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## Implementing the Constructed Scaffold Model: Hands-On Activity Units for Advanced Placement Calculus

Susan Scott  
*University of South Carolina*

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IMPLEMENTING THE CONSTRUCTED SCAFFOLD MODEL:  
HANDS-ON ACTIVITY UNITS FOR ADVANCED PLACEMENT CALCULUS

by

Susan Scott

Bachelor of Science  
North Carolina State University, 1986

Master of Education  
Coastal Carolina University, 2002

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Submitted in Partial Fulfillment of the Requirements

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Accepted by:

Susan Schramm-Pate, Major Professor

Richard Lussier, Committee Member

Russell Conrath, Committee Member

Kenneth Vogler, Committee Member

Cheryl L. Addy, Vice Provost and Dean of the Graduate School

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## **DEDICATION**

To my wonderful husband Willie and my incredible children Cassidy, Cameron, and Caleb, I would like to thank each of you for all of the support, encouragement, and love that you have given me as I have embarked on this journey. I really could not have done this without the sacrifices that you made for me. I hope that my experiences will serve as a reminder that you should always pursue your dreams and never let obstacles stand in the way.

To the educators who work tirelessly to ensure that every student feels safe, accepted, and is challenged to achieve, I commend you.

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## ABSTRACT

The purpose of the present action research study is to describe a hands-on activity model, named the Constructed Scaffold Model (CSM), used in an Advanced Placement Calculus class in a southeastern United States suburban high school. Data were collected over an 8-week period during the spring 2017 semester. The teacher-researcher developed a pretest and posttest to describe the CSM's impact on 10 student-participants in her Advanced Placement classroom. Repeated measures t-tests were used to determine the effect of the CSM on the student-participants' test scores. Additional qualitative data were collected from field notes, student surveys, a focus group, and student work samples to determine student-participants' perceptions of the CSM. Findings include the class average score on the 20-question pretest was 1.5 questions correct, and the posttest average score was 13.4 for a net gain of 11.9 points. Based on an analysis of the qualitative data the following themes emerged: 1) developing conceptual understanding, 2) fostering interpersonal dynamics, and 3) improving model efficacy. The teacher-researcher developed an action plan that included meeting with school leaders, district leaders, and fellow teachers to share the results of this action research study and begin a new cycle of the CSM in the fall of 2017.

*Keywords:* analytical representation, conceptual understanding, graphical representation, hands-on activity, manipulative, mathematical modeling, process, numerical representation, rote memorization, theoretical understanding, verbal representation

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## **CHAPTER ONE: RESEARCH OVERVIEW**

### **Introduction**

The purpose of Chapter One is to introduce the present action research study, which involved an Advanced Placement (AP) Calculus BC class offered through the College Board (2016a) at County High School (pseudonym). County High School (CHS) was a large, suburban high school located in the southeastern United States. After studying other research that utilized hands-on activities in mathematics, the teacher-researcher designed a hands-on activity model, named the Constructed Scaffold Model (CSM), that was implemented during the 2017 spring semester with her AP Calculus students (Couture, 2012; Cruse, 2012; Pfaff & Weinberg, 2009; Schunk, Pintrich, & Meece, 2007). In an effort to enable student-participants at CHS to make critical connections within the calculus content and foster enduring understandings, the teacher-researcher designed the CSM. The teacher-researcher worked with the student-participants to develop an action plan to improve the effectiveness and utility of the CSM for future AP Calculus courses in order to continually and positively affect her students' scholastic achievement.

### **Background of the Advanced Placement Calculus Program**

The AP Mathematics program at CHS is offered through the College Board (2016a) to give these high school students a chance to earn college credit and to study at

a rigorous level. The College Board (2016a) offers two calculus courses, AP Calculus AB which is one semester of calculus and AP Calculus BC which is two semesters of calculus. AP Calculus BC provides the participant-researcher's students the vehicle to learn college-level calculus while still in high school with the added benefit of no cost to the student. AP Calculus BC is the equivalent of Calculus 1 and Calculus 2 at many college campuses and consequently the participant-researcher's students who scored at least 3 out of a possible 5 points on the AP Exam received up to eight credit hours before their freshman year in college (College Board, 2016a). This is equal to approximately \$2,600 in tuition savings at the University of South Carolina (University of South Carolina-Columbia, n.d.). The AP Calculus BC Exam is a demanding test that assesses student comprehension of calculus content. The College Board (2016a) has indicated that AP Calculus "is designed to develop mathematical knowledge conceptually, guiding students to connect topics and representations throughout each course and to apply strategies and techniques to accurately solve diverse types of problems" (p. 4).

Students needed to make connections within the course material and not simply perform procedural calculations from memory to understand class concepts and be successful in the course. Nao (2008) concluded that the goal for students enrolled in AP classes was to pass the AP Exam and as a result AP teachers must present the curriculum in a manner that prepared their students for the rigor of the AP Exam. Based on the results of Nao's (2008) research, the teacher-researcher in the present action research study determined that for her student-participants enrolled in AP Calculus BC their objectives were twofold: they had a desire to study calculus at a high level, and they wanted to pass the AP Calculus BC Exam so that they earned credit for Calculus 1 and

Calculus 2 before college. The research study by Nao (2008) indicated that most AP teachers reviewed and reflected on the scores of their students in order to help their students achieve the goal of passing the AP Exam. If the scores were not at an appropriate level then the educator usually had a desire to modify classroom instruction in an attempt to improve students' AP Exam performance. Lamattina (2008) found that 70% of AP teachers who were participants in her research believed that "the rigors of the AP curriculum call for learning new teaching methods and practices" (p. 85). Following Nao (2008) and Lamattina (2008), the purpose of the present action research study was to determine the impact of the CSM, a hands-on activity model, on calculus achievement of AP Calculus BC students at CHS.

### **Problem of Practice**

The Problem of Practice (PoP) for the present action research study was that students at CHS were scoring below their global peers on the AP Calculus Exam and struggling to build connections from basic mathematical operations to complex calculus topics. This was especially a problem for CHS students from groups that have been historically marginalized in advanced mathematics such as females, Blacks, and Hispanics (Caviness, 2014). After reviewing the AP Calculus Exam scores of her former students, the teacher-researcher found that her male students had the highest average score followed by White students, students of color, and lastly female students. In 2017, the AP Calculus Exam consisted of multiple-choice and free-response questions, which required students to apply and synthesize multiple concepts (College Board, 2016a). Based on a review of the previous AP Calculus BC Exam scores at CHS, the teacher-researcher discovered that students at CHS had scored below their global peers on the

free-response questions and had difficulty modeling mathematics. The teacher-researcher believed that helping her student participants make connections and scaffold their prior knowledge to new calculus content would increase the AP Calculus Exam scores of her student-participants so that the individual scores met or exceeded the global mean score of all examinees.

### **Background of the Problem of Practice**

High school students in the United States performed worse than their counterparts from around the world according to the Organisation for Economic Co-operation and Development (OECD, 2012b), and the Programme for International Student Assessment (PISA, 2012) that tested 510,000 students in developed nations in 2012. These 15-year-old students were assessed in reading, mathematics, and science (OECD, 2012b) with a focus not on the recall of learned facts but the application of knowledge in real-world situations. The OECD (2012a) provided a sample exam problem that was based on a “real world” scenario. In this sample problem, students were given a map of a city and required to find a location in which three people could meet in a place that would not require any of them to travel for more than 15 minutes (OECD, 2012a). The results of the 2012 assessment found that U.S. students ranked 27<sup>th</sup> out of the 34 OECD nations in mathematics (OECD, 2012b). Students in the U.S. scored well when they were required to make a single calculation or deduce simple explanations from charts and graphs but struggled when they had to create a mathematical model to represent a problem or make mathematical interpretations from various contexts (OECD, 2012b). Students were able to perform mathematical processes in isolation, but they did not understand how to

represent mathematics concepts in multiple contexts and struggled to create mathematical models in order to solve problems.

The AP Calculus BC Exam results for CHS students indicated that they did not perform as well as their global peers on questions that required them to model calculus content. Educators must understand that their life experiences can be quite different from those of their students and subsequently find ways to relate curriculum to students' lived experiences. Brazilian educator Freire believed that traditional schools were not meeting the needs of the illiterate peasants (Ryan & Cooper, 1998). Freire (2013) noted, "one cannot expect positive results from an educational or political action program which fails to respect the particular view of the world held by the people" (p. 161). Based on the ideas of Freire, educators must take the time to understand the lived experiences of their students if they are going to be able to develop activities that will relate to students' lives. Hands-on activities can provide students with the opportunity to explore and learn from new experiences.

Many educators have worked to bridge the gap so that students can scaffold from prior knowledge when learning new material. For example, Dewey (1938/1997), a leader of the progressive education movement, believed that students must understand not only the finished product but also how this finished product was built or can be modified in order to fully learn about the product, otherwise a disconnect in learning would exist. Fowler (2010) noted the importance of connecting learning to the lived experiences of the learner. The results of research by Cruse (2012) indicated that the utilization of a hands-on activity model helped students form more concrete understanding of abstract ideas. However, as teachers develop hands-on activity models for students, these processes



must have a purpose and need to be linked within the curriculum so that they enhance and expand student understanding (Dewey, 1938/1997). A research study by White (2012) emphasized that educators must be careful to avoid using hands-on activities to simply entertain students, but should methodically select activities that provide learning connections for students.

With the global economy and instant access to information, students must develop skills to process and discern everything that is going on around them. Research by Buoncristiani and Buoncristiani (2012) highlighted the role that globalization played in the new ideas that have emerged on the way that people think and learn, and underscored the importance of developing thinking skills. Based on this evidence the teacher-researcher's student-participants needed to develop problem-solving skills to be competitive in the global economy, and rote memorization of mathematical procedures would not adequately prepare them for the future. Furthermore, the results of the PISA (OECD, 2012b) assessment showed that mathematics students in the U.S. were falling behind students in other countries. Mathematics teachers cannot rely simply on the lecture method of instruction in order to maximize student learning. Research by Schunk et al. (2007) determined that learning for students might become meaningless when teachers rely solely on the lecture method of instruction. Students had better comprehension when they were actively involved in the learning process. Hands-on activities provided students with the opportunity to be actively engaged in the learning process, which led to deeper conceptual understanding (Cruse, 2012).

Based on the results of research studies by White (2012) and Cruse (2012), students developed a greater understanding when they made connections from prior

knowledge to the current material within the course. Scholastic achievement in AP Calculus BC requires students to develop a multirepresentational approach to problem solving (College Board, 2016a), which is built through connections that are made within the course. An occurrence that might be seen in some mathematics classes across the U.S. is for teachers to be actively involved in demonstrating how to solve a particular problem and the students passively following along. Schunk et al. (2007) concluded that when teachers rely solely on the lecture method of instruction, students do not become personally involved in the lesson and the learning becomes meaningless. A focus of the present action research study was to explore the effect of the hands-on activity model in an AP Calculus BC class, including students from groups that have been historically marginalized in advanced mathematics such as females, Blacks, and Hispanics (Caviness, 2014).

### **Research Question and Objectives**

The research question for the present action research study was, “What is the impact of the hands-on activity model on secondary student achievement in Advanced Placement Calculus?” The teacher-researcher’s primary objective was to establish a continuous and positive impact on her students-participants’ scholastic achievement in AP Calculus as a result of critical connections that were made within the course content through implementation of the CSM. An additional objective was for the teacher-researcher’s student-participants to increase their AP Calculus Exam scores after participating in the CSM.

## **Purpose of the Study**

The primary purpose of the present action research study was to describe the implementation of the CSM, a hands-on activity model (Couture, 2012; Cruse, 2012; Pfaff & Weinberg, 2009; Schunk et al., 2007) designed by the teacher-researcher for an AP Calculus class and used over an 8-week span. The teacher-researcher developed the CSM in order to help her student-participants increase their AP Calculus Exam scores by enabling them to build connections from basic mathematical operations to complex calculus content. The secondary purpose of the present study was to design an action plan in conjunction with the student-participants to improve the effectiveness and utility of the hands-on activity model in providing critical connections and enduring understanding within the course content for future AP Calculus students at CHS.

## **Scholarly Literature**

### **Rationale**

A more detailed literature review appears in Chapter Two of the present Dissertation in Practice (DiP). A major justification for designing the CSM for the present action research study involved improving the curriculum and pedagogy for an AP Calculus BC class which was offered by the College Board (2016a) to provide CHS students with the opportunity to take college-level Calculus 1 and Calculus 2 while still in high school. This impacted the teacher-researcher's action research study because upon completion of her AP Calculus course, her student-participants were required to take the AP Calculus Exam to assess their knowledge of the course content. The teacher-

researcher wanted her student-participants to earn AP Calculus Exam scores that met or exceeded the global mean score of all examinees.

### **Description of the AP Calculus BC Exam**

The AP Calculus BC Exam consists of 45 multiple-choice questions and six free-response questions, all of which must be completed in 195 minutes (College Board, 2016a). The College Board (2016a) has developed a curriculum framework for each AP course to ensure that rigorous standards are met and that the course content is consistent all over the world. AP Calculus BC is designed to be challenging to students using the premise identified by the College Board (2016a) that “building enduring mathematical understanding requires students to understand the *why* and *how* of mathematics in addition to mastering the necessary procedures and skills” (p. 4). It is the objective of the College Board (2016a) to teach students how to apply the mathematical concepts that have been learned and to not simply mimic operations that have been shown or rely on rote memorization of facts. For students to be successful on the AP Calculus Exam they must be able to understand the theoretical aspects of calculus as well as have the ability to develop multirepresentational approaches to the content. Mathematics educators have strived to help students become better problem solvers and become more comfortable working through complex operations.

Additionally, the National Council of Teachers of Mathematics (2015), commonly referred to as NCTM, has indicated that students need to gain proficiency in the process standards of “problem solving, reasoning and proof, communication, connections, and representation” (p. 1). Proficiency in problem solving indicates that

students are able to apply what they have learned to solve real-world problems and develop a greater understanding of mathematics, the NCTM (2015) also notes that proficiency in making connections allows students to “understand how mathematical ideas interconnect and build on one another to produce a coherent whole” (p. 1) and proficiency in representation indicates that students are able to use correct mathematical representations for problem solving. Students needed to utilize each of these NCTM (2015) process standards in order to pass the AP Calculus Exam. If a student received at least 3 out of a possible 5 points on the AP Calculus BC Exam then they received credit for Calculus 1 and Calculus 2 at most colleges and universities (College Board, 2016a).

### **Progressive Educational Theory**

Progressive educational theory was used to create the CSM for the present action research study. The early progressive education movement, led by John Dewey, advocated for learning through hands-on activities so that students would gain more understanding of concepts (Dewey, 1938/1997). Hands-on activities allow students to actively engage and become responsible for their own learning, whereas relying on lecture is a passive form of learning (Dewey, 1938/1997). Utilizing Dewey’s (1938/1997) theories of learning, the teacher-researcher in the present action research study believed that it was important for her AP course to incorporate components of a variety of instructional methods to prepare students for the challenge of the AP Calculus Exam. Additionally, the results of a study by Lamattina (2008) indicated that AP teachers must develop new teaching methods in order to prepare students for the rigor of the AP Exam. It was the goal of the teacher-researcher in the present action research

study to implement the CSM to gain an understanding of the impact of hands-on activities on students' scholastic achievement in an AP Calculus BC class at a CHS.

### **Background Theories**

As a mathematics instructor, the teacher-researcher initially believed in the essentialist ideal that students would learn best through direct instruction of a standards-driven curriculum (McNergney & Herbert, 1998). Furthermore, due to concern about her students not performing as well in the area of mathematics as students from other countries, the teacher-researcher leaned toward the ideas presented in *A Nation at Risk* (U.S. National Commission on Excellence in Education, 1983) and the importance of excellence in education. The U.S. National Commission on Excellence in Education (1983) highlighted that “excellence characterizes a *school* or *college* that sets high expectations and goals for all learners, then tries in every way possible to help students reach them” (p. 12). As the teacher-researcher gained more experience in the classroom, a better understanding of the needs of students as learners developed. Having high expectations for students was crucial, but believing that every student learned the same material using an identical method was not realistic and failed to differentiate the needs of the individual.

The teacher-researcher began to understand the importance of the progressive theory of education whereby students are, as McNergney and Herbert (1998) wrote, “responsible for maximizing their own potential, with the teacher’s role being that of a helper and guide” (p. 104). This corresponded to the ideas of Buoncristiani and Buoncristiani (2012) who concluded, “we learn best when we are actively engaged in the

process of our own learning” (p. 5) and that students are better able to learn new material when they are able to connect it to past experiences and understanding. The premise that her students could become responsible for their own learning in a calculus classroom became an intriguing prospect for the teacher-researcher. The goal of the present action research study was to develop and implement the CSM within an AP Calculus BC classroom in order to enable students to gain a better understanding of complex calculus topics and increase their scholarly achievement on the AP Calculus Exam.

Research in the scholarly literature on the topic of hands-on activities such as O’Shea, Heilbronner, and Reis (2010) indicated that high-achieving female students prefer teachers who allow them to work collaboratively and provide academic challenge. O’Shea et al. (2010) found “educators who model a passion for mathematics, who provide challenging learning activities, and who occasionally enable their female students to work cooperatively may encourage these students’ talents” (p. 262). Ladson-Billings (1997), an advocate for mathematics achievement of African American students noted, “prominent in the reform of mathematics education is the call for students not merely to memorize formulas and rules and apply procedure but rather to engage in the *processes* of mathematical thinking” (p. 697). After studying scholarly literature, the teacher-researcher designed the CSM, which enabled her course to be constructed as a scaffold where her student-participants’ prior mathematical knowledge built connections within the AP course in order to impact her students’ scholarly achievement on the AP Calculus Exam.

## **Data Sources**

The teacher-researcher collected the quantitative data from two sources. The first was the pretest (see Appendix A) and second data source was the posttest administered after implementation of the CSM in her AP Calculus BC classroom during the spring 2017 semester. The teacher-researcher used a pre- and posttest assessment to measure growth in calculus ability and to answer the research question. Students took the pretest prior to implementation of the CSM, which provided the teacher-researcher with data on students' knowledge of calculus. After the students completed the CSM they took a posttest to measure their achievement in calculus. The results were disaggregated by race, gender, and socioeconomic status. Using field notes, the teacher-researcher recorded student engagement and interactions while completing the hands-on activities (see Appendix B). Student surveys (see Appendices C and D) and student work samples were also collected. These anecdotal data, as well as documents from the College Board (2016a) and the high school where the teacher-researcher teaches, were used to triangulate (Mertler, 2014) the quantitative data. After the data were collected and the student-participants completed the AP Calculus Exam, the teacher-researcher used a focus group discussion (see Appendix E) to reflect on the data with her student-participants in order to form an action plan designed to continually improve the CSM for the AP Calculus BC classroom at CHS.

## **Glossary of Keywords**

Definitions of keywords that are included in the study are listed below.



*Action research*: “any systematic inquiry conducted by educators for the purpose of gathering information about how their particular schools operate, how they teach, and how their students learn” (Mertler, 2014, p. 32).

*Analytical (algebraic) representation*: representing a function “by an equation in two variables” (Larson, 2007, p. 40).

*Conceptual understanding*: “comprehension of mathematical concepts, operations and relations” (Common Core State Standards Initiative, 2015b, p. 1).

*Criterion-referenced test*: an assessment that measures “a student’s performance by comparing it to some clearly defined criterion for mastering a learning task or skill” (McNergney & Herbert, 1998, p. 406).

*Direct instruction*: a teaching method that “consists of four stages: teacher analysis and definition of goals, with selection of appropriate materials; teacher instruction; teacher demonstration; and student practice” (Unger, 2007a, p. 350).

*Graphical representation*: representing a function “by points on a graph in a coordinate plane in which the input values are represented by the horizontal axis and the output values are represented by the vertical axis” (Larson, 2007, p. 40).

*Hands-on activity*: a “teaching method in which a student is an active rather than passive participant” (Unger, 2007b, p. 530).

*Likert scale*: “statements provided on surveys or questionnaires to respondents where individuals are asked to respond on an agree-disagree continuum” (Mertler, 2014, p. 309).

*Manipulative*: “defined as any tangible object, tool, model, or mechanism that may be used to clearly demonstrate a depth of understanding, while problem solving, about a specified mathematical topic or topics” (Kelly, 2006, p. 184).

*Mathematical modeling*: the Common Core State Standards Initiative (2015a) identifies modeling as “the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions” (p. 1).

*(Mathematical) process*: as defined in *Webster’s New World College Dictionary* (Agnes, 2000) is “a particular method of doing something, generally involving a number of steps or operations” (p. 1144).

*Numerical representation*: representing a function “by a table or a list of ordered pairs that matches input values with output values” (Larson, 2007, p. 40).

*Rote memorization*: “the acquisition of essential information through the process of memorization” (Unger, 2007c, p. 952).

*Theoretical understanding (knowledge) of mathematics*: according to *Webster’s New World College Dictionary* (Agnes, 2000) is knowledge of mathematics that is “limited to or based on theory; not practical or applied; hypothetical” (p. 1485).

*Verbal representation*: representing a function “by a sentence that describes how the input variable is related to the output variable” (Larson, 2007, p. 40).

## Potential Weaknesses

During implementation of the CSM, the teacher-researcher made the assumption that the student-participants would put forth their best effort in all of the activities as well as on the pre- and posttest. This assumption was critical since the student-participants completed most of the collected data. A second assumption that the teacher-researcher made was that the student-participants would be honest and candid in their responses and not feel compelled to provide responses that they believed that the teacher-researcher wanted. The teacher-researcher continually reminded the student-participants to answer questions openly and honestly without fear of repercussions, and the assumption was made that the student-participants were candid in their responses.

The primary limitation of the study was the multiple-choice style of pre- and posttest questions used for data collection. The teacher-researcher made the decision to use a multiple-choice format because the AP Calculus Exam includes multiple-choice sections. Student-participants could conceivably guess when answering a multiple-choice question and get a correct answer for knowledge that they really do not have. On the other hand, students could make a minor error and get a multiple-choice question wrong when they actually had knowledge of the topic. Although the teacher-researcher did not see evidence of the student-participants guessing on the pre- and posttest, there was the possibility that they could have guessed on some of the questions if they were experiencing difficulty with the topic.

The teacher-researcher delimited the study to include only AP Calculus BC student-participants. The teacher-researcher wanted her student-participants to improve

their AP Calculus Exam scores and make critical connections within the mathematical content. While the teacher-researcher could have chosen student-participants from her Precalculus Honors class since the majority of them enroll in her AP Calculus class the following school year, she believed that using her AP Calculus class would be the most impactful for the present action research study.

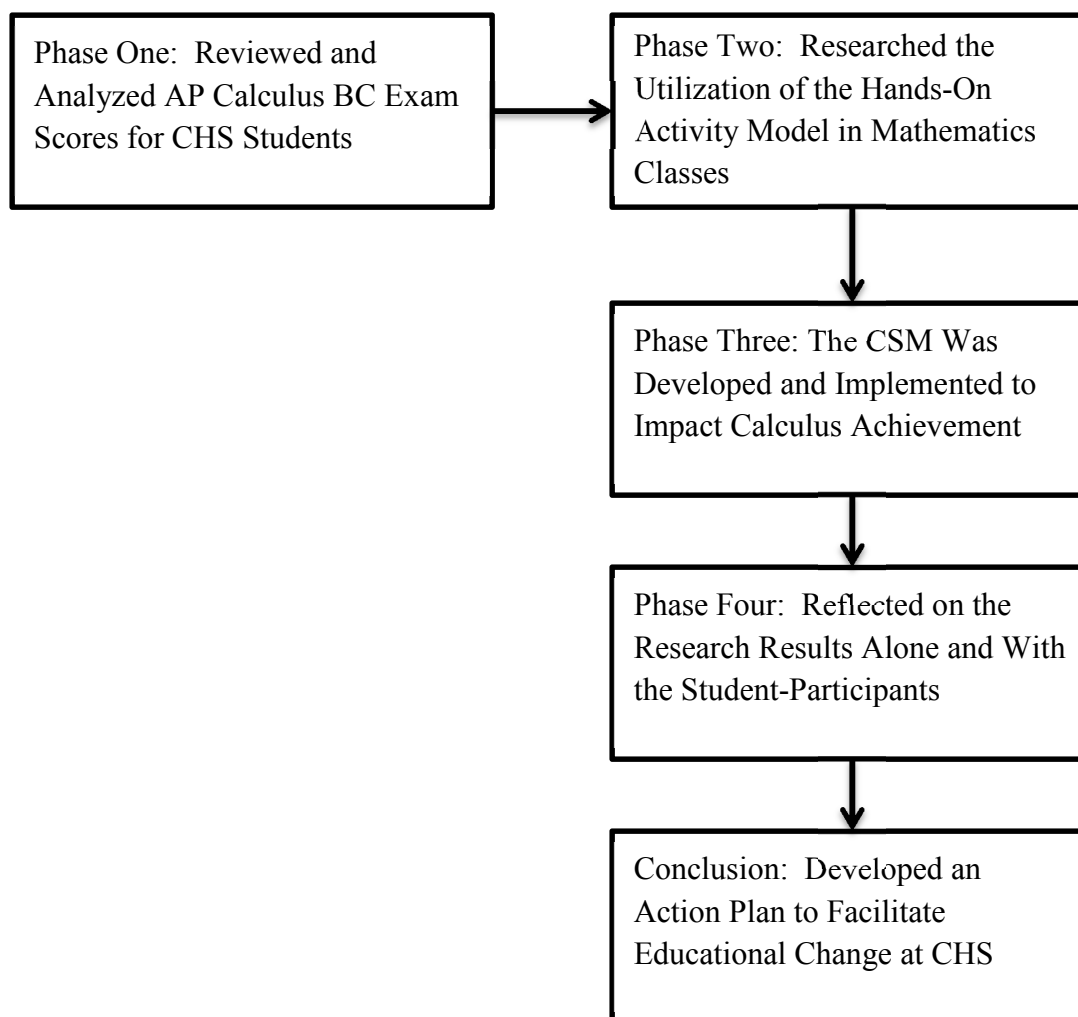
### **The Significance of the Study**

The present action research study is significant and adds to the body of scholarly literature because the purpose was for the student-participants to increase their academic achievement on the AP Calculus Exam. All schools in the state in which CHS was located receive a report card and rating based on the report card scores. Each high school was required to report the percentage of the student body “Enrolled in AP/IB programs” and “Successful in AP/IB programs” (South Carolina Department of Education, 2017). Consequently, any educational research that can lead to improvement of the report card scores for the school was important. Additionally, if the student-participants have positive experiences when encountering the CSM then they might tell their peers about the classroom activities and encourage them to enroll in AP Calculus. While improving the report card scores was beneficial, researchers have also found that students who take AP Calculus in high school have higher college retention rates than those who do not (Mattern, Shaw, & Xiong, 2009). The teacher-researcher has used the student-participants’ work samples that were created during the CSM to show other students the work that was completed in her AP Calculus class.

This study is also significant because of the impact that the CSM had on groups that have been historically marginalized in advanced mathematics such as female, Black, and Hispanic students. Prior research studies have shown the benefits that female, Black, and Hispanic students receive from engaging in hands-on activities in small groups (Ladson-Billings, 1997; O’Shea et al., 2010). Other research has shown that high school students who take AP courses are better prepared for the rigor of college work than students who take other types of math courses (Mattern et al., 2009). The problem of practice for the present action research study was the need for the student-participants to increase their AP Calculus Exam scores as a result of critical connections that were made from basic mathematical operations to complex calculus content. If the student-participants earned a passing score on the AP Calculus BC Exam then they would receive credit for Calculus 1 and Calculus 2 at all of the colleges and universities in their state. This credit for two classes is a substantial financial benefit for the student-participants and their families who have to pay their tuition or take out student loans. As a result, the student-participants who encountered the CSM in AP Calculus have received numerous benefits, which have a direct effect on their future college studies. Any study that can positively impact students is significant for educators and the present action research study is no exception.

### **Conclusion of Chapter One**

As a result of what Mertler (2014) identified as the “cyclical, iterative process” (p. xv) of action research, the teacher-researcher developed a conceptual framework for the present action research study. Figure 1.1 shows the conceptual diagram of this study.



*Figure 1.1.* Conceptual diagram.

The teacher-researcher used quantitative data to answer the research question. The student-participants took a pretest prior to implementation of the CSM and the same assessment also served as the posttest that was given after 8 weeks. The student-participants earned a mean score on the 20-question multiple-choice pretest of 1.5 correct with a standard deviation (S.D.) of 1.02, and 13.4 correct with S.D. of 4.61 on the posttest for a net mean gain of 11.9 correct. Every student-participant had a score increase from

the pre- to posttest. Calculations from the t-tests indicated statistically significant growth from the pretest mean score to the mean score of the posttest for these particular student-participants. Qualitative data were used to triangulate (Mertler, 2014) the quantitative findings. The teacher-researcher transcribed the student-participants' responses to a survey and highlighted data that fit into specific themes. After studying the results, the teacher-researcher determined that three themes emerged from the data. The themes were: 1) developing conceptual understanding, 2) fostering interpersonal dynamics, and 3) improving model efficacy.

### **Dissertation Overview**

This dissertation is composed of the following chapters:

- 1) CHAPTER ONE: RESEARCH OVERVIEW. This chapter introduces the research by describing the problem of practice, the research question, the purpose of the research, and an overview of the research design and methodology.
- 2) CHAPTER TWO: LITERATURE REVIEW. A thorough review of the literature from both theoretical and historical contexts is presented.
- 3) CHAPTER THREE: METHODOLOGY. The research question, action research design, and data collection strategies are included in this chapter.
- 4) CHAPTER FOUR: FINDINGS AND IMPLICATIONS. The results of the research are provided in this chapter along with a discussion of their implications.
- 5) CHAPTER FIVE: SUMMARY, CONCLUSIONS, AND ACTION PLAN. This chapter includes interpretation of the research results, discusses the implications

of the results for continuous improvement in the teacher-researcher's AP Calculus class, and provides an action plan.



## **CHAPTER TWO: LITERATURE REVIEW**

### **Introduction**

The purpose of Chapter Two is to present a detailed review of the related literature on the current action research study of the impact of the Constructed Scaffold Model (CSM), a hands-on activity model designed by the teacher-researcher, for use with her AP Calculus BC class. This literature review provides a thorough overview of the literature related to AP Calculus instruction as well as key information about prior research on the use of manipulatives and hands-on activities in mathematics classes and studies on participation in advanced mathematics courses based on race, gender, and socioeconomic status. A review of literature on the historical background of hands-on activities is included. The prior research in this area showed that implementing hands-on activities in mathematics made classes more enjoyable for students, had an impact on student confidence levels, and increased student understanding of the material by allowing students to scaffold from previous learning to new content.

The related literature on the identified problem of practice for the present action research study also gave the teacher-researcher a tremendous amount of information on the guiding principles of the AP program and a better understanding of the desired learning outcomes for the AP Calculus BC course. For example, a review of the use of the hands-on activity model in education was needed to provide a historical context for

the problem of practice. By reviewing past literature, the teacher-researcher was able to identify a problem of practice for the current study that built on the scholarly work that has already been completed as well as to evaluate the quality and depth of work that existed on the relationship between hands-on activities and mathematics achievement. In order to conduct a review of the literature, the teacher-researcher searched through scholarly literature of textbooks and peer-reviewed journals. While the teacher-researcher found scholarly literature related to using manipulatives and hands-on activities in mathematics, increasing equitable access to advanced mathematics, and programs to increase calculus achievement, she did not find any studies on the use of hands-on activities in AP Calculus classes. This literature review was important because it provided broad knowledge of the research that has been done with educators using hands-on activities in mathematics classes and the manner in which teachers worked to build student learning and understanding of mathematics. A review of demographic information on historically marginalized groups gave a context to the need for equity in advanced mathematics classes. The theories and methodologies used in prior research helped to guide the present action research study so that it added to the community of scholarly work by providing data on the effectiveness of using the CSM in an AP Calculus BC class.

## **Themes and Ideas**

### **Definition of Hands-On Activities**

Unger (2007b) described hands-on activities as a “teaching method in which a student is an active rather than passive participant” (p. 530). Hands-on activities allow

students to construct their own understanding through the completion of a task rather than passively listening to a lecture by the teacher. Hands-on activities include using manipulative resources that serve as tools, which allow students to model or represent abstract mathematical concepts. Algebra tiles, rulers, protractors, compasses, paper, and scissors are all examples of manipulatives that provide students with the opportunity to understand abstract mathematical concepts using tangible learning tools. Graphing calculators are important technological tools because they give students the ability to see visual models of algebraic expressions. In the mathematics classroom hands-on activities can include using manipulatives and creating either graphical or physical models of algebraic expressions. A research study by Pfaff and Weinberg (2009) developed the following design guidelines for hands-on activities:

The activity should be motivated by a “natural” question and students should explore the situation by gathering and interpreting data. The teacher should ask scaffolding questions, solicit conjectures, and ask students to explain their reasoning. Multiple representations (numerical descriptions, tables, and graphs) and modes (kinesthetic, verbal, and written) should be integrated into the activity and the teacher should facilitate making connections between the representations. The context should build on students’ intuitions and experiences by using situations with which students are familiar and asking questions that students see as non-abstract. This should enable students to make predictions before engaging in the activity. Students should be asked to reflect on their predictions after they have collected data and again during a guided class discussion to articulate the differences between their predictions and the results. (p. 2)

Each of these guidelines helped the teacher-researcher enable students to construct their own understanding from the hands-on activities. In AP Calculus, a graphical or physical model can be essential in providing students with the ability see a concrete representation of an abstract concept.

Several studies have investigated the effectiveness of hands-on activities in mathematics. A research study showed that utilizing paper as a manipulative tool in which students used origami to fold the paper helped students understand the concept of planes in geometry and thus develop a better sense of spatial reasoning (Sze, 2005). In addition to the aforementioned learning tools, researcher Munley (2003) also identified compasses, geoboards, tangrams, and other household items that can be used to model mathematics as manipulatives. Educators do not need to invest in expensive props to engage students, but can find any number of readily available items to provide students with enriching learning experiences. Hands-on activities can be completed by an individual, small group, or whole group based on the requirements of the lesson. Mathematics educators have found many ways to provide active learning experiences for their students.

### **Problem of Practice**

#### **Hands-On Activities Affect Student Understanding in Mathematics**

Many students and teachers believe that rote memorization of facts is vital for success in mathematics. A study was conducted by Tsao (2004) to compare the attitudes toward mathematics between fifth graders in the United States and Taiwan. The results indicated that American students relied on memorization in mathematics and did not

place emphasis on understanding the concepts related to the procedures while Taiwanese students placed more focus on the process of solving the problem to develop a deeper level of understanding. The Taiwanese students believed that mathematical knowledge is useful in life at a much higher rate than students from the United States (Tsao, 2004). This study highlighted some of the problems that American students have in understanding how to apply what they learn in mathematics classes to real-world problem solving. Based on these results, it seems as if students in the United States do not see the relevance of mathematics in their lives.

The mathematics classroom of today is much different from that of the past. Students need to develop problem-solving skills to be competitive in the global economy and rote memorization of mathematical procedures will not adequately prepare students for the future job market. Mathematics teachers can no longer rely on simply lecturing students if they want to maximize student understanding because students comprehend more when they are actively involved in the learning process. Hands-on activities give students the opportunity to apply what is learned for deeper conceptual understanding.

One of the ways in which hands-on activities help students understand mathematical concepts was identified in a research study conducted by Kelly (2006) on the effects of using manipulatives in cooperative learning groups in mathematics problem solving. The study data revealed that when students used manipulatives in cooperative learning groups they learned to use mathematical language, which helped to build a foundation for conceptual understanding and the ability to use abstract math skills.

The use of manipulatives helped students construct their own knowledge base and showed that hands-on activities can be used in the mathematics classroom to enhance instruction. In order to realize the benefits of hands-on activities, educators must make class time available for students to engage in them. However, one study of the use of hands-on activities in high school geometry classes did not provide definitive results (Munley, 2003). In the study, one geometry teacher from eight different high schools was selected to be interviewed on the use of hands-on instruction in his or her respective class. Most of the teachers did not use manipulatives on a regular basis for various reasons including not feeling comfortable using the manipulatives, fear that introducing hands-on activities would take away valuable classroom time when there is already a struggle to find time to finish teaching the standards, lack of available supplies, or students not properly engaging in the activities.

University researchers Awang and Zakaria (2012) were asked to evaluate a computer program that used hands-on activities to explore calculus topics. The purpose of the hands-on activities was to allow students to gain a better understanding of the calculus content while at the same time applying the concepts. The evaluators found that the computer program was most effective when students were asked reflective questions after the hands-on activities so that this would trigger their metacognitive awareness. The reflection component of the activity offered students the opportunity to make the connection between the hands-on activities and the mathematical content (Awang & Zakaria, 2012).

In another study focused on the use of hands-on activities to enhance the understanding of calculus, researchers Yoon, Dreyfus, and Thomas (2010) used modeling

activities to help students get a better grasp of the content. Educators would agree that most calculus students are able to perform the required mathematical processes but struggle with conceptual understanding. The focus of the study was to determine if hands-on activities were more beneficial for students before or after instruction on the topic. When students were engaged in hands-on activities before formal instruction on a topic, then the hands-on activities fostered mathematical thought; but when hands-on activities were used after instruction this led to a deeper conceptual understanding (Yoon et al., 2010). The previous research indicated that hands-on activities must be well planned. Teachers cannot mindlessly assign hands-on activities but they must carefully integrate them into course content to optimize student learning (Dewey 1938/1997). These studies were related to the current research because the purpose was to determine the impact of hands-on activities on AP Calculus BC Exam scores. Since the current research focused on hands-on activities with high school AP Calculus BC students it extends beyond previous studies conducted at the university level.

### **Points of View**

#### **Students in the United States**

Students in the United States ranked 27<sup>th</sup> out of 34 nations in the area of mathematics on the 2012 Programme for International Student Assessment (OECD, 2012b). If the U.S. wants to be a global leader in science and technology then students must improve their knowledge and understanding of mathematics. Mathematics teachers must find instructional methods that increase student achievement. Lamattina (2008) concluded that new teaching methods should be employed when preparing students for

rigorous course content. In order for students to develop enduring understanding of mathematical content, educators must help students create critical connections to new learning using prior knowledge.

In order for American students to be able to compete in the global economy, it has become necessary for them to be proficient in the areas of mathematics and science. Students must understand mathematics and science for many of the high-technology jobs of today. Unfortunately, American students are falling behind students from other countries in these subjects. Mathematics teachers must find instructional methods that help students understand how to solve higher-level mathematics problems (Ladson-Billings, 1997). Kent and Caron (2008) argued that:

[Before] the development of national standards for mathematics teaching and learning, there was a tradition of keeping mathematics concepts and procedures abstract and obscure so that only those who can “master” such ideas can succeed and progress in the field. (p. 246)

They also noted that, similarly, when students were taught mathematical processes in isolation, they were unable to see the relevance and importance of mathematics in their lives although the study of higher-level mathematics is required for careers in engineering and science. Mathematical competency is essential in helping students prepare for the future job market.

Ladson-Billings (1997) underscored the idea that in American society while few would admit that they struggle to read or write people readily claim that they do not understand mathematics. Parents often tell their children stories about how much they



hated mathematics when they were students. Mathematical aptitude is often viewed as “nerdy” and therefore leads many students away from the study of higher-level mathematics (Ladson-Billings, 1997). As a result of these negative opinions, many students have developed a dislike for mathematics and a lack of belief in their abilities to be successful mathematics learners. In the 2002 National Science Foundation Authorization Act (P.L. 107-368), congress mandated the funding for more laboratories in U.S. schools to enhance instruction in mathematics and science (H. R. No. 107-488, 2002). Congress has placed emphasis on funding laboratories because results of prior research have shown that hands-on experiences can help student understanding, allow students to develop practical problem-solving skills, learn to work as a team, and help students become more interested in mathematics and science. Additionally, a key focus of research has been on identifying methodologies that change students’ attitudes about mathematics and make mathematics more enjoyable and accessible for learners.

### **Summaries of Literature**

#### **The Development of the Hands-On Activity Model to Make Calculus More Enjoyable for High School Students**

As previously noted, a significant amount of research has been conducted to identify instructional methods that change learners’ outlooks about mathematics in order to make mathematics more pleasurable and manageable for learners. Mukhopadhyay, Resnick, and Schnauble (1990) conducted a research study to determine if there was a difference in children’s ability to solve mathematics problems when presented within the context of a story and when given as an algebraic equation. The investigation showed that elementary school students were able to solve the mathematical calculations in a

narrative story with a great degree of fluency, but were unable to perform the same calculations when they were in the form of an equation. These results pointed to the importance of relevance in the study of mathematics.

A study of high school students by Mensah, Okyere, and Kuranchie (2013) was designed to determine if a relationship existed between students' attitudes about mathematics and their achievement. Students were given a questionnaire to record their attitudes about mathematics. At the end of the term the researchers reviewed the final mathematics course grade for each participant. Using a Pearson Correlation coefficient, researchers found a positive correlation between student attitude and achievement in mathematics. If research showed that a positive attitude about mathematics increased achievement then educators must utilize methods that help students find mathematics more enjoyable.

In a previous study, Miller (2003) evaluated the use of cooperative learning groups in two Algebra 2 classes. Students were put into cooperative groups in which they had to create a daily group journal, work practice problems on a review carousel, make a group presentation of practice work, and complete a cooperative review activity. The classes engaged in cooperative learning activities at least three times over a 3-week period. Students were surveyed prior to and at the conclusion of the 3-week period on their views about mathematics and cooperative learning. The results showed a slight increase in favorable attitudes towards mathematics at the conclusion of the 3-week period, although the results were not statistically significant. Even though these results were not statistically significant, any methodology that causes students to find mathematics more enjoyable can be deemed beneficial.

The outcome of a study of fourth-grade students revealed more positive results (Couture, 2012). In that research, the teacher supplemented instruction on fractions with math manipulative activities. Fractions usually are considered difficult and most mathematics students would rather not have to perform arithmetic operations involving fractions. Students often struggle to remember the rules that must be applied when completing arithmetic operations on fractions. Finding instructional approaches that make working with fractions a less daunting task for students can be instrumental in changing students' attitudes toward mathematics. After the hands-on activity, students were given a Likert scale survey to record their views on the activity's effectiveness. The survey data revealed that 89.7% of students indicated that using math manipulatives made learning fun. Additionally, the results identified an improvement in achievement.

The effectiveness of using hands-on activities in mathematics was shown in a different study of fifth-grade students' understanding and attitudes toward mathematics before and after engaging in a hands-on activity. The students in that study had an improved level of achievement, increased understanding, and a positive attitude toward mathematics after taking part in a hands-on activity (Allen, 2007). The aforementioned research study was important because creating positive attitudes toward mathematics so that more students study mathematics and science was one of the goals of the congressional mandate. Furthermore, it is critical for educators to make connections within the content of mathematics so that students use prior knowledge to develop new concepts. The results of these previous studies have proven to be beneficial for the present action research because they provided a foundation for the study into the impact of the CSM in an AP Calculus BC class.

## **Hands-On Activities Impact Student Confidence in Mathematics**

Psychologist Albert Bandura has studied the concept of self-efficacy, which is a person's belief in his or her ability to be successful (Bandura, 1982). Consequently, a student with a great deal of self-efficacy has more belief in him or herself and thus is more confident in his or her ability. Teachers can help students feel more assured in their abilities by giving students assignments that offer a challenge without being too difficult, as well as assignments that have a real-world context. In a study of African American and Latino college students, DeFreitas and Bravo, Jr. (2012) found a relationship between self-efficacy and academic achievement. The researchers measured the students' academic self-efficacy using the Self-Regulated Learning Scale of the Multidimensional Scales of Perceived Self-Efficacy that was developed by Bandura in 1990. It was determined that students with higher self-efficacy had better achievement academically than those with less self-efficacy (DeFreitas & Bravo, Jr., 2012). This outcome leads to the conclusion that confidence is a vital trait for students and therefore it is critical for educators to find methods to help students gain confidence.

A study of university-level calculus and linear algebra students indicated that when students felt more confident in their mathematical skill level then they developed a positive attitude and were willing to work harder (Garcia-Santillan, Escalera-Chavez, Lopez-Morales, & Rangel, 2014). The data for that research were collected using a Likert scale survey. Additionally, a study by Jennison and Beswick (2010) was conducted to determine if a focused intervention would help to build confidence in students who suffered from math anxiety. The eight participants were eighth-grade students at a boys' school. The participants were given an intervention on fractions that

included mental computational materials, hands-on learning materials, and work in collaborative groups. The results of a Likert scale survey showed that 50% of the students were confident in their mathematical ability after the intervention. In interviews the students indicated that the intervention taught them how to solve problems on their own without relying on others for help (Jennison & Beswick, 2010). After reviewing the literature, the results indicated that hands-on activities can help students build confidence and increase proficiency in their mathematical abilities.

### **Equitable Access to Advanced Mathematics**

In the *AP Calculus AB and AP Calculus BC: Course and Exam Description*, the College Board (2016a) stated a commitment to equitable access in Advanced Placement classes:

We encourage the elimination of barriers that restrict access to AP for students from ethnic, racial, and socioeconomic groups that have been traditionally underrepresented. Schools should make every effort to ensure their AP classes reflect the diversity of their student population. (p. 2)

While this goal is admirable, research showed that minority students were not represented in equal proportions in AP mathematics courses. Research by the National Science Foundation (2014) determined that “Black students were particularly underrepresented in the exam-taking population for AP exams in calculus BC” (para. 17). AP Calculus BC has the second-lowest percentage of African American participants, with only AP Physics C having a lower percentage of African American students (National Science Foundation, 2014).

A report by Toppo (2013) found that only about one-third of African American and Hispanic students who had Preliminary Scholastic Aptitude Test (PSAT) scores that indicated readiness for AP mathematics classes actually enrolled in the courses. One reason that enrollment in AP courses by qualified African American and Hispanic students was lower than other races was because many of the schools in less affluent areas do not offer as many AP classes (Smith, 2012). For schools that do offer a wide variety of AP classes, educators need to make students more aware of the prerequisite courses that prepare them for AP mathematics courses. For students to have the required courses to enroll in AP Calculus BC at CHS, they have to take two mathematics classes in at least one school year. If students and their parents are unaware of this need for taking more than one mathematics class in a school year then the students will not be able to enroll in AP Calculus BC.

Another issue that results in lowering the number of students taking AP courses is that some schools restrict the courses to only the top students because of the pressure to maintain a high rate of students passing their AP Exams. Although participation in AP courses can help prepare high school students for the rigor of college, some schools do not allow students to enroll in AP classes, even if the students have the prerequisite coursework, if they believe that the student will not be successful on the AP Exam and thus lower the school's AP Exam passing rate (Matthews, 2014). Educators must maintain high expectations for all students and provide qualified students with the opportunity to take AP classes even if the students do not pass the AP Exam. In *The 10<sup>th</sup> Annual Report to the Nation: Calculus BC Subject Supplement*, the College Board (2014) listed racial demographics of 2013 AP Calculus BC Exam takers as 56.1% White, 28.8%

Asian/Asian American/Pacific Islander, 8.3% Hispanic/Latino, 2.9% Black/African American, 0.4% American Indian/Alaska Native, and 3.5% Other/No Response. Based on these data it is apparent that Hispanic, African American, and Native American students were underrepresented in AP Calculus BC classes. The College Board (2014) also indicated that 59% of 2013 AP Calculus BC Exam takers were male and 41% female. Similarly, female students were underrepresented in AP Calculus BC.

Scores presented in *AP Data – Archived Data 2014: National Report* (College Board, 2016b) indicated students in the United States earned a mean score of 3.82 points out of a possible 5 points on the 2014 AP Calculus BC Exam. Additionally, 81.5% of students who took the 2014 AP Calculus BC Exam made a score between 3 and 5 points which was considered a passing mark and resulted in credit for Calculus 1 and Calculus 2 at most colleges and universities (College Board, 2016b). When the scores were disaggregated by race then disparity became evident. Asian students performed the best on the 2014 AP Calculus BC Exam, earning a mean score of 4.05 points and the results showed that 85.9% Asian examinees passed the exam (College Board, 2016b). The mean score of students classified as Other was 3.88 points and 82.8% of these students passed the exam (College Board, 2016b). White students had a mean score of 3.84 points and passed the exam at a rate of 82.6% (College Board, 2016b). The mean score of Puerto Rican students was 3.41 points and 72.6% passed the exam (College Board, 2016b). American Indian students had a mean score of 3.33 points and 72.1% of this group earned a passing mark (College Board, 2016b). Sixty-nine percent of students in the Other Hispanic category passed the exam and achieved a mean score of 3.28 points (College Board, 2016b). Sixty-two percent of Mexican American students achieved a

passing score on the exam and their mean score was 3.02 points (College Board, 2016b). Black students earned a mean score of 2.99 points and 61.8% passed the 2014 AP Calculus BC Exam (College Board, 2016b). The results indicated that African American students scored the lowest of any group on the 2014 AP Calculus BC Exam, and the mean score for African American students was 0.83 points less than the national mean for all examinees while the percentage of African American students who passed the exam lagged the national average by 19.7%. The archived data for the 2014 AP Calculus BC Exam also indicated that male students made a mean score of 3.91 points and 83.6% passed the exam (College Board, 2016b). Female students achieved a mean score of 3.68 points and 78.6% earned a passing mark (College Board, 2016b). Male students earned a mean score that was above the national average by 0.09 points while the score for female students was 0.14 points below the national average.

Boys outnumber girls in AP Calculus BC classes by 18%. Research by Dentith (2008) revealed that girls were more willing to take AP classes when they had positive feelings toward the teacher's ability to teach and support them academically. Developing a positive relationship with a teacher and believing that the teacher encourages students were both characteristics that were important to girls. Female students were more willing to take AP courses when they received additional weight in the calculation of their grade point average for the classes (Dentith, 2008). Unfortunately, that study also showed female students felt less confident in their mathematical ability than their male peers even when taking AP Calculus and had a fear of being perceived as "the dumb girl" (Dentith, 2008). Additionally, research by O'Shea et al. (2010) indicated that high-achieving female students preferred teachers who allowed them to work collaboratively and



provided academic challenge. The disparity between the number of male and female students taking AP Calculus BC can be eased by selecting quality teachers who have the ability to design lessons that encourage students to maximize their learning as well as providing the incentive of adding weight when calculating grade point averages for students taking AP courses.

The College Board (2016a) has made an attempt to expand the AP program by offering courses in urban and rural schools (Sadler, 2010). Schools in less affluent neighborhoods need teachers who are prepared to teach AP courses which are comparable to college-level classes and the students in these schools need the prerequisite coursework for AP classes (Smith, 2012). Some schools offer a large variety of AP courses but restrict enrollment to only the top-achieving students in order to maintain a high percentage of students with passing exam scores. By increasing access to AP courses, typically underserved groups are able to study at a more rigorous level. Smith (2012) argued that the format of AP Exams gave an advantage to wealthy students. A research study by Oates (2003) concluded that many African American students do not enroll in AP classes for fear of being ostracized by their peers for being smart and acting “White” (Oates, 2003).

## **Variables**

### **The Advanced Placement (AP) Exam as a Measure of Calculus Achievement**

The AP Calculus BC Exam is used to measure the calculus achievement of students. The AP Calculus BC Exam is a standardized test that is “criterion-referenced and scored from 1 through 5, with a 5 representing a score of ‘extremely well-qualified’”

(Mattern, Shaw, & Xiong, 2009, p. 1) which has been accepted by the academic community as a valid instrument for the measurement of calculus achievement. Research has shown that students who took the AP Calculus Exam had a higher retention rate after the first year of college than those who took rigorous courses that were not AP (Mattern et al., 2009).

Some researchers had less-than-favorable opinions of the AP program. Sadler (2010) indicated “criticism that AP courses cover too much material, too superficially, and too quickly have surfaced from blue-ribbon panels studying advanced high school coursework, like that of the National Research Council” (p. 4). A study by Geiser and Santelices (2004) identified that some colleges use participation in AP courses as a criterion for admission. This policy created issues of equity because students in less affluent schools were penalized simply for having less access to AP classes.

### **Improving AP Exam Scores**

A great deal of effort has been placed on increasing demographic diversity among students who take AP courses. Many states have developed initiatives to ensure that AP courses reflect the demographics of the school population. Multiple studies have been conducted in an attempt to increase diversity, whether gender, race, or socioeconomic status, in AP classes. Unfortunately, few studies have been conducted on developing methods to improve AP Exam scores.

In a study on the characteristics of AP classes in California, researchers Furry and Hecsh (2001) presented an overview of the statistics of AP classes in California. Their research determined that the schools that had the highest percentage of students passing

AP Exams were schools that had supplies available for hands-on instruction, including graphing calculators available for all students in AP Calculus. Supplies such as graphing calculators are expensive and some school districts do not have the funds for this expenditure. This could lead to a disparity for students in less affluent schools. The researchers indicated that group activities should be an integral part of classroom instruction in order to help students gain a better comprehension of the course material.

Bokern (n.d.) identified strategies to improve AP Exam scores. In that study, emphasis was placed on providing students with active learning opportunities and improving rigor. He emphasized that rote memorization, an excessive amount of homework, and superficially difficult assessments do not constitute rigor. Rigor occurred when students used their skills to analyze solutions and synthesize to create products. Based on the research it was concluded that hands-on activities gave students the opportunity to develop a level of conceptual understanding that helped them to be more successful on the AP Calculus BC Exam.

### **Primary and Secondary Sources**

#### **Issues of Equity in the Mathematics Classroom**

While it has been noted that students in the United States are not performing as well as students from other countries in the area of mathematics, the results of research indicated that there were disparities within the mathematics achievement levels of students in the United States. Based on results of mathematics standardized tests, Kent and Caron (2008) stated, “the gap between middle-class white students and working-class poor and students of color continues to exist at all levels” (p. 246). Unfortunately race,

gender, and socioeconomic status factor into a student's ability to achieve in mathematics. Research by Aud, Fox, and KewalRamani (2010) reported data from the National Assessment of Educational Progress (2005) mathematics assessment of 12<sup>th</sup> grade students in 2005 that showed 70% of Black students scored at the Below Basic level and 60% of Hispanic students scored at the Below Basic level while only 30% of White students scored at the Below Basic level. This provided evidence of an achievement gap for African American and Hispanic students in mathematics. In 1998, 11.0% of all high school graduates took calculus in high school (Aud et al., 2010). If these data were disaggregated by race, 12.1% of White, 6.6% of African American, and 6.2% of Hispanic high school graduates took calculus in high school. By 2005, 13.6% of all high school graduates took calculus in high school with 15.3% of White, 5.5% of African American, and 6.3% of Hispanic high school graduates taking calculus in high school (Aud et al., 2010). The data indicated that the percentage of African American students taking calculus in high school dropped from 1998 to 2005 while there was an increase for White and Hispanic students. Educators must determine the causes of the disparity in enrollment in advanced mathematics courses by certain minority students and work to increase diversity in upper-level mathematics.

Oakes (1990) researched the systemic causes for lower mathematics achievement of African American students, Hispanic students, and children in lower socioeconomic groups. The research determined that ability grouping tended to result in disproportionate numbers of minority and low-income students being placed in low-ability classes. Once a student was placed on the low-ability track, it was almost impossible for that student to participate in higher-level mathematics. Schools in low-

income or high-minority neighborhoods often offered fewer higher-level mathematics courses. As a result, the brightest students at these schools were unable to take the same level of mathematics as students in more affluent areas. Additionally, less affluent schools tended to have less-qualified teachers and even when the schools have highly qualified teachers, the less-experienced teachers tended to be assigned the lower-ability classes. Schools in disadvantaged areas had fewer resources such as computers and laboratories to enhance instruction. This puts minority and impoverished students at a disadvantage.

Specific curriculum and instruction techniques have been suggested to help African American students become more successful in mathematics. Research by Ladson-Billings (1997) found that teachers who have high expectations for student achievement, use prior knowledge to scaffold instruction, maximize instructional time, extend students' capacity to learn at higher levels, and have strong content and pedagogical knowledge are able to help their students perform better in mathematics. Finally, Oakes (1990) noted that teachers in schools with high minority enrollment often placed less emphasis on "inquiry and problem-solving skills" (p. x). These data were corroborated by Aud et al. (2010) who found that in secondary schools with a majority of African American students about 25% of mathematics teachers were neither certified nor mathematics majors in college, while for secondary schools with a majority of White students only 8% of mathematics teachers were neither certified nor college mathematics majors. To achieve parity, all students must be instructed by highly qualified teachers regardless of the race or socioeconomic status of the student.

There was also a disparity based on gender in the number of students taking advanced mathematics courses. Research showed that female students responded to teachers that encouraged them to take upper level mathematics classes and worked to form positive relationships in which students were valued as learners (Dentith, 2008). Gender bias causes some girls to shy away from advanced mathematics and science courses even if they excel in these areas. The founder and CEO of GoldieBlox, Debbie Sterling, graduated in engineering and was frustrated with the gender bias of toys that were available to girls and became inspired “to use GoldieBlox to help close the gender gap for young people” and inspired girls to pursue careers in engineering (Noble, 2014, para. 3). GoldieBlox sets allowed children to construct machines in order to solve problems and to learn that building machines was fun. It is critical for all students to be encouraged to take mathematics and not fall into stereotypical gender roles.

The study of advanced mathematics in high school is significant for many reasons. Research indicated, “completing advanced math courses in high school has a greater influence on whether students will graduate from college than any other factor – including family background” (Achieve, 2013, p. 1). Students who graduated from high school with coursework that included upper-level mathematics classes had higher salaries 10 years after graduation than students who did not and, as an example, the International Brotherhood of Electricians required applicants to complete algebra problems in a test for apprenticeships (Achieve, 2013). Research at a university was conducted to determine if enrolling female students in a single-gender Calculus 1 class would increase the number of female students taking advanced mathematics courses and help overcome some of the gender bias that some women face when studying upper-level mathematics.

Collaborative problem-solving activities were an integral component of instruction in the calculus class. The female students who participated in the program earned higher grades than other Calculus 1 students at the university and a larger percentage of program participants took Calculus 2 than nonparticipants (Steele, Levin, Blecksmith, & Shahverdian, 2005). Educators must find methods to keep students interested in studying mathematics.

### **Methodologies**

This action research study used a quantitative research design. This research method afforded the most effective means to address the problem of practice because it provided quantitative data to answer the research question. The purpose of the present action research study was to determine the impact of the CSM on student achievement in an AP Calculus BC class. Student-participants completed hands-on activities as a portion of the classroom instruction for an 8-week period. The activities ranged in the time that they took to complete from 30 minutes to several class periods. Some of the hands-on activities provided student-participants with a physical model of an abstract concept so that they were able to understand how the calculus processes were developed and others gave students the opportunity to study graphical relationships.

The goal of the present action research study was to determine if the CSM provided the teacher-researcher's students with an increased comprehension of the calculus topics. In order to do this, the student-participants were given a pretest (see Appendix A) prior to introduction of the CSM. After completion of the CSM, student-participants were given a posttest. The results of a pre- and posttest assessments were

reviewed to provide the teacher-researcher with information on each student-participant's calculus achievement. These quantitative measures provided the teacher-researcher with data to determine the effect of the hands-on activity model on student achievement in calculus. Furthermore, the pre- and posttest data were disaggregated based on the race, gender, and socioeconomic status of the student. Throughout the action research process the teacher-researcher used field notes (see Appendix B) to record any observations, thoughts, and reflections as the students were participating in the CSM. Student surveys (see Appendices C and D), formative assessments, and student work samples were collected. After the student-participants took the AP Calculus Exam, the teacher-researcher used a focus group (see Appendix E) to allow them to reflect on their experiences during the exam and to modify the CSM for the next cycle that will begin in the fall of 2017. These anecdotal data along with data from the College Board (2016a) and the high school where the research occurred, served to triangulate (Mertler, 2014) the quantitative data.

Based on a review of literature, research studies with identified problems of practice involving increasing student understanding in mathematics, increasing minority student participation in higher level mathematics courses, expanding the participation of female students in advanced mathematics classes, and implementing hands-on activities in mathematics have been conducted. These studies have utilized a variety of methodologies and designs.

In a study of fourth grade students, a researcher considered the effects of math manipulatives on the enjoyment and understanding of fractions (Couture, 2012). Students were given a Likert scale to record their levels of satisfaction during the hands-



on activities. Students were given a questionnaire to record their feelings toward the effectiveness of the manipulatives. The teacher maintained a log to document student engagement during the hands-on activities. Students were given a pretest on fractions before the treatment and a posttest afterward to determine the change in student understanding. This mixed-methods research design was very effective for the study because it enabled the researcher to gain a better understanding of the problem of practice. The mixed-methods design provided both qualitative and quantitative data. The present action research study differs from the aforementioned study logistically because the students were in 11<sup>th</sup> and 12<sup>th</sup> grades instead of 4<sup>th</sup> graders, the CSM was conducted in an AP Calculus BC class, and only quantitative data were collected. A quantitative research design was effective in the present action research study because it enabled the teacher-researcher to utilize quantitative data to answer the research question.

### **Action Research Design**

In reviewing prior research, the most significant design issues appeared to be with researchers using a very small number of participants. For example, a study was implemented to determine whether students gained a better understanding when they were given modeling activities before formal instruction to encourage mathematical dialogue and interpretation, or were given modeling activities after formal instruction to encourage the application of knowledge (Yoon et al., 2010). This qualitative study videotaped the participants as they were completing the modeling activities and analyzed the participants' written work to assess their understanding of the calculus content. This design allowed the researchers to gain a significant amount of information on the participants' conceptual understanding as a result of the modeling activities. The

problem with this research design was that the study was conducted using only two university students and two high school teachers. It was difficult to reach conclusions that allowed for inferences to be made from the results with only four participants.

However, a small sample size is not necessarily a major flaw in educational research. One example was a study implemented to improve student attitudes toward and increase understanding of fractions (Jennison & Beswick, 2010). The number of participants in this study was eight. While this number might seem low, the participants were given an extensive treatment which included mental computational work to develop conceptual understanding, hands-on learning materials to lead to the visualization of fractions, and collaborative group work. Had more students been included this could have caused the resources to be stretched in a manner that would lead to ineffective treatment of the participants. Researchers need to ensure that an appropriate number of participants are used so that data can be effectively gathered to address the problem of practice. The number of participants in the current research study was 10 student-participants and was limited by the course enrollment.

## **Conclusion**

### **A Brief History of the Hands-On Activity Model in Education**

Throughout the history of schooling in the United States, many educators have extolled the virtues of using hands-on activities to help students attain comprehension. These educators understood that students not only required theoretical textbook knowledge but students also needed to understand how to apply this knowledge in practical applications. There was emphasis placed on students actively participating in

their learning and not passively receiving information from teachers. Even in many of the early schools of colonial America there was a desire for students to learn through active experiences. Benjamin Franklin identified what he considered to be an ideal academy in *Proposals Relating to the Education of Youth in Pensilvania [sic]*. Franklin (1749) understood that students need more than just lectures from teachers when he proposed that the academy have “some mathematical instruments, [and] an Apparatus for Experiments in Natural Philosophy, and for Mechanics” (p. 23). Furthermore, Franklin (1749) acknowledged that school curriculum was limited in time and it was critical for educators to use the time to best meet the needs of students by writing:

As to their Studies, it would be well if they could be taught *every thing* that is useful, and *every thing* that is ornamental; But Art is long, and their Time is short. It is therefore propos'd that they learn those Things that are likely to be *most useful* and *most ornamental*. Regard being had to the several Professions for which they are intended. (p. 23)

It is evident that Franklin wanted education to be practical and for students to have hands-on experiences to facilitate learning. Another educator who advocated for students discovering knowledge through hands-on activities was Johann Pestalozzi. As a Swiss educator, Pestalozzi's ideas were instrumental in reforming European schools and his philosophies ultimately gained popularity in the United States in the mid-1800s. It was Pestalozzi's belief that students should not simply memorize information such as multiplication facts, but use physical objects to model the problem so that they can discover the fundamental principles of multiplication for themselves (McNergney &

Herbert, 1998). When students made discoveries on their own as opposed to simply memorizing facts, they were able to apply the concepts in new situations.

Understanding the importance of students learning through hands-on activities, the progressive education movement advocated for a student-centered approach to schooling in which teachers used class discussions, demonstrations, and debates instead of lectures (McNergney & Herbert, 1998). Progressivism gained popularity in the 1920s as a result of the declining quality of schools in the United States. As a leader in the progressive movement, John Dewey did not want students to learn subjects in isolation and believed that there should be less emphasis on rote memorization and more focus on students learning through their own experiences (McNergney & Herbert, 1998). Dewey (1938/1997) expressed a concern that when students were only taught information in the form of a finished product without understanding how the finished product was built or how to modify the finished product to make it better, then the students were being denied the opportunity to fully learn. Dewey (1938/1997) theorized that students have a deeper level of comprehension when they are responsible for their own learning through hands-on experiences. While Dewey (1938/1997) was a proponent of students learning by doing, he also warned that educators need to ensure that learning experiences are connected in a manner that works to create a cumulative effect so that each experience builds from past knowledge to help students fully understand concepts. It is not enough to have students participate in hands-on activities, but the activities must be tied in to the learning objectives and designed at a level that builds on the students' prior knowledge to serve as a foundation for new and better understanding.

Dewey's (1938/1997) theories corresponded to the idea presented by Buoncristiani and Buoncristiani (2012) that "we learn best when we are actively engaged in the process of our own learning" (p. 5) and that students are better able to learn new material when they are allowed to connect it to past experiences and understanding. Jean Piaget's work on cognitive development of children became popular in the 1920s through 1940s. Piaget (1948/1972) stated that in order to teach mathematics and science:

The use of active methods which give broad scope to the spontaneous research of the child or adolescent and require that every new truth to be learned be rediscovered or at least reconstructed by the student, and not simply imparted to him. (pp. 15-16)

This approach led to the use of more manipulatives and hands-on activities in the classroom (McNergney & Herbert, 1998). Based on the results of this cognitive research, constructivism as an educational approach through which students used active learning to build their knowledge was born. It is not the teacher's role to give students knowledge, but to help students to construct their own understanding (McNergney & Herbert, 1998). This is difficult for some teachers because they have to stop using the lecture method of instruction but develop lessons that provide students with the opportunity to learn through problem solving (Piaget, 1948/1973). Real learning involves more than the ability to memorize facts, but occurs when students understand how to use the information to solve problems (Ryan & Cooper, 1998). Furthermore, Piaget (1948/1973) emphasized that hands-on activities provided students with an "interdisciplinary nature at every level of the subjects taught as opposed to the compartmentalization still so widely prevalent both in the universities and secondary schools" (p. 12). Additionally, hands-on activities help

students gain a better level of understanding. Piaget (1948/1973) theorized that a “student who achieves a certain knowledge through free investigation and spontaneous effort will later be able to retain it” (p. 93). It was the belief of Piaget that all students were capable of successfully learning mathematics if they were given the opportunity to participate in activities that make the abstract mathematical concepts personally relevant and concrete (Piaget, 1948/1973).

In the 1960s and 1970s, Jerome Bruner was influential in changing the curriculum based on his ideas of the structure of disciplines, which encouraged students to construct their own understanding using the inquiry method (McNergney & Herbert, 1998). Buoncristiani and Buoncristiani (2012) studied how students learn and theorized, “new standards of learning are being developed worldwide to accommodate globalization and the changing view of how people think and learn. Often these standards emphasize the vital role of thinking skills” (p. 4). The Common Core State Standards (CCSS) were introduced in 2009 to create consistent learning of content and problem-solving skills throughout the United States. A component of the CCSS (2009) in mathematics was the development of Standards for Mathematical Practice, which consists of behaviors that are necessary for mathematics proficiency. Two of the practices “model with mathematics” and “use appropriate tools strategically” are specifically related to hands-on activities (Common Core State Standards Initiative, 2015b, p. 1). Hands-on activities are an essential component of the constructivist classroom because students develop a deeper conceptual understanding when they are able to construct their own knowledge and are not simply “spoon-fed” information by the teacher.

## **The Hands-On Activity Model Is a Constructivist Approach**

Whether students work in collaborative groups or alone, hands-on activities are an important instructional tool in mathematics classrooms. When students are active participants in learning, this provides them with the opportunity to construct their own understanding. Numerous academic studies have been completed using hands-on activities in mathematics and have confirmed the constructivist theory that hands-on activities create greater understanding.

A research study using origami construction in geometry class was based on the work of Piaget, because during the activities that included origami, the teacher's role changed from that of a lecturer to a guide (Sze, 2005). The study highlighted the work of Bruner as well, because the origami construction led to higher-level thinking by the students (Sze, 2005). Using the inquiry method to help students develop a greater understanding was evident in research that demonstrated that the use of manipulatives enabled even kindergarten children to explore algebra concepts such as patterns and functions (Kelly, 2006). Students of any age level can benefit from a constructivist classroom. Two studies that used hands-on activities to supplement instruction in university calculus courses revealed that students were able to develop a deeper conceptual understanding as a result of the activities (Awang & Zakaria, 2012; Yoon et al., 2010). These studies have shown the benefits of adopting a constructivist approach and using hands-on activities in the mathematics classroom; however, some educators choose not to use them.

One focus of a research study was to determine why some teachers do not use hands-on activities. The conclusion of the study indicated that many teachers did not allow their students to engage in hands-on activities because they believed that hands-on activities were a waste of time and did not benefit students (Munley, 2003). Some teachers did not feel comfortable relinquishing control in the classroom and therefore did not incorporate hands-on activities. Other teachers were unable to develop effective activities that allowed students to construct their own understanding. This relates back to Dewey's (1938/1997) philosophy that learning experiences must be connected to prior knowledge in order to create a cumulative effect that maximizes understanding. In conclusion, hands-on activities provide students with the opportunity to construct a deeper level of conceptual understanding and have been shown to be an effective instructional tool. The present action research study focused on the impact of the CSM in an AP Calculus BC class, which added to the scholarly knowledge of the effectiveness of hands-on activities as an instructional tool.



## **CHAPTER THREE: METHODOLOGY**

### **Introduction**

This chapter of this Dissertation in Practice (DiP) describes the action research methods that were used in the present study. This study was implemented to determine the impact of the Constructed Scaffold Model (CSM), a hands-on activity model developed by the teacher-researcher, on AP Calculus students in a large, suburban high school that was given the pseudonym here of County High School (CHS). The College Board (2016a) has identified that AP Calculus “is designed to develop mathematical knowledge conceptually, guiding students to connect topics and representations throughout each course and to apply strategies and techniques to accurately solve diverse types of problems” (p. 4). In order for student-participants to make critical connections within the AP Calculus BC course content and develop the ability to represent the material in numerous contexts, the teacher-researcher explored methods of instruction to bridge the gap.

In the results of a previous study, students indicated an increase in favorable opinions toward mathematics after participating in cooperative learning activities (Miller, 2003) in their Algebra 2 class. While a study of the use of hands-on activities in high school geometry classes did not provide results of increased student understanding (Munley, 2003), a different study of fifth-graders revealed that the students had improved levels of achievement, increased understanding, and positive attitudes toward

mathematics after taking part in hands-on activities (Allen, 2007). In a study of university-level calculus students, researchers wanted to decide if hands-on activities were more beneficial when introduced before or after instruction on the material. When introduced before instruction hands-on activities helped students think about the content, and when introduced after instruction hands-on activities provided students with a deeper understanding (Yoon et al., 2010).

The teacher-researcher reviewed related literature and found that studies have been conducted to examine if hands-on activity models increased student comprehension and student attitudes toward mathematics at the elementary, middle, high school, and university levels. The present action research study adds to the current body of research because it was conducted in an AP Calculus BC class. The teacher-researcher determined the optimal research design to address the research question after reviewing related literature (Allen, 2007; Miller, 2003; Munley, 2003). A quantitative research design was utilized in the present action research study to provide the teacher-researcher with quantifiable data on the calculus achievement of the student-participants. This design choice was corroborated by Mertler (2014) who wrote, “quantitative data provide information that can be analyzed statistically and can offer useful information” (p. 104). The teacher-researcher decided that qualitative data such as student surveys (see Appendices C and D) and a focus group (see Appendix E) would be better utilized for making sense of the quantitative results.

A teacher-made pretest (see Appendix A) was given to the student-participants prior to introduction of the CSM as an instructional method and the student-participants took the same instrument as a posttest at the culmination of the 8-week period. Field

notes (see Appendix B), student surveys (see Appendices C and D), and student work samples were also collected throughout the implementation of the CSM. These anecdotal data, as well as documents from the College Board (2016a) and the high school where the teacher-researcher teaches, were used to triangulate (Mertler, 2014) the quantitative data. These data were used to answer the research question. After the student-participants took the AP Calculus Exam, the teacher-researcher used a focus group (see Appendix E) to gather additional information about their experiences in order to develop the action plan.

### **Problem of Practice**

The identified Problem of Practice (PoP) for the present action research study was that students at CHS were scoring below their global peers on the AP Calculus Exam and experiencing difficulty building connections from basic mathematical operations to complex calculus topics. This was especially a problem for CHS students from groups that have been historically marginalized in advanced mathematics such as females, Blacks, and Hispanics (Caviness, 2014). AP Calculus students were required to solve six free-response questions on the 2017 AP Calculus Exam and these problems typically involved the need for students to apply and synthesize several concepts. The teacher-researcher determined that her student-participants were able to solve problems that involved isolated concepts, but struggled to solve problems that required them to apply and synthesize unfamiliar contexts.

The teacher-researcher reviewed the AP Calculus Exam scores from her prior years teaching the course and found that students at CHS scored below their global peers

on the free-response questions. The teacher-researcher found that young women and people of color at CHS have historically scored lower on the AP Calculus Exam according to school data. Studies have shown that these students typically learn better with hands-on, constructivist curriculum (Ladson-Billings, 1997). Research by O'Shea et al. (2010) indicated that high-achieving female students prefer teachers who allow them to work collaboratively and provide academic challenge. Therefore, to enable these students to access prior knowledge and apply that knowledge to more complex calculus equations, the CSM hands-on activity model was designed by the teacher-researcher, who is an AP Calculus teacher, to enable her students to increase their scores on the AP Calculus Exam. The CSM has not been previously studied at CHS. Moreover, the perceptions of AP students of color and young women are unknown and are important for the design of an action plan to implement and improve the CSM so it can effectively be used in future AP Calculus classes at CHS.

**Background of the Problem of Practice.** The results of the 2012 Programme for International Student Assessment (PISA) found that U.S. students ranked 27<sup>th</sup> out of the 34 Organisation for Economic Co-operation and Development (OECD) nations in mathematics (OECD, 2012b). Students in the U.S. scored well when they were required to make a single calculation or deduce simple explanations from charts and graphs but struggled when they had to create a mathematical model to represent a problem or make mathematical interpretations from various contexts (OECD, 2012b). Many educators have worked to bridge the gap so that students can scaffold from prior knowledge when learning new material. John Dewey (1938/1997), a leader of the progressive education movement, believed that students must understand not only the finished product but also

how this finished product was built or can be modified in order to fully learn.

Additionally, the results of research by Cruse (2012) indicated that hands-on activities helped students form more concrete understanding of abstract ideas. However, as teachers develop hands-on activity models for students, these processes must have a purpose and need to be linked within the curriculum so that they enhance and expand student understanding (Dewey, 1938/1997).

### **Research Question**

Based on the results of previously conducted research studies, achievement in AP Calculus BC requires students to make connections from prior knowledge to current material within the course (Lamattina, 2008; Nao, 2008). The hands-on activity model is an instructional method that mathematics educators use to provide students with the opportunity to apply the mathematical concepts in a manner that helps them gain a better understanding of the relevance of what they are learning. A research study by Cruse (2012) found that students who participated in a hands-on activity model had higher gains in solving problems that required them to make connections from prior knowledge than those who did not participate in the activities. Hands-on activities made mathematical concepts less theoretical and more concrete for students (Dewey, 1938/1997). The objective of this action research study was to explore the effect of the CSM in an AP Calculus BC class. Through the use of hands-on activities, the teacher-researcher answered the following research question:

What is the impact of the hands-on activity model on secondary student achievement in Advanced Placement Calculus?

## **Purpose of the Study**

The purpose of the present action research study was to determine the impact of the CSM, a hands-on activity model designed by the teacher-researcher, on the calculus achievement of her AP Calculus BC students at CHS. An additional purpose of the present study was for the teacher-researcher to design an action plan from her work with the student-participants in order to improve the effectiveness and utility of the CSM in providing critical course connections and enduring understanding within the course content, especially for young women and students of color.

## **Action Research Method/Design**

First, the teacher-researcher designed and developed the CSM, a hands-on activity model, for her AP Calculus classroom (Couture, 2012; Cruse, 2012; Pfaff & Weinberg, 2009; Schunk et al., 2007). The teacher-researcher developed the following criteria for the CSM:

- Each activity was based on a primary course standard.
- Each activity required the student-participants to make a physical and/or graphical representation of a calculus concept.
- Some activities were implemented before instruction on a topic to provide student-participants with the opportunity to construct their own understanding and other activities took place after instruction on a topic to allow student-participants to build a higher level of understanding.
- Some activities required students to independently predict outcomes and relationships and other activities allowed students to answer questions that were

provided by the teacher-researcher to help the students scaffold from prior knowledge to predict outcomes.

- After the student-participants completed the activity independently or in small groups, the teacher-researcher initiated a class discussion to review the results that were obtained and clarify any of the student-participants' misconceptions.
- Each activity took between 30 minutes and several class periods to complete.

The teacher-researcher believed that it was critical for her student-participants to have the opportunity to visualize the content that they were studying. Consequently, several of the activities within the CSM allowed the student-participants to create physical models of calculus topics. Student-participants used tissue paper spheres to model the solid that was formed when a region is revolved about an axis and calculated the volume of the solid that was formed. In another activity, student-participants cut out cardboard geometric shapes of varying sizes and attached the shapes to a graph to model the solid that was formed from these cross-sections and calculated the volume. Student-participants simulated a pulley lifting a chain and weight vertically using models made from wooden craft sticks, yarn, and cardboard and calculated the work required to lift the chain and weight. This activity provided the student-participants of a visual representation of constant and variable forces.

The College Board (2016a) emphasized, “connecting multiple representations” (p. 9) which means students must be able to represent concepts in more than one way. As a result, many of the activities that the teacher-researcher developed for the CSM required the student-participants to create graphical models to represent calculus concepts.

Student-participants used their past knowledge of calculating area from geometry to

model two graphs and calculate the area between the two graphs. Another activity required the student-participants to model the logistic growth differential equation and then adjust the model based on various initial population sizes to learn the concept of carrying capacity. Student-participants were given the opportunity to match slope field graphs and equations during an activity. In a different activity, student-participants placed dot stickers on a graph to model bounded and monotonic sequences. Finally, student-participants utilized graphing calculators to model Taylor polynomial approximations of functions and used these graphs to gain a better comprehension of the importance of Taylor polynomials in the study of calculus.

Some of the activities in the CSM allowed the student-participants to derive formulas in order to construct their own understanding. Student-participants were given the graph of a curve and required to use their knowledge of the distance formula in geometry to calculate the length of the curve and connect that knowledge to the formula for arc length. In another activity student-participants used separation of variables to derive the formula for exponential growth and decay that they used in Algebra 2 and Precalculus. Throughout implementation of the CSM, student-participants created either physical or graphical models of calculus concepts and used the models to build connections from past mathematical knowledge. Information on all of the CSM activities is provided (see Appendix F).

Next, the teacher-researcher designed and developed a pretest (see Appendix A) that was administered to the student-participants prior to implementation of the CSM in the spring of 2017 and used the same instrument as a posttest that was administered at the culmination of the 8-week study period. The teacher-researcher used a repeated



measures t-test to compare the mean scores from both tests. The results were disaggregated by race, gender, and socioeconomic status. These data incorporated a new instructional method in the AP Calculus BC classroom in CHS and was new to the students and to other school personnel. Students at CHS were accustomed to a traditional essentialist pedagogical classroom for AP Calculus BC. Within the present action research study the student-participants were involved with the CSM, which is a progressivist inquiry-based model of student learning and meaning-making. Since the teacher-researcher incorporated a new instructional method, adequate time was allotted for student-participants to become acclimated to the change and to understand how to work within the CSM. The teacher-researcher used field notes (see Appendix B) to record student engagement, interactions, and perceptions throughout the CSM. Student surveys (see Appendices C and D) were completed and student work samples were collected during the implementation of the CSM. These anecdotal data as well as documents from the College Board (2016a) and CHS where the teacher-researcher teaches were used to triangulate (Mertler, 2014) the data. After data collection and the 2017 AP Calculus Exam, the teacher-researcher reflected on the data with her student-participants during a class period by discussing the results and using a focus group (see Appendix E) and formed an action plan designed to continually improve the CSM for her AP Calculus BC classroom at CHS.

### **Role of the Teacher-Researcher**

The teacher-researcher assumed multiple roles during the course of this action research study. The primary role was that of participant-researcher because the teacher-researcher participated in all of the classroom activities. Mertler (2014) noted that a

participant-researcher “continues to observe and take notes on what is observed but also has the opportunity to interact with the participants in the study” (p. 94). The teacher-researcher used her role as a participant-researcher, which made her an insider, to build trust with the student-participants and help them understand her expectations during the action research study (Termini, 2013). In an effort to determine how the CSM impacted her student-participants’ calculus achievement and their abilities to relate calculus content to their prior knowledge, the teacher-researcher reflected with her student-participants during implementation of the CSM to determine their views on this new instructional approach. The teacher-researcher reflected reciprocally with her student-participants by asking them what they learned during individual activities as well as asking them ways in which the activities could be improved and sharing her observations with the student-participants. With her knowledge of the student-participants’ personalities and prior academic achievement, when the teacher-researcher took on the role of researcher, which gave her an outsider position with her student-participants, she ensured that her data collection was unbiased, objective, and depicted an accurate portrayal of classroom events. The teacher-researcher’s knowledge of her students helped her as she reflected on the student-participants’ engagement and comprehension of calculus content during implementation of the CSM.

### **Sample**

The participants in the present action research study were students enrolled in the teacher-researcher’s AP Calculus BC class at CHS during the 2016-2017 school year and came from the population of all AP Calculus BC students. There were 10 students enrolled in the class of which six students were female and four students were male. All

of the enrolled students were included in the sample of student-participants in the present action research study and therefore a cluster sampling method was used. Bluman (2004) summarized cluster sampling as “subjects are selected by using an intact group that is representative of the population” (p. 13). Although the teacher-researcher used the students from her AP Calculus class at CHS, the sample was similar to the population of all AP Calculus students in age, grade level, and mathematical ability. The sample differed slightly from the population in terms of race, ethnicity, and gender. While a sample size of 10 student-participants was not large, researchers have found that having fewer students allowed the teacher-researcher to have more interactions with individuals (Jennison & Beswick, 2010). There were eight White students, one Black student, and one Hispanic student in the class. Nine of the student-participants were in the 12<sup>th</sup> grade and one was in the 11<sup>th</sup> grade. Two of the student-participants received free or reduced lunch. Data collection took place during the spring 2017 semester.

## **Ethics**

Prior to conducting the present action research study, the teacher-researcher received approval from the Research Committee of the school district where she teaches. Proposals for research were required to include an Institutional Review Board approval/exemption letter, rationale for the study, data collection and analysis methodology, copies of informed consent forms that comply with Family Educational Rights and Privacy Act (1974), and draft copies of any letters, and other documents that were provided to participants (see Appendix G) or their parents/guardians (see Appendix H). Additionally, a final requirement for conducting research was receiving approval from the teacher-researcher’s school principal.

The teacher-researcher kept all of the documents associated with the present action research study in a locked file cabinet in her classroom. She developed a random numbering system to identify each student-participant and worked to confirm that this numbering system did not inadvertently coincide with the alphabetical class roster. Throughout implementation of the CSM, the teacher-researcher worked to help the student-participants feel comfortable with the action research process. The student-participants were continually reminded of the importance of honest and open discourse, and the teacher-researcher informed the student-participants that they would not be penalized or judged for responses that they provided.

### **The Setting**

The present action research study took place at CHS, which is a pseudonym for a large, suburban high school of approximately 2,000 students located in the southeastern United States. Based on data from the 2012-2013 school year the racial composition of CHS was 74% White, 15% Black, 5% Hispanic, 4% Two or More Races, 1% Asian, and less than 1% other races (High-Schools, 2015). Approximately 40.3% of students at the school qualified for free or reduced lunch (High-Schools, 2015).

### **Technology**

All of the student-participants at the teacher-researcher's school were issued an Apple iPad ® tablet, a computer-technology device popular in 2017 classrooms. The student-participants had access to this technology, which was utilized in the AP Calculus BC classroom when student-participants encountered the CSM implemented by the teacher-researcher. The teacher-researcher also had a classroom set of Texas Instruments

TI-84 Plus ® graphing calculators so that all of her student-participants were able to use them when working on CSM.

### **Building Trust**

All of the student-participants except one in the AP Calculus BC class had the teacher-researcher as a teacher for Precalculus Honors in the previous school year. As a result, students were familiar with the teacher-researcher and how information was presented in the class. Classroom instruction utilizing the CSM was a new experience for both the teacher-researcher and the student-participants. The teacher-researcher took time to ensure that her student-participants understood how the CSM was employed.

### **Instrumentation and Materials**

Prior to implementation of the CSM, the student-participants were given a pretest (see Appendix A) and were subsequently given the same instrument as a posttest at the culmination of data collection. The pre- and posttest consisted of 20 multiple-choice questions. Using the *AP Calculus Multiple-Choice Question Collection 1969-1998* (College Board, 2005) as a guide, the teacher-researcher developed questions that were similar to multiple-choice questions on the AP Calculus Exam. The teacher-researcher followed the guidelines for classroom use of AP materials that were provided by the College Board (2016a). As a result of copyright laws, the actual questions on the pre- and posttest were not reproduced in this document, but a replica test with similar questions covering the same calculus content was provided (see Appendix A). Since the problem of practice for the present action research study involved the need for the student-participants to increase their scores on the AP Calculus Exam and make

connections between prior knowledge and calculus topics, the teacher-researcher believed that selecting an instrument that contained content similar to the AP Calculus Exam would be most effective.

The pre- and posttest formally assessed the student-participants' knowledge of the following calculus concepts: definite integrals, differentiation of trigonometric and transcendental functions, area between two curves, graph behavior, area under the curve, average value, the Midpoint Rule, total distance, volume using the washer method, the Fundamental Theorem of Calculus, related rates, logistic growth, volume using known cross-sections, sum of a geometric series, and arc length. The scores were calculated by tallying the number of correct answers for each student-participant. The teacher-researcher determined that if a student-participant answered a question correctly then that student had knowledge of the calculus content that the question assessed. Mertler (2014) described validity as "an essential quality in quantitative research data and has to do with whether the data are, in fact, what they are believed or purported to be – in other words, did we *actually* measure what we intended to measure" (p. 149) and with a reliable instrument, "you get similar results each time you administer the test" (p. 150). Since the pre- and posttest questions were based on the AP Calculus Exam, which is a standardized, "criterion referenced" (Mattern, et al., 2009, p. 2) test that has been used for decades to measure students' calculus achievement, the teacher-researcher surmised that the pre- and posttest were valid and reliable measures of calculus achievement for her study.

The student-participants took the pretest prior to implementation of the CSM and the posttest at the conclusion of 8 weeks of classroom instruction using the CSM.

Student-participants were not allowed to use a calculator or any other resource when completing the pre- and posttest. Both assessments were taken in the teacher-researcher's classroom during the student-participants' scheduled AP Calculus class period. The teacher-researcher asked the student-participants not to guess if they did not know how to solve a problem because she did not want the student-participants to appear to have calculus knowledge on content that they do not know. In addition to the pre- and posttest, student-participants were given a Likert scale survey and questionnaire to record their perceptions of the CSM. The student-participants were given time in class to complete the survey, but three of the student-participants did not have enough time in class and finished the survey at home.

### **Data Collection**

The teacher-researcher designed and developed the CSM, a hands-on activity model. The teacher-researcher also designed and developed a pretest that was administered to the student-participants prior to implementation of the CSM, and the same instrument was used as a posttest that was administered to the student-participants after the 8-week period. The quantitative data from the pre- and posttests were used to answer the research question. Additionally, the results were disaggregated by race, gender, and socioeconomic status. The teacher-researcher used field notes to record student engagement throughout implementation of the CSM as well as information on her perceptions of the overall effectiveness of each activity.

Research by Lau (2014) found that field notes and student surveys provide critical information on student engagement and perceptions during action research data

collection. Field notes, student surveys, and student work samples were completed during the data collection phase of the present action research study. After the student-participants took the AP Calculus Exam the teacher-researcher used a focus group to gather their impressions based on their experiences with the CSM. These anecdotal data, along with data from the College Board (2016a) and CHS where the teacher-researcher teaches, were used to triangulate (Mertler, 2014) the primary quantitative data set from the pre- and posttest.

### **Data Analysis and Reflection**

After the student-participants completed the pre- and posttest, the teacher-researcher used repeated measures t-tests to compare the mean score of the pretest to that of the posttest. These data were used by the teacher-researcher to measure the calculus achievement of the student-participants. In order to gain a better understanding of the data, the teacher-researcher disaggregated the pre- and posttest data based on race, gender, and socioeconomic status. Even though some of the student-participants did not score as well as others in the class, the teacher-researcher found that these discrepant cases still provided important information for analysis. The teacher-researcher reviewed her reflections that were maintained in the field notes, compared them with the results of the t-tests, and looked for trends in her data. The teacher-researcher also studied the field notes and made comparisons between student-participant engagement and the results of the t-tests. Formative assessments and student work samples were used by the teacher-researcher to help her determine the student-participants' ability to make connections within the course content. Finally the teacher-researcher analyzed the results of the student surveys. The teacher-researcher transcribed all of the survey responses and used



different colored highlighter markers to code the data. After reviewing the coded data, the teacher-researcher found that specific themes emerged. The teacher-researcher used a summative data analysis (Dana & Yendol-Hoppey, 2014) to discover trends in her data.

### **Plan for Reflecting With Participants on Data**

Prior to the data collection process, the teacher-researcher talked to her student-participants about the action research study and the role that they would play. After these discussions occurred, the student-participants were interested in the study and eager to begin. The teacher-researcher shared some of her findings with her student-participants during the data collection process to confirm that the collected data was consistent with their classroom experiences. For instance, after collecting and analyzing the data, the teacher-researcher shared most of the results with her student-participants. After the student-participants took the posttest, the teacher-researcher allowed them to see their graded pre- and posttests. During this time, the teacher-researcher and student-participants discussed specific questions on the pre- and posttest and reflected on the processes that were used to answer specific questions. The teacher-researcher verbally provided her student-participants with the results of the repeated-measures t-tests that served to compare the mean score of the pretest with the mean score of the posttest. Additionally, the teacher-researcher gave her students the results of the student surveys. The teacher-researcher made the decision not to share the results that were disaggregated by gender, race, or socioeconomic status with her student-participants in order to protect the anonymity of individuals.

## **Plan for Devising an Action Plan**

Action research is cyclical and Mertler (2014) wrote that the “action plan is essentially a proposed strategy for implementing the results of your action research project” (p. 43). The student-participants played a key role in the development of an action plan within the current action research study. The student-participants took the AP Calculus Exam in the beginning of May. The teacher-researcher had the student-participants in class until the end of the school year in late May. Since all of the instruction for the course standards was completed prior to the AP Exam, the teacher-researcher used a class period for a focus group to reflect with her student-participants on their experiences with the CSM. The results of the focus group were instrumental in developing the next cycle of the CSM for the fall of 2017. The data collection and analysis were completed at this time and the teacher-researcher shared the results with her student-participants. The teacher-researcher took time to reflect alone on all aspects of the research and results before collaborating with her student-participants.

## **Conclusion**

This chapter described the methodology that was used in the present action research study. AP Calculus BC students at CHS were not performing as well as their global peers on the AP Calculus Exam and struggling to make connections from basic mathematical operations to complex calculus concepts. The teacher-researcher developed the CSM as an instructional method and worked to determine the impact that the model had on her student-participants’ calculus achievement. The teacher-researcher used repeated measures t-tests to compare the pre- and posttest mean scores. In addition

to these quantitative data, the teacher-researcher collected field notes, student surveys, formative assessments, and student work samples along with a focus group to triangulate (Mertler, 2014) the quantitative data. The teacher-researcher reflected alone on the data as well as with her student-participants. After the data were collected and analyzed, the teacher-researcher collaborated with her student-participants to create an action plan that would serve to improve instruction with her future AP Calculus students.

## **CHAPTER FOUR: FINDINGS AND IMPLICATIONS**

### **Introduction**

The purpose of Chapter Four is to report the findings of the present action research study, which explored the impact of the Constructed Scaffold Model (CSM), a hands-on activity model designed by the teacher-researcher, on AP Calculus students at County High School (pseudonym). County High School (CHS) was a large, suburban high school located in the southeastern United States. Student-participants were given a pretest (see Appendix A) prior to deployment of the CSM and a posttest at the end of the 8-week period to determine the impact of the model on student-participants' calculus achievement. The teacher-researcher used the CSM as the primary instructional method for 8 weeks. In addition to the data from the pre- and posttest, the teacher-researcher collected field notes (see Appendix B), student surveys (see Appendices C and D), student work samples, and a focus group (see Appendix E) to provide information on the student-participants' perceptions, understanding, and engagement during the CSM. Once all of the data were collected, the teacher-researcher completed a summative data analysis, which Dana and Yendol-Hoppey (2014) described as based on the four steps of "description, sense making, interpretation, and implication drawing" (p. 169), to understand the experiences of the student-participants as they encountered the CSM.

## **Problem of Practice**

The Problem of Practice (PoP) for the present action research study was that students at CHS were scoring below their global peers on the AP Calculus Exam and struggling to build connections from basic mathematical operations to complex calculus topics. This was especially a problem for CHS students from groups that have been historically marginalized in advanced mathematics such as females, Blacks, and Hispanics (Caviness, 2014). The teacher-researcher reviewed the AP Calculus Exam scores from her prior years teaching the course and found that students at CHS scored below their global peers on the free-response questions. The teacher-researcher found that young women and people of color at CHS have generally scored lower on the AP Calculus Exam according to school data. Consequently, the need to make connections from prior course content was especially true for her student-participants, including those from groups that have been historically marginalized in advanced mathematics such as females, Blacks, and Hispanics (Caviness, 2014), at CHS where the teacher-researcher is an AP Calculus teacher. The teacher-researcher designed the CSM, which enabled her course to be constructed as a scaffold where her students-participants' prior mathematical knowledge was used to build connections within the course and subsequently impacted her students' scholarly achievement on the AP Calculus Exam.

## **Research Question**

What is the impact of the hands-on activity model on secondary student achievement in Advanced Placement Calculus?

## **Purpose Statement**

The purpose of the present action research study was to describe the implementation of the CSM, a hands-on activity model (Couture, 2012; Cruse, 2012; Pfaff & Weinberg, 2009; Schunk et al., 2007), designed by the teacher-researcher in an AP Calculus class over an 8-week span. The secondary purpose of the present study was to design an action plan in conjunction with the student-participants in order to improve the effectiveness and utility of the CSM in providing critical connections and enduring understanding within the course content for future AP Calculus students at CHS. Consequently there was a continuous and positive effect on the teacher-researcher's future students' scholastic achievement in AP Calculus as a result of these critical connections that were made within the course content based on her use of the CSM and the impact that it had on her student-participants in the present action research study.

## **Findings of the Study**

The student-participants were given a 20-question multiple-choice pretest (see Appendix A) that was designed by the teacher-researcher prior to implementation of the CSM and the same instrument served as a posttest at the conclusion of the 8-week period. The pre- and posttest scores for the individual student-participants are provided in Table 4.1. Due to the fact that there were only two student-participants in the class that were Black or Hispanic and two who received free or reduced lunch, this information was omitted from the individual student-participant's data to avoid the possible identification of specific individuals. The class average score on the pretest was 1.5 out of 20 questions

correct and the posttest average score was 13.4 out of 20 questions correct. All of the student-participants increased their scores from the pre- to posttest.

Table 4.1

*Student Pretest and Posttest Scores*

Student Number	Gender	Pretest Score	Posttest Score	Net Change
1	Male	2	13	11
2	Female	0	11	11
3	Female	3	18	15
4	Male	0	7	7
5	Male	2	17	15
6	Female	1	6	5
7	Female	2	18	16
8	Female	3	20	17
9	Female	1	10	9
10	Male	1	14	13

The teacher-researcher disaggregated the pre- and posttest mean scores by gender, race, and socioeconomic status, those results are shown in Table 4.2. After a thorough review of the disaggregated data, the teacher-researcher found that Black and Hispanic student-participants had the highest posttest mean score gain of 13.0 questions correct, student-participants that did not receive free or reduced lunch had the next

highest posttest mean score gain of 12.38, and female student-participants had the third highest posttest mean score gain of 12.16.

Table 4.2

*Comparison of Pretest and Posttest Scores*

Student Group	Pretest Mean Score	Posttest Mean Score	Net Change
Whole Class	1.50	13.40	11.90
Female	1.67	13.83	12.16
Male	1.25	12.75	11.50
Black or Hispanic	1.00	14.00	13.00
White	1.63	13.25	11.62
Free or Reduced Lunch	0.50	10.50	10.00
Not Free or Reduced Lunch	1.75	14.13	12.38

In order to understand these quantitative results, the teacher-researcher studied her field notes, student surveys, and student work samples. The teacher-researcher compared the mean score that was earned by the student-participants on the pretest to the mean score on the posttest using repeated measures t-tests to determine the impact of the CSM on her students' calculus achievement in Table 4.3. Based on the results of the t-tests, the teacher-researcher found statistically significant growth for her particular student-participants from the pre- to posttest.



Table 4.3

*T-tests of the Difference Between the Pretest and Posttest Means*

Student Group	t-value	p-value	Mean	Standard Deviation	n
Whole Class	9.38	0.00000304	11.90	4.01	10
Female	6.39	0.000696	12.16	4.67	6
Male	6.73	0.00334	11.50	3.41	4
Black or Hispanic	6.50	0.0486	13.00	2.83	2
White	7.52	0.0000676	11.62	4.37	8
Free or Reduced Lunch	10.00	0.0317	10.00	1.41	2
Not Free or Reduced Lunch	8.00	0.0000454	12.38	4.37	8

At the conclusion of the CSM, the student-participants were given a Likert scale survey and questionnaire to record their perceptions of the model. The Likert scale survey responses were tallied and the results are given in Table 4.4. From an analysis of the qualitative data, the following themes emerged: 1) developing conceptual understanding, 2) fostering interpersonal dynamics, and 3) improving model efficacy.

Table 4.4

*Frequency Distribution of Student Survey Responses*

Statement	Strongly Agree	Agree	Neither Agree Nor Disagree	Disagree	Strongly Disagree
The activities increased my understanding.	3	6	1	0	0
The activities were enjoyable.	4	4	2	0	0
I believe that I will perform better on the test as a result of the activities.	3	3	2	2	0
The activities were a good use of class time.	3	6	1	0	0

**Data Collection Strategy**

**Quantitative Data**

The teacher-researcher used quantitative data to answer the research question. The teacher-researcher developed a 20-question multiple-choice assessment that served as a pre- and posttest to measure calculus achievement. Using the *AP Calculus Multiple-Choice Question Collection 1969-1998* (College Board, 2005) as a guide, the teacher-researcher developed questions that were similar to multiple-choice questions on the AP Calculus Exam. The teacher-researcher followed the guidelines for classroom use of AP materials that were provided by the College Board (2016a). As a result of copyright laws, the actual questions on the pre- and posttest were not reproduced in the present document, but a replica test with similar questions covering the same calculus content has been provided (see Appendix A). The student-participants took the pretest during their

normal class period prior to implementation of the CSM. The teacher-researcher advised the student-participants not to make random guesses when completing the pretest in order to avoid giving the false impression of possessing calculus knowledge that they did not actually have. This served as the beginning of the CSM and the commencement of student-centered classroom instruction through hands-on activities. After 8 weeks of instruction through the CSM, student-participants completed the posttest during their regularly scheduled class period.

### **Qualitative Data**

Qualitative data were collected by the teacher-researcher to triangulate the quantitative results. Mertler (2014) noted triangulation as “a process of relating multiple sources of data in order to establish their trustworthiness or verification of the consistency of the facts while trying to account for their inherent biases” (p. 11). Furthermore, when there is more than one data source, Mertler (2014) suggested, “perhaps a more appropriate term would be ‘*polyangulation*’ (since the prefix *poly-* is defined as ‘more than one or many’)” (p. 11). The student-participants were given a survey prior to introduction of the CSM to determine their past experiences with hands-on activities (see Appendix D). These data provided the teacher-researcher with information on the existing perceptions of the student-participants before implementation of the CSM. The teacher-researcher used field notes (see Appendix B) throughout employment of the CSM to record student-participant engagement and her reflections of the efficacy of each activity. Slight modifications to the activities were made based on reflections of the teacher-researcher throughout the process of applying the CSM. At the culmination of the CSM student-participants were given a survey (see Appendix C) to

share their reactions to the CSM and the utility of the CSM for comprehension of calculus concepts. Finally, after the student-participants took the AP Calculus Exam, the teacher-researcher used a focus group (see Appendix E) to serve as a guide when developing the next cycle of the CSM.

### **Ongoing Analysis and Reflection**

Prior to introduction of the CSM, the student-participants completed a survey to record their past experiences with hands-on activities. All of the student-participants indicated that they utilized hands-on activities in other subject areas, but had minimal experience with hands-on activities in mathematics. Consequently, when the student-participants were exposed to the CSM, this was a new instructional approach. The student-participants were accustomed to a teacher-centered mathematics classroom and the CSM was a change for them.

Upon inception of classroom instruction using the CSM, the teacher-researcher continuously monitored the student-participants' actions and interactions. In order for student-participants to be impacted by the CSM they had to fully participate in the activities. Researchers have determined that students who were more engaged in classroom activities had higher levels of achievement than those who were less focused. Moller, Stearns, Mickelson, Bottia, and Banerjee (2014) underscored the importance of classroom participation and found "students who are more attentive, organized, and focused in school are undeniably the students who have the greatest academic success" (p. 22). The teacher-researcher found that student-participants were actively engaged and appeared to have positive interactions while working on the CSM activities. As a result

of the above-mentioned observations, the teacher-researcher believed that the CSM would help the student-participants gain a greater understanding of the calculus topics that they were learning.

The teacher-researcher continuously reviewed formative and summative assessments from the student-participants in order to reflect on their calculus understanding. After grading a quiz that was composed of problems that required calculating area and volume the teacher-researcher realized that all of the student-participants earned a perfect score. The calculus content on this quiz represented one of the first topics taught using the CSM. The teacher-researcher had never given any assessment in her AP Calculus class in which every student earned a perfect score. This result indicated that there was a substantial level of growth on all of the assignments that these student-participants completed on the topic of calculating area and volume.

As the student-participants completed the activities, their conversations indicated understanding of the calculus content as well as the utility of the tasks. Prior to implementing the CSM, the teacher-researcher was unsure as to whether it would be effective to place students into groups based on her knowledge of their abilities, personalities, pace of work, and friendships. The teacher-researcher decided to use random methods of forming groups such as having the student-participants pick a card with a classmate's name or pick a group number or a convenience method of having each student-participant work with the person to whom they were seated closest. Throughout the CSM, the teacher-researcher reflected alone and also had several discussions with a colleague on the choices that she made to group the student-participants. The teacher-researcher thought about the group dynamics and interactions of the student-participants

and described them to her colleague in addition to the concerns that she had about some of the groups. As a result of these reflections, the teacher-researcher concluded that in the future it would be more beneficial for her to form the groups.

Based on these observations and reflections of the classroom dynamics during the CSM, the teacher-researcher believed that not only would all of the student-participants increase their scores from the pre- to posttest, but also that all of the student-participants would pass the posttest. While all of the student-participants increased their scores from the pre- to posttest indicating growth in calculus knowledge, the teacher-researcher found that all of her student-participants did not pass the posttest.

### **Reflective Stance**

The teacher-researcher learned a great deal through the action research process. In order to develop and design the CSM, it was necessary to study other hands-on activity models and determine the best methods for utilizing hands-on activities in an AP Calculus class (Couture, 2012; Cruse, 2012; Pfaff & Weinberg, 2009; Schunk et al., 2007). Additionally, the teacher-researcher had to step out of her pedagogical comfort zone of teacher-centered lectures during the CSM. Upon reflection of the process, the teacher-researcher learned that she had the ability to create hands-on activities that provided her student-participants with the opportunity to construct their own understanding. For example, during an activity in which students used pulleys to model the work that was required to lift a rope and attached weight vertically, Student 7 discovered the relationship between the force and amount of rope asking, "Isn't the force going to change based on the amount of rope?" While completing an activity in which

the student-participants worked in groups to create graphical models of Taylor polynomials, Student 6 noted, “The activity was more work, but it made the concept make sense.”

As a member of the learning community in her classroom, the teacher-researcher served a dual role. She participated with the students in all of the activities in the role of teacher. At the same time it was necessary for her to assume the role of researcher. The teacher-researcher wanted to make sure that the student-participants made the vital connections within the calculus content while engaged in the CSM. There were instances when the student-participants became frustrated while working on the activities. In her dual role, the teacher-researcher understood that her students would have to work within their groups and not simply be given the answers. It became necessary to encourage and guide the student-participants but to still give them the opportunity to figure out concepts for themselves. This was especially difficult for the teacher-researcher when the activities took longer than expected because she had course pacing standards to maintain. When the activities required additional time, the teacher-researcher believed that the proposed gains in conceptual understanding for the student-participants would make the utilization of additional class time worthwhile.

### **Data Analysis**

All 10 student-participants who were enrolled in the AP Calculus class during the spring 2017 semester took the pretest prior to introduction of the CSM. The pre- and posttest consisted of 20 multiple-choice questions and was made by the teacher-researcher using questions similar to those found in the *AP Calculus Multiple-Choice*

*Question Collection 1969-1998* (College Board, 2005). Due to copyright laws, the actual questions on the pre- and posttest cannot be reproduced in the present document, but a replica test with similar content has been provided (see Appendix A). Classroom instruction through the CSM culminated after 8 weeks, when the student-participants subsequently took the posttest, and data analysis began. The average pretest score for the class was 1.5 questions correct and the posttest mean score was 13.4 questions correct for a net gain of 11.9 questions. The pre- and posttest scores for the individual student-participants are provided in Table 4.1. The teacher-researcher disaggregated the pre- and posttest mean scores by gender, race, and socioeconomic status, those results are shown in Table 4.2.

While all of the student-participants improved their scores from the pre- to posttest, Students 2, 4, 6, and 9 did not earn a passing score of at least 60% correct on the posttest. After disaggregating the pre- and posttest scores by gender, race, and socioeconomic status, the teacher-researcher discovered that the females in the class increased an average of 12.16 points on the posttest compared with an average gain of 11.50 points for the males. These results contradict the finding that 75% of the student-participants who did not pass the posttest were female. Of the females, 50% passed the posttest and 50% did not while 75% of the males passed the posttest. As a group, Black and Hispanic students had the highest average gain of 13.00 points compared to an average gain of 11.62 points for White students. Of the students who did not pass the posttest, 75% were White and 25% were Black or Hispanic. Overall, 50% of the Black or Hispanic student-participants passed the posttest and 62.5% of those who passed were White.



The difference between the pre- and posttest score for each of the student-participants was analyzed using a repeated-measures t-test. Mertler (2014) wrote that the repeated-measures t-test “is appropriate for designs where, for example, students are pretested, exposed to some intervention, and then posttested” (p. 176). Since the same student-participants took the pre- and posttest, the sample was dependent (Bluman, 2004). Furthermore, Bluman (2004) noted that this t-test was appropriate for “small dependent samples” (p. 464). Each student-participant’s pretest score was subtracted from his or her posttest score to get a net change. Since all of the student-participants had a score increase from the pre- to posttest, the net change was a net gain for them all. The null hypothesis for the t-test was that the net change between the pre- and posttest means would be zero and the alternative hypothesis was that the pretest score subtracted from the posttest score would result in a mean value greater than zero. The results of the t-test are presented in Table 4.3. Additional t-tests were performed based on the demographic groups of the student-participants and those outcomes are also provided in Table 4.3.

The results of the t-tests indicated that the student-participants had a statistically significant increase from pre- to posttest mean score. In education research, a p-value of less than 0.05 typically indicates that a result is statistically significant (Mertler, 2014). While the teacher-researcher did not use the calculated p-values to make inferences from the sample of her student-participants to the population of all AP Calculus students, the p-values that were found still provided critical information. The difference between the means of the pre- and posttests for all demographic groups of her particular student-participants were statistically significant.

## **Implications of the Findings**

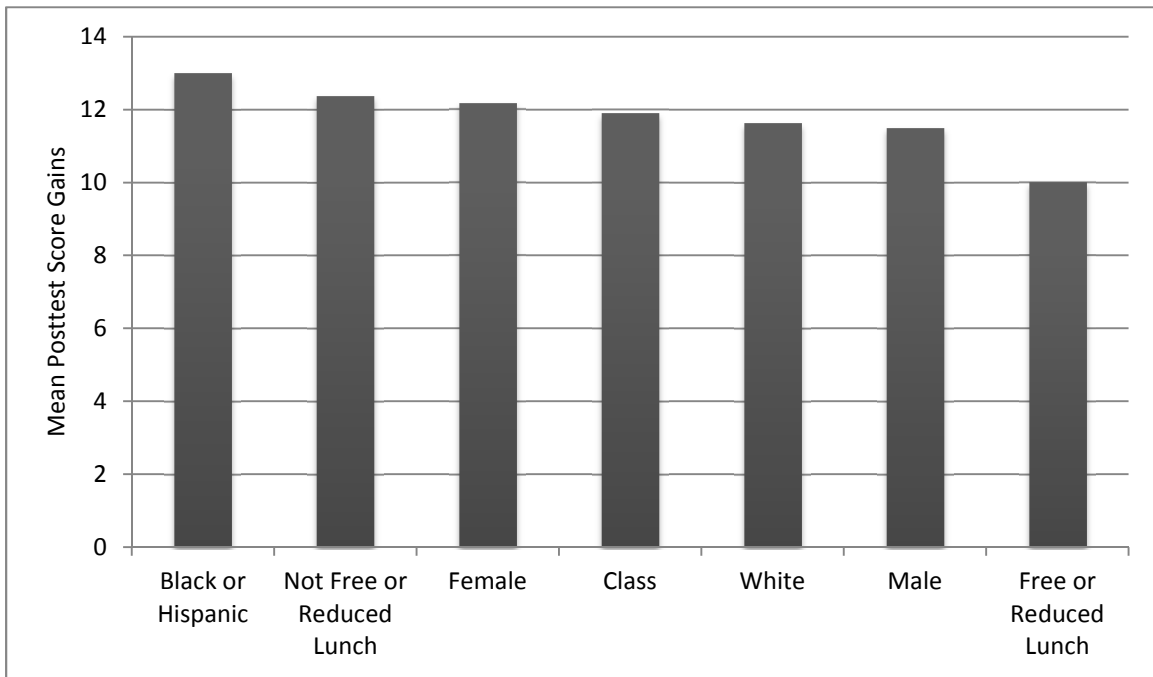
While the pre- and posttest scores gave crucial information on the student-participants' understanding of calculus, collected qualitative data provided the teacher-researcher with more information on their perceptions of the CSM. At the conclusion of the CSM, the student-participants were given a Likert scale survey and questionnaire to record their perceptions of the model. The Likert scale survey responses were tallied and the results are given in Table 4.4.

The teacher-researcher transcribed all of the survey responses from the student-participants in order to make sense of and interpret their replies. The responses were coded with different colors to indicate common ideas. Based on an analysis of the coded qualitative data, three themes emerged. The teacher-researcher found that a majority of the student-participants believed that the hands-on activities increased their conceptual understanding of calculus. Many of the student-participants indicated that the CSM improved the interpersonal dynamics in the classroom and helped them feel that they were members of a team. Finally, most of the student-participants indicated that there were changes that could be made to improve the efficacy of the CSM for future AP Calculus classes at CHS.

## **Data Interpretation**

While all of the student-participants increased their scores from the pre- to posttest, four of them did not earn a passing score on the posttest. The mean posttest score gains for the student-participants are provided in Figure 4.1. Based on the disaggregated mean posttest score gains, the historically marginalized groups of student-

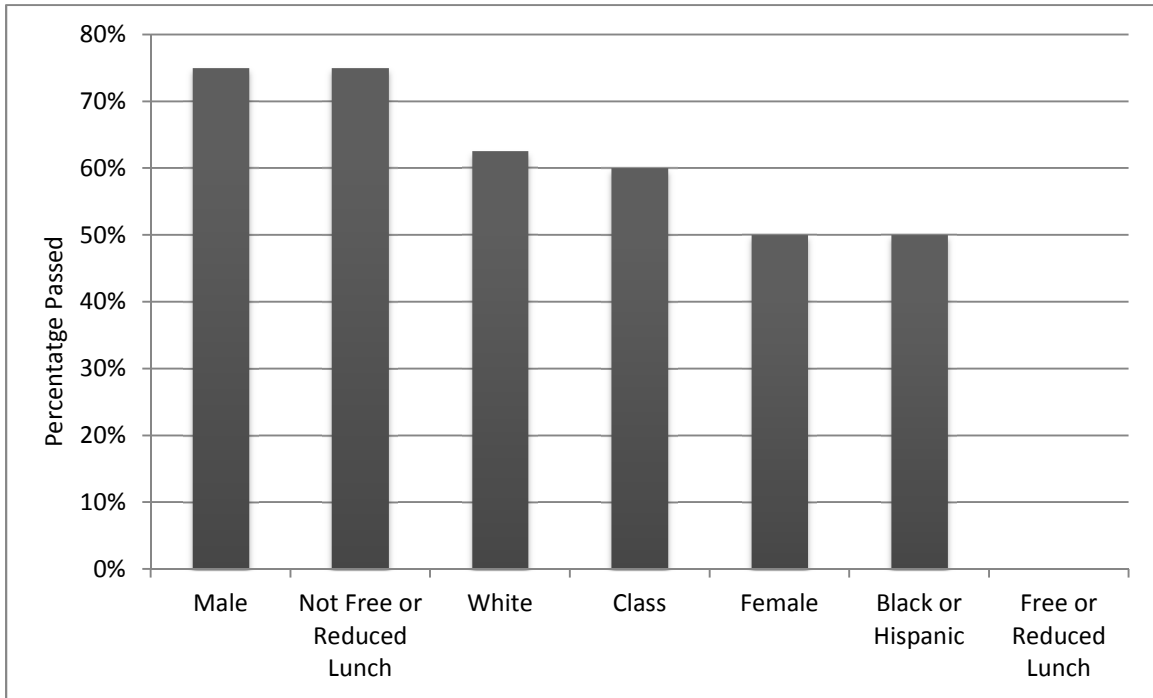
participants had the greatest gains. The Black or Hispanic student-participants had the highest mean gain of 13.00 points on the posttest, student-participants not receiving free or reduced lunch had the second highest mean gain on the posttest of 12.38 points, and female student-participants had the third highest mean gain of 12.16 points.



*Figure 4.1.* Mean gain from pretest to posttest for the class and demographic groups.

The percentage of student-participants earning a passing score on the posttest differed somewhat from the data for the mean gains from the pre- to posttest. The percentage of student-participants earning a passing score on the posttest is shown in Figure 4.2. Seventy-five percent of both males and student-participants who do not receive free or reduced lunch passed the posttest. This represented the highest percentage for any demographic group of student-participants. Of the White student-participants, 62.5% passed the posttest. Both 50% of females and Black or Hispanic student-

participants passed the posttest. None of the student-participants who receive free or reduced lunch passed the posttest.

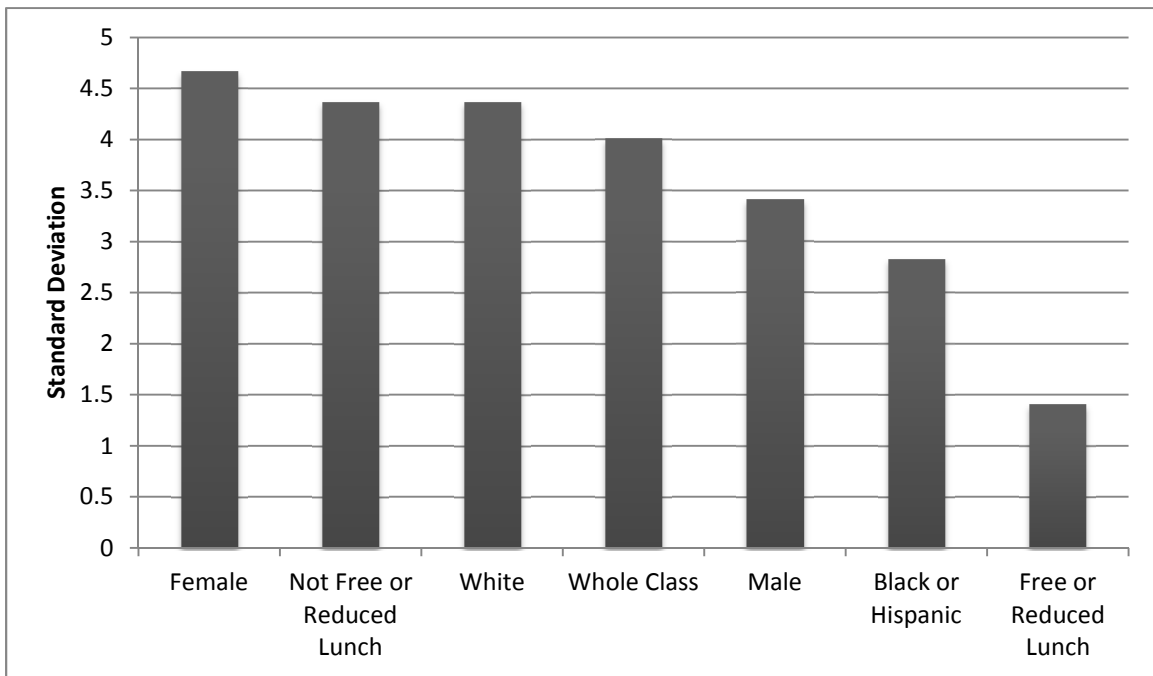


*Figure 4.2.* Percentage of student-participants who earned a passing score on the posttest.

All of the student-participants had net gain scores from the pre- to posttest. Some of the demographic groups performed better than others. Furthermore, based on the results of the t-tests, all of the demographic groups of student-participants had statistically significant growth from the pre- to posttest. While the results of the t-tests are specific to this particular group of student-participants only, the net gain shows that everyone increased knowledge of calculus topics during the CSM.

The standard deviation data are shown in Figure 4.3. Mertler (2014) defined standard deviation as “the average distance of scores away from the mean” (p. 171). Therefore, a smaller standard deviation value indicated that the scores within that group

were closer to the group mean and a larger standard deviation meant that the scores were farther away from the group mean. The female student-participants had the greatest standard deviation within their scores. Of the six female student-participants, three earned the highest scores on the posttest answering 20, 18, and 18 questions correctly and the other three were in the group of four student-participants who did not pass the posttest. Neither of the student-participants who received free or reduced lunch passed the posttest, which led to a lower amount of standard deviation.



*Figure 4.3.* Standard deviation of the difference between the pretest and posttest for all demographic groups.

In addition to the quantitative pre- and posttest data, the teacher-researcher collected qualitative data from field notes, student surveys, and student work samples in order to triangulate (Mertler, 2014) the results. After studying and interpreting the qualitative data through a summative data analysis, the teacher-researcher found that the following three themes emerged: 1) developing conceptual understanding, 2) fostering

interpersonal dynamics, and 3) improving model efficacy. These themes are expanded in the following sections.

### **Developing Conceptual Understanding**

Although all of the student-participants did not pass the posttest, they all improved from the pretest. This growth from pre- to posttest gave evidence of the development of conceptual understanding of calculus during implementation of the CSM. Furthermore, the student-participants indicated that the CSM helped them develop conceptual understanding. When asked to respond to the statement, “The activities increased my understanding” on the Likert scale student survey, three of the student-participants responded *Strongly Agree*, six answered *Agree*, and one replied *Neither Agree Nor Disagree*. These responses were consistent with the observations made by the teacher-researcher as well as results from prior research (Allen, 2007; Awang & Zakaria, 2012). During an activity in which the student-participants created graph models and calculated the area under the curve, the teacher-researcher reflected that Students 2 and 4 made critical connections to prior knowledge before any of their classmates. Based on their conversations while working within their groups, these two student-participants concluded that the area under the curve would be calculated by taking the integral of the lower bounded curve subtracted from the upper bounded curve earlier than any of their classmates. Although these two student-participants did not earn passing scores on the posttest, they did show evidence of the conceptual understanding that they gained through the CSM.

While completing instructional activities through the CSM, the student-participants made comments about the manner in which they were able to gain conceptual understanding of calculus topics. During an activity that necessitated that the student-participants use prior knowledge to solve separable differential equations and the exponential growth model, the student-participants were required to make connections from Precalculus. During this activity Student 7 asked, “Isn’t this the  $Pe^{rt}$  equation that we used in Precalculus?” Her question confirmed that student-participants were able to make connections to prior learning during the activity. This evidence of conceptual understanding through the CSM was corroborated during an activity in which student-participants were playing a matching game with slope fields of differential equations and Student 3 highlighted, “I knew which slope field to pick because  $x$  is the only variable and the slope field is vertically not changing.” She was able to apply her previous knowledge of horizontal and vertical movement on a graph to the new concept of slope fields. Throughout implementation of the CSM, where a constructivist approach to learning was established, the student-participants showed the conceptual understanding that they gained. Ryan and Cooper (1998) highlighted that in a constructivist classroom, students learn by “building on the base of prior knowledge, attitudes, and values” (p. 414). The student-participants were able to develop conceptual understanding by constructing their own knowledge through the CSM.

Based on their survey responses, student-participants indicated that they were able to gain more conceptual understanding when the activities were presented after they had been given some direct information on the content before engaging in the hands-on activity. Many noted that they became frustrated when they completed activities that

required them to make connections to past knowledge without any instruction from the teacher-researcher while others believed that this helped them. Student 5 commented on his feelings that the activities would be more effective after instruction with his survey response:

If all of these activities were given after the material was taught, I believe it would make them more effective. For example, the activity involving cross-sections would have made a lot more sense and would have been much less infuriating if I knew beforehand what cross-sections were.

However, Student 3 viewed a lack of instruction prior to the activities positively and wrote, “They made it easier to remember concepts by allowing us to kind of figure it out on our own and make connections ourselves rather than the teacher giving us the notes.” The statements by these student-participants are consistent with prior research that found hands-on activities given prior to instruction on a topic fostered mathematical thought and hands-on activities after instruction promoted deeper conceptual understanding (Yoon et al., 2010). The teacher-researcher required some activities to be completed prior to instruction and others after some instruction in order to provide the student-participants with the opportunity to encounter both during implementation of the CSM.

### **Fostering Interpersonal Dynamics**

The activities that were completed during the CSM often required the student-participants to work in small groups or with a partner. Consequently, the student-participants had the opportunity for more communication than the teacher-centered lectures to which they were accustomed. A key component of the constructivist



classroom is that the teacher takes on more of a facilitator role and the students are discussing solutions to problems (Brewer & Daane, 2002; Sheppard, 2008). The student-participants benefited from student-centered instruction. For example, during the focus group Student 5 stated, “The best part of the CSM is how student-driven the activities are, as it means that I actually have to pay full attention, thus making me learn and retain the lesson better.” In the focus group Student 2 reiterated the benefits of the constructivist classroom noting, “The CSM helped because peers were able to answer questions in a different way from the teacher.” Then Student 1 added, “The teacher took a back seat and allowed students to make discoveries for themselves.” After reflecting on the classroom events during implementation of the CSM, the student-participants noticed that the classroom became student-centered.

During the CSM, the teacher-researcher noticed that the student-participants were taking control of their own learning and many of the female student-participants were much more vocal in their responses. This was especially true for Students 8 and 9. Both of these female student-participants were very quiet in class prior to implementation of the CSM and were often apprehensive about answering questions even when they knew the answers. Researchers have found that girls, even those in advanced courses, often have less confidence in their mathematical ability than boys (Dentith, 2008). The demeanors of Students 8 and 9 changed throughout the CSM and they began to take leadership roles within their small groups when completing hands-on activities. While all of the student-participants gained confidence and were more willing to answer questions during the CSM, the teacher-researcher found that this was especially true for Students 8 and 9. This finding was consistent with results of prior research that determined that girls

gain confidence when they work collaboratively (O'Shea et al., 2010). The teacher-researcher reflected on the impact that the CSM had in relationship to increasing the confidence levels of these student-participants.

The student-participants developed camaraderie while encountering the CSM. Student 3 noted the change in classroom dynamics with her statement about the hands-on activities. She remarked, "They were enjoyable because you got to work with a team and have (somewhat) fun instead of just taking notes." Student 9 also emphasized the importance of working together during the CSM and commented, "they were enjoyable because they were hands-on and grouped activities". The teacher-researcher noticed that student-participants engaged in friendly banter and developed a sense of teamwork while completing the tasks. Wei and Ford (2015) found "working collaboratively provides students with experience solving problems and negotiating the dynamics of working with others from different perspectives, a skill that will be crucial to students" (p. 51). The student-participants learned to cooperate and work as a team during the CSM and became actively involved in the problem solving process.

### **Improving Model Efficacy**

The teacher-researcher designed and developed the CSM primarily to help student-participants visualize calculus concepts and make connections to prior learning. The student-participants indicated that the CSM helped them visualize the concepts they were learning. Many of the student-participants stated that they found the activity in which they cut out paper spheres to represent a graph and opened the paper sphere to

create a three-dimensional solid that modeled the disk method of calculating volume very helpful. When asked to reflect on the CSM, Student 1 wrote:

The activities were a good use of time because they taught me things words are not always able to. For example, the activity using 3D shapes through revolving showed the different shapes 2D graphs can revolve into and give an idea of how they will generally look, something words could not truly teach me.

The student-participants found it especially helpful to have tangible objects that served to represent calculus concepts. After completing a different activity, Student 2 stated, “Some of the activities such as the cross section activity helped me visualize how creating the cross sections made one large shape and it helped me see what was happening when we took cross sections of a shape.” Researchers have found that students benefit when they are able to visualize the concepts that they are learning (Yoon et al., 2010). When asked to respond to the statement, “I believe that I will perform better on the test as a result of the activities” on the Likert scale student survey, three of the student-participants responded *Strongly Agree*, three answered *Agree*, two replied *Neither Agree nor Disagree*, and two *Disagree*. Based on all of the data that were collected, the teacher-researcher determined that the CSM gave her student-participants the opportunity to visualize calculus concepts, which could serve to increase achievement.

In order to maximize the effectiveness of the CSM, the teacher-researcher worked to ensure that each activity had a direct correlation to the course standards, was based on a concept that her student-participants had previously studied, and/or made a connection to a real-world application. Dewey (1938/1997) underscored that hands-on activities

must be carefully planned and scaffold onto prior knowledge to increase learning. When asked to respond to the statement, “The activities were a good use of class time” on the Likert scale student survey, three of the student-participants responded *Strongly Agree*, six answered *Agree*, and one replied *Neither Agree nor Disagree*. The student-participants in the present action research study believed that the teacher-researcher carefully planned the activities in a manner that facilitated learning. When asked to discuss the hands-on activities, Student 10 wrote, “They were a good use of class time because they pertained to the material. During none of the activities were there time-wasting, irrelevant content.” The student-participants also found that the CSM helped them connect their learning to real-world applications. Student 6 indicated, “These activities allowed me to visualize the material so I could see it in a real life example rather than just numbers.” Student 8 corroborated this with her statement, “The activities helped me see a more ‘real-world’ example of the concept. Instead of looking at equations and numbers, the concept was modeled in a hands-on way, allowing me to see what was actually happening.” During the focus group the student-participants also indicated that the hands-on activities of the CSM prepared them for the AP Calculus Exam. For example, when asked, “In what ways did the CSM prepare you for the AP Calculus Exam?” Student 7 responded:

I would say that they did help me be prepared for the Exam in that they were more real-life problems. It was something where it did have an application to the real-world, similar to the FRQs [Free-Response Questions]. We were able to create something or see something that we could then perform computations on to draw conclusions.

Based on the statements of her student-participants, the teacher-researcher determined that the CSM had a positive impact because the activities were based on the course material and made connections to real-world applications.

While the student-participants indicated many positive aspects of the CSM, difficulties were still encountered. At the onset of the CSM the teacher-researcher spent a great deal of time thinking about the optimal way to group her student-participants for the hands-on activities. Researchers have found that when teachers select groups this helps to ensure that diverse students are given the opportunity to work together, which fosters acceptance of different points of view, but when the groups are selected by other methods this can also be beneficial as students learn to navigate working relationships with their peers (Mitchell, Reilly, Bramwell, Lilly, & Solnosky, 2004; Wei & Ford, 2015). As the teacher-researcher, she was unsure whether or not to create the student groups based on her knowledge of the student-participants' ability levels, gender, and personalities or through random selections. The teacher-researcher decided to use either random techniques to place the student-participants in groups or convenience methods in which the student-participants were grouped with those seated in closest proximity. This decision by the teacher-researcher led to a student-participant becoming frustrated during an activity. Student 2 noted:

With some of the partnered activities such as the Taylor polynomial activity, it was difficult to understand what I was doing and when trying to ask partners, they were trying to get it done so I was behind and confused.

After reflecting on this statement, the teacher-researcher remembered noticing that this student was not interacting with her group members as much during the aforementioned activity, but did not realize the extent of her frustration. The teacher-researcher discussed this event with a colleague at the time and was still concerned about the use of random or convenience grouping of student-participants. Based on the comments of this student-participant, the teacher-researcher determined that she should reevaluate her method of grouping student-participants and include some teacher-selected groups in the future.

After reviewing the student-participant surveys, the teacher-researcher found that instruction with the CSM increased enjoyment in her class. When asked to respond to the statement, “The activities were enjoyable” on the Likert scale student survey, four of the student-participants responded *Strongly Agree*, four answered *Agree*, and two replied *Neither Agree nor Disagree*. Student 1 wrote, “The activities were enjoyable because they weren’t the normal write problems and solve. They were a step away from the norm, like the activity where we made cross sections and saw what they looked like.” The positive feelings that the student-participants had while encountering the CSM were also corroborated with the comment by Student 7, “I think these activities were enjoyable because it wasn’t just us taking notes and listening to the lesson.” The student-participants indicated that they enjoyed the change from the teacher-centered lecture to the student-centered approach to learning through the CSM. Researchers have determined that students who enjoy and have a positive attitude about mathematics have increased achievement (Allen, 2007; Couture, 2012; Mensah et al., 2013) and that a constructivist approach to mathematics instruction enables students to have improved attitudes (Sheppard, 2008).

## **Answering the Research Question**

The purpose of the present action research study was to improve the calculus achievement of student-participants through a shift in the primary instructional method from teacher-centered lectures to student-centered learning through the CSM. The research question was, “What is the impact of the hands-on activity model on secondary student achievement in Advanced Placement Calculus?” Quantitative pre- and posttest data were used to answer the research question while qualitative data were used to triangulate (Mertler, 2014) the quantitative findings. The mean score on the 20-question multiple-choice pretest for the student-participants was 1.5 correct (S.D. 1.02) and that of the posttest was 13.4 correct (S.D. 4.61) for a net mean gain of 11.9. Every student-participant had a score increase from the pre- to posttest. The results of the t-tests identified statistically significant growth from the mean of the pretest to that of the posttest. The quantitative pre- and posttest data indicated the level of growth in calculus understanding by the student-participants during implementation of the CSM.

While all of the student-participants gained from the pre- to posttest, the amount of the increase varied. Of the 10 student-participants, six earned a passing score on the posttest and four did not. Although four of the student-participants did not pass the posttest, they did grow in calculus achievement while encountering the CSM. All of the student-participants earned a perfect score on a quiz that assessed their understanding of calculating area and volume. Additionally, two of the student-participants who did not earn a passing score on the posttest gave evidence of their understanding through the connection they made to their prior knowledge of area during a hands-on activity.

Another impact on calculus achievement during implementation of the CSM was that all of the student-participants were given equal access to learning. With a focus on student-centered instruction, student-participants who were normally less vocal were able to take on leadership roles in their small groups. Researchers have shown that high-achieving female students benefitted from working collaboratively in small groups (O'Shea et al., 2010). The teacher-researcher observed that her female and Black or Hispanic student-participants were willing to take risks within the calculus content when encountering the CSM. This was a change from their normal classroom behavior in which they were much more apprehensive to ask and answer questions.

During implementation of the CSM, the teacher-researcher changed from teacher-centered lectures to student-centered hands-on activities in which the student-participants used their prior knowledge to construct understanding of calculus topics. Many of the student-participants indicated they found instruction through the CSM much more enjoyable than lectures. As a result of the positive experiences that the student-participants had during the CSM and a review of prior studies, the teacher-researcher believed that the enjoyment led to an increase in calculus achievement (Allen, 2007; Couture, 2012; Mensah et al., 2013).

The teacher-researcher determined through her review of the pre- and posttest scores that the student-participants had an increase in calculus achievement during the CSM. It is unclear as to the exact amount of growth in calculus understanding that her student-participants gained which can be attributed to the CSM. While the results of the present action research study have provided the teacher-researcher with evidence of growth by her student-participants in calculus understanding, further research can be



conducted in the future to quantify the differences. The teacher-researcher would like to conduct future research on her other classes to determine the impact that the CSM would have on students of different grade levels and mathematical ability levels.

### **Conclusion**

The problem of practice for the present action research study involved the need for the student-participants to build connections from basic mathematical operations to complex calculus topics in order to improve their scores on the AP Calculus Exam. Through their interactions while completing the hands-on activities in the CSM, the student-participants showed that they made important connections within the calculus content. The student-participants referred to their prior knowledge to build new understanding during the CSM. Every student-participant made a perfect score on a quiz that covered the topic of calculating area and volume. Since the teacher-researcher had never experienced a previous assessment in which everyone made a perfect score, she attributed the student-participants' success on this quiz to the conceptual understanding that they gained through the CSM. The quiz that the student-participants took was of a similar format to the AP Calculus Exam free-response questions. The teacher-researcher selected this format in order to prepare the student-participants for the rigor of the AP Calculus Exam.

The purpose of the present action research study was to describe implementation of the CSM, a hands-on activity model (Couture, 2012; Cruse, 2012; Pfaff & Weinberg, 2009; Schunk et al., 2007), designed by the teacher-researcher in an AP Calculus class over an 8-week span in the spring of 2017. Additionally, the purpose of the present study

was to design an action plan in conjunction with the student-participants in order to improve the effectiveness and utility of the CSM in providing critical connections and enduring understanding within the course content for future AP Calculus students at CHS.

The teacher-researcher used quantitative data to answer the research question. The research question was, “What is the impact of the hands-on activity model on secondary student achievement in Advanced Placement Calculus?” The student-participants took a pretest prior to implementation of the CSM and the same assessment served as the posttest that was given after 8 weeks. The student-participants earned a mean score on the 20-question multiple-choice pretest of 1.5 correct (S.D. 1.02) and 13.4 correct (S.D. 4.61) on the posttest for a net mean gain of 11.9 questions. Every student-participant had a score increase from the pre- to posttest. Calculations from the analytic t-tests indicated statistically significant growth from the pretest mean score to the mean score of the posttest for this particular group of student-participants.

Qualitative data were used to triangulate (Mertler, 2014) the quantitative findings. The teacher-researcher transcribed the student-participants’ responses to a survey and highlighted data that fit into specific themes. After studying the results, the teacher-researcher determined that three themes emerged from the data. The themes were: 1) developing conceptual understanding, 2) fostering interpersonal dynamics, and 3) improving model efficacy. These themes provided the teacher-researcher with a better understanding of the quantitative outcomes.

The teacher-researcher has consistently worked to ensure that students in her AP Calculus classes represent a full cross-section of the student population at CHS. At the end of the previous school year the teacher-researcher met with students who had taken the coursework that was required to enroll in AP Calculus to encourage them to take the class. This included a focus on females and underrepresented minority groups. Consequently, the demographics in the teacher-researcher's AP Calculus class represented a greater level of diversity than national averages. Sixty percent of the student-participants were female while nationally only 41% of the students who took the AP Calculus Exam were female (College Board, 2014). The teacher-researcher's student population consisted of 10% Black and 10% Hispanic compared with 2.9% Black and 8.3% Hispanic nationally (College Board, 2014).

Based on the aforementioned information, the teacher-researcher was able to create a demographically diverse calculus classroom. This was consistent with the high expectations that the teacher-researcher maintained for all of her students. Even if all of the students that were encouraged to take the class were not successful on the AP Calculus Exam, the teacher-researcher understood the importance of exposing students to advanced mathematics. Researchers have found that many teachers do not want students who they believe will perform poorly on the AP Exam to take their AP courses (Matthews, 2014). The teacher-researcher in the present action research study worked to create an inclusive atmosphere in her AP classroom and developed the CSM in an attempt to help her student-participants develop conceptual understanding of calculus content.

Students who take the AP Calculus Exam while in high school have a higher retention rate in college (Mattern et al., 2009). The teacher-researcher in the present action research study wanted to ensure that her students were prepared for the rigor of collegiate academics. It was the teacher-researcher's desire for her student-participants to challenge themselves academically during the CSM. Consequently, she worked to ensure that the student-participants made their own connections and guided them through the process without simply providing them with answers. The teacher-researcher believed that the student-participants would be better equipped to handle the rigor of college after encountering the CSM. The student-participants will be impacted by the CSM even after they are no longer in the teacher-researcher's AP Calculus class.

## **CHAPTER FIVE: SUMMARY, CONCLUSIONS, AND ACTION PLAN**

### **Introduction**

The purpose of Chapter Five is to present the summary, conclusions, and action plan developed in conjunction with the current action research study of the impact of the Constructed Scaffold Model (CSM), a hands-on activity model designed by the teacher-researcher, on her Advanced Placement (AP) Calculus class. The Problem of Practice (PoP) for the present action research study was that students at County High School (pseudonym) were scoring below their global peers on the AP Calculus Exam and facing difficulty when required to build connections from basic mathematical operations to complex calculus topics. The participants in the present action research study were students enrolled in the AP Calculus BC class at County High School (CHS) during the 2016-2017 school year. There were 10 student-participants in the class of which six students were female and four students were male. There were eight White students, one Black student, and one Hispanic student in the class. Nine of the student-participants were in the 12<sup>th</sup> grade and one was in the 11<sup>th</sup> grade. Two of the student-participants received free or reduced lunch.

The present action research study took place at CHS, a large, suburban high school of approximately 2,000 students located in the southeastern United States. Based on data from the 2012-2013 school year the racial composition of CHS was 74% White,

15% Black, 5% Hispanic, 4% Two or More Races, 1% Asian, and less than 1% other races (High-Schools, 2015). Approximately 40.3% of students at the school qualified for free or reduced lunch (High-Schools, 2015).

The teacher-researcher designed and developed the CSM, a hands-on activity model. The teacher-researcher also designed and developed a pretest (see Appendix A) that was administered to student-participants prior to implementation of the CSM and the same instrument was used as a posttest completed by the student-participants after an 8-week period during the spring of 2017. The teacher-researcher tested the difference between the pre- and posttest means using a t-test.

The quantitative data from the pre- and posttest were used to answer the research question. The research question was, “What is the impact of the hands-on activity model on secondary student achievement in Advanced Placement Calculus?” Additionally, the results were disaggregated by race, gender, and socioeconomic status. The teacher-researcher used field notes (see Appendix B) to record student engagement throughout implementation of the CSM as well as information on her perceptions of the overall effectiveness of each activity. Research by Lau (2014) found that field notes and student surveys provide critical information on student engagement and perceptions during action research data collection. Student surveys (see Appendices C and D) were completed and student work samples were reviewed during the data collection phase of the present action research study. These anecdotal data, along with data from the College Board (2016a) and CHS where the teacher-researcher teaches, were used to triangulate (Mertler, 2014) the quantitative data. After student-participants completed the AP

Calculus Exam, the teacher-researcher used a focus group (see Appendix E) to assist with the development of an action plan.

All of the student-participants increased their scores from the pre- to posttest. The average pretest score for the class was 1.5 questions correct and the posttest mean score was 13.4 questions correct for a net gain of 11.9 questions. Additionally, calculations from the difference of two means t-tests indicated statistically significant growth from the pretest mean score to the mean score of the posttest for these particular student-participants. The qualitative data were used to triangulate (Mertler, 2014) the quantitative data using a summative data analysis (Dana & Yendol-Hoppey, 2014). After studying the results, the teacher-researcher determined that three themes emerged from the data. The themes were: 1) developing conceptual understanding, 2) fostering interpersonal dynamics, and 3) improving model efficacy.

The increase in scores from the pre- to posttest provided evidence of the conceptual understanding that the student-participants gained in calculus when receiving instruction through the CSM. The student-participants made comments while completing tasks in the CSM as well as on the survey at the culmination of the CSM that indicated the conceptual understanding they developed. The student-participants gained confidence throughout implementation of the CSM. Females and historically underrepresented minorities became more vocal and confident when they worked in their small groups to complete the hands-on activities.

## Key Questions

While the findings of the present action research study indicate that the student-participants gained conceptual understanding of calculus through the CSM, key questions have emerged for the teacher-researcher. A key question that emerged was “Would the teacher-researcher have the ability to design hands-on activities that provided the student-participants with the required scaffolding to build conceptual understanding?” As the CSM was designed, there was not an abundance of prior research in calculus classes, therefore the teacher-researcher had to design and develop the activities within the CSM using her knowledge of curriculum and the difficulties that her previous students faced with the course content. Based on survey responses and comments while working on the hands-on activities, the student-participants made the required connections within the course content and found the CSM much more enjoyable than lectures by the teacher-researcher. The teacher-researcher developed the criteria for the activities in the CSM after studying prior research using hands-on activities (Couture, 2012; Cruse, 2007; Pfaff & Weinberg, 2009; Schunk et al., 2007). The teacher-researcher’s need to develop more hands-on activities for her AP Calculus students led to an additional key question, which was “How can other AP Calculus teachers at CHS and in the school district collaborate to create additional hands-on activities for their students?” The AP Calculus teachers in the teacher-researcher’s school district regularly share instructional resources and work to maximize their students’ understanding of calculus. In order to provide the opportunity for hands-on activities in other AP Calculus classes at CHS and in her school district, the teacher-researcher would like to collaborate with other AP Calculus teachers to create more hands-on activity resources.



Prior to implementation of the CSM, the teacher-researcher questioned the impact that hands-on activities could have on advanced mathematics students. After completing the first cycle of the CSM, the teacher-researcher has found that utilizing hands-on activities can increase students' comprehension of calculus topics. As a result of her action research study, the teacher-researcher would like to share her findings with other teachers at CHS. A key question was "How can the teacher-researcher share the results of her study with other teachers at CHS?"

Research by Mattern et al. (2009) found that students who take AP Calculus in high school have a higher retention rate in college. Consequently, students who take AP Calculus can have an advantage over students who do not. Enrolling in AP Calculus is beneficial for students. The teacher-researcher determined that a key question was "What steps can the teacher-researcher take to increase enrollment in her AP Calculus class, especially for historically marginalized groups in advanced mathematics such as female, Black, and/or Hispanic students?"

The teacher-researcher found that all of her student-participants increased their scores from the pre- to posttest. Although every student-participant improved on the posttest, four of the 10 student-participants did not earn a passing score on the posttest. The teacher-researcher wanted all of her student-participants to pass the posttest and would like to determine what changes need to be made in the future so that all of her student-participants earn passing posttest grades. The problem of practice for the present action research study involved the need to build connections from basic mathematical operations to complex calculus topics as well as for student-participants to increase their AP Calculus Exam scores. The AP Calculus Exam consists of multiple-choice and free-

response questions. The posttest that was used consisted of multiple-choice questions only. Researchers have found that students will guess when they are unsure of an answer on a multiple-choice test and this can lead to scores that do not accurately reflect the students' knowledge (Bush, 2015). A key question that emerged was "Should the format of the pre- and posttest be modified to include both multiple-choice and free-response questions to provide better replication of the AP Calculus Exam?" The teacher-researcher plans to modify the pre- and posttest to include both types of questions when she begins a new cycle of the CSM in the fall of 2017. After completion of the next cycle of the CSM, the teacher-researcher will have experience with both test formats and be able to determine if there is a difference in the AP Calculus Exam scores of the student-participants based on the format of the pre- and posttest.

### **Action Researcher**

The teacher-researcher assumed several roles throughout the present action research study. A primary role for the teacher-researcher was that of curriculum leader. The teacher-researcher used her knowledge of AP Calculus course standards and her knowledge of her students to design and develop the CSM. As a curriculum leader the teacher-researcher was able to anticipate the content within the course for which student-participants would benefit from hands-on activities. When she developed CSM activities she was able to accurately predict the time that it would take for the student-participants to complete the work and foresee difficulties and obstacles that they would face. As a leader in her classroom, the teacher-researcher had to allow her student-participants to struggle at times so that they could learn from their own experiences and not simply be provided with answers. The teacher-researcher closely monitored her student-

participants' interactions while they worked within the CSM, although she was not privy to all of their conversations, so that she could gain a better understanding of their experiences and be receptive to their perceived difficulties. It was important to the teacher-researcher that her student-participants had a voice in classroom events during the action research study. As the teacher-researcher analyzed the data, she understood that while the quantitative pre- and posttest scores were the primary data source, her student-participants' experiences and perceptions while encountering the CSM provided additional information. The teacher-researcher reflected alone on the classroom events throughout implementation of the CSM and reflected with her student-participants in order to make sure that they were formulating the connections within the course content to develop conceptual understanding. The teacher-researcher provided her student-participants with a survey so that they had the opportunity to think about the classroom events and give their opinions of the CSM. As a curriculum leader, the teacher-researcher was able to design and develop the CSM, analyze the collected data, and reflect alone and with her student-participants.

As an insider, the teacher-researcher used her role as a participant-researcher to build trust with the student-participants and help them understand her expectations during the action research study (Termini, 2013). The teacher-researcher reflected with her student-participants during implementation of the CSM to determine their views on this new student-centered instructional approach, how the CSM impacted her student-participants' calculus achievement, and their ability to relate the calculus content to their prior knowledge. As a result of the insider status of the teacher-researcher, her student-participants were comfortable enough to provide honest feedback throughout the action

research study. With her knowledge of the student-participants' personalities and prior academic achievement, when the teacher-researcher took on the role of researcher, which gave her an outsider position with her student-participants, she ensured that her data collection was unbiased, objective, and depicted an accurate portrayal of classroom events. As an outsider, the teacher-researcher did not have access to all of the conversations and interactions that took place within the groups when the student-participants were working on the activities. Consequently, when the student-participants mentioned certain events on their surveys, she was somewhat surprised. The teacher-researcher's knowledge of her students helped her as she reflected on the student-participants' engagement and comprehension of calculus content during implementation of the CSM.

One of the greatest personal challenges that the teacher-researcher faced during this study was changing to progressive and constructivist education approaches in her classroom. After studying the work of Dewey (1938/1997), the teacher-researcher believed that the student-centered progressive approach helped students learn how to solve problems and gain a better understanding of the content in many academic disciplines. She was unsure of the benefits of progressivism in an advanced mathematics course. Consequently, the teacher-researcher had to rethink her pedagogical beliefs. Through her implementation of the CSM, the teacher-researcher was able to see how her student-participants were able to make important connections within the calculus content for themselves and increase their scores from the pre- to posttest.

Prior to implementation of the CSM, the teacher-researcher believed that she had an equitable classroom in which all students were given equal opportunities to speak and

interact. As the teacher-researcher watched the student-participants work through CSM activities she realized that all of her student-participants were engaged in the learning. This was a change from her teacher-centered classroom in which the student-participants had fewer opportunities to speak and the more vocal students tended to dominate the conversations. She noticed that two female student-participants who seldom spoke during teacher lectures were much more vocal when participating in small groups. The teacher-researcher realized that some students thrived when they were given the opportunity to work collaboratively and this result has been corroborated in prior research (O'Shea et al., 2010). In order to create equitable classrooms, educators must design lessons that will encourage all students to participate.

Another personal challenge that the teacher-researcher faced when implementing the CSM was the time that was required for the student-participants to complete the activities. As a curriculum leader in her school district, the teacher-researcher collaborated with an AP Calculus teacher from a different school in the district to develop a pacing guide for the course several years ago. Therefore, the teacher-researcher knew the time that was allotted for each course standard. Many of the CSM activities took additional time to complete. The teacher-researcher became concerned that the pacing of the course would suffer. Ironically, the teacher-researcher noticed that because her student-participants developed a deep understanding of the calculus topics during the CSM, she was able to make up the extra time spent on the activities as a result of the decrease in time spent answering questions on formative assessments.

## **Developing an Action Plan**

The teacher-researcher learned a great deal from her investigation. Based on results of the pre- and posttest, her student-participants increased their understanding of calculus topics while encountering the CSM. Through their conversations while completing activities, the student-participants gave evidence of conceptual understanding that they developed. Several of the student-participants stated that the activities in the CSM gave them the opportunity to visualize concepts, which led to improved understanding. Therefore, the teacher-researcher learned that progressive pedagogical methods were effective in her higher-level mathematics class.

While she never considered the possibility that her student-participants were tired of teacher lectures as the primary instructional method prior to implementation of the CSM, the teacher-researcher learned that her student-participants enjoyed being able to learn calculus topics through hands-on activities. She learned that the CSM activities gave all of her student-participants the opportunity to share their ideas and conjectures. While some of her student-participants were apprehensive about asking and answering questions during teacher lectures, they were much more vocal and even assumed leadership roles in the small groups.

Prior to implementing the CSM, the teacher-researcher was unsure as to what would be the ideal method to form the small groups of student-participants as they worked on hands-on activities. The teacher-researcher believed that it might be more beneficial for her to form the groups based on her knowledge of their personalities, abilities, and gender, but used random methods such as drawing a name or number or

allowed the student-participants to form their own groups. Researchers have found that the advantage of teacher-selected groups was that diverse students were given the opportunity to work together which fostered acceptance of different points of view and when the groups were selected by other methods this could be beneficial as students learned to navigate working relationships with their peers (Mitchell, Reilly, Bramwell, Lilly, & Solnosky, 2004; Wei & Ford, 2015). Based on prior research and her experiences with grouping the student-participants during the CSM, the teacher-researcher has concluded that she will use a variety of grouping methods, including teacher-selected groups for the next cycle of the CSM in the fall of 2017. As more time is spent with student-participants grouped using different methods, the teacher-researcher will determine methods that will allow student-participants to work most effectively so that they will be able to build connections within the course content and increase their scores on the AP Calculus Exam.

In order to ensure that the student-participants' perceptions and experiences about the CSM were captured for development of the action plan, the teacher-researcher used a focus group. Mertler (2014) noted, "interactions among the focus group participants may be extremely informative because of the tendency for people to feed off others' comments" (p. 132). Since the problem of practice for the present action researcher study was the need to help the student-participants increase their AP Calculus Exam scores and build connections between basic mathematical processes and complex calculus topics, the teacher-researcher believed that the focus group would be best after the student-participants took the AP Calculus Exam. As a result of the experiences that the student-

participants had while encountering the CSM, the teacher-researcher thought that their opinions were a critical component needed for the development of an action plan.

During the focus group, the student-participants were given the opportunity to reflect on classroom instruction with the CSM and offered several suggestions that they believed would improve the CSM for future AP Calculus classes at CHS. Overall, the student-participants enjoyed the student-centered classroom during the CSM. For example, Student 4 stated, “I prefer hands-on activities over notes. I am more visual. The activities help me remember more.” However Student 6 believed that the student-participants would be better prepared for the AP Calculus Exam if questions similar to those on the exam were incorporated into the activities. Student 3 stated, “The CSM would be better if every activity resulted in a tangible object.” Students 2, 5, and 8 agreed that they preferred it when the teacher-researcher explained the calculus content and used the hands-on activity to reinforce and expand on the topic as opposed to when they completed the hands-on activity prior to any instruction on the material. As a result of the time that the teacher-researcher spent reflecting with her student-participants, she was able to determine ways to improve the CSM for the new cycle that will begin in the fall of 2017.

### **Action Plan**

Upon reviewing the findings from the present action research study, the teacher-researcher developed an action plan (see Appendix I) to ensure that future AP Calculus students at CHS as well as other students at her school and district will benefit from a student-centered constructivist approach to mathematics instruction. In July 2017, the



teacher-researcher will receive the student-participants' AP Calculus Exam scores. Since the problem of practice for the teacher-researcher was that her student-participants were not scoring as well as their global peers on the AP Calculus Exam, the teacher-researcher will compare the scores of her student-participants with those of all examinees. The teacher-researcher will study these scores and reflect on the results. One component of the reflection will be to compare the AP Calculus Exam score of each student-participant to his or her pre- and posttest scores. A second component will be to review each student-participant's survey responses to look for connections between each AP Calculus Exam score and that individual's perceptions of the CSM. The student-participants' AP Calculus Exam scores should help the teacher-researcher decide if the CSM requires additional modifications for the next cycle.

In August 2017, the teacher-researcher will meet with the district mathematics coordinator and the other AP Calculus teachers in the district to discuss her action research study and the findings related to calculus understanding and interpersonal dynamics. She will provide an overview of all of the activities that her student-participants completed during implementation of the CSM. The teacher-researcher will invite these colleagues to collaborate via an email group, that the teacher-researcher will form, throughout the school year to develop and share new hands-on activities for their students. The teacher-researcher will encourage the other AP Calculus teachers to try some of the activities that were used during the CSM in order to compare the perceptions of their students with those of her student-participants. This email communication will continue throughout the 2017-2018 school year and at the end of the school year the

teacher-researcher will survey the other teachers to determine the next steps in this process.

In an attempt to share her findings with other teachers at CHS, the teacher-researcher will meet with her school principal and the instructional resource coordinator in the fall of 2017. The administration at CHS encourages teacher-led professional development and offers numerous opportunities for teachers to share ideas and instructional methods with their colleagues. The instructional resource coordinator will schedule a date for the teacher-researcher to lead a professional development session to share her action research study with her colleagues. During this professional development session the teacher-researcher will form a professional learning community (PLC) with any teachers who are interested in using hands-on activities in their classrooms. This PLC will meet once a month during scheduled collaborative planning time throughout the 2017-2018 school year. The purpose of the PLC will be to share ideas, activities, and data from the individual classrooms as well as to encourage members to develop and implement student-centered activities. The teacher-researcher will determine the next steps beyond the 2017-2018 based on recommendations from the PLC members.

The College Board (2016a) has stated a commitment to ensure equitable access to rigorous coursework noting, “schools should make every effort to ensure their AP classes reflect the diversity of their student population (p. 2). Similarly, the teacher-researcher would like to increase the number of female, Black, Hispanic, and lower socioeconomic status students in her AP Calculus class to reflect the school demographics. In order for students to have the prerequisite coursework to enroll in AP Calculus, they must take two

mathematics classes in the same school year prior to their senior year. Many students are not aware of this course sequencing and many parents and students do not know about the AP courses that are offered at CHS. The teacher-researcher will meet with all of the Algebra 2 Honors teachers at CHS during a mathematics department meeting and provide them with information to give to their students about mathematics course sequencing so that they will be on track to enroll in AP Calculus during their senior year in high school. The teacher-researcher will personally meet with any students, especially those from historically marginalized groups in advanced mathematics such as female, Black, and Hispanic students, to tell them about AP Calculus and encourage them to enroll. The teacher-researcher has found that many CHS students are apprehensive about taking AP Calculus because of the perceived difficulty level of the course and believes that meeting with Algebra 2 Honors students will help them feel less intimidated and more excited about the class.

A final component of the action plan will be to implement a new cycle of the CSM in the fall of 2017. The teacher-researcher will use the student survey and focus group responses from her current student-participants to continue to make modifications to the CSM. The student-participants and the teacher-researcher have indicated that the pre- and posttest should have a similar format as the AP Calculus Exam. Consequently, for the next cycle of the CSM the pre- and posttest will have both multiple-choice and free-response questions. While her student-participants indicated that they benefitted from visualizing calculus concepts during the CSM, several noted that they especially preferred activities that produced tangible objects. The teacher-researcher will modify some of the current hands-on activities and include new activities for the next cycle of the

CSM so that more of the activities will include tangible objects. Based on her own reflections as well as the responses by the student-participants, the teacher-researcher will use a variety of methods for placing students in small groups to complete the hands-on activities during the next cycle of the CSM. While many of the student-participants stated that they would prefer some instruction on a topic before beginning an activity, based on prior research by Yoon et al. (2010) the teacher-researcher will provide some instruction before many activities but structure other activities with no introductory instruction in order for the student-participants to build understanding from prior knowledge. As a result of the reciprocal reflection with her student-participants, the teacher-researcher has made modifications to the CSM for the next cycle that will begin in the fall of 2017.

### **Facilitating Educational Change**

The teacher-researcher's action plan will create positive change at CHS and other schools in her school district. The teacher-researcher plans to collaborate with other AP Calculus teachers at her school and district to implement more student-centered, progressive, and constructivist approaches to AP Calculus instruction. These changes will allow other AP Calculus students to become actively engaged in their own learning. Based on her essentialist beliefs of the past, the teacher-researcher knows that many higher-level mathematics teachers are hesitant to use student-centered instruction. In order to address those challenges the teacher-researcher will describe the positive reactions of her student-participants during the CSM. The teacher-researcher knows that finding time to develop hands-on activities can also be a challenge for some educators. It is for this reason that the teacher-researcher believes that utilizing an email group for the

AP Calculus teachers in her school and district will alleviate some of the challenge that these other educators might face. The teacher-researcher wants to create a scholarly community between these educators where they are able to collaborate and learn from her experiences.

After leading a professional development session to discuss the results of her action research study, all of the teachers at CHS will be introduced to the teacher-researcher's constructivist approach to AP Calculus instruction. Even if some teachers do not want to change their methods of instruction at least they will have been given the opportunity to learn from the teacher-researcher. The teacher-researcher plans to form a PLC at CHS for teachers who are interested in developing hands-on activities for their students. When teachers of all academic disciplines are allowed to collaborate in the PLC, then they will be able to learn and grow from the experiences of others in the group. The teacher-researcher believes that a PLC will be an ideal way for teachers to collaborate and improve the educational opportunities for more CHS students. The main challenge that will be faced is finding a time for the PLC to meet. The teacher-researcher believes that the optimal time will be during the monthly collaborative planning sessions when students are dismissed early from school. Another challenge will be to keep teachers from feeling overwhelmed by creating hands-on activities for their students and the teacher-researcher believes that with the collaborative nature of the PLC, teachers will be able to share resources to reduce the load for any individual.

Research has shown that students who take AP Calculus in high school have higher retention rates in college (Mattern et al., 2009). Consequently, the teacher-researcher would like to increase the enrollment in her AP Calculus class so that these

students will possibly earn college credit for Calculus 1 and Calculus 2 and be prepared for the academic rigor of college. In order to increase the number of students taking AP Calculus at CHS, the teacher-researcher needs to meet with the Algebra 2 Honors teachers to get them to encourage their students to take two mathematics courses in a single year so that they will have the prerequisite courses needed. The teacher-researcher believes that it is especially important for Algebra 2 Honors teachers to make students aware of the mathematics course sequence needed in order to take AP Calculus. Some students might not be aware of the need to take two math courses in one school year. The teacher-researcher would also like to talk to the Algebra 2 Honors classes personally and tell them about the CSM and help them learn more about AP Calculus. The teacher-researcher has found that many students at CHS feel intimidated by upper-level mathematics courses such as AP Calculus and believes that meeting with Algebra 2 Honors students personally will help them learn more about the class and feel less apprehensive about taking AP Calculus. She will use this time to meet with students individually if they have additional questions or concerns about taking AP Calculus. The teacher-researcher believes that this outreach will help increase the number of historically underrepresented minorities in advanced mathematics such as female, Black, and Hispanic students. The biggest challenge for this outreach program is that some teachers might feel that they are giving up instructional time for the teacher-researcher to speak with their students. In order to address this challenge, the teacher-researcher will schedule times with the Algebra 2 Honors classes that cause the least disruption.

## **Summary of Research Findings**

An objective of the present action research study was to describe implementation of the CSM, a hands-on activity model (Couture, 2012; Cruse; Pfaff & Weinberg, 2009; Schunk et al., 2007), designed and developed by the teacher-researcher in an AP Calculus class at CHS. Based on progressive and constructivist education theories, the teacher-researcher modified her instruction from teacher-centered to student-centered for the 8 weeks of the CSM. This student-centered instruction was new to the AP Calculus students at CHS. The teacher-researcher found that the student-participants increased their scores from pre- to posttest after encountering the CSM.

The present action research is important because it involved a new instructional method for AP Calculus at CHS. Through implementation of the CSM the student-participants developed conceptual understanding of calculus content. From the creation of physical and graphical models of calculus concepts, the student-participants were given the opportunity to visualize the material. Many of the student-participants stated that they better understood the concepts because they were able to visualize them. The student-participants were able to construct their own understanding of the calculus content through conversations that they had while completing the CSM activities.

## **Suggestions for Future Research**

The problem of practice for the present action research study was that students at CHS were scoring below their global peers on the AP Calculus Exam and struggling to build connections between mathematical concepts. The teacher-researcher studied the impact of a hands-on activity model on the student-participants' calculus achievement.

For future research, educators could study the impact of other instructional methods such as project-based learning or computer-based learning in an AP Calculus class. This future research would provide more insight into the instructional methods that prepare students for the rigor of the AP Calculus Exam.

The College Board (2016a) has maintained a commitment to diversity in AP classes. Many groups that have been historically marginalized in advanced mathematics such as female, Black, and Hispanic students need to be encouraged to enroll in AP Calculus. Future research needs to be conducted to determine how to increase enrollment of female, Black, Hispanic, and lower socioeconomic status students in AP Calculus. Research has shown that students who take AP classes in high school are better prepared for college (Mattern et al., 2009) so educators must find effective methods to increase enrollment in AP Calculus for all students, but especially those from historically underrepresented groups.

In addition to research studies designed to impact students' AP Calculus Exam scores, future research can be conducted using hands-on activities with students of other grade and course levels. While much more scholarly research exists with mathematics classes other than AP Calculus, students in the U.S. are still not performing as well as other students from around the world (OECD, 2012b) and thus any study that can lead to improvements in mathematics achievement is an important contribution to the scholarly literature that exists.



## Conclusion

This chapter included the summary, conclusions, and action plan for the present action research study. AP Calculus BC students at CHS were not performing as well as their global peers on the AP Calculus Exam and struggling to make connections from basic mathematical operations to complex calculus concepts. The teacher-researcher developed the CSM as an instructional method and used it to determine the impact that the model had on her student-participants' calculus achievement. Prior to implementation of the CSM the student-participants took a pretest (see Appendix A). The same assessment was used as a posttest after an 8-week period. The student-participants earned a mean score on the 20-question multiple-choice pretest of 1.5 correct with a standard deviation (S.D.) of 1.02, and 13.4 correct with S.D. of 4.61 on the posttest for a net mean gain of 11.9 correct questions. The teacher-researcher used repeated measures t-tests to compare the pre- and posttest mean scores.

In addition to these quantitative data, the teacher-researcher collected field notes (see Appendix B), student surveys (see Appendices C and D), formative assessments, and student work samples along with a focus group (see Appendix E) to triangulate (Mertler, 2014) the quantitative data. The teacher-researcher transcribed the student-participants' responses to a survey and highlighted data that fit into specific themes. After studying the results, the teacher-researcher determined that three themes emerged from the data. The themes were: 1) developing conceptual understanding, 2) fostering interpersonal dynamics, and 3) improving model efficacy. The teacher-researcher reflected alone on the data as well as with her student-participants. After the data were collected and analyzed, the teacher-researcher collaborated with her student-participants

to create an action plan that would serve to improve instruction for her future AP Calculus students.

There were five components of the action plan that the teacher-researcher created. The first part of the action plan will take place in July 2017 after the teacher-researcher receives her student-participants' AP Calculus Exam scores. She will compare the score of each individual student-participant to his or her pre- and posttest scores to determine whether the assessments provided similar outcomes. In August 2017, the teacher-researcher will begin the second component of the action plan by meeting with the district mathematics coordinator and other AP Calculus teachers in her school and district to share her action research study and to form an email group to share ideas and resources for additional hands-on activities. The third component of the action plan will occur in the fall of 2017 when the teacher-researcher leads a professional development session at CHS to share her action research study. As a result of this professional development, the teacher-researcher will form a PLC with other CHS teachers who want to develop and use hands-on activities in their classrooms. In order to increase enrollment in her AP Calculus class, the teacher-researcher will begin the fourth part of her action plan in the fall of 2017. She will begin an outreach program with Algebra 2 Honors classes to introduce these students to AP Calculus, review the course sequencing that is required to be able to enroll in AP Calculus, and encourage these students to take AP Calculus when they are high school seniors. In the fall of 2017 the teacher-researcher will begin a new cycle of the CSM as the fifth component of the action plan.

The teacher-researcher has learned a great deal about herself and her students as a result of this action research study. The teacher-researcher realized the importance of

providing her student-participants with the opportunity to construct their own understanding of calculus concepts. As a result of the CSM, the student-participants developed conceptual understanding of calculus. After studying the benefits that students received from taking AP Calculus in high school, the teacher-researcher recognized the importance of increasing the enrollment of female, Black, Hispanic, and lower socioeconomic status students in her class. The teacher-researcher would like to see a continued and positive impact on the community of learners in her classroom, at CHS, and in other schools in the district as a result of this action research study.

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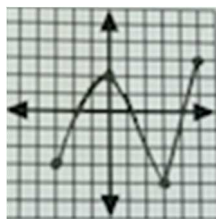
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**APPENDIX A: PRETEST/POSTTEST DATA COLLECTION INSTRUMENT  
REPLICA**

Name \_\_\_\_\_

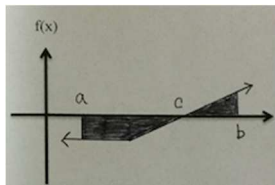
**AP Calculus BC – Pretest/Posttest Replica (No Calculator)**

- 1) Find:  $\int_1^{27} -\frac{1}{3}x^{-\frac{2}{3}} dx$   
(A) -6            (B) -2            (C) 1            (D) 2            (E) 6
- 2) Given  $f(x) = -2e^{-x} - \cos x$ , find  $f''(0) =$   
(A) -3            (B) -2            (C) -1            (D) 1            (E) 3
- 3) Find:  $[\ln(\sin 5x)]'$   
(A)  $5 \cos 5x$    (B)  $5 \cos 5x \sin 5x$    (C)  $5 \cot 5x$    (D)  $5 \sin 5x$    (E)  $5 \tan 5x$
- 4) Calculate the area of the region bounded by curves  $y = x - 2$  and  $y = \sqrt{x - 2}$   
(A)  $\frac{1}{6}$             (B)  $\frac{1}{5}$             (C)  $\frac{1}{4}$             (D) 1            (E) 0
- 5) The x-coordinate of the inflection point of  $y = 2x^3 + 3x^2 - 72x + 95$  is  
(A) -4            (B)  $-\frac{1}{2}$             (C) 0            (D) 2            (E) 3
- 6) The graph of  $f'$  is shown below. Which of the following statements is true for the coordinate  $x = 3$ ?



- (A)  $f$  has a relative minimum            (B)  $f$  has an inflection point  
(C)  $f$  is discontinuous            (D)  $f$  is not differentiable but  $f$  is continuous  
(E) none of the statements are true

- 7) The graph of continuous function  $f$  is shown in the diagram below. Which of the expressions represents the area of the shaded region?



- (A)  $\int_a^c f(x)dx - \int_c^b f(x)dx$                       (B)  $\int_a^c f(x)dx + \int_c^b f(x)dx$
- (C)  $\int_c^b f(x)dx - \int_a^c f(x)dx$     (D)  $\int_a^b f(x)dx - \int_c^b f(x)dx$     (E)  $\int_a^b f(x)dx$
- 8) Find the average value of  $g(x) = \sqrt[4]{2x}$  on the interval  $[0, 8]$
- (A)  $-\frac{64}{5}$       (B)  $-\frac{8}{5}$       (C)  $\frac{4}{5}$       (D)  $\frac{8}{5}$       (E)  $\frac{64}{5}$
- 9) Use the Midpoint Rule to approximate  $\int_1^9 3x \, dx$  with 4 subintervals
- (A) 40            (B) 96            (C) 100            (D) 120            (E) 144
- 10) The velocity of a particle moving in a straight line is  $v(t) = \sin t$  in feet per second. Find the total distance the particle travels between time  $t = \frac{\pi}{2}$  and  $t = \frac{3\pi}{2}$  seconds.
- (A) -2 ft      (B) 0 ft      (C)  $\frac{1}{2}$  ft      (D) 1ft      (E) 2 ft

- 11) Find the volume of the solid formed when the region bounded by graphs  $y = 3x^2$  and  $y = 3$  is rotated about the x-axis.
- (A)  $\frac{18\pi}{5}$       (B)  $\frac{72\pi}{5}$       (C)  $\frac{96\pi}{5}$       (D)  $\frac{108\pi}{5}$       (E)  $\frac{144\pi}{5}$



12) If  $g(x) = \int_{x^2}^1 \ln(7+t) dt$ , then  $g'(x) =$

- (A)  $-2x \ln(7+x^2)$       (B)  $-\ln(7+x^2)$       (C)  $\frac{1}{7+t}$   
(D)  $2x \ln(7+x^2)$       (E)  $\ln(7+x^2)$

13) Find:  $\int_0^1 12xe^{3x^2} dx$

- (A)  $2(e-1)$       (B)  $2(e^2-1)$       (C)  $2(e^3-1)$   
(D)  $12(e^3-1)$       (E)  $12e^3$

14) The edges of a cube are increasing at a rate of 3 inches per second. How fast is the surface area of the cube changing when the area of the cube is  $16 \text{ in}^2$ ?

- (A) 36      (B) 48      (C) 72      (D) 96      (E) 144

15) Find:  $\frac{d}{dx}(3^{5x})$

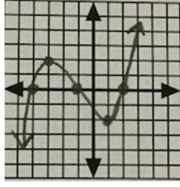
- (A)  $3^{5x}$       (B)  $15^x$       (C)  $(\ln 3)3^{5x}$       (D)  $5(\ln 3)3^{5x}$       (E)  $5(\ln 5)3^{5x}$

16) A conservationist releases 3,500 trout into a lake. The population  $P(t)$  of trout is modeled by the logistical differential equation  $\frac{dP}{dt} = P\left(3 - \frac{P}{1000}\right)$ . What is

$\lim_{x \rightarrow \infty} P(t)$ ?

- (A) 333      (B) 1,000      (C) 3,000      (D) 3,500      (E) 7,000

17) The graph  $f'(x)$  is shown below. Find the x-coordinate of the relative maximum.



- (A) -4      (B) -3      (C) -1      (D) 1      (E) 2

18) Find the volume of the solid with a base bounded by the graphs  $y = x^2 + 2$  and  $y = 3$  using square cross-sections perpendicular to the x-axis.

- (A) 1      (B)  $\frac{4}{3}$       (C)  $\frac{12}{15}$       (D)  $\frac{16}{15}$       (E) 9

19) Find the sum of the given infinite geometric series:  $2 - \frac{8}{5} + \frac{32}{25} - \frac{128}{125} + \frac{512}{625} - \dots$

- (A)  $\frac{4}{5}$       (B)  $\frac{10}{9}$       (C) 2      (D) 4      (E) 10

20) Which expression represents the arc length of the curve  $y = \frac{2}{5}x^{5/2}$  from  $x = 1$  to  $x = 4$ ?

- (A)  $\int_1^4 \sqrt{1+x^3} dx$       (B)  $\int_1^4 \sqrt{1-x^3} dx$       (C)  $\int_1^4 \sqrt{1+\frac{2}{5}x^{5/2}} dx$   
 (D)  $\int_1^4 \sqrt{1+\frac{4}{25}x^5} dx$       (E)  $\int_1^4 \sqrt{1-\frac{4}{25}x^5} dx$

## APPENDIX B: FIELD NOTES

Field Notes Page

Date:

Lesson Learning Target(s):

Overview of the Hands-on Activity:

Duration of the Activity:

Students Working: individually      small groups of \_\_\_ students      whole group

Key Connections Made By Students During the Activity:

Difficulties or Confusion Faced By Students During the Activity:

Level of Student Engagement During the Activity:

Other Notes or Reflections:

### APPENDIX C: HANDS-ON ACTIVITY SURVEY

Please complete the following survey giving your honest opinion. Your responses will not affect your grade or class standing in any way.

	Strongly Agree	Agree	Neither Agree Nor Disagree	Disagree	Strongly Disagree
The activities increased my understanding.					
The activities were enjoyable.					
I believe that I will perform better on the test as a result of the activities.					
The activities were a good use of class time.					

1) How did these activities change your understanding of the topics?

---

---

2) Why or why not were these activities enjoyable?

---

---

3) How do you believe that these activities will change your results on the test?

---

---

4) Why or why not were these activities a good use of class time?

---

---

5) Would these activities be more effective at a different time?

---

---

6) In what ways, if any, have these hands-on activities increased your understanding of the course content? \_\_\_\_\_

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7) In what ways, if any, have you found the hands-on activities to not be beneficial?

---

---

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8) What changes could make to the hands-on activities more meaningful?

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**APPENDIX D: STUDENTS' PAST EXPERIENCES WITH HANDS-ON  
ACTIVITIES IN MATH**

The purpose of this questionnaire is to understand your past experiences with hands-on activities in your previous math classes. There are no correct or incorrect answers. Please do not include your name on this form.

Male \_\_\_\_\_ Female \_\_\_\_\_

1) I learn math best when:

2) How frequently have you participated in hands-on activities in math class?

Never            Infrequently            Occasionally            Often

3) I find hands-on activities helpful for understanding class material.

Strongly Disagree    Disagree    No Opinion    Agree    Strongly Agree

Why?

4) In what ways have hands-on activities increased your understanding of course content?

5) In what ways have you found hands-on activities to not be beneficial?

## **APPENDIX E: FOCUS GROUP QUESTIONS**

- 1) In what ways did the classroom instruction change during the implementation of the Constructed Scaffold Model (CSM)?
- 2) Explain how these changes impacted your understanding of calculus?
- 3) Explain how these changes impacted your feelings about the class?
- 4) In what ways did the CSM prepare you for the AP Calculus Exam?
- 5) How could the CSM be improved to better prepare you for the AP Calculus Exam?
- 6) In what ways did the CSM help you build connections within the calculus content?
- 7) What did you like most about the CSM?
- 8) What did you like least about the CSM?
- 9) If you could change anything about the CSM, what would it be?

## APPENDIX F: CSM ACTIVITIES

Table F.1

### *CSM Activities*

CSM Activity	Hands-on Activity Description	Class Periods Used
1	With a partner, students modeled two graphs and used past knowledge of calculating area in geometry to calculate the area between the two graphs.	1
2	Students and their partners used a tissue paper sphere to model the solid that was formed when a region was revolved about an axis and calculated the volume of the solid that was formed.	2
3	Working in small groups, students cut out cardboard geometric shapes and attached them to a graph to model the solid that formed from the cross-sections and calculated the volume.	1
4	Students were given a curve and required to use their knowledge of the distance formula in geometry to calculate the length of the curve and connect that knowledge to the formula for arc length. Completed as a whole class.	1
5	With a partner, students used pulleys made from wooden craft sticks, yarn, and cardboard to model the process of lifting a chain and weight and calculated the work required to lift the chain and weight.	1
6	Students played a quiz-type game that was created on the computer to review for a test. Completed as a whole class.	0.5
7	Working in small groups, students were given index cards with the graph and equation of slope fields and were required to match the correct graph and equation.	1
8	Students worked with a partner to use separation of variables to derive the equation for exponential growth and decay that they used in Algebra 2 and Precalculus.	1
9	In small groups, students created a graph to model the logistic growth differential equation and adjusted the model for various initial population sizes to learn the concept of carrying capacity.	1
10	Students played a quiz-type game that was created on the computer to review for a test. Completed as a whole class.	0.5



11	Working in small groups, students placed dot stickers on a graph to model bounded and monotonic sequences.	1
12	Students worked individually to create a book that contained all of the tests for series convergence or divergence.	5
13	Working in small groups, students found $n$ th-degree Taylor polynomial approximations and then compared the graphs of Taylor polynomial approximations to the original function to see what it meant to be centered at a value.	1
14	Students played a quiz-type game that was created on the computer to review for a test. Completed as a whole class.	0.5

## APPENDIX G: STUDENT ASSENT FORM

July 10, 2016

Dear Student:

I am a doctoral student at the University of South Carolina, which requires me to complete an action research project. I will compile the completed project as a written dissertation, which will be the culminating assignment for my degree. The results of this action research study could be presented at a professional conference and/or published in a professional journal. My action research project will study the impact of hands-on activities in an AP Calculus BC class.

During the school year, I will use activities, assignments, questionnaires, assessments, demographic studies and surveys to gather data or measure achievement related to this topic. All of the sample project materials will be available for your review upon request. If you do not want to participate in this action research study, there will be no penalty and your grade will not be adversely affected. Your participation in this action research study is voluntary and you have the right to change your mind and stop participating in this study at any time. Your name or image will not appear in any of the material that is presented or published. All data will be stored securely during the study and destroyed upon completion.

I would appreciate your participation in this research. If you have any questions, please feel free to contact me.

Thank you,

Susan S Scott

\_\_\_\_\_ **YES.** I agree to participate in this action research study. I understand that this study will be completed during class time and that even if I agree to participate, I can change my mind later.

\_\_\_\_\_ **NO.** I do not want to participate in the study.

Student Name: \_\_\_\_\_

Student Signature: \_\_\_\_\_ Date: \_\_\_\_\_

## **APPENDIX H: PARENT CONSENT FORM**

July 10, 2016

Dear Parent or Guardian:

I am your student's teacher, Susan S Scott. I am a doctoral student at the University of South Carolina. My program of study requires me to complete an action research project. I will compile the completed project as a written dissertation, which will be the culminating assignment for my degree. The results of this action research study could be presented at a professional conference and/or published in a professional journal.

My action research project will study the impact of hands-on activities in an AP Calculus BC class.

During the school year, I will use activities, assignments, questionnaires, assessments, demographic studies and surveys to gather data or measure achievement related to this topic. All of the sample project materials will be available for your review upon request.

The purpose of this letter is to ask for your permission to include data gathered from your child in my proposed action research project.

Your child will not be named or shown in any material presented or published, and all information will be kept absolutely confidential and anonymous. All data will be stored securely during the study and destroyed upon completion.

I would appreciate your child's participation in this research. If you have any questions, please feel free to contact me.

Please return the attached permission form with your signature by September 20, 2016.

Thank you for your help.

Sincerely,

Mrs. Susan S Scott  
Mathematics Teacher

Mrs. Susan S Scott:

I understand that you are enrolled in a program that requires an action research project that will be compiled into a dissertation and which could be presented at a professional conference and/or published in a professional journal.

I understand that you are asking for my permission to include my child's data in your research and that no child will be named or shown in any resulting presentation or publication.

Choose one:

\_\_\_\_\_ I **GIVE** my permission for my child, \_\_\_\_\_,  
to participate in your research during the 2016–2017 school year.

\_\_\_\_\_ I **DO NOT GIVE** my permission for my child, \_\_\_\_\_,  
to participate in your research during the 2016–2017 school year.

Parent or Guardian's Signature: \_\_\_\_\_

Date: \_\_\_\_\_

## APPENDIX I: DETAILED ACTION PLAN

Table I.1

*Detailed Action Plan*

<b>Target</b>	<b>Action</b>	<b>People</b>	<b>Timeframe</b>
Reflect on the Problem of Practice	<ul style="list-style-type: none"> <li>• Review AP Exam Scores</li> </ul>	<ul style="list-style-type: none"> <li>• Teacher-Researcher</li> </ul>	July 5, 2017 to July 19, 2017
Increase Student-Centered Instruction in AP Calculus	<ul style="list-style-type: none"> <li>• Share Research Findings</li> <li>• Determine the Interest in Sharing Resources and Ideas for Hands-On Activities</li> <li>• Form an Email Group to Share Ideas</li> </ul>	<ul style="list-style-type: none"> <li>• Teacher-Researcher</li> <li>• District Mathematics Coordinator</li> <li>• Other AP Calculus Teachers</li> </ul>	August 2017 to May 2018
Lead Professional Development	<ul style="list-style-type: none"> <li>• Share Research Findings</li> </ul>	<ul style="list-style-type: none"> <li>• Teacher-Researcher</li> <li>• CHS Principal</li> <li>• CHS Instructional Resource Coordinator</li> <li>• CHS Teachers</li> </ul>	September 2017
Form Professional Learning Community	<ul style="list-style-type: none"> <li>• Share Ideas for Hands-On Activities</li> <li>• Collaborate With Teachers of Other Academic Disciplines to Share Teaching Ideas and Resources</li> </ul>	<ul style="list-style-type: none"> <li>• Teacher-Researcher</li> <li>• CHS Teachers</li> </ul>	October 2017 to May 2018 During Collaborative Planning Time

<p>Increase Enrollment in AP Calculus – Especially Among Female, Black, Hispanic, and Lower Socioeconomic Status Students</p>	<ul style="list-style-type: none"> <li>• Meet With Algebra 2 Honors Teachers</li> <li>• Visit Algebra 2 Honors Classes</li> </ul>	<ul style="list-style-type: none"> <li>• Teacher-Researcher</li> <li>• Math Department Head</li> <li>• Algebra 2 Honors Teachers</li> <li>• Algebra 2 Honors Students</li> </ul>	<p>October 2017 and March 2018</p>
<p>Future CSM Cycle</p>	<ul style="list-style-type: none"> <li>• Begin New Cycle</li> </ul>	<ul style="list-style-type: none"> <li>• Teacher-Researcher</li> <li>• Student-Participants</li> </ul>	<p>September 2017 to November 2017</p>