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Measurement of New Observables from the $\pi^+\pi^-$ Electroproduction off the Proton

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Measurement of New Observables from the $\pi^+\pi^-$ Electroproduction off the Proton

by

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Abstract

Knowledge of the Universe as constructed by human beings, in order to tackle its complexity, can be thought to be organized at varying scales at which it is observed. Implicit in such an approach is the idea of a smooth evolution of knowledge between scales and, therefore, access to how Nature constructs the visible Universe beginning from its most fundamental constituents. New and, in a sense, fundamental phenomena may typically be emergent as the scale of observation changes. The study of the Strong Interaction, which is responsible for the construction of the bulk of the visible matter in the Universe (98% by mass), in this sense, is a labor of exploring evolutions and unifying aspects of its knowledge found at varying scales ranging from interaction of quarks and gluons as represented by the theory of Quantum Chromodynamics (QCD) at small space-time scale to emerging dressed quark and even meson-baryon degrees of freedom mostly described by effective models as the space-time scale increases. A direct effort to study the Strong Interaction over this scale forms the basis of an international collaborative effort often referred to as the N* program. The core work of this thesis is an experimental analysis prompted by the need to measure experimental observables that are of particular interest to the theory-experiment epistemological framework of this collaboration. While the core of this thesis, therefore, discusses the nature of the experimental analysis and presents its results which will serve as input to the N* program’s epistemological framework, the particular nature of this framework in the context of not only the Strong Interaction, but also that of the physical science and human knowledge in general will be used to motivate and introduce the experimental analysis and its related observables.
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1.1 Motivation

Human experience in the observable Universe forms the context upon which human beings construct knowledge, referred to as *human knowledge* in this thesis. This context is arguably and for practical purposes, infinite. This is in the sense that neither is it possible, thus far, to quantify this context in any analytical language nor encapsulate a feeling for this context in any qualitative expression.

Just as this context is infinite, so is human knowledge, with its uncountable quantitative and qualitative expressions of its understanding of this infinite context.

A perspective that can attempt to contain, and then imagine this infinite human knowledge based on an infinite context, can be formed by thinking of the experience of human beings at various points in this context giving rise to domains of human knowledge centered at each point of experience. Each domain can be thought to have a finite extent which can be called the *sub-context*. In this perspective, an infinitely complex context-space can be thought to contain an infinitely complex configuration of domains or sub-contexts, within which is contained human knowledge specific to the domain, which we can call *sub-knowledge*.

In order to continue motivating this thesis and present its most immediate motivation, one of the dimensions of this context-space along which several domains of human knowledge can be more specifically conceptualized will be isolated. This context-space is that of space-time and can be represented by a single dimension

---

1
stretching to infinitely large extents of space-time at one end, and that of infinitesimally small extents on the other end. The infinitely large extents can be thought to be the domain of the sub-knowledge fields of Cosmology and Astronomy, and the infinitesimally small extents that of the fields of Nuclear and Particle physics, and the range in between can be broadly thought to be the domain of Life, Social, and Earth sciences in order of ascending order of their space-time extent. This specific representation of the abstracted perspective is based largely upon the endeavor of science and therein the space-time extent of each sub-context is often thought to represent the *scale* at which the sub-knowledge field observes the Universe. The span of the scale ranges from the largest to the smallest space-time extents.

Implicit in the ordering of the various scientific disciplines in this scale is the effect of a powerful idea that is a part of the epistemology of each scientific domain. It is the idea that the largest level of observable complexity arises from the smaller lying fundamental parts, where large and small are used in the context of the scale. This idea is so powerful that it can be thought to be one of the ideas that unifies the scientific endeavor in that it is common to all domains of science. Even further, this idea overarches the entire scale in the sense that the field at the smallest level of this scale is thought to provide access to *the fundamental* parts of the observable Universe, and going up the scale all the way to its largest extents, the complexity of the Universe can be reconstructed.

Nuclear and Particle physics are sub-knowledge fields where this idea can be explored since as per this idea, here the fundamental parts that build up the large scale complexity of the Universe can be discovered. Additionally, the range of the scale of this sub-knowledge is large enough to begin to reconstruct the complexity of phenomenon seen at this scale from the fundamental parts discovered. In exploring this idea at this scale, a comparative approach will be taken, in that two phenomena will be compared: one that can be described by a *fundamental theory* which sets the
standards, within reasonable limits in the subliminal context of this unifying idea, where all the large scale complexity can be understood from the fundamental parts and the other where this unifying idea, to say the least, seems to be reaching its limit. In another perspective, in trying to explain this second phenomenon, this unifying idea instead seems to be instead giving rise to what appears to be, thus far, a series of disconnected and distinct frameworks of knowledge. These two phenomena are those of Electrodynamics and the Strong Interaction, respectively, and in the next few paragraphs they will be studied in a comparative manner.

The phenomenon of Electrodynamics, at the scale of Nuclear and Particle physics is described by the theory of Quantum Electrodynamics (QED). Within reasonable limits and especially within the context of a subliminal unifying idea of all complexity arising from fundamentals, QED may well set the standards for such a fundamental theory. It not only describes most electromagnetic phenomenon at this scale, but even moving on beyond the limits of the scale of Nuclear and Particle physics to macroscopic scales of Newtonian physics, elements of this fundamental theory of QED can be traceable, in as smooth a manner as possible, again within limits, to the classical theory of Electrodynamics described by Maxwell’s equations. A key feature of this evolution is that the fundamental degrees of freedom (d.o.f.) that are a part of QED at the quantum scale, continue to remain the same, again within limits, at the larger macroscopic scale of classical Electrodynamics.

In contrast to Electrodynamics is the phenomenon of the Strong Interactions, which operates within the nucleus of the atom. Here, compared to the unifying manner in which Electrodynamics can be thought of, there appears to instead be, thus far, fragmentation of its knowledge. These fragmented pieces of its knowledge can be laid out as disconnected pieces of its knowledge on the scale at which the Strong Interaction operates. At its smallest limits, it is the theory of Quantum Chromodynamics (QCD) that describes its phenomenon, and at the larger limits,
it is the various models, as for example the Constituent Quark Model (CQM) and beyond that the Yukawa potential, that are most suitable to describe its phenomena at increasing levels of complexity. In contrast to the key feature of QED, where the d.o.f. remain the same, here the d.o.f change as the knowledge domain changes: the d.o.f. for QCD, CQM, and Yukawa potential are the current quarks, constituent quarks, and the pions and nucleons that comprise the nucleus, respectively. Each of these d.o.f. are significantly different and do not evolve smoothly, as one would expect in the unifying picture of complexity arising from fundamentals, where by its implicit assumption, the theory of QCD should be the fundamental theory, but thus far, it has not proved to be. This is as far as this comparative approach with QED, which is arguably the most successful fundamental theory, can tell us anything about QCD. In fact in this approach, the knowledge of QCD only appears fragmentary.

The limitations of this comparative approach is directly limited to the limitations of the ideas of such a fundamental theory. Here is a good place to introduce another idea that can be as profound, which is that of emergence. According to this idea, not all complexity (arguably most of the complexity of the observable universe) can be obtained from a fundamental theory. Instead, complexity emerges at increasing levels of complexity in a manner that cannot be understood from a fundamental theory, and in this sense the idea of a fundamental theory needs to be questioned because immanent within the idea of emergence is another perspective of fundamental that, in contrast to that of a fundamental theory, exists at various points along a scale as complex phenomena emerge. This interplay between these two key ideas is the subject of extensive debates often encapsulated under the philosophical disciplines of Reductionism and Emergence, respectively.

The lens of emergence may be better suited to layout the scale dependent phenomenon of the Strong Interaction, where in contrast to QED, the fact that its d.o.f. freedom change actually point to its own two overarching key features that relate to
the idea of emergence. The first relates to its d.o.f in the sense that unlike in QED, cannot be isolated and observed in isolation. This is because its d.o.f. are inseparably contained within a bound system. This is the emergent phenomenon of Confinement and is one of the key features that make the study the Strong Interactions interesting and challenging because Nature presents an emergently complex whole that thus far eludes a full understanding based purely on a reductionist approach that attempts to understand it purely by uncovering its parts. The parts and the whole, in this sense, have to be studied together. The second key feature is also related to its d.o.f. in the sense that the mass of these d.o.f are vastly different at various points of the scale. At the level of QCD, the mass of the current up-quark is $2.3 \, \text{MeV}$ and at the level of CQM, the mass of the constituent up-quark is $336 \, \text{MeV}$. This increase in mass at increasing scales of space-time is due to the result of an emergent dynamical phenomenon called Dynamical Chiral Symmetry Breaking (DCSB) and the mass it generates is responsible for generation of $98\%$ of the mass of the visible Universe.

This interplay between the idea of reduction and emergence forms the core motivation of this thesis. The experimental data that is obtained from this analysis will directly probe the scale dependent phenomena of the Strong Interaction within the simplest bound system, the simplest emergent whole, that it presents, because a full understanding of this whole can only come from such an investigation. The understanding gained from probing such emergent complexity, whose fundamental parts are bound inseparably within a whole, at the smallest extent of the space-time scale, can have widespread consequences not just for abstract thought and philosophy of the nature of human knowledge, but even more immediately, to inform the exploration of emergence of complexity at the larger extents of the space-time scale that are the domain of the other sciences, for example that of Life, Social, and Earth sciences, where it is becoming more and more compelling to understand phenomena based on the idea of emergence, rather than that of reduction.
The experimental approach to probe the scale dependent phenomena of the Strong Interaction within a bound nucleon, which is the simplest case of an emergent whole that makes up matter at the time scale at which life abounds, necessitates the use of a probe with not only a variable spatial-time extent, but additionally the need for this probe on its own to be well understood and not additionally complicating the experiment. For that purpose, it is the photon, a d.o.f. of QED that is best suited. Its spatial extent is tunable in electro-production experiments where an electron is scattered of a bound Strongly Interacting system and in the process, in the language of a Quantum Field Theory, a virtual photon is exchanged between the scattered electron and the bound system. The knowledge of such an interaction at the electron vertex is related to QED, which is one of the most successful theories of modern physics, and in that sense, most of the unknowns in this interaction are related to the unknown nature of this interaction at the hadronic vertex, which can be written symbolically, in the language of Field Theory, as $\gamma^* NN^*$. Here the symbol $N$ represents the bound initial state Strongly Interacting system and $N^*$ the resulting excitation of this bound system. The space-time extent of this virtual photon is directly related to the variable $Q^2$, which is the ubiquitous scale-probing variable used in Nuclear and Particle physics. It is defined as the square of the four-momentum carried by the virtual photon, $\gamma^*$,

$$Q^2 = -q^\mu q_\mu,$$

where

$$q^\mu = e^\mu - e'^\mu$$

and $e$ and $e'$ are the initial and scattered electron four-momenta, respectively.

The resulting energy configuration of the initial state bound system due its interaction with the virtual photon is encapsulated by the variable $W$, which is the
invariant mass of the photon and initial nucleon system, and defined as

\[ W = \sqrt{s} = \sqrt{(q^\mu + P^\mu)(q_\mu + P_\mu)}, \]

where \( s \) is the Mandelstam variable that represents the square of the invariant mass of the photon and initial nucleon system, \( q \) and \( P \) are their respective four momenta.

This variable \( W \), thus far, has been studied extensively using photo-production reactions (photo-production reactions in contrast to electro-production operate exclusively at the photon point, that is \( Q^2 = 0 \)) in the field of Hadron Spectroscopy. This subfield of Nuclear and Particle physics has and continues to contribute to the understanding of the Strong Interaction. However, operating at the photon point, \( Q^2 = 0 \), it is not sensitive to scale based phenomena of the Strong Interactions. Its full extension to electro-production is relatively new and can be thought to be the convergence of the fields of Scattering and Spectroscopy to extract the knowledge of the \( \gamma^* NN^* \) as a function of \( Q^2 \) and \( W \), using the well understood virtual photon probe from QED and the already existing knowledge of the Strong Interaction contained in the \( W \) spectrum as a result of the extensive Hadron Spectroscopy studies, by uncovering the scale dependence of Strong Interaction phenomena, which is encapsulated in this \( \gamma^* NN^* \) vertex that is a function of both \( Q^2 \) and \( W \). The information contained within this interaction vertex is often referred to as Electrocouplings or Transition Form Factors, which basically probe a level deeper into this scale based interaction in the sense of being additionally sensitive to the possible spin degrees of freedom of the virtual photon and the hadronic bound system. A more detailed overview of Electrocouplings and Transition Form Factors can be found in [1] and [2].

In the next section this motivation in the context of the resultant collaborative effort, often referred to as the \( N^* \) program, will be described.
1.2 N* program

The N* program is an international collaborative effort that uses, amongst other probes, the electromagnetic excitation and the subsequent decay of nucleon resonances as a basis to investigate the dynamics of the Strong Interaction. Experimentally measured cross-sections (or observables) from photo- \( (Q^2 = 0 \text{ GeV}^2) \) and electro-production reactions \( (Q^2 > 0 \text{ GeV}^2) \) off the nucleon, at various values of \( W \) (\( \sqrt{s} \); the invariant mass of the photon and nucleon system) serve as an input to reaction models that strive to extract all contributing, independent resonant and non-resonant reaction amplitudes that encapsulate the dynamics of the Strong Interaction. The information contained in the resonant reaction amplitudes provides a basis for comparison with models and QCD based calculations. Figure 1.1 illustrates this process.

![Diagram](image.png)

Figure 1.1 Epistemological framework of the N* program [1].

A key goal of the process is to be able to define and measure a complete set of observables such that all independent resonant reaction amplitudes can be extracted as unambiguously as possible.

Tables II and I in [12] list the observables defined for the simplest case of single pseudoscalar meson photo- and electro-production, respectively. The definitions are
based on the various combinations of photon-/beam- (photo-/electro-production),
target-, and recoil-polarizations that can, at least in theory, be determined in the
initial and final state of the reaction. Note that not all observables need to be mea-
sured: the number of complex independent resonant reaction amplitudes for photo-
and electro-production are 4 and 6, respectively, corresponding to a minimal set of
8 and 12 real observables, respectively. Often in the literature experiments in which
the minimal set of observables can be measured are called complete experiments.

Photo- in contrast to electro-production experiments are closer to measuring the
minimal set of observables and a significant part of the collaborative effort is dedicated
towards this end. While photo-production experiments at the real photon point have
been vital in the area of Baryon Spectroscopy and in establishing resonant reaction
amplitudes at $Q^2 = 0 \text{ GeV}^2$, it is the more recently successful electro-production
experiments with tunable $Q^2$ that serve to probe the evolving dynamics of the Strong
Interaction, the importance of which is used as a motivation for this thesis.

The next section presents the set of observables that are measured as a part of this
thesis from electro-production of the double (charged) pseudoscalar meson electro-
production channel off the proton ($e p \to e' p' \pi^+ \pi^-$) that will serve as an input to the
Jefferson Laboratory-Moscow State University (JM) reaction model [13].

1.3 Observables from the electro-production of $p\pi^+\pi^-$ off the proton

For the first time photon polarization dependent observables in the double (charged)
meson electro-production: $R_{2T}^{00} + R_{2L}^{00}$, $R_{2LT}^{c,00}$, $R_{2TT}^{c2,00}$, $R_{2LT}^{s,00}$ and $R_{2TT}^{s2,00}$
[15] are measured in the reaction channel $p\pi^+\pi^-$ and will add to the single-differential
cross-sections that are defined irrespective of polarization [13] and that have served,
thus far, as the only input for the JM reaction model. (Note that the nomenclature
of the photon polarization dependent observables in the double meson electro-
production channel is based on their single meson electro-production counterparts [12] by replacing “R” with “R2.”

In contrast to polarization observables for single meson electro-production listed in Table II of [12], the presence of an additional particle in the final state adds $R_{2LT}^{s,00}$ and $R_{2TT}^{s2,00}$ to the list of observables. Note that the superscript “00” in this nomenclature of the observables, which refers to the fact that both the beam and target are unpolarized and as is the case for the experimental data used for this thesis, will from hereon be dropped when referring to these observables.

In addition to providing, for the first time, photon polarization dependent observables in the double (charged) meson electro-production channel, this analysis also extends the $Q^2 - W$ coverage of the $p\pi^+\pi^-$ reaction channel to hitherto unexplored regions: $Q^2$ is extended to be between 2.00 $GeV^2$ and 5.00 $GeV^2$, and $W$ to be between 1.400 $GeV$ and 2.125 $GeV$ [16].

While the detailed process of extracting these observables will be given Chapter 2, the following provides an overview of the observables based on the perspective of the motivation of this thesis.

As already noted, $Q^2$ and $W$ provide the general kinematical landscape within which photo- and electro-production observables are extracted. In this thesis, these observables are related to the final state of $p\pi^+\pi^-$ that results from the interaction of the photon with the proton within this $Q^2 - W$ landscape. This hadronic final state can be described by 3 possible assignments of its 5 independent kinematical d.o.f. expressed in the center of mass system (CMS) of the reaction [13]:

1. $M_{p\pi^+}, M_{\pi^+\pi^-}, \theta_{\pi^-}, \phi_{\pi^-}, \alpha_{\pi^+\pi^-}$
2. $M_{p\pi^+}, M_{\pi^+\pi^-}, \theta_p, \phi_p, \alpha_{\pi^+\pi^-}$
3. $M_{p\pi^+}, M_{p\pi^-}, \theta_{\pi^+}, \phi_{\pi^+}, \alpha_{\pi^+\pi^-}$

These final state variables are collectively referred to by $X^{ij}$, where the index ‘i’
represents one of the 3 variable sets and the index ‘j’ the variable within the set.

Often in this thesis these $X^{ij}$ may also be referred to as the hadronic variables or the kinematical hadronic d.o.f.. Since $Q^2$ and $W$ are obtained using the initial and final electron kinematics, they may also be referred to as the electronic variables or the kinematical electron d.o.f..

Figure 1.2 illustrates the angular kinematics of variable set 1.

![Figure 1.2 Illustration of the angular kinematics of variable set 1. Left side: $\theta_{\pi^-, \phi_{\pi^-}}$. Right side: $\alpha[p_{\pi^+}][p_{\pi^-}]$.](image)

In the perspective of the motivation for this thesis, it is insightful to think of the $X^{ij}$ forming a 5 dimensional Phase Space (PS) ($\tau^5$) within the 2 dimensional $Q^2 - W$ PS. Each observable is finally a one dimensional differential cross-section in bins of one of the variables in $X^{ij}$ obtained within a single $Q^2 - W$ bin. In the sense of the motivation, using this way of thinking, the cross-section in $X^{ij}$ provide data to further explore scale dependent phenomena, where the scale and the phenomena are encapsulated in $Q^2$ and $W$, respectively.

There are 51 observables that are extracted within a $Q^2 - W$ bin: 9 are the common single-differential cross-sections and 42 are the first-time measured observables that depend on the photon polarization, of which 30 provide additional constraints to the JM reaction model while extracting Electrocouplings. All of this will be discussed in detail in Chapter 2.
Chapter 2

Extraction of Observables from Experimental Data

This chapter describes in detail the process of extracting the observables from the experimental data. The experiment itself will be described in detail in Chapter 3.

The 51 observables that are extracted within the 2-dimensional $Q^2 - W$ bin space from the various projections of the 5-dimensional differential cross-section represent observables from the $\gamma^*p \rightarrow p'\pi^+\pi^-$ reaction. However, the experimental is configured to be able to directly measure cross-sections for the $ep \rightarrow e'p'\pi^+\pi^-$ which is 7-dimensional because of the additional 2 d.o.f. of the electron. The 5-dimensional cross-section is extracted from this directly measured 7-dimensional cross-section within $Q^2 - W$ bins, thus providing access to deeper exploration of scale dependent phenomenon. The relevant description, including the mathematical details, for this directly measurable 7-dimensional cross-section and the extraction from it of the 5-dimension cross-section within $Q^2 - W$ bins will be provided in detail starting in the subsequent paragraph. Cross-sections related to the reaction $\gamma^*p \rightarrow p'\pi^+\pi^-$, in this section for the sake of clarity, are denoted with the subscript $v$, and those related to the reaction $ep \rightarrow e'p'\pi^+\pi^-$ have no special denotation. In the later chapters, where the observables are discussed, this denotation is dropped since there all cross-sections relate to $\gamma^*p \rightarrow p'\pi^+\pi^-$. The description begins by expressing the directly measurable 7-dimensional cross-section in terms of all the experimental measurable data, constants, and correction
factors, all of which will be described in detail in the subsequent chapters, but for
now only an overview is listed. Before proceeding, note that the processing of such 7-
dimensional data is done using a 7-dimensional histogram, and therefore the following
will contain references to the bins of such a histogram, which are basically the data
points of this analysis. The binning of the histogram is setup to optimally extract
cross-sections given not only resolution of the measurement process, but also the
resolution of the resonances (affecting binning in \(W\)) and the general need of using
an optimal number of data points (bins) with enough statistical significance for the
JM model to fit to. Therefore the mathematical expression of this 7-dimensional
cross-section uses the \(\Delta\) symbol in reference to the bin width of the histogram, viz-a-
viz the \(d\) of Differential Calculus, which is a symbol for the infinitesimal change, that
is used in theoretical expression of the cross-section. It is an implicit assumption that
the integration of the theoretical expression over the bin width will be compared to
the experimental cross-sections obtained within these bins that is obtained using the
following formula:

\[
\frac{\Delta^{7} \sigma}{\Delta W \Delta Q^{2} \Delta \tau^{5}} = \frac{1}{L} \left( \frac{\Delta^{7} N_{ER}}{A \epsilon_{PMT}} + \Delta^{7} N_{EH} \right) \frac{\Delta W \Delta Q^{2} \Delta \tau^{5}}{\Delta W \Delta Q^{2} \Delta \tau^{5}} \tag{2.1}
\]

In this formula:

\(\Delta^{7} N_{ER}\) = Total \(ep \rightarrow e'p'\pi^{+}\pi^{-}\) events in an experimentally accessible 7D bin

\(\Delta^{7} N_{EH}\) = Total \(ep \rightarrow e'p'\pi^{+}\pi^{-}\) events in an experimentally inaccessible 7D bin

\(\Delta W \Delta Q^{2} \Delta \tau^{5}\) = Bin volume in the 7-dimensional space

\(A\) = Acceptance factor in the 7D bin obtained using simulation

\(\epsilon_{PMT}\) = Efficiency factor in a 7D cell for the Cherenkov detector

\(R\) = Radiative correction factor in the 7D bin

\(L\) = Integrated luminosity for the experiment
Once the 7-dimensional differential cross-section in Equation 2.1 is obtained, the 5-dimensional differential cross-section of the reaction $\gamma^* p \rightarrow p'\pi^+\pi^-$ in a $Q^2 - W$ bin, which is used to extract the observables, can be factored out using well known factorization factor known as the virtual photon flux, $\Gamma_v$, that separates the electronic part of interaction from the hadronic [13]:

$$\frac{\Delta^5\sigma^v}{\Delta \tau^5}(\Delta Q^2, \Delta W) = \frac{1}{\Gamma_v} \frac{\Delta^7\sigma}{\Delta W \Delta Q^2 \Delta \tau^5}$$ (2.2)

where

$$\Delta \tau^5 = \text{bin volume in the 5-dimensional space}$$

$$\Gamma_v = \text{virtual photon flux}$$

Equations 2.1 and 2.2 are the two main formulas into which experimental measurables, constants and correction factors are inserted to obtain first the 7-dimensional and then the 5-dimensional cross-sections, respectively. All of these experimental measurables, constants and measurables will be described in separate chapters because they form the core of the experimental analysis and for now they are noted in the sense of providing an overview to the process of extracting observables.

### 2.1 Single-differential observables

The single differential cross-sections are directly obtained by making projections of the 5-dimensional differential cross-sections onto each dimension. As noted previously in Chapter 1, this 5-dimensional cross-section is obtained in 3 variable sets with 5 variables in each. Therefore, there can be a total of 15 single differential cross-sections, however not all of them are used.

It can be directly seen that variables related to the various two-particle mass combinations of the particles in the final state are repeated in the variable sets.
Therefore only the 3 unique mass distributions, $M_{p\pi^+}$, $M_{\pi^+\pi^-}$ and $M_{p\pi^-}$, are finally used. This reduces the 15 possible combinations to 12.

Additionally, none of the $\phi$ angle variables are used which reduces the total observables to 9. This is because after fitting to the cross-sections in the 9 observables, the additional $\phi$ angle cross-sections provide little additional constraint to the JM model in comparison to the photon-polarization dependent observables whose starting point is exploiting this $\phi$ degree of freedom (see next section).

In summary, within a $Q^2-W$ bin there are 9 single-differential observables that are obtained from a direct projection of the 5-dimensional cross-section. The 9 variables used for obtaining the single-differential cross-sections are:

1. $M_{p\pi^+}, \theta_{\pi^-}, \alpha_{[p'\pi^+][p\pi^-]}$
2. $M_{\pi^+\pi^-}, \theta_p, \alpha_{[\pi^+\pi^-][p'p]}$
3. $M_{p\pi^-}, \theta_{\pi^+}, \alpha_{[p'\pi^-][p\pi^+]}$

2.2 Photon polarization dependent cross-sections

The reason that the $\phi$ cross-section provide little additional constraint to the model after fitting to the 9 single-differential observables can be inferred from the following equation which describes the 2-dimensional projection of the 5-dimensional cross-section where one of the dimension is always the $\phi$ angle [15] (In the following formula $\phi_i$ to the $\phi$ angle from the i-th variable set and $X^{ij} \neq \phi_i$. The rest of the terms will be explained in detail over the course of this section.):

$$\left( \frac{d^2\sigma}{dX_{ij}d\phi_i} \right) = R_{2T}X_{ij} + R_{2L}\phi_i + R_{2LT}\phi_i \cos \phi_i + R_{2TT}\phi_i \cos 2\phi_i + \delta_{X_{ij}\alpha_i} \left( R_{2LTs,\phi_i} \sin \phi_i + R_{2TTs,\phi_i} \sin 2\phi_i \right)$$

(2.3)
and the fact that each of the coefficients of the sinusoid functions, which will be described later in this section, in comparison to the constant term, \( R_{2T}X_{ij} + R_{2L}X_{ij} \) are significantly small. Therefore it can be directly seen that the \( \phi \) integrated cross-section gives the single-differential cross-sections in \( X_{ij} \) whose strength is equal to \( R_{2T}X_{ij} + R_{2L}X_{ij} \cdot 2\pi \). In contrast, integration over \( X_{ij} \), results in the single-differential \( \phi \) cross-sections, whose strength is dominated by \( R_{2T}X_{ij} + R_{2L}X_{ij} \) because the coefficients of sinusoid distributions are significantly smaller in comparison. Therefore the single-differential \( \phi \) cross-sections provide little additional constraint to the model after making use of 9 single-differential cross-sections.

However, even if significantly smaller than the dominant constant term, these coefficients do contain additional information about the nature of interaction of the virtual photon with the proton: The “L” and “T” in this equation refer to the longitudinal and transverse spin polarization of the virtual photon, and the “R2” coefficients encapsulate the photon polarization dependent interaction of the photon with the proton. The 9 single differential cross-sections, due to their integration over \( \phi \), contain only the dominant \( (R_{2T}X_{ij} + R_{2L}X_{ij}) \) contribution to the cross-section when the photon is either longitudinally or transversely polarized, and the contribution from the remaining interference terms \( (R_{2LT}X_{ij}, R_{2TT}X_{ij}, R_{2LTs}X_{ij}, \text{ and } R_{2TTs}X_{ij}) \), even if negligible in comparison, are missed. The polarization dependent variables, by isolating these smaller interference contributions from the dominant contributions encapsulated in the single differential cross-sections maximize the additional constraint to the model. These photon polarization dependent cross-sections are extracted for the first time in this thesis work.

The process of extracting these begins by making 2-dimensional projections of the cross-section, where one of the dimensions is the \( \phi \) angle. Then these cross-sections are fitted using the functional form in equation 2.3 and from the fit function that best describes the data, the observables, which are constant term and the various
coefficients of the sinusoidal functions, are extracted. The nomenclature of the observables therefore includes in the superscript a "c", "c2", "s", or "s2" depending on the sinusoid it was a coefficient of. The constant term has no such superscript.

As compared to the 9 single-differential cross-sections that are measured, the total number of polarization observables that are measured are significantly more. This is because not only are there 5 polarization observables \((R_2^{X_{ij}} + R_2^{L_{ij}}, R_2^{c_{X_{ij}}}, R_2^{c_{X_{ij}}}, R_2^{c_{X_{ij}}}, R_2^{c_{X_{ij}}}, R_2^{c_{X_{ij}}})\), but also because each of them is extracted for all the relevant \(X_{ij}\), which are more than what was relevant for the 9 single differential cross-sections. This is because the discounted redundancy in the mass variable for the case of the single-differential cross-section is no longer applicable here because each observable depends not only on the \(X_{ij}\), but also the corresponding \(\phi\) angle \((:=\phi_i, \text{ where } "i", \text{ as usual, is the variable set index})\), hence the nomenclature that includes \(X_{ij}\) and \(\phi_i\) in the superscript and subscript, respectively (Note that it is implicit that \(X_{ij} \neq \phi_i\)). Therefore the redundant mass variables now provide unique information because each is with respect to a distinct \(\phi\) angle.

Before noting the total number of polarization observables that are possible, another feature of equation 2.3 needs to considered, which is that the observables that are the coefficients of the sine function \((R_2^{s_{X_{ij}}} \text{ and } R_2^{s_{2X_{ij}}})\) are non-zero only when \(X_{ij} = \alpha_i\), where \(\alpha_i\) is the respective \(\alpha\) angle in the \(i\)-th variable set. This is denoted by the \(\delta_{X_{ij}}\alpha_i\) term in the equation.

Therefore, there are 42 possible polarization observables:

- 36 from the possible \(R_2^{X_{ij}}, R_2^{L_{ij}}, R_2^{c_{X_{ij}}}, R_2^{c_{X_{ij}}}, R_2^{c_{X_{ij}}}, R_2^{c_{X_{ij}}}\): 3 observables x 3 variable sets x 4 variables/variable set \((X_{ij} \neq \phi_i)\)

- 6 from the possible \(R_2^{s_{X_{ij}}} \text{ and } R_2^{s_{2X_{ij}}}\): 2 observables x 3 variable sets x 1 variable/variable set \((X_{ij} = \alpha_i\) only)

Of these 42, only 30 provide additional information to further constrain the JM
model because the 12 related to $R_{2T}^{X_{ij}} + R_{2L}^{X_{ij}}$ already constrain the model as a part of the 9 single-differential cross-sections: as noted earlier in this section, integrating equation 2.3 over $\phi_i$ results in the 9 single-differential cross-sections in $X^{ij}$ scaled by a factor $\frac{1}{2\pi}$. Nevertheless, they are still extracted and their consistency with their counterpart in the 9 single-differential cross-section is used as a consistency check in this analysis.

In summary, 42 photon polarization observables are extracted. The relevant $X^{ij}$ for them are (Note that for $R_{2LT}^{s,X_{ij}}$ and $R_{2TT}^{s_2,X_{ij}}$ only $\alpha_i$ are valid):

1. $M_{p\pi^+}, M_{\pi^+\pi^-}, \theta_{\pi^-}, \alpha_{[p'\pi^+]}|p\pi^-|

2. $M_{p\pi^+}, M_{\pi^+\pi^-}, \theta_{p}, \alpha_{[\pi^+\pi^-]}|p'p|

3. $M_{p\pi^+}, M_{p\pi^-}, \theta_{\pi^+}, \alpha_{[p'\pi^-]}|p\pi^+|

2.3 Summary

In summary, this chapter provides an overview of the core analysis pieces that when put together in Equations 2.1 and 2.2 give, within $Q^2 - W$ bins, the 5-differential cross-sections for the $\gamma^*p \to p'\pi^+\pi^-$. Further, this chapter lists in detail the steps using which the 51 observables – 9 single differential cross-section and 42 photon polarization observables – are obtained from this 5-dimensional cross-section. Of these 51 observables, 42 are measured for the first time and of which 30 will provide additional constraints to the JM reaction model[13] in extracting the Electrocoupling parameters.

In the subsequent chapters these core analyses pieces will be discussed in detail, but before that the experiment and its apparatus will be discussed in detail. Following the core analyses details, the results will be discussed in Chapter 13.
CHAPTER 3

EXPERIMENT

As noted in Chapter 2, the observables are extracted from the reaction $ep \rightarrow e'p'\pi^+\pi^-$ whose kinematics can be directly measured from the experimental configuration. In this chapter the details of this experimental configuration will be described.

The experiment that provides data for this analysis was carried out Jefferson Laboratory (JLab). The main experimental configuration consists of directing a beam of electrons, using JLab’s Continuous Electron Beam Accelerator Facility (CEBAF) [22], on various target materials that need to be probed. The interaction point of the electron beam with the hydrogen target is contained within the CEBAF Large Angle Spectrometer (CLAS) [23] that serves to identify and note the kinematics of the various final states produced from the interaction. In general the beam and target can be have various configurations that includes their respective polarization states, the energy of the beam and the dimensions of the target.

For the experimental run that provided data for this analysis, called the E16 experimental run, the target is a Kapton cell filled with liquefied hydrogen. Its length and diameter is 5 cm and 1.4 cm, respectively, and it is located $-4$ cm off-center, along the direction of the beam line, relative to the center of the CLAS detector. Both the target and the beam are unpolarized. The beam energy is 5.754 GeV.

Figures 3.1 and 3.2 provide a schematic overview of CEBAF and CLAS, respectively. A detailed description of the experimental configuration can be found in the references for CEBAF and CLAS, which are [22] and [23], respectively. In this chapter a high level description of the experimental configuration, along with details relevant
to understanding the full process of identifying and measuring the kinematics of the $ep \to e'p'\pi^+\pi^-$, will be provided.

Figure 3.1  Schematic overview of CEBAF reproduced from [22].

Figure 3.2  Schematic overview of CLAS reproduced from [23].

The CLAS detector consists of the following main subsystems: The Drift Cham-

20
ber(DC), the Cherenkov Counters (CC), Time-of-Flight Counters which are also referred to as the Scintillation Counters (SC), and the Electromagnetic Calorimeter (EC). These can also be directly seen in Fig 3.2. The sections below provide an overview followed by the relevant, high level technical details for each of the subsystems.

For additional detail see [22] and [23].

3.1 The Drift Chambers (DC)

The DC system is the primary means to determine the kinematics of the particles that emerge from the interaction point. This measurement process is based on the knowledge when a charged particle encounters a magnetic field perpendicular to its direction of motion, its original path will bend to follow a curved path. The direction in which the path bends and the radius of curvature of its path are directly proportional to its charge and momentum that is perpendicular to the magnetic field, respectively. Therefore the key components of the DC is a magnetic field configuration that provides such a deflection to all charged particles emerging from the interaction point and a gaseous ionization detector system that can track this deflected passage of the particle, both of which will be described starting in the subsequent paragraph. Based on these two pieces of information a sophisticated track fitting software is developed to extract the kinematics of the particle’s track [10].

The CLAS detector consists of a toroidal magnetic field generated by the main torus coils within the three tracking regions of the DC. This can be seen in Figure 3.2. It can also be seen in this figure that it is because of these coils that the CLAS detector is split into six sectors. Note that the mini torus is not a part of the DC and instead is used to prevent Moller electrons from the target from reaching the innermost layer of the DC.

As with the beam and target, the setting of the magnetic field, which are its
magnitude and direction, can vary depending on the needs of the experimental configuration. The superconducting coils that generate the magnetic field can tolerate currents up to 3860 A, which can generate up to $2.5 \, T \, m$ of integrated magnetic field. The toroidal field direction is oriented along $\phi$ such that positively (or negatively) charged particles are bent away from (or toward the) beam line, or vice versa. Note that there is no bending in the axial ($\phi$) direction. This integrated field strength, due to the nature of the toroidal configuration, can vary from $2 \, T \, m$ for tracks that go in the forward direction to about $0.5 \, T \, m$ for tracks beyond 90 degrees [19]. For the E16 run the current in coils is set to 3375 A and the direction of the field was such that negatively charged particles were bent towards the beam line.

Each of the three tracking regions of the DC, within each sector, contains the gaseous ionization detector. Each region is further broken down into two superlayers, which can be seen in Figure 3.3. Each layer is a gaseous ionization detector made up of a configuration of sense and field wires within a gaseous mixture made up of 90% Argon and 10% $CO_2$. In one of these superlayers the wires are axial to the magnetic field and in the other they are tilted at an angle. In combination, the axial and tilted layers provide polar and azimuthal tracking information, respectively.

For additional detail see Reference [19].

3.2 Cherenkov Counters (CC)

Along with the Electromagnetic Calorimeter (EC), described in section 3.3, the CC is an important subsystem dedicated to not only identifying electrons, but along with the EC also forms a part of the hardware system called the trigger, described in detail in [23], which is able to note the presence of an electron candidate in an event and thereupon prompt the readout of data from all the subsystems of CLAS.

The value of this identification is based on the design idea that only electrons should register any signal in the detector. For this end, the CC operates on the
principle of Cherenkov radiation that states that light is emitted by charged particles when they move faster than the speed of light in the particular medium. The gas used in the CC is $C_4H_{10}$ and in it only pions with momentum greater than $\approx 2.5 \text{ GeV}$ can generate Cherenkov light, and this greatly suppresses any potential misidentification of electrons with pions, and additionally provides a clean signal to detect the presence of an electron and trigger the CLAS detector.

The light generated by the passage of electrons is collected in photomultiplier tubes (PMT) situated at the end of each CC detector element. Figure 3.4 illustrates the construction of one such detector element and how the light generated is collected by the PMT.

There are 18 such elements of the CC detector within each of the 6 sectors in CLAS. These elements are called segments and cover the polar angle ranging from 8 degrees to 42 degrees. More information for the CC can be found in [18].
3.3 Time-of-Flight Counter (SC)

The Time-of-Flight counters, also called Scintillation Counters, work in tandem with the DC to provide information needed to identify charged hadrons. As the name suggests, they provide the time it takes for a particle to travel from the interaction point to the SC, $t_{SC}$. An independent measure of this time can also be obtained from the DC’s measurement of the particle’s momentum, $p_{DC}$, its flight length to the SC, $l_{DC}$ (which is a sum of its trajectory directly measured in the DC and its projected path length, estimated by the DC, from the DC to the SC), under an assumed mass hypothesis for the particle, using the formula

$$t_{DC} = \frac{l_{DC}}{\sqrt{\frac{p_{DC}^2}{m^2} + p_{DC}^2}} \cdot c$$

If the assumed mass hypothesis is true, then both these independent time mea-
surements should agree within errors that arise due to the combined resolution of the individual measurements: $t_{SC}$, $p_{DC}$, and $l_{DC}$. The detector elements are designed and built in such a way that this combined time resolution is good enough to identify charged hadrons, especially within limits of higher momentum where there is increasingly little difference between the time of flight of heavy and light hadrons, for example that of the pion and the proton, which often need to be identified within the same event.

The basic element of the SC is a plastic scintillation counter with a PMT at either ends to detect and note the time of arrival of the scintillation light. This thickness of each counter is 5 cm and the length of the counters vary from 32 cm at the forward angle to 450 cm at the backward angles. This overall geometry for the SC in a particular sector can be seen in Figure 3.5.

![Figure 3.5 Illustration of the overall geometry of the SC within a sector taken from [20].](image-url)

For more details see [20].
3.4 Electromagnetic Calorimeter (EC)

The EC forms the last layer of the CLAS detector, that along with CC, forms a part of the CLAS trigger. This is because of its design capabilities of suppressing signals from hadrons and triggering on signals only from electrons. The EC is also designed to be able to reconstruct neutral decays of the $\pi$ and $\eta$ mesons into photons and to detect neutrons.

The operating principles are the following: Electrons lose all their energy within an electromagnetic calorimeter via an electromagnetic shower that is generated by the process of Bremsstrahlung and subsequent pair production. Hadrons, on the other hand, traverse the EC either as Minimum Ionizing Particles (MIP) and deposit only a fraction of their energy which is largely independent of their momentum or due to Strong Interactions where they deposit less energy per depth but continue depositing energy deep into the calorimeter. These principles can be used to identify electrons and separate them from hadrons, first at the hardware and subsequently, at the software level. At the hardware level this is accomplished by setting the minimum energy threshold in the hardware trigger to be twice the minimum ionizing energy deposition. While this largely suppresses hadrons from triggering the EC, signals from hadrons still make their way into the data. One way this could happen is if a hadron and an electron are co-incident within the same EC detector and another way, if the EC from a sector (EC detectors, like other detectors, are also divided into 6 sectors) triggers the event and that causes the readout of the other EC sectors too where a hadron may have deposited its energy. Such cases are handled at the level of software analysis of data from EC.

In order to make use of these principles, the EC, should have, amongst other requirements, good spatial and depth resolution of the energy deposit signature. The large level geometric construction of the EC can be understood on this basis and noting that the EC is a sampling calorimeter made up of alternating layers of active
and passive layers made up of scintillation strips and lead sheets, respectively. The fraction of energy deposited in the passive layers is recorded in the active layer. Therefore, only a fraction of the energy deposited is directly measured and is called the sampling fraction (SF), and is equal to roughly 0.3.

Each EC detector within a sector is made up of 39 layers and each layer is a triangle that contains a sandwich of this passive-active arrangement. While each layer is the same in this sense, the difference is that strips of the scintillation layer are oriented at an angle with respect to each other giving rise to 3 different orientations, where in each the strips are parallel to one of the 3 sides of the triangular geometry. In this manner stereo information on the transverse location of a energy deposition signal can be obtained. The 3 orientations are U,V and W, and each has 13 layers. This arrangement is made clear in Figure 3.6.

![Figure 3.6 Illustration of the U,V and W layers that comprise the 39 passive-active sandwiched layers of the EC detector within a sector. The figure is reproduced from [17].](image)
In order to prove additional longitudinal depth resolution, the 39 layers are additionally divided into an inner and outer stack, each containing 15 and 24 layers, respectively. The active-passive layer geometry is such that for electron energies between 0.5 GeV and 4.5 GeV the longitudinal shower shape peaks between layers 6 and 12, and therefore this can be used to discriminate against hadrons whose energy loss will typically continue deep into the outer layer.

For additional details see [17].

3.5 Experimental Data

The description, in the previous sections, of the different subsystems of CLAS is inspired by the data that is finally needed to perform a physics analysis. The data in this final state has to be obtained from the more basic data, often called raw data, that is directly measured by each of the detector elements, which is basically various forms of digitized electronic signals. The process of obtaining finally usable physics data from the raw data directly measured by each detector is done in the process of cooking which is described in detail in Reference [24]. This cooked data consists of reconstructed data from all of the detector subsystems that “are deemed by the [CLAS]collaboration to be suitable as input to publishable physics analysis” [24]. The cooked data files for E16 experiment were used as the starting point of the analysis in this thesis.
Chapter 4

Core Analysis Details

Most of the core analysis steps are prompted by the need for the information that needs to go into Equations 2.1 and 2.2, which can be categorized under the following categories:

4.1 Experimental measurables

This includes the number of measured reaction events, $\Delta^7N_{ER}$ and for each event, its associated 7-dimensional kinematics, $Q^2, W, \tau^5$. The total number of $ep \rightarrow e'p'\pi^+\pi^-$ events are counted using the CLAS detector and for each event, the associated 7-dimensional kinematics is measured. This is the foremost specialized core task and consists of specialized subtasks that are described in detail in Chapters 5, 6, 7, 8 and 9.

4.2 Correction factors

In the description of the specialized task of counting the number of reaction events, $\Delta^7N_{ER}$, it will be noted that in order to select just the $ep \rightarrow e'p'\pi^+\pi^-$ events from all the possible events that can result from the interaction of the electron with the proton and remove any regions of the detector that may be inefficient or non-functional, several selection criteria have to be applied. While these selection criteria keep most of the reaction events, some of them are lost in the process. To recover these lost events the a correction factor, called the acceptance, $A$ in Equation 2.1, needs to
be applied. This factor is obtained from the simulation process and is described in Chapter 10.

This acceptance factor, however, is not able to account for the loss of events from one selection criterion which is related to identifying electrons using information from the Cherenkov detector. This correction factor, $\epsilon^{PMT}$, has to be obtained using experimental data and will be discussed in detail in Section 5.1.

The Radiative correction factor, $R$, also obtained from the simulation, relates to obtaining cross-sections that are corrected for radiative effects at the electronic vertex of the reaction from the experimentally measured cross-sections that include this radiative effect. This process will be described in Chapter 11.

The last correction factor that goes into Equation 2.1, $\Delta^7N_{EH}$, is related to obtaining the estimate of the number of $ep \rightarrow e'p'\pi^+\pi^-$ events in an experimentally inaccessible kinematic 7D bin. Such a kinematic bin is called a kinematical hole in this thesis. This will be discussed in Chapter 12.

### 4.3 Constants

The constants that are used are: Integrated luminosity, $L$ and the virtual photon flux, $\Gamma_v$. The integrated luminosity is used to normalize the total number of acceptance corrected events in a 7-dimensional bin to obtain the 7-dimensional differential cross-section. The formula for obtaining the luminosity using experimental measurables and related constants is:

$$\frac{Q_{tot}l_t D_t N_A}{q_e M_H}$$

where $Q_{tot}$ is the total incident charge on the target, $l_t$ is the length of the target, $D_t$ is the density of liquid hydrogen, $N_A$ is the Avogadro’s number, $q_e$ is the elementary charge, and $M_H$ is the molar mass of hydrogen. Of these $Q_{tot}$ is obtained from the experiment and is the total charge that is accumulated in the Faraday cup. For the
E16 experiment this is equal to $2.129 mC$ [6], and when used in the formula with the values of the constants as noted below, gives a total integrated luminosity of $28 fb^{-1}$.

\[
l_t = 5 \text{ cm for E16 target}
\]

\[
D_t = 0.073 \text{ g/cm}^3
\]

\[
N_A = 6.022e23 \text{ mol}^{-1}
\]

\[
q_e = 1.602e - 19 \text{ C}
\]

\[
M_H = 1.007 \text{ g/mole}
\]

The virtual photon flux is used to extract the 5-dimensional hadronic cross section of the $\gamma^*p \rightarrow p'\pi^+\pi^-$ from the $ep \rightarrow e'p'\pi^+\pi^-$ and is defined mathematically as [14]:

\[
\Gamma_v = \frac{\alpha}{4\pi} \frac{1}{E^2M_p^2} \frac{W(W^2 - M_p^2)}{(1 - \epsilon)Q^2}
\]

where $\alpha$ is the fine structure constant and is equal to $\frac{1}{137}$, $E$ is the energy of the E16 beam, $W$ and $Q^2$ are the invariant mass of the photon-nucleon system and the square of the four-momentum transferred by the virtual photon, respectively, and $\epsilon$ is the virtual photon transverse polarization given by [14]:

\[
\epsilon = \frac{1}{1 + \frac{2(Q^2 + \omega^2)}{4EE' - Q^2}}
\]

where $E$ and $E'$ are the energies of the incident and scattered electron beam, respectively, and $\omega = E - E'$. In the the formulae for $\Gamma_v$ and $\epsilon$ the values of $Q^2$ and $W$ are obtained from the center of their respective bins.
Chapter 5

Particle Identification

The task of binning the kinematics of the $ep \rightarrow e'p'^{+}\pi^{-}$ begins by identifying the final state particles from the list of all the tracks that have been reconstructed for an event during the cooking process. In general, track information from all of the sub-systems of the CLAS detector – Drift Chamber (DC), Cherenkov Counter (CC), TOF system (SC) and Electromagnetic Calorimeter (EC) – is used in identifying particles, the details of which are already described in Chapter 3. Depending on the particle, some of the sub-systems that are designed specifically for its detection play a specialized role, which for the electron, for example, are the CC and EC. The process for identifying any particle, in that sense, involves a series of general and specialized selection criteria, which at the level of this analysis’ software, are called general cuts and specialized cuts, respectively. These cuts are applied to each track within an event and the track that passes all cut conditions for a particular final state particle is retained for further analysis.

These cuts can further be classified as those whose parameters are directly encoded in the cooked data and those whose parameters need to be obtained by various levels of further analysis of the cooked data.

Lastly, cuts generally are independent of each other and unless noted otherwise can be applied in any given sequence. The sequence in which they are noted below are a reflection of the particular sequence used in the analysis code.
5.1 Electron Identification

General Cuts

All of the following items refer to potential electron candidates.

1. Its track should be the first track in the event.

2. Its track’s charge should be -1.

3. Its track’s trajectory should have registered a sub-track in the DC, SC and EC.

4. Its track’s status from time-based tracking (TBT) [19] should be good.

5. Its track’s status from hit-based tracking (HBT) [19] should be good.

6. Its track’s $z$-vertex position at the interaction point should be within $[-8.0 \text{cm}, 0.8 \text{cm}]$.

Cuts 1 to 5 are direct cuts since their parameters are already encoded in the cooked data.

The parameters of cut 6, that is the $z$-vertex cut, are established based on the knowledge of the target and is used to remove any electrons that are scattered off the target window which is located 2 cm downstream of the target. However, before the parameters of this cut can be established, the $z$-vertex position of the event in the data has to be corrected. This is because during the experimental run of E16 the beam position was not centered on the target’s position in the $x - y$ place. Its central position in this plane, instead of being at $x, y = 0 \text{ cm}, 0 \text{ cm}$ was at $x, y = 0.090 \text{ cm}, -0.345 \text{ cm}$ [6]. Because the vertex reconstruction process during cooking assumes this central position, its estimate of the $z$ position is therefore affected and needs to be corrected. This correction is done using the standard procedure developed for E16 [6] following which the $z$-vertex cut is applied which removes events whose $z$-vertex position is either less than -8.0cm or greater than 0.8cm. Figure 5.1 shows
the reconstructed $z$-vertex position for electrons that are detected by each of the 6 sectors. The entries within the green cut-lines at $-8.0\ cm$ and $-8.0\ cm$, respectively, arise from the interaction of the electron with the protons within the target. With this cut, the entries that arise from the interaction of the electron with the target window, the smaller peak offset $2\ cm$ from the upper edge of the target, are removed from the analysis. The blue histogram is obtained after applying vertex corrections to the uncorrected vertex positions, which are plotted in the black histogram.

Figure 5.1 Illustration of the reconstructed $z$-vertex position before (black) and after (blue) $z$-vertex corrections, and the standard $z$-vertex cut (green) at $-8.0\ cm$ and $-8.0\ cm$. 

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Specialized Cuts

The following series of specialized cuts, based on the CC and EC are applied to tracks that pass the general cuts and lead to selection of the electron in the event:

1. Its tracks should have registered a signal in the CC.
2. Its track’s momentum should should be larger then 0.70 GeV.
3. Its track’s minimum energy deposited in the EC should be greater then 0.06 GeV.
4. Its track in the EC should lie within its fiducial volume.
5. Its track’s photoelectrons in the CC’s PMTs should be larger than the determined threshold.
6. Its track’s Sampling Fraction (SF) in the EC should be within the values determined for electrons.

Cut 1 is a direct cut since its parameter is directly encoded in the cooked data. The CC is designed to register a signal only from electrons and thus help in separating them from pions, for example, whose threshold to trigger the CC is approximately 2.5 GeV, which is a relatively large value for the final state pions. [4].

The value of 0.70 GeV for cut 2 is a direct consequence of the fact that the EC forms a part of the electron trigger system in CLAS and therefore a minimum energy threshold is set in the hardware such that energy deposited from pions and other minimum ionizing particles (MIPs) is below this threshold and therefore does not trigger the EC. However, this hardware cut, due to the inherent statistical nature of energy deposition, the detector resolution and other possible effects, can produce artifacts in the analysis and in order to deal with this a procedure was developed to remove electrons whose energy deposited is within to this threshold. This details of this procedure are contained in Reference [3]. Equation 1 from this reference that is
used to calculate the value of 0.70 GeV is reproduced below.

\[ EEC(\text{in MeV}) = 214 + 2.47 \times E_{\text{C threshold}}(\text{in mV}) \]

Here EEC stands for electron energy cut. For E16 experiment the $E_{\text{C threshold}}$ is set to 172 mV \[6\] and putting this value in the above equation gives EEC of 0.63884 GeV. Given the fluctuations in energy deposition, a conservative, upper bound value of 0.70 GeV is finally used. The range of this possible fluctuation is also directly inferred from the following statement in \[3\]: "For instance, if the threshold energy is 500 MeV then it should be expected that in energy region $\leq 500 - 600 MeV$ the cross sections will be distorted by the threshold effects [...]".

Even with this hardware threshold in the EC to reject MIPs from triggering an event, often a MIP can still be registered in the data as the triggering particle. This can happen, for example, when a MIP is coincident with an electron in the same EC sector \[17\]. Cut 3 is therefore used to remove such particles \[9\].

As already described in Section 3.4, electrons deposit a known fraction of their energy in the EC by means of an electromagnetic (EM) shower and this known fraction, roughly equal to 0.3, forms the basis of the main specialized cut for the electron: the Sampling Fraction cut, which is cut 6 and which will be discussed later. This EM shower has a longitudinal and transverse extent in the detector and if it occurs close to the edges, then the shower can leak out of the detector and the energy deposited cannot be fully reconstructed and this can lead to misidentification of electrons. In order to deal with this effect, a method was established to determine the centroid of the electron hit using the U,V,W coordinate system of the EC and requiring this centroid to be at least 10 cm away from the U,V,W edges of the EC \[5\]. Effectively this cut, which is cut 4, requires that the U,V,W hit positions of the electron satisfy: $20 \text{cm} \leq U \leq 400 \text{cm}, V \leq 375 \text{cm}$ and $W \leq 420 \text{cm}$.

While cut 1, because of the specialized task of CC of triggering only on electrons and thereby separating them from pions \[18\], works very well in identifying
the electrons within the kinematic range of this analysis, there is, however, some contamination from non-electrons, which are mostly pions. This contamination can be seen in Fig. 5.2 where the photoelectron distribution in the left PMT for each of the 18 segments of the CC in sector 1 is shown: while the photoelectron distribution due to tracks from the electron fits a parametrized Poisson distribution [6], which is shown by the green dotted fit line to the plotted distribution (blue data points), the contamination due to pions stands out from this distribution as a “noise-peak” at low photoelectron values. This main contribution to this noise-peak comes from pion tracks that are in coincidence with accidental noise in the PMT [21]. In specialized cut 4, also referred to as the photoelectron cut this noise-peak is cut away: the two solid lines in the plots, green and red, represent the loose and tight cut values, respectively. For extracting the observables the loose cut is used and the difference between the two cuts in the sense of extracting observables is at the 1% level.

Note that the effect of this cut on the acceptance cannot be obtained from the simulation. This is because in the simulation the distribution of the photoelectrons is not the same as that in the experiment. Therefore the fraction of events lost to this cut have to be estimated and corrected for differently. This process involves calculating the efficiency of this photoelectron cut for every PMT, $\epsilon(PMT)$, of the CC by dividing the integral of the parametrized Poisson distribution [6] from the cut-value determined to infinity by the entire integral of this distribution. Note that the parametrized Poisson distribution and the cut-value are obtained separately for each PMT of the CC as shown in Figure 5.2. The mathematical expression for the efficiency is

$$\epsilon(PMT) = \frac{\int_{\text{pho-cut}}^{\infty} f(x)dx}{\int_{0}^{\infty} f(x)dx},$$

where $f$ is the parametrized Poisson distribution reproduced from [6]

$$f = A \left( \frac{L^{x/p}}{\Gamma(x/p + 1)} \right) e^{-L}$$

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and $A$, $L$ and $p$ are its variable parameters.

This $\epsilon(PMT)$ is then used to correct for electrons that are lost because of this cut by noting the PMT for electron candidates that survive this cut and giving them a weight factor of $\frac{1}{\epsilon(PMT)}$.

Additionally, note that in Figure 5.2 the noise-peak for segments 1, 15, 16, 17 and 18 appears to be enhanced with respect to all other sectors. This is only an artifact of this cut’s analysis because these segments, within the constraints of the kinematics of the reaction $ep \rightarrow e'p'\pi^+\pi^-$, have very little data. In order to obtain more data in these segments this constraint had to be loosened, thereby allowing not only more data due to the electrons, but also due to the accidental pions. It has been observed that this noise-peak is significantly reduced within the kinematic constraints of this analysis, mainly due to the upper limit of $W = 2.125 GeV$. Removing this upper limit on $W$ by removing this kinematic constraints allows for lower energy electrons to enter the analysis and it is observed that there is a significantly higher contamination with accidental-pions for these electrons.

Cut 6 is the main specialized cut and is called the Sampling Fraction (SF) cut. As already described in Chapter 3, due to the physical construction of the EC, electrons deposit only a fraction of their energy in the active layers of the DC, which is roughly equal to 0.3. The SF is calculated by dividing the total energy deposited by a track in the calorimeter by the momentum of the track:

$$SF = \frac{etot}{p}$$

$etot$ and $p$ are directly obtain from the cooked data as part of the information from the EC and DC, respectively. The SF for electrons of all momentums is $\approx 0.3$ and this feature is used to remove any tracks that are not electrons.

Due to the inherent nature of the energy deposition process and the resolution of the detector, this SF is Gaussian distributed. A procedure is developed to estimate the parameters of this Gaussian histogram in various momentum bins. The $\pm 3-\sigma$
Figure 5.2 Illustration of the photoelectron cut all the left PMTs of CC in sector 1: The blue points represent that data points and the green dotted line the fit to the parametrized Poisson function. The solid green and red lines represent the loose and hard thresholds of this cut, respectively.

limits, from the $\mu$ of this distribution, is fitted as a 3rd order polynomial function of momentum to obtain the finally used SF cuts. Figure 5.3 contains plots, from each of the 6 CLAS sectors, for the SF of electron candidates versus their momentum. Figure 5.4 shows an example of a one dimensional projection of the SF in momentum bin $[1.64 \ GeV, 1.74 \ GeV]$ from sector 1 that is fitted to a Gaussian distribution. The $\pm 3\sigma$ limits from the fits to such projections is then overlaid on the respective momentum
bins in Figure 5.3 and fitted to a 3rd order polynomial as a function of momentum to obtain the final parameters for the SF cut versus momentum.

Note that two sets of SF cuts are shown in the following figures: one obtained by fitting the bins in the peak of the Gaussian SF distribution (magenta lines) and the other by fitting the maximal extent of the distribution that was visually found to be free of events from non-electrons (black lines). The fit parameters obtained from fitting the maximal extent were found to best describe the data and were finally used for the analysis.

Figure 5.3 Illustration of the SF cut for all sectors: The magenta and black lines represent cuts obtained by optimizing the fits to fit at least the peak-bins and maximal-bins of the SF distribution, respectively, in each momentum bin.
Figure 5.4 Illustration of the Gaussian fit to SF for momentum bin [1.64 GeV, 1.74 GeV).

5.2 Hadron Identification

In this analysis only the proton and positive pion are required to be directly measured in the detector. The remaining particle, which is the negative pion, is indirectly measured using the missing mass technique, which will be discussed in Chapter 9.

Unless noted, all of the following cuts are applicable for selecting either the proton or the positive pion.

General Cuts

The following series of general cuts are directly applied to all particles in the cooked data:

1. Its track’s charge should be 1.

2. Its track’s trajectory should have registered a sub-track in the DC and SC.
These cuts are direct cuts since their parameters are already encoded in the cooked data.

**Specialized Cuts**

The following series of specialized cuts, based on the DC and SC, are applied to particles that pass the general cuts, and lead to the identification of the hadron in the event:

1. Its track passes the $\Delta t$-cut.

$\Delta t$ here refers to the difference between the time-of-flight as obtained indirectly from the DC ($t_{DC}$), using the particle’s directly measured path-length ($l_{DC}$) and momentum ($p_{DC}$) and an assumed value for its mass ($m_a$), and obtained directly from the SC ($t_{SC}$). If this mass assumption is valid, this $\Delta t$ should belong to a distribution centered at zero for all momentum, and if not, it should belong to distributions offset from zero depending on the falseness of this mass assumption. In the former case, the mass assumption leads to the identification of the particle.

Note, that $\Delta t$ is adjusted for any fixed time offset that may be present between $t_{DC}$ and $t_{SC}$. This offset is obtained using the electron which by this point is already identified using the CC and EC. Therefore, if this time-of-flight difference is also calculated for these identified elections, $\Delta t_e$, its value should belong to a distribution around zero, and any offset of this distribution is due to the presence of this offset.

The following set of equations are a direct mathematical expression of the above:

$$\Delta t = t_{DC} - t_{SC} - \Delta t_e,$$
where:

\[ t_{DC} = \frac{l_{DC}}{\sqrt{\frac{p_{DC}^2}{m^2+p_{DC}^2} \cdot c}} \]

\[ \Delta t_e = \frac{l_{e,SC}}{c} - t_{e,SC} \]

Note that for the electrons, the velocity is directly set to the speed of light because the electrons are relativistic.

Figure 5.5 contains a plot of \( \Delta t \) versus momentum for protons and positive pions, respectively. Superimposed on this plot is the cut line that was obtained similarly to the SF cut: the \( \Delta t \) distribution in each momentum bin is fitted to a Gaussian and the \( \pm 3\sigma \) limits from these fits is fitted to a 3rd order polynomial functional form to obtain the parameters for the \( \Delta t \)-cut as function of momentum for the proton and positive pions. This cut line along with the \( \pm 3\sigma \) is also shown in Figure 5.5. Figures 5.6 and 5.7 shows low and high momentum projections of \( \Delta t \) and their respective fits for the protons and the positive pions, respectively.
Figure 5.5 Illustration of the $\Delta t$-cut for the protons (top) and positive pions (bottom).
Figure 5.6 Illustration of the low and high momentum projections of $\Delta t$ and their respective fits for the protons.
Figure 5.7 Illustration of the low and high momentum projections of $\Delta t$ and their respective fits for the positive pions.
Chapter 6

Fiducial Boundary Identification

The geometry of the CLAS detector is such that it does not fully cover the $4\pi$ angular area subtended from the center of interaction point of the electron beam with the target. Because of the design constraint of the detector, there are a few physical gaps, sometimes also referred to as holes or dead areas of the detector:

- Sector gaps: gaps between the six CLAS sectors at respective $\phi$ angles
- Forward angle hole: hole to accommodate the beam line at $\theta = 0$
- Backward angle hole: hole as determined by the polar angle coverage required of the CLAS detector [8].

As noted earlier Chapter 2, the process of extracting observables requires estimating the acceptance of the CLAS detector using simulation data. While it is possible to fine tune the simulation process to, as closely as possible, simulate the response of the physical detector, this can only be done so for the active, in contrast to the dead, areas of the detector. The transitional edge separating the active and dead areas of the detector is not sharp and cannot be well defined in the simulation, and therefore the simulation process cannot accurately model the detector response close to these transitional edges. Therefore an analysis needs to be done that defines these edges within which the simulation response is the same as the detector. The definition of these edges make up the fiducial cuts that remove from the analysis any particles falling outside of them.
While each of the detector’s subsystems contributes to these edges, the total contribution can be studied if the two dimensional (2D) distribution of the kinematic $\theta$ and $\phi$ angles of the particles emerging from the interaction point is observed. This is because the points where the particle’s trajectory traverses the detector, in addition to its momentum which causes the trajectory to bend in the DC, is correlated to these angles and therefore whenever a particle passes through a hole, its corresponding $\theta - \phi$ information is not measured and this manifests itself as empty points in the corresponding 2D space. This momentum dependence of the topography of this 2D space is found to have a significant impact only for the electrons because their $\theta$ angle is strongly correlated with momentum: high momentum electrons typically have a small $\theta$ angle and strike the forward region of the detector where the azimuthal extent of this 2 dimensional space is narrower, and the low momentum electrons typically have a large $\theta$ angle and strike the backward region of the detector where the azimuthal extent of this 2 dimensional space is wider. The hadrons directly measured in the analysis, namely the $p$ and the $\pi^+$, on the other hand, show no such momentum dependence and the determination of their fiducial boundary in this 2D space can be done independent of momentum.

In this analysis, fiducial cuts already developed for electrons and hadrons for E1-6 in [9] are used and will be illustrated in the following sections.

6.1 Electron Fiducial Cuts

Figures 6.1, 6.2, 6.3, 6.4 and 6.5 illustrate fiducial cuts for electrons from [9] of momentum in range [2.20 GeV, 2.40 GeV], [2.50 GeV, 2.80 GeV], [2.80 GeV, 3.00 GeV], [3.00 GeV, 3.20 GeV] and [3.40 GeV, 3.60 GeV], respectively. The colored points represent the data points and the black lines the cut.
Figure 6.1 Illustration of fiducial cuts for electrons of momentum in range [2.20 GeV, 2.40 GeV].

Figure 6.2 Illustration of fiducial cuts for electrons of momentum in range [2.50 GeV, 2.80 GeV].
Figure 6.3  Illustration of fiducial cuts for electrons of momentum in range $[2.80\,\text{GeV}, 3.00\,\text{GeV}]$.

Figure 6.4  Illustration of fiducial cuts for electrons of momentum in range $[3.00\,\text{GeV}, 3.20\,\text{GeV}]$. 

Figure 6.5 Illustration of fiducial cuts for electrons of momentum in range [3.40 \text{ GeV}, 3.60 \text{ GeV}].
6.2 Hadron Fiducial Cuts

Figures 6.6 and 6.7 illustrate fiducial cuts for the protons and positive pions from [9], respectively. As with the electron fiducial cuts, the colored points represent the data, but here more than one cut line is shown. The black cut line is the one from [9] and that is, thus far, used in this analysis. The red and blue cut lines are obtained from [26] and appear to be, at least visually, better than the cuts from [9]. As a part of the further and continuing refinement of this analysis, which will be done after this thesis is written, observables using cuts from [26] will also be obtained. Till this further analysis is done, the the potential impact to the observables from these variations is estimated to be $\approx 1\%$.

Figure 6.6 Illustration of fiducial cuts for protons.
Figure 6.7 Illustration of fiducial cuts for positive pions.
Chapter 7

Detector Inefficiency Identification

This section describes cuts that are similar to those described in Chapter 6. While in that chapter these inefficient regions are directly related with physical gaps in the detector, here they are related to areas within the detector where a particular detector subsystem is found to be inefficient at various levels: from being partly to fully inefficient.

In this analysis, some of these inefficient regions have been found to be directly correlated with bad SC paddles. The rest, are hypothesized to be due to inefficiencies in the DC, but this has not been directly established. In the case of the former, these paddles are directly removed from the analysis, and for the latter, kinematic cuts are used.

The effects of the inefficient regions in the detector are visible as areas of depletion in kinematic plots where the polar angle and momentum of the particles, both in the lab frame, are plotted on the $y$-axis and $x$-axis, respectively. To illustrate cuts that remove the inefficient regions, two sets of these kinematic plots are shown for each particle (in each sector): the first directly containing this kinematic information and the second containing the same plot but with the cuts superimposed. In these plots, the black shaded areas represent the effect of removing a bad paddle from the analysis and the red lines represent the kinematic cuts. The full list removed bad paddles is listed at the end of this section.

Figures 7.1, 7.2 and 7.3 illustrate these cuts for the electron, proton and postive pion respectively. The top two rows represent the first set of plots and the bottom
two rows, the second.

A list of all the bad paddles that were removed from the analysis follows these plots.

Figure 7.1 Cuts to remove inefficient detector regions for the electron.
Figure 7.2  Cuts to remove inefficient detector regions for the proton.
Figure 7.3  Cuts to remove inefficient detector regions for the positive pion.
The following paddles were removed from the analysis. The first digit in the paddle number corresponds to the sector number, and the subsequent digits to the paddle number in that sector: 145, 205, 245, 311, 324, 337, 338, 340, 342, 345, 346, 347, 505, 520, 532, 540, 542, 632, 633, 634, 635, 636, 637, 638, 639, 644.

While most of them were identified from the experimental data, the ones denoted by an asterisk symbol were actually found to be turned off in the Simulation process, and thus had to be removed from the analysis too.
Chapter 8

Momentum and Energy Loss Corrections

Measurement of a track’s kinematics at the interaction point relies on a standard technique of fitting its curved trajectory in the tracking sub-system of the detector, the DC, in the presence of a magnetic field to that predicted by a track model whose parameters are directly related to the kinematics at the interaction point [10]. Any irregularities in the magnetic field or the tracking detector, the DC in this case, will cause related irregularities in the reconstruction of the track’s momentum, and these irregularities will be reflected in parts of the the analysis that uses the momentum of tracks: for example the invariant mass distributions will appear to be shifted and wider beyond what may be expected from the known detector resolution.

If these irregularities could be modeled in the simulation, correction factors could be obtained from there. However, just like the limitation of using simulation to accurately model the areas of the detector near the transitional edge of the fiducial boundaries, such irregularities, too, cannot be modeled in the simulation and empirical means need to be employed to obtain relevant correction factors [6].

Reference [6] was directly used to obtain such momentum correction factors for the electron and positive pion. No such correction factors are available for the proton and therefore their measured momentum, in this analysis, is left uncorrected.

The protons, even so reconstructed, are within the kinematic range where they suffer significant energy loss when traversing the material of the DC. Therefore, their energy loss is corrected using [11]. Note that for the electron and positive pion, the empirical momentum corrections, because of the direct relation between momentum
and energy, also end up correcting for their energy loss, which are expected to be not as significant as those for the proton in the kinematic range of the analysis.

In summary, the electron and positive pion momentum is corrected using empirically obtained correction factors, which because of the correlation between energy and momentum implicitly corrects for their energy loss as well. No such empirical momentum corrections are available for the proton and their measured momentum is left uncorrected. However, since the protons do suffer significant energy loss, this is corrected.

Figures 8.1 and 8.2 show the effect of these corrections on the Elastic peak and the missing mass (MM) distributions, in W bins from low, middle and high W range of the analysis (integrated over all $Q^2$), that are used to apply the final cut that isolates the $ep \rightarrow e'p'\pi^+\pi^-$ events. This missing mass(MM) distribution, which will be discussed in detail in the following chapter on Event Selection, Chapter 9, is a kinematic construct used to isolate events in which a negative pion was produced in the $ep \rightarrow e'p'\pi^+\pi^-$ that either is or is not measured in the detector. Most of the negative pions from this reaction, due to the nature of the magnetic field, are bent into the forward hole and are therefore undetected. Thus the MM distribution is required to isolate most of the events from this channel and since the case where the negative pion is measured in the detector is relatively much smaller, it is also selected using the same method.)
Figure 8.1 Effect of momentum corrections for electrons on the Elastic peak.

Figure 8.2 Effect of momentum correction for electron and positive pion, and energy loss correction for the proton, on the missing mass distribution for various W bins (integrated over all $Q^2$ bins).
CHAPTER 9

EVENT SELECTION

This is the last step in the analysis that uses the kinematic construct of missing mass ($MM$) to identify events that belong exclusively to the reaction channel $\gamma^* p \rightarrow p' \pi^\pm \pi^-$ irrespective of if the negatively charged pion is measured or not. As already noted, most of the negative pions from this reaction, due to the nature of the magnetic field, are bent into the forward hole and are therefore undetected. This prompts the nomenclature of the missing mass which is undetected in the reaction channel and can be constructed using energy and momentum conservation from the kinematics of the detected particles as shown below.

$$MM^2 = (\gamma'^\mu + p'^\mu - p^\mu - \pi^\mu) \cdot (\gamma^\mu + p^\mu - p'_\mu - \pi^+_\mu).$$

$MM$ can be obtained by taking the square root of the $MM^2$. In this equation $\gamma^*$, $p$, $p'$ and $\pi^+$ are four-momenta of the virtual photon, initial state proton, final state proton and final state positive pion, respectively. For the events of interest, the $MM$ is equal to the mass of the negative pion and this fact can be used to establish a cut that removes from the analysis events that are not of interest. Additionally, since the case where the negative pion is measured in the detector is relatively much smaller, it is also selected using the same method.

In practice, it is often better to study the $MM^2$ distribution instead to obtain the $MM$ cut. This is especially the case when the distribution spans values in the range between 0 and 1, where the effect of taking the square produces mathematical artifacts that can obscure the presence of physics and detector related effects.
Figures 9.1 and 9.2 show the $MM^2$ and $MM$ distributions, respectively, in $W$ bins from the low, middle and high $W$ range of the analysis (integrated over all $Q^2$), and the upper and lower bound of the $MM^2$ and $MM$ cuts (in green), respectively, that is used to isolate the events. For reference, the $MM$ distribution from simulation is directly shown in the same plots to justify the selection of these $MM$-cuts in the sense that maximizes the signal while rejecting background. The systematic uncertainties from this cut are estimated to be 2.9% (Table 13.1).

Figure 9.1  $MM^2$ distribution from experimental (black) and simulation (red) data in $W$ bins from the low, middle and high $W$ range of the analysis (integrated over all $Q^2$).
Figure 9.2  $M_M$ distribution from experimental (black) and simulation (red) data in $W$ bins from the low, middle and high $W$ range of the analysis (integrated over all $Q^2$).
Chapter 10

Acceptance Calculation

All the cuts applied are necessary for not just selecting just the $ep \rightarrow e'p'\pi^+\pi^-$ events from all the possible events from the reaction of the electron with the proton, but also to use only the fully functional areas of the detector. In this process some events are lost and these need to be recovered in order to obtain the cross-sections.

The correction factor, called the acceptance, that recovers these lost events is obtained from the simulation process where the $ep \rightarrow e'p'\pi^+\pi^-$ reaction is simulated in accordance with experimental conditions. Cuts defined using the experimental data are then applied to the simulated data and the fraction of events lost is obtained. This process of simulation, therefore, plays a key role in the extraction of cross-sections and as such represents in-depth knowledge of the physics of the reaction being simulated and its response in the detector. In general both these aspects of the process represent significant effort, not just at the level of local experimental collaborations, for example the CLAS collaboration, where knowledge specific to physics reactions is constructed, but also at the level of the larger community of experimental physics that need to simulate the passage of particles through matter –for example particle detectors as used in Medium and High Energy Physics and also in related applications in the medical industry –and therefore the need for a common technology core like GEANT (GEometry ANd Tracking) [25] whose knowledge base represents the collective efforts of Modern Particle and Nuclear Physics.

The simulation used in this thesis is based on such knowledge and technology. It is beyond the scope of this thesis to go into all the details and in this section only
the higher level knowledge presented to the end user of such a simulation process will be provided in Section 10.2. Quantitative values of the calculated acceptance and related quantities from which it is calculated will be shown in Section 10.3. But foremost a key fact of the acceptance factor, the fact that it is extracted in a model independent way, will be described in Section 10.1.

10.1 Model Independent Extraction of Acceptance

The acceptance factor obtained in a manner that is independent of the model used because it is obtained in each of the 7D PS bins of the $ep \rightarrow e'p'\pi^+\pi^-$. The experimental yield, which as already noted is also obtained in these 7D PS bins, and is therefore corrected at this level. This is possible because the simulated $ep \rightarrow e'p'\pi^+\pi^-$ events are able to reproduce the 7D kinematic correlation as seen in the experimental data. The simulation process is able to do this because the event generator used, genev, which is a Monte Carlo event generated of the Genova group based on the JM model [13], is knowledgeable about the experimental 7D PS, and the detector simulation accurately represents the real detector.

The agreement between experiment and simulation of this 7D kinematic correlation can be seen in Figures 10.1 and 10.2 where for the kinematics for the final state particles for the $ep \rightarrow e'p'\pi^+\pi^-$ from the experiment and simulation are directly compared by plotting them together, normalized to the same area. For the simulation both the generated events, also called the thrown events, and the reconstructed thrown events are shown (the blue, red and green lines represent experimental, simulated-reconstructed and simulated-thrown data, respectively). The top, middle and bottom rows compare the magnitude, polar and azimuthal angle of the particle’s momentum in the laboratory frame, respectively. Within each row, the four plots show the kinematics for each of the final state particles: $e, p, \pi^+$ and $\pi^-$. In order to efficiently simulate the large $Q^2$ range of the analysis, which is between
2.0 \, GeV^2 \text{ and } 5.0 \, GeV^2, \text{ the simulation was separated into the low limit of this range, which is between } 2.0 \, GeV^2 \text{ and } 3.0 \, GeV^2, \text{ and the high limit of this range, which is between } 3.0 \, GeV^2 \text{ and } 5.0 \, GeV^2. \text{ Therefore this kinematic comparison has to be done separately, hence the two figures: Figure 10.1 does this comparison for the low limit of the simulated } Q^2 \text{ and Figure 10.2 for the high limit.}

Figure 10.1  \text{ Comparison of the final state kinematics for } Q^2 = [2.0 \, GeV^2, 3.0 \, GeV^2): \text{ the blue, red and green lines represent experimental, simulated-reconstructed and simulated-thrown data, respectively.}
Figure 10.2  Comparison of the final state kinematics for $Q^2 = [3.0 \text{ GeV}^2, 5.0 \text{ GeV}^2])$: the blue, red and green lines represent experimental, simulated-reconstructed and simulated-thrown data, respectively.
It can be inferred from Figures 10.1 and 10.2 that not only is the extent of the kinematic coverage between experiment and simulation the same, but the correlations are also largely in agreement. It can also be seen that the resolution of the reconstructed momenta is also in good agreement, though a stronger illustration of this fact can be had from Figures 9.1 and 9.2 where the width of the $MM^2$ and $MM$ distributions, respectively, that takes into account the four-momenta of all final state particles and therefore their respective resolutions, are shown to be in good agreement between experiment and simulation. Therefore, because of these requirements that are fulfilled by the simulation, is a model independent extraction of the acceptance factor possible.

Additionally, in these plots can also be seen kinematical areas where the detector does not provide coverage and therefore an acceptance factor cannot be obtained. In order to obtain cross-section in such kinematical-hole bins a model dependent process is used which is discussed in detail in Chapter 12. However, in the finally measured cross-sections, the contributions from such kinematical holes is relatively small and is on the average at the 10% level, and because of this a conservative systematic uncertainty of 5% is added to the list of the other systematic uncertainties of this analysis (Table 13.1).

10.2 Simulation Process

The simulation process consist of four steps: event generation, detector simulation, post processing of detector simulation and reconstruction. During event generation, the $ep \rightarrow e'p'\pi^+\pi^-$ events are generated within the kinematic regime for this analysis’ using genev, which generates events based on the Monte Carlo technique using its current knowledge of the process encapsulated in the 5D cross-sections. Where the knowledge of genev is not current, as with this analysis’ kinematic range, its knowledge is extrapolated. The final state particles from the event generated by genev
are then put through the CLAS detector simulation program called \textit{GSIM}, which is
the simulation framework used for simulating the CLAS detector based on GEANT.
Often the detector response modeled by GSIM is not enough and further tuning de-
pending on the particular nature of the experimental conditions is required. This
tuning may involve, for example, providing parameters to tune the level of detector
smearing of the reconstructed kinematic quantities so that it matches the experimen-
tal resolution, or turning off elements of the detector that were not functional in the
experiment. This is accomplished by the next step of \textit{GSIM Post Processing} (GPP).

The response of the detector to the generated $ep \rightarrow e'p'\pi^+\pi^-$ events, modeled in
GSIM and further tuned by GPP, is put through the same reconstruction program
that the experimental events are process by, the details of which are described in
Section 3.5.

10.3 Acceptance

After reconstruction, the data is subject to the same sets of cuts and the kinematics
of the surviving events are binned similarly as the experimental data. The only
addition is that for the simulation, the generated data, also called the \textit{thrown} data,
is also binned similarly because together with the cut-surviving reconstructed data,
it is used to extract the acceptance.

The acceptance is obtained in all the 7-dimensional bins of the analysis. Therefore,
the calculation involves dividing the total number of reconstructed events that survive
all cuts by the total number of generated events in each 7-dimensional bin:

$$SA^7 = \frac{SR^7}{ST^7}$$  \hspace{1cm} (10.1)

where $ST^7$, $SR^7$ and $SA^7$ are total the number of \textit{thrown} (generated) events,
the total number of thrown events that make it through the detector simulation
and survive all the cuts, and the acceptance, respectively, in each 7-dimensional
bin. The letter ‘S’ in this nomenclature is used to denote simulation (This part of the nomenclature is important to note because in Appendix A it is used to in the mathematical expression of the formula that is used in filling out kinematical holes in the experiment using the simulation process.)

Figures 10.3 and 10.4 show, respectively, the averaged number of thrown and cut-surviving $ep \rightarrow e'p'\pi^+\pi^-$ events within a 5D cell within a $Q^2 - W$ bin, labeled $< ST^5 >$ and $< SR^5 >$, respectively. Figure 10.5 shows the average 5D acceptance within these $Q^2 - W$ bins, labeled $< SA^5 >$, where the acceptance within each 5D bin is calculated using Equation 10.1. (Note that the relative number of events in Figures 10.3 and 10.4 appear to have a discontinuity at $Q^2 = 3.00$ GeV$^2$. This is related to the fact that the simulation of the full $Q^2$ region is separated into its low and high limits of this range as already described earlier).

![Figure 10.3](image)

Figure 10.3 Average number of thrown $ep \rightarrow e'p'\pi^+\pi^-$ events within a 5D cell within a $Q^2 - W$ bin, labeled $< ST^5 >$. }

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Figure 10.4  Average number of cut-surviving thrown $ep \rightarrow e'p'\pi^+\pi^-$ events within a 5D cell within a $Q^2 - W$ bin, labeled $<SR^5>$.

Figure 10.5  Average 5D acceptance within $Q^2 - W$ bins, labeled $<SA^5>$.
This section describes the process of obtaining the radiative effects correction factor, \( R \), that is used in Equation 2.1.

The cross section without this factor \( R \) includes effects of radiation at the electronic vertex of the \( ep \rightarrow e'p'\pi^+\pi^- \) interaction. The cross section that is to be finally extracted has to be independent of such effects, and therefore the directly measured cross section, which includes radiative effects, has to be corrected. For this purpose this correction factor, \( R \), needs to be estimated.

Note that the radiative effects and their related correction at the hadronic vertex are not directly investigated in this analysis. The possible systematic effect from this on the measured cross-section is 5% [7].

In order to describe the process for obtaining this correction factor, it is insightful to consider the correction factor as \( 1/R \) multiplying the radiative-effects included cross-section, which results from the rest of the terms in Equation 2.1, instead of \( R \) that enters in the denominator. In that sense, \( 1/R \) is multiplicative correction factor that estimates the ratio of the cross-section with no radiative effects to that with radiative-effects:

\[
\frac{1}{R} = \frac{\Delta^7 \sigma_{\text{no-rad}}(\Delta Q^2, \Delta W, \Delta \tau^5)}{\Delta^7 \sigma_{\text{rad}}(\Delta Q^2, \Delta W, \Delta Q^2 \tau^5)}
\]

Since this correction factor depends only on the electronic vertex, it is only a function of \( Q^2 \) and \( W \), and therefore Equation 11.1 can be written more simply as:

\[
\frac{1}{R} = \frac{\Delta^2 \sigma_{\text{no-rad}}(\Delta Q^2, \Delta W)}{\Delta^2 \sigma_{\text{rad}}(\Delta Q^2, \Delta W)}
\]
Following Equation 11.2, the correction factor within a $Q^2 - W$ bin is the ratio of number of events from a distribution that follow the cross-section that includes radiative effects to that from a cross section that is without radiative effects. This can be done by using an event generator that is capable of generating both these kind of events.

Genev, the event generator that is also used to obtain acceptance factors, where it generated events with radiative-effects, is such an event generator. It incorporates radiative effects at the electronic vertex based on the well known knowledge contained in Reference [27]. For this purpose it was used to generate a fixed number of events with and without radiative effects and the correction factor was obtained as:

$$\frac{1}{R} = \frac{N_{\text{no-rad}}(\Delta Q^2, \Delta W)}{N_{\text{rad}}(\Delta Q^2, \Delta W)}$$

(11.3)

Figure 11.1 is a plot of this 2-dimensional correction factor as projections on the $W$ axis for various $Q^2$ bins.

Figure 11.1  Radiative effects correction factor plotted as a function of $W$ for various $Q^2$ bins.
Chapter 12

Estimating Experimental Yield in the Kinematical Holes

While the yield in most of the 7D bins is directly filled using the experimental data, a part of these 7D kinematic bins are not filled because the kinematics of these bins are correlated with physical holes in the detector. In order to extract the fully differential, i.e. 7D cross-section, experimental yields in all possible 7D bins need to be obtained. The yield in these kinematical holes, $\Delta^7N_{EH}$ from Equation 2.1, has be estimated using the simulation.

Kinematical holes are identified using simulation data and the simulated yield in them, $\Delta^7N_{SH}$, is noted. The experimental yield in these kinematical holes, $\Delta^7N_{EH}$, is then obtained from $\Delta^7N_{SH}$ using a scale factor (sf), which is defined in the full listing of this procedure in Appendix A. The idea behind this process is that while the thrown $e^p \rightarrow e'p'\pi^+\pi^-$ events cover the maximum allowed 7D binned Phase Space (PS), due to the physical holes in the detector which is modeled in the simulation’s GSIM software, the events in some of these 7D bins, after passing through GSIM, will not register a track in the detector. Therefore, some of the 7D bins, post GSIM, will be empty due to physical holes in the detector and are noted as the kinematical holes. The yield in these kinematical holes are obtained from the thrown data and are appropriately scaled to fill the experimental data. The idea behind the scale factor is to obtain, using 7D bins that are not kinematical holes, a ratio between the acceptance corrected experimental and simulated yield. This ratio can then be used
to estimate experimental yields in 7D kinematical-hole bins from yields already noted in them using simulation.

Figure 12.1 shows, within each $Q^2 - W$ bin, the fraction of 5D bins that are identified to be kinematical holes.

![5D Hole fraction($Q^2,W$)](image)

Figure 12.1 Fraction of 5D bins within a $Q^2 - W$ bin that are identified to be kinematical holes.

It can be directly seen that in the lowest W-bin, $1.400 GeV - 1.425 GeV$, the kinematic-hole fraction increases significantly. This is related to the fact that the 2 pion cross-section in this region, being closer to the 2 pion production threshold, is relatively lower, resulting in significantly lower simulation statistics, which can be directly seen in Figures 10.3 and 10.4. In order to obtain a final number on the number of kinematical holes, the simulation statistics need to be high enough such that with any further increase in statistics, the number of kinematical holes do not decrease any further. The simulation is complete when this steady state for the number of kinematical holes is reached. At the point of writing this thesis, the simulation data thus far processed is close enough to this steady state in the sense that for all W-bins except for $1.400 GeV - 1.425 GeV$, the additional increase in the cross-
Section through kinematical hole filling is becoming insignificant. In this manner, the simulation statistics does have a systematic effect on the measured cross sections and is estimated to be less than 1%.

Figure 12.2 illustrates the effect of kinematic-hole filling on the integrated cross-sections. The increased effect for the lowest W-bin, 1.400GeV − 1.425GeV, is directly visible here too. Additionally, it can be seen that this contribution increases with W and that is because as W increases, the scattered electron momentum decreases, causing it to bend even more strongly in the magnetic field into the forward hole.

Figure 12.2 Illustration of the kinematical-hole filling effect: Integrated cross-sections before and after kinematic-hole filling (top), and the relative contribution of this process to the measured cross-sections (below).
Chapter 13

Results

In this chapter the 51 observables that are extracted within the 2-dimensional $Q^2 - W$ bin will be presented and discussed with the aid of the results for a specific $Q^2 - W$ bin, which is illustrative of the general results for all $Q^2 - W$ bins. The results from all of the $Q^2 - W$ bins are presented in appendices B, C, D, E, F, and G for the single-differential cross-sections, $R^T_{X_{ij}}$, $R^L_{X_{ij}}$, $R^{cT}_{X_{ij}}$, $R^{cL}_{X_{ij}}$, $R^{sT}_{X_{ij}}$, and $R^{sL}_{X_{ij}}$, respectively.

Additionally, a note on the nature of the systematic uncertainties present in the analysis and their current estimate will be briefly discussed.

13.1 Systematic Uncertainties

It was observed during the process of estimating the systematic uncertainties that any changes in the observables due to systematic effects present in the analysis, which will be noted below, are contained within their statistical uncertainties, which are dominantly a function of the experimental yield. Therefore, for the practical purpose of putting a final error bound on the observables, the statistical uncertainty is all that is needed.

Nevertheless, an understanding and estimate of the systematic uncertainties present in the analysis is still important even if for this analysis it is not valuable at a practical level. This importance is at the level of the general importance of performing a systematic study of all the procedures that make up a complex experimental measurement: the process not only deepens the understanding and the degree of confidence
at all levels of complexity, but also provides insights and raises concerns that can be otherwise be missed. It is in this spirit that the systematic uncertainties are estimated for this analysis.

All of the tasks identified to be the core of this analysis can possibly contribute to the systematic uncertainty. They are listed again in Table 13.1 along with their systematic uncertainty. In addition to the core tasks, there is the additional effect of “Variable set dependent extraction of Observables”, item 12 in the table, that can only be considered only after the observables have been extracted using the core tasks, which are the item numbers 1-11 in the list. This additional effect is observed by noting that the integrated cross-section for the $\gamma^* p \rightarrow p' \pi^+ \pi^-$ for a $Q^2 - W$ bin obtained from each of the 3 variable sets, instead of being the same, is different. This difference can be attributed to the fact that 5-dimension PS in each of the variable set is not the same and this may lead to various effects like the fraction of kinematical holes in each PS being different, which in turn can lead to estimating different yields from the simulation in each of the 3 variable sets.

Table 13.1 List of estimated systematic uncertainties for the analysis.

<table>
<thead>
<tr>
<th>Number</th>
<th>Effect</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Electron Identification</td>
<td>1%</td>
</tr>
<tr>
<td>2</td>
<td>Electron fiducial boundary selection</td>
<td>1%</td>
</tr>
<tr>
<td>3</td>
<td>Hadron identification</td>
<td>2.5%</td>
</tr>
<tr>
<td>4</td>
<td>Hadron fiducial boundary selection</td>
<td>1%</td>
</tr>
<tr>
<td>5</td>
<td>Detector Inefficiency Identification</td>
<td>1%</td>
</tr>
<tr>
<td>6</td>
<td>Momentum and Energy Loss Corrections</td>
<td>1%</td>
</tr>
<tr>
<td>7</td>
<td>Event Selection</td>
<td>2.9%</td>
</tr>
<tr>
<td>8</td>
<td>Acceptance Calculation</td>
<td>1%</td>
</tr>
<tr>
<td>9</td>
<td>Radiative Effects correction</td>
<td>5%</td>
</tr>
<tr>
<td>10</td>
<td>Estimation of Experimental Yields in Kinematical Holes</td>
<td>5%</td>
</tr>
<tr>
<td>11</td>
<td>Luminosity measurement</td>
<td>5%</td>
</tr>
<tr>
<td>12</td>
<td>Variable set dependent extraction of Observables</td>
<td>5.3%</td>
</tr>
</tbody>
</table>

The estimate of systematic uncertainties fall into two categories: Those analyzed as a part of this thesis and those who estimates are taken from other studies (refer-
ences listed alongside). Assuming the uncertainties to be Gaussian and using error propagation based on that brings the total systematic uncertainty estimate to 11.1%.

13.2 Overview of the $Q^2 - W$ kinematic coverage of the E16 experiment

Figure 13.1 shows the $Q^2 - W$ coverage of the E16 data (Note that the data presented in this plot is not acceptance corrected). The $Q^2 - W$ bins in which the analysis is done is shown by the black grid. The W projection of the 2-dimensional plot is overlaid to directly bring out the resonance structure in the W spectrum.

In this chapter, to discuss the typical features of the results, results from just one $Q^2 - W$ bin will be presented. This bin is for $Q^2 = [2.40 \, GeV^2, 3.00 \, GeV^2]$ and $W = [1.725 \, GeV, 1.750 \, GeV]$ and is also highlighted in Figure 13.1.

![Figure 13.1 $Q^2 - W$ kinematic coverage of experiment E16.](image)

13.3 Single-differential cross-sections

Figure 13.2 shows the 9 single differential cross-sections for $Q^2 = [2.40 \, GeV^2, 3.00 \, GeV^2]$ and $W = [1.725 \, GeV, 1.750 \, GeV]$). The cyan and blue points show experimentally...
measured cross-sections without and with kinematical holes filled using the simulation, respectively. Also shown, for comparison, are yields from the JM model, whose integral is normalized to the integral of the kinematical-hole filled experimental cross-section. This is done to show the difference in shape of the distribution between the experiment and the model. Relative to the photon polarization dependent cross-sections, the agreement here is good.

Figure 13.2  Single Differential Cross-sections for $Q^2 = [2.40 \text{ GeV}^2, 3.00 \text{ GeV}^2)$ and $W = [1.725 \text{ GeV}, 1.750 \text{ GeV})$.

13.4 PHOTON POLARIZATION DEPENDENT CROSS-SECTIONS

In this section the 42 photon polarization dependent cross-sections will be presented. In these plots too, the cyan and blue points represent experimental cross-sections without and with kinematical holes filled from simulation, and the red points show the integral-normalized yields from the model.

Figure shows $R_{T\phi_i}^{X_{ij}} + R_{L\phi_i}^{X_{ij}}$ for $Q^2 = [2.40 \text{ GeV}^2, 3.00 \text{ GeV}^2)$ and $W = [1.725 \text{ GeV}, \ldots$
1.750 GeV). In comparison to the single-differential cross-sections shown in Figure 13.2, it can be be seen that the only difference is the factor of $2 \cdot \pi$.

Figure 13.3 $R_{2T}x_{ij} + R_{2L}x_{ij}$ for $Q^2 = [2.40 \text{ GeV}^2, 3.00 \text{ GeV}^2]$ and $W = [1.725 \text{ GeV}, 1.750 \text{ GeV})$.

Figures 13.4, 13.5, 13.6 and 13.7 show $R_{2LT}^{c,X_{ij}}, R_{2TT}^{c,X_{ij}}, R_{2LT}^{s,X_{ij}}$ and $R_{2TT}^{s,2X_{ij}}$, respectively for $Q^2 = [2.40 \text{ GeV}^2, 3.00 \text{ GeV}^2]$ and $W = [1.725 \text{ GeV}, 1.750 \text{ GeV})$. As compared to the single-differential and the directly correlated $R_{2T}x_{ij} + R_{2L}x_{ij}$, here there is significant difference in shape between model and experimental data.
Figure 13.4 $R_{LT}^{c.X_{ij}}$ for $Q^2 = [2.40 \text{ GeV}^2, 3.00 \text{ GeV}^2]$ and $W = [1.725 \text{ GeV}, 1.750 \text{ GeV})$.

Figure 13.5 $R_{TT}^{c.X_{ij}}$ for $Q^2 = [2.40 \text{ GeV}^2, 3.00 \text{ GeV}^2]$ and $W = [1.725 \text{ GeV}, 1.750 \text{ GeV})$. 

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Figure 13.6 $R_{LT}^{s,X_{ij}}$ for $Q^2 = [2.40 \text{ GeV}^2, 3.00 \text{ GeV}^2)$ and $W = [1.725 \text{ GeV}, 1.750 \text{ GeV})$.

Figure 13.7 $R_{TT}^{s,X_{ij}}$ for $Q^2 = [2.40 \text{ GeV}^2, 3.00 \text{ GeV}^2)$ and $W = [1.725 \text{ GeV}, 1.750 \text{ GeV})$. 
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Appendix A

Process of filling holes in experimental data

Before listing the full process the technical terms used therein are described below.

- $h^5 = 5D$ histogram containing binned data over 5D kinematic phase space (PS).
- In simulation:
  1. $h^5-ST = \text{Thrown yield}$
  2. $h^5-SR = \text{Thrown yield reconstructed in simulated detector.}$
  3. $h^5-SA = \text{Acceptance } (h^5-SA = h^5-SR / h^5-ST)$
  4. $h^5-SC = \text{Acceptance corrected yield. } (h^5-SC = h^5-SR / h^5-SA)$
  5. $h^5-SH = \text{Hole yield } (h^5-SH = h^5-ST - h^5-SC)$
  6. $h^5-SF = \text{Yield in full (PS) } (h^5-SF = h^5-SC + h^5-SH)$
- In experiment:
  1. $h^5-ER = \text{Natural yield reconstructed in actual detector.}$
  2. $h^5-EC = \text{Acceptance corrected yield } (h^5-EC = h^5-ER / h^5-SA)$
  3. $h^5-EH = \text{Hole yield. } (h^5-EH = 'sf'x h^5-SH)$
  4. $h^5-EF = \text{Yield in full (PS) } (h^5-EF = h^5-EC + h^5-EH)$

Listing of full process:

1. Obtain $h^5-SH$: $h^5-SH = h^5-ST - h^5-SC$
2. Obtain $h^5-EH$: 
i Set $h_{5-EH}$ equal to $h_{5-SH}$: $h_{5-EH} = h_{5-SH}$

ii Obtain $sf$ as the ratio of total yield in $h_{5-EC}$ and total yield in $h_{5-SC}$.

Note that for both, the total yield is integrated over $h_{5-SC}$’s PS bins that are filled (i.e. their bin content $> 0$).

$$sf = \frac{\sum_{i=1}^{N} h_{5-EC_i}}{\sum_{i=1}^{N} h_{5-SC_i}}$$

where $i=1,...,N$ are the filled PS bins filled in $h_{5-SC}$.

In other words, given that we have to use $h_{5-SH}$ to fill $h_{5-EH}$, this scale factor uses the ratio of integrated yields of $h_{5-SC}$ and $h_{5-EC}$ to give us an estimate of the proportionality factor between them.

3. Scale $h_{5-EH}$: $h_{5-EH} = sf \times h_{5-EH}$

4. Obtain experimental yield in full PS: $h_{5-EF} = h_{5-EC} + h_{5-EH}$

5. Normalize $h_{5-EF}$ and $h_{5-EC}$ using Luminosity, virtual photon flux and bin-width factors, to obtain Hole-filled and Hole-not-filled cross-sections.
Appendix B

Results: Single Differential Cross-sections

Figure B.1 Single Differential Cross-sections for $Q^2 \ [GeV^2], W \ [GeV]$ bins: $[2.00,2.40),[1.400,1.425)$ (a), $[2.00,2.40),[1.425,1.450)$ (b), $[2.00,2.40),[1.450,1.475)$ (c) and $[2.00,2.40),[1.475,1.500)$ (d)
Figure B.2  Single Differential Cross-sections for $Q^2 \ [GeV^2], W \ [GeV] \ bins: \ [2.00,2.40),[1.500,1.525) \ (a), \ [2.00,2.40),[1.525,1.550) \ (b), \ [2.00,2.40),[1.550,1.575) \ (c) \ and \ [2.00,2.40),[1.575,1.600) \ (d)$
Figure B.3  Single Differential Cross-sections for $Q^2 \ [GeV^2]$, $W \ [GeV]$ bins: $[2.00,2.40),[1.600,1.625)$ (a), $[2.00,2.40),[1.625,1.650)$ (b), $[2.00,2.40),[1.650,1.675)$ (c) and $[2.00,2.40),[1.675,1.700)$ (d)
Figure B.4  Single Differential Cross-sections for $Q^2\ [GeV^2], W\ [GeV]$ bins:
[2.00,2.40),[1.700,1.725) (a), [2.00,2.40),[1.725,1.750) (b), [2.00,2.40),[1.750,1.775) (c) and [2.00,2.40),[1.775,1.800) (d)
Figure B.5  Single Differential Cross-sections for $Q^2 \ [GeV^2], W \ [GeV]$ bins: 
[2.00,2.40),[1.80,1.825) (a), [2.00,2.40),[1.825,1.850) (b), [2.00,2.40),[1.850,1.875) (c) and [2.00,2.40),[1.875,1.900) (d)
Figure B.6  Single Differential Cross-sections for $Q^2 \ [GeV^2], W \ [GeV]$ bins: 
[2.00,2.40),[1.900,1.925) (a), [2.00,2.40),[1.925,1.950) (b), [2.00,2.40),[1.950,1.975) (c) and [2.00,2.40),[1.975,2.000) (d)
Figure B.7 Single Differential Cross-sections for $Q^2 [GeV^2], W [GeV]$ bins: $[2.00, 2.40), [2.00, 2.025)$ (a), $[2.00, 2.40), [2.025, 2.050)$ (b), $[2.00, 2.40), [2.050, 2.075)$ (c) and $[2.00, 2.40), [2.075, 2.100)$ (d)
Figure B.8  Single Differential Cross-sections for $Q^2 [GeV^2], W [GeV]$ bins: $[2.00,2.40),[2.40,2.100,2.125] (a), [2.40,3.00),[1.400,1.425] (b), [2.40,3.00),[1.425,1.450] (c)$ and $[2.40,3.00),[1.450,1.475] (d)$
Figure B.9  Single Differential Cross-sections for $Q^2 [GeV^2], W [GeV]$ bins: 
[2.40,3.00),[1.475,1.500) (a), [2.40,3.00),[1.500,1.525) (b), [2.40,3.00),[1.525,1.550) (c) and [2.40,3.00),[1.550,1.575) (d)
Figure B.10  Single Differential Cross-sections for $Q^2 [GeV^2], W [GeV]$ bins: $[2.40,3.00),[1.575,1.600)$ (a), $[2.40,3.00),[1.600,1.625)$ (b), $[2.40,3.00),[1.625,1.650)$ (c) and $[2.40,3.00),[1.650,1.675)$ (d)
Figure B.11  Single Differential Cross-sections for $Q^2 [GeV^2], W [GeV]$ bins: [2.40,3.00),[1.675,1.700) (a), [2.40,3.00),[1.700,1.725) (b), [2.40,3.00),[1.725,1.750) (c) and [2.40,3.00),[1.750,1.775) (d)
Figure B.12  Single Differential Cross-sections for $Q^2 \ [GeV^2], W \ [GeV]$ bins: 
[2.40,3.00),[1.775,1.800) (a), [2.40,3.00),[1.800,1.825) (b), [2.40,3.00),[1.825,1.850) (c) and [2.40,3.00),[1.850,1.875) (d)
Figure B.13  Single Differential Cross-sections for $Q^2 [GeV^2], W [GeV]$ bins: [2.40,3.00),[1.875,1.900] (a), [2.40,3.00),[1.900,1.925] (b), [2.40,3.00),[1.925,1.950] (c) and [2.40,3.00),[1.950,1.975] (d)
Figure B.14 Single Differential Cross-sections for $Q^2 \text{[GeV}^2]\text{, } W \text{[GeV]}$ bins: [2.40,3.00),[1.975,2.000) (a), [2.40,3.00),[2.000,2.025) (b), [2.40,3.00),[2.025,2.050) (c) and [2.40,3.00),[2.050,2.075) (d)
Figure B.15  Single Differential Cross-sections for $Q^2 [GeV^2], W [GeV]$ bins: 
[2.40,3.00),[2.075,2.100) (a), [2.40,3.00),[2.100,2.125) (b), [3.00,3.50),[1.400,1.425) (c) and [3.00,3.50),[1.425,1.450) (d)
Figure B.16  Single Differential Cross-sections for $Q^2 [GeV^2], W [GeV]$ bins: $[3.00,3.50),[1.450,1.475)$ (a), $[3.00,3.50),[1.475,1.500)$ (b), $[3.00,3.50),[1.500,1.525)$ (c) and $[3.00,3.50),[1.525,1.550)$ (d)
Figure B.17  Single Differential Cross-sections for $Q^2 [GeV^2], W [GeV]$ bins: [3.00,3.50],[1.550,1.575) (a), [3.00,3.50],[1.575,1.600) (b), [3.00,3.50],[1.600,1.625) (c) and [3.00,3.50],[1.625,1.650) (d)
Figure B.18  Single Differential Cross-sections for $Q^2 [GeV^2], W [GeV]$ bins: $[3.00,3.50),[1.650,1.675)$ (a), $[3.00,3.50),[1.675,1.700)$ (b), $[3.00,3.50),[1.700,1.725)$ (c) and $[3.00,3.50),[1.725,1.750)$ (d)
Figure B.19  Single Differential Cross-sections for $Q^2 \ [GeV^2], W \ [GeV]$ bins: $[3.00,3.50),[1.750,1.775)$ (a), $[3.00,3.50),[1.775,1.800)$ (b), $[3.00,3.50),[1.800,1.825)$ (c) and $[3.00,3.50),[1.825,1.850)$ (d)
Figure B.20  Single Differential Cross-sections for $Q^2 [GeV^2], W [GeV]$ bins: $[3.00, 3.50), [1.850, 1.875)$ (a), $[3.00, 3.50), [1.875, 1.900)$ (b), $[3.00, 3.50), [1.900, 1.925)$ (c) and $[3.00, 3.50), [1.925, 1.950)$ (d)
Figure B.21  Single Differential Cross-sections for $Q^2$ $[GeV^2]$, $W$ $[GeV]$ bins: $[3.00,3.50),[1.950,1.975)$ (a), $[3.00,3.50),[1.975,2.000)$ (b), $[3.00,3.50),[2.000,2.025)$ (c) and $[3.00,3.50),[2.025,2.050)$ (d)
Figure B.22  Single Differential Cross-sections for $Q^2 [GeV^2], W [GeV]$ bins: $[3.00,3.50),[2.050,2.075)$ (a), $[3.00,3.50),[2.075,2.100)$ (b), $[3.00,3.50),[2.100,2.125)$ (c) and $[3.50,4.20),[1.400,1.425)$ (d)
Figure B.23  Single Differential Cross-sections for $Q^2 [GeV^2]$, $W [GeV]$ bins: 
[3.50,4.20),[1.425,1.450] (a), [3.50,4.20),[1.450,1.475] (b), [3.50,4.20),[1.475,1.500] (c) and [3.50,4.20),[1.500,1.525] (d)
Figure B.24  Single Differential Cross-sections for $Q^2 [GeV^2], W [GeV]$ bins: $[3.50,4.20),[1.525,1.550)$ (a), $[3.50,4.20),[1.550,1.575)$ (b), $[3.50,4.20),[1.575,1.600)$ (c) and $[3.50,4.20),[1.600,1.625)$ (d)
Figure B.25  Single Differential Cross-sections for $Q^2 [GeV^2], W [GeV]$ bins:
[3.50,4.20),[1.625,1.650) (a), [3.50,4.20),[1.650,1.675) (b), [3.50,4.20),[1.675,1.700) (c) and [3.50,4.20),[1.700,1.725) (d)
Figure B.26  Single Differential Cross-sections for $Q^2 [GeV^2], W [GeV]$ bins: $[3.50,4.20),[1.725,1.750)$ (a), $[3.50,4.20),[1.750,1.775)$ (b), $[3.50,4.20),[1.775,1.800)$ (c) and $[3.50,4.20),[1.800,1.825)$ (d)
Figure B.27  Single Differential Cross-sections for $Q^2$ [$GeV^2$], $W$ [$GeV$] bins:
[3.50,4.20),[1.825,1.850) (a), [3.50,4.20),[1.850,1.875) (b), [3.50,4.20),[1.875,1.900) (c)
and [3.50,4.20),[1.900,1.925) (d)
Figure B.28  Single Differential Cross-sections for $Q^2 \text{[GeV}^2\text{]}, W \text{[GeV]}$ bins:
[3.50,4.20),[1.925,1.950) (a), [3.50,4.20),[1.950,1.975) (b), [3.50,4.20),[1.975,2.000) (c) and [3.50,4.20),[2.000,2.025) (d)
Figure B.29  Single Differential Cross-sections for $Q^2 [GeV^2], W [GeV]$ bins: $[3.50,4.20),[2.025,2.050)$ (a), $[3.50,4.20),[2.050,2.075)$ (b), $[3.50,4.20),[2.075,2.100)$ (c) and $[3.50,4.20),[2.100,2.125)$ (d)
Figure B.30  Single Differential Cross-sections for $Q^2 \ [GeV^2], W \ [GeV]$ bins: 
[4.20,5.00), [1.400,1.425) (a), [4.20,5.00), [1.425,1.450) (b), [4.20,5.00), [1.450,1.475) (c) and [4.20,5.00), [1.475,1.500) (d)
Figure B.31  Single Differential Cross-sections for $Q^2 [GeV^2], W [GeV]$ bins: [4.20,5.00),[1.500,1.525) (a), [4.20,5.00),[1.525,1.550) (b), [4.20,5.00),[1.550,1.575) (c) and [4.20,5.00),[1.575,1.600) (d)
Figure B.32  Single Differential Cross-sections for $Q^2 [GeV^2], W [GeV]$ bins:
[4.20,5.00),[1.600,1.625) (a), [4.20,5.00),[1.625,1.650) (b), [4.20,5.00),[1.650,1.675) (c)
and [4.20,5.00),[1.675,1.700) (d)
Figure B.33  Single Differential Cross-sections for $Q^2 [GeV^2], W [GeV]$ bins: [4.20,5.00),[1.700,1.725) (a), [4.20,5.00),[1.725,1.750) (b), [4.20,5.00),[1.750,1.775) (c) and [4.20,5.00),[1.775,1.800) (d)
Figure B.34  Single Differential Cross-sections for $Q^2 [GeV^2], W [GeV]$ bins:
[4.20,5.00),[1.800,1.825) (a), [4.20,5.00),[1.825,1.850) (b), [4.20,5.00),[1.850,1.875) (c) and [4.20,5.00),[1.875,1.900) (d)
Figure B.35  Single Differential Cross-sections for $Q^2 \ [GeV^2], W \ [GeV]$ bins: 
[4.20,5.00),[1.900,1.925) (a), [4.20,5.00),[1.925,1.950) (b), [4.20,5.00),[1.950,1.975) (c) and [4.20,5.00),[1.975,2.000) (d)
Figure B.36 Single Differential Cross-sections for $Q^2 \ [GeV^2], W \ [GeV]$ bins: $[4.20,5.00), [2.000,2.025)$ (a), $[4.20,5.00), [2.025,2.050)$ (b), $[4.20,5.00), [2.050,2.075)$ (c) and $[4.20,5.00), [2.075,2.100)$ (d)
Figure B.37  Single Differential Cross-sections for $Q^2 \, [GeV^2], W \, [GeV]$ bin: [4.20,5.00),[2.100,2.125) (a)
Appendix C

Results: $R_{T,\phi_i}^X_{ij} + R_{L,\phi_i}^X_{ij}$

Figure C.1 $R_{T,\phi_i}^X_{ij} + R_{L,\phi_i}^X_{ij}$ for $Q^2 \geq \text{[GeV]}^2$, $W \geq \text{[GeV]}$ bins: (a) [2.00,2.40),[1.400,1.425) (b) [2.00,2.40),[1.425,1.450) (c) [2.00,2.40),[1.450,1.475) (d) [2.00,2.40),[1.475,1.500)
Figure C.2  $R^{X_{ij}}_T + R^{X_{ij}}_{L \phi_i}$ for $Q^2 \ [GeV^2], W \ [GeV]$ bins: [2.00,2.40),[1.500,1.525) (a), [2.00,2.40),[1.525,1.550) (b), [2.00,2.40),[1.550,1.575) (c) and [2.00,2.40),[1.575,1.600) (d)
Figure C.3 \( R_{T_{X_{ij}}} + R_{L_{X_{ij}}} \) for \( Q^2 [GeV^2], W [GeV] \) bins: [2.00,2.40),[1.600,1.625) (a), [2.00,2.40),[1.625,1.650) (b), [2.00,2.40),[1.650,1.675) (c) and [2.00,2.40),[1.675,1.700) (d)
Figure C.4  $R_2^{X_{ij}} + R_2^{X_{ij}}$ for $Q^2$ [GeV$^2$], $W$ [GeV] bins: [2.00,2.40), [1.700,1.725) (a), [2.00,2.40), [1.725,1.750) (b), [2.00,2.40), [1.750,1.775) (c) and [2.00,2.40), [1.775,1.800) (d)
Figure C.5  $R_2^{i,j}X_{ij} + R_2^{L,j}X_{ij}$ for $Q^2 \ [GeV^2], W \ [GeV]$ bins: [2.00,2.40),[1.800,1.825) (a), [2.00,2.40),[1.825,1.850) (b), [2.00,2.40),[1.850,1.875) (c) and [2.00,2.40),[1.875,1.900) (d)
Figure C.6  $R_2^{X_{ij}} + R_2^{L_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [2.00,2.40),[1.900,1.925) (a), [2.00,2.40),[1.925,1.950) (b), [2.00,2.40),[1.950,1.975) (c) and [2.00,2.40),[1.975,2.000) (d)
Figure C.7 \( R_{2T^{X_{ij}}} + R_{2L^{X_{ij}}} \) for \( Q^2 \ [GeV^2], W \ [GeV] \) bins: [2.00,2.40), [2.000,2.025) (a), [2.00,2.40), [2.025,2.050) (b), [2.00,2.40), [2.050,2.075) (c) and [2.00,2.40), [2.075,2.100) (d)
Figure C.8  \( R_{2T}^{X_{ij}} + R_{2L}^{X_{ij}} \) for \( Q^2 \ [GeV^2], W \ [GeV] \) bins: [2.00,2.40),[2.100,2.125) (a), [2.40,3.00),[1.400,1.425) (b), [2.40,3.00),[1.425,1.450) (c) and [2.40,3.00),[1.450,1.475) (d)
Figure C.9  $R_{2T}^{X_{ij}} + R_{2L}^{X_{ij}}$ for $Q^2 \ [GeV^2], W \ [GeV]$ bins: [2.40,3.00),[1.475,1.500) (a), [2.40,3.00),[1.500,1.525) (b), [2.40,3.00),[1.525,1.550) (c) and [2.40,3.00),[1.550,1.575) (d)
Figure C.10 \( R_{2T}^{X_{ij}} + R_{2L}^{X_{ij}} \) for \( Q^2 \) [GeV\(^2\)], \( W \) [GeV] bins: [2.40,3.00),[1.575,1.600) (a), [2.40,3.00),[1.600,1.625) (b), [2.40,3.00),[1.625,1.650) (c) and [2.40,3.00),[1.650,1.675) (d)
Figure C.11 \( R_{T_{\phi}X_{ij}} + R_{L_{\phi}X_{ij}} \) for \( Q^2 \ [GeV^2], W \ [GeV] \) bins: 

- \([2.40,3.00), [1.675,1.700) \) (a), 
- \([2.40,3.00), [1.700,1.725) \) (b), 
- \([2.40,3.00), [1.725,1.750) \) (c), and 
- \([2.40,3.00), [1.750,1.775) \) (d)
Figure C.12  $R_{2T}^{X_{ij}} + R_{2L}^{X_{ij}}$ for $Q^2 [GeV^2]$, $W [GeV]$ bins: $[2.40,3.00),[1.775,1.800)$ (a), $[2.40,3.00),[1.800,1.825)$ (b), $[2.40,3.00),[1.825,1.850)$ (c) and $[2.40,3.00),[1.850,1.875)$ (d)
Figure C.13 $R_{2T \phi_{ij}^X} + R_{2T \phi_{ij}^X}$ for $Q^2 [GeV^2], W [GeV]$ bins: [2.40,3.00),[1.875,1.900) (a), [2.40,3.00),[1.900,1.925) (b), [2.40,3.00),[1.925,1.950) (c) and [2.40,3.00),[1.950,1.975) (d)
Figure C.14 $R_{T'}^{X_{ij}} + R_{L'}^{X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins:
[2.40,3.00),[1.975,2.000) (a), [2.40,3.00),[2.000,2.025) (b), [2.40,3.00),[2.025,2.050) (c) and [2.40,3.00),[2.050,2.075) (d)
Figure C.15  $R_{2T}X_{ij} + R_{2T}X_{ij}$ for $Q^2 [GeV^2], W [GeV]$ bins:

- [2.40,3.00),[2.075,2.100) (a),
- [2.40,3.00),[2.100,2.125) (b),
- [3.00,3.50),[1.400,1.425) (c) and
- [3.00,3.50),[1.425,1.450) (d)
Figure C.16 \( R_{2T}^{X_{ij}} + R_{2L}^{X_{ij}} \) for \( Q^2 \) [\( GeV^2 \)], \( W \) [\( GeV \)] bins: [3.00,3.50),[1.450,1.475) (a), [3.00,3.50),[1.475,1.500) (b), [3.00,3.50),[1.500,1.525) (c) and [3.00,3.50),[1.525,1.550) (d)
Figure C.17 $R_{2T^X_{\phi}} + R_{2T^Y_{\phi}}$ for $Q^2 \,[GeV^2], W \, [GeV]$ bins: $[3.00,3.50),[1.550,1.575)$ (a), $[3.00,3.50),[1.575,1.600)$ (b), $[3.00,3.50),[1.600,1.625)$ (c) and $[3.00,3.50),[1.625,1.650)$ (d)
Figure C.18  \( R_{2T}^{X_{ij}} + R_{2T}^{X_{ij}} \) for \( Q^2 \) [GeV^2], \( W \) [GeV] bins: [3.00,3.50),[1.650,1.675) (a), [3.00,3.50),[1.675,1.700) (b), [3.00,3.50),[1.700,1.725) (c) and [3.00,3.50),[1.725,1.750) (d)
Figure C.19  \( R^2_{T, \phi} + R^2_{T, \phi^{'}} \) for \( Q^2 [\text{GeV}^2], W [\text{GeV}] \) bins: 

(a) \([3.00,3.50),[1.750,1.775)\), (b) \([3.00,3.50),[1.775,1.800)\), (c) \([3.00,3.50),[1.800,1.825)\) and (d) \([3.00,3.50),[1.825,1.850)\)
Figure C.20  \( R_{2T \phi}^{X_{ij}} + R_{2L \phi}^{X_{ij}} \) for \( Q^2 [GeV^2], W [GeV] \) bins:  
\[3.00,3.50),[1.850,1.875)\] (a), \([3.00,3.50),[1.875,1.900)\] (b), \([3.00,3.50),[1.900,1.925)\] (c) and \([3.00,3.50),[1.925,1.950)\) (d)
Figure C.21  \( R_{2T}^{X_{ij}} + R_{2L}^{X_{ij}} \) for \( Q^2 \) [GeV\(^2\)], \( W \) [GeV] bins: \([3.00,3.50),[1.950,1.975)\) (a), \([3.00,3.50),[1.975,2.000)\) (b), \([3.00,3.50),[2.000,2.025)\) (c) and \([3.00,3.50),[2.025,2.050)\) (d)
Figure C.22 $R_{T\phi}^{X_{ij}} + R_{T\phi}^{X_{ij}}$ for $Q^2 \ [GeV^2], W \ [GeV]$ bins: 

- [3.00,3.50), [2.050,2.075) (a), 
- [3.00,3.50), [2.075,2.100) (b), 
- [3.00,3.50), [2.100,2.125) (c) 
- and [3.50,4.20), [1.400,1.425) (d)
Figure C.23 $R^{X_{ij}}_T + R^{X_{ij}}_L$ for $Q^2 [GeV^2], W [GeV]$ bins: $[3.50,4.20),[1.425,1.450)$ (a), $[3.50,4.20),[1.450,1.475)$ (b), $[3.50,4.20),[1.475,1.500)$ (c) and $[3.50,4.20),[1.500,1.525)$ (d)
Figure C.24 $R_{2T}^{X_{ij}} + R_{2L}^{X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [3.50,4.20),[1.525,1.550) (a), [3.50,4.20),[1.550,1.575) (b), [3.50,4.20),[1.575,1.600) (c) and [3.50,4.20),[1.600,1.625) (d)
Figure C.25  \( R_{2T_{\phi_i}} \times X_{ij} + R_{2T_{\phi_i}} \times X_{ij} \) for \( Q^2 [GeV^2], W [GeV] \) bins: [3.50,4.20),[1.625,1.650) (a), [3.50,4.20),[1.650,1.675) (b), [3.50,4.20),[1.675,1.700) (c) and [3.50,4.20),[1.700,1.725) (d)
Figure C.26 $R_{2T}X_{ij} + R_{1T}X_{ij}$ for $Q^2$ [GeV$^2$], $W$ [GeV] bins: [3.50,4.20),[1.725,1.750) (a), [3.50,4.20),[1.750,1.775) (b), [3.50,4.20),[1.800,1.825) (c) and [3.50,4.20),[1.800,1.825) (d)
Figure C.27 \( R_{2T_{\phi}}^{X_{ij}} + R_{2T_{\phi}}^{X_{ij}} \) for \( Q^2 [GeV^2], W [GeV] \) bins: 
[3.50,4.20),[1.825,1.850) (a), [3.50,4.20),[1.850,1.875) (b), [3.50,4.20),[1.875,1.900) (c) and [3.50,4.20),[1.900,1.925) (d)
Figure C.28  $R_{2T}^{X_{ij}} + R_{2L}^{X_{ij}}$ for $Q^2 \ [GeV^2], W \ [GeV]$ bins:
[3.50,4.20),[1.925,1.950) (a), [3.50,4.20),[1.950,1.975) (b), [3.50,4.20),[1.975,2.000) (c)
and [3.50,4.20),[2.000,2.025) (d)
Figure C.29 $R_{T,R}^{X_{ij}}$ for $Q^2\ [GeV^2], W\ [GeV]$ bins:

- $[3.50,4.20], [2.025,2.050]$ (a)
- $[3.50,4.20], [2.050,2.075]$ (b)
- $[3.50,4.20], [2.075,2.100]$ (c)
- $[3.50,4.20], [2.100,2.125]$ (d)
Figure C.30  $R_{2T}^{X_{ij}} + R_{2T}^{X_{ij}}$ for $Q^2 \text{[GeV}^2]\), W \text{[GeV]}$ bins:

- (a) $[4.20,5.00),[1.400,1.425)$
- (b) $[4.20,5.00),[1.425,1.450)$
- (c) $[4.20,5.00),[1.450,1.475)$
- (d) $[4.20,5.00),[1.475,1.500)$

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Figure C.31 $R_{2T_{\phi}}^{X_{ij}} + R_{2L_{\phi}}^{X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins:

- [4.20,5.00),[1.500,1.525) (a)
- [4.20,5.00),[1.525,1.550) (b)
- [4.20,5.00),[1.550,1.575) (c)
- and [4.20,5.00),[1.575,1.600) (d)
Figure C.32  \( R_{2T}^{X_{ij}} + R_{2L}^{X_{ij}} \) for \( Q^2 \) [GeV\(^2\)], \( W \) [GeV] bins: 
\([4.20, 5.00), [1.600, 1.625)\) (a), \([4.20, 5.00), [1.625, 1.650)\) (b), \([4.20, 5.00), [1.650, 1.675)\) (c) and \([4.20, 5.00), [1.675, 1.700)\) (d)

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Figure C.33 $R_{2T,ij}^{X_{ij}} + R_{2L,ij}^{X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [4.20,5.00),[1.700,1.725) (a), [4.20,5.00),[1.725,1.750) (b), [4.20,5.00),[1.750,1.775) (c) and [4.20,5.00),[1.775,1.800) (d)
Figure C.34 $R_{2T_1}^{X_{ij}} + R_{2L_1}^{X_{ij}}$ for $Q^2$ [GeV$^2$], $W$ [GeV] bins: [4.20,5.00),[1.800,1.825) (a), [4.20,5.00),[1.825,1.850) (b), [4.20,5.00),[1.850,1.875) (c) and [4.20,5.00),[1.875,1.900) (d)
Figure C.35 $R_{T_{X_{ij}}}^2 + R_{L_{X_{ij}}}^2$ for $Q^2 [GeV^2], W [GeV]$ bins: 
$[4.20,5.00),[1.900,1.925)$ (a), $[4.20,5.00),[1.925,1.950)$ (b), $[4.20,5.00),[1.950,1.975)$ (c) and $[4.20,5.00),[1.975,2.000)$ (d)
Figure C.36 $R_2^{X_{ij}} + R_2^{X_{ij}}$ for $Q^2 \ [GeV^2], W \ [GeV]$ bins:

- $[4.20,5.00),[2.000,2.025)$ (a), $[4.20,5.00),[2.025,2.050)$ (b), $[4.20,5.00),[2.050,2.075)$ (c) and $[4.20,5.00),[2.075,2.100)$ (d)
Figure C.37 $R_{T\phi_i}^{X_{ij}} + R_{L\phi_i}^{X_{ij}}$ for $Q^2 \ [GeV^2], W \ [GeV]$ bin: [4.20,5.00),[2.100,2.125) (a)
Appendix D

Results: $R_{2LT}^{c,X_{ij}}$

Figure D.1 $R_{2LT}^{c,X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [2.00,2.40),[1.400,1.425) (a), [2.00,2.40),[1.425,1.450) (b), [2.00,2.40),[1.450,1.475) (c) and [2.00,2.40),[1.475,1.500) (d)
Figure D.2 $R_{LT}^{c,X_{ij}}$ for $Q^2$ [GeV$^2$], $W$ [GeV] bins: [2.00,2.40),[1.500,1.525) (a), [2.00,2.40),[1.525,1.550) (b), [2.00,2.40),[1.550,1.575) (c) and [2.00,2.40),[1.575,1.600) (d)
Figure D.3  $R_{LT}^{c,X_{ij}}$ for $Q^2 \ [GeV^2]$, $W \ [GeV]$ bins: $[2.00,2.40),[1.600,1.625)$ (a), $[2.00,2.40),[1.625,1.650)$ (b), $[2.00,2.40),[1.650,1.675)$ (c) and $[2.00,2.40),[1.675,1.700)$ (d)
Figure D.4  $R_{LT}^{cX_{ij}}$ for $Q^2$ [$GeV^2$], $W$ [$GeV$] bins: [2.00,2.40),[1.700,1.725) (a), [2.00,2.40),[1.725,1.750) (b), [2.00,2.40),[1.750,1.775) (c) and [2.00,2.40),[1.775,1.800) (d)
Figure D.5  \( R^{LT}_{c,X_{ij}} \) for \( Q^2 \) [GeV]\(^2\), \( W \) [GeV] bins: [2.00,2.40),[1.800,1.825) (a), [2.00,2.40),[1.825,1.850) (b), [2.00,2.40),[1.850,1.875) (c) and [2.00,2.40),[1.875,1.900) (d)
Figure D.6 $R_{LT}^c, X_{ij}$ for $Q^2 [\text{GeV}^2], W [\text{GeV}]$ bins: [2.00,2.40),[1.900,1.925) (a), [2.00,2.40),[1.925,1.950) (b), [2.00,2.40),[1.950,1.975) (c) and [2.00,2.40),[1.975,2.000) (d)
Figure D.7  $R_{2LT}^{c,X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [2.00,2.40),[2.000,2.025) (a), [2.00,2.40),[2.025,2.050) (b), [2.00,2.40),[2.050,2.075) (c) and [2.00,2.40),[2.075,2.100) (d)
Figure D.8  $R_{LT}^{c,X_{ij}}$ for $Q^2$ [GeV$^2$], $W$ [GeV] bins: [2.00,2.40),[2.100,2.125) (a), [2.40,3.00),[1.400,1.425) (b), [2.40,3.00),[1.425,1.450) (c) and [2.40,3.00),[1.450,1.475) (d)
Figure D.9 \( R^{c_{i}}_{L,T,j} \) for \( Q^2 \) [\( GeV^2 \)], \( W \) [\( GeV \)] bins: [2.40,3.00),[1.475,1.500) (a), [2.40,3.00),[1.500,1.525) (b), [2.40,3.00),[1.525,1.550) (c) and [2.40,3.00),[1.550,1.575) (d)
Figure D.10  $R_{LT}^{c_{X^{ij}}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [2.40,3.00),[1.575,1.600) (a), [2.40,3.00),[1.600,1.625) (b), [2.40,3.00),[1.625,1.650) (c) and [2.40,3.00),[1.650,1.675) (d)
Figure D.11 \( R_{LT_cX_{ij}} \) for \( Q^2 \ [GeV^2] \), \( W \ [GeV] \) bins: [2.40,3.00),[1.675,1.700) (a), [2.40,3.00),[1.700,1.725) (b), [2.40,3.00),[1.725,1.750) (c) and [2.40,3.00),[1.750,1.775) (d)
Figure D.12  $R_{LT}^{c_X}$ for $Q^2 [GeV^2], W [GeV]$ bins: [2.40,3.00),[1.775,1.800) (a), [2.40,3.00),[1.800,1.825) (b), [2.40,3.00),[1.825,1.850) (c) and [2.40,3.00),[1.850,1.875) (d)
Figure D.13 \( R_{LT}^{c,X_{ij}} \) for \( Q^2 [GeV^2] \), \( W [GeV] \) bins: [2.40,3.00), [1.875,1.900) (a), [2.40,3.00), [1.900,1.925) (b), [2.40,3.00), [1.925,1.950) (c) and [2.40,3.00), [1.950,1.975) (d)
Figure D.14  $R_{2LT}^{c,X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [2.40,3.00), [1.975,2.000) (a), [2.40,3.00), [2.000,2.025) (b), [2.40,3.00), [2.025,2.050) (c) and [2.40,3.00), [2.050,2.075) (d)
Figure D.15 $R_{2LT}^{c,X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [2.40,3.00),[2.075,2.100) (a), [2.40,3.00),[2.100,2.125) (b), [3.00,3.50),[1.400,1.425) (c) and [3.00,3.50),[1.425,1.450) (d)
Figure D.16  $R_{LT_i^c, X^{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [3.00,3.50),[1.450,1.475) (a), [3.00,3.50),[1.475,1.500) (b), [3.00,3.50),[1.500,1.525) (c) and [3.00,3.50),[1.525,1.550) (d)
Figure D.17  \( R_{LT_i}^{c,X_{ij}} \) for \( Q^2 \ [GeV^2], W \ [GeV] \) bins: [3.00,3.50],[1.550,1.575) (a), [3.00,3.50],[1.575,1.600) (b), [3.00,3.50],[1.600,1.625) (c) and [3.00,3.50],[1.625,1.650) (d)
Figure D.18 $R^{2LT}_{c,Xij}$ for $Q^2 [GeV^2], W [GeV]$ bins: [3.00,3.50),[1.650,1.675) (a), [3.00,3.50),[1.675,1.700) (b), [3.00,3.50),[1.700,1.725) (c) and [3.00,3.50),[1.725,1.750) (d)
Figure D.19 $R_{LT_{\phi}}^{c_{X_{ij}}}$ for $Q^2 \ [GeV^2], W \ [GeV] \ bins: \ [3.00,3.50],[1.750,1.775) \ (a), \ [3.00,3.50],[1.775,1.800) \ (b), \ [3.00,3.50],[1.800,1.825) \ (c) \ and \ [3.00,3.50],[1.825,1.850) \ (d)$
Figure D.20  $R_{LT}^{c,X_{ij}}$ for $Q^2 \ [GeV^2], W \ [GeV]$ bins: [3.00,3.50),[1.850,1.875) (a), [3.00,3.50),[1.875,1.900) (b), [3.00,3.50),[1.900,1.925) (c) and [3.00,3.50),[1.925,1.950) (d)
Figure D.21  $R^{2}_{LT,\phi^{c_{ij}}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [3.00,3.50),[1.950,1.975) (a), [3.00,3.50),[1.975,2.000) (b), [3.00,3.50),[2.000,2.025) (c) and [3.00,3.50),[2.025,2.050) (d)
Figure D.22 \( R^{2}_{LT_{C,X_{ij}}} \) for \( Q^2 \ [GeV^2] \), \( W \ [GeV] \) bins: [3.00,3.50),[2.050,2.075) (a), [3.00,3.50),[2.075,2.100) (b), [3.00,3.50),[2.100,2.125) (c) and [3.50,4.20),[1.400,1.425) (d)
Figure D.23 $R_{LT}^{c,X_{ij}}$ for $Q^2$ $[GeV^2], W [GeV]$ bins: $[3.50,4.20),[1.425,1.450)$ (a), $[3.50,4.20),[1.450,1.475)$ (b), $[3.50,4.20),[1.475,1.500)$ (c) and $[3.50,4.20),[1.500,1.525)$ (d)
Figure D.24 $R_{\phi}^{LT_c X_{ij}}$ for $Q^2$ [GeV$^2$], $W$ [GeV] bins: [3.50,4.20),[1.525,1.550) (a), [3.50,4.20),[1.550,1.575) (b), [3.50,4.20),[1.575,1.600) (c) and [3.50,4.20),[1.600,1.625) (d)
Figure D.25 $R_{LT,c,\phi,ij}^2$ for $Q^2 [GeV^2], W [GeV]$ bins: [3.50,4.20),[1.625,1.650) (a), [3.50,4.20),[1.650,1.675) (b), [3.50,4.20),[1.675,1.700) (c) and [3.50,4.20),[1.700,1.725) (d)
Figure D.26 $R_{LT}^{c_{X_{ij}}}$ for $Q^2 \ [GeV^2]$, $W \ [GeV]$ bins: [3.50,4.20),[1.725,1.750) (a), [3.50,4.20),[1.750,1.775) (b), [3.50,4.20),[1.775,1.800) (c) and [3.50,4.20),[1.800,1.825) (d)
Figure D.27  \( R_{LT}^{\phi_i} \) for \( Q_2^2 \) [\( GeV^2 \)], \( W \) [\( GeV \)] bins: [3.50,4.20),[1.825,1.850) (a), [3.50,4.20),[1.850,1.875) (b), [3.50,4.20),[1.875,1.900) (c) and [3.50,4.20),[1.900,1.925) (d)
Figure D.28  $R_{LT}^{c,X\mid ij}$ for $Q^2 [GeV^2], W [GeV]$ bins: [3.50,4.20),[1.925,1.950) (a), [3.50,4.20),[1.950,1.975) (b), [3.50,4.20),[1.975,2.000) (c) and [3.50,4.20),[2.000,2.025) (d)
Figure D.29 $R_{2LT}^{cX_j}$ for $Q^2 \text{ [GeV}^2\text{]}, W \text{ [GeV]}$ bins: [3.50,4.20),[2.025,2.050) (a), [3.50,4.20),[2.050,2.075) (b), [3.50,4.20),[2.075,2.100) (c) and [3.50,4.20),[2.100,2.125) (d)
Figure D.30  $R_{LT}^{c,X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [4.20,5.00),[1.400,1.425) (a), [4.20,5.00),[1.425,1.450) (b), [4.20,5.00),[1.450,1.475) (c) and [4.20,5.00),[1.475,1.500) (d)
Figure D.31 \( R^{2LT}_{c,Xi} \) for \( Q^2 \) [GeV\(^2\)], \( W \) [GeV] bins: [4.20,5.00),[1.500,1.525) (a), [4.20,5.00),[1.525,1.550) (b), [4.20,5.00),[1.550,1.575) (c) and [4.20,5.00),[1.575,1.600) (d)
Figure D.32 $R_{2LT}^{c_{Xij}}$ for $Q^2 \ [GeV^2]$, $W \ [GeV]$ bins: [4.20,5.00),[1.600,1.625) (a), [4.20,5.00),[1.625,1.650) (b), [4.20,5.00),[1.650,1.675) (c) and [4.20,5.00),[1.675,1.700) (d)
Figure D.33 $R_{LT,ij}^{c,X}$ for $Q^2$ [GeV$^2$], $W$ [GeV] bins: [4.20,5.00),[1.700,1.725) (a), [4.20,5.00),[1.725,1.750) (b), [4.20,5.00),[1.750,1.775) (c) and [4.20,5.00),[1.775,1.800) (d)
Figure D.34 $R_{2LT}^{c,X_{ij}}$ for $Q^2 [GeV^2]$, $W [GeV]$ bins: [4.20,5.00),[1.800,1.825) (a), [4.20,5.00),[1.825,1.850) (b), [4.20,5.00),[1.850,1.875) (c) and [4.20,5.00),[1.875,1.900) (d)
Figure D.35  $R^{2}_LT^{c,X_{ij}}$ for $Q^2$ [GeV$^2$], $W$ [GeV] bins: [4.20,5.00),[1.900,1.925) (a), [4.20,5.00),[1.925,1.950) (b), [4.20,5.00),[1.950,1.975) (c) and [4.20,5.00),[1.975,2.000) (d)
Figure D.36  $R_{LT}^{c,X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [4.20,5.00),[2.000,2.025) (a), [4.20,5.00),[2.025,2.050) (b), [4.20,5.00),[2.050,2.075) (c) and [4.20,5.00),[2.075,2.100) (d)
Figure D.37  $R_{LT}^{c,X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bin: [4.20,5.00),[2.100,2.125] (a)
Appendix E

Results: $R2_{TT}^{c,X_{ij}}$

Figure E.1  $R2_{TT}^{c,X_{ij}}$ for $Q^2 \ [GeV^2]$, $W \ [GeV]$ bins: [2.00,2.40),[1.400,1.425) (a), [2.00,2.40),[1.425,1.450) (b), [2.00,2.40),[1.450,1.475) (c) and [2.00,2.40),[1.475,1.500) (d)
Figure E.2 $R_{2TTc,X_{ij}}$ for $Q^2 \, [GeV^2], W \, [GeV]$ bins: [2.00, 2.40), [1.500, 1.525) (a), [2.00, 2.40), [1.525, 1.550) (b), [2.00, 2.40), [1.550, 1.575) (c) and [2.00, 2.40), [1.575, 1.600) (d)
Figure E.3 \( R_{TT}^{\phi_{ij}} \) for \( Q^2 \) [GeV\(^2\)], \( W \) [GeV] bins: [2.00,2.40),[1.600,1.625) (a), [2.00,2.40),[1.625,1.650) (b), [2.00,2.40),[1.650,1.675) (c) and [2.00,2.40),[1.675,1.700) (d)
Figure E.4 $R_{2TT}^{C_{Xij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [2.00,2.40],[1.700,1.725] (a), [2.00,2.40],[1.725,1.750] (b), [2.00,2.40],[1.750,1.775] (c) and [2.00,2.40],[1.775,1.800] (d)
Figure E.5 $R^{2\gamma T\phi}_{\gamma \phi}$ for $Q^2 [GeV^2], W [GeV]$ bins: $[2.00, 2.40), [1.800, 1.825)$ (a), $[2.00, 2.40), [1.825, 1.850)$ (b), $[2.00, 2.40), [1.850, 1.875)$ (c) and $[2.00, 2.40), [1.875, 1.900)$ (d)
Figure E.6  $R_{2TT_{\phi}}^{C,X_{ij}}$ for $Q^2 [GeV^2]$, $W [GeV]$ bins: [2.00,2.40),[1.900,1.925) (a), [2.00,2.40),[1.925,1.950) (b), [2.00,2.40),[1.950,1.975) (c) and [2.00,2.40),[1.975,2.000) (d)
Figure E.7 $R_{2TT}^{c_i X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [2.00,2.40),[2.000,2.025) (a), [2.00,2.40),[2.025,2.050) (b), [2.00,2.40),[2.050,2.075) (c) and [2.00,2.40),[2.075,2.100) (d)
Figure E.8  $R_{2TT, C_{X_{ij}}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [2.00,2.40),[2.100,2.125) (a), [2.40,3.00),[1.400,1.425) (b), [2.40,3.00),[1.425,1.450) (c) and [2.40,3.00),[1.450,1.475) (d)
Figure E.9 $R_{TT}^{c,X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [2.40,3.00),[1.475,1.500) (a), [2.40,3.00),[1.500,1.525) (b), [2.40,3.00),[1.525,1.550) (c) and [2.40,3.00),[1.550,1.575) (d)
Figure E.10  $R_{2TT}^{c,X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [2.40,3.00),[1.575,1.600) (a), [2.40,3.00),[1.600,1.625) (b), [2.40,3.00),[1.625,1.650) (c) and [2.40,3.00),[1.650,1.675) (d)
Figure E.11  \( R^{2}_{TT^{c,X_{ij}}} \) for \( Q^2 \ [GeV^2] \), \( W \ [GeV] \) bins: [2.40,3.00),[1.675,1.700) (a), [2.40,3.00),[1.700,1.725) (b), [2.40,3.00),[1.725,1.750) (c) and [2.40,3.00),[1.750,1.775) (d)
Figure E.12 $R_{TT}^{c,X_i}$ for $Q^2 \, [GeV^2], W \, [GeV]$ bins: $[2.40,3.00),[1.775,1.800)$ (a), $[2.40,3.00),[1.800,1.825)$ (b), $[2.40,3.00),[1.825,1.850)$ (c) and $[2.40,3.00),[1.850,1.875)$ (d)
Figure E.13  $R_{TTTT}^{c,Xij}$ for $Q^2 [GeV^2], W [GeV]$ bins: [2.40,3.00),[1.875,1.900) (a), [2.40,3.00),[1.900,1.925) (b), [2.40,3.00),[1.925,1.950) (c) and [2.40,3.00),[1.950,1.975) (d)
Figure E.14 $R_2^{TT\phi, c^\gamma} c_{\gamma}$ for $Q^2$ [GeV$^2$], $W$ [GeV] bins: [2.40,3.00],[1.975,2.000) (a), [2.40,3.00],[2.000,2.025) (b), [2.40,3.00],[2.025,2.050) (c) and [2.40,3.00],[2.050,2.075) (d)
Figure E.15  $R_{TT}^{c,Y_i}$ for $Q^2 [GeV^2], W [GeV]$ bins: [2.40,3.00],[2.075,2.100] (a), [2.40,3.00],[2.100,2.125] (b), [3.00,3.50],[1.400,1.425] (c) and [3.00,3.50],[1.425,1.450] (d)
Figure E.16  $R_{TT_{c,X_i}}$ for $Q^2 \ [GeV^2], W \ [GeV]$ bins: [3.00,3.50],[1.450,1.475) (a), [3.00,3.50],[1.475,1.500) (b), [3.00,3.50],[1.500,1.525) (c) and [3.00,3.50],[1.525,1.550) (d)
Figure E.17  $R_{TT_i}^{c,N(i)}$ for $Q^2 [GeV^2], W [GeV]$ bins: [3.00,3.50),[1.550,1.575) (a), [3.00,3.50),[1.575,1.600) (b), [3.00,3.50),[1.600,1.625) (c) and [3.00,3.50),[1.625,1.650) (d)
Figure E.18 $R_{TT}^{c,Xij}$ for $Q^2 [GeV^2], W [GeV]$ bins: [3.00,3.50),[1.650,1.675) (a), [3.00,3.50),[1.675,1.700) (b), [3.00,3.50),[1.700,1.725) (c) and [3.00,3.50),[1.725,1.750) (d)
Figure E.19  $R_{TT}^{c,X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [3.00,3.50),[1.750,1.775) (a), [3.00,3.50),[1.775,1.800) (b), [3.00,3.50),[1.800,1.825) (c) and [3.00,3.50),[1.825,1.850) (d)
Figure E.20 $R_{2TT_c,Xij}$ for $Q^2 \ [GeV^2], W \ [GeV]$ bins: [3.00,3.50),[1.850,1.875) (a), [3.00,3.50),[1.875,1.900) (b), [3.00,3.50),[1.900,1.925) (c) and [3.00,3.50),[1.925,1.950) (d)
Figure E.21 $R_{TT,c,Xij}^{Q^2}$ for $Q^2$ [GeV$^2$], $W$ [GeV] bins: [3.00,3.50],[1.950,1.975) (a), [3.00,3.50],[1.975,2.000) (b), [3.00,3.50],[2.000,2.025) (c) and [3.00,3.50],[2.025,2.050) (d)
Figure E.22  $R_{2TTc,Xij}^{Q^2}$ for $Q^2$ [GeV$^2$], $W$ [GeV] bins: [3.00,3.50],[2.050,2.075) (a), [3.00,3.50],[2.075,2.100) (b), [3.00,3.50],[2.100,2.125) (c) and [3.50,4.20],[1.400,1.425) (d)
Figure E.23  $R_{TCP}^{c_iX_j}$ for $Q^2 [GeV^2], W [GeV]$ bins: $[3.50,4.20],[1.425,1.450)$ (a), $[3.50,4.20],[1.450,1.475)$ (b), $[3.50,4.20],[1.475,1.500)$ (c) and $[3.50,4.20],[1.500,1.525)$ (d)
Figure E.24 \( R_{2TT \phi_{i}}^{c_{X_{ij}}} \) for \( Q^{2} \ [GeV^{2}], W \ [GeV] \) bins: [3.50,4.20),[1.525,1.550) (a), [3.50,4.20),[1.550,1.575) (b), [3.50,4.20),[1.575,1.600) (c) and [3.50,4.20),[1.600,1.625) (d)
Figure E.25  $R_{TT}^{c,X_{ij}}$ for $Q^2 [GeV^2]$, $W [GeV]$ bins: [3.50,4.20),[1.625,1.650) (a), [3.50,4.20),[1.650,1.675) (b), [3.50,4.20),[1.675,1.700) (c) and [3.50,4.20),[1.700,1.725) (d)
Figure E.26 $R_{TT}^{c,X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [3.50,4.20),[1.725,1.750) (a), [3.50,4.20),[1.750,1.775) (b), [3.50,4.20),[1.775,1.800) (c) and [3.50,4.20),[1.800,1.825) (d)
Figure E.27 \( R_{TT}^{c,X_{ij}} \) for \( Q^2 \) [GeV\(^2\)], \( W \) [GeV] bins: [3.50,4.20),[1.825,1.850) (a), [3.50,4.20),[1.850,1.875) (b), [3.50,4.20),[1.875,1.900) (c) and [3.50,4.20),[1.900,1.925) (d)
Figure E.28 \[ R_{TT_i}^{\phi, X_{ij}} \] for \( Q^2 \) [GeV^2], \( W \) [GeV] bins: [3.50,4.20],[1.925,1.950] (a), [3.50,4.20],[1.950,1.975] (b), [3.50,4.20],[1.975,2.000] (c) and [3.50,4.20],[2.000,2.025] (d)
Figure E.29  $R_{\text{TT}}^{c,X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: $[3.50,4.20),[2.05,2.050)$ (a), $[3.50,4.20),[2.05,2.075)$ (b), $[3.50,4.20),[2.075,2.100)$ (c) and $[3.50,4.20),[2.100,2.125)$ (d)
Figure E.30 $R_{TTc,X}^{c,Xij}$ for $Q^2 [GeV^2], W [GeV]$ bins: [4.20,5.00),[1.400,1.425) (a), [4.20,5.00),[1.425,1.450) (b), [4.20,5.00),[1.450,1.475) (c) and [4.20,5.00),[1.475,1.500) (d)
Figure E.31 $R_{TT}^{c,X_{ij}}$ for $Q^2 \ [GeV^2], W \ [GeV]$ bins: $[4.20,5.00) \, [1.500,1.525)$ (a), $[4.20,5.00) \, [1.525,1.550)$ (b), $[4.20,5.00) \, [1.550,1.575)$ (c) and $[4.20,5.00) \, [1.575,1.600)$ (d).
Figure E.32 $R_{TT}^{c,X_{ij}}$ for $Q^2 \ [GeV^2], W \ [GeV]$ bins: \([4.20,5.00),[1.600,1.625]\) (a), \([4.20,5.00),[1.625,1.650]\) (b), \([4.20,5.00),[1.650,1.675]\) (c) and \([4.20,5.00),[1.675,1.700]\) (d)
Figure E.33 $R_{TT}^{c,X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [4.20,5.00),[1.700,1.725) (a), [4.20,5.00),[1.725,1.750) (b), [4.20,5.00),[1.750,1.775) (c) and [4.20,5.00),[1.775,1.800) (d)
Figure E.34 $R_{TT}^{c,X;j}$ for $Q^2 \ [GeV^2], W \ [GeV]$ bins: [4.20,5.00),[1.800,1.825) (a), [4.20,5.00),[1.825,1.850) (b), [4.20,5.00),[1.850,1.875) (c) and [4.20,5.00),[1.875,1.900) (d)
Figure E.35  $R_{TT_{ij}}^{c,X_{ij}}$ for $Q^2 \ [GeV^2], W \ [GeV]$ bins: [4.20,5.00),[1.900,1.925) (a), [4.20,5.00),[1.925,1.950) (b), [4.20,5.00),[1.950,1.975) (c) and [4.20,5.00),[1.975,2.000) (d)
Figure E.36 \( R_{2TT}^{c,X_{ij}} \) for \( Q^2 \ [GeV^2], W \ [GeV] \) bins: \([4.20,5.00],[2.000,2.025]\) (a), \([4.20,5.00],[2.025,2.050]\) (b), \([4.20,5.00],[2.050,2.075]\) (c) and \([4.20,5.00],[2.075,2.100]\) (d)
Figure E.37  $R_{2TT}^{cX_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bin: [4.20,5.00),[2.100,2.125) (a)


Appendix F

Results: $R_{\phi_i}^{s,X_{ij}}$

Figure F.1 $R_{\phi_i}^{s,X_{ij}}$ for $Q^2$ [GeV$^2$], $W$ [GeV] bins: [2.00,2.40),[1.400,1.425) (a), [2.00,2.40),[1.425,1.450) (b), [2.00,2.40),[1.450,1.475) (c) and [2.00,2.40),[1.475,1.500) (d)
Figure F.2  $R_{LT}^{s,X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [2.00,2.40),[1.500,1.525) (a), [2.00,2.40),[1.525,1.550) (b), [2.00,2.40),[1.550,1.575) (c) and [2.00,2.40),[1.575,1.600) (d)
Figure F.3 $R^{s_{X_{ij}}}_{LT \phi_i}$ for $Q^2 [GeV^2], W [GeV]$ bins: [2.00,2.40), [1.600,1.625) (a), [2.00,2.40), [1.625,1.650) (b), [2.00,2.40), [1.650,1.675) (c) and [2.00,2.40), [1.675,1.700) (d)
Figure F.4  $R_{LT, \phi}^{X_{ij}}$ for $Q^2 \text{ [GeV}^2\text{]}, W \text{ [GeV]}$ bins: [2.00,2.40),[1.700,1.725) (a), [2.00,2.40),[1.725,1.750) (b), [2.00,2.40),[1.750,1.775) (c) and [2.00,2.40),[1.775,1.800) (d)
Figure F.5 \( R_{LT, \phi_i}^{X_{ij}} \) for \( Q^2 [GeV^2], W [GeV] \) bins: \([2.00, 2.40), [1.800, 1.825)\) (a), \([2.00, 2.40), [1.825, 1.850)\) (b), \([2.00, 2.40), [1.850, 1.875)\) (c) and \([2.00, 2.40), [1.875, 1.900)\) (d)
Figure F.6  $R_{2LT}^{s,H_{ij}}$ for $Q^2$ [GeV$^2$], $W$ [GeV] bins: [2.00,2.40),[1.900,1.925) (a), [2.00,2.40),[1.925,1.950) (b), [2.00,2.40),[1.950,1.975) (c) and [2.00,2.40),[1.975,2.000) (d)
Figure F.7  \( R_{LT,\phi_{ij}} \) for \( Q^2 \) [GeV\(^2\)], \( W \) [GeV] bins: [2.00,2.40], [2.000,2.025) (a), [2.00,2.40), [2.025,2.050) (b), [2.00,2.40), [2.050,2.075) (c) and [2.00,2.40), [2.075,2.100) (d)
Figure F.8 \( R_{LT}^{s_{X_{ij}}} \) for \( Q^2 [GeV^2], W [GeV] \) bins: \([2.00,2.40],[2.100,2.125]\) (a), \([2.40,3.00],[1.400,1.425]\) (b), \([2.40,3.00],[1.425,1.450]\) (c) and \([2.40,3.00],[1.450,1.475]\) (d)
Figure F.9 $R_{LT}^{\phi_i}$ for $Q^2 \ [GeV^2], W \ [GeV]$ bins: [2.40,3.00),[1.475,1.500) (a), [2.40,3.00),[1.500,1.525) (b), [2.40,3.00),[1.525,1.550) (c) and [2.40,3.00),[1.550,1.575) (d)
Figure F.10  $R_{LT}^{S_{X_{ij}}, Q^2}$ for $Q^2$ [GeV$^2$], $W$ [GeV] bins: [2.40,3.00],[1.575,1.600) (a), [2.40,3.00],[1.600,1.625) (b), [2.40,3.00],[1.625,1.650) (c) and [2.40,3.00],[1.650,1.675) (d)
Figure F.11  \( R_{LT,\phi}^{S_{X_{ij}}} \) for \( Q^2 \ [GeV^2], W \ [GeV] \) bins: \([2.40,3.00),[1.675,1.700)\) (a), \([2.40,3.00),[1.700,1.725)\) (b), \([2.40,3.00),[1.725,1.750)\) (c) and \([2.40,3.00),[1.750,1.775)\) (d)
Figure F.12  $R_{LT}^{s,X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [2.40,3.00],[1.775,1.800] (a), [2.40,3.00],[1.800,1.825] (b), [2.40,3.00],[1.825,1.850] (c) and [2.40,3.00],[1.850,1.875] (d)
Figure F.13 \( R_{LT}^{s,X_{ij}} \) for \( Q^2 \ [GeV^2] \), \( W \ [GeV] \) bins: [2.40,3.00), [1.875,1.900) (a), [2.40,3.00), [1.900,1.925) (b), [2.40,3.00), [1.925,1.950) (c) and [2.40,3.00), [1.950,1.975) (d)
Figure F.14 $R_{2LT_{\phi_i}}^{s,X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [2.40,3.00),[1.975,2.000) (a), [2.40,3.00),[2.000,2.025) (b), [2.40,3.00),[2.025,2.050) (c) and [2.40,3.00),[2.050,2.075) (d)
Figure F.15  $R_{LT}^{2_{\text{s,xij}}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [2.40,3.00],[2.075,2.100) (a), [2.40,3.00],[2.100,2.125) (b), [3.00,3.50],[1.400,1.425) (c) and [3.00,3.50],[1.425,1.450) (d)
Figure F.16  \( R_{LT}^{S,X_{ij}} \) for \( Q^2 [GeV^2] \), \( W [GeV] \) bins: [3.00,3.50],[1.450,1.475) (a), [3.00,3.50],[1.475,1.500) (b), [3.00,3.50],[1.500,1.525) (c) and [3.00,3.50],[1.525,1.550) (d)
Figure F.17  \( R_{LT}^{s_{X_{i}}^{j}} \) for \( Q^2 \ [GeV^2] \), \( W \ [GeV] \) bins: [3.00,3.50),[1.550,1.575) (a), [3.00,3.50),[1.575,1.600) (b), [3.00,3.50),[1.600,1.625) (c) and [3.00,3.50),[1.625,1.650) (d)
Figure F.18  $R_{LT}^{s,X_{ij}}$ for $Q^2$ [GeV$^2$], $W$ [GeV] bins: [3.00,3.50],[1.650,1.675) (a), [3.00,3.50],[1.675,1.700) (b), [3.00,3.50],[1.700,1.725) (c) and [3.00,3.50],[1.725,1.750) (d)
Figure F.19  $R_{LT}^{s,X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [3.00,3.50),[1.750,1.775) (a), [3.00,3.50),[1.775,1.800) (b), [3.00,3.50),[1.800,1.825) (c) and [3.00,3.50),[1.825,1.850) (d)
Figure F.20 $R_{LT}^{s,X_{ij}}$ for $Q^2 \ [GeV^2], W \ [GeV]$ bins: 
3.00,3.50,1.875-1.925 (a), 
3.00,3.50,1.875,1.900 (b), 
3.00,3.50,1.900,1.925 (c) and 
3.00,3.50,1.925,1.950 (d)
Figure F.21 \( R_{LT}^{s_{X_{ij}}} \) for \( Q^{2} \ [GeV^{2}] \), \( W \ [GeV] \) bins: \([3.00,3.50],[1.950,1.975]\) (a), \([3.00,3.50],[1.975,2.000]\) (b), \([3.00,3.50],[2.000,2.025]\) (c) and \([3.00,3.50],[2.025,2.050]\) (d)
Figure F.22 \( R_{2LT}^{s, X_{ij}} \) for \( Q^2 [GeV^2] \), \( W [GeV] \) bins: [3.00,3.50),[2.050,2.075) (a), [3.00,3.50),[2.075,2.100) (b), [3.00,3.50),[2.100,2.125) (c) and [3.50,4.20),[1.400,1.425) (d)
Figure F.23 $R_{LT}^{s,X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [3.50,4.20),[1.425,1.450) (a), [3.50,4.20),[1.450,1.475) (b), [3.50,4.20),[1.475,1.500) (c) and [3.50,4.20),[1.500,1.525) (d)
Figure F.24  \( R_{LT}^{s,X_{ij}} \) for \( Q^2 \ [GeV^2] \), \( W \ [GeV] \) bins: [3.50,4.20],[1.525,1.550) (a), [3.50,4.20],[1.550,1.575) (b), [3.50,4.20],[1.575,1.600) (c) and [3.50,4.20],[1.600,1.625) (d)
Figure F.25 $R_{2LT}^{S, X_{ij}}$ for $Q^2 \ [GeV^2], W \ [GeV]$ bins: $[3.50,4.20),[1.625,1.650)$ (a), $[3.50,4.20),[1.650,1.675)$ (b), $[3.50,4.20),[1.675,1.700)$ (c) and $[3.50,4.20),[1.700,1.725)$ (d)
Figure F.26 \( R_{2LT_{s,X_{ij}}} \) for \( Q^2 [GeV^2], W [GeV] \) bins: [3.50,4.20),[1.725,1.750) (a), [3.50,4.20),[1.750,1.775) (b), [3.50,4.20),[1.775,1.800) (c) and [3.50,4.20),[1.800,1.825) (d)
Figure F.27  $R_{LT_s}^{S_{X_{ij}}}$ for $Q^2 \text{ [GeV}^2\text{]}, W \text{ [GeV]}$ bins: [3.50,4.20],[1.825,1.850) (a), [3.50,4.20],[1.850,1.875) (b), [3.50,4.20],[1.875,1.900) (c) and [3.50,4.20],[1.900,1.925) (d)
Figure F.28  $R_{LT_{s,X_{ij}}}^{Q^2,W}$ for $Q^2 \ [GeV^2], W \ [GeV]$ bins: [3.50,4.20],[1.925,1.950] (a), [3.50,4.20],[1.950,1.975] (b), [3.50,4.20],[1.975,2.000] (c) and [3.50,4.20],[2.000,2.025] (d)
Figure F.29  $R_{L,T}^{s_i,X_{ij}}$ for $Q^2 \ [GeV^2], W \ [GeV]$ bins: [3.50,4.20],[2.025,2.050) (a), [3.50,4.20],[2.050,2.075) (b), [3.50,4.20],[2.075,2.100) (c) and [3.50,4.20],[2.100,2.125) (d)
Figure F.30  $R_{sLT,ij}^{2, X_i}$ for $Q^2$ [GeV$^2$], $W$ [GeV] bins: [4.20,5.00),[1.400,1.425) (a), [4.20,5.00),[1.425,1.450) (b), [4.20,5.00),[1.450,1.475) (c) and [4.20,5.00),[1.475,1.500) (d)
Figure F.31  $R_{2LT_{s,X_{ij}}}^{Q^2}$ for $Q^2 [GeV^2], W [GeV]$ bins: [4.20,5.00),[1.500,1.525) (a), [4.20,5.00),[1.525,1.550) (b), [4.20,5.00),[1.550,1.575) (c) and [4.20,5.00),[1.575,1.600) (d)
Figure F.32  $R_{LT}$ for $Q^2 [GeV^2], W [GeV]$ bins: [4.20,5.00),[1.600,1.625) (a), [4.20,5.00),[1.625,1.650) (b), [4.20,5.00),[1.650,1.675) (c) and [4.20,5.00),[1.675,1.700) (d)
Figure F.33 $R_{LT_{\phi_{ij}}}$ for $Q^2$ [GeV$^2$], $W$ [GeV] bins: [4.20,5.00),[1.700,1.725) (a), [4.20,5.00),[1.725,1.750) (b), [4.20,5.00),[1.750,1.775) (c) and [4.20,5.00),[1.775,1.800) (d)
Figure F.34  $R_{LT}^{S,X_{ij}}$ for $Q^2 \text{ [GeV}^2\text{]}, W \text{ [GeV]}$ bins: $[4.20,5.00),[1.800,1.825)$ (a), $[4.20,5.00),[1.825,1.850)$ (b), $[4.20,5.00),[1.850,1.875)$ (c) and $[4.20,5.00),[1.875,1.900)$ (d)
Figure F.35  $R^{LT}_{\phi}^{X_{ij}}$ for $Q^2$ [GeV$^2$], $W$ [GeV] bins: [4.20,5.00),[1.900,1.925) (a), [4.20,5.00),[1.925,1.950) (b), [4.20,5.00),[1.950,1.975) (c) and [4.20,5.00),[1.975,2.000) (d)
Figure F.36 $R_{LT}^{s,x_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [4.20,5.00),[2.00,2.025) (a), [4.20,5.00),[2.025,2.050) (b), [4.20,5.00),[2.050,2.075) (c) and [4.20,5.00),[2.075,2.100) (d)
Figure F.37  $R_{2LT_{\phi_1}}^{s,X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bin: [4.20,5.00),[2.100,2.125) (a)
Appendix G

Results: $R_{TT}^{s2,X_{ij}}$ for $Q^2$ [GeV$^2$], $W$ [GeV] bins: [2.00,2.40),[1.400,1.425) (a), [2.00,2.40),[1.425,1.450) (b), [2.00,2.40),[1.450,1.475) (c) and [2.00,2.40),[1.475,1.500) (d)
Figure G.2  \(R_{TT}^{X^2,\phi_i}\) for \(Q^2, W\) bins: [2.00,2.40),[1.500,1.525) (a), [2.00,2.40),[1.525,1.550) (b), [2.00,2.40),[1.550,1.575) (c) and [2.00,2.40),[1.575,1.600) (d)
Figure G.3  $R_{TT}^{X^2, ij}$ for $Q^2$ [GeV$^2$], $W$ [GeV] bins: [2.00,2.40),[1.600,1.625) (a), [2.00,2.40),[1.625,1.650) (b), [2.00,2.40),[1.650,1.675) (c) and [2.00,2.40),[1.675,1.700) (d)
Figure G.4  $R_{TT}^{Q^2,X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [2.00,2.40),[1.700,1.725) (a), [2.00,2.40),[1.725,1.750) (b), [2.00,2.40),[1.750,1.775) (c) and [2.00,2.40),[1.775,1.800) (d)
Figure G.5 \( R_{TT}^{s^2,X_{ij}} \) for \( Q^2 [\text{GeV}^2] \), \( W [\text{GeV}] \) bins: [2.00,2.40),[1.800,1.825) (a), [2.00,2.40),[1.825,1.850) (b), [2.00,2.40),[1.850,1.875) (c) and [2.00,2.40),[1.875,1.900) (d)
Figure G.6  $R_{2TT}^{\phi_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [2.00,2.40),[1.900,1.925) (a), [2.00,2.40),[1.925,1.950) (b), [2.00,2.40),[1.950,1.975) (c) and [2.00,2.40),[1.975,2.000) (d)
Figure G.7 \( R_{TT,\phi_i}^{Q^2,\chi_{ij}} \) for \( Q^2 [GeV^2], W [GeV] \) bins: [2.00,2.40),[2.000,2.025) (a), [2.00,2.40),[2.025,2.050) (b), [2.00,2.40),[2.050,2.075) (c) and [2.00,2.40),[2.075,2.100) (d)
Figure G.8  $R_{TT}^{s^2 \times \sigma_{ij}}$ for $Q^2 [\text{GeV}^2], W [\text{GeV}]$ bins: [2.00,2.40),[2.100,2.125) (a), [2.40,3.00),[1.400,1.425) (b), [2.40,3.00),[1.425,1.450) (c) and [2.40,3.00),[1.450,1.475) (d)
Figure G.9 \( R_{TT}^{s2, X_{ij}} \) for \( Q^2 [GeV^2], W [GeV] \) bins: [2.40,3.00),[1.475,1.500) (a), [2.40,3.00),[1.500,1.525) (b), [2.40,3.00),[1.525,1.550) (c) and [2.40,3.00),[1.550,1.575) (d)
Figure G.10  $R^{2_{TT_{\phi_i}}} Q^2 [GeV^2], W [GeV]$ bins: [2.40,3.00),[1.575,1.600) (a), [2.40,3.00),[1.600,1.625) (b), [2.40,3.00),[1.625,1.650) (c) and [2.40,3.00),[1.650,1.675) (d)
Figure G.11 $R_{TT\phi_i}^{s_2 X_{ij}}$ for $Q^2 \, [GeV^2], W \, [GeV]$ bins: [2.40,3.00),[1.675,1.700) (a), [2.40,3.00),[1.700,1.725) (b), [2.40,3.00),[1.725,1.750) (c) and [2.40,3.00),[1.750,1.775) (d)
Figure G.12 $R_{TT,s^{2},X_{ij}}^{s^{2},X_{ij}}$ for $Q^{2}$ [GeV$^2$], $W$ [GeV] bins: [2.40,3.00),[1.775,1.800) (a), [2.40,3.00),[1.800,1.825) (b), [2.40,3.00),[1.825,1.850) (c) and [2.40,3.00),[1.850,1.875) (d)
Figure G.13 $R_{2TT_{\phi_i}}^{X_{ij}}$ for $Q^2 \ [GeV^2], W \ [GeV]$ bins: [2.40,3.00),[1.875,1.900) (a), [2.40,3.00),[1.900,1.925) (b), [2.40,3.00),[1.925,1.950) (c) and [2.40,3.00),[1.950,1.975) (d)
Figure G.14  $R^{2TT_{\phi_i}}_{X_{ij}}$ for $Q^2 \ [GeV^2], W \ [GeV]$ bins: [2.40,3.00],[1.975,2.000) (a), [2.40,3.00],[2.000,2.025) (b), [2.40,3.00],[2.025,2.050) (c) and [2.40,3.00],[2.050,2.075) (d)
Figure G.15  \( R_{TTs}^{s2,X_{ij}} \) for \( Q^2 \ [GeV^2], W \ [GeV] \) bins: \([2.40,3.00),[2.075,2.100)\) (a), \([2.40,3.00),[2.100,2.125)\) (b), \([3.00,3.50),[1.400,1.425)\) (c) and \([3.00,3.50),[1.425,1.450)\) (d)
Figure G.16  \( R_{TT,\phi}^{2, X_{ij}} \) for \( Q^2 \ [GeV^2], W \ [GeV] \) bins: \([3.00,3.50),[1.450,1.475)\) (a), \([3.00,3.50),[1.475,1.500)\) (b), \([3.00,3.50),[1.500,1.525)\) (c) and \([3.00,3.50),[1.525,1.550)\) (d)
Figure G.17  \( R_{TT}^{s2X_{ij}} \) for \( Q^2 [GeV^2] \), \( W [GeV] \) bins: [3.00,3.50], [1.550,1.575) (a), [3.00,3.50],[1.575,1.600) (b), [3.00,3.50],[1.600,1.625) (c) and [3.00,3.50],[1.625,1.650) (d)
Figure G.18  \( R_{TT}^{s2, X_{ij}} \) for \( Q^2 [\text{GeV}^2], W [\text{GeV}] \) bins: [3.00,3.50),[1.650,1.675) (a), [3.00,3.50),[1.675,1.700) (b), [3.00,3.50),[1.700,1.725) (c) and [3.00,3.50),[1.725,1.750) (d)
Figure G.19  $R_{TT}^{s_2 X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [3.00,3.50],[1.750,1.775) (a), [3.00,3.50],[1.775,1.800) (b), [3.00,3.50],[1.800,1.825) (c) and [3.00,3.50],[1.825,1.850) (d)
Figure G.20  $R_{TT,\phi_i}^{s_2 X_{ij}}$ for $Q^2$ [GeV$^2$], $W$ [GeV] bins: [3.00,3.50],[1.850,1.875) (a), [3.00,3.50],[1.875,1.900) (b), [3.00,3.50],[1.900,1.925) (c) and [3.00,3.50],[1.925,1.950) (d)
Figure G.21 $R_{TT\phi_i}^{s_i^2X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [3.00,3.50],[1.950,1.975) (a), [3.00,3.50],[1.975,2.000) (b), [3.00,3.50],[2.000,2.025) (c) and [3.00,3.50],[2.025,2.050) (d)
Figure G.22 $R_{TT}^{s_2,X_{ij}}$ for $Q^2 \ [GeV^2], W \ [GeV]$ bins: [3.00,3.50],[2.050,2.075) (a), [3.00,3.50],[2.075,2.100) (b), [3.00,3.50],[2.100,2.125) (c) and [3.50,4.20],[1.400,1.425) (d)
Figure G.23  $R_{TT}^{2,Q^2,X_i,j}$ for $Q^2 [GeV^2], W [GeV]$ bins: [3.50,4.20],[1.425,1.450) (a), [3.50,4.20],[1.450,1.475) (b), [3.50,4.20],[1.475,1.500) (c) and [3.50,4.20],[1.500,1.525) (d)
Figure G.24  $R_{TT,\phi_i}^{X_{ij}}$ for $Q^2 [GeV^2], W [GeV]$ bins: [3.50,4.20],[1.525,1.550) (a), [3.50,4.20],[1.550,1.575) (b), [3.50,4.20],[1.575,1.600) (c) and [3.50,4.20],[1.600,1.625) (d)
Figure G.25 $R_{TT,\phi_i}^{s_2}X_{ij}$ for $Q^2\ [GeV^2], W\ [GeV]$ bins: $[3.50,4.20),[1.625,1.650)$ (a), $[3.50,4.20),[1.650,1.675)$ (b), $[3.50,4.20),[1.675,1.700)$ (c) and $[3.50,4.20),[1.700,1.725)$ (d)
Figure G.26  $R_{TT}^{X_i \phi_i} s^2_{X_i}$ for $Q^2 [GeV^2], W [GeV]$ bins: [3.50,4.20],[1.725,1.750) (a), [3.50,4.20],[1.750,1.775) (b), [3.50,4.20],[1.775,1.800) (c) and [3.50,4.20],[1.800,1.825) (d)
Figure G.27 \( R_{TT,ij}^{s2,X_i} \) for \( Q^2 \ [GeV^2], W \ [GeV] \) bins: [3.50,4.20],[1.825,1.850) (a), [3.50,4.20],[1.850,1.875) (b), [3.50,4.20],[1.875,1.900) (c) and [3.50,4.20],[1.900,1.925) (d)
Figure G.28  $R^2_{TT,\phi_i} s^2_{X_i}$ for $Q^2 [GeV^2], W [GeV]$ bins: [3.50,4.20],[1.925,1.950) (a), [3.50,4.20],[1.950,1.975) (b), [3.50,4.20],[1.975,2.000) (c) and [3.50,4.20],[2.000,2.025) (d)
Figure G.29 \( R_{TT,\phi_i}^{s2,X_{ij}} \) for \( Q^2 \ [GeV^2] \), \( W \ [GeV] \) bins: [3.50,4.20),[2.025,2.050) (a), [3.50,4.20),[2.050,2.075) (b), [3.50,4.20),[2.075,2.100) (c) and [3.50,4.20),[2.100,2.125) (d)
Figure G.30  $R^2_{TT,y} s^2 X_{ij}$ for $Q^2 [GeV^2], W [GeV]$ bins: [4.20,5.00),[1.400,1.425) (a), [4.20,5.00),[1.425,1.450) (b), [4.20,5.00),[1.450,1.475) (c) and [4.20,5.00),[1.475,1.500) (d)
Figure G.31  \( R_{TT,2}^{X_{ij}} \) for \( Q^2 [GeV^2], W [GeV] \) bins: [4.20,5.00),[1.500,1.525) (a), [4.20,5.00),[1.525,1.550) (b), [4.20,5.00),[1.550,1.575) (c) and [4.20,5.00),[1.575,1.600) (d)
Figure G.32  \( R_{TT,\phi_i}^{S2,X_{ij}} \) for \( Q^2 \) [GeV\(^2\)], \( W \) [GeV] bins: [4.20,5.00),[1.600,1.625) (a), [4.20,5.00),[1.625,1.650) (b), [4.20,5.00),[1.650,1.675) (c) and [4.20,5.00),[1.675,1.700) (d)
Figure G.33 \( R_{TT,\phi_i}^{s2,X_{ij}} \) for \( Q^2 \ [GeV^2], W \ [GeV] \) bins: [4.20,5.00),[1.700,1.725) (a), [4.20,5.00),[1.725,1.750) (b), [4.20,5.00),[1.750,1.775) (c) and [4.20,5.00),[1.775,1.800) (d)
Figure G.34  $R_{TT,\phi_i}^{s2,X_{ij}}$ for $Q^2$ [GeV$^2$], $W$ [GeV] bins: [4.20,5.00),[1.800,1.825) (a),
[4.20,5.00),[1.825,1.850) (b), [4.20,5.00),[1.850,1.875) (c) and
[4.20,5.00),[1.875,1.900) (d)
Figure G.35  $R_{TT,\phi_i}^{s_{X_i}^2}$ for $Q^2$ [GeV$^2$], $W$ [GeV] bins: [4.20,5.00),[1.900,1.925) (a), [4.20,5.00),[1.925,1.950) (b), [4.20,5.00),[1.950,1.975) (c) and [4.20,5.00),[1.975,2.000) (d)
Figure G.36  $R_{TT, ij}^{s2, X_{ij}}$ for $Q^2\ [GeV^2]$, $W\ [GeV]$ bins: $[4.20,5.00],[2.000,2.025)$ (a), $[4.20,5.00],[2.025,2.050)$ (b), $[4.20,5.00],[2.050,2.075)$ (c) and $[4.20,5.00],[2.075,2.100)$ (d)
Figure G.37  \( R_{TT}^{s2,\phi_i} \) for \( Q^2 \) [GeV\(^2\)], \( W \) [GeV] bin: [4.20,5.00),[2.100,2.125) (a)