cm} (4.5)

and it occurs at:

$$t_{\text{max}} = \frac{t_l + t_{l-1}}{2}.$$  \hspace{1cm} (4.6)
Hence, the amplitude of the wrinkle is computed using:

\[ a = z_w(t_{max}, 1) = w \sin(2k). \]  
(4.7)

The computation of the amplitude (4.7) is carried for several mode shapes, and their values with respect to the parameter \( t \) of the reference curve \( C(t) \) (4.1) and the number of wrinkles is shown in Figure 4.2 and Figure 4.3.

**Figure 4.2** Wrinkles amplitude as function of the wrinkles number and location

**Figure 4.3** Amplitude of wrinkles as function of the wrinkle number
The conclusions drawn by visual inspection of Figure 4.1 are numerically proved in Figure 4.2 and Figure 4.3 that increasing the number of wrinkles decreases the resulting amplitude (Figure 4.3), and the amplitude within a mode shape are the same since the path has a constant curvature (Figure 4.2). Relating these results to the actual fiber placement process, higher number of wrinkles can only be achieved by the presence of the adhesion between the tow and the mold surface. Hence, with better surface adhesion, the spots where the tow stick to the surface are increased, therefore the number of wrinkles increases resulting in smaller wrinkles amplitude.

Another important geometrical feature to investigate is the effect of tow width on the severity of the wrinkles formation. To accomplish that, the same reference path $C(t)$ defined in equation (4.1) is considered for analysis, with different tow width ranging from $w = 0.05$ to $w = 0.3$, resulting in different parallel paths $C_p(t)$. Four of these different tows with different tow width are shown in Figure 4.4.

To obtain a fair comparison between different tow widths, the same mode shape is applied for all the paths. Visual interpretation of Figure 4.4 shows that increasing the tow width increases the amplitude of the wrinkle. Numerical investigation of the maximum amplitude (4.7) is carried for several tow widths and presented in Figure 4.5 and Figure 4.6. The same conclusion can be obtained, that reducing the tow width reduces the amplitude of the wrinkles.
Figure 4.4 Effect of tow width on the wrinkles’ amplitude

Figure 4.5 Wrinkles’ amplitude as function of location and tow width
4.1.2 A NURBS Path Analysis

After investigating the effect of the tow width and the mode shape on the amplitude of the wrinkle, another important geometrical feature to account for is the curvature of this path. To do so, a Non-Uniform Rational B-Spline (NURBS) curve is considered for this study due to the high flexibility of such curves, and their importance in Computer Aided Design (CAD) software. A NURBS curve \( C(t) \) is defined by a set of control points \( P_i \) and corresponding weights \( w_i \), a degree \( p \), and a knot vector \( KV \). A parametrization of the NURBS curve can be obtained using the following equation:

\[
C(t) = \frac{\sum_{i=0}^{n} N_{i,p}(t) w_i P_i}{\sum_{i=0}^{n} N_{i,p}(t) w_i},
\]

where \( N_{i,p}(t) \) are the \( p^{th} \)-degree B-spline basis function defined on the non-uniform knot vector \( KV \). For more details about the NURBS parametrization and the B-Spline basis functions, reference can be found in [38].
A random curvilinear reference path is defined in NURBS form with the following parameters:

\[
P_i = \{(0,0), (2,-1), (5,-1), (4,7)\}, \quad (4.9)
\]

\[
K_V = \{0,0,0,0,1,1,1,1\}, \quad (4.10)
\]

\[
w_i = \{1,1.5,1,1\}, \quad (4.11)
\]

\[
\text{and } p = 3 \quad (4.12)
\]

The resulting parametric equations of the curve using [38] is:

\[
C(t) = \begin{cases} 
  x(t) = \frac{t \left(6 - 2t - \frac{4}{3}t^2\right)}{\left(\frac{2}{3} + t - 2t^2 + t^3\right)}, & 0 \leq t \leq 1. \\
  y(t) = \frac{t \left(-3 + 4t + \frac{11}{3}t^2\right)}{\left(\frac{2}{3} + t - 2t^2 + t^3\right)} 
\end{cases} \quad (4.13)
\]
A graphical representation of the reference path $C(t)$, the control points $P_i$ and the parallel path $C_p(t)$ using a tow width of $w = 0.5$ is shown in Figure 4.7. The curvature of the reference path is also computed using (3.41) and is shown in Figure 4.8.

The implementation of the wrinkling equations to the reference path $C(t)$ and the obtained shapes of the wrinkled tow for the first four mode shapes are visualized in Figure 4.9. It is hard to notice the effect of the curvature on the amplitude for the first three mode shapes, however, in the fourth one the first two amplitudes are slightly larger than the remaining two. Also, note that for the first mode shape, and due to the extreme curvature, the tow is rotated for more than 90° around the reference path.

![Figure 4.9 Deformed tow placed on a NURBS reference path: 4 different mode shapes](image)

To fully understand the effect of the curvature on the amplitude of the wrinkles, numerical implementation of equation (4.7) as function of the numbers of wrinkles and the corresponding $t_i$ (4.6) are presented in Figure 4.10. For a small number of assumed
wrinkles, the effect of the wrinkles is somehow negligible. It is only for high number of wrinkles that the importance of the curvature emerges, and the amplitude of the wrinkles for $0.2 < t < 0.6$ are the highest (Figure 4.10). This coincides with the value of the curvature presented earlier in Figure 4.8, where the peak occurs for $0.2 < t < 0.6$. An additional note to point out for Figure 4.10 is that the concentration of the $t_{\text{max}}$ values is higher for $t > 0.6$. The reason behind that is that the parametrization of the reference curve $C(t)$ is not by arc length (as the case for the circular arc presented earlier), and the parameter $t$ does not necessarily reflect the distance. On the other hand, the length of the wrinkles occurring for that are assumed to be equal, hence explaining the difference between the arc-length parametrization and the actual NURBS parametrization of the considered reference curve. Besides that, the shape of the amplitude curve for high number of wrinkles (more than five for this case) can be considered very close to the shape of the curvature plot.

![Figure 4.10 Amplitude of wrinkles as function of number of wrinkles](image)

Figure 4.10 Amplitude of wrinkles as function of number of wrinkles
4.1.3 Curves with Inflection Point

One possible issue that might arise and was not discussed the previous examples is when the curve changes concavity from upward to downward or vice versa at an inflection point. At this point, the curvature \( k_g(t) \) (or also it can be referred to as the signed curvature) changes signs from positive to negative or the other way around. For the case where the signed curvature is positive as presented earlier in the first two examples, the curve is concave upward and the parallel curve \( C_p(t) \) computed using equation (3.40) is shorter than the reference curve \( C(t) \). This fact does not hold true when the curve is concave downward or the curvature is negative: the parallel curve is longer than the reference one.

To solve this issue, it is important to first detect the inflection point where the curvature is zero (as discussed previously in Section 3.4). Second, each section of the curve is considered separately for analysis, and the length of the edges are determined. Then, the previous assumption that the shorter edge will lift from the surface to form wrinkles by rotating about the other edge is carried on to determine the shape of the wrinkle.

The following spline curve (a special case of the NURBS curve where all the weights are equal to 1) is considered for analysis, with the given parameters:

\[
P_t = \{(0,0),(2,2),(4,-3),(7,-2.5)\},
\]

\[
KV = \{0,0,0,1,1,1,1\},
\]

\[
w_t = \{1,1,1,1\},
\]

and \( p = 3 \).

Hence, the resulting parametric equation of the reference path \( C(t) \) is found to be:

\[
C(t) = \begin{cases} 
  x(t) = t (6 + t^2) \\
  y(t) = t (6 - 21 t + 12.5 t^2)
\end{cases}, \quad 0 \leq t \leq 1.
\]
A graphical representation of the reference path $C(t)$, the control points $P_i$ and the parallel path $C_p(t)$ using a tow width of $w = 0.3$ (non-dimensional units) is shown in Figure 4.11. The curvature of the reference path is also computed using (3.41) and is shown in Figure 4.11. The inflection point for a given curve can be found by equating $k_g(t) = 0$ and solving for the parameter $t$. For this given curve, the inflection point occurs at $t = 0.52486$ and no other inflection points were found for $0 \leq t \leq 1$. Therefore, the analysis
of this curve is carried separately over two boundaries: the first one for \(0 \leq t \leq 0.52486\) and the second one for \(0.52486 \leq t \leq\)

Since the first section of the reference path has a negative curvature (or concave downward) as shown in Figure 4.12, the obtained parallel curve using equation (3.40) has a longer length (Figure 4.11). In this case, the reference curve will lift from the surface and form the wrinkle by rotating about the parallel one. In the second section of the curve where the curvature is positive, the previous analysis presented in the first two example applies where the parallel path lifts from the surface to form the wrinkle. Numerical implementation of the wrinkling equations is carried, and the obtained shape of the wrinkled tow is shown in Figure 4.13 for the first mode shape, and in Figure 4.14 for the second mode shape.

![Figure 4.13 Wrinkled tow on a spline with inflection point for \(N = 1\) wrinkle](image)

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4.1.4 Wrinkling Analysis for a Variable Stiffness Laminate

To expand the wrinkling analysis from a single curve to a full layup, a variable stiffness laminate is considered. Using a previously developed algorithm at the McNair Center [39], a 2D optimized variable stiffness layup of a plate with a hole is obtained. Then, the output from the ode is translated to NC code using an modular conversion tool [40] so that the output can be manufactured using a fiber placement machine. For the wrinkling analysis considered here, only the cloud of points in the \((x,y)\) coordinate system are considered. A plot of these point is shown in Figure 4.15: a total of 71 different paths and 77,978 points will be considered for analysis.

The second step in the analysis process is to reconstruct 71 reference paths from the given points. To accomplish that, the built-in “Interpolation” function in Mathematica is used to create a reference path \(C(t)\) for each of the 71 sets of points. This interpolation function fits \(3^{rd}\) order polynomial curves between successive data points and agree with the

Figure 4.14 Wrinkled tow on a spline with inflection point for \(N = 2\) wrinkles
data at every point explicitly [41]. Those reference paths are considered as the centerline of the towpath, and are shown as dashed blue curves in Figure 4.16. The other 2 parallel paths representing the edges of the towpath and shown in Figure 4.16 are generated using equation (3.40) at a distance \( d = \pm 5 \, mm \) resulting in a total tow width of \( w = 10 \, mm \).

Once the path for each tow to be placed is found, wrinkling analysis can be carried to determine the critical location for this layup. For this specific case, finding a global inflection point was difficult, since the provided converted points are a doubly linked nodal list [40] inferring that there is an inflection point between every 2 or 3 provided points along the path. The presence of that many inflection points along the path makes the distance at which the wrinkling analysis is carried on very small, hence the wrinkles amplitude cannot be noticed. An alternate method is used for this specific case to accommodate for this issue: an approximate wrinkle length of \( l = 30 \, mm \) is assumed, and the number of wrinkles is computed by:

\[
N = \text{Floor} \left[ \frac{L}{l} \right]
\]

(4.19)

where \( L \) is the length of the reference curve, and the \text{Floor} function gives the largest integer less than or equal to \( L/l \). Once the number of wrinkles for each path is computed, the boundary points are found by solving for their corresponding parameters \( t_i \) and \( t_{i-1} \) using equation (3.44). Then, the length of the boundary curves for each section is determined where the shortest curve has to lift from the surface and form the wrinkle. The results of the wrinkling analysis for the whole layup are shown in Figure 4.17. The tows are colored based on the height (amplitude) of the wrinkle in the z-direction. The critical regions can be detected where the tows are colored in red corresponding to an amplitude greater than or equal to 1 mm.
Figure 4.15 Obtained cloud of points (units in mm)
Figure 4.16 Reproduced tow paths (units in mm)
Figure 4.17 Wrinkling map for the variable stiffness layup (units in mm)
Based on the wrinkling color map shown in Figure 4.17, process parameters during the manufacturing process can be changed to alleviate wrinkling formation especially in the severe areas where wrinkling is predicted to occur.

4.2 PATHS ON GENERAL SURFACES

This section demonstrates the implementation of the developed wrinkling equations for paths on general surfaces. The first example consists of analyzing the wrinkled shape of tow placed on a quasi- NURBS surface, and the effect of the geodesic curvature on the wrinkling amplitude. Then, several different layups with fixed-angle reference curves are considered for wrinkling analysis, where the results are presented in a color map showing potential regions for wrinkling. A second example for a tow placed on a geodesic path of a sphere is analyzed for wrinkling. Finally, the surface of an industrial scale mold is considered for analysis where arbitrary variable angle paths where analyzed.

4.2.1 Layup on a NURBS Surface

The first step in the analysis of a 3D towpath is to specify the surface on which the tow is laid down. To do so, the NURBS form of a surface is used since it is compatible with and implemented in most CAD software packages and offers a wide flexibility for the user. A NURBS surface is given by:

\[
S(u, v) = \frac{\sum_{i=0}^{n} \sum_{p=0}^{m} N_{i,p}(u) N_{j,q}(v) w_{i,j} P_{i,j}}{\sum_{i=0}^{n} \sum_{p=0}^{m} N_{i,p}(u) N_{j,q}(v) w_{i,j}}.
\] (4.20)

The \(P_{i,j}\) are the control points (control net), \(w_{i,j}\) are the corresponding weights, \(N_{i,p}(u)\) and \(N_{j,q}(v)\) are the non-uniform rational B-Splines basis functions of degree \(p\) and \(q\), respectively, which are defined over the knot vectors \(U\) and \(V\) (see [38] for details on NURBS surface parameterization).
To illustrate the calculation presented previously for a general surface, a quasi-random NURBS surface (Figure 4.18) is considered with the following parameters:

\[
P_{i,j} = \begin{cases} 
\{(1,1, -0.643), (1,2, -0.404), (1,3, -0.795), (1,4, -0.513)\}, \\
\{(2,1,0.692), (2,2,0.281), (2,3, -0.458), (2,4, -0.14)\}, \\
\{(3,1,0.085), (3,2,0.986), (3,3, -0.693), (3,4,0.678)\}, \\
\{(4,1,0.502), (4,2, -0.999), (4,3,0.804), (4,4,0.475)\} 
\end{cases} \quad (4.21)
\]

\[
w_{ij} = \{(1,1,1,1), (1,1,1,1), (1,1,1,1), (1,1,1,1)\} \quad (4.22)
\]

\[
U = V = \{0, 0, 0, 1, 1, 1, 1\} \quad (4.23)
\]

\[
p = q = 3 \quad (4.24)
\]

Figure 4.18 Example NURBS surface

The Z-coordinate of the control points is a randomly generated real number, whereas the X and Y coordinates are equally spaced to form a square mesh between 1 and 4 (units). The knot vectors are uniform, and the weights are equal to 1. In this case, the
NURBS surface is a special case of a B-Spline surface. This case is used for demonstration purpose and is displayed in Figure 4.18.

For demonstration purposes, the path to be analyzed is chosen to have the following functional form parameters:

\[ u_c(t) = \sin\left(\frac{\pi}{2} t\right), \quad (4.25) \]

\[ v_c(t) = t. \quad (4.26) \]

Therefore, the reference path \( C(t) \) has the following equation:

\[ C(t) = S(u_c(t), v_c(t)) = S\left(\sin\frac{\pi}{2} t, t\right), \quad 0 \leq t \leq 1. \quad (4.27) \]

Calculation of the geodesic curvature (3.23) along the length of this curve on the NURBS surface is shown in Figure 4.19.

![Figure 4.19 Geodesic curvature of the reference path on the NURBS surface](image)

Inspection of the geodesic curvature indicates that the amplitude of the wrinkles will be higher in the first section of the path and lower in the later section. The first four mode shapes generated by the software are shown in Figure 4.20 which agrees with the prediction that the wrinkles are proportional to the geodesic curvature.
Another feature that this method can offer is a general color map of the tows indicating the locations where wrinkles might appear and their severity, as demonstrated in (Figure 4.21). Based on the differential length between the two sides of the tow (directly related to the geodesic curvature), the calculations can be used to detect the possible location of the wrinkle. A positive differential length (colored in blue) indicates that wrinkles might appear on the dashed blue side of the represented tow, whereas a negative differential length (colored in yellow-red) indicates that the wrinkles might appear on the solid orange side of the tow. The green region is the best from manufacturing perspective since it is the least portion where wrinkling defect might appear.

Figure 4.20 First 4 mode shapes of a tow placed on a NURBS surface
4.2.2 Layup on a sphere

To investigate wrinkling of a tow placed on a geodesic path, the basic case of a placing tows on a dome shape is considered. The simple parametrization of a sphere is therefore considered to mimic the effect of surface curvature on wrinkling initiation, even for geodesic paths. A parametrization of a sphere with a radius $r$ is given by:

$$\textbf{Sphere}(u, v) = \begin{cases} X(u, v) = r \cos(2\pi u) \cos(2\pi v) \\ Y(u, v) = r \cos(2\pi u) \sin(2\pi v) \\ Z(u, v) = r \sin(2\pi u) \end{cases} , \quad 0 \leq u, v \leq 1$$

(4.28)

A geodesic path on a sphere is a great circle such as the equator. Hence, the reference path to be analyzed has the following parameters:

$$u_c(t) = 0.5 ,$$

(4.29)

$$v_c(t) = t .$$

(4.30)
Therefore, the reference path $C(t)$ has the following equation:

$$C(t) = \textit{Sphere}(u_c(t), v_c(t)) = \textit{Sphere}(0.5, t), \quad 0 \leq t \leq 1$$

(4.31)

or explicitly:

$$C(t) = \{-r \cos(2\pi t), -r \sin(2\pi t), 0\}, \quad 0 \leq t \leq 1.$$  

(4.32)

Note that for this given parametrization, the reference path starts and ends at the same location on the sphere. Implementation of equation (3.23) to compute the value of the geodesic curvature of $C(t)$ results in a value of zero, which reassures that a great circle on a sphere is a geodesic path.

The sphere radius for the following computation is chosen to be $r = 2$ (non-dimensional unit), and the tow width is considered to be $w = 1$. Scaling these non-dimensional numbers for instance would result of placing a $\frac{1}{4}$ in tow on a 1 in diameter sphere, which is technically impossible given the scale of fiber placement machines. However, these numbers are used for demonstration and visualization purposes.

The wrinkling equations are applied to the described tow placed on the sphere, and several mode shapes are visualized in Figure 4.22. Even though the reference path is a geodesic one, the parallel path on the sphere is not due to the finite width of the tow resulting in a smaller circle, hence the other edge of the tow will be under compression and producing the wrinkles as visualized in Figure 4.22.
4.2.3 Case study of wrinkling on an industrial scale part

To fully exploit the capability of the developed analysis, an industrial scale mold is considered (Figure 4.23). The surface of the mold has a doubly curved, i.e. the Gaussian curvature of the surface (3.9) varies from positive to negative depending on the location.
The first step for the analysis is to define the surface of interest where the paths are generated. To do so, a CAD model of the mold (Figure 4.24) is used in CATIA™ to determine the boundaries of interest, and to extract the surface where the fiber placement will occur. Another feature that can be used in CATIA™ is the ability to add or remove control points without changing the shape of the surface. This feature is based on the Knot Removal algorithm, which determines the possibility if a knot is removable, how many times, and the resulting new control points [38]. If a knot can be removed, the resulting control points are fewer, which makes the parametrization of the surface easier therefore reducing the complexity and the computational time of the analysis. The selected surface of interest in CATIA™ is refined by removing unnecessary control points, then the file is saved as a STEP file (.stp extension) so that the embedded information can be shared.

Once the surface is saved in the STEP format, the control points are be easily accessed by using Notepad++ [42]. Then, the control points were extracted manually and reordered to generate the exact shape of the surface in Mathematica. The obtained surface and the control points are shown in Figure 4.25.
Figure 4.25 Regenerated tool surface and corresponding control points in Mathematica (scale in mm)

For demonstration purposes, the center region of the surface is only considered for path analysis, where several paths with different parametrizations are generated and shown in Figure 4.26. Actual ¼ in tows (6.35mm) are modeled for the given reference path, and computation of the geodesic curvature is carried and presented in Figure 4.27 in the form of a color map of the actual tows. From a geometrical perspective, wrinkling will occur anytime there is a change in the curvature of the path, or surface curvature. Based on the color map shown in Figure 4.27, wrinkling will occur on the solid edge of the tow for the regions colored in blue where the geodesic curvature is positive, and on the dashed edge of the tow for the regions colored in yellow. As for the green areas, wrinkling is not a concerning issue. In addition, since actual scale is used (mm), the critical value of the geodesic curvature occurring in the bottom corner area shown in Figure 4.27, can be related
to a physical radius of curvature. For the case presented here, the value of the geodesic curvature is around $2.5 \times 10^{-3}$, or a steering radius of 400 mm.

Figure 4.26 Variable angle reference paths on the tool surface

Figure 4.27 Color map of $\frac{1}{4}$ in tows on doubly curved surface indicating possible regions of wrinkling initiation
4.3 SUMMARY

The implementation of the wrinkling governing equations within Mathematica™ to several examples is presented in this chapter. The case of a constant curvature circular path placed on a flat surface is first considered where the effect of the tow width and wrinkle numbers is studied. Numerical results show that increasing the tow width will lead to higher amplitude of wrinkles. However, increasing the number of wrinkles within a prescribed path results in a lower wrinkling amplitude. Higher number of wrinkle can be achieved in the presence of adhesion and good compaction pressure. In addition, the effect of the path curvature on the wrinkles amplitude is also investigated for an arbitrary NURBS path placed on a flat surface. Numerical results show that higher path curvature (smaller turning radius) leads to a higher amplitude of wrinkles. The special case of a curve with inflection points is also presented and analyzed for wrinkling. A variable stiffness layup of a plate with a hole is also analyzed for wrinkling, and a contour map indicating possible regions for wrinkles and their amplitudes is also presented.

For the case of paths on a general surface, an arbitrary path on a quasi-random NURBS surface is analyzed. The results show that the wrinkles amplitude is proportional to the geodesic curvature of the path. In addition, four different constant angle layups ($\pm 45^\circ, 0^\circ, \text{and} 90^\circ$) on the same NURBS surface are analyzed and results are shown in a color map indicating possible regions of wrinkling. For the same placement surface, different angle layups have different critical regions for wrinkling. Investigation of a geodesic path on a sphere is also carried on, showing that even for a path with zero geodesic curvature, wrinkling still occurs on a general surface due to the surface curvature. Finally, the developed equations are tested on an industrial scale mold, where the placement surface
is extracted from a CAD file, and reproduced within Mathematica™. Arbitrary variable angle reference paths are considered on the placement surface, the wrinkling analysis for these paths is carried, and the results are shown in color map indicating possible regions for wrinkling.
CHAPTER 5

EXPERIMENTAL VALIDATION

The purpose of this chapter is to compare and validate the developed equations for wrinkling by experimental measurement of the out-of-plane displacement of tows placed on curved paths using Digital Image Correlation (DIC). Experiments are conducted on a rigid thermoplastic tape for several radii of curvatures and different tow width. A description of the experimental procedures and the obtained results are summarized and presented in this chapter [43]. Then the experimental results are compared to the developed model for several tow width and curvatures. Improvements to the presented model are made based on the limitation of the developed testing fixture and a comparison between the experimental data and all presented models is presented.

5.1 EXPERIMENTAL PROCEDURE

For accurate out-of-plane displacement measurements, a stereo DIC setup is considered for the experimental validation. A stereo DIC setup consists of a pair of stereo cameras mounted on a rigid frame attached to a robust tripod, a calibration grid for the stereo DIC system, a dedicated computer with digital image correlation software installed (VIC 3D [44]) and a suitable high contrast speckle pattern on the surface of the tow so that the surface deformations incurred by the tow can be measured through post-processing of the data.
5.1.1 Stereo vision system

Two identical Point Grey CCD 9.1MP digital cameras (GS3-U3-91S6M-C) are used for the stereo vision system and are firmly attached to a rigid mounting bar as shown in Figure 5.1. The resolution of each camera is $3376 \times 2704$ Pixel and the physical size of each sensing element (pixel) is $3.69 \, \mu m$. The camera lenses used for imaging are a matched pair of compact Schneider-Kreuznach $35 \, mm \, f/1.9$ with a C-Mount for attachment to the camera body. The field of view obtained by using these cameras is $305 \, mm \times 254 \, mm$. The rigid platform shown in Figure 5.1 is attached to a tripod with rubber foot pads to dampen vibrations. Illumination is obtained by using a LED light source which emits minimal radiation resulting in minimal thermal strains on the tow due to lighting. Removal of reflections from the images of the relatively shiny tow surface is essential so that the image matching process can be performed accurately and with minimal error. To accomplish that, the light from the LED is linearly polarized by attaching a polarizing film to the front of the light source, and the axis on the polarizing filter on the camera lens is adjusted relative to the LED film.

![Figure 5.1 Stereo vision system set up](image-url)
5.1.2 Speckle pattern for Stereo DIC

The requirements for a good speckle pattern to accurately measure the shape and deformation of the tow are summarized as follows [45]:

- The speckle pattern should maintain adherence to the tow surface without deterioration during the deformation process.
- An even distribution of the speckle pattern is required for good contrast throughout the field of view.
- The size of the speckle pattern should be large enough to be sufficiently sampled by the camera sensors (at least \(3 \times 3\) pixels).

It is observed that the polarizing filters eliminated most of the reflections from the tow surface which gives a nice black background. To obtain good contrast speckle, white paint particles are sprayed using an airbrush on the black surface of the carbon fiber tow. The airbrush nozzle diameter is selected to deliver a fine mist with an average speckle size of 0.3 mm. Since the StereoDIC system magnification for the experiment is 90 \(\mu m/pixel\), the average speckle (0.3 mm) is sampled by 3.33 pixels.

5.1.3 Calibration of the stereo vision system

Calibration of the stereo vision system is performed by using a standard calibration grid shown in Figure 5.2. The size of the calibration grid is chosen such that it covers more than two third of the field of view of both the cameras of the stereo vision system. The calibration grid dot size and dot spacing are 6.35 mm and 18 mm, respectively. More than 120 synchronized stereo image pairs of the calibration grid are acquired for calibrating the stereo system. Each pair of synchronized calibration images is acquired after rotating the calibration grid by different angles in 3D space while keeping the grid within the depth of
field of both cameras. A 3rd order radial distortion correction model is used to correct typical lens distortions [45].

![Standard calibration grid for stereo vision system](image)

Figure 5.2 Standard calibration grid for stereo vision system [43]

5.1.4 Developed fixture and Material

A fixture is manufactured in order to speculate different fiber paths and compare them with the theoretical model. This fixture (shown in Figure 5.3) consists of 5 different paths printed on a paper and sandwiched between a Plexiglas and a wooden plate. The first path is considered as the reference path with zero curvature (straight line) and with length $L = 10 \text{ in}$. The 4 remaining paths have different radii of curvature starting from a radius of $100 \text{ in}$, then $50 \text{ in}$, $15 \text{ in}$ and finally $12 \text{ in}$. The carbon fiber tow is clamped on both sides and fixed in specific drilled holes in the wooden plate. The left clamp stays in place while the right clamp can change position on the plate to change between different paths while keeping the length constant (the path length used is 10 in).

To obtain a good comparison between the theoretical model and the experimental results, the material to be used in this experiment has to adhere to the modeling assumptions. For the case of an uncured thermoset prepreg tow, several deformation
mechanisms can appear while steering the material which were described in the first Chapter and shown in Figure 1.4. Hence, a UD carbon fiber thermoplastic tape (TenCate Cetex TC1200 PEEK [46]) is used in this experiment, since it is stronger in the transverse direction compared to an uncured thermoset tow, thus avoiding other deformation mechanisms. The original tape of 6 in width is cut into several widths to investigate the effect of the tow width on the wrinkling behavior.

Figure 5.3 Developed mounting fixture (Tow position at $R = 50$ in)

5.2 OBTAINED EXPERIMENTAL RESULTS

Stereo DIC analysis of the speckle pattern applied to the thermoplastic tow is performed using a commercial DIC software, VIC 3D [44]. A subset size of $25 \times 25$ pixels and a step size of 7 pixels are used in all DIC analyses.

Three tow widths are tested in this work: 1.375 in, 0.625 in, and 0.375 in. The results for the several tow widths and radii of curvature are shown in Figure 5.4, Figure 5.5, and Figure 5.6. A first observation concerning these results is that there is only 1 wrinkle/buckle of the tape for all cases. This was predicted, since in the absence of the
Figure 5.4 Shape and out of plane displacement measurement for several radii of curvature, and for 1.375 in wide tow [43]
Figure 5.5 Shape and out of plane displacement measurement for several radii of curvature, and for 0.625 in wide tow [43]
Figure 5.6 Shape and out of plane displacement measurement for several radii of curvature, and for 0.375 in wide tow [43]
adhesion, there is nothing to hold down the tow while steering it except the clamped boundaries. Another observation is that the clamping mechanism did not hold the tow accurately in place for the small steering radii (12 and 15 in), and the tow deviates from the actual path, indicating some inaccuracies in the manufacturing of the mounting fixture.

The maximum displacement in the z-direction is recorded from the DIC measurements for each tested case and is shown in Table 5.1. Note that this maximum measured value is slightly less than the actual maximum value in the tow since the selected area of interest during the DIC analysis did not cover the whole tow surface and some spatial resolution is lost. However, the distance from the edge of the tow to where the maximum measurement occurs is recorded for each tested tow, and it corresponds to 0.1974 in, 0.1132 in and 0.1555 in for the corresponding tow width of 0.375 in, 0.625 in and 1.375 in respectively. A first conclusion from Table 5.1 and the figures shown above is that increasing the tow width will increase the amplitude/elevation of the wrinkle/buckle. This conclusion agrees well with the circular path example discussed in the previous chapter. Another conclusion is that steering at a smaller radius of curvature will result in a higher wrinkle amplitude. Two outliers are identified here for the tight steering radius of 12 in and for the smallest two values of tow width. A possible reason behind that can be related to the small inaccuracies in the clamping mechanism at this extreme curvature. Another reason for that is that for only 1 wrinkle and for high curvatures, the tow can rotate more than 90° leading to a smaller elevation in the z-direction. This result is observed earlier in the NURBS path analysis section where tow in the first wrinkling mode rotates more than 90° and is shown in Figure 4.9 part (a).
Table 5.1 Maximum measured displacement in the z-direction (in in) of the buckled tow using DIC

<table>
<thead>
<tr>
<th>R = 12 in</th>
<th>R = 15 in</th>
<th>R = 50 in</th>
<th>R = 100 in</th>
<th>Distance from the tow edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>w = 0.375 in</td>
<td>0.6287</td>
<td>0.7412</td>
<td>0.5631</td>
<td>0.4424</td>
</tr>
<tr>
<td>w = 0.625 in</td>
<td>1.0179</td>
<td>1.0203</td>
<td>0.6770</td>
<td>0.5080</td>
</tr>
<tr>
<td>w = 1.375 in</td>
<td>1.6455</td>
<td>1.6118</td>
<td>1.0006</td>
<td>0.7379</td>
</tr>
</tbody>
</table>

5.3 COMPARISON WITH ANALYTICAL MODEL

To compare the obtained results from DIC to the developed analytical model, the following reference path is considered:

\[ C(t) = \begin{cases} 
  x(t) = u_c(t) = R \cos \left( \frac{L}{R} t - \frac{\pi}{2} \right) + v_x, & 0 \leq t \leq 1 \\
  y(t) = v_c(t) = R \sin \left( \frac{L}{R} t - \frac{\pi}{2} \right) + R + v_y 
\end{cases} \]

where \( R \) corresponds to the radius of curvature of the tow, \( L = 10 \text{ in} \) is the length of the tow, and the vector \( \mathbf{v} = \{v_x, v_y\} \) is a translation vector necessary to shift the path in the 2D plane so it coincides with the coordinate system of the experimental data. The parallel path \( C_p(t) \) can be computed using equation (3.40) at a distance \( d \) equal to the tow width. The curvature of this path (3.41) is constant and equal to \( 1/R \).

The shape of the wrinkled tow for \( N = 1 \) plotted up to the distance of interest from the parallel edge (Table 5.1) is shown along with the data points obtained from DIC in Figure 5.7 for a radius of 100 in and for all tow widths. Also, the maximum displacement in the z-direction is recorded at the distance of interest from the tow edge for all radii of curvature and tow widths and the results are shown in Table 5.2.
Figure 5.7 Analytical model (orange) and DIC data points (blue dots) for a radius of 100 in and for (a) $w = 0.375 \text{ in}$, (b) $w = 0.625 \text{ in}$ and (b) $w = 1.375 \text{ in}$

The major difference between the analytical model and the data points from Figure 5.7 is that the outer edge of the tow lifts up from the surface in the actual experiment,
whereas the analytical model assumes that this edge remains on the surface. This behavior was observed for all experiments and is also reflected in Table 5.2 where all the obtained results are significantly smaller than the experimental ones. This leads to the following remarks: (1) either the fixture is not accurate leading to the problem that the whole tow is under compression when moved to the curved path causing it to buckle and lift from the surface from both sides, (2) or the assumption that the outer edge remains on the surface is not correct even with a perfect fixture and the problem formulation has to be adjusted accordingly. This behavior can be justified since in the absence of the adhesion, there is no force to keep the tow on the surface leading to all these out-of-plane deformations.

Table 5.2 Maximum displacement in the z-direction (in \( \text{in} \)) of the buckled tow using the analytical model

<table>
<thead>
<tr>
<th>( w = 0.375 \text{ in} )</th>
<th>( R = 12 \text{ in} )</th>
<th>( R = 15 \text{ in} )</th>
<th>( R = 50 \text{ in} )</th>
<th>( R = 100 \text{ in} )</th>
<th>Distance from the tow edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w = 0.375 \text{ in} )</td>
<td>0.2001</td>
<td>0.2072</td>
<td>0.2103</td>
<td>0.1850</td>
<td>0.1325</td>
</tr>
<tr>
<td>( w = 0.625 \text{ in} )</td>
<td>0.5077</td>
<td>0.5117</td>
<td>0.4499</td>
<td>0.3671</td>
<td>0.1072</td>
</tr>
<tr>
<td>( w = 1.375 \text{ in} )</td>
<td>1.1751</td>
<td>1.1412</td>
<td>0.8446</td>
<td>0.6443</td>
<td>0.1178</td>
</tr>
</tbody>
</table>

5.4 COMPARISON WITH IMPROVED MODEL

The improved model of the wrinkling formulation consists of adding an assumed function \( w_c(t) \) to the z-direction in equation (3.43) to better describe the deformation of the outer edge due to the inaccuracies in the mounting mechanism. The adjusted formulation of the wrinkled tow is as follows:

\[
S_{w,\text{adj}}(t,n) = \begin{cases} 
    x_w(t,n) = u_c(t) - n \, d \, \hat{v}^c(t) \cos(\beta(t)) \\
    y_w(t,n) = v_c(t) + n \, d \, \hat{u}^c(t) \cos(\beta(t)) \\
    z_w(t,n) = w_c(t) + n \, d \, \sin(\beta(t)) 
\end{cases}
\]
where \( w_c(t) \) is assumed to be a cosine function based on the clamped conditions with the following equation:

\[
w_c(t) = \frac{a_0}{2} \left( 1 - \cos \left( \frac{2\pi (t - t_0)}{t_1 - t_0} \right) \right),
\]

with \( a_0 \) is the amplitude of the cosine shape extracted from the experimental data points, and \( t_1 \) and \( t_0 \) are the start and end points of the path. The values of \( a_0 \) extracted from the DIC data for each radius of curvature and tow width are shown in Table 5.3.

Table 5.3 Amplitude \( a_0 \) (in in) of the outer edge deformation

<table>
<thead>
<tr>
<th></th>
<th>( R = 12 \text{ in} )</th>
<th>( R = 15 \text{ in} )</th>
<th>( R = 50 \text{ in} )</th>
<th>( R = 100 \text{ in} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w = 0.375 \text{ in} )</td>
<td>0.4646</td>
<td>0.5157</td>
<td>0.4134</td>
<td>0.3492</td>
</tr>
<tr>
<td>( w = 0.625 \text{ in} )</td>
<td>0.5827</td>
<td>0.6142</td>
<td>0.3858</td>
<td>0.3032</td>
</tr>
<tr>
<td>( w = 1.375 \text{ in} )</td>
<td>0.5709</td>
<td>0.6752</td>
<td>0.4252</td>
<td>0.3031</td>
</tr>
</tbody>
</table>

The results of the new deformed model plotted along with the experimental data points for a radius of 100 in and for all tow widths are shown in Figure 5.8. Similarly, the generated models are plotted up to the distance of interest from the parallel edge which is the same for all presented models. The shape of the wrinkled tow for \( N = 1 \) in the adjusted model shows a very good agreement with the measured shape using DIC where most data points (shown in blue in Figure 5.8) are overlapping with the tow surface (shown in orange). A common behavior observed for all tested tows is that the middle section in the adjusted model is over predicting the amplitude of the wrinkle compared to the actual DIC data. The maximum elevation in the z-direction corresponding to the new amplitude of the wrinkle in the adjusted model is reported in Table 5.4. Comparing these results with the experimental data (Table 5.1) shows much better agreement compared to the non-adjusted
model (Table 5.2). The percentage difference between the maximum amplitude of the adjusted model and the actual experiment is computed and shown in Table 5.5.

Figure 5.8 Improved analytical model (orange) and data points (blue dots) for a radius of 100 in and for (a) \( w = 0.375 \text{ in} \), (b) \( w = 0.625 \text{ in} \) and (c) \( w = 1.375 \text{ in} \)
Table 5.4 Maximum displacement in the z-direction (in \textit{in}) of the buckled tow using the adjusted analytical model

<table>
<thead>
<tr>
<th>( w ) = 0.375 in</th>
<th>( R ) = 12 in</th>
<th>( R ) = 15 in</th>
<th>( R ) = 50 in</th>
<th>( R ) = 100 in</th>
<th>Distance from the tow edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6808</td>
<td>0.7303</td>
<td>0.5688</td>
<td>0.4585</td>
<td>0.1325</td>
<td></td>
</tr>
<tr>
<td>1.0818</td>
<td>1.0887</td>
<td>0.7195</td>
<td>0.5427</td>
<td>0.1072</td>
<td></td>
</tr>
<tr>
<td>1.6421</td>
<td>1.6527</td>
<td>1.0580</td>
<td>0.7663</td>
<td>0.1178</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.5 Percentage difference between the adjusted model and the measured data at the maximum displacement point

<table>
<thead>
<tr>
<th>( w ) = 0.375 in</th>
<th>( R ) = 12 in</th>
<th>( R ) = 15 in</th>
<th>( R ) = 50 in</th>
<th>( R ) = 100 in</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.6536 %</td>
<td>-1.4900 %</td>
<td>1.0154 %</td>
<td>3.5156 %</td>
<td></td>
</tr>
<tr>
<td>5.9061 %</td>
<td>6.2759 %</td>
<td>5.9071 %</td>
<td>6.3875 %</td>
<td></td>
</tr>
<tr>
<td>-0.2033 %</td>
<td>2.4770 %</td>
<td>5.4250 %</td>
<td>3.7110 %</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.9 Summary of maximum amplitude of the wrinkles for various radii of curvature and tow width, and for the DIC data, analytical and adjusted models
On average, the adjusted model is over-predicting the maximum amplitude by 4.12%. The main reason behind this over-prediction is that compressive strains are not accounted for in this model, and the outer edge’s length remains the same. Other deviation from the actual experimental data can be related to the inaccuracies of the mounting mechanism, other shear deformations in the tow, possible slippage in the clamping mechanism, and the small errors in the measuring technique.

A summary for all obtained amplitudes of the wrinkled tow shape for all conducted experiments, computed analytical and adjusted models, and for all radii of curvature and tow width is shown in Figure 5.9. The improved model shows an excellent agreement with the experimental data, with a capability of over-predicting the results. The effect of the tow width and curvature of the path can be also depicted from Figure 5.9: the amplitude of the wrinkles increases with wider tows and decreases with higher radius of curvature. Also, the amplitude of the wrinkles is more sensitive to the change in the tow width compared to the radius of curvature.

5.5 CONCLUSION

Experimental validation of the developed wrinkling equations is conducted by measuring the out-of-plane displacement of thermoplastic carbon fiber tows using Digital Image Correlation (DIC). 10 in long thermoplastic tapes at three different tow width of 0.375 in, 0.625 in, and 1.375 in are speckled and loaded in a mounting fixture. The manufactured fixture has five different loading locations: the first one is a straight path to acquire the reference images for the DIC, and the remaining ones are constant curvature paths with a radius of 100, 50, 15, and 12 in.
The acquired DIC data points show that the mounting fixture is slightly inaccurate making the loaded tows under compression and lifting up from the surface from both sides. This behavior is also due to the absence of any adhesion between the tow and the surface of the mounting fixture. Comparing the results of the theoretical model to the mentioned experimental result does not yield in a good agreement since the outer edge of the tow is assumed to remain on the placement surface.

An adjustment to the developed wrinkling model is made by adding a term that accounts for the initial imperfections of the loading mechanism. The shapes of wrinkled tows from the adjusted model are compared to the experimental results and show a good agreement for the first buckling mode. The maximum amplitude is also recorded for all tested cases, where the adjusted model seems to over-predict the actual experimental results by 4.12%. This over-prediction in the amplitude is due to the fact that the theoretical model does account for any compressive strain in the tow, and assumed that the length of the inner edge will remain the same. In addition, the trend of the effect of tow width and radius of curvature on the amplitude of the wrinkle is depicted in the experimental results, in the analytical and in the adjusted models. The amplitude of the wrinkles increases with increasing tow width and decreasing radius of curvature, with the amplitude being more sensitive to the variation in the tow width.
CHAPTER 6

CONCLUSIONS AND FUTURE WORK

Several defects might arise during manufacturing of fiber reinforced composite structures using the Automated Fiber Placement process, one of them is tow wrinkling. The main reason behind the occurrence of tow wrinkling is the mismatch in length between the tow-path on the placement surface and the actual length of the tow delivered from the machine head. Modeling tow wrinkling is a complex task that involves process parameters, material properties and geometry. In the presented work, geometrical parameters affecting tow wrinkling were only investigated, neglecting the elastic properties, tackiness of the tow and other process parameters. A worst case scenario is assumed where the tow has a very large in-plane stiffness but a negligible bending stiffness, causing the tow to buckle and form wrinkles in the out-of-plane direction when steered or placed on a general curved surface.

Based on these assumptions, governing equations for tow wrinkling along an arbitrary path on a general surface were derived and presented in Chapter 3. The tow is assumed to wrinkle on its compressive edge where the length of the tow-path is smaller than the length of the actual tow leading the outer edge to lift totally from the surface to form a wrinkle. The shape of the wrinkle is assumed to be a cosine function similar to the buckling of a beam with clamped-ends conditions. In addition, the shape of the tow in the transverse direction is assumed to remain the same as the placement surface. Further
simplifications of the relevant equations were also presented in Chapter 3 to account for the special case of curved paths placed on a flat surface.

The governing equations of wrinkling were implemented within Mathematica™ and applied to several examples in Chapter 4. The effect of the curvature of the path, the number of wrinkles, and the tow width on the amplitude of the obtained wrinkles were investigated. Numerical results showed that decreasing the tow width, decreasing the curvature, and increasing the number of wrinkles within a path leads to a smaller wrinkle amplitude. As for the case of paths on general surfaces, it was determined that the amplitude of the wrinkles is mainly proportional to the geodesic curvature. For the case of geodesic paths, it was determined that wrinkling still occurs due to the surface curvature. Finally, color maps indicating possible regions of wrinkling were also generated for a variable angle layup of a plate with a hole, for different constant angle layups on a NURBS surface, and for a variable angle layup on a doubly curved surface of an industrial scale mold.

Experimental validation of the developed wrinkling equations was conducted in Chapter 5 by measuring the out-of-plane displacement of thermoplastic carbon fiber tows using Digital Image Correlation. 10 in long thermoplastic tapes at three different tow width were speckled and loaded in a mounting fixture at several radii of curvature. Experimental results showed a good agreement with the adjusted theoretical model that accounted for initial imperfections in the mounting fixture. A comparison of the wrinkles amplitude between the adjusted model and the experimental results showed on average that the model over-predicted the amplitude by 4.12%. This over-prediction is due to the fact that the theoretical model does account for any compressive strain in the tow, and assumed that the
length of the inner edge will remain the same. In addition, the trend of the effect of tow width and radius of curvature on the amplitude of the wrinkle was depicted in all the results where the amplitude increased with increasing tow width and decreasing radius of curvature, with the amplitude being more sensitive to the variation in the tow width.

Further work in the field of modeling tow wrinkling during AFP process should focus on implementing elastic properties of the material, the tackiness of the interface between the tow and the placement surface, and the effect of the process parameters on these properties. The effect of material properties on the wrinkling behavior can be critical. For instance, other mechanisms of length absorption (tensile/compressive/shear strains, fiber waviness, bunching, and folding) can occur simultaneously with tow wrinkling for materials that are weak in the transverse direction such as uncured prepreg tows and dry fiber tows. Experimental investigation of such mechanisms is crucial to understand the behavior of steered tows, and to improve the modeling capabilities. DIC can be used as a measuring technique to obtain the deformations and strains applied to a tow placed on a curved path using an AFP machine, hence classifying which mechanism is more likely to occur. Also, measuring the strains and deformations of the tow over a certain period of time is important to understand the viscoelastic behavior of the material. In addition, obtaining accurate measurements of the material properties of the uncured state of the tows such as the modulus of elasticity in the longitudinal and transverse direction, the shear modulus, and Poisson’s ratio, is critical for the modeling part and for the understanding of which deformation mechanism is the most energetically favorable.

To further improve the modeling capabilities, future work should also include modeling of the tackiness of the interface between the tow and the underlying surface. The
below surface can be either the tool surface, or most likely other carbon fiber layers. The direction of the carbon fiber tows in the underlying surface might also have an important effect on the tackiness properties. A separation law at this interface should be established as a function of the material properties, and other process parameters such temperature, compaction pressure, and layup speed. At a critical load induced by the compressive strains due to steering, the interface will fail resulting in the initiation of a wrinkle. The propagation of the wrinkle after failure initiation should be monitored until a stable solution is found resulting in a final shape of the wrinkle. Several experiments should be conducted by changing the process parameters to understand their effect on the strength of the bonding interface. Finally, the effect of initial imperfections in the placement surface such as the presence of minor gaps or overlaps between adjacent tows should be included in the overall model. The presence of such minor defects leads to a weaker interface and favorable critical sites for wrinkling initiation.
REFERENCES


