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Formative Assessment Strategies for Mathematical Thinking: A Qualitative Action Research Study

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ABSTRACT

Developing Formative Assessment Strategies for Mathematical Thinking is a qualitative action research study that investigates how one mathematics teacher implements formative assessment strategies aimed at impacting students' mathematical thinking in two geometry classes a southern Title I public high school. This study is predicated on the notion that when students share their mathematical thinking in class, it stimulates classroom discussion and discourse and provides evidence vis-à-vis formative assessments that the teacher can use to improve mathematics curriculum and instruction. The study is theoretically grounded in Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies that include: 1. Clarifying and Sharing; 2. Engineering Effective Classroom Discussions; 3. Providing Feedback; 4. Activating Students as Instructional Resources; and 5. Activating Students as Owners. These assessment strategies were implemented and qualitative data on student mathematical thinking was collected in the form of the written reflections of student-participants. Other data included samples of student-participants' work and a teacher journal. Data was examined using the constant comparative analysis method in the spirit of a qualitative educational action research paradigm in order to provide a rich, thick description of the different ways that formative assessment strategies impacted these high school students' mathematical thinking. The findings of the study include how the teacher-researcher's implementation of the Five Key Formative Assessment Strategies

(Wiliam & Thompson, 2007) impacted student-participant mathematical thinking; student-participants: 1) used mathematical thinking to think about and learn mathematics while problem solving, 2) engaged in metacognitive processes while solving problems, 3) discussed and engaged in mathematical thinking while working and learning collaboratively, 4) described the importance of presence of mind, motivation and emotion to mathematical thinking, and 5) described their beliefs about actions they could take to improve their mathematical thinking. The findings were reflected upon and shared with student-participants and other teachers in the teacher-researcher's school district. The Action Plan that was derived from the present study will be put into action in the teacher-researcher's classroom and findings will be disseminated at regional and national mathematics professional conferences.

Keywords: formative assessment, action research, constant comparative method, field notes, student work, mathematical thinking

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Introduction

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Problem solving and mathematical reasoning is currently a focus of mathematics education in the United States (NCTM, 2000, 2009, 2014; CCSSM, 2009). This is evidenced by two of the Common Core State Standards for Mathematics' (CCSSM, 2009) eight Standards of Mathematical Practice: 1) "make sense of problems and persevere in solving them" and 2) "construct viable arguments and critique the reasoning of others" (CCSSM, 2009, p. 6). At the same time that these standards were released, the National Council of Teachers of Mathematics called for a focus on reasoning and sense making in secondary mathematics classrooms ((NCTM, 2009). In *Principles to Actions* (NCTM, 2014), the National Council of Teacher of Mathematics:

> identified a number of principles of learning that provide the foundation for effective mathematics teaching. Specifically, learners [of mathematics] should have experiences that enable them to—
>
> - engage with challenging tasks that involve active meaning making and support meaningful learning;
> - connect new learning with prior knowledge and informal reasoning and, in the process, address preconceptions and misconceptions;
> - acquire conceptual knowledge as well as procedural knowledge, so that

they can meaningfully organize their knowledge, acquire new knowledge, and transfer and apply knowledge to new situations;

- construct knowledge socially, through discourse, activity, and interaction related to meaningful problems;
- receive descriptive and timely feedback so that they can reflect on and revise their work, thinking, and understandings; and
- develop metacognitive awareness of themselves as learners, thinkers, and problem solvers, and learn to monitor their learning and performance. (p. 8)

These principles of learning are at the heart of creating a classroom environment where students can better learn to do mathematics and to explain their mathematical thinking. The National Council of Teachers of Mathematics links the aforementioned principles of learning to eight effective mathematical teaching practices:

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Facilitate meaningful mathematical discourse.
- Pose purposeful questions.
- Build procedural fluency from conceptual understanding.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking. (NCTM, 2014)

The National Council of Teachers of Mathematics (2000, 2009, 2014) has identified and made clear the types of experiences students should have as learners and the types of teaching practices that should be taking place in effective mathematics classrooms. The Council points out that effective implementation is "possible only when school mathematics programs have in place a commitment to access and equity" (NCTM, 2014, p. 59) and that "all students have access to a high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential" (p. 59). The question that arises from this is: What can the teacher do to help *all* students develop and express how they think about mathematics? As a teacher leader, the teacher-researcher is "determined to become the architect of vibrant professional communities in which teachers take the lead in inventing new possibilities for their students and for themselves" (Lieberman & Miller, 2004/2013, p. 422). The teacher-researcher shares NCTM's vision of access and equity that "includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement" (NCTM, 2014, p. 60).

The Problem of Practice

The problem of practice in this dissertation is that high school geometry students routinely do not show evidence of mathematical thinking while working on or after working on mathematical tasks. The mathematical pedagogical strategies that the students in this study were exposí to before they entered high school geometry were more focused on memorization and the application of mathematical procedures rather than on how to make sense of and reason with mathematics – to think mathematically. Students were rarely asked to explain their mathematical thinking on assignments or assessments and often had limited opportunities to learn mathematics via social interactions with other students. In short, students were not enabled to explicitly show and develop their mathematical thinking in prior mathematics classes. In the years leading up to this study, the teacher-researcher had struggled to get his students to show evidence of mathematical thinking and to engage in thinking mathematically. The teacher-researcher aimed to bring his mathematics classroom into the 21st century where students would not only learn how to do mathematics but also how to learn mathematics and to think mathematically. To accomplish this, the teacher-researcher sought to improve his own practice and elected to use Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies during instruction to enable students to develop and show evidence of mathematical thinking.

Consider the following three hypothetical vignettes representing everyday interactions from secondary mathematics classrooms:

Vignette One. Mr. Salois asks his geometry students to estimate the surface area of the walls in his classroom. He tells them that they need a mathematical explanation that goes with their estimate. Within a few seconds, some groups are out of their seats and recording the number of square tiles on the floor. Another group is looking up at the ceiling tiles and talking about how big they are. The group sitting by the window is silent and still. Mr. Salois walks over and asks the group by the window if they understand the problem. They nod that they do and remain still. He offers them a yardstick to help them measure the room. They take it, and the group walks to the corner with a notebook and holds the yardstick to the wall. A few more minutes pass while the teacher checks in with the other groups. The teacher tells all of the students to go back to their seats and discuss how they estimated the area and to provide a written mathematical explanation of their method. The room fills immediately with the voices of students talking about the task. Sandra, one of the students in the group by the window is writing in her notebook. The rest of the students in her group shake their heads in agreement and Sandra puts her pencil down. Sandra's paper says, "We used a yardstick and got 800." The actual surface area of the walls was approximately 825 square feet.

Vignette Two. Mrs. Howorth asked her students to write a rule that describes each of the following patterns:

- Pattern 1: 3, 5, 7, 9, …
- Pattern 2: 6, 12, 18, 24, …
- Pattern 3: 30, 60, 90, 120, …

Students were asked to work quietly through the problem for three minutes and then asked to share their thinking with their partner. The teacher observed the work students had on their papers and listened to the students talk about mathematics. It appeared to her that everyone had solved the problem. During the whole class discussion, students shared their rules and provided sound mathematical explanations. To finish the task, Mrs. Howorth asked the class if anybody had any other rules. Jennifer raised her hand and said that "multiplying by two works for all of them." After much debate between Jennifer, the teacher and other students about her rule, she came to the board and pointed at the nine in Pattern 1. Jennifer said: "to get the nine, multiply the seven by two and subtract the number that comes before it [she points to the 5]. Two times seven minus five equals nine. This works for all of them." She was correct.

Vignette 3. Ms. Miller brings in a box of ice cream cones during the final week of school and asks her students to work in pairs to find the volume of one of the ice cream cones. As she observed students measuring the cones, she saw that Dalton, whose partner was absent from class, was measuring the thickness of the cone to calculate the interior radius and the interior height of the cone. During the class discussion, the teacher found that the rest of the students in the class had used two methods: 1) they estimated the interior height and radius of the ice cone to find out how much ice cream would fit in the cone or 2) they found height and radius of the ice cream cone and found its volume as if it were a geometric solid. She knew that Dalton had found the volume of the ice cream cone (the baked part) itself. She asked him to explain how he found the volume of the ice cream cone. Dalton explained, "I found the thickness of the ice cream cone and subtracted." Ms. Miller pressed him for more explanation, but he offered no more. She finished the class by showing, on the board, how Dalton had solved the problem on his paper.

These vignettes illustrate everyday interactions in typical high school mathematics classrooms, and each scenario raises questions about student engagement in mathematical explanations. In Vignette One, the group by the window had trouble getting started (making a plan) with solving the problem. They were "not inclined to communicate their thinking" (Szetela & Nicol, 1992, p. 43). However, the group by the window appears to have arrived at a correct answer and then wrote a sentence about their answer that lacked mathematical explanation. What could the teacher do next to get this group to explain their reasoning?

In Vignette Two, the teacher seems satisfied with her lesson, perhaps because her mathematical goals had been met. Though the teacher might have been ready to move on, Jennifer's explanation of her method for solving the problem caused additional discussion about her idea. Jennifer's explanation appears incorrect at first, but after some discussion, Jennifer describes a rule that works for all three patterns. What was the impact of the teacher asking students for 'other rules' on their mathematical explanations? If the teacher had dismissed Jennifer's method by providing a counterexample (*i.e.* in Pattern 1, three times two does not equal five) and merely re-stated the previous rules, would the class ever have gotten to hear Jennifer's powerful mathematical explanation?

Dalton (Vignette Three*)* was the only student in the class to find the volume of the ice cream cone itself. He subtracted the inner volume (air) from the total volume (air and the cone itself). His method combined the two methods shared by other students in the class. The teacher shared Dalton's method for solving the problem with the class. How does the teacher know if the students understand Dalton's method? Do the students know that Dalton's solution was the only correct method that was presented to the class? What moves could the teacher have made to move the thinking in the class forward on this problem?

The teachers portrayed in each vignette promoted reasoning through mathematical explanations by observing the written work of students, monitoring task completion, and facilitating classroom discourse. Mathematics teachers require great skill in order to be able to make sense of student thinking and to make *in vivo* adjustments to their curriculum and pedagogy to optimize student learning. We know that a teacher's "attention to minute-by-minute and day-by-day formative assessment is likely to have the biggest impact on student outcomes" (Wiliam, 2011, p. 27). Formative assessment in the form of observations of student interactions in class dialogue requires a certain teacher skill set. Postmodern curriculum studies value qualitative data and the promotion of class dialogue and discourse. Furthermore, class dialogue and discourse are part of the foundation for creating an equitable mathematics classroom (Horn, 2012; Moschkovich, 2013; NCTM, 2014). Enabling mathematics teachers to engage in a pedagogical approach whereby students feel free to articulate the issues they are having in the mathematics classroom is challenging and a worthwhile endeavor. Sfard's (2007; 2008a) 'commognitive' framework allows us to make the connections between all of the teaching and learning interactions that are happening in the classroom because they are discursive. Her theory accounts for the thoughts of both the students and the teacher and includes them as part of the classroom discourse. This is particularly important to formative assessment since it is in those "moments of contingency" (Black & Wiliam, 2009, p. 10) in the classroom during which the teacher must decide what choices to make to foster learning. William and Thompson (2007) have identified the following Five Key Formative Assessment Strategies that the teacher can use to implement formative assessment in their teaching:

1. *Clarifying and Sharing* learning intentions and criteria for success;
2. *Engineering Effective Classroom Discussions* and other learning tasks that elicit evidence of student understanding;

3. *Providing Feedback* that moves learners forward;
4. *Activating Students as Instructional Resources* for one another; and
5. *Activating Students as Owners* of their own learning. (Wiliam & Thompson, 2007, p. 53)

The use of any of Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies during instruction creates an environment in which students are given opportunities to modify or extend their discourse (learning) and to "make sense of the world around them" (Sfard, 2007, p. 575). The teacher-researcher in this action research study adopted these Five Key Formative Assessment Strategies into his teaching practices in an effort to get students to show evidence of mathematical thinking while working on or after working on mathematical tasks.

Research Question

1. How does the use of Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies impact students' mathematical thinking?

Purpose Statement

The purpose of this study is to investigate how one mathematics teacher's implementation of the Five Key Formative Assessment Strategies (William & Thompson, 2007) impacted student-participants' mathematical thinking in his two high school geometry classes. The teacher-researcher used the Five Key Formative Assessment Strategies while students worked on mathematical tasks. The teacher-researcher used these strategies to enable the classroom to be more constructivist and progressivist in nature where students were encouraged to engage in discussion and discourse about mathematics and mathematical thinking throughout the eight week data collection process. The teacher-researcher used these strategies to "elicit evidence of students' current mathematical understanding" (NCTM, 2014, p. 53) and to develop the mathematical thinking of the student-participants.

This qualitative action research study took place in two geometry classrooms where students were free to articulate their perceptions and feelings about mathematics content and instruction. This was possible because the teacher-researcher took a progressivist approach to his instruction. This enabled the teacher-researcher to use this feedback as a formative assessment strategy in order to alter pedagogical practices to meet students "where they are." Meeting students "where they are" is more than just a cliché, it is a "key characteristic of equitable classrooms" (Horn, 2012, p. 15) and "fits in the broader landscape of *equitable mathematics teaching*: modes of instruction that optimally support meaningful learning for all students" (p. 9). Data was collected in the form of a teacher journal, student-participant reflections and student-participant work and was analyzed using the constant-comparative method. This DiP describes what happened to the mathematical thinking of the students in this action research study.

The teacher-researcher employed action research methodology in his classroom in order to implement Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies. The use of these Five Key Formative Assessment Strategies in mathematics education is an area of active research (Swan & Burkhardt, 2014; Evans & Swan, 2014). Evans and Swan (2014) report that there are "early indications" (p. 27) that formative assessment strategies can help improve student mathematical thinking and indicate that this is an area for further study. The teacher-researcher used Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies while the students worked on mathematical tasks to further his pedagogical goal of assessing student progress in “live” time (Black *et al.*, 2004). These formative strategies are aligned to “equitable mathematics teaching” (Horn, 2012, p. 9) and will be carried out during “moments of contingency” (Black & Wiliam, 2009, p. 10) which are those times during the instructional process where the teacher makes a decision based on what is happening in the class with the intention of improving learning (2009). In the present action research study, one of the instructional goals, in addition to any geometry-specific goals (*i.e.* applying the Pythagorean Theorem), was to “elicit evidence of students’ current mathematical understanding” (NCTM, 2014, p. 53) in the form of mathematical explanations. The teacher-researcher plans to share his findings with the student-participants and with other teachers in his school, his district and with other teachers at the National Council of Teachers of Mathematics’ professional development institutes and conferences. This will provide “even greater opportunities for professional dialogue, reflection and brainstorming” (Mertler, 2014, p. 249) and help the teacher-researcher to lead others by making public his “understandings about students, strategies for learning, and the organization of the curriculum” (Lieberman & Miller, 2004/2013, p. 421).

Rationale

The importance of formative assessment on student learning has been well documented (Black & Wiliam, 1998, 1999, 2009; Swan & Burkhardt, 2014; Wiliam, 2011; Wiliam & Thompson, 2007). Black and Wiliam (2009) define formative assessment as:

> Practice in a classroom is formative to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers,

> to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited. (p. 9)

Black and Wiliam's (2009) work towards a theory of formative assessment includes a focus on "issues of psychology, curriculum and pedagogy [because] such a focus allows the teacher to engage with these issues in a way that is directly and immediately relevant to their practice" (p. 28). Their theory attempts to "bring into relationship the three spheres: the teacher's agenda, the internal world of each student, and the inter-subjective" (Black & William, 2009, p. 26). This metaphor, in combination with the vignettes described earlier, help us to think about the complex interactions that occurred in the classroom where this study took place. Black and Wiliam's work toward developing a theory of formative assessment provides a theoretical framework that ties together theoretical work on curriculum, pedagogy, social learning theory, cognition and classroom discourse that provides the justification for the use of the Five Key Formative Assessment Strategies in this DiP. What follows in the next section are the specifics about curriculum, pedagogy, social learning theory, cognition and classroom discourse as they relate to the Five Key Formative Assessment Strategies and student mathematical explanations.

Pedagogy and mathematics. The National Council of Teachers of Mathematics (2014) writes the following about teaching mathematics:

> The teaching of mathematics is complex. It requires teachers to have a deep understanding of the content that they are expected to teach and a clear view of how student learning of that mathematics develops and progresses across grades.

> It also calls for teachers to be skilled at using instructional practices that are effective in engaging students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically. (p. 7)

This description helps to further justify the use of the Five Key Formative Assessment Strategies because the strategies align with what it means to be an effective mathematics teacher (NCTM, 2014). This alignment will be further described in the review of the related literature in Chapter 2. The importance of student mathematical explanations is evidenced in the above description of mathematics teaching when it refers to students being able "to make sense of mathematical ideas and reason mathematically" (NCTM, 2014, p. 7).

Civil rights leader Robert Moses states that "the absence of math literacy in urban and rural communities throughout this country is an issue as urgent as the lack of black voters was in Mississippi in 1961" (Moses & Cobb, 2001, p. 5). Effective mathematics teaching requires a "commitment to access and equity" (NCTM, 2014, p. 59) and this commitment can be translated into practice using Horn's (2012) "four principles for equitable mathematics teaching" (p. 13). These principles are:

- Principle 1: Learning is not the same as achievement.
- Principle 2: Achievement gaps often represent gaps in opportunities to learn.
- Principle 3: All students can be pushed to learn mathematics more deeply.
- Principle 4: Students need to see themselves in mathematics. (Horn, 2012, p. 19)

A description of how these principles relate to formative assessment and a review of the related literature are provided in Chapter 2.

Mathematical explanations in the curriculum. Student mathematical explanations in the classroom are usually explanations provided by students in response to thinking about solving, solving or attempting to solve mathematical tasks or problems. The importance of problem solving, what it looks like and ways to measure it are well researched topics in mathematics education (Charles, Lester & O'Daffer, 1987; Polya, 1945; Schoenfeld, 1985; Schoenfeld, 1992). The Common Core State Standards for Mathematics describe Standards for Mathematical Practice (CCSSM, 2009, p. 6) that stress the importance of problem solving and thinking mathematically. The National Council of Teachers of Mathematics (2014), describes the importance of the effective mathematical teaching practice labeled "facilitate meaningful mathematical discourse" (p. 10) using a quotation from Carpenter, Franke, and Levi (2003):

> Students who learn to articulate and justify their own mathematical ideas, reason through their own and others' mathematical explanations, and provide a rationale for their answers develop a deep understanding that is critical to their future success in mathematics and related fields. (p. 3)

The direct connection of student mathematical explanations to the Key Formative Assessment Strategy of "engineering effective classroom discussions and other learning tasks that elicit student understanding" (Wiliam & Thompson, 2007, p. 53) provides further justification for this action research study's focus on formative assessment strategies and student mathematical explanations.

Social learning theory, cognition and classroom discourse in mathematics. The interplay between feedback and learning as a social process is well documented in Vygotsky's theory of social constructivism (1978). Effective feedback can help to engage students in productive mathematical struggle and to become more successful at solving problems (Kilpatrick, Swafford, & Findell, 2001; Carpenter *et al.*, 2003; Sfard, 2007). Sfard (2007) tells us that:

> Without other people's examples, children may have no incentive for changing their discursive ways. From the children's point of view, the discourse in which they are fluent does not seem to have any particular weaknesses as a tool for making sense of the world around them. (p. 577)

When students share their mathematical explanations, they are acting as "instructional resources for one another" (Wiliam & Thompson, 2007). Sfard's (2007) theory of 'commognition' helps us to further understand the connections between the Five Key Formative Assessment Strategies and mathematical explanations. The neologic 'commognition' is a blend word that combines communication and cognition (thinking). Commognition theory operates under the premise "that thinking is a form of communication and that learning mathematics is tantamount to modifying and extending one's discourse" (Sfard, 2007, p. 565). This theory helps us to understand what "activating students as the owners of their learning" (Wiliam & Thompson, 2007, p. 53) means. Owning one's learning implies that the individual has learned something and Sfard (2007) argues that this learning, albeit intrapersonal (thinking), is inherently linked to the interpersonal communication (*i.e.*, classroom discourse) that influenced it. This theory of commognition, in particular, the view of "thinking as communication" (Sfard, 2008a) helps underline the importance of "engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding" (Wiliam & Thompson, 2007). When a student expresses a mathematical explanation (verbally or non-verbally) in a classroom environment, they are making their own intrapersonal discourse (thinking) become part of the larger interpersonal discourse taking place throughout the classroom. Sfard (2007) also posits that the "learning-teaching agreement" is a "condition for learning" and that "routines of mathematizing" (p. 605) need to be agreed upon between the teacher and the learner. This provides a theoretical context for the importance of William and Thompson's (2007) strategy of "clarifying and sharing learning intentions and criteria for success." The teacher and learner must engage in a "social agreement" (Sfard, 2007, p. 605) for learning to flourish (Yackel & Cobb, 1996). Sfard's (2007) theory of commognition provides an interpretive framework with which to connect Wiliam and Black's (2009) theory of formative assessment (rationale for using the Five Key Formative Assessment Strategies) directly to mathematical learning (mathematical explanations as an extension of one's discourse (thoughts and utterances) (Sfard, 2007, p. 565)).

Glossary

Action Research: "any systematic inquiry conducted by teachers, administrators, counselors, or others with a vested interest in the teaching and learning process or environment for the purpose of gathering information about how their particular schools operate, how they teach, and how their students learn" (Mertler, 2014, p. 305).

Acquisitionism: "A research discourse grounded in the "metaphor of learning as an act of increasing individual possession - as an acquisition of entities such as concepts, knowledge, skills, mental schemas – comes to this scholarly discourse directly from everyday expressions, such as acquiring knowledge, forming concepts or constructing meaning" (Sfard, 2006, p. 153).

Commognition: "term that encompasses thinking (individual cognition) and (interpersonal) communicating; as a combination of the words communication and cognition, it stresses the fact that these two processes are different (intrapersonal and interpersonal) manifestations of the same phenomenon" (Sfard, 2008a, p. 296).

Communication: "a collectively performed rules-driven activity that mediates and coordinates other activities of the collective" (Sfard, 2007, p. 567).

Constant Comparative Method: "a research design for studies involving multiple data sources, where data analysis begins early in the study and is nearly completed by the end of data collection" (Mertler, 2014, p. 306).

Discourse: "rule-governed mulitmodal communicational activity" (Sfard & Cobb, 2014, p. 558).

Equitable Mathematics Teaching: teaching that "involves using modes of instruction that optimally support meaningful mathematical learning for all students" (Horn, 2012, p. 9). Formative assessment is "consonant with the four principles for equitable mathematics teaching" (p. 56).

Formative Assessment: *"*Practice in a classroom is formative to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited" (Black & Wiliam, 2009, p. 9).

Learning: "change of discourse" (Sfard, 2008a, p. 255; Sfard & Cobb, 2014, p. 558).

Metacognition: “the ability to monitor one’s current level of understanding and decide when it is not adequate” (Bransford *et al.*, 2000, p. 47).

Moments of Contingency: those times during the instructional process where the teacher makes a decision based what is happening in the class with the intention of improving learning (Black & Wiliam, 2009).

Participationism: “research discourse grounded in the metaphor of learning as improving participation in historically established forms of activity” (Sfard, 2008a, p. 301).

Reification: “the process of giving form to our experience by producing objects that congeal this experience into ‘thingness’” (Wenger, 2000, p. 58). It includes cases when “aspects of the human experience and practice are congealed into fixed forms and given the status of an object” (Wenger, 2000, p. 59).

Thinking: “an individualized form of (intrapersonal) communicating” (Sfard, 2008a, p. 81).

Methodology

The research tradition used in this classroom-based action research study was a qualitative research design (Mertler, 2014). The qualitative approach that was taken in this action research study was an observational study in which the teacher-researcher was a “full participant” (Mertler, 2014, p. 95). Participant observation allowed the teacher-researcher to “learn firsthand how the actions of the participants correspond with their words, to see patterns of behavior, [and] to experience that which was unexpected” (Mertler, 2014, p. 93). The data in this study was analyzed using the constant comparative method (Merriam, 2009; Mertler, 2014).

Participant Selection and Research Site

This action research study took place in two geometry classes at a southern Title I public high school of approximately 800 students. The mathematics courses taught at this high school follow a traditional sequence (Algebra I, Geometry, Algebra 2). Students attend classes that meet for 88 minutes on every other day and all students have access to personal Chromebooks provided by the school and excellent internet access during the school day. This action research study took place with one teacher, the teacher-researcher, and the students from two of his geometry classes over the course of eight weeks during the fall of the 2016-2017 school year. Improving mathematical explanations was one of the instructional goals for the classes. Consenting students from these two classes were the participants selected for this study and they represent a convenience sample. The teacher-researcher is currently teaching several different courses at his school and has taught a variety of others over the last several years. The rationale for selecting two geometry classes is that geometry represents the class that the teacher-researcher has taught the most consistently at his school. The rationale for selecting two classes is that this number represents the total number of geometry classes that the teacher-researcher teaches. The number of students in each class is 28 and 24 respectively. The teacher-researcher is in his fifteenth year of teaching, is a National Board certified teacher, holds a Master's of Education degree in mathematics education and a Bachelor of Arts degree in Biology. He is a regular speaker at mathematics education conferences, provides professional development at the local, state and national level and is a teacher leader in mathematics education. These roles and activities will provide an opportunity for the teacher-researcher to disseminate more broadly the findings of this DiP (Mertler, 2014).

Sources of Data Collection

The primary source of data was the written reflections of the student-participants to the following prompts:

1. Describe how you came up with your mathematical explanation or your mathematical understanding of today's problem. What helped you? What didn't?
2. Reflect on anything that you found important, challenging or otherwise notable about the problem solving process that you or your classmate(s) used to work through today's mathematical task.
3. What could help you get better at coming up with a mathematical explanation?

This source of data served a dual-purpose for the teacher-researcher: 1) it provided data that related directly to the research question of this study, and 2) the prompt itself and the resulting reflections provided evidence of one of the Five Key Formative Assessment Strategies in action. The second source of data was classroom artifacts in the form of written mathematical explanations that students produced as part of routine classroom activities. The third and final data source was a teacher journal containing field notes about this teacher-researcher's use of the Five Key Formative Assessment Strategies and relevant observations of student interactions. The teacher-researcher also used the teacher journal to reflect about his use of the Five Key Formative Assessment Strategies in regards to student mathematical explanations at the end of each class period, when appropriate and feasible.

Summary of the Findings

The findings of this study fall under four themes: 1) student-participant participation in problem solving, 2) social and intrapersonal factors in the classroom environment, 3) pedagogical factors in the classroom environment, and 4) future learning needs. This first theme provides evidence that student-participants: 1) made sense of problems using prior knowledge and figured out how to get started solving the problem, 2) solved problems (and thought about) solving problems in a variety of ways while using a variety of tools, 3) explained or made sense of their mathematical solution or solution path in different ways, and 4) monitored their progress during the problem solving process. The second theme found evidence that reciprocity (working together), presence of mind (attention), metacognition, motivation and affect were social and intrapersonal factors important to mathematical thinking. The third theme found evidence that instructional decisions (pedagogy) and time (timing) had an impact on mathematical thinking. Finally, theme four found that student-participants identified that the following four activities could help them to better explain their mathematical thinking in the future: 1) practicing more, 2) learning more mathematical vocabulary and terminology, 3) learning more mathematical content, and 4) studying. The following section provides an overview of this DiP.

Dissertation Overview

This section provides an overview of the remaining chapters of this DiP. Chapter Two, related literature review, serves to situate this DiP in the literature. It includes a discussion of literature related to formative assessment and the participationist perspective as part of the theoretical framework. In addition, it includes a discussion of the literature related to action research and the constant comparative method. Chapter Three, methodology, describes the qualitative action research methodology used in this dissertation. In addition, it describes the plan for data analysis, plan for reflecting with participants on data, and the plan for devising an Action Plan. Chapter Four, findings and implications, provides a detailed description of the findings of this DiP. In addition, it describes the implications of the findings as they relate to the related literature, to the teacher-researcher, and to the student-participants. Chapter Five, summary and conclusions, includes a summary of the study, the focus of the study, the major points of the study, a detailed Action Plan including sharing data with student-participants and a timeline of actions to be taken by the teacher-researcher, suggestions for future research, and a conclusion. The next chapter provides a review of the related literature.

CHAPTER TWO: RELATED LITERATURE REVIEW

Introduction

Throughout his teaching career, the teacher-researcher has observed that students find it difficult to express their mathematical thinking. To be an effective mathematics teacher, it is essential that students express their mathematical thinking and that the teacher establishes an environment where the expression of mathematical thinking is fostered and elicited (NCTM, 2014). Formative assessment requires teachers to monitor and solicit evidence of student mathematical thinking during instruction. By observing the written work of students, monitoring task completion, and facilitating classroom discourse, teachers can employ formative assessment strategies that gauge student progress and help ascertain how pedagogy needs to be altered to meet students "where they are." Problems arise when teachers are not able to use formative assessment to enable their students to talk with each other or the teacher about the issues they are having in the classroom regarding mathematics. The question that arises from this problem is: What can the teacher do to help students develop and express how they think about mathematics?

Chapter Two of this Dissertation in Practice (DiP) involves a literature review of the theoretical foundation and the historical contextualization of formative assessment strategies in United States schooling, specifically mathematics classrooms. The Chapter begins with a discussion of the importance of a literature review and is followed by a brief overview of the methodology of the action research study and the theoretical and historical foundations for the study. The Chapter culminates in a study of keywords associated with this DiP and the identified 'problem of practice' surrounding the impacts of the use of formative assessment in secondary mathematics classrooms.

The Problem of Practice

The problem of practice in this dissertation is that high school geometry students routinely do not show evidence of mathematical thinking while working on or after working on mathematical tasks. The mathematical pedagogical strategies that the students in this study were exposed to before they entered high school geometry were more focused on memorization and the application of mathematical procedures rather than on how to make sense of and reason with mathematics – to think mathematically. Students were rarely asked to explain their mathematical thinking on assignments or assessments and often had limited opportunities to learn mathematics via social interactions with other students. In short, students were not enabled to explicitly show and develop their mathematical thinking in prior mathematics classes. In the years leading up to this study, the teacher-researcher had struggled to get his students to show evidence of mathematical thinking and to engage in thinking mathematically. The teacher-researcher aimed to bring his mathematics classroom into the 21st century where students would not only learn how to do mathematics but also how to learn mathematics and to think mathematically. To accomplish this, the teacher-researcher sought to improve his own practice and elected to use Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies during instruction to enable students to develop and show evidence of mathematical thinking.

Research Question

1. How does the use of Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies impact students' mathematical thinking?

Purpose Statement

The purpose of this study is to investigate how one mathematics teacher's implementation of the Five Key Formative Assessment Strategies (William and Thompson, 2007) impacted student-participants' mathematical thinking in his two high school geometry classes. The teacher-researcher used the Five Key Formative Assessment Strategies while students worked on mathematical tasks. The teacher-researcher used these strategies to enable the classroom to be more constructivist and progressivist in nature where students were encouraged to engage in discussion and discourse about mathematics and mathematical thinking throughout the eight week data collection process. The teacher-researcher used these strategies to "elicit evidence of students' current mathematical understanding" (NCTM, 2014, p. 53) and to develop the mathematical thinking of the student-participants.

Importance of a Literature Review

The literature review is important to this DiP because it "is the foundation and inspiration for substantial, useful research" (Boote & Biele, 2005, p. 3) and it is "an essential component of research" (Maxwell, 2006, p. 31). Boote and Biele (2005) state that "a substantive, thorough, sophisticated literature review is a precondition for doing substantive, thorough, sophisticated research" (p. 3) and that "a researcher cannot perform significant research without first understanding the literature in the field" (p. 3). The literature review process is critical to "constructing a foundation on which research can be built" (Boote & Biele, 2005, p. 4) and allows the teacher-researcher to differentiate between “what has been done from what needs to be done” (Hart, 1999, p. 27). The literature review assisted the teacher-researcher in “choosing a productive dissertation topic and appropriating fruitful methods of data collection and analysis” (Boote & Biele, 2005, p. 3). It also traces the evolution of various theories that have allowed the teacher-researcher to establish the theoretical framework of this DiP. In particular, this related literature review discusses Wiliam and Thompson’s (2007) Five Key Formative Assessment Strategies which include: 1. “Clarifying and Sharing”; 2. “Engineering Effective Classroom Discussions”; 3. “Providing Feedback”; 4. “Activating Students as Instructional Resources”; and 5. “Activating Students as Owners” (p. 53).

These Five Key Formative Assessment Strategies were employed in order to supply the qualitative data on student mathematical thinking that was collected in the form of teacher field notes, sample student-participant work, and student-participant reflections. Within the spirit of action research, this related literature review allows the teacher-researcher to share and reflect upon mathematical curriculum pedagogy with his colleagues, in the scholarly literature, and in contemporary discourse on formative assessment in mathematics education.

Theoretical Base

The related literature review of this DiP describes “theoretical perspectives and previous research findings regarding the problem at hand” (Leedy & Ormrod, 2005, p. 64). It defines the theoretical framework of this study and situates it in the larger context of educational research on formative assessment, learning, and student mathematical thinking. By organizing and synthesizing the literature on formative assessment, learning, and student mathematical thinking, the teacher-researcher provides the reader with an understanding of how this action research study connects with and contributes to knowledge in the field. The theoretical base section of this dissertation conceptualizes and provides a justification for the theoretical framework of this DiP.

Methodology

The review of the literature related to action research (Herr & Anderson, 2005; Mertler, 2014) and the constant comparative method (Glaser & Strauss, 1967; Merriam, 2009) provides a justification of the qualitative methods of data collection and data analysis that were used in this action research study. The teacher-researcher uses supporting literature to describe the methods that were used in this action research study to help prepare the reader for Chapter 3 (methodology) of this DiP. In addition, the teacher-researcher provides examples of other qualitative action research studies that have used the constant comparative method of data analysis in contexts that are similar to the context of the present action research study.

Historical Context

This literature review situates this action research study in a historical context to:

1. Explain the evolution of the theories and concepts that laid the groundwork for this study;
2. Demonstrate the state-of-the-art developments in formative assessments in mathematics; and
3. Suggest avenues for further research upon completion of the present action research study.

By providing a historical context, the teacher-researcher demonstrates to the reader his understanding of previous research related to the topic of this DiP (Hart, 1999). The historical literature review serves “not only to summarize the existing literature but also to synthesize it in a way that permits a new perspective” (Boote & Biele, 2005, p. 4). More specifically, it provides historical background for the participationist theory of learning in order to establish the significance of and to provide additional framing for the theoretical base of this research.

Glossary

The glossary provides the reader with definitions of key terms that are specific to the problem of practice in this DiP. The definitions serve to inform the reader and to situate the terms in the context of this DiP. The glossary allows the reader easy access to the definitions of words or phrases that may have different or nuanced meanings for different readers. It also serves to clarify for the reader what is meant when certain terms are used. Citations of and the sources for the definitions in the glossary are also provided.

METHODOLOGY

This section will review the literature on action research and the constant comparative method to position this study in educational research.

Action Research

Action research is defined by Mertler (2014) as:

> Any systematic inquiry conducted by teachers, administrators, counselors, or others with a vested interest in the teaching and learning process or environment for the purpose of gathering information about how their particular schools operate, how they teach, and how their students learn. (p. 305)

Research has shown that “the place to find out about classroom practices is the naturalistic setting of the classroom and from the lived experiences of teachers”

(Gladson-Billings, 1995, p. 163). This action research DiP functions "to deepen their [the teacher-researcher's] own practice toward problem solving and professional development" (Herr & Anderson, 2005, p. 29). "Unlike traditional dissertations that insist on a dispassionate, distanced attitude toward one's research, action research is often chosen by doctoral students because they are passionate about their topic, their setting and co-participants" (p. xvii). It is that passion that has drawn the teacher-researcher to action research and to this topic. It is also this passion that helped drive the teacher-researcher to take action in the classroom and to share results of this DiP in his school, district, state and beyond.

The problem of practice in this dissertation is that high school geometry students routinely do not show evidence of mathematical thinking while working on or after working on mathematical tasks. The solution practice for this DiP is to use the Five Key Formative Assessment Strategies during instruction to help students improve (Wiliam & Thompson, 2007). As the teacher-researcher engaged in the reflective and iterative process of carrying out this action research study, the routines became "intimately embedded in [his] practice" (Rust, 2009, p. 1883) as his result instruction likely improved. This belief is supported by the work of Rust and Myers (2006) who have "shown us that when teachers question their practice and gather and analyze data using tools easily incorporated into everyday teaching, an improvement in practice is a logical outcome" (p. 73). In addition, the findings of this DiP have the potential to reach many others through the teacher-researcher's extensive professional outreach and leadership activities in the field of mathematics education. The focus of this study is on student learning, not on teacher learning.

The teacher-researcher took up the qualitative research tradition to approach this action research study. Teacher action research "describes a form of qualitative inquiry that draws on techniques that are generally already part of the instructional tool kit of most practitioners" (Rust, 2009, p. 1883). The qualitative nature of the inquiry that takes place in regular classroom interactions makes the qualitative research tradition a natural fit (Herr and Anderson, 2005; Mertler, 2014). Data was collected in the form of student-participant written reflections, samples of student-participant work, and a teacher journal (field notes) and the constant comparative method was used to analyze the data. The next sub-section reviews the literature on the constant comparative method.

Constant Comparative Method

The data collected in this study was classroom artifacts (student-participant work and student-participant reflections) and observations in the form of field notes (a teacher journal). The rationale for using these qualitative data sources is that they were likely to yield evidence of student mathematical thinking (Merriam 2009; Mertler, 2014). The constant comparative method was used to analyze the data from these sources. The constant comparative method was originally developed as the basis of grounded theory research by Glaser and Strauss (1967) and now "the constant comparative method of data analysis is widely used in all kinds of qualitative studies" (Merriam, 2009, p. 31). The constant comparative method is defined as "a research design for studies involving multiple data sources, where data analysis begins early in the study and is nearly completed by the end of data collection" (Mertler, 2014, p. 306). This method involves, as its name indicates, constantly comparing one type of data to another (*i.e.*, field notes to student reflections) or comparing data within one type (*i.e.*, comparing two different student's reflection on the same day or one student's reflections from one day to another) (Saldana, 2009). The constant comparative method was appropriate for use in this study due to the multiple qualitative data sources that were used in this study and because the "apparent cyclical nature" (Mertler, 2014, p. 95) of the constant comparative method "epitomizes classroom-based action research" (Mertler, 2014, p. 95).

Merriam (2009) provides us with a description of how the constant comparative method is operationalized:

> Basically, the constant comparative method involves comparing one segment of data with another to determine the similarities and differences. Data are grouped together on similar dimension. The dimension is tentatively given a name; it then becomes a category. The overall object of this analysis is to identify patterns in the data. (p. 30)

As these categories are defined, they are given a code, and this code is assigned to the portion of the data in which it occurs. Glaser and Strauss (1967) give a "defining rule for the constant comparative method: while coding an incident for a category, compare it with previous incidents in the same and different groups coded in the same category" (p. 106). Throughout the data collection process, the researcher reviews the data and the codes to search for patterns and insight into the data (Fram, 2013; Glaser and Strauss, 1967; Merriam, 2009). In a study of a mathematics classroom, Staples (2007) used "coding and constant comparison to identify initial patterns and themes" about "students' participation in mathematical activities" (p. 169). This coding process is subject to researcher bias and subjectivity that are "natural and acceptable in action research as long as they are critically examined rather than ignored" (Herr & Anderson, 2005, p. 60). The teacher-researcher attended to these areas of potential bias and subjectivity by validating his coding procedure and categories with critical friends (Lomax et al., 1996; Herr & Anderson, 2005). To make the coding and data analysis process as transparent as possible, the teacher-researcher also recorded notes of his thinking throughout the process.

Lo and Wheatley (1994) have also used the constant comparative method to study how "the negotiation of social norms makes possible the negotiation of mathematical meaning" (p. 145). The work of Staples (2007) and Lo and Wheatley (1994) illustrates the constant comparative method being used to help the researcher understand and analyze interactions in the classroom and their relationship to the learning of mathematics. This action research study uses the constant comparative method to help understand the impact of the use of the Five Key Formative Assessment Strategies on student mathematical thinking.

Theoretical Base

The purpose of this study is to investigate how one mathematics teacher's implementation of the Five Key Formative Assessment Strategies impacts student mathematical thinking in his two high school geometry classes. What follows is a review of the literature on formative assessment, mathematical thinking, and learning as it relates to the purpose of this study.

Formative Assessment

One of NCTM's (2014) eight effective mathematical teaching practices is to "elicit and use evidence of student [mathematical] thinking" (p. 53). NCTM (2014) describes this teaching practice as using "evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continuously in ways that support and extend student learning” (p. 10) and calls to our attention studies of formative assessment (Black & William, 2009; William, 2007; Leahy, Lyon, Thompson & Wiliam, 2005). Supporting and extending learning for all students aligns with Horn’s (2012) third principle for “equitable mathematics teaching” (p. 9) which states that “all students can be pushed to learn mathematics more deeply” (p. 14). Black and Wiliam (2009) give the following definition of formative assessment:

> Practice in a classroom is formative to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited. (p. 9)

Chapman and King (2012) tell us that “the goal of formative assessment is to use the most appropriate tool [strategies] for the learner to demonstrate knowledge and skills” (p. 7). Wiliam and Thompson (2007) identify the following Five Key Formative Assessment Strategies that the mathematics teacher can use to implement formative assessment in their teaching:

1. *Clarifying and Sharing* learning intentions and criteria for success;
2. *Engineering Effective Classroom Discussions* and other learning tasks that elicit evidence of student understanding;
3. *Providing Feedback* that moves learners forward;
4. *Activating Students as Instructional Resources* for one another; and
5. *Activating Students as Owners* of their own learning. (p. 53)

While the second of these strategies certainly shows a specific connection to the mathematical teaching practice of "elicit and use evidence of student thinking" (NCTM, 2014, p. 53), all five are aligned with NCTM's (2014) larger list of eight effective mathematical teaching practices. The Five Key Formative Assessment Strategies align with NCTM's effective mathematics teaching practices (see Figure 2.1). Evans and Swan (2014) describe how engaging students as "instructional resources for one another" and "as owners of their own learning" (Wiliam & Thompson, 2007, p. 53) can result in "metacognitive acts in which students reflect on their own decisions and planning actions during mathematical problem solving" (p. 1).

Equity and formative assessment. Black and Wiliam's (2009) definition of formative assessment and the Five Key Formative Assessment Strategies are "consonant with the [following] four principles for equitable mathematics teaching" (Horn, 2012, p. 56):

- Principle 1: Learning is not the same as achievement.
- Principle 2: Achievement gaps often represent gaps in opportunities to learn.
- Principle 3: All students can be pushed to learn mathematics more deeply.
- Principle 4: Students need to see themselves in mathematics. (Horn, 2012, p. 19)

Horn (2012) elaborates on this consonance in the following statements:

> Firstly, by emphasizing learning, they [formative assessments] distinguish learning and achievement. Second, by becoming a part of the feedback teachers use in their instructional design, these forms of assessment do not punish students for having missed opportunities. Third, the focus on learning over summative evaluations pushes all students to learn mathematics more deeply. Finally,

Five Key Formative Assessment Strategies	NCTM Effective Teaching Practices
1. Clarifying and sharing learning intentions and criteria for success.	1. Establish mathematics goals to focus learning.
2. Engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding.	2. Implement tasks that promote reasoning and problem solving. 3. Use and connect mathematical representations. 4. Facilitate meaningful mathematical discourse 5. Pose purposeful questions. 8. Elicit and use evidence of student thinking.
3. Providing feedback that moves learners forward.	3. Use and connect mathematical representations. 5. Pose purposeful questions. 7. Support productive struggle in learning mathematics.
4. Activating students as instructional resources for one another.	4. Facilitate meaningful mathematical discourse. 6. Build procedural fluency from conceptual understanding. 8. Elicit and use evidence of student thinking.
5. Activating students as owners of their own learning	4. Facilitate meaningful mathematical discourse. 6. Build procedural fluency from conceptual understanding. 8. Elicit and use evidence of student thinking

Figure 2.1 Alignment of the Five Key Formative Assessment Strategies with NCTM's effective teaching practices.

> assessments for learning give students a role in classroom design when teachers adapt lessons to students' learning needs. Likewise, opportunities for self-assessment give students the chance to participate in the evaluation of their own learning. (pp. 56-57)

The consonance of formative assessment and "equitable mathematics teaching" practices (Horn, 2012, p. 9) supports the rationale for using the Five Key Formative Assessment Strategies to study mathematical thinking. In today's climate of accountability, it is notable that standardized testing data is not mentioned in Black and Wiliam's (2009) definition of formative assessment or in the principles of "equitable mathematics teaching" (Horn, 2012, p. 9). Gutiérrez (2008) points out that many researchers (Ilana Horn is not one of those many) focus "on a single [equity] issue – the 'achievement gap' – to the exclusion of others" (p. 357) and that "it is a moral imperative to move beyond this 'gap-gazing' fetish" (p. 357). The formative assessment focus of this DiP allows the teacher-researcher to focus on both the longitudinal mathematical achievement of individuals and their progress towards meeting and exceeding high expectations (excellence). "A focus on excellence alongside gains is important to avoid a false sense of security in seeing student scores increase if that increase does not also accompany a shift in one's relative position in society" (Gutiérrez, 2008, p. 362). Along these lines, it is important to note that the teaching of mathematics has the potential to have a positive impact on social justice issues (Gutiérrez, 2008; Martin, 2006; Moses & Cobb, 2001). Horn (2012) challenges us to "re-create our classrooms and departments in ways that will increase the opportunities for students across achievement levels to learn by thinking mathematically" (p. 14).

Mathematical Thinking and Learning

Schoenfeld (1992) conceptualizes, provides a literature review for, and discusses the implications of thinking mathematically. In addition, he reminds us that "the absence of math literacy in urban and rural communities throughout this country is an issue as urgent as the lack of black voters was in Mississippi in 1961" (Moses & Cobb, 2001, p. 5) and that "metacognition, belief and mathematical practices are considered critical aspects of thinking mathematically" (Schoenfeld, 1992, p. 363). The Common Core

State Standards for Mathematics' (CCSSM, 2009) eight Standards of Mathematical Practice are:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning. (p. 6)

Schoenfeld (1992) would describe the eight standards above as "mathematical practices" (p. 363) that are meant to help incorporate mathematical thinking into the teaching of content standards. The importance of problem solving to the teaching of mathematics and to the field of mathematics is well documented (Halmos, 1980; Polya, 1945; Schoenfeld, 1985; NCTM, 2000, 2009, 2014). The importance of mathematical thinking in learning mathematics can be seen in the work involving student conceptual learning in mathematics and its practical applications (Neimi, 1994; Neimi, 1996; Smith & Stein, 2011; Horn, 2012; NCTM, 2014). To understand the connections between problem solving, formative assessment, mathematical thinking and learning it is necessary to review the literature related to theories of learning. Schoenfeld (1992) directs us to sociocultural theories of learning when he talks about mathematical learning in relation to "interactions with others" (p. 363) and "in terms of the mathematical communities in which students live" (p. 363).

The classroom is a complex and dynamic system of interactions between the teacher, students, mathematical tasks and mathematical thinking that result in mathematical learning (NCTM, 2014). This study looks at what happens to student-participant mathematical thinking when a teacher-researcher uses the Five Key Formative Assessment Strategies during instruction. When one adopts a participationist view of learning (a historical context for the participationist view of learning is provided in the next section), one sees that it is discourse that links together the complex interactions taking place in the teaching, learning and assessment that happens in the classroom (Bakhtin, 1981; Sfard, 2007; Sfard & Cobb, 2014).

Participationism is a "research discourse grounded in the metaphor of learning as improving participation in historically established forms of activity" (Sfard, 2008a, p. 301) that includes discourse as one of those 'forms of activity.' Discourse can be described as a "rule-governed mulitmodal communicational activity" (Sfard & Cobb, 2014, p. 558). Since "participationists do not mean transformations in individuals, but rather in what and how people are doing patterned human processes, both individual and collective" (p. 568), it is clear that learning is being described in a social context. Bakhtin's (1981) dialogic theory of learning sheds light onto why discourse is social when he states that "verbal discourse is a social phenomenon – social throughout its entire range and in each and every one of its factors, from the sound image to the furthest reaches of abstract meaning" (p. 270). Sfard's 'commognitive' standpoint provides a theoretical framework of learning, from a participationist perspective, for understanding the complex discourse that takes place in the classroom by connecting communication and thinking to learning. Commognition is a "term that encompasses thinking (individual cognition) and (interpersonal) communicating; as a combination of the words communication and cognition, it stresses the fact that these two processes are different manifestations (intrapersonal and interpersonal) of the same phenomenon" (Sfard, 2008a, p. 296). Sfard (2007), in fact, "regards discourse as the very object of learning" (p. 576) and argues against "not just the split between thinking and speech, but the more general one, between thinking and communicating" (p. 577). This viewpoint "leads to the conclusion that learning mathematics is equivalent to changing patterns of participation in discourse" (Sfard & Cobb, 2014, p. 553)." To be even more clear, "learning mathematics is a change of discourse" (Sfard, 2008a, p. 255; Sfard & Cobb, 2014, p. 558). Thinking is defined by Sfard (2008a) as "an individualized form of (interpersonal) communicating" (p. 81). Sfard and Cobb (2014) describe the implications of this perspective on learning:

> Recognition of the discursive nature of mathematics and its learning necessarily affects the very foundation of research on learning. It directly challenges the strict ontological divide between what is going on 'inside' the human mind and what is happening 'outside.' This ontological unification has a least two implications. First, it completes the Vygotskian solution to the puzzle of the human ability to constantly build on previous achievements: human communication, most of which happens in language, serves as the main repository of complexity and the principal carrier of invention. Second, the statement that learning means changes in discourse creates an opportunity to operationalize research vocabulary and to refine methodology. (p. 559)

Sfard's (2007; 2008a) 'commognitive' framework allows us to make the connections between all of the teaching and learning interactions that are happening in the classroom because they are discursive. Her theory accounts for the thoughts of both the students and the teacher and includes them as part of the classroom discourse. This is particularly important to formative assessment since it is in those "moments of contingency" (Black & Wiliam, 2009, p. 10) in the classroom during which the teacher must decide what choices to make to foster learning. The use of any of Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies during instruction creates an environment in which students are given opportunities to modify or extend their discourse (to learn) and to "make sense of the world around them" (Sfard, 2007, p. 575). By collecting and analyzing the student-participant written reflections, student-participant work and the teacher journal over time, the teacher-researcher will describe the mathematical thinking of the student-participants in this action research study.

Historical Context

This section provides a historical context for formative assessment, mathematical thinking (and problem solving) and participationism. To help locate this study of mathematics education in the larger education field, this DiP approaches the teaching of school mathematics from a reform mathematics perspective (Kilpatrick, Swafford & Findell, 2001; National Research Council, 1989; NCTM, 1989, 1991).

Formative Assessment

Dunn and Mulvenon (2009) and Wiliam (2011) trace the origins of formative assessment back to the term "formative evaluation" that was first used by Scriven (1967). Bloom (1969) also uses "formative evaluation" to discuss the assessment that goes on during the teacher-learning process. Wiliam (2011) states that, “although the term *formative* was little used over the following twenty years [that followed 1969], a number of studies investigated ways of integrating assessment with instruction, the best known of which is probably cognitively guided instruction (CGI)” (p. 34). Fennema, Carpenter, Franke, Levi and Jacobs (1996) used the CGI model to help “teachers to make the fundamental changes” (p. 403) to their practice that take into account student mathematical thinking. Franke and Kazemi (2001) have used CGI to gain perspective on teacher and student learning in the classroom. This action research study does not seek to change the practice of the teacher-researcher, but, to align and focus the teacher-researcher’s existing practices with formative assessment pedagogy (the Five Key Formative Assessment Strategies). Black and William (1998) reviewed more than forty studies on classroom assessment and found that “formative assessment is an essential component of classroom work and that its development can raise standards of achievement” (p. 148). In an era where achievement and accountability often take center stage, adopting a formative assessment approach to the teaching-learning process has benefits for both the students (learning) and the teacher (added job-security) (Siskin, 2013). In addition, the “ongoing work by educators attempting to increase the use of formative assessment to support learning and engagement in schools could help inform policymakers and researchers” (Nolen, 2011, p. 325).

Since the 1990s, many researchers have investigated formative assessment and the term itself has taken on different meanings in different contexts (Black, 2011; Dunn & Mulvenon, 2009). For example, Black and Wiliam (2009) define formative assessment as:

> Practice in a classroom is formative to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited. (p. 9)

Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies describe how to put formative assessment into practice in the classroom. In Wiliam's (2011) *Embedded Formative Assessment*, separate chapters are dedicated to detailing the importance of and practical methods for implementing each of the five key strategies. Sleep and Boerst (2012) have shown that "formative assessment practices" can be used "specifically to elicit and interpret students' mathematical thinking" (p. 1039) and Evans and Swan (2014) have indicated that studying formative assessment as it relates to mathematical thinking is an area where more research is needed. Wiliam (2011) describes formative assessment as the "bridge between teaching and learning" (p. 46) and it is evidence of student learning that will be analyzed in this dissertation. The next section provides a historical context for mathematical thinking and problem solving.

Mathematical Thinking and Problem Solving

It seems almost impossible to describe mathematical thinking outside of the context of problem solving. This section begins with a description of Polya's (1957) problem solving process as described in his famous book, *How To Solve It*, in the context of formative assessment and NCTM's (2014) effective teaching practices (see Figure 2.1). Then, the connections between problem solving and mathematical thinking are made explicit, followed by a discussion of what constitutes mathematical thinking and its importance.

Problem solving. Polya (1957) describes problem solving as the four phase process of "understanding the problem, devising a plan, carrying out the plan and looking back" (p. xvii). "Understanding the problem" amounts to understanding what the question is asking, knowing the important information in the problem and what is "unknown" (Polya, 1957, p. 6). By being clear about "learning intentions" (Wiliam & Thompson. 2007, p. 53), educators can help students better understand the mathematical context for mathematical problems. Without this first phase, students are not able to move forward in the problem solving process because they do not know what they need to be solving. Many mathematics education researchers have proposed the use of classroom and sociomathematical norms to help create an environment where students can actively contribute to learning (Cobb, Yackel, & Wood, 1989; Yackel & Cobb, 1996; Cobb, Stephan, McClain, & Gravemeijer, 2001; Horn, 2012; NCTM 2014). Sociomathematical norms align with Wiliam and Thompson's (2007) strategy of "clarifying and sharing learning intentions and criteria for success" (p. 53) because sociomathematical norms establish classroom routines and expectations for classroom participation around the learning of mathematics. Another method for helping students who struggle with Polya's first phase of problem solving is to use a think-pair-share activity to help "students feel a sense of belonging in the classroom where mathematical discussions are prevalent" (Bostic & Jacobbe, 2010, p. 32) and to use those discussions to help students understand what the problem is asking and the important features of the problem. The think-pair-share activity also serves to "activate learners as instructional resources for one another" (Wiliam & Thompson, 2007, p. 53). It also provides an opportunity for students to explain the problem to others and to listen to it being explained to them so that they can move onto the second phase of the process: "devising a plan" (Polya, 1957, p. 7).

A student's problem solving "plan" might consist of writing an equation, making a graph, or trial and error (Polya, 1957, p.7). Students need to be supported in this phase of problem solving so that they can make an attempt at solving the problem even if they are unsuccessful the first time. Polya (1957) points out that failure is part of the problem solving process (p. 6). One of the eight mathematical teaching practices that the NCTM has highlighted is to "support productive struggle in learning mathematics" (NCTM, 2014, p. 10). In this practice, NCTM elaborates that the "effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships" (2014, p. 10). Applying what is known about effective mathematical teaching practices to Polya's problem solving process yields a practical way to think about how to incorporate pedagogical strategies that support students problem solving.

"Carrying out the plan" (Polya, 1957, p. 12) is the third phase of the problem solving process and is the phase where the teacher can employ pedagogical strategies to facilitate students in solving or attempting to solve a problem. The Common Core State Standards for Mathematics (CCSSM) list one of the eight standards of mathematical practice to be: "make sense of problems and persevere in solving them" (CCSSM, 2009, p. 6). This standard of mathematical practice really takes into account the first three of

Polya's phases and even touches on the fourth by mentioning in its description that students should "reflect on whether the results make sense" (CCSSM, 2009, p. 6). Teachers can help students to "carry out the plan" (Polya, 1957, p. 12) by knowing that "learning can be enhanced by respecting and encouraging children to try out the ideas and strategies that they bring to school-based learning in classrooms" (Bransford *et al.*, 2010, p. 171-172) and creating a classroom culture reflective of that knowledge. Teachers can allow students to use their prior knowledge to solve problems rather than just use a prescribed algorithm (Bransford *et al.*, 2010, p. 172). These activities help students to become "owners of their own learning" (Wiliam & Thompson, 2007).

The final phase of problem solving is "looking back" and is where the student "reconsiders and reexamines the result and the path that led to it" so they can "consolidate their knowledge and develop their ability to problem solve" (Polya, 1957, p. 14-15). It is this fourth phase where a student reflects on what he/she has done so that he/she can refine his/her skills and learn from his/her thinking. When a student completes this phase, he/she is developing "what is known as 'metacognition' – the ability to monitor one's current level of understanding and decide when it is not adequate" (Bransford *et al.*, 2000, p. 47). Metacognition helps students to "develop the ability to teach themselves" (p. 58) and this ability is what helps to separate "novice" (p. 29) problem solvers from "expert" (p. 29) problem solvers. The teacher can serve an active role at this critical point to push students towards the answer with skillfully asked questions or by offering examples to help lead the students to a particular solution path and to help them become "owners of their own learning" (Wiliam & Thompson, 2007, p. 53).

Linking problem solving and mathematical thinking. As stated earlier in this chapter, Schoenfeld (1992) conceptualizes, provides a literature review for, and discusses implications for thinking mathematically. He states that "metacognition, belief and mathematical practices are considered critical aspects of thinking mathematically" (Schoenfeld, 1992, p. 363). Cuoco, Goldenberg and Mark (1996) describe these 'mathematical practices' as "habits of mind" (pp. 375-376) that are used by people (including students and research mathematicians) when they think about and do mathematics. Cuoco *et al.* (1996) tell us that for students to develop their mathematical thinking ("habits of mind"), students should be engaged in experiences that allow them to be: 1) "Pattern Sniffers", 2) "Experimenters", 3) "Describers", 4) "Tinkerers", 5) "Inventors", 6) "Visualizers", 7) "Conjecturers" and 8) "Guessers" (pp. 377-383). Cuoco *et al.* (1996) explain that "students should be describers" who "argue" in mathematics class and "should be able to convince their classmates that a particular result is true or plausible by giving precise descriptions of good evidence or (even better) by showing generic calculations that actually constitute proofs" (p. 379). These "habits of mind" (Cuoco *et al.*, 1996) can be thought of through the lens of the Five Key Formative Assessment Strategies to help provide a framework that helps the teacher to design and facilitate instruction so that the teacher's students can improve in their ability to think mathematically. In Horn's (2012) discussion of equitable teaching principles, she references Cuoco *et al.* (1996) when she states that "when students understand the relationships between mathematical ideas and have developed habits of mind" (p. 15), the students will be able to "see themselves in mathematics" (p. 15), and will come to view themselves as mathematical thinkers. But what does it mean to think mathematically? Certainly, the "habits of mind" (Cuoco *et al.*, 1996) offer the reader some insight.

Mathematical thinking. This section discusses ways to think about mathematical thinking as a process and not as a strictly cognitive activity. Describing mathematical thinking as a process is important to this DiP. It gives the teacher-researcher a way to know that he had evidence of mathematical thinking when he saw it during data collection and analysis, and to convince the reader that what he found was evidence of mathematical thinking. The process of mathematical thinking, from Sfard's (2007) participationist commognitive perspective, follows the same rules as interpersonal discourse (communication) and can be regarded as part of that discourse even though thinking, itself, is necessarily intrapersonal (Sfard, 2007; Sfard, 2008a). Sfard (2008a) lends clarity to this theoretical standpoint on mathematical thinking and on thinking in general:

> In participationist narratives, individual and collective forms of doing are presented as different manifestations of the same type of processes. Within this perspective, the historical change in forms of human doing becomes fully accountable. It now seems not unreasonable to assume that patterned, collective forms of distinctly human activities are developmentally prior to the activities of the individual. Thinking, although it seems inherently private, should not be any different. Cognitive processes may thus be defined as individualized forms of interpersonal communication, whereas communication itself is described as a collectively performed rule-driven activity that mediates and coordinates other activities of actors. The term *commognition* was coined to encompass thinking

and (interpersonal) communicating to stress the unity of these two types of processes. (p. 91)

The descriptions of mathematical thinking that follow include descriptions that support a participationist approach to learning and exclude descriptions that support an acquisitionist approach (see Sfard & Cobb, 2014) because they do not fit into the theoretical framework of this DiP. The following example is included to provide justification for the exclusion of the acquisitionist approach. Wood, Williams and McNeal (2006) take an acquisitionist approach to learning and define "mathematical thinking as the mental activity involved in the abstraction and generalization of mathematical ideas" (p. 226). When one conceives of thinking as communicating and not cognitive (or acquisitionist) it becomes clear that this example and others like it are not part of a participationist framework.

Regardless of the approach to learning, it can be a trying task to establish the historical context for mathematical thinking when "the earliest documents available are the clay tablets from Mesopotamia (Uruk, 3000 BC), in which mathematics appears as a necessary and useful tool for solving problems of agriculture and economic administration" (Keitel, 2006, p. 12-13). While these documents do not represent a window into the mathematical thinking of an individual, the tablets are artifacts from a society that used mathematics and mathematical thinking to solve problems. Since antiquity, mathematical thinking has been a topic written and thought about by Plato, Aristotle, Descartes and many others (Keitel, 2006). In this DiP, the teacher-researcher is concerned with the mathematical thinking of students. As such, this section will focus on current definitions of mathematical thinking that have emerged as a result of the historical development and growth of mathematical discourse (Sfard, 2008a).

Previously, Cuoco *et al.*'s (1996) "habits of mind" were used help make the transition from problem solving to mathematical thinking. Stein, Grover and Hennsington (1996) describe mathematical thinking in a similar way to that of Cuoco *et al.* (1996); they describe the "processes of mathematical thinking as, in essence, doing what makers and users of mathematics do: framing and solving problems, looking for patterns, making conjectures, examining constraints, making inferences from data, abstracting, inventing, explaining, justifying, challenging, and so on" (p. 456). Tall (2011) provides the following explanation of mathematical thinking that is intertwined with teaching and learning:

> For those of us who teach mathematics to learners, whether we see it as our purpose to introduce them to the wonders of mathematics or to inspire them to discover mathematics by their own efforts, we surely need to encourage them to think in ways that gives them power in operation and pleasure in success. This involves not only being aware of their current development and how they might profit by exploring new ideas in ways that are appropriate for them at the time, but also to seek a broader understanding of the crystalline structures of mathematics itself. (p. 8)

Tall (2011) speaks to mathematical thinking in a way that parallels the philosophy that went into NCTM's (2009) book on reasoning and sense making.

Mathematical thinking is a process that is challenging to define because it happens on the inside, within one's mind. The teacher-researcher does not have a window into the thinking (intrapersonal communication) of his students, but he can hope to attempt to understand what it is that they might be thinking by the way he asks questions, engages students in classroom discourse, the feedback he gives, and by the mathematical tasks that he chooses (Jacobs, Lamb & Phillips, 2010). Once again, this work relates back to Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies that the teacher-researcher will use in this action research study. Earlier in this DiP, thinking was framed (defined) as "an individualized form of (intrapersonal) communicating." This framing helps the reader to think about mathematical thinking as "an individualized form of (intrapersonal) communicating" (Sfard, 2008a, p. 81) about mathematics. If one thinks about mathematical thinking as an inseparable part of discourse (ways of communicating that can include talking, gesturing, writing, and drawing (Sfard, 2008b; Horn 2012; Sfard & Cobb, 2014)) and learning as a "change in discourse" (Sfard & Cobb, 2014, p. 558), then it becomes possible to accept that a change in the thinking of an individual can be considered as a "change in discourse" (p. 558) and, therefore, as learning.

Learning

This DiP takes on a participationist approach to learning (Cobb *et al.*, 2001; Sfard, 2007; Sfard, 2008a; Sfard & Cobb, 2014). The participationist perspective emerged in the 1990s when "some researchers concluded that it would not be possible to answer certain core questions about mathematical learning" without addressing the limitations of the acquisitionist approach to learning (Sfard & Cobb, 2014, p. 551). Piaget's theory of learning is focused on cognition and learning being 'acquired' in the mind of the individual, a viewpoint that is fundamental to the acquisitionist approach (Piaget, 1952).

The focus on learning being ‘acquired’ in the individual poses difficulties for analyzing learning in social settings (in this study, the classroom). The participationist framework “portrays mathematics as a form of human activity rather than as something to be ‘acquired,’ and therefore a view of learning mathematics as the process of becoming a participant in this distinct type of activity” (Sfard & Cobb, 2014, p. 547) emerges. The participationist approach, as the name implies, situates learning in a social context in which the learners are active ‘participants’ in the process. Dewey’s (1929) declaration of his belief that “all education proceeds by the participation of the individual in the social consciousness of the race” (p. 291) and that “the school is primarily a social institution” (p. 292) provides added depth to this study’s participationist approach to learning. The participationist approach has its foundations in Vygotsky’s sociocultural learning theory that places learning in a social context (Vygotsky, 1978). Later, it was Lave and Wenger (1991) who laid the groundwork for and who would be credited with introducing participationism (Sfard, 2008a; Sfard & Cobb, 2014).

Within the participationist approach to learning, Sfard and Cobb (2014) describe “two overlapping lines of study” (p. 547). One “line of study conducts investigations in classrooms, where mathematics learning can be seen as part and parcel of changing *learning-teaching practices*,” while the second line of study “focuses on ways of communicating (or discourses) as the primary objects of change in the learning of mathematics” (Cobb & Sfard, 2014, p. 547). This DiP’s focus is on the ‘second line of study’ and, in particular, uses Sfard’s (2007; 2008a) commognitive view of participationism as part of its theoretical framework.

To help explain the thinking behind her commognitive framework, Sfard (2008a) describes Vygotsky's sociocultural view of learning as "whatever name is given to what is being learned by an individual – *knowledge, concept* or *higher mental function* – all of these terms refer to culturally produced and constantly modified outcomes of collective human efforts" (Sfard, 2008a, p. 77; Vygotsky, 1987). She creates a carefully crafted argument to include thinking as part of communication that uses the thinking of Wittgenstein (1953). Wittgenstein's (1953) discussion of "language games" refers to one's thinking about whether it is thinking that comes before talking or whether it is one's talking that comes before thinking. Sfard (2008a) states: "For Wittgenstein, meaning was neither a thing in the world, nor a private entity in one's mind: It was an aspect of human discursive activity and, as such was public and fully investigable" (p. 73). Sfard (2008a) presents a convincing case for thinking as communication in her book entitled *Thinking as Communicating: Human Development, the Growth of Discourses, and Mathematizing* that builds on the work of Vygotsky and Wittgenstein.

The data that were collected and analyzed in this study were a teacher journal, student-participant work and student-participant reflections. These data sources are reifications of student mathematical thinking (Wenger, 1998). Reification is the "process of giving form to our experience by producing objects that congeal this experience into 'thingness'" (Wenger, 2000, p. 58). Sfard's commognitive participationist framework (Lave & Wenger, 1991; Sfard, 2007; 2008a) gives us a way to include reifications of mathematical thinking (Wenger, 1989) as part of discourse. Bruner's (2013) statement that "the nature of man's world view, whether formulated in myth or in science, depends upon, and is constrained by, the nature of human language" (p. 80) helps us to further understand the significance of the reification of mathematical thinking. One of the weaknesses of Sfard's (2007; 2008a) commognitive framework is that it does not provide practical methods for analyzing discourse in a classroom setting. One of the strengths of Sfard's framework is that it provides a powerful way to conceptualize thinking as communication and as part of discourse. This study will contribute to the literature on teaching, learning and thinking mathematically by applying Sfard's framework to the classroom in a practical way.

CHAPTER THREE: METHODOLOGY

Introduction

Chapter Three describes the qualitative action research methods used to investigate the impact of the teacher-researcher's use of Wiliam and Thompson's (2007) Five Formative Assessment Strategies on the mathematical thinking of high school students. This qualitative research study is action research. Action research is defined by Mertler (2014) as "any systematic inquiry conducted by teachers, administrators, counselors, or others with a vested interest in the teaching and learning process or environment for the purpose of gathering information about how their particular schools operate, how they teach, and how their students learn" (p. 306). This study investigated, analyzed and described the impact of the Five Formative Assessment Strategies on the mathematical thinking of the student-participants in the teacher-researcher's classrooms for the purposes of improving student learning.

The teacher-researcher took up the qualitative research tradition to approach this action research study. He used the constant comparative method to analyze how the teacher-researcher's implementation of the Five Key Formative Assessment Strategies impacted the student-participants' mathematical thinking in the classroom (Merriam, 2009; Mertler, 2014). The constant comparative method is defined as "a research design for studies involving multiple data sources, where data analysis begins early in the study and is nearly completed by the end of data collection" (Mertler, 2014, p. 306). Merriam

(2009) provides us with a description of how the constant comparative method is operationalized:

> Basically, the constant comparative method involves comparing one segment of data with another to determine the similarities and differences. Data are grouped together on similar dimension. The dimension is tentatively given a name; it then becomes a category. The overall object of this analysis is to identify patterns in the data. (p. 30)

The Chapter begins with a restatement of the problem of practice, the research question, and purpose of this Dissertation in Practice (DiP). It then describes the DiP's qualitative action research design including the: 1) participant selection and research site, 2) data sources, 3) data collection, 4) data analysis, and 5) reflection strategy used to design the Action Plan.

The Problem of Practice

The problem of practice in this dissertation is that high school geometry students routinely do not show evidence of mathematical thinking while working on or after working on mathematical tasks. The mathematical pedagogical strategies that the students in this study were exposed to before they entered high school geometry were more focused on memorization and the application of mathematical procedures rather than on how to make sense of and reason with mathematics – to think mathematically. Students were rarely asked to explain their mathematical thinking on assignments or assessments and often had limited opportunities to learn mathematics via social interactions with other students. In short, students were not enabled to explicitly show and develop their mathematical thinking in prior mathematics classes. In the years leading up to this study, the teacher-researcher had struggled to get his students to show evidence of mathematical thinking and to engage in thinking mathematically. The teacher-researcher aimed to bring his mathematics classroom into the 21st century where students would not only learn how to do mathematics but also how to learn mathematics and to think mathematically. To accomplish this, the teacher-researcher sought to improve his own practice and elected to use Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies during instruction to enable students to develop and show evidence of mathematical thinking.

Research Question

1. How does the use of Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies impact students' mathematical thinking?

Purpose Statement

The purpose of this study is to investigate how one mathematics teacher's implementation of the Five Key Formative Assessment Strategies (William and Thompson, 2007) impacted student-participants' mathematical thinking in his two high school geometry classes. The teacher-researcher used the Five Key Formative Assessment Strategies while students worked on mathematical tasks. The teacher-researcher used these strategies to enable the classroom to be more constructivist and progressivist in nature where students were encouraged to engage in discussion and discourse about mathematics and mathematical thinking throughout the eight week data collection process. The teacher-researcher used these strategies to "elicit evidence of students' current mathematical understanding" (NCTM, 2014, p. 53) and to develop the mathematical thinking of the student-participants.

Qualitative Action Research Design

The teacher-researcher used qualitative action research methodology to address the problem of practice and to answer the research question. Action research is defined by Mertler (2014) as:

> Any systematic inquiry conducted by teachers, administrators, counselors, or others with a vested interest in the teaching and learning process or environment for the purpose of gathering information about how their particular schools operate, how they teach, and how their students learn. (p. 305)

Action research methodology is further justified for use in this study because research has shown that “the place to find out about classroom practices is the naturalistic setting of the classroom and from the lived experiences of teachers” (Gladson-Billings, 1995, p. 163). This action research DiP functions to “to deepen their [the teacher-researcher’s] own practice toward problem solving and professional development” (Herr & Anderson, 2005, p. 29). “Unlike traditional dissertations that insist on a dispassionate, distanced attitude toward one’s research, action research is often chosen by doctoral students because they are passionate about their topic, their setting and co-participants” (p. xvii). It is that passion that has drawn the teacher-researcher to action research and to this topic. It is also this passion that helped drive the teacher-researcher to take action in the classroom and to share results of this DiP in his classroom, school, district, state and beyond. The teacher-researcher informally shared his insights about the findings of this study with his student-participants throughout the duration of this study and offered the student-participants opportunities to provide feedback, offer suggestions and to ask questions. In addition, at the conclusion of this study, the teacher-researcher formally presented the findings to the student-participants. He used this formal presentation and subsequent discussion as an opportunity to engage in a reciprocal reflection process with the student-participants. This process allowed the student-participants to provide additional confirmation and validity to the findings of the study, to provide additional insights, and to identify additional questions or needs that they had related to mathematical thinking.

Participant Selection and Research Site

The present action research study took place in two geometry classes at a southern Title I public high school of approximately 800 students. The mathematics courses taught at this school follow a traditional sequence (Algebra I, Geometry, Algebra 2). Students attend classes that meet for 88 minutes on every other day and all have access to personal Chromebooks provided by the school and excellent internet access during the school day. The teacher-researcher is in his sixteenth year of teaching, is a National Board certified teacher, holds a Master's of Education degree in mathematics education and a Bachelor of Arts degree in Biology and is a teacher-leader in mathematics education at the state and national level.

The study took place with students in two of the teacher-researcher's honor's geometry classes over the course of eight weeks (see Appendix A for the research planning schedule) during the fall of the 2016-2017 school year. Assented students from each of the two classes were the participants selected for this study and they represent a convenience sample (see Appendix B for a copy of the assent document). During the study, the teacher-researcher was teaching several different courses at his school and he has taught a variety of other courses through his teaching career. The rationale for selecting the honors geometry classes is that honors geometry represents the class that the teacher-researcher has taught the most consistently. The rationale for not selecting all of his mathematics classes is to allow sufficient time for planning and reflection for the teacher-researcher since this study is taking place *in situ*. The numbers of students in the classes studied were 28 and 26, respectively, and all students were assented. The teacher-researcher collected data six times during the eight weeks of this study.

Data Sources

This section describes the three sources of data that were collected and analyzed in this DiP. The three data sources are student-participant written reflections, student-participant work, and a teacher journal (field notes).

The primary source of data was classroom artifacts in the form of written reflections of the student-participants to the following prompts:

1. Describe how you came up with your mathematical explanation or your mathematical understanding of today's problem. What helped you? What didn't?
2. Reflect on anything that you found important, challenging or otherwise notable about the problem solving process that you or your classmate(s) used to work through today's mathematical task.
3. What could help you get better at coming up with a mathematical explanation?

The student-participant written reflections were collected and compiled using Google Forms six times during the eight weeks of the study. The written reflections served a dual-purpose for the teacher-researcher: 1) they provided data that related directly to the research question of this study, and 2) they served as a way for the teacher-research to put formative assessment into practice using two of the Five Key Formative Assessment Strategies (Wiliam & Thompson, 2007). The two formative assessment strategies put into practice by the use of student-participant written reflections were as follows: 1) they served as “learning tasks that elicit[ed] evidence of student understanding” (Wiliam & Thompson, 2007, p. 53), and 2) they served as a way of “activating students as owners of their own learning” (p. 53).

The second of the three data sources was the student-participant work. The student-participant work are the classroom artifacts that took the form of the written mathematical explanations that students-participants wrote each week in response to mathematical tasks that were part of the teacher-researcher’s weekly instruction. Student-participant work was collected six times during the eight weeks of the study.

The final data source was a teacher journal kept by the teacher-researcher containing field notes (see Appendix C for the field note template) about his use of Wiliam and Thompson’s (2007) key formative assessment strategies in order to record relevant observations of student-participant interactions. For example, the teacher-researcher reflected on his use of the formative assessment strategies as they related to student mathematical explanations at the end of each class, when appropriate and feasible. While it was the teacher-researcher who authored this journal, the journal is referred to as a ‘teacher journal’ since the teacher journal is meant to provide a record of the observations and thoughts had by the teacher-researcher about instruction and the instructional process the he engaged in as the teacher. This included planning, teaching, assessing, reflecting on practice, and, any other practices that were a part of teaching. Another reason for referring to this data source as a ‘teacher journal’ was to take a step to make sure that it was not mistaken for the "thoughts, musings, speculations, and hunches" (Merriam, 2009, p. 174) that were recorded by the teacher-researcher while collecting and analyzing the data in this study. The teacher journal contains a record of teacher moves, notable classroom discourse and "moments of contingency" (Black & Wiliam, 2009, p. 10). "Moments of contingency" (Black & Wiliam, 2009, p. 10) are those times during the instructional process where the teacher makes a decision based on what is happening in the class with the intention of improving learning. The teacher journal also served as a record to help explain observed classroom interactions (teacher to student, student to student, or student to teacher). As such, "moments of contingency" (Black & Wiliam, 2009, p. 10) provide a category that is research-based and has a specific meaning in the context of formative assessment.

Data Analysis

The data was analyzed using the constant comparative method (Merriam, 2009; Mertler, 2014). For example, classroom artifacts such as 'student-participant work' and 'student-participant written reflections' and field notes taken in a 'teacher journal' were documented and analyzed during the ongoing data collection in the two geometry classrooms. These data were transcribed and managed using the computer assisted qualitative data analysis software program (CAQDAS) called NVivo. The student-participant written reflections that were collected using Google Forms did not need to be transcribed.

Whenever possible, the teacher-researcher analyzed the data constantly during the study using an "inductive and comparative analysis strategy" (Merriam, 2009, p. 269). The teacher-researcher used the constant comparative method of data analysis (Glaser &

Strauss, 1967) including an "inductive, concept-building orientation" (Merriam, 2009, p. 199). The constant comparative method involved, as its name indicates, constantly comparing one type of data to another (*i.e.*, teacher journal to student-participant written reflections) or comparing data within one type (*i.e.*, comparing two different student-participant's reflections on the same day or one student's reflections from one day to another) (Saldana, 2009). During this process, the teacher-researcher coded the data with "a word or short phrase that symbolically assigns a summative, salient, essence-capturing, and/or evocative attribute for a portion" (Saldana, 2009, p. 3) of the transcribed data. As a novice coder, the teacher-researcher began coding by hand, because "there is something about manipulating qualitative data on paper and writing codes in pencil that give you more control over and ownership of the work" (Saldana, 2009).

Once the data collection and a first round of coding was complete, the teacher-researcher identified patterns or themes in the data and assigned different codes to categories resulting from those patterns or themes. The categories coded were "mutually exclusive" (Merriam, 2009, p. 185) so that a particular datum did not fall into more than one category. After identifying categories, the teacher-researcher analyzed the data again based on the categories and codes that resulted from the initial codification of the data (Saldana, 2009). When coding was complete, the teacher-researcher inputted all of the codes into the CAQDAS program NVivo. The teacher-researcher then looked for patterns or themes to emerge and, again, created categories to "interpret the meaning of the data" (Merriam, 2009. p. 193). These categories are "the findings of the study" (Merriam, 2009, p. 193) and will be discussed in the next chapter.

Ethical data collection. The aforementioned coding process is subject to researcher bias and subjectivity that are "natural and acceptable in action research as long as they are critically examined rather than ignored" (Herr & Anderson, 2005, p. 60). The teacher-researcher attended to these areas of potential bias and subjectivity by validating his coding procedure and categories with critical friends (Lomax et al., 1996; Herr & Anderson, 2005). To make the coding and data analysis process as transparent as possible, the teacher-researcher also recorded notes of his thinking throughout the process in case any ethical questions were to arise.

All students and parents/guardians of students in the target population received an invitation letter that informed them about the study and which invited the subjects to participate. One invitation (see Appendix B) served as the parental consent form and another, parallel form, served as the child assent form. The subjects of this study could have voluntarily withdrawn at any time without any negative consequences. All that the parent/guardian of the subject or the subject him/herself would have needed to do was to notify the teacher researcher of his/her withdrawal (no subjects withdrew from the study). All electronic data is kept on the teacher researcher's password protected laptop computer and backed-up on a password protected external storage device. Hard copies of data are kept in a locked file in the teacher researcher's classroom. Student-participant names present in the data were recoded to keep the identity of the student-participants confidential. Files will be kept for up to five years and will be destroyed at the end of the five year period. Also, any conference presentations and manuscripts will use pseudonyms to protect the identity of participants.

Validity and reliability. Triangulation was used to increase the validity of this study. The data were triangulated by collecting and analyzing multiple data sources and methods (Denzin, 1978). One way the data from multiple data sources was triangulated was by looking at the codes on student-participant work over time and to verify that they were consistent. Another way that the teacher-researcher used triangulation was by determining if codes present in the student-participant written reflections were present in the sample student-participant work and the teacher journal. This study did not triangulate using the traditional educational, social science, qualitative tradition but it did so within the action research tradition (Mertler, 2014). The primary mode of data collection was the student-participant written reflections. As themes emerged (as part of the constant comparative method) in the student-participant written reflection data, they were cross-referenced (triangulated) with the student-participant work and the teacher journal. This was done to ensure that the student-participant work data and the teacher journal data did not contradict the themes. In addition, by cross-referencing the themes that emerged from the student-participant written reflections with corresponding information from the other two data sources, the teacher-researcher was able to strengthen and provide additional support for the narrative (the results) that emerged from the data analysis.

In order to strengthen the reliability of this study, the teacher-researcher kept a record, an audit trail (detailing "how data were collected, how categories were derived, and how decisions were made throughout the inquiry" (Merriam, 2009, p. 213). Keeping this log served as a record of the specific methods used in the study so that the results could be replicable. In addition, the teacher-researcher recorded his interactions with the data in a log to help provide further detail in the audit trail.

A detailed and descriptive account of the findings will be provided in the results chapter (Chapter Four). The attention to detail in this description allows the reader to potentially compare the findings of this DiP to their own experiences, to their own study, and to other studies (Lincoln & Guba, 1985). In other words, the external validity of this is study is bolstered by this rich description since other researchers can then use it to frame or interpret the results of her or his own research.

Research bias and assumptions. The teacher-researcher in this study was the teacher of the students who were the participants in this study. As a teacher, the teacher-researcher's bias was to help the student-participants meet the goals of his instruction. In a quantitative study, this may have biased the teacher-researcher toward findings that show that his method of instruction (using the Five Key Formative Assessment Strategies (Wiliam & Thompson, 2007)) caused student achievement gains (improved student mathematical explanations). However, in this qualitative study, the teacher-researcher was attempting to capture what it was about those assessment strategies that impacted student mathematical explanations and how it was taking place. The teacher-researcher was interested in what worked, what did not work, and what did not matter, so that he could improve his own practice and share those effective practices with the larger mathematics education community. In this way, the teacher-researcher's bias as a teacher served to help him be a better observer and gave him an overall sense of the larger context from which the data came. As a researcher, the teacher-researcher may have been biased by what he knew about particular students and other external factors (*i.e.*, a fight at lunch or classroom disruptions) that could impact how he looked at the data. Because of this bias, and also to maintain internal validity, it was important that the teacher-researcher read student-participant work and student-participant explanations anonymously. To aid this, student-participants were assigned an identification code so that data from a particular student-participant could still be tracked, but, the teacher-researcher was blind to the identity of the individual student. That said, the teacher-researcher could occasionally recognize student-participant work and student-participant written reflections as coming from certain individuals. To control for this, the teacher-researcher was fastidious in the procedures he used for coding, and, to the best of his ability, did not allow his personal bias to affect his coding of the data. The teacher-researcher also needed to be sensitive to patterns or themes in the data which may have reflected negatively on him as a teacher or as a person, and to code those using the same procedures and rationale as the rest of the data. The teacher-researcher's goals were to do what was best for his students and to reflect on ways to improve his practice. In his role as the teacher-researcher in this study, the teacher-researcher had the opportunity to meet those two goals by approaching the data in an objective manner to help his students and to find ways to improve his own teaching practice.

Action Research Reflection Plan

The teacher-researcher plans to engage in ongoing self-reflection and to engage in reciprocal reflection with the student-participants both during and after this DiP study. The sections that follow describe the actions that the teacher-researcher planned to take as a teacher and the actions that he planned to take with student-participants based on this DiP study.

Teacher actions. This section focuses on the actions that the teacher-researcher planned to take to improve his own practice as a result of this DiP study. Storey *et al.* (2014) have indicated that it is important to show "clear evidence of impact on practice" (Discussion section, para. 4) in a DiP and that impact can include "decisions" and "changed practices" (Key Principles and Components of an Innovative DiP section, para. 5). As part of the Action Plan, the teacher-researcher planned to look for evidence in the teacher journal of changes in practice as a result of his engagement as the teacher-researcher in this study. In addition, he reflected on ways to improve his teaching practices as a result of his participation in this DiP study. These two actions are aligned to Storey, Caskey, Hesbol, Marshall and Maughan's (2014) study about action research methodology that results in "motivating and guiding change with evidence, arguments and values" (p. 326).

The teacher-researcher also plans to include evidence of any changes to decisions made in his classroom (or as a professional development provider) and changes to his teaching practice at the conclusion of this study. As a practical matter, even though data collection for this DiP study will end, the teacher-researcher's teaching will continue and his teaching practices may change as he continues to reflect on the findings of this study. In accordance with the action research methodology, "because practitioner-scholars are client-centric and perform research *in situ*, they are positioned to use a methodology that allows iterative experimentation" (Storey & Maughan, 2016, p. 225). For this reason, the teacher-researcher will continue to keep a teacher journal (field notes) until data analysis is complete, the findings are determined, and the implications of those findings are established. In the spirit of action research, the teacher-researcher plans to disseminate these early findings through: 1) professional conferences (the teacher-researcher will be presenting a session at the National Council of Teachers of Mathematics' (NCTM) 2017 annual conference), 2) professional development workshops for high school mathematics teachers at NCTM's 2017 winter institute, and 3) his leadership roles with the NCTM and the College Board (Mertler, 2014).

Reflecting with students. The previous section described the ways in which the teacher-researcher planned to reflect on his own teaching practice (he was a study participant) based on the results of this action research study. This section will discuss how the teacher-researcher planned to share the results of this study with his students. When the teacher-researcher creates a classroom environment that "encourages and facilitates reflection" (Wheatley, 1992, p. 540), the teacher-researcher is providing students with an environment where effective instruction can occur (Wheatley, 1992; Ladson-Billings, 1995; NCTM, 2014).

In this study, student-participants provided written reflections and mathematical explanations (in the form of student-participant work) which was then analyzed by the teacher-researcher. Using the aforementioned two data sources and a teacher journal, the teacher-researcher looked for evidence of mathematical thinking while using the following Five Key Formative Assessment Strategies in his instructional practice:

1. *Clarifying and Sharing* learning intentions and criteria for success;
2. *Engineering Effective Classroom Discussions* and other learning tasks that elicit evidence of student understanding;
3. *Providing Feedback* that moves learners forward;
4. *Activating Students as Instructional Resources* for one another; and

5. *Activating Students as Owners* of their own learning. (Wiliam & Thompson, 2007, p. 53)

The teacher-researcher shared these formative assessment strategies with the student-participants and had them posted in his classroom for the duration of the study. The purpose of this was four-fold: 1) to help students be aware of the teaching methods that were being used; 2) to let students know what the teacher-researcher was attempting to do in his pedagogical approach; 3) to help students reflect on their own practice as learners; and 4) to help prepare students to learn and to engage them in learning about the results of this action research study.

The constant comparative approach to this study allowed the teacher-researcher to reflect on the results of this study while the study was ongoing. Just as the teacher-researcher was able to reflect on and make changes to his teaching practice during the study, he was also able to share those changes or the rationale for those changes with the student-participants. In addition, during the study, he shared any emergent results about the Five Key Formative Assessment Strategies with the student-participants. This had the potential to help the students be more reflective in their reflections, create better mathematical explanations or to engage in peer-to-peer discourse that helped move their learning forward or move the teacher-researcher's pedagogical practice forward. By sharing the results with students, the teacher-researcher hoped to gain (*i.e.*, from student-participant written reflections or during classroom discussions) insight into his own practice. In addition, when the analysis and interpretation of the data in this study are complete, the teacher- researcher had planned to present the results, in an age-appropriate way, with his student-participants.

The teacher-researcher planned for the focus of the presentation to student-participants to be the two formative assessment strategies of “activating students as instructional resources for one another and activating students as owners of their own learning” (Wiliam & Thompson, 2007, p. 53). The rationale for this is that these two strategies are the strategies that are directly relevant to the student-participants in this study. The teacher-researcher also planned to attempt to relay any useful strategies, suggestions, advice or tools that emerge from the findings of this study to the student-participants. Furthermore, the teacher-researcher planned to take actions on any of the implications relevant to student-participants in this study during the regular course of his instruction for the remainder of the school year (*i.e*., he may develop graphic organizers related to the findings or reference specific ideas from the findings or engage in other pedagogical decisions that stem directly from the findings of this study). While planning to make instructional decisions may represent a plan of indirect actions to be taken by the teacher-researcher to share the results of this study with his students, the fact remains that these actions will represent another way that the teacher planned to share his findings with his student-participants (through his actions). The implications section of the next chapter will focus on both implications for teacher practices and student practices that can help move learning forward in the classroom. In essence, the implications will, themselves, form the basis for a plan of action.

Conclusion

This DiP study investigated how the use of Wiliam and Thompson’s (2007) Five Key Formative Assessment Strategies impact students’ mathematical thinking using qualitative action research methodology. The teacher-researcher chose to use the Five Key Formative Assessment Strategies (Wiliam & Thompson, 2007) in his instruction as a way to support students as they learned mathematics and learned to explain their mathematical thinking. The qualitative nature of this study sought to identify specific ways that the use of formative assessment strategies impacted students' thinking and to provide evidence for the teacher-researcher to improve his own practice. The use of action research methods allowed the teacher-researcher to study his own practice and how it impacted the mathematical thinking of student-participants in his own classroom. In addition, action research methodology provided the teacher-researcher with a mechanism to enact change both during the implementation of and after the implementation of this study. Further, by doing research in his own classroom and with his own students, the teacher-researcher is able to authentically learn from this study as both a teacher and as a researcher; the student-participants are also able to benefit from the teacher-researcher actions in both of these roles through the reciprocal reflection process. The next chapter, Chapter Four, describes the findings and the implications of the findings of this DiP.

CHAPTER FOUR: FINDINGS AND IMPLICATIONS

Introduction

Chapter Four presents the findings of a teacher-researcher conducted qualitative action research study of two of his high school geometry classes as a way to understand the impact of Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies on the mathematical thinking of his students. The identified problem of practice for the study centered on his high school geometry students who routinely did not show evidence of their mathematical thinking on mathematical tasks and/or had not been provided with opportunities to show evidence of their mathematical thinking while working on or after working on mathematical tasks. To conduct the research, the teacher-researcher changed his instructional practice to include the Five Key Formative Assessment Strategies (2007) as his pedagogical approach. He collected data to study the impact of this pedagogical approach on the mathematical thinking of his student-participants in the fall of 2016. He used the constant comparative method to code and to analyze the data (Merriam, 2009; Mertler, 2014). The themes that emerged from this analysis include: 1) Student participation in problem solving, 2) Social and intrapersonal factors in the classroom environment, 3) Pedagogical factors in the classroom environment, and 4) Future learning needs.

Research Question

1. How does the use of Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies impact students' mathematical thinking?

Findings of the Study

This section describes the participants in this qualitative action research study, the data analysis strategy and the findings of the study. The interpretation of the findings of the study is provided in the 'interpretation of results of the study' section that follows immediately after this one.

Participants

The present action research study took place in two geometry classes at a southern Title I public high school of approximately 800 students. The mathematics courses taught at this school follow a traditional sequence (Algebra I, Geometry, Algebra 2) sequence. Students attended classes that met for 88 minutes on every other day and all have access to personal computers (Chromebooks) provided by the school and had excellent internet access during the school day.

The study took place with students in the teacher-researcher's two honor's geometry classes over the course of eight weeks (see Appendix A for the research planning schedule) during the fall of the 2016-2017 school year. All students from each of the two classes gave assent students and are referred to as the student-participants in this study (see Appendix B for a copy of the assent document). The teacher-researcher was teaching several different courses at this school and has taught a variety of other courses throughout his teaching career. The rationale for selecting the honors geometry classes is that honors geometry represents the class that the teacher-researcher has taught the most frequently. The rationale for not selecting all of his mathematics classes was to allow sufficient time for planning and reflection for the teacher-researcher since this study is taking place *in situ*. The numbers of students in the classes studied were 28 and

24, respectively, and all students were assented – all of the students in the two classes are the student-participants in this study. The teacher-researcher collected student-participant written reflections and sample student-participant work on six occasions during the eight weeks of this study. The teacher-researcher also kept field notes in a teacher journal for the duration of the study. Student-participant written reflections, typed by student-participants in Google Forms, were the primary source of data. He collected a total of 295 (an average of 49 during each of the 6 rounds of data collection) written reflections out of a possible 312 (52 student-participants times 6 rounds of data collection). The reason that 17 written reflections were missing was due to student-participants being absent from class (there were 17 total student-participant absences). The teacher-researcher collected an identical amount of sample student-participant work. Six student-participants were absent during the first round of data collection, zero student-participants in the second round, three in each of the third and fourth rounds, two in the fifth round and three in the sixth round. Overall, the attendance rate was just about 94.5% and the teacher-researcher collected 100% of the student-participant written reflections and sample student-participant work from the students that were present in class.

Data Analysis and Coding Strategy

The primary data source in this study was student-participant written reflections and secondary sources were sample student-participant work and the teacher journal (field notes) (see Chapter Three for a detailed description of each data source). The constant comparative method was used to analyze the data (Merriam, 2009; Mertler, 2014). This method involved ongoing and continuous comparison of data sets throughout the collection and analysis phases. Student-participant written reflections were compared to teacher journal entries from classroom observations on the same day and to student-participants' written reflections from each of the different days within the 8-week time frame of data collection over the Fall 2016 semester (Saldana, 2009). Throughout the constant comparative data analysis process (Merriam, 2009), the teacher-researcher coded the data line by line using Saldana's (2009) method of combining two structural coding methods (the First Cycle Initial Coding method and the First Cycle Descriptive Coding method). He then used the Second Cycle Pattern Coding method (Saldana, 2009) to organize the data into patterns which eventually became the themes of this study.

During First Cycle Coding, he used a "word or short phrase" (Saldana, 2009, p. 73) to describe the data or to note a particular "process" (p. 84). The data was coded line by line to allow the teacher-researcher to "reflect deeply on the contents and nuances" (p. 81) in the data. The teacher-researcher then recorded these codes (see Appendix D) in a code book and read again and reread the coded data corpus in an attempt to gain "an organizational grasp of the data" (Saldana, 2009, p. 73). During this process, he recorded analytic memos which included any patterns (*i.e*., categories) that he observed. He repeated this process several times. After this cyclical process was complete, he reviewed his memos and the codes were then organized into categories (see Appendix D) and compared to each other. He then imported the data into a computer program NVivo 11 and recoded the data using categories (see Appendix D). Both during and after the coding process, he used text queries and word frequency queries to help look for patterns both within and between categories and to focus on keywords to verify the completeness of the First Cycle Coding categories. In addition, he used NVivo 11's "coding stripe" feature to visually identify and view overlap between the categories. He recorded any patterns and other notable features of the data in analytic memos.

The categories that resulted from the First Cycle Coding methods were then organized into Pattern Codes (see Table 4.1) using the Second Cycle Pattern Coding

Table 4.1 First Cycle Coding Categories Organized by Pattern Code

Participating in Problem Solving	Social & Intrapersonal Factors	Pedagogical Factors in the Classroom Environment	Future Learning Needs
Being precise	Metacognition	Teaching concerns	Mathematical content
Justifying thinking	Motivation & affect	Time	Practice
Patterns	Presence of mind		Vocabulary & terminology
Understanding & solving the problem	Reciprocity		
Using Tools			

method (Saldana, 2009). To aid in this process, the teacher-researcher created a concept map of the categories to better see the relationships both within and between the data. In the Second Cycle Pattern Coding process, the data (already First Cycle-coded in NVivo 11) was recoded with the pattern codes in an attempt to “develop a coherent synthesis of the data corpus” (Saldana, 2009, p. 149). He looked for themes and patterns in the data coded with each pattern code and compared it to data in the other pattern codes. He kept analytic memos during this process to capture themes and relationships as they emerged in the data. The themes that emerged from the three sources of data and the analytic memos were reviewed and compared, and then they were triangulated. Four interrelated themes about student-participant mathematical thinking emerged from the data analysis. The four themes are: 1) student-participant participation in problem solving, 2) social and intrapersonal factors in the classroom environment, 3) pedagogical factors in the classroom environment and 4) future learning needs. The four themes and detailed descriptions of the four themes that follow constitute the findings of this DiP. The ‘findings’ sub-section below presents an organized description of the analyzed data by theme and is follow by a an ‘interpretation of results of the study’ section that serves to relate the analyzed data presented in the ‘findings’ sections to the study’s identified problem of practice, to the study’s research question and to the purpose of the study.

Findings

This section presents the four themes that represent the findings of this study using a conceptual model (see Figure 4.1) to visually represent how the themes are interrelated followed by a written description of the data. In Figure 4.1, ‘student participation in problem solving’ is in the middle and is surrounded by the ‘social and interpersonal factors’ (Reciprocity, Metacognition, Presence of Mind, Motivation and Affect) to indicate that problem solving happens within the context of those factors. ‘Student participation in problem solving’ is connected with two-way arrows to the ‘social and intrapersonal factors’ to indicate that the themes are influenced by each other. Reciprocity; Metacognition; Presence of Mind; and Motivation and Affect are connected with dashed two-way arrows to indicate that there is preliminary evidence to show that they are related. The shading (the light gray area) in the top part of Figure 4.1 serves to indicate that there were ‘pedagogical factors in the classroom environment’ that were happening while student-participants were problem solving and thinking mathematically. The ‘future learning needs’ in the lower right corner summarizes what student-participants wrote in their written reflections and could help them explain their reciprocal and metacognitive mathematical thinking in the future. A detailed description of each theme follows.

Theme one: Student participation in problem solving. Theme one, student

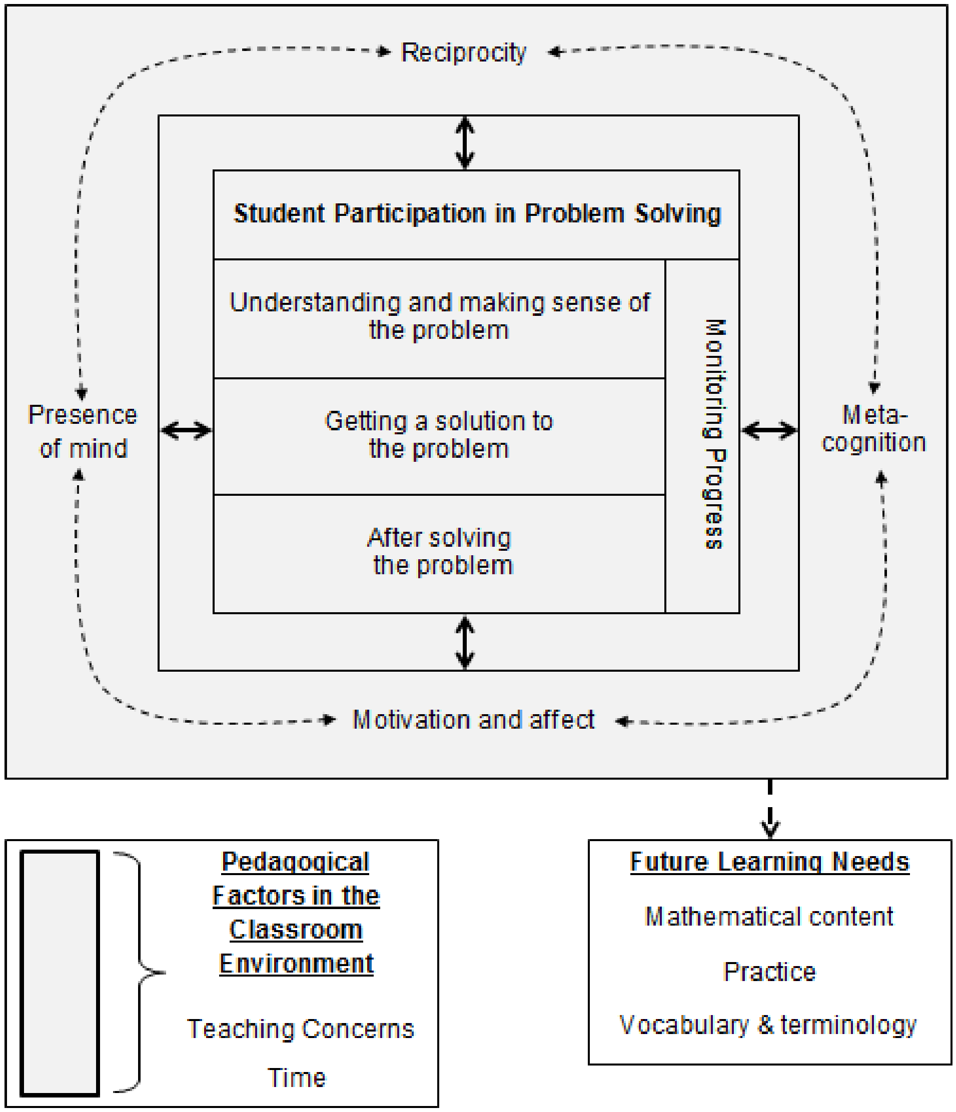

Figure 4.1 Problem solving and mathematical thinking in the classroom. The relationships between four themes: 1) student-participant participation in problem solving, 2) social and intrapersonal factors in the classroom environment (reciprocity, metacognition, presence of mind, and motivation and affect), 3) pedagogical factors in the classroom environment, and 4) future learning needs.

participation in problem solving, emerged during Second Cycle Coding as the result of analyses which stemmed from assembling the following categories into a Pattern Code: being precise, justifying thinking, patterns (finding, looking for and using), understanding and solving the problem, and using tools. This theme is organized into four sub-themes. The four sub-themes are that student-participants: 1) made sense of problems using prior knowledge and figured out how to get started solving the problem, 2) solved problems (and thought about) solving problems in a variety of ways while using a variety of tools, 3) explained or made sense of their mathematical solution or solution path in a different ways, and 4) monitored their progress during the problem solving process.

It is important to note in the findings for this theme that the categories used in this theme came from an analysis of more than forty different codes (see Appendix D) from the application of First Cycle Coding methods (Saldana, 2009). In addition, this theme appeared throughout the student-participant written reflections and sample student-participant work for almost every one of the student-participants (97.2%) during each of the six weeks of data collection. Out of the 295 opportunities for this theme to appear in student-participant written reflections (52 student-participant written responses each week times 6 equals 312, minus the 17 missing responses due to absences gives 295), it appeared in 287 of them. The eight exceptions were student-participant written reflections that did not specifically relate to the mathematics (*i.e.*, “we took notes” or “you reviewing helped me”). When the teacher triangulated this with the sample student-participant work, the theme was found to be applicable in all 295 samples (there was evidence of problem solving in all of the sample student-participant work). ‘Student participation in problem solving’ is a robust finding and because it is based on the presence of more than 40 First Cycle codes it covers substantial conceptual territory. The description of the data for ‘student participation in problem solving’ that follows is organized by the four sub-themes described above. The data from student-participant written reflections are summarized in Appendix E.

Understanding and making sense of the problem. To understand and make sense of the problem, student-participants reported using prior knowledge and having to figure out how to get started (see Appendix E for a summary of the data). In their written reflections, student-participants reported several ways of using their prior knowledge of mathematics. Student-participant prior knowledge was found to come from both internal sources ('recalling and remembering facts' and 'thinking') and external sources (notes). Student-participants also described mathematical content that they used as prior knowledge (see Appendix E for descriptions). While understanding and making sense of the problems, student-participants also indicated, in their written reflections, that they had to figure out how to get started (see Appendix E for descriptions of how student-participants got started). Student-participants talked about using a variety of tools to help make sense of the problem. These included the use of color, pencil and paper, dynamic geometry software, tables, lists, physical models and drawings. It is important to point out that no (zero) student-participants wrote in their written reflections that they used their peers or the teacher to help them understand or make sense of the problem. The teacher field journal indicates that student-participants did ask the teacher-researcher clarifying questions while beginning to solve problems and that he either addressed those directly to the individual or to the whole class.

Getting a solution to the problem. While getting a solution to the problem, student-participants reported that they figured out how to get started, used tools to solve problems, and took a variety of actions to solve problems – they thought about and solved the problem in different ways (see Appendix E for a summary of the data). The data includes details about the various ways that student-participants figured out how to get started on solving a problem. The various ways were: deciding what tools to use, deciding what to look for in the problem, devising a plan for solving the problem, organizing thoughts, and using critical thinking. There is some overlap between how the student-participants were 'figuring out how to get started' and how the student-participants 'figured out how to get started' since the former refers to the data where student-participants wrote about what they were thinking about doing and the latter refers to the data where student-participants wrote about what they actually did.

Student-participants used a variety of different tools to solve problems (see Appendix E for a list of the tools). The different tools that student-participants reported using in the student-participant written reflections were confirmed through triangulation with the teacher journal and the sample student-participant work. In addition, student-participants took a variety of actions to solve problems (see Appendix E for a list of the actions). The following 5 actions reported by student-participants were actions that could not be confirmed in the sample student-participant work because they are internal (hidden) processes: 'looking beyond the obvious,' 'thinking critically,' 'thinking from a different perspective,' 'thinking outside the box,' and 'visualizing'. The other 12 actions reported by student-participants (as listed in Appendix E) were confirmed, by triangulation, using both sample student-participant work and the teacher journal. Student-participants also reported that while they were taking the actions to solve problems that they were also engaged in some 'other processes' (*i.e.*, 'being careful,' 'concentrating,' 'organizing') related to mathematical thinking (see Appendix E for a complete listing). ‘Being methodical’ is one of the ‘other processes’ reported by student-participants and it includes the following descriptions: counting clockwise (or counter-clockwise), creating a color scheme, counting the big part then the little part, keeping a tally, and making an organized list. It is important to note that the ‘’other processes’ come from the descriptions provided by student-participants in their written reflections and represent reifications of internal (intrapersonal) processes, with one exception. The one exception to this is ‘reciprocity’ which is a social (interpersonal) process and, as such, was verified in the teacher journal. ‘Reciprocity’ included: asking for help from another student, discussing what one is communicating with peers, explaining one’s thinking (to the teacher or other students), getting feedback from the teacher, learning about someone else’s method, and looking at someone else’s work.

After solving the problem. In their written reflections, student-participants reported a variety of actions taken (*i.e.*, checking, reviewing, reflecting, thinking) to explain or make sense of their mathematical solution or solution path (see Appendix E for a complete summary of the actions taken). Student-participants were also found to engage with their own solution and, at times, with the solutions of other student-participants in the class. The actions student-participants took after solving problems took the form of individual practices (*i.e.*, going back, making sure) and social practices (*i.e.*, comparing with others, discussing). Some student-participants reported doing a combination of both individual and social practices. This subsection provides evidence for the claim that student-participants explained or made sense of their mathematical solution or solution path in different ways.

***Monitoring progress*.** This subsection, monitoring progress, discusses evidence (see Appendix E for a summary of the data) that student-participants monitored their progress while getting started, during, and after problem solving. While doing the analysis and reporting the findings for this theme 'student participation in problem solving,' the teacher-researcher began to notice another trend in the data. He noticed that student-participants described different parts of and actions they took during the problem solving process as "hard" or "easy." He recorded this trend in an analytic memo for future review. After completing an analysis of the other themes, he revisited the data already coded as 'student participation in problem solving' and coded it for "hard" and "easy." Student-participants, in their written reflections, described different parts of the problem solving process as "being hard" or "getting harder" on 31 occasions. They talked about it "being easy" or "getting easier" on 29 occasions. The student-participants made statements about how things were going during the problem solving process in either absolute terms (hard/easy) or relative terms (harder/easier). This finding, 'monitoring progress,' provides evidence for the claim that student-participants were monitoring their progress during the problem solving process.

Summary of theme one. In sum, the theme 'student participation in problem solving' found that student-participants: 1) made sense of problems using prior knowledge and figured out how to get started solving problems, 2) solved problems (and thought about) solving problems in a variety of ways while using a variety of tools, 3) explained or made sense of their mathematical solution or solution path in different ways, and 4) monitored their progress during the problem solving process.

Theme two: Social and intrapersonal factors in the classroom environment. This theme, 'social and intrapersonal factors in the classroom environment,' emerged during the Second Cycle Coding process as the result of analyses which stemmed from assembling the following categories into a pattern code: Presence of Mind, Motivation and Affect, Metacognition, and Reciprocity. Theme two was coded 181 times in the student-participant written reflections and the four categories emerged, upon analysis, into four sub-themes with the same names: 1) Reciprocity, 2) Motivation, 3) Presence of Mind, and 4) Motivation and Affect.

***Reciprocity*.** Student-participants reported working with their peers and the teacher-researcher to solve problems (see Appendix F for a summary of the data). The student-participant written reflections were coded with the sub-theme Reciprocity 63 times. Of these 63 instances, 51 of them referenced working with peers (other student-participants) and 12 of them referenced working with the teacher-researcher. Evidence of Reciprocity in the form of working together with peers and working together with the teacher-researcher was also found in the teacher journal. Throughout this study, student-participants were regularly instructed by the teacher-researcher to talk to each other in structured ways (*i.e.*, think-pair-share, turn-and-talk), to look back at the problem, explain their mathematical thinking, and to discuss their work or their mathematical thinking with other student-participants.

Student-participants reflected on Reciprocity with peers in many ways (see Appendix F for a complete summary of the ways). Student-participants reported Reciprocity with peers in three major ways: 1) 'looking at work,' 2) 'talking,' and 3) 'listening.' Overall, these social interactions (Reciprocity with peers) around problem solving and mathematical thinking were described in the written reflections as beneficial (helpful) to the student-participants. Of the 51 times that Reciprocity with peers was coded, only twice was Reciprocity with peers reported as not being beneficial. The two instances were ‘being annoyed and discouraged by what others say’ and ‘being confused about peers all having different answers and not agreeing on one.’ The teacher-researcher also wants to draw attention to the directionality of the peer interactions listed above. There were three main ways that the interactions occurred: 1) from one peer to another peer or peers (*i.e.*, ‘talking to’ or ‘looking at’), 2) to one peer from another peer or peers (*i.e.*, ‘getting help from’), and 3) as a bi-directional interaction to/from one to/from another peer or peers (*i.e.*, ‘sharing answers with peers’ and ‘discussing with peers’). ‘Discussing solving the problem with peers then revising one’s own approach’ and ‘deciding if one’s own way or a peer’s way is better’ are examples that show student-participants taking action (‘revising one’s own approach’) or comparing methods (ways) that resulted from interactions (*i.e.*, talking to, listening to, seeing the work of) with peers. This finding, Reciprocity with peers, provides evidence for the claim that student-participants interacted with a peer or peers while problem solving and thinking mathematically.

Student-participants also described instances of Reciprocity with the teacher-researcher (see Appendix F for descriptions). Upon triangulation of the data, the teacher journal confirmed these findings and also noted that the teacher-researcher also directed student-participants to look at their own work, to think about the problem again (or more closely) and to discuss the problem with a peer or peers. This finding, Reciprocity with the teacher-researcher, provides evidence for the claim that student-participants interacted with the teacher-researcher while problem solving and thinking mathematically.

Metacognition. This sub-theme, Metacognition, of theme two is focused on those occasions ('metacognition' was coded 72 times in the student-participant written reflections) during which student-participants wrote about thinking or the thinking process. The first theme presented in this dissertation was 'student participation in problem solving' which detailed what student-participants did while they were problem solving. The first theme provided evidence from the student-participant written reflections, the sample student-participant work and the teacher journal that student-participants were thinking about the problems they were solving – the findings in theme one are evidence that student-participants were engaged in mathematical thinking (mathematical thinking is defined in this DiP as "an individualized form of (intrapersonal) communicating" (Sfard, 2008a, p. 81) about mathematics). This sub-theme, Metacognition, describes evidence of student-participants thinking about their mathematical thinking (see Appendix F for a summary of the data).

Student-participants wrote about their Metacognition in a variety of ways (see Appendix F). The ways that student-participants described Metacognition in a variety of ways that can be categorized as follows: 1) describing what they thought about, 2) thinking about one's problem solving process, 3) using one's prior knowledge, 4) thinking about what one knows or does not know, 5) thinking in different ways, 6) thinking differently, 7) getting one's thoughts into words or on paper, 8) describing the type of thinking one did, and 9) the intensity of one's thinking. An outlier was that one student-participant expressed that it was important to have "your own logical way of thinking."

In general, the student-participant responses included in the sub-theme Metacognition were almost exclusively about the intrapersonal thoughts of the individual student-participants, but, sometimes the responses involved the interpersonal (social). The student-participant that wrote "to ask questions so I know I am doing it right" clearly looked outside the intrapersonal to the social (the interpersonal) by asking questions to confirm (the intrapersonal) that she was "doing it right." Another student-participant wrote that she "gave a logical explanation that could help someone else find the same thing and or come to the same conclusion based on my reasoning." While this statement refers to the student-participant's intrapersonal thoughts, it does so in a way that invokes the interpersonal (social). This student-participant wrote that she thought about how someone else would interpret her explanation while she wrote it. The teacher-researcher recorded in his teacher journal that he had told students that: "someone else should be able to read it [the mathematical explanation] and arrive at the same solution using the same steps as you did." He did this to clarify his learning intentions for student-participants and to help the student-participants understand his expectations for how they should write their mathematical explanations. "Clarifying and sharing learning intentions" (Wiliam & Thompson, 2007, p. 53) key formative assessment strategies the teacher-researcher used in his instruction with the goal of helping student-participants to develop their mathematical thinking. Another student-participant wrote that "multiple and creative ways of thinking helped us solve problems." This student-participant appears to have been thinking about what helped him to solve problems and what could help others to solve problems. The three examples of student-participant thoughts about mathematical thinking detailed above offer a glimpse into the connectedness of the social (interpersonal) and the intrapersonal.

To summarize, the sub-theme Metacognition provides evidence for the finding that student-participants thought about their mathematical thinking while solving problems and that social factors played a role in the mathematical thinking of some student-participants.

Presence of mind. Student-participants reported that they showed Presence of Mind while working through the problem solving process (see Appendix F for a summary). In the student-participant written reflections Presence of Mind was coded 27 times. The student-participants wrote about Presence of Mind as being important or helpful while working through mathematical tasks. The data indicate that student-participants viewed concentration, focus, paying attention and reading carefully as an important or helpful part of solving problems and thinking about mathematics. The teacher journal was used to triangulate this finding. It indicated that the teacher-researcher had to quiet down the class on several occasions because student-participants indicated that they needed quiet or needed the class to be less loud – this provides anecdotal support for the finding that Presence of Mind was important to student-participants during the problem solving process and while engaging in mathematical thinking. It also suggests that the social nature of the classroom (*i.e.*, students talking in the classroom) plays a role in the Presence of Mind of student-participants during the problem solving process and while thinking mathematically – the social (interpersonal) nature of the classroom impacts the mathematical thinking of student-participants.

Motivation and affect. The findings in this theme, 'social and intrapersonal factors,' which have been discussed to this point are related to the sub-themes Reciprocity, Metacognition, and Presence of Mind. This sub-section discusses the portions of the data that were coded in the sub-theme Motivation and Affect.' This code appeared 16 times in the student-participant written reflections. The data for this sub-theme (see Appendix F) includes all 16 student-participant responses in an effort to preserve both the nuance and the diversity of the student-participant responses.

Before giving more details about the 16 student-participant responses, it must be noted that the teacher-researcher originally located the code "being challenged" (51 occurrences in the student-participant written reflections) in the category (now sub-theme) Motivation and Affect, but, later removed it from the category. The rationale for this was that the word 'challenging' was used in the prompt for the student-participant written reflections and the nature of how student-participants were using the word was unclear to the teacher-researcher and to critical friends. In addition, the teacher-researcher also asked several student-participants about their use of the word 'challenge' (this process is referred to as member checking (Mertler, 2014)) and this confirmed that student-participants were using 'challenging' in different ways. One student-participant indicated that the only reason she used it was because it was in the prompt. However, one student-participant did appear to use 'challenging' in a way that was relevant to Motivation and Affect when she said "it was challenging to do some things alone."

The summary of each of the 16 occurrences of Motivation and Affect (see Appendix F) touch on a variety of human experiences that did not fit into Reciprocity, Metacognition or Presence of Mind, but, which were still relevant to the theme, 'social and intrapersonal factors.' In 3 of the 16 occurrences of Motivation and Affect, there appears to be evidence of the role of social and intrapersonal factors. The three occurrences are: 1) being annoyed and discouraged by what others say, 2) noticing problem solving getting easier while working with others, and 3) thinking it was "cool" that people used color. The other 13 occurrences of Motivation and Affect have only evidence of intrapersonal factors (note: the teacher-researcher is not saying that social factors are not involved in the 'other 13,' but, that there is no evidence for it in the 'other 13'). The first occurrence that suggests evidence of the role of social and interpersonal factors comes from a student-participant reporting 'being annoyed' (this relates to affect) and 'discouraged' (this relates to motivation) by others (this is social). The student-participant wrote "it honestly annoys me when people whine, 'I don't get it!!!' because the rest of us our trying to work and we really don't need your out loud exclamation of not getting it, because that just gets me even more discouraged if I can't get it." This quotation, on its own, speaks to how complicated the emotional landscape (sociocultural landscape) of the classroom (a social setting) can be and how it can have an affective and motivational impact on the individuals within that setting. The second occurrence, comes from a student-participant who reported "problem solving getting easier while working with others," deals with motivation in a social setting because it implies that the student-participant changes her perception (important to motivation) when she works with her peers. Finally, in the third occurrence, another student-participant writes that she thought another student-participants' method was "cool." This statement implies that the actions of another student-participant had a positive impact on her mood (affect) or outlook (this is related to motivation and possibly outlook (Schwarz & Clore, 2007)).

This sub-theme, Motivation and Affect, offers evidence that motivation and affect played a role in the social and intrapersonal factors that impact the problem solving process and mathematical thinking of student-participants.

Theme three: The role of pedagogical factors in the classroom environment. Theme three, the role of pedagogical factors in the classroom environment, discusses data (see Appendix G for a summary of the data) related to pedagogical factors in the classroom environment that student-participants reported as having an impact on mathematical thinking. These factors included issues related to instructional decisions (pedagogy) and timing. This theme emerged during Second Cycle Coding as the result of analyses which stemmed from assembling the following categories into a pattern code: teaching concerns and time (or timing). As a category, 'teaching concerns,' refers to an instance or instances (18 total occurrences) in the classroom where an instructional (or pedagogical) decision made by the teacher-researcher impacted the mathematical thinking of student-participants. The category 'time' refers to portions of the data (6 total occurrences) in which student-participants indicated that time or timing had an impact on the mathematical thinking of student-participants. The categories, 'teaching concerns' and 'time' are the two sub-themes of theme three.

Teaching concerns. Student-participants wrote about 'teaching concerns' in their written reflections 18 times (see Appendix G for a summary of the data). The student-participants indicated that the teacher-researcher did not give clear directions, that 'taking notes' was reported as helpful on 9 occasions, that three student-participants reported needing a process, problem or topic illustrated (or explained), that one student-participant indicated that he needed to follow the teacher-researcher's directions, and that one student-participant indicated that the teacher-researcher could have done more explaining. The teacher journal indicates that the teacher-researcher did have to interrupt the whole class to clarify directions on two different occasions on one of the days (September 30, 2016) that student-participants wrote written reflections and this is the same day that 2 of the 3 comments about ‘directions’ were made. This finding indicates that some student-participants reported that they had difficulty with the instruction or pedagogy used in the classroom.

Time. The following statements describe what student-participants wrote in their reflections about ‘time’ (see Appendix G for a summary of the data). The teacher journal also indicates that time was an issue; sometimes this was expected or planned by the teacher-researcher and other times it was not. This finding provides evidence that ‘time’ impacted student-participants while solving problems or thinking mathematically – they indicated they needed more of it, needed information to be presented slower, or had other concerns with knowing that there were time limits.

Summary of theme three. *Theme* three, the role of pedagogical factors in the classroom environment, deals with factors (‘teaching concerns’ and ‘time’) in the classroom environment (instructional and pedagogical decisions, issues of time and timing) that are more structural (and external) in nature than those in the prior two themes, but, they are still relevant to the purpose of this study since there is evidence that they impacted the mathematical thinking of student-participants. In addition, this study did not collect data from student-participants about the classroom environment, nor did this study ask student-participants to comment on the instructional choices being made by the teacher-researcher. The fact that these findings emerged organically from the student-participants serves to highlight their importance in this study. This finding provides evidence for the claim that student-participants reported that instructional decisions (pedagogy) and time (or timing) impacted their mathematical thinking.

Theme four: Future learning needs. Theme four, future learning needs, emerged during Second Cycle Coding as the result of analyses which stemmed from assembling the following categories into a pattern code: mathematical content, practice, and vocabulary and terminology. After assembling these three categories, future learning needs was coded 93 times in the student-participant written reflections. The findings in this theme are that student-participants reported three undertakings (three sub-themes) that could help them to explain their own mathematical thinking in the future: 1) practicing more (n = 67), 2) learning mathematical vocabulary and terminology (n = 20), and 3) learning mathematical content (n = 5). In addition, four student-participants also included "studying" as something which could help them to explain their mathematical thinking in the future. The teacher-researcher did not include "studying" in the findings above because it could apply more broadly or narrowly to any of the three categories listed. A summary of the data for each of the three sub-themes is included in Appendix H.

Practicing more. Student-participant written reflections indicated that 'practicing more' could help them, in the future, to explain their mathematical thinking (see Appendix H for a summary of the data). "Practicing GeoGebra" was mentioned by just one student-participant and is included in this finding as an outlier. The teacher journal notes that several student-participants used GeoGebra software (free dynamic geometry software) to solve problems and that several student-participants expressed frustration about using it, but, there is no mention in the teacher journal about student-participants saying it could help them explain their mathematical thinking. This finding provides evidence (see appendix H) that student-participants indicated that practicing, practicing explaining, practicing solving problems, practicing writing, and practicing writing mathematical explanations could help them to explain their mathematical thinking in the future.

Learning mathematical vocabulary and terminology. The analysis of student-participant written reflections (see Appendix H for a summary of the data) indicated that student-participants reported that 'learning mathematical vocabulary and terminology' could help them to explain their mathematical thinking in the future. Student-participants' written reflections about 'learning mathematical vocabulary and terminology' communicate what it means to learn mathematical vocabulary and terminology; it means: 1) knowing more of it, 2) using it in one's own thinking, 3) remembering it, 4) understanding it or understanding it better, and 5) using it or using it more. This finding provides evidence (see Appendix H) that student-participants indicated that remembering, understanding and knowing more mathematical vocabulary or terminology could be important to explaining their mathematical thinking in the future.

Learning mathematical content. Student-participants, in their written reflections, indicated that 'learning mathematical content' would help them to explain their mathematical thinking in the future. This finding provides evidence (see Appendix H) that student-participants indicated that knowing or learning more mathematical content could help them to explain their mathematical thinking in the future.

Summary of theme four. The findings in this theme, future learning needs, provide evidence for the claim that student-participants identified practicing more, learning more mathematical vocabulary and terminology, learning more mathematical content and studying as undertakings which could help them to better explain their mathematical thinking in the future.

Summary of the findings by theme. Theme one, student-participation participation in problem solving, found that student-participants: 1) made sense of problems using prior knowledge and figured out how to get started solving the problem, 2) solved problems (and thought about) solving problems in a variety of ways using a variety of tools, 3) explained or made sense of their mathematical solution or solution path in different ways, and 4) monitored their progress during the problem solving process.

Theme two, social and intrapersonal factors in the classroom environment, found that student-participants: 1) interacted with a peer or peers while thinking mathematically, 2) thought about their mathematical thinking while solving problems, 3) reported that presence of mind was important during the problem solving process and while engaging in mathematical thinking, and 4) identified motivation and affect as having an impact on their mathematical thinking.

Theme three, pedagogical factors in the classroom environment, found that student-participants reported that instructional decisions (pedagogy) and time (or timing) impacted their mathematical thinking.

Theme four, future learning needs, found that student-participants identified that the following four activities could help them to better explain their mathematical thinking in the future: 1) practicing more, 2) learning more mathematical vocabulary and terminology, 3) learning more mathematical content, and 4) studying.

The next section of Chapter Four describes the implication of the findings and provides an interpretation of the findings of this DiP.

Interpretation of Results of the Study

The results of this study are interpreted by theme in this section for the purpose of answering the research question of this DiP. The findings in each of the themes will: 1) be discussed in relation to the existing research, 2) be contextualized in the teacher-researcher's instructional practice (his use of the Five Key Formative Assessment Strategies (Wiliam & Thompson, 2007)), 3) be explained in relation to this DiP's research question, purpose and problem of practice, and 4) include the actions that the teacher-researcher plans to take based on the interpretation of the results of the study.

Theme one: Student-participant Participation in Problem Solving

Theme one, student-participant participation in problem solving, found evidence that student-participants: 1) made sense of problems using prior knowledge and figured out how to get started solving the problem, 2) solved problems (and thought about) solving problems in a variety of ways while using a variety of tools, 3) explained or made sense of their mathematical solution or solution path in different ways, and 4) monitored their progress during the problem solving process.

The finding that student-participants participated in problem solving is critical to this dissertation's purpose because problem solving requires mathematical thinking. Wilson, Fernandez and Hadaway (1993) wrote:

> To many mathematically literate people, mathematics is synonymous with solving problems -- doing word problems, creating patterns, interpreting figures, developing geometric constructions, proving theorems, etc. On the other hand, persons not enthralled with mathematics may describe any mathematics activity as problem solving. (p. 57)

Problem solving is at the core of doing mathematics and thinking mathematically (Charles, Lester & O'Daffer, 1987; NCTM, 2014; Polya, 1945; Schoenfeld, 1985, 1992). The actions, reported in the findings of the study, that students took while solving problems are consistent with the "habits of mind" (Cuoco *et al.*, 1996, p. 377) that students (and mathematicians) use to develop their mathematical thinking. Cuoco *et al.* (1996) explain that students (and mathematicians) should engage in experiences that allow them to be: 1) "Pattern Sniffers", 2) "Experimenters", 3) "Describers", 4) "Tinkerers", 5) "Inventors", 6) "Visualizers", 7) "Conjecturers" and 8) "Guessers" (pp. 377-383). The descriptors that student-participants used to describe the actions they took while solving problems are also consistent with other findings related to mathematical thinking that were reviewed in Chapter Two of this DiP (NCTM, 2009; Schoenfeld, 1992; Stein, Grover, &Hennsington, 1996; Tall, 2011).

Throughout this study, student-participants engaged in problem solving and were found, as stated previously, to monitor their progress while making sense of the problem, solving the problem and reviewing the problem. This finding is evidence that student-participants were thinking mathematically and is consistent with Polya's (1957) four-phase problem solving process of "understanding the problem, devising a plan, carrying out the plan and looking back" (p. xvii). A difference between the findings of this DiP and Polya's four-phase process is that 'devising a plan' did not emerge in the findings of this DiP. It may be inferred that for a student-participant to solve a problem that the student-participant must have followed some type of plan, however haphazard it may or may not have been. In Carlson and Bloom's (2005) study of problem solving behavior, they found that evidence of 'devising a plan' (planning) was often expressed with non-verbal communication. This study found that students reflected on their solutions and this is consistent with Polya's fourth phase, 'looking back.' However, this study's findings related to 'monitoring progress' (being easy, being hard, getting easier, getting harder) during the problem solving process is about more than just 'looking back.' It is about keeping track of how the problem solving process is going while solving – it is metacognitive in nature (Bransford *et al.*, 2000; Schoenfeld, 1985, 1992). Eflkides (2011) discusses monitoring progress during problem solving as being "indicative of metacognitive awareness" (p. 9). The student-participants in this study were thinking mathematically and engaging in metacognitive processes while problem solving. Metacognition will be discussed in further detail in the sub-section on 'metacognition' as part of the interpretation of the results of theme two.

Theme one's findings include details (*i.e.*, checking, drawing, looking for a pattern, thinking critically, and thinking outside the box) about how students went about solving problems. The findings are evidence of students engaging in the Common Core State Standards for Mathematics' (CCSSM, 2009) eight Standards of Mathematical Practice:

1. Make sense of problems and persevere in solving them;
2. Reason abstractly and quantitatively;

3. Construct viable arguments and critique the reasoning of others;
4. Model with mathematics;
5. Use appropriate tools strategically;
6. Attend to precision;
7. Look for and make use of structure; and
8. Look for and express regularity in repeated reasoning. (p. 6)

These practices are described as “varieties of expertise that mathematics educators at all levels should seek to develop in their students” (CCSSM, 2009, p. 6). The implication that student-participants are engaging in the Standards of Mathematical Practice (CCSSM, 2009) during the problem solving process adds to the significance of the findings in theme one.

The student-participants in this study showed evidence of their mathematical thinking while problem solving during geometry class. This did not happen by chance – the teacher-researcher intentionally created a classroom environment where this could take place. He used Wiliam and Thompson’s (2007) Five Key Formative Assessment Strategies to design and carry out his instruction and to create an environment with the explicit intention of getting students learning about and showing evidence of thinking about mathematics. He took on this approach as a means to work towards a solution to this DiP’s problem of practice. The Five Key Formative Assessment Strategies he used in his instruction are:

1. *Clarifying and Sharing* learning intentions and criteria for success;
2. *Engineering Effective Classroom Discussions* and other learning tasks that elicit evidence of student understanding;

3. *Providing Feedback* that moves learners forward;
4. *Activating Students as Instructional Resources* for one another; and
5. *Activating Students as Owners* of their own learning. (Wiliam & Thompson, 2007, p. 53)

As part of his daily instruction, the teacher-researcher set a mathematical goal to have students justify their reasoning (*i.e.*, showing work, writing a mathematical explanation, or using mathematical terminology to explain one's thinking) and informed his students of the goal. He also provided various examples of what justifying one's reasoning looks like; he was "*Clarifying and Sharing* learning intentions and criteria for success" (p. 53). The teacher-researcher also selected tasks that could be solved in a variety of ways or that had multiple correct solutions and created a classroom environment to support student-participants as they worked through the tasks. The teacher-researcher structured the classroom environment so that student-participants had opportunities to share evidence of their mathematical thinking in the form of mathematical explanations. Their mathematical explanations were shared in writing, individually (in pairs or in small groups) with the teacher-researcher, with each other (in pairs or in small groups) and in whole class discussions; the teacher-researcher was "*Engineering Effective Classroom Discussions* and other learning tasks that elicit[ed] evidence of student understanding" (p. 53). He also allowed students to revise (and share again) their mathematical explanations; the sharing of mathematical explanations was one of the main ways that the teacher-researcher and the student-participants engaged in "*Providing Feedback* that moves learners forward" (p. 53). At the same time, he was "*Activating Students as Instructional Resources* for one another" (p. 53) and "*Activating Students as Owners* of their own learning." The teacher-researcher's intentional use of the Five Key Formative Assessment Strategies as the focus of his instructional practice is an inseparable part of the research design which was responsible for creating the data that yielded the result that student-participants participated in problem solving – that the student-participants showed evidence of thinking mathematically while problem solving. The teacher-researcher plans to continue to use the Five Key Formative Assessment Strategies in his practice.

Theme two: Social and Intrapersonal Factors in the Classroom Environment

Theme two, social and intrapersonal factors in the classroom environment, found evidence that student-participants: 1) interacted with a peer or peers while thinking mathematically (Reciprocity), 2) thought about their mathematical thinking while solving problems (Metacognition), 3) reported that presence of mind was important during the problem solving process and while engaging in mathematical thinking (Presence of Mind'), and 4) identified motivation and affect as having an impact on their mathematical thinking (Motivation and Affect). The implications of each of the four findings within this theme, enumerated above, are discussed in a separate subsection below. A discussion of the theme as a whole will follow the four subsections.

Reciprocity. This study found evidence that student-participants interacted with a peer or peers while thinking mathematically. This section discusses how this result fits into existing research in mathematics education, how the teacher researcher took action to facilitate Reciprocity (working together), and provides evidence that student-participant engagement in Reciprocity resulted in learning.

Having students work together (in "an ideal collaboration" (Horn, 2012, p. 6)) is the topic of Horn's (2012) book entitled *Strength in numbers: Collaborative learning in secondary mathematics*. She indicates that "collaborative learning environments" (Horn, 2012, p. 9) are part of the "broader landscape of equitable mathematics teaching: modes of instruction that optimally support meaningful learning for all students" (p. 9). The finding that students are engaging in acts of reciprocity (working together) is consistent with Horn's description of collaborative learning environments where students are "actively involved in making sense of the mathematics" (p. 5) and where "their confusions and disagreements become the basis for the instructional dialogue" (p. 5). Boaler (2016) recommends having students work together is a teaching strategy that helps pave the "path to equity" (p. 93).

The teacher-researcher, in an effort to get students to show evidence of their mathematical thinking (to address the problem of practice in this DiP), used the instructional strategy "*activating students as instructional resources*" (Wiliam & Thompson, 2007, p. 53). He consciously designed his instruction to allow student-to-student interactions and whole group discussions. To accomplish this he moved from a "focus on teacher talk during instruction" (Horn, 2012, p. 4) to a focus on student talk – he focused on having students "construct knowledge socially, through discourse, activity, and interaction related to meaningful problems" (NCTM, 2014, p. 8). He accomplished this by providing opportunities for student-participants to share their ideas in writing, by providing opportunities for small group and whole group conversations, and through the creation of classroom norms and routines that supported learning. The establishment and maintenance of classroom and sociomathematical norms is a research-based practice that helps to create an environment where students can actively contribute to learning (Cobb, Yackel, &Wood, 1989; Yackel & Cobb, 1996; Cobb, Stephan, McClain, & Gravemeijer, 2001; Horn, 2012; NCTM 2014). The teacher-researcher's intentional decision to structure social interactions in the classroom is an inseparable part of the research design which was responsible for creating the data that yielded the result that student-participants interacted with a peer or peers while thinking mathematically – they used each other as resources. How did this impact student-participant learning?

The research question in this DiP is: How does the use of Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies impact students' mathematical thinking? While this research question is not specifically about learning to think mathematically, there is preliminary evidence in the findings that learning occurred as a result of student-participants working together (Reciprocity). Evidence of student-participant learning represents a significant impact on student mathematical thinking. Before delving into the details of this implication, the reader will be provided with an overview of the theoretical framework used to approach learning in this DiP.

This DiP takes a participationist approach to learning that has its foundations in Vygotsky's sociocultural learning theory that places learning in a social context (Vygotsky, 1978). Dewey's (1929) declaration of his belief that "all education proceeds by the participation of the individual in the social consciousness of the race" (p. 291) and that "the school is primarily a social institution" (p. 292) provides added depth to this study's participationist approach to learning. The participationist approach, as the name implies, situates learning in a social context in which the learners are active 'participants' in the process. Participationism is a "research discourse grounded in the metaphor of learning as improving participation in historically established forms of activity" (Sfard, 2008a, p. 301). In this case, that form of 'historically established form of activity' is mathematical thinking (see Chapter Two of this DiP for the rationale for including thinking as part of discourse; it draws heavily on Sfard's (2008a) book entitled *Thinking as Communicating: Human Development, the Growth of Discourses, and Mathematizing*). This DiP defines learning as a "change in discourse" (Sfard & Cobb, 2014, p. 558) and includes thinking as a part of that discourse. In essence, if a student-participant experiences a change in how they think about mathematics or how they think about mathematical thinking then learning has occurred. To be clear, the teacher-researcher does not claim to know the thinking of student-participants. Rather, it is the student-participants' written reflections that were used as evidence (reifications) of the thinking of student-participants.

The findings section of this chapter described, with few exceptions, student-participants who reported, in their written reflections, that Reciprocity (*i.e.*, "working together helped me understand" or "it helped me to see what other people did to show their work") was beneficial to their thinking about mathematics or their explanations of the mathematics. While Reciprocity being beneficial seems to imply that Reciprocity helped students learn, it does not provide sufficient evidence to say that learning has occurred; it does not represent a "change in discourse" (Sfard & Cobb, 2014, p. 558). To find evidence of learning, one needs to look at the exact words (the discourse) that student-participants used in their written reflections.

One student wrote, "I also found that if you compare answers with your friends and you both explain your answer then you can see how others did the work and think their way may be better." This statement describes that the student worked with another student and then compared the other students' answer to hers to see which way (of solving) she thought was better. Without the work of another, she would not have been able to compare methods (ways). The implication that this student thought about whose method ("way") was better is evidence that learning took place – that there was a "change in discourse" (Sfard & Cobb, 2014, p. 558). Another student-participant wrote, "I noticed I started to add to little or too many lines. When sharing with others I had different answers. I revised my approach to drawing and counting lines. I eventually felt confident with my answer." One can make an argument isometric to the one above that learning occurred.

Another student-participant wrote, "my friends helped me fully understand the problem." This statement implies that it was by working together ('reciprocity') with his friends that this student went from a state of not understanding to a state of understanding. It seems more difficult to argue that this student had a "change in discourse" (Sfard & Cobb, 2014, p. 558) without including what this student seems to be saying about his own thoughts – his own understanding. Using this lens, it seems clear that he is saying that his thoughts (his understanding of the problem) changed after working with peers – that he had a "change in [intrapersonal] discourse" (Sfard & Cobb, 2014, p. 558).

A different student-participant wrote that "when you see how many triangles someone else got and you see that you had less, you open your eyes more to find more triangles." While this statement is open to interpretation, it seems to imply that the student-participant went back to the problem (thought about the problem again) and found more triangles (the goal of the task was to explain how many triangles there were and to justify how to find them). It appears that going back to the problem to think about it again (with eyes more opened) was prompted by an interaction with another student (or the work of another student). The student-participant quoted above appeared to have changed her behavior and recommitted to thinking about the problem with a new or renewed perspective. While this different perspective may not be evidence of a "change in discourse" (Sfard & Cobb, 2014, p. 558) or learning, it certainly is evidence of her taking an action that resulted in learning taking place.

This study found that student-participants interacted with a peer or peers while thinking mathematically – they used each other as resources for understanding and learning mathematics. It is important for students to work together while learning mathematics because it has the potential to make "inequities disappear" (Boaler, 2016, p. 104) and because it provides opportunities, as evidenced in this study, for the learning of mathematics and the development of mathematical thinking to take place. The teacher-researcher plans to continue having students work together and to continue "activating students as instructional resources for one another" (Wiliam & Thompson, 2007, p. 53). Boaler (2016) and Horn (2012) offer specific suggestions about effective strategies for getting students to work together and to learn collaboratively. The teacher-researcher plans to review the work of Boaler (2016) and Horn (2012) as part of the Action Plan for this DiP.

Metacognition. This study found evidence that student-participants thought about their mathematical thinking while solving problems – student-participants were engaged in acts of Metacognition. Metacognition took the form of student-participants:

1) describing what they thought about, 2) thinking about one's problem solving process, 3) using one's prior knowledge, 4) thinking about what one knows or does not know, 5) thinking in different ways, 6) thinking differently, 7) getting one's thoughts into words or on paper, 8) describing the type of thinking one did, and 9) the intensity of one's thinking. In addition, one of the findings from theme one was that students engaged in the metacognitive act of monitoring their progress during problem solving.

The finding, Metacognition, refers to what one knows (thinks) about one's own thinking. Metacognition is defined by Bransford *et al*. (2002) as "the ability to monitor one's current level of understanding and decide when it is not adequate" (p. 47) and is a much discussed topic in mathematics education (Evans & Swan, 2014; NCTM, 2009, 2014; Schoenfeld 1992; Wilson, Fernandez, & Hadaway, 1993). Schoenfeld (1992) writes that "metacognition, belief and mathematical practices are considered critical aspects of thinking mathematically" (p. 363). The NCTM (2014) writes that effective mathematics instruction gives students opportunities which allow them to "develop metacognitive awareness of themselves as learners, thinkers, and problem solvers, and learn to monitor their learning and performance" (NCTM, 2014, p. 8). The finding that students thought about their thinking (engaged in metacognition) during and after problem solving was not a surprising finding, but, it is a significant one and it is relevant to research question of this DiP. Engaging students as "instructional resources for one another" and "as owners of their own learning" (Wiliam & Thompson, 2007, p. 53) can result in "metacognitive acts in which students reflect on their own decisions and planning actions during mathematical problem solving" (Evans & Swan, 2014, p. 1). This finding (that student-participants thought about their mathematical thinking while solving problems) provides further evidence for that result. Metacognition is one of the significant ways, as evidenced by the findings, that student-participants were activated as "owners of their own learning" (Wiliam & Thompson, 2007, p. 53) – they were thinking about their mathematical thinking.

The teacher-researcher used Wiliam and Thomson's (2007) Five Key Formative Assessment Strategies to elicit evidence of student-participant mathematical thinking and that evidence included evidence of student-participants engaging in metacognitive activities. The findings of this DiP detailed the relationship of student-participant 'metacognition' with the interpersonal (social) and the intrapersonal. While this may be evidence that student-participants engaged in metacognition through acts of reciprocity (*i.e.*, working together), there is not enough evidence to make such a claim, though such a claim would be supported in the research about social metacognition (Huntsinger and Clore, 2011; Iiskala, Vauras and Lehtine, 2004).

As part of the teacher-researcher's implementation of the Five Key Formative Assessment Strategies (Wiliam & Thomson, 2007), he had student-participants solve problems, write mathematical explanations, share their thinking (in small groups and whole groups), give and receive feedback about their thinking and explanations (from peers or the teacher), revise their thinking and explanations, and reflect on what they learned (the student-participant written reflections). It is the last part, having student-participants reflect on what they learned, that is particularly relevant to this sub-section because it literally prompted student-participants to think about their thinking – writing a reflection about how one solves a problem literally involves explicitly thinking about and writing down what one has thought about. The student-participant written reflections: 1) provided evidence for the teacher-researcher about the mathematical thinking and mathematical understanding of the student-participants, and 2) engaged the student-participants in metacognitive thinking. Writing the written reflections helped the student-participants to become "owners of their own learning" (William & Thompson, 2007, p. 53). The teacher-researcher asserts that by having student-participants respond to the prompts in the written reflections that he was helping students to learn to think using metacognition. Kramarski and Mevarech (2003) found that students who had been trained, individually or in small groups, to apply metacognitive strategies while solving problems outperformed those students without training on measures of mathematical reasoning. In addition, they were better able to explain their reasoning in writing (Kramarski & Mevarech, 2003). To summarize, this subsection provides evidence for the finding that student-participants engaged in metacognitive processes while solving problems and while explaining their mathematical thinking. The teacher-researcher plans to continue to have students write written reflections and he plans to research other strategies for getting students to engage in metacognitive acts and will include these plans as part of the Action Plan for this DiP.

Presence of mind. This study found that student-participants reported that Presence of Mind (paying attention and focusing) was important during the problem solving process and while thinking mathematically. Presence of Mind is a significant finding because attention is regulated by emotion (Brosch, Scherer, Grandjean, & Sander, 2013; Huntsinger & Clore, 2011). Mathematical thinking is often approached from a cognitive perspective that does not make any reference to affect (emotion, mood and valence (Schwarz & Clore, 2007)). Debellis and Goldin (2006) discuss affect and the "interaction between affect and cognition" as being "fundamental to powerful mathematical problem solving" (p. 131). Mathematical thinking is defined in this DiP as "an individualized form of (intrapersonal) communicating" (Sfard, 2008a, p. 81) about mathematics. To help place attention (and emotion) in the context of mathematical thinking, one can think of emotion as having a role in what gets paid attention to (thought about) during the problem solving process and in how much attention (how much thought) is given to different aspects of mathematical thinking. While the previous statement is an oversimplification of a complex process, it serves as a way to describe the role of emotion (in this case, through attention (Presence of Mind)) in mathematical thinking. A short description of the importance of Presence of Mind to mathematical thinking and a brief description of the interactions between affect and thinking will follow in the next subsection, Motivation and Affect.

The Standards of Mathematical Practice (CCSSM, 2009) include references to attention as important to thinking about and doing mathematics; three of the eight Standards ("attend to precision" (p. 6), "look for and make use of structure" (p. 6), and "look for and express regularity in repeated reasoning" (p. 6)) have verbs in their stems ('attend to' and 'look for') that deal directly with the importance of 'presence of mind.' Huntsinger and Clore (2011) write about "the critical role of affect as a guide to cognition – a metacognitive guide" (p. 121) and explain that "affective reactions regulate attention" (p. 121). Brosch *et al.* (2013) explain that "emotion and cognition are closely intertwined, complex human behaviour emerges from dynamic interactions between multiple processes and brain networks. Emotion determines how we perceive our world, how we remember it, and which decisions we take" (p. 6). Mathematical thinking is steeped in emotion. Student-participants reported that paying attention and focusing were important to the problem solving process and this finding suggests that affect plays an important role in mathematical thinking. The next subsection, Motivation and Affect, delves further into the importance of affect in mathematical thinking. The actions the teacher-researcher will take based on this finding will be discussed at the conclusion of the next sub-section.

M**otivation and affect.** This study found evidence that Motivation and Affect had an impact on the mathematical thinking of student-participants. This sub-section begins with a description of affect and motivation as they relate to thinking mathematically. The purpose of this description is to ground the importance of this finding (Motivation and Affect) in the literature. This sub-section concludes with a discussion about actions, stemming from the results of this study, that the teacher-researcher plans to take to further develop the mathematical thinking of his student-participants.

DeBellis and Goldin (2006) use the phrase "mathematical intimacy" (p. 137) to name a construct that refers to the ways that affect plays a role in mathematical thinking and explain that "mathematical intimacy involves deeply-rooted emotional engagement, vulnerability, and the building of mathematical meaning and purpose for the learner" (p. 137). Bosch *et al.* (2013) define emotion:

> as an event-focused process consisting of (a) specific elicitation mechanisms based on the relevance of a stimulus that (b) shape an emotional response

> instantaneously across several organismic subsystems, including motivational changes (changes in action tendency, such as approach versus withdrawal), physiological changes (e.g., heart rate, skin conductance), changes in motor expression (in face, voice, and body), and changes in subjective feeling. (p. 2)

Motivation can be "seen as the inclination to do certain things and avoid doing some others" (Hannula, 2006, p. 165). Hannula (2006) writes that motivation can be defined as "a potential to direct behaviour through the mechanisms that control emotion." (p. 175) and operationalizes his definition and makes it relevant to this DiP when he states that "this potential is structured through needs and goals" (p. 175). Brosch *et al.*'s (2013) statement that "emotions are elicited as the individual continuously evaluates objects, events and situations with respect to their relevance for his/her needs, goals, values, and general well-being (appraisal)" (p. 2) also points to the importance of emotions to motivation when they reference 'needs' and 'goals.'

The findings of this DiP provide evidence that Motivation and Affect play an active role in the mathematical thinking of student participants. The discussion that follows describes how this finding can be put into practice by the teacher-researcher for the benefit of the student-participants.

The teacher-researcher used "engineering effective classroom discussions and other learning tasks" (Wiliam & Thompson, 2007, p. 53) as a formative assessment strategy to "elicit evidence of student understanding" (p. 53). As stated above, this DiP has found that affect and motivation (fueled by emotion) are relevant to how students understand and think about mathematics. The role of motivation in the learning of mathematics has been a field of active study in mathematics education (Boekaerts and

Niemvirta, 2000; Heyd-Metzuyanim, 2015; Hannula, 2006; Nolen, Horn and Ward, 2015). Boekaert and Niemvirta (2000) discuss "attention, metacognition, motivation, emotion, action, and volition control" (p. 445) as interactive processes that help students to self-regulate (to own their own learning) and that students need to be made aware that "their attempts at self-regulation are situated within a social context with changing personal and social goals" (p. 446) in order to "regulate their learning in an optimal way" (p. 446). Just as was the case with metacognition, teaching student-participants about regulating presence of mind, emotion and motivation is important to the learning of mathematics – to student-participants thinking mathematically. As part of the Action Plan for this DiP, the teacher-researcher will attempt to distill findings from emergent research to improve his practice. In particular, he will look to Nolen, Horn and Ward (2015) who point to the "role of identity" (p. 235) and the "relationship between the context and the individual" (p. 236) as important to motivating students to learn. Heyd-Metzuyanim (2015) discusses how the "emotional, social, and cognitive aspects of learning mathematics interact with one another" (p. 504) and offers the reader a new way to conceptualize learning by describing "a new lens: that of looking at mathematical as well as emotional activity through the lens of communicative actions" (p. 544). "This lens is derived from viewing discourse not as a reflection of mental processes but as constitutive of them" (p. 544) and makes the claim that "once discourse is seen as constituting and shaping the mind, rather than only reflecting it, one can dispense with the dichotomy of emotion and cognition, as this dichotomy does not, in fact, exist in human communication" (p. 544). With this in mind, one can look back to the shaded area (the light gray area) of Figure 4.1 and conceptualize (redefine) it to include emotion (see Figure 4.2) as an inseparable part of mathematical thinking in the classroom. The

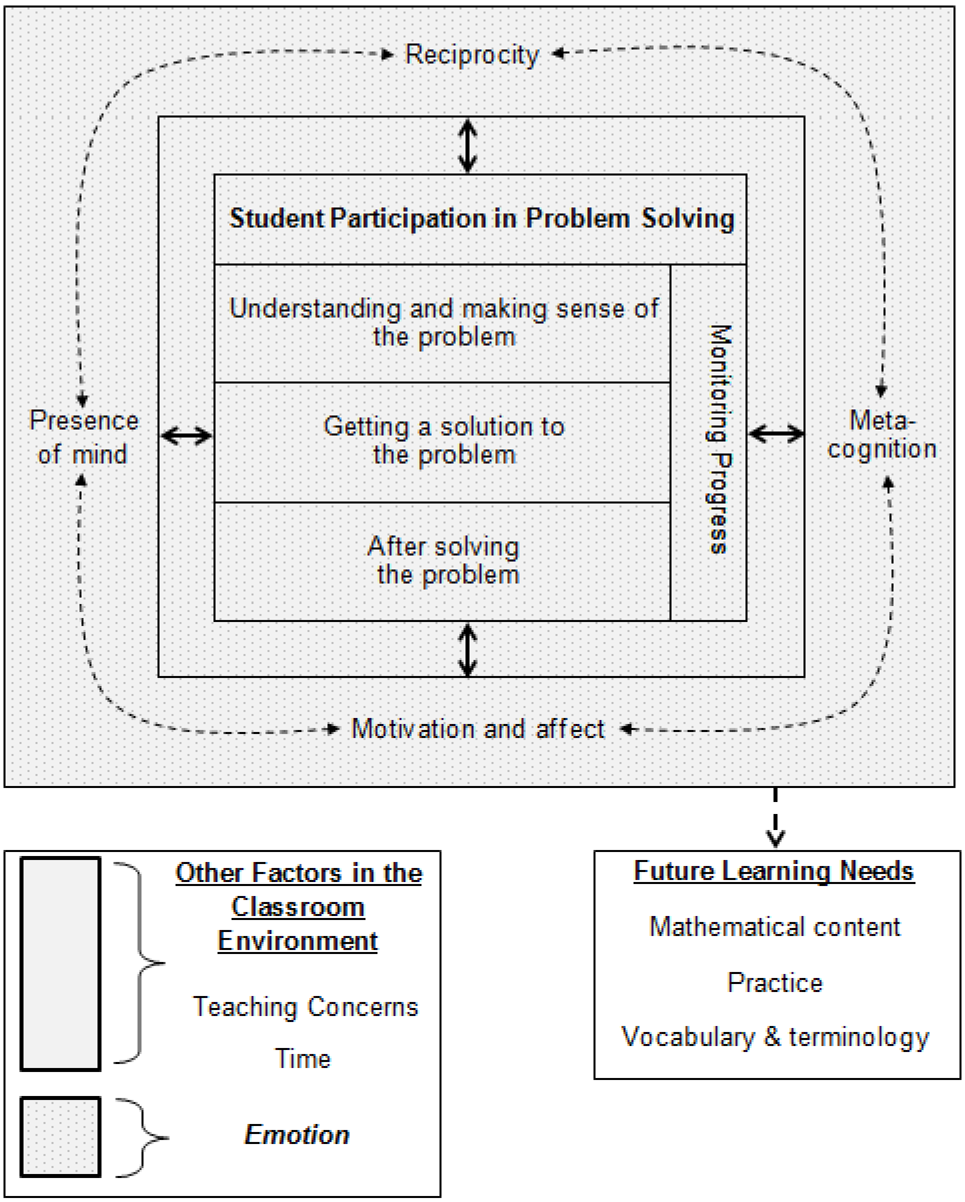

Figure 4.2 Problem solving, mathematical thinking and emotion in the classroom. The relationships between emotion and four themes: 1) student-participant participation in problem solving, 2) social and intrapersonal factors in the classroom environment (reciprocity, metacognition, presence of mind, and motivation and affect), 3) other factors in the classroom environment, and 4) future learning needs.

teacher-researcher plans to work towards developing concrete ways of attending to emotion in the mathematical thinking (intrapersonal communication) of students. Within the framework of the Five Key Formative Assessment Strategies (Wiliam & Thompson, 2007), the teacher plans to address this concern in the ways that he goes about "providing feedback that moves learners forward" (p. 53) that take into account the importance of motivation, presence of mind and emotion to the mathematical thinking of student participants. In particular, he will reflect deeply about how he gives feedback knowing that: "the wrong kind of praise creates self-defeating behavior" (Dweck, 2007, p. 34) and that "the right kind motivates students to learn" (p. 34). As part of his Action Plan, he will review the work of Horn (2012) and Boaler (2016) to help him reflect and develop strategies to improve how he gives feedback.

Summary of theme two. In summary, this theme, social and interpersonal factors in the classroom environment, found that: 1) student-participants worked together to understand, think about and learn mathematics, 2) student-participants engaged in metacognitive processes while thinking about mathematics during and after the problem solving process, and 3) that attention (Presence of Mind), motivation and emotions of student-participants are inseparable parts of mathematical thinking. These findings have implications for how the teacher-researcher will take action to change his teaching to develop student mathematical thinking. A discussion of the teacher-researcher's Action Plan is detailed in Chapter 5 of this dissertation.

Theme three: Pedagogical Factors in the Classroom Environment

Theme three, pedagogical factors in the classroom environment, found evidence that student-participants reported that instructional decisions and time (or timing) impacted their mathematical thinking. The focus of this DiP has been on student learning, and this finding indicates the importance of the role of the teacher in this process. This finding provides evidence that the teacher-researcher's use of the Five Key Formative Assessment Practices (Wiliam & Thompson, 2007) in his instructional practice (his pedagogical approach) was a factor in how student-participants thought about mathematics and that the teacher-researcher's instructional practice could be improved. As such, the teacher-researcher will take action to reflect on his own practice and will draw from the work of Horn (2012) and Boaler (2016) in an attempt to improve his teaching practice as a means to further develop the mathematical thinking of the student-participants (to continue to address the problem of practice). Boaler (2016) includes a number of mathematical tasks that the teacher-researcher plans to use to develop the mathematical thinking of student-participants. Horn (2012) includes timing ("appropriate amount of time" (p. 11)) as part of the "pedagogical practices" (p. 12) that are part of a larger set of "equitable mathematics teaching practices" (p. 12) and suggests an adaptation of Smith, Bill and Hughes' (2008) "Thinking Through a Lesson Protocol" (p. 132) as a way to refine one's teaching practice. The teacher-researcher plans to adopt this protocol to improve his instruction. As part of the action research process, the teacher-researcher will detail the actions that he plans to take to address his own instructional practices in the Action Plan of this DiP. He has already begun this process by reading Boaler's (2016) "Mathematical Mindsets" to both inform and inspire his teaching practice. In addition, he has shared the findings of this DiP with another mathematics teacher at his high school.

Theme four: Future Learning Needs.

Theme four, future learning needs, found evidence that student-participants stated that the following four actions could help them to better explain their mathematical thinking in the future: 1) practicing more, 2) learning more mathematical vocabulary and terminology, 3) learning more mathematical content, and 4) studying. This finding emerged from the data in the student-participant written reflections. In the spirit of action research, the teacher-researcher sought to both study the impact of his instructional practice on student-participant mathematical thinking and the improvement of his own instructional practice (Mertler, 2014). As stated above, this theme focuses on four actions that student-participants reported could be helpful to them in the future. This section will discuss the importance of this finding to the teacher-researcher's future instructional practice and its implications for the mathematical thinking of students.

Student-participants wrote that practice, vocabulary and terminology, content and studying could help them to better explain their mathematical thinking in the future. These four actions can be conceptualized as predictions. Whether or not these predictions will actually serve student-participants well is outside of the scope of this DiP. Rather, what is important to this DiP, is that "predictions reflect and contribute to people's current self-conceptions" (Schryer & Ross, 2011, p. 141). Predictions are "influenced by preferences, goals [motivation], and metacognitive processes such as theories, norms, and feelings of accessibility" (p. 141). The influence of 'norms' on predictions relates directly to the research question in this dissertation. Norms can be thought of as result of the teacher-researcher "clarifying and sharing learning intentions and criteria for success" (Wiliam & Thompson, 2007, p. 53). The maintenance of classroom and sociomathematical norms is a research-based practice that helps to create an environment where students can actively contribute to learning (Cobb, Yackel, & Wood, 1989; Cobb, Stephan, McClain, & Gravemeijer, 2001; Horn, 2012; NCTM 2014; Yackel & Cobb, 1996). This finding provides preliminary evidence for the claim that student-participants believed that practice, learning mathematical vocabulary and terminology, learning mathematical content and studying were important to the development of their explanations of their mathematical thinking. This finding is also important because it caused the teacher-researcher to reflect about ways to maintain existing norms and to establish new norms in his classroom, and to support student-participant beliefs about what it means to "be smart" (Horn, 2012, p. 19) as a learner of mathematics. The teacher-researcher plans to learn more about the beliefs of the student-participants in an effort to improve his practice. In particular, he plans to: 1) continue to reflect about the current and future use of norms in his classroom, and 2) investigate interventions that address "what 'being smart' means" (Horn, 2012, p. 19) in his mathematics classes as a way to work towards equity.

Conclusion

The teacher-researcher conducted a qualitative action research study with two of his high school geometry classes as a way to understand the impact of Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies on the mathematical thinking of students. He did this to address a problem of practice, that high school geometry students routinely do not show and have not been provided with opportunities to show evidence of mathematical thinking while working on or after working on mathematical tasks. The research question in this DiP is: how does the use of Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies impact students' mathematical thinking?

The findings presented above describe the different ways that the teacher-researcher's use of the Five Key Formative Assessment Strategies (Wiliam & Thompson, 2007) impacted the mathematical thinking of the student-participants. The first of four themes, participation in problem solving, found evidence that student-participants: 1) made sense of problems using prior knowledge and figured out how to get started solving the problem, 2) solved problems (and thought about) solving problems in a variety of ways while using a variety of tools, 3) explained or made sense of their mathematical solution or solution path in different ways, and 4) monitored their progress during the problem solving process. The second theme, social and interpersonal factors in the classroom environment, found evidence that: 1) student-participants worked together to understand, think about and learn mathematics, 2) student-participants engaged in metacognitive processes while thinking about mathematics during and after the problem solving process, and 3) that attention ('presence of mind'), motivation and emotions are critical to the mathematical thinking of student-participants. Theme three, pedagogical factors in the classroom environment, found evidence that student-participants indicated that the teacher could improve his instructional practice in ways that would benefit their mathematical thinking. Theme four, future learning needs, found evidence that students believed that practicing more, learning more mathematical vocabulary and terminology, learning more mathematical content and studying could improve how they explain their mathematical thinking. A detailed description of the actions that the teacher-researcher plans to take based on the findings of this DiP study and the interpretation of those findings are included in the Action Plan described in Chapter Five of this DiP.

CHAPTER FIVE: SUMMARY AND CONCLUSIONS

Introduction

Chapter Five presents the summary and conclusions of a teacher-researcher conducted qualitative action research study with two high school geometry classes that experienced Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies and the subsequent Action Plan on teacher planning and pedagogy as a result. Chapter Five includes a restatement of the identified problem of practice statement; the research question; the purpose of the study; a description of the focus of the study; an overview of the major points of the study and an Action Plan to improve mathematics curriculum and pedagogy at a southern Title I public high school; suggestions for future research; and a conclusion.

The high school geometry students at this school routinely did not show nor were they provided with opportunities to show evidence of their mathematical thinking while working on mathematical tasks. Therefore, the teacher-researcher altered his instructional practice to include the Five Key Formative Assessment Strategies (2007) in his pedagogical approach. While using these assessment strategies, the teacher-researcher collected and analyzed student-participants' written reflections about their mathematical thinking; the written reflections serve as the primary source of data in this dissertation in practice (DiP). The teacher-researcher also collected and analyzed a teacher journal (field notes) and sample student-participant mathematics work as secondary data sources. The teacher-researcher used the constant comparative method and polyangulation to analyze the data (Mertler, 2014; Merriam, 2009). During the data analysis process, the teacher-researcher coded the data line-by-line using a combination of two Structural Coding methods: the First Cycle Initial Coding method and the First Cycle Descriptive Coding method (Saldana, 2009). The teacher-researcher engaged in an iterative process to look for patterns in the data and to assign codes to categories; he kept analytic memos during the process. The categories that resulted from the First Cycle Coding methods were then organized into pattern codes (see Table 4.1) using several iterations of the Second Cycle Pattern Coding method (2009).

The pattern codes resulted in the identification of four themes, the findings of this DiP. The four themes are:

1. Student-participant participation in problem solving;
2. Social and intrapersonal factors in the classroom environment;
3. Pedagogical factors in the classroom environment; and
4. Future learning needs.

The implications of the findings describe how the teacher-researcher's implementation of the *Five Key Formative Assessment Strategies* (Wiliam & Thompson, 2007) impacted student-participant mathematical thinking. Student-participants:

1. Used mathematical thinking to think about and learn mathematics while problem solving;
2. Engaged in metacognitive processes while solving problems;
3. Discussed and engaged in mathematical thinking while working and learning collaboratively;

4. Described the importance of presence of mind, motivation and emotion to mathematical thinking; and
5. Described their beliefs about actions they could take to improve their mathematical thinking.

The teacher-researcher shared the findings and implications with the student-participants in order to design and implement an Action Plan that includes:

1. Reflecting with student-participants about the study's results;
2. Improving his teaching practice based on the findings; and
3. Disseminating the findings of the study to other mathematics teachers.

Focus of the Study

The teacher-researcher began this study identifying a problem of practice in his high school geometry students class with his students who routinely did not show and had not been provided with opportunities to show evidence of their mathematical thinking while working on or after working on mathematical tasks. This problem of practice caused the teacher-researcher to reflect on his teaching practice and to think about ways to improve his practice as he looked for ways to take action to solve the problem of practice. He decided to implement the William and Thompson's (2007) Five Key Formative Assessment Strategies to determine how and if they would impact his students' mathematical thinking.

The teacher-researcher found that his implementation of the Five Key Formative Assessment Strategies (Wiliam & Thompson, 2007) impacted his student-participants' mathematical thinking in several ways:

1. Students' mathematical thinking skills while problem solving increased;

2. Students engaged in metacognitive processes while solving problems;
3. Students discussed and engaged in mathematical thinking while working and learning collaboratively;
4. Students described the importance of presence of mind, motivation and emotion to mathematical thinking;
5. Students described their beliefs about actions they could take to improve their mathematical thinking. The findings of the teacher-researcher's action research study caused the teacher-researcher to reflect on his teaching practice and his understanding of how the student-participants learn to think mathematically.

The teacher-researcher engaged in a process of self-reflection as he collected data, analyzed data and generated the findings of this study for the purposes of developing an Action Plan. The questions below helped to guide his thinking while he worked to formulate an Action Plan:

1. What teaching practices are currently working in the teacher-researcher's classroom?
2. What teaching practices could be improved or changed?
3. What new or different teaching practices might the teacher-researcher consider?
4. What can be done to support the presence of mind, motivation and emotional needs of student-participants?
5. What do the student-participants think and feel about the findings of this study?

6. What do the student-participants believe about themselves as learners of mathematics?
7. What can be done to help student-participants to enjoy learning and thinking about mathematics?
8. What opportunities exist to disseminate the findings of this study?
9. What lines of future research are suggested by this study?

The above questions have the potential to be taken up by the teacher-researcher – they are questions that have actionable answers. This is important to the action research methodology used in this DiP because action research is "the type of research that really puts the action into action research" (Mertler, 2014, p. 211). Action research is a "cyclical and iterative" process (Mertler, 2014, p. 32) that is done with, not to, the participants (Herr and Anderson, 2005). The teacher-researcher has plans to address the questions above in his teaching and professional practices. The steps and actions he plans to take to address the questions above are presented as an Action Plan in the next section of Chapter Five, the summary of the study. The summary of the study will discuss the major points of the study and the Action Plan.

Summary of the Study

The major points of the study and an Action Plan are discussed in this section of Chapter Five.

Major Points of the Study

The purpose of this study was to investigate how one mathematics teacher's implementation of the Five Key Formative Assessment Strategies (William & Thompson, 2007) impacted student-participants' mathematical thinking. The teacher-researcher changed his instructional practice to include the five key strategies in an effort to "elicit evidence of students' current mathematical understanding" (NCTM, 2014, p. 53) and to better understand the mathematical thinking of the student-participants. The changes he made to his instruction included:

1. Setting a goal of improving students' mathematical explanations;
2. Being explicit about what constitutes a satisfactory mathematical explanation;
3. Using tasks which lent themselves to students working together and which allowed students multiple opportunities to explain their thinking;
4. Giving specific feedback to students about their mathematical explanations;
5. Structuring small-group and whole group class discussions for the purposes of discussing evidence of student s' mathematical thinking; and
6. Engaging students in metacognitive activities (*i.e.*writing mathematical explanations).

The teacher-researcher found that he was engaging student-participants in problem solving (*i.e.*, modeling the maximum number of regions in a circle versus the number of diagonals) as a result of his focus on mathematical thinking in his pedagogical approach. In essence, the student-participants were thinking mathematically. At the same time, the teacher-researcher's attempts to "elicit evidence of student understanding" (Wiliam & Thompson, 2007, p. 53) and to find evidence of the mathematical thinking of the student-participants caused him to have student-participants write and reflect (via student-participant written reflections) on their mathematical explanations of the problem solving process. By writing (doing) the student-participant written reflections and through their participation in and monitoring of the problem solving process, the student-participants were found to engage in metacognitive processes. For example, one student-participant wrote that "[I] gave a logical explanation that could help someone else find the same thing and or come to the same conclusion based on my reasoning." Another student-participant wrote that she had to "think about it [solving the problem] in a different way than I normally would." The student-participants were being activated as "owners of their learning" (p. 53). For example, one student-participant wrote that it was important to "ask questions so I know that I'm doing it right." Another student-participant stated that "I came up with my explanation by learning from my past mistake." The teacher-researcher also structured opportunities for students to work together (*i.e.*, in small heterogeneous groups or in pairs) to solve problems and to discuss their mathematical thinking (*i.e.*, a mathematical explanation of how a problem was solved). These collaborations resulted in a pushing forward of the mathematical thinking and learning of all of the student-participants – they were activated "as instructional resources for one another" (p. 53). The data analyzed in this study further support Horn's (2012) claim that collaborative learning can help produce equitable outcomes in the classroom. One student-participant explained that "I found out making an explanation was difficult for a problem like this and I talked to a friend to see how they explained it." The student-participants also indicated that presence of mind, motivation and emotion were important to mathematical thinking. For example, one student-participant stated that he believed it was important to become "more comfortable with thinking outside the box and paying more attention to things you don't see first." Another student-participant wrote that "the thing that helped me was that I was having fun."

The finding that the affective needs of student-participants were important to mathematical thinking is significant because giving attention to the emotional needs of students deserves to be considered when designing mathematics instruction and is often overlooked (Debellis & Goldin, 2006). The study by Debellis and Golden impacted the present Action Research study because it found that students predicted that practicing more, learning more mathematical vocabulary and terminology, learning more mathematical content, and studying more would help them to explain their mathematical thinking. The teacher-researcher used this finding to predict and reflect on what the student-participants believed about what it means to do, learn and think about mathematics. For example, one student-participant wrote that “lots of practice could help me get better at coming up with mathematical explanations.” Another student-participant wrote that “learning more mathematical vocabulary, and paying a little bit better attention to details” would her to come up with better mathematical explanations. This finding is significant because student beliefs about mathematics play an important role in mathematical thinking (Yackel & Cobb, 1996).

During the course of this action research study, from identifying a problem of practice to interpreting the findings of the study, the teacher-researcher was able to reflect about his use of the Five Key Formative Assessment Strategies (Wiliam & Thompson, 2007) in his instructional practice. The teacher-researcher reflected about what worked and what did not work in his practice and how he could further improve his practice. He reflected about ways to share his findings with the student-participants and how to disseminate the findings of this study more broadly. The following sub-section describes the actions that the teacher-researcher plans to take based on the findings of this study.

Action Plan

The Action Plan is designed to assist the teacher-researcher and colleagues in implementing the Five Key Formative Assessment Strategies (Wiliam & Thompson, 2007) in the classroom. It includes details about his plans for the actions that he will take in his teaching practice, actions he will take to reflect on the findings of the study with student-participants, and his plans for outreach (actions he will take to share the findings of this study at his school and with the larger mathematics education community). The actions that the teacher-researcher plans to take as part of the Action Plan, a proposed timeline for the actions, and a list of necessary resources are summarized in an Action Plan Chart (see Appendix I). It is important to note that a focus on the processes of teaching and student learning has the potential to be at odds with the prevailing culture of accountability at the teacher-researcher's school. The teacher-researcher includes evidence (*i.e.*, research-based claims, equitable teaching strategies) for the actions he plans to take as a way to mitigate any problems that arise as a result of the obstacles posed by a culture of accountability.

Teaching action plan. This sub-section describes the teaching actions, as a part of the Action Plan, that the teacher-researcher plans to take in his future instructional practices based on the findings of this study. The timeline for the Action Plan and any resources necessary are included in Appendix I. The teacher-researcher plans to continue to use Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies in his instructional practice because this study found student-participants engaged in problem solving (ergo, mathematical thinking) and progress monitoring (a metacognitive process) as a result. He will continue to make student-participant mathematical thinking a focus of his instruction and improving student-participant mathematical explanations a goal of his instruction. Student-participants will continue to complete written reflections during class using Google Forms. The rationale for this is that the student-participants' written reflections were found to engage student-participants in metacognitive processes.

The teacher-researcher plans to continue to have students work together ('reciprocity') in his classes, however, he plans to improve this practice and to make it more intentional. The rationale for this is that 'reciprocity' was found to help student-participants think mathematically and to learn mathematics. He plans to use Smith, Bill and Hughes' (2008) "Thinking Through a Lesson Protocol" (p. 132) to account for group dynamics and group interactions in the lesson planning process. In addition, this protocol addresses a host of pedagogical decisions that get made in the classroom including presenting mathematical content (*i.e.* giving notes or lecturing), giving directions and timing. The use of this protocol directly addresses the finding of this study dealing with 'teaching concerns' and 'time.' The teacher-researcher plans to locate or design a "groupworthy task" (Horn, 2012, p. 35) to use in his instruction because "groupworthy tasks" (p. 35) are designed to "support students' engagement with challenging mathematical content" (p. 44). In addition, if his pilot of a "groupworthy task" (p. 44) is successful, he will locate or design more of them for use in his future instruction. The rationale for using "groupworthy tasks" (p. 44) is that 'reciprocity' was found to help student-participants think mathematically and to learn mathematics and that the teacher-researcher is seeking to refine his instructional practice.

The teacher-researcher also plans to establish and maintain classroom and sociomathematical norms that support the affective needs of the student-participants and to create a classroom culture that, as has been shown in the research, supports mathematical thinking (Boaler, 2016; Horn, 2012; Yackel & Cobb, 1996). The rationale for establishing and maintaining norms is based on the finding that 'presence of mind,' motivation and emotions are important to the mathematical thinking of student-participants and that there is research-evidence that supports the use of sociomathematical norms to support student mathematical thinking (Boaler, 2016; Horn, 2012; Yackel & Cobb, 1996). Horn (2012) suggests the following norms:

- Take turns.
- Listen to others' ideas.
- Disagree with ideas, not people.
- Be respectful.
- Helping is not the same as giving answers.
- Confusion is part of learning.
- Say your "becauses." (p. 28)

Boaler's (2016) classroom norms include statements about her beliefs in a growth mindset, that "every time they [students] make a mistake their brain grows" (p. 172) and other norms that send clear, positive, emotionally-charged messages to students about classroom and mathematical expectations. The teacher-researcher plans to review the literature for other research-based ways to support the emotional growth of students in his classroom to determine how the emotional needs of his students can be better served. To further address the emotional needs of students in class he plans to begin "assigning competence, a form of praise where teachers catch students being smart. The praise is public, specific to the task, and intellectually meaningful" (Horn, 2012, p. 31). The teacher-researcher will engage in "assigning competence" (p. 31) with student-participants to positively "shift their self-concepts and their ideas about others" (p. 31) "so that they know their own strengths and can work confidently on hard problems" (p. 31). The teacher-researcher also plans to "give growth praise and help" (Boaler, 2016, p. 178) in order to support the emotional and motivational needs of students and to value their mathematical thinking. The plans described above (norms, "assigning competence" (Horn, 2012, p. 31), "give growth praise and help" (Boaler, 2016, p. 178)) are concrete ways that the teacher-researcher can attend to 'presence of mind,' motivation and emotion in the classroom. In addition, they offer ways to establish or create productive beliefs about mathematics and about what constitutes mathematical thinking and are equitable mathematics teaching practices (Horn, 2012; Boaler, 2016). The teacher-researcher includes student beliefs in his Action Plan because the finding, 'future learning needs,' introduced preliminary findings about the mathematical beliefs of the student-participants. Actions that the teacher-researcher plans to take to better identify and further address the mathematical beliefs of student-participants are included in the following sub-section, 'plans to reflect with student-participants.'

Plans to reflect with student-participants. This sub-section describes actions that the teacher-researcher plans to take to share and reflect on the findings of this study with student-participants. The timeline and the resources necessary for these actions are included in Appendix I. The teacher-researcher plans to create a presentation, using Google Slides, which he will share with the student-participants to present the findings of the study and the implications of the findings. The presentation will focus on the findings related to two of the five formative assessment strategies: “activating students as instructional resources for one another and activating students as owners of their own learning” (Wiliam & Thompson, 2007, p. 53). Those two strategies are the focus because they deal most directly with the student-participants themselves and not the teacher-researcher. The teacher-researcher will discuss the findings related to problem solving, reciprocity, metacognition, presence of mind, motivation and affect. He plans to include a ‘word cloud’ (see Figure 4.3 for a sample ‘word cloud’) produced by the nVivo coding

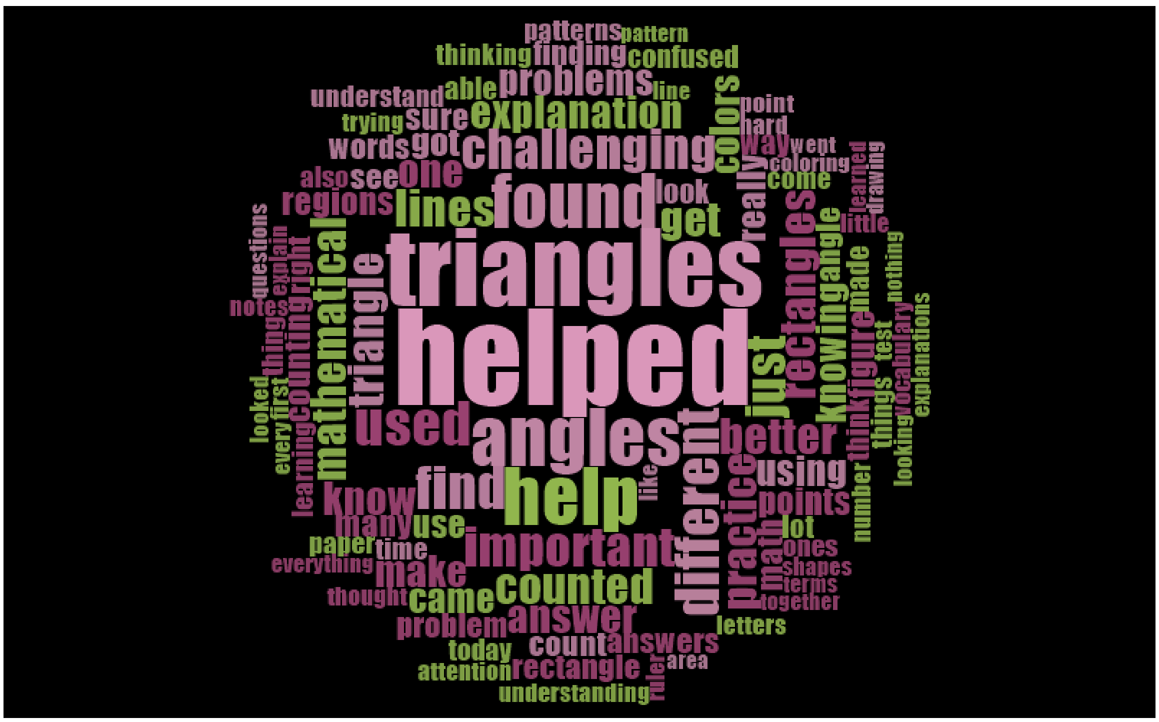

Figure 4.3 Sample ‘word cloud’ for presenting findings to student-participants.

software to help students understand the coding process and to help students get an idea of the findings of the study in a visual and creative way. The teacher-researcher plans to give the presentation to the student-participants in an informal way where student-participants are allowed to ask questions and offer both comments and criticism. In addition, the teacher-researcher plans to provide student-participants with opportunities to reflect on his presentation of the findings and to share their thoughts and feelings with the teacher-researcher. The teacher-researcher recognizes that his dual role as teacher and researcher may cause students to be less likely to offer criticism. In an attempt to receive authentic feedback, the teacher-research will provide multiple and varied (*i.e.*, in-person, in writing, in public or in private) opportunities for the student-participants to share their thoughts and feelings. He plans to share his results with student-participants, because "sharing the results – either formally or informally – is the real activity that helps bridge the divide between research and application" (Mertler, 2014, p. 245). He also plans to share the findings as a way to verify the trustworthiness of the findings – to engage in member checking of the findings (Mertler, 2014). Furthermore, since action research is interactive and involves an exchange of ideas between the teacher-researcher and the student-participants, it is possible that the student-participants may suggest interpretations (or implications) of the findings or plans of action that had not previously occurred to the teacher-researcher (Herr & Anderson, 2005). In addition to giving the presentation allowing time for reflection and feedback, the teacher-researcher plans to ask the student-participants (individually and then as a whole class) to share their beliefs about mathematics and any feelings (emotions) that they have about problem solving or thinking mathematically using interview protocol or a written reflection. The teacher-researcher plans to ask this of the student-participants to help him to better understand their beliefs about mathematics and their emotional needs based on the findings of this study related to 'future learning needs,' 'presence of mind' and 'motivation and affect.' He plans to use this information to improve his instruction.

Plans for outreach. This sub-section describes the actions that the teacher-researcher plans to take to share and reflect on the findings of this study at his school and in the larger mathematics education community. The timeline and the resources necessary for these actions are included in Appendix I. The teacher-researcher plans to share the findings of this study, informally, with other members of his high school mathematics department during collegial conversations. In addition, he presented his use of the student-participant written reflections as a strategy for developing the metacognition of students at the December 2016 mathematics department meeting at his high school. The teacher-researcher in this study is a teacher-leader in mathematics education. He will share relevant portions of his findings at state and national presentations. In addition, he will include relevant findings as part of the professional development he provides at the local, state and national level. He has already worked with his principal and district superintendent to secure release time and funding to attend NCTM's annual meeting. He will be presenting a portion of the findings from this study at the annual meeting and will be hosting discussion session immediately following the presentation. These outreach activities of the teacher-researcher allow "opportunities for professional dialogue, reflection, and brainstorming" (Mertler, 2014, p. 249) and are a way of "communicating the results of action research" (p. 245), an important step in the cyclical action research process (Mertler, 2014).

Suggestions for Future Research

This action research study investigated the impact of the teacher-researcher's use of Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies on the mathematical thinking of high school geometry students. The teacher-researcher found evidence of students learning from each other during the problem solving process and suggests that further research be undertaken to describe how this learning occurs and how to cultivate it in the classroom. Work by Horn (2012) and Boaler (2016) indicates that having students working collaboratively works toward the goals of social justice – specifically towards creating equity in the learning of mathematics. The teacher-researcher found evidence suggesting the importance of emotion to the mathematical thinking of participants. Investigating the connections between emotions and thinking as well as 'presence of mind' and motivation is another avenue of suggested future research. Heyd-Metzuyanim (2015) has offered a theoretical lens through which to study emotion and thinking in the context of mathematics by viewing both emotion and thinking (cognition) as elements of communication, of discourse. Lastly, the teacher-researcher found evidence that one of the student-participants reported "having fun" while thinking about mathematics and the teacher-researcher suggests that future research take place to investigate how formative assessment or other pedagogical approaches could help students to enjoy doing and thinking about mathematics. Wolk's (2008) article "Joy in School" offers some steps that could be taken toward "joyful learning" (p. 8).

Conclusion

In this study, the teacher-researcher made changes to his own teaching practice for the purpose of investigating its impact on the mathematical thinking of his students. To accomplish this, he used Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies as the focus of his instructional practice and studied how using the strategies impacted the mathematical thinking of the student-participants. A description of how these strategies were found to impact student-participant mathematical thinking provides the answer to the study's research question: how does the use of Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies impact students' mathematical thinking? The use of the strategies impacted the mathematical thinking of student-participants by enabling them to: 1) use mathematical thinking to think about and learn mathematics while problem solving, 2) engage in metacognitive processes while solving problems, 3) discuss and engage in mathematical thinking while working and learning collaboratively, 4) describe the importance of presence of mind, motivation and emotion to mathematical thinking, and 5) describe their beliefs about actions they could take to improve their mathematical thinking in the future. The teacher-researcher: 1) allowed student-participants opportunities to work together, 2) provided them with opportunities to ask questions, 3) supported student-participants while engaging in problem solving and mathematical thinking, 4) asked them to write and reflect on mathematical explanations, and 5) encouraged student-participants to think about their mathematical thinking and mathematical explanations for the purpose of supporting and developing their mathematical thinking. He continues to work on ways to use the formative assessment strategies and to look for other pedagogical strategies that focus on the processes that are at the core of how students learn to think mathematically. In 1808, the famous mathematician Carl Friedrich Gauss wrote that "it is not knowledge, but the act of learning, not possession but the act of getting there, which grants the greatest enjoyment" (as cited in Dunnington, 1955/2004). The "act of learning" mathematics (student-participant mathematical thinking) and the process of learning mathematics (*i.e.*, problem solving, reciprocity) were at the heart of this dissertation. The teacher-researcher hopes to continue working towards a classroom environment where all students enjoy the “act of learning.” The teacher-researcher plans to: 1) continue to support and encourage the mathematical learning of his students by improving his own practice, 2) help other teachers (and their students) learn to do the same by sharing the results of this research, and 3) continue to an open dialogue with his own students to help them enjoy learning to think mathematically.

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Appendix A

Research Planning Schedule Sheet

Activity to be Completed	Estimated Amount of Time Needed	Target Date for Completion	Task Completion Date
Distribute and assent and informed consent forms	1 week	August 26, 2016	August 17, 2016
Collect student work, student reflections. Take notes in teacher journal.	1 day	September 2, 2016	August 29, 2016
Code data.	1 week window	September 9 2016	September 6, 2016
Collect student work, student reflections. Take notes in teacher journal.	1 day	September 16, 2016	September 7, 2016
Code data.	1 week window	September 16 2016	September 14, 2016
Collect student work, student reflections. Take notes in teacher journal.	1 day	September 23, 2016	September 15, 2016
Code data. Look back and previous 2 weeks. Recode data.	1 week window	September 23 2016	September 22, 2016
Collect student work, student reflections. Take notes in teacher journal.	1 day	September 30, 2016	September 23, 2016
Code data. Look back and previous 3	1 week window	September 30, 2016	September 22, 2016

weeks. Recode data.			
Collect student work, student reflections. Take notes in teacher journal.	1 day	October 7, 2016	September 30, 2016
Code data. Look back and previous 4 weeks. Recode data.	1 week window	October 7, 2016	October 5, 2016
Collect student work, student reflections. Take notes in teacher journal.	1 day	October 14, 2016	October 6, 2016
Code data. Look back and previous 5 weeks. Recode data.	1 week window	October 14, 2016	October 16, 2016
Finalize coding categories. Analyze trends	2 weeks	November 11, 2016	October 23, 2016
Select and transcribe significant instances related to the research question	2 weeks	November 25, 2016	October 30, 2016

Appendix B

Assent to be a Research Subject

Dear Student,

My name is Benjamin Sinwell. I am your mathematics teacher. I am also a graduate student in the Education Department at the University of South Carolina. I am conducting a research study as part of the requirements of my doctoral degree in Curriculum and Instruction, and I would like to invite you to participate. The purpose of this research is to investigate how formative assessment impacts student mathematical thinking.

Your part in this study will be a natural part of your classroom learning. You will be asked to reflect on your learning during class. I will also collect examples of your classwork and scan it into electronic documents. I will also record teaching notes in a journal. This study will take place during ten class periods over the course of the next ten to twelve weeks. Your participation in this study will not take any additional work nor take time from your classroom learning.

Participation is confidential. Study information will be kept in a secure location on my school-issued laptop and backed up on a password protected external storage device. The results of the study may be published or presented at professional meetings, but your identity will not be revealed. I will do everything I can to protect your privacy and confidentiality.

You do not have to be in this research study. You may tell me at any time that you do not want to be in the study anymore. There will be no negative consequences if you decide not to be in the study or if your parent/guardian does not want you to be in the study. I will also inform your parent/guardian that you are being invited to participate in this study. Your parent/guardian may also refuse to allow you to be in the study or to withdraw you from the study at any time. You may also decide not to answer any question you are not comfortable answering. You can decide not to be in the study at any time without any negative consequences.

I will be happy to answer any questions you may have about the study. You may contact me at bsinwell@anderson4.org (864-403-2100) or my faculty advisor, Dr. Susan

Schramm-Pate, sschramm@mailbox.sc.edu (803-777-3087), if you have study related questions or problems. If you have any questions about your rights as a research participant, you may contact the Office of Research Compliance at the University of South Carolina at 803-777-7095.

By being in this study, you are saying that: 1) you have read this form and have asked any questions that you may have, 2) all of your questions have been answered and you understand what you are being asked to do, and 3) you are willing and would like to be in this study.

With kind regards,

Benjamin J Sinwell

Benjamin Sinwell
Pendleton High School
864-403-2100
bsinwell@anderson4.org

Appendix C

Field Note Template

Observation # ___ **Date:** **Time Period:** **Class:**	**Observations**	**Observer's Comments**
This space is to write down specific times or to identify the group or the student talking/gesturing. Use a coded seating chart if needed. **Reminder for the teacher-researcher:** protect the identity of students in his dissertation by giving them pseudonyms or using a code to preserve anonymity.		

Appendix D

First Cycle Coding Categories Followed By Initial and Descriptive Codes

Understanding and Solving the Problem

Checking, Coloring, Comparing with others, Confused about, Counting, Creating, Devising a plan, Directions, Doing, Equations, Formula, Going back, Ignoring, Labeling, Listing, Making Sense, Modeling, Outlining, Organizing, Persisting, Planning, Recording, Reflecting, Relating, Thinking about the explanation, Using notes, Visualizing, Vocabulary

Justifying Thinking

Describing steps, Discussing with other students, Explaining, Thinking, Thoughts to words, Vocabulary

Patterns

Finding patterns, Looking for patterns, Using a pattern

Using Tools

Coloring, Formula, Measuring, Using tools

Being Precise

Checking, Coloring, Counting, Labeling, Listing, Measuring, Organizing, Precision, Recording, Terminology, Vocabulary

Metacognition

Self-assessing, Reflecting, Thinking, Thinking differently, Thoughts to Paper

Practice

Practice

Teaching Concern

Teaching Concern, Teacher Help

Mathematical Content

Mathematical Content

Time

Timing

Vocabulary & Terminology

Mathematical language, Terminology, Vocabulary, Words

Presence of Mind

Attention, Concentration, Focus

Reciprocity

Student help, Discussing with other students, Asking others, Comparing with others

Motivation & Affect

Being sure, Effort, Feelings, Motivation

Appendix E

Summarized Data for Theme One Organized by Four Sub-Themes

Theme One: Student Participation in Problem Solving	
Sub-theme: Understanding and Making Sense of the Problem	
Grouping	Summarized Data
Ways of Using Prior Knowledge	Recalling prior mathematical content. Looking at notes. Remembering to use mathematical knowledge to solve the problem. Thinking about solving similar or analogous problems.
Mathematical Content of the Prior Knowledge	Definitions and mathematical terminology. Equations. Formulas. Mathematical concepts (i.e., algebra, angle relationships).
Ways of Getting Started	Being sure that they knew what the problem was asking. Deciding what they needed to do to get started (i.e., choosing an appropriate mathematical tool, engaging in critical or logical thinking). Deciding what they needed to do next to get started (i.e., making a plan). Figuring out what they knew (or did not know). Figuring out what the problem was asking.
Sub-theme: Getting a Solution to the Problem	
Grouping	Summarized Data
Tools Used	Different colors. Drawing software. Dynamic geometry software. Equations. Formulas. Graphs. Lists. Pencil and paper. Physical models. Rulers.

	Tables.
Actions Taken	Applying prior knowledge. Calculating. Coloring. Counting. Drawing. Inspecting. Labeling. Looking for patterns. Looking beyond the obvious. Noticing patterns. Outlining. Recording. Thinking critically. Thinking from a different perspective. Thinking outside the box. Using patterns. Visualizing.
Other Processes	Being careful. Being methodical. Being neat. Concentrating. Getting one's thoughts together. Not forgetting details. Not rushing. Ordering. Organizing information. Organizing to avoid confusion. Organizing to be sure about the answer. Paying attention. Paying attention to details. Remembering not to forget. Using tools precisely. Working together.
Sub-theme: After Solving the Problem	
Grouping	Summarized Data
Actions Taken	Checking one's solution. Comparing one's solutions with other students. Convincing oneself that one indeed has a solution. Deciding if one's own way or a peer's way is better. Discussing solutions with others to make sure. Double checking. Going back and doing something different to make sure. Going back and repeating the steps one used to solve.

	Having a reason for a pattern. Making predictions. Making sure. Recognizing patterns. Reviewing one's solution. Taking on a different geometric perspective (physically looking at the problem differently). Taking time to reflect on one's solution. Thinking about a more efficient solving method. Thinking about explaining one's solution differently. Thinking about how to explain one's solution to someone else. Trying to relate mathematical ideas.
<u>Sub-theme: Monitoring Progress</u>	
Grouping	Summarized Data
Statements About the Problem Solving Process	Being easy. Being hard. Getting easier. Getting harder.

Appendix F

Summarized Data for Theme Two Organized by Four Sub-Themes

Theme Two: Social and Intrapersonal Factors in the Classroom Environment	
	Sub-theme: Reciprocity
Grouping	Summarized Data
Reciprocity With Peers	Asking a peer questions. Being annoyed and discouraged by what others say. Being confused about peers all having different answers and not agreeing on one. Bouncing ideas around. Comparing answers with peers. Discussing solving the problem with peers then revising one's own approach. Deciding if one's own way or a peer's way is better. Discussing with peers. Getting help from a peer or peers. Having multiple group conversations. Helping each other. Listening to how peers solved the problem. Looking at another student's work. Reviewing with peers. Seeing what other peers did. Sharing answers with peers. Talking to peers about getting the solution. Working as a group. Working together. Working with others.
Reciprocity With the Teacher-Researcher	Asking the teacher questions. Getting help from the teacher. Listening to the teacher explain.
	Sub-theme: Metacognition
Grouping	Summarized Data
Statements about Metacognition	Asking questions to know if one is doing it right. Being observant.

	Brainstorming ideas. Deciding if one’s own way or a peer’s way is better. Doing a lot of thinking. Finding it difficult to explain one’s thinking. Figuring out how to put words to one’s problem solving process. Getting better at explaining one’s problem solving process. Giving a logical explanation that allows someone else to draw the same conclusion(s). Having a more open imagination. Having a more open way of thinking. Having one’s own logical way of thinking. Having to get one’s thoughts together. Knowing how to word one’s thoughts correctly. Knowing that one is not sure of one’s findings. Knowing that one did not understand. Knowing that one often overlooks important details while solving problems. Knowing what one is doing. Knowing words for what one is trying to say. Learning from past mistakes. Learning how to improve putting one’s thoughts on paper. Looking at the problem in every possible way. Looking more than skin deep at the problem. Monitoring one’s actions while solving a problem. Revising one’s approach to solving a problem. Taking time to reflect after solving a problem. Thinking from a different view. Thinking critically. Thinking harder. Thinking in a different way than normal. Thinking more logically and critically. Thinking of the easiest way to explain one’s thinking. Thinking outside of the box. Using multiple and creative ways of thinking. Using prior learning. Using visual interpretation.
Sub-theme: Presence of Mind	
Grouping	Summarized Data
Statements About Presence of Mind	Bolding lines to help focus on certain regions. Concentrating. Focusing.

	Focusing more. Focusing on patterns. Paying attention. Paying attention to detail. Paying attention to not forgetting. Paying close attention. Paying more attention to things not seen at first. Reading the problem closely and carefully. Really really looking.
<u>Sub-theme: Motivation and Affect</u>	
Grouping	Summarized Data
Statements About Motivation and Affect	Being annoyed and discouraged by others. Being lazy. Being successful. Being tired. Feeling confident eventually. Feeling confused. Feeling more confident. Having fun. Hoping the answer is right. Liking triangles. Not getting confused today. Noticing problem solving getting easier. Noticing problem solving getting easier while working with others. Thinking it was “cool” that people used color. Trying to succeed more. Trying hard.

Appendix G

Summarized Data for Theme Three Organized by Two Sub-Themes

Theme Three: The Role of Pedagogical Factors in the Classroom Environment	
Sub-theme: Teaching Concerns	
Grouping	Summarized Data
Statements About Teaching Concerns	Doing what the teacher-researcher said to do. Getting more detail about the topic. Needing a more detailed picture of the things to do in each step. Needing an illustration of the process. Needing better directions. Needing clear directions. Needing more detailed directions. Needing more explanation from the teacher-researcher. Needing more help from the teacher-researcher. Taking notes helped (n = 9).
Sub-theme: Time	
Grouping	Summarized Data
Statements About Time	Spending more time on one thing before moving on to another. Being slightly bothered by having a time schedule. Spending more time on the problem. Getting used to the time limit. Being challenged time wise. Explaining mathematical content slower.

Appendix H

Summarized Data for Theme Four Organized by Three Sub-Themes

Theme Four: Future Learning Needs	
Sub-theme: Practicing More	
Grouping	Summarized Data
Statements About Practicing More	Doing more problems. More practicing. Practicing. Practicing explaining. Practicing GeoGebra (n = 1). Practicing writing. Practicing writing mathematical explanations.
Sub-theme: Learning mathematical vocabulary and terminology	
Grouping	Summarized Data
Statements About Learning Mathematical Vocabulary and Terminology	Being more mathematical with one's words. Knowing more terminology. Knowing the terminology better. Knowing what mathematical terms to use. Learning more mathematical terminology. Thinking of "math words." Remembering terminology. Understanding mathematical terminology. Using more mathematical vocabulary.
Sub-theme: Learning Mathematical Content	
Grouping	Summarized Data
Statements About Learning Mathematical Content	Knowing how to calculate area. Knowing more formulas. Learning more about angles and mathematics in general. Learning more about triangles. Learning more and understanding more about angles.

Appendix I

Action Plan Chart

Recommended Action	Timeline	Resources Needed
Teaching Action Plan		
The teacher-researcher will continue to use Wiliam and Thompson's (2007) Five Key Formative Assessment Strategies	Ongoing	N/A
Student-participants will continue to complete written reflections (student-participant written reflections)	Biweekly	Creation of a new Google Form for each reflection
The teacher-researcher will plan for group interactions using the *Thinking Through a Lesson Protocol* (Smith, Bill and Hughes, 2008)	March 15, 2017	Locate or create a TTLP template
Implement a groupworthy task	By April 15, 2017	Locate or create a groupworthy task. Read Horn (2012) and Boaler (2016) for more information on design and implementation
Plan and design groupworthy tasks for future use	Summer 2017	Locate or create a groupworthy tasks
Establishing classroom and sociomathematical norms	Ongoing and 2017-2018 School Year	Synthesize the research to determine classroom and sociomathematical norms.
The teacher-researcher will engage in "Assigning Competence" (Horn, 2012, p. 31)	Ongoing	Horn, 2012, p. 31 Boaler, 2016, pp. 134-135
The teacher-researcher will "give growth praise and help" (Boaler, 2016, p. 178)	Ongoing	Boaler, 2016, p. 178
The teacher-researcher will review the literature to identify ways to support the emotional growth of student-participants	By February 15, 2017	Access to online databases (via University of South Carolina libraries)

Plans to reflect with student-participants		
The teacher-researcher will present findings to student-participants (allow time for reflection and feedback)	November 22, 2017	Create Google Slide Presentation
Student-participants will discuss (or write about) their mathematical beliefs and feelings about mathematics	To Be Determined	Further IRB Approval? Development of interview protocol or reflection questions
Plans for Outreach		
The teacher-researcher will share findings with members of the mathematics department	Ongoing	Planning time or online (email or Google Classroom)
The teacher-researcher will share a metacognitive strategy (written reflections) with other teachers.	December 13, 2017 (after school)	Create slides for a presentation
The teacher-researcher will present findings at conferences and in professional development settings	February 3-4, 2017 (NCTM Winter Institute, San Diego, California) April 7, 2017 (NCTM Annual Meeting, San Antonio, Texas) Additional dates and venues to be determined	Creating a presentation Travel funds Release time from teaching