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Predictive Modeling of a Paradigm Mechanical Cooling Tower Model: II. Optimal Best-Estimate Results with Reduced Predicted Uncertainties

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Abstract: This work uses the adjoint sensitivity model of the counter-flow cooling tower derived in the accompanying PART I to obtain the expressions and relative numerical rankings of the sensitivities, to all model parameters, of the following model responses: (i) outlet air temperature; (ii) outlet water temperature; (iii) outlet water mass flow rate; and (iv) air outlet relative humidity. These sensitivities are subsequently used within the "predictive modeling for coupled multi-physics systems" (PM_CMPS) methodology to obtain explicit formulas for the predicted optimal nominal values for the model responses and parameters, along with reduced predicted standard deviations for the predicted model parameters and responses. These explicit formulas embody the assimilation of experimental data and the "calibration" of the model's parameters. The results presented in this work demonstrate that the PM_CMPS methodology reduces the predicted standard deviations to values that are smaller than either the computed or the experimentally measured ones, even for responses (e.g., the outlet water flow rate) for which no measurements are available. These improvements stem from the global characteristics of the PM_CMPS methodology, which combines all of the available information simultaneously in phase-space, as opposed to combining it sequentially, as in current data assimilation procedures.

Keywords: adjoint sensitivity analysis; data assimilation; model calibration; best-estimate predictions; reduced predicted uncertainties

1. Introduction

In the present work, the predictive modeling of the counter-flow cooling tower presented in [1] is further developed by applying the "predictive modeling for coupled multi-physics systems" (PM_CMPS) methodology recently developed in [2]. The PM_CMPS methodology constructs a prior distribution for the parameters and responses by using all of the available computational and experimental information, and by relying on the maximum entropy principle to maximize the impact of all available information and minimize the impact of ignorance. Subsequently, the PM_CMPS methodology [2] constructs formally the posterior distribution using Bayes' theorem, and then evaluates asymptotically, to first-order sensitivities, the posterior distribution using the saddle-point method to obtain explicit formulas for the predicted optimal nominal values for the model responses and parameters, along with reduced predicted uncertainties (i.e., reduced predicted standard deviations) for the predicted model parameters and responses. The PM_CMPS methodology has been successfully applied to the analysis of large-scale experiments and the experimental validation of reactor design codes of interest to reactor physics [3,4], light water reactors [5] and sodium-cooled fast reactors [6].

The PM_CMPS methodology relies fundamentally on the sensitivities to model parameters of the measured model responses, which, in this work, are as follows: (i) the outlet air temperature; (ii) the outlet water temperature; (iii) the outlet water mass flow rate; and (iv) the air outlet relative humidity. The expressions, numerical results, and relative rankings of the sensitivities of these responses are presented in Section 2.1. These sensitivities are subsequently used in Section 2.2 for assimilating experimental data in order to "calibrate" the model parameters, and for obtaining best-estimate predicted results with reduced predicted uncertainties. Section 3 concludes this work by discussing the significance of the results presented herein in the context of ongoing work aimed at further applications and generalization of the adjoint sensitivity analysis and PM_CMPS methodologies.

2. Results

It has been shown in the accompanying PART I [1] that the total sensitivity of a model response $R\left(\mathbf{m}_w,\ \mathbf{T}_w,\ \mathbf{T}_a,\ \boldsymbol{\omega};\ \boldsymbol{\alpha}\right)$ to arbitrary variations in the model's parameters $\delta\boldsymbol{\alpha} \equiv \left(\delta\alpha_1,\ \ldots,\ \delta\alpha_{N_\alpha}\right)$ and state functions $\delta\mathbf{m}_w,\ \delta\mathbf{T}_w,\ \delta\mathbf{T}_a,\ \delta\boldsymbol{\omega}$, around the nominal values $\left(\mathbf{m}_w^0,\ \mathbf{T}_w^0,\ \mathbf{T}_a^0,\ \boldsymbol{\omega}^0;\ \boldsymbol{\alpha}^0\right)$ of the parameters and state functions, is provided by the G-differential of the model's response to these variations. This G-differential was denoted as $DR\left(\mathbf{m}_w^0,\ \mathbf{T}_w^0,\ \mathbf{T}_a^0,\ \boldsymbol{\omega}^0;\ \boldsymbol{\alpha}^0;\ \delta\mathbf{m}_w,\ \delta\mathbf{T}_w,\ \delta\mathbf{T}_a,\ \delta\boldsymbol{\omega};\ \delta\boldsymbol{\alpha}\right)$, and was expressed in terms of the adjoint sensitivity functions as follows:

$$DR\left(\mathbf{m}_w^0,\ \mathbf{T}_w^0,\ \mathbf{T}_a^0,\ \boldsymbol{\omega}^0;\ \boldsymbol{\alpha}^0;\ \delta\mathbf{m}_w,\ \delta\mathbf{T}_w,\ \delta\mathbf{T}_a,\ \delta\boldsymbol{\omega};\ \delta\boldsymbol{\alpha}\right) = \sum_{i=1}^{N_\alpha}\left(\frac{\partial R}{\partial\alpha_i}\delta\alpha_i\right) + DR_{indirect}, \tag{1}$$

where the so-called "indirect effect" term, $DR_{indirect}$, is given by:

$$DR_{indirect} \equiv \boldsymbol{\mu}_w\cdot\mathbf{Q}_1 + \boldsymbol{\tau}_w\cdot\mathbf{Q}_2 + \boldsymbol{\tau}_a\cdot\mathbf{Q}_3 + \mathbf{o}\cdot\mathbf{Q}_4 \tag{2}$$

and where the vector $\left[\boldsymbol{\mu}_w,\ \boldsymbol{\tau}_w,\ \boldsymbol{\tau}_a,\ \mathbf{o}\right]^+$ is the solution of the following *adjoint sensitivity system*:

$$\begin{pmatrix} \mathbf{A}_1^+ & \mathbf{A}_2^+ & \mathbf{A}_3^+ & \mathbf{A}_4^+ \\ \mathbf{B}_1^+ & \mathbf{B}_2^+ & \mathbf{B}_3^+ & \mathbf{B}_4^+ \\ \mathbf{C}_1^+ & \mathbf{C}_2^+ & \mathbf{C}_3^+ & \mathbf{C}_4^+ \\ \mathbf{D}_1^+ & \mathbf{D}_2^+ & \mathbf{D}_3^+ & \mathbf{D}_4^+ \end{pmatrix}\begin{pmatrix} \boldsymbol{\mu}_w \\ \boldsymbol{\tau}_w \\ \boldsymbol{\tau}_a \\ \mathbf{o} \end{pmatrix} = \begin{pmatrix} \mathbf{R}_1 \\ \mathbf{R}_2 \\ \mathbf{R}_3 \\ \mathbf{R}_4 \end{pmatrix}. \tag{3}$$

Furthermore, the sources $\mathbf{R}_\ell \equiv \left(r_\ell^{(1)},\ldots,r_\ell^{(I)}\right),\ \ell = 1,2,3,4$, for the adjoint sensitivity system represented by Equation (3) are the functional derivatives of the model responses with respect to the state functions, i.e.:

$$r_1^{(i)} \equiv \frac{\partial R}{\partial m_w^{(i+1)}};\ r_2^{(i)} \equiv \frac{\partial R}{\partial T_w^{(i+1)}};\ r_3^{(i)} \equiv \frac{\partial R}{\partial T_a^{(i)}};\ r_4^{(i)} \equiv \frac{\partial R}{\partial\omega^{(i)}};\quad i = 1,\ldots,\ I \tag{4}$$

while the components of the vectors $\mathbf{Q}_\ell \equiv \left(q_\ell^{(1)},\ldots,q_\ell^{(I)}\right),\ \ell = 1,2,3,4$, in Equation (2) are the derivatives of the model's equations with respect to model parameters, namely:

$$q_\ell^{(i)} \equiv \sum_{j=1}^{N_\alpha}\left(\frac{\partial N_\ell^{(i)}}{\partial\alpha_j}\delta\alpha_j\right);\ i = 1,\ldots,I;\ \ell = 1,\ 2,\ 3,\ 4. \tag{5}$$

The explicit expressions of the vectors $\mathbf{Q}_\ell \equiv \left(q_\ell^{(1)},\ldots,q_\ell^{(I)}\right),\ \ell = 1,2,3,4$ are provided in Appendix A. The model responses of interest in this work are the following quantities: (i) the outlet air temperature, $T_a^{(1)}$; (ii) the outlet water temperature, $T_w^{(50)}$; (iii) the outlet water flow rate, $m_w^{(50)}$; and (iv) the outlet air relative humidity, $RH^{(1)}$. Except for the water outlet flow rate $m_w^{(50)}$, these responses have been measured experimentally [7,8], and the first four moments of their respective statistical distributions have been quantified in [1].

2.1. Sensitivity Analysis Results and Rankings

As has been discussed in the accompanying PART I [1], there are a total of 8079 measured benchmark data sets for the cooling tower model with the "fan-on," with a drafted air exit velocity at 10 m/s at the shroud. For this velocity (and corresponding air flow rate), the Reynolds number is around 4500, which means that the flow within the cooling tower is in the "transitional flow and heat transfer" regime. As has also been discussed in [1], 7668 benchmark data sets (out of the total of 8079 data sets) are considered to correspond to the "unsaturated conditions" which are analyzed in this work. The nominal values for boundary and atmospheric conditions used in this work were obtained, as described in [1], from the statistics of these 7668 benchmark data sets corresponding to "unsaturated conditions." In turn, these "unsaturated" boundary and atmospheric conditions were used to obtain the sensitivity results reported, below, in this Subsection. Sub-subsections 2.1.1 through 2.1.4, below, provide the numerical values and rankings, in descending order, of the relative sensitivities computed using the adjoint sensitivity analysis methodology for the four model responses $T_a^{(1)}$, $T_w^{(50)}$, $m_w^{(50)}$ and $RH^{(1)}$. Note that the relative sensitivity, $RS\left(\alpha_i\right)$, of a response $R\left(\alpha_i\right)$ to a parameter α_i is defined as $RS\left(\alpha_i\right) \equiv \left[dR\left(\alpha_i\right)/d\alpha_i\right]\left[\alpha_i/R\left(\alpha_i\right)\right]$. Thus, the relative sensitivities are unit-less and are very useful in ranking the sensitivities to highlight their relative importance for the respective response. Thus, a relative sensitivity of 1.00 indicates that a change of 1% in the respective parameter will induce a 1% change in a response that is linear in the respective sensitivity. The higher the relative sensitivity, the more important the respective parameter to the respective response.

2.1.1. Relative Sensitivities of the Outlet Air Temperature, $T_a^{(1)}$

The sensitivities of the air outlet temperature with respect to all of the model's parameters have been computed using Equations (1) and (2). The numerical results and ranking of the relative sensitivities, in descending order of their magnitudes, are provided in Table 1 below, along with their respective relative standard deviations.

Table 1. Ranked relative sensitivities of the outlet air temperature $T_a^{(1)}$.

Rank #	Parameter (α_i)	Nominal Value	Relative Sensitivity $RS\left(\alpha_i\right)$	Relative Standard Deviation (%)
1	Inlet air temperature, $T_{a,in}$	299.11 K	0.4858	1.39
2	Air temperature (dry bulb), T_{db}	299.11 K	0.4829	1.39
3	Inlet water temperature, $T_{w,in}$	298.79 K	0.2756	0.57
4	Dew point temperature, T_{dp}	292.05 K	0.1834	0.81
5	$P_{vs}(T)$ parameter, a_0	25.5943	−0.0945	0.04
6	$P_{vs}(T)$ parameter, a_1	−5229.89	0.0618	0.08
7	Inlet air humidity ratio, ω_{in}	0.0138	0.0100	14.93
8	Fan shroud inner diameter, D_{fan}	4.1 m	−0.0056	1.00
9	Water enthalpy $h_f(T)$ parameter, a_{1f}	4186.51	0.0050	0.04
10	Wetted fraction of fill surface area, w_{tsa}	1.0	−0.0049	0.00
11	Nusselt number, Nu	14.94	−0.0049	34.0
12	Fill section surface area, A_{surf}	14221 m^2	−0.0049	25.0
13	Dynamic viscosity of air at T = 300 K, μ	1.983×10^{-5} kg/(m·s)	0.0045	4.88
14	Nu parameter, $a_{1,Nu}$	0.0031498	−0.0045	31.75
15	Reynolds number, Re_d	4428	−0.0045	15.17
16	Fill section flow area, A_{fill}	67.29 m^2	0.0045	10.0
17	$C_{pa}(T)$ parameter, $a_{0,cpa}$	1030.5	0.0032	0.03
18	Inlet water mass flow rate, $m_{w,in}$	44.02 kg/s	0.0031	5.0
19	$h_g(T)$ parameter, a_{0g}	2005744	−0.0030	0.05
20	$D_{av}(T)$ parameter, $a_{1,dav}$	2.65322	0.0028	0.11
21	Exit air speed at the shroud, V_{exit}	10.0 m/s	−0.0028	10.0
22	Inlet air mass flow rate, m_a	155.07 kg/s	−0.0028	10.26
23	Heat transfer coefficient multiplier, f_{ht}	1.0	−0.0026	50.0
24	Thermal conductivity of air at T = 300 K, k_{air}	0.02624 W/(m·K)	−0.0026	6.04
25	Mass transfer coefficient multiplier, f_{mt}	1.0	−0.0022	50.0
26	Sherwood number, Sh	14.13	−0.0022	34.25
27	$D_{av}(T)$ parameter, $a_{2,dav}$	-6.1681×10^{-3}	−0.0019	0.37
28	$h_f(T)$ parameter, a_{0f}	−1,143,423	−0.0017	0.05
29	$D_{av}(T)$ parameter, $a_{0,dav}$	7.06085×10^{-9}	−0.0015	0
30	Atmospheric pressure, P_{atm}	100,586 Pa	−0.0013	0.40
31	Kinematic viscosity of air at 300 K, ν	1.568×10^{-5} m^2/s	−0.00074	12.09
32	Prandlt number of air at T = 80 C, Pr	0.708	0.00074	0.71
33	Schmidt number, Sc	0.60	−0.00074	12.41
34	$h_g(T)$ parameter, a_{1g}	1815.437	−0.00074	0.19

Table 1. *Cont.*

Rank #	Parameter (α_i)	Nominal Value	Relative Sensitivity $RS(\alpha_i)$	Relative Standard Deviation (%)
35	D_{av}(T) parameter, $a_{3,dav}$	6.55265×10^{-6}	0.00063	0.58
36	Nu parameter, $a_{2,Nu}$	0.9902987	−0.00032	33.02
37	Fill section equivalent diameter, D_h	0.0381 m	0.00032	1.0
38	C_{pa}(T) parameter, $a_{1,cpa}$	−0.19975	−0.00018	1.0
39	C_{pa} (T) parameter, $a_{2,cpa}$	3.9734×10^{-4}	0.00010	0.84
40	Sum of loss coefficients above fill, k_{sum}	10.0	0.000	50.0
41	Fill section frictional loss multiplier, f	4.0	0.000	50.0
42	Nu parameter, $a_{0,Nu}$	8.235	0.000	25.0
43	Nu parameter, $a_{3,Nu}$	0.023	0.000	38.26
44	Cooling tower deck width in x-dir, W_{dkx}	8.5 m	0.000	1.0
45	Cooling tower deck width in y-dir, W_{dky}	8.5 m	0.000	1.0
46	Cooling tower deck height above ground, Δz_{dk}	10.0 m	0.000	1.0
47	Fan shroud height, Δz_{fan}	3.0 m	0.000	1.0
48	Fill section height, Δz_{fill}	2.013 m	0.000	1.0
49	Rain section height, Δz_{rain}	1.633 m	0.000	1.0
50	Basin section height, Δz_{bs}	1.168 m	0.000	1.0
51	Drift eliminator thickness, Δz_{de}	0.1524 m	0.000	1.0
52	Wind speed, V_w	1.80 m/s	0.000	51.1

As the results in Table 1 indicate, the first five parameters (i.e., $T_{a,in}$, T_{db}, $T_{w,in}$, T_{dp}, a_0) have relative sensitivities between ca. 10% and 50%, and are therefore the most important for the air outlet temperature response, $T_a^{(1)}$. The two largest sensitivities have values of 48%, which means that a 1% change in $T_{a,in}$ or T_{db} would induce a 0.48% change in $T_a^{(1)}$. The next two parameters (i.e., a_1 and ω_{in}) have relative sensitivities between 1% and 6%, and are therefore somewhat important. Parameters #8 through #16 (i.e., D_{fan}, a_{1f}, w_{tsa}, Nu, A_{surf}, μ, $a_{1,Nu}$, Re_d, A_{fill}) have relative sensitivities of the order of 0.5%. The remaining 36 parameters are relatively unimportant for this response, having relative sensitivities smaller than 1% of the largest relative sensitivity (with respect to $T_{a,in}$) for this response. Positive sensitivities imply that a positive change in the respective parameter would cause an increase in the response, while negative sensitivities imply that a positive change in the respective parameter would cause a decrease in the response.

2.1.2. Relative Sensitivities of the Outlet Water Temperature, $T_w^{(50)}$

The results and ranking of the relative sensitivities of the outlet water temperature with respect to the most important 12 parameters for this response are listed in Table 2.

Table 2. Most important relative sensitivities of the outlet water temperature, $T_w^{(50)}$.

Rank #	Parameter (α_i)	Nominal Value	Relative Sensitivity $RS(\alpha_i)$	Relative Standard Deviation (%)
1	Dew point temperature, T_{dp}	292.05 K	0.5482	0.81
2	Inlet air temperature, $T_{a,in}$	299.11 K	0.2318	1.39
3	Air temperature (dry bulb), T_{db}	299.11 K	0.2244	1.39
4	P_{vs}(T) parameters, a_0	25.5943	−0.1949	0.04
5	P_{vs}(T) parameters, a_1	−5229.89	0.1282	0.08
6	Inlet water temperature, $T_{w,in}$	298.79 K	0.1066	0.57
7	Inlet air humidity ratio, ω_{in}	0.0138	0.0299	14.93
8	Fan shroud inner diameter, D_{fan}	4.1 m	−0.0085	1.00
9	Water enthalpy hf(T) parameter, a_{1f}	4186.51	0.0082	0.04
10	$D_{av}(T_{db})$ parameter, $a_{1,dav}$	2.653	0.0071	0.11
11	Enthalpy h_g(T) parameter, a_{0g}	2,005,744	−0.0062	0.05
12	Sherwood number, Sh	14.13	−0.0056	34.25

The largest sensitivity of $T_w^{(50)}$ is to the parameter T_{dp}, and has the value of 0.548; this means that a 1% increase in T_{db} would induce a 0.548% increase in $T_w^{(50)}$. The sensitivities to the remaining 40 model parameters have not been listed since they are smaller than 1% of the largest sensitivity (with respect to T_{dp}) for this response.

2.1.3. Relative Sensitivities of the Outlet Water Mass Flow Rate, $m_w^{(50)}$

The results and ranking of the relative sensitivities of the outlet water mass flow rate with respect to the most important 10 parameters for this response are listed in Table 3. This response is most sensitive to $m_{w,in}$ (a 1% increase in this parameter would cause a 1.01% increase in the response) and the second largest sensitivity is to the parameter $T_{w,in}$ (a 1% increase in this parameter would cause a 0.447% decrease in the response). The sensitivities to the remaining 42 model parameters have not been listed since they are smaller than 1% of the largest sensitivity (with respect to $m_{w,in}$) for this response.

Table 3. Most important relative sensitivities of the outlet water mass flow rate, $m_w^{(50)}$.

Rank #	Parameter (α_i)	Nominal Value	Relative Sensitivity RS(α_i)	Relative Standard Deviation (%)
1	Inlet water mass flow rate, $m_{w,in}$	44.02 kg/s	1.0060	5.00
2	Inlet water temperature, $T_{w,in}$	298.79 K	−0.4474	0.57
3	Dew point temperature, T_{dp}	292.05 K	0.3560	0.81
4	Pvs(T) parameters, a_0	25.5943	−0.1416	0.04
5	Air temperature (dry bulb), T_{db}	299.11 K	−0.1184	1.39
6	Inlet air temperature, $T_{a,in}$	299.11 K	−0.1134	1.39
7	Pvs(T) parameters, a_1	−5229.89	0.0930	0.08
8	Inlet air humidity ratio, ω_{in}	0.0138	0.0195	14.93
9	Fan shroud inner diameter, D_{fan}	4.1 m	−0.0117	1.00
10	Inlet air mass flow rate, m_a	155.07 kg/s	−0.0058	10.26

2.1.4. Relative Sensitivities of the Outlet Air Relative Humidity, $RH^{(1)}$

The results and ranking of the relative sensitivities of the outlet air relative humidity with respect to the most important 20 parameters for this response are listed in Table 4. The first three sensitivities of this response are quite large (relative sensitivities larger than unity are customarily considered to be very significant). In particular, an increase of 1% in $T_{a,in}$ or T_{db} would cause a decrease in the response of 6.66% or 6.525%, respectively. On the other hand, an increase of 1% in T_{dp} would cause an increase of 5.75% in the response. The sensitivities to the remaining 32 model parameters have not been listed since they are smaller than 1% of the largest sensitivity (with respect to $T_{a,in}$) for this response.

Table 4. Most important relative sensitivities of the outlet air relative humidity, $RH^{(1)}$.

Rank #	Parameter (α_i)	Nominal Value	Relative Sensitivity RS (α_i)	Relative Standard Deviation (%)
1	Inlet air temperature, $T_{a,in}$	299.11 K	−6.660	1.39
2	Air temperature (dry bulb), T_{db}	299.11 K	−6.525	1.39
3	Dew point temperature , T_{dp}	292.05 K	5.750	0.81
4	Inlet water temperature, $T_{w,in}$	298.79 K	0.747	0.57
5	Inlet air humidity ratio, ω_{in}	0.0138	0.3141	14.93
6	P_{vs}(T) parameters, a_0	25.5943	−0.3123	0.04
7	Wetted fraction of fill surface area, w_{tsa}	1.0	0.1487	0.00
8	Fill section surface area, A_{surf}	14,221 m^2	0.1487	25.0
9	Nusselt number, Nu	14.94	0.1487	34.0
10	Dynamic viscosity of air at T = 300 K, μ	1.983×10^{-5} kg/(m·s)	−0.1388	4.88
11	Nu parameters, $a_{1,Nu}$	0.0031498	0.1388	31.75
12	Fill section flow area, A_{fill}	67.29 m^2	−0.1388	10.0
13	Reynold's number, Re	4428	0.1388	15.17
14	$D_{av}(T_{db})$ parameter, $a_{1,dav}$	2.65322	−0.1297	0.11
15	Mass transfer coefficient multiplier, f_{mt}	1.0	0.1023	50.0
16	Sherwood number, Sh	14.13	0.1023	34.25
17	Atmosphere pressure, P_{atm}	100,586 Pa	0.0992	0.40
18	$D_{av}(T_{db})$ parameter, $a_{2,dav}$	-6.1681×10^{-3}	0.0902	0.37
19	$D_{av}(T_{db})$ parameter, $a_{0,dav}$	7.06085×10^{-9}	0.0682	0.00
20	P_{vs}(T) parameters, a_1	−5229.89	0.0681	0.08

Overall, the outlet air relative humidity, $RH^{(1)}$, displays the largest sensitivities, so this response is the most sensitive to parameter variations. The other responses, namely the outlet air temperature, the outlet water temperature, and the outlet water mass flow rate display sensitivities of comparable magnitudes.

2.2. Experimental Data Assimilation, Model Calibration and Best-Estimate Predicted Results with Reduced Predicted Uncertainties

This subsection presents the results of applying the Predictive Modeling of Coupled Multi-Physics Systems (PM_CMPS) methodology [2] to the counter-flow cooling tower model. The PM_CMPS methodology [2] encompasses into a unified conceptual and mathematical framework, the concepts of both "forward" and "inverse" modeling, including data assimilation, model calibration and prediction of best-estimate values for model parameters and responses, with reduced predicted uncertainties. For the simplest case of a single computational model, such as the counter-flow cooling tower model analyzed in this work, the PM_CMPS methodology considers the following a priori information:

1. A model comprising N_α imprecisely known system (model) parameters, α_n, considered as the components of a (column) vector, $\boldsymbol{\alpha}$, defined as:

$$\boldsymbol{\alpha} = \{\alpha_n |\, n = 1, \ldots, N_\alpha\} \tag{6}$$

The mean values of the model parameters α_n are denoted as $\alpha_n^0 \equiv \langle \alpha_n \rangle$, and the covariances between two parameters α_i and α_j are denoted as $\mathrm{cov}(\alpha_i,\alpha_j)$. The mean values α_n^0 are considered to be known a priori, so that the vector $\boldsymbol{\alpha}^0$, defined as $\boldsymbol{\alpha}^0 = \{\alpha_n^0 | n = 1, \ldots, N_\alpha\}$ is considered to be known a priori. The covariances $\mathrm{cov}(\alpha_i,\alpha_j)$ are also considered to be a priori known; these covariances are considered to be the elements of the a priori known parameter covariance matrix, denoted as $\mathbf{C}_{\alpha\alpha}^{(N_\alpha \times N_\alpha)}$ and defined as:

$$\mathbf{C}_{\alpha\alpha}^{(N_\alpha \times N_\alpha)} \equiv \left[\mathrm{cov}\left(\alpha_i, \alpha_j\right)\right]_{N_\alpha \times N_\alpha} \equiv \left\langle \left(\alpha_i - \alpha_i^0\right)\left(\alpha_j - \alpha_j^0\right) \right\rangle_{N_\alpha \times N_\alpha};\ i, j = 1, \ldots, N_\alpha \tag{7}$$

2. Also associated with the model are N_r experimentally measured responses, r_i, considered to be components of the column vector:

$$\mathbf{r} = \{r_i | i = 1, \ldots, N_r\} \tag{8}$$

The mean values, denoted as r_i^m, of the measured responses, r_i, and the covariances, denoted as $\langle (r_i - r_i^m)(r_j - r_j^m) \rangle$, between two measured responses, r_i and r_j, are also considered to be known a priori. The mean measured values r_i^m will be considered to constitute the components of the vector $\mathbf{r}^m$ defined as:

$$\mathbf{r}^m = \{r_i^m | i = 1, \ldots, N_r\}, \quad r_i^m \equiv \langle r_i \rangle,\ i = 1, \ldots, N_r, \tag{9}$$

and the covariances $\langle (r_i - r_i^m)(r_j - r_j^m) \rangle$ of the measured responses are considered to be components of the a priori known measured covariance matrix, denoted as $\mathbf{C}_{rr}^{(N_r \times N_r)}$, and defined as:

$$\mathbf{C}_{rr}^{(N_r \times N_r)} \equiv \left\langle (r_i - r_i^m)\left(r_j - r_j^m\right) \right\rangle_{N_r \times N_r},\ i, j = 1, \ldots, N_r. \tag{10}$$

3. In the most general case, correlations may also exist among all parameters and responses. Such correlations are quantified through a priori known *parameter-response matrices*, denoted as $\mathbf{C}_{\alpha r}^{(N_\alpha \times N_r)}$, and defined as follows:

$$\mathbf{C}_{\alpha r}^{(N_\alpha \times N_r)} \equiv \left\langle \left(\boldsymbol{\alpha} - \boldsymbol{\alpha}^0\right)(\mathbf{r} - \mathbf{r}^m)^+ \right\rangle = \left[\mathbf{C}_{r\alpha}^{(N_r \times N_\alpha)}\right]^+ \tag{11}$$

To keep the notation simple, the dimensions of the various vectors and matrices will not be shown in subsequent formulas. For a single multi-physics system, as is the case of the cooling tower model under consideration in this work, the quantities predicted by the PM_CMPS methodology [2] are as follows:

A. Optimally predicted "best-estimate" nominal values, $\boldsymbol{\alpha}^{pred}$, for the model parameters:

$$\boldsymbol{\alpha}^{pred} = \boldsymbol{\alpha}^0 - \left(\mathbf{C}_{\alpha\alpha}\mathbf{S}_{r\alpha}^+ - \mathbf{C}_{\alpha r}\right)[\mathbf{D}_{rr}]^{-1}\left[\mathbf{r}^c\left(\boldsymbol{\alpha}^0, \boldsymbol{\beta}^0\right) - \mathbf{r}^m\right], \tag{12}$$

where the matrix $\mathbf{D}_{rr}$ is defined as:

$$\mathbf{D}_{rr} = \mathbf{S}_{r\alpha}\mathbf{C}_{\alpha\alpha}\mathbf{S}_{r\alpha}^+ - \mathbf{S}_{r\alpha}\mathbf{C}_{\alpha r} - \mathbf{C}_{\alpha r}^+\mathbf{S}_{r\alpha}^+ + \mathbf{C}_{rr}, \tag{13}$$

and the components of the matrix $\mathbf{S}_{r\alpha}^{(N_r \times N_\alpha)}$ are the first-order sensitivities (i.e., functional derivatives) of all responses with respect to all model parameters, defined as follows:

$$\mathbf{S}_{r\alpha}^{N_r \times N_\alpha} \equiv \begin{pmatrix} \frac{\partial r_1}{\partial \alpha_1} & \cdots & \frac{\partial r_1}{\partial \alpha_{N_\alpha}} \\ \vdots & \ddots & \vdots \\ \frac{\partial r_{N_r}}{\partial \alpha_1} & \cdots & \frac{\partial r_{N_r}}{\partial \alpha_{N_\alpha}} \end{pmatrix}. \tag{14}$$

It is important to note that the first term on the right side of Equation (13) is the covariance matrix of the computed responses, $\mathbf{C}_{rr}^{comp}$, when only the first-order sensitivities are taken into account, i.e.:

$$\mathbf{C}_{rr}^{comp} = \mathbf{S}_{r\alpha}\mathbf{C}_{\alpha\alpha}\mathbf{S}_{r\alpha}^{+}. \tag{15}$$

B. Reduced predicted uncertainties, $\mathbf{C}_{\alpha\alpha}^{pred}$, for the predicted nominal parameter values, given by the expression below:

$$\mathbf{C}_{\alpha\alpha}^{pred} = \mathbf{C}_{\alpha\alpha} - \left(\mathbf{C}_{\alpha\alpha}\mathbf{S}_{r\alpha}^{+} - \mathbf{C}_{\alpha r}\right)\left[\mathbf{D}_{rr}\right]^{-1}\left(\mathbf{C}_{\alpha\alpha}\mathbf{S}_{r\alpha}^{+} - \mathbf{C}_{\alpha r}\right)^{+}; \tag{16}$$

C. Optimally predicted "best-estimate" nominal values, $\mathbf{r}^{pred}$, for the model responses, given by the expression below:

$$\mathbf{r}^{pred} = \mathbf{r}^{m} - \left(\mathbf{C}_{\alpha r}^{+}\mathbf{S}_{r\alpha}^{+} - \mathbf{C}_{rr}\right)\left[\mathbf{D}_{rr}\right]^{-1}\left[\mathbf{r}^{c}\left(\boldsymbol{\alpha}^{0}, \boldsymbol{\beta}^{0}\right) - \mathbf{r}^{m}\right]; \tag{17}$$

D. Reduced predicted uncertainties, $\mathbf{C}_{rr}^{pred}$, for the predicted nominal response values, given by the expression below:

$$\mathbf{C}_{rr}^{pred} = \mathbf{C}_{rr} - \left(\mathbf{C}_{\alpha r}^{+}\mathbf{S}_{r\alpha}^{+} - \mathbf{C}_{rr}\right)\left[\mathbf{D}_{rr}\right]^{-1}\left(\mathbf{C}_{\alpha r}^{+}\mathbf{S}_{r\alpha}^{+} - \mathbf{C}_{rr}\right)^{+}; \tag{18}$$

E. Predicted correlations, $\mathbf{C}_{\alpha r}^{pred}$, between the predicted model parameters and responses, given by the expression below:

$$\mathbf{C}_{\alpha r}^{pred} = \mathbf{C}_{\alpha r} - \left(\mathbf{C}_{\alpha\alpha}\mathbf{S}_{r\alpha}^{+} - \mathbf{C}_{\alpha r}\right)\left[\mathbf{D}_{rr}\right]^{-1}\left(\mathbf{C}_{\alpha r}^{+}\mathbf{S}_{r\alpha}^{+} - \mathbf{C}_{rr}\right)^{+}. \tag{19}$$

The expressions given in Equations (6) through (19) can also be obtained from the results presented originally in [9] for the particular case of a time-independent single multi-physics system. Note that if the model is perfect (which means that $\mathbf{C}_{\alpha\alpha}$ = 0 and $\mathbf{C}_{\alpha r}$ = 0), Equations (6) through (19) would yield $\boldsymbol{\alpha}^{pred}$ = $\boldsymbol{\alpha}^{0}$ and $\mathbf{r}^{pred}$ = $\mathbf{r}^{c}(\boldsymbol{\alpha}^{0},\boldsymbol{\beta}^{0})$, without any accompanying uncertainties (i.e., $\mathbf{C}_{rr}^{pred} = 0$, $\mathbf{C}_{\alpha\alpha}^{pred} = 0$, $\mathbf{C}_{\alpha r}^{pred} = 0$). In other words, for a perfect model, the PM_CMPS methodology predicts values for the responses and the parameters that would coincide with the model's original corresponding parameter and computed responses (assumed to be perfect), and the experimental measurements would have no effect on the predictions (as would be expected, since imperfect measurements could not possibly improve a "perfect" model's predictions). On the other hand, if the measurements were perfect, (i.e., $\mathbf{C}_{rr}$ = 0 and $\mathbf{C}_{\alpha r}$ = 0), but the model were imperfect, then Equations (6) through (19) would yield $\boldsymbol{\alpha}^{pred} = \alpha^{0} - \mathbf{C}_{\alpha\alpha}\mathbf{S}_{r\alpha}^{+}\left[\mathbf{S}_{r\alpha}\mathbf{C}_{\alpha\alpha}\mathbf{S}_{r\alpha}^{+}\right]^{-1}\mathbf{r}^{d}\left(\boldsymbol{\alpha}^{0}\right)$, $\mathbf{C}_{\alpha\alpha}^{pred} = \mathbf{C}_{\alpha\alpha} - \mathbf{C}_{\alpha\alpha}\mathbf{S}_{r\alpha}^{+}\left[\mathbf{S}_{r\alpha}\mathbf{C}_{\alpha\alpha}\mathbf{S}_{r\alpha}^{+}\right]^{-1}\mathbf{S}_{r\alpha}\mathbf{C}_{\alpha\alpha}$, $\mathbf{r}^{pred} = \mathbf{r}^{m}$, $\mathbf{C}_{rr}^{pred} = 0$, $\mathbf{C}_{\alpha r}^{pred} = 0$. In other words, in the case of perfect measurements, the PM_CMPS predicted values for the responses would coincide with the measured values (assumed to be perfect), while the model's uncertain parameters would be calibrated by taking the respective measurements into account to yield improved nominal values and reduced parameters uncertainties.

The a priori response-parameter covariance matrix, $\mathbf{C}_{r\alpha}$, has been already computed in [1], Equation (A5), and is reproduced below:

$$Cov\left(T_{a,\,out}^{meas},\ T_{w,\,out}^{meas},\ RH^{meas},\ \alpha_1,\ \ldots,\ \alpha_{52}\right) \triangleq \mathbf{C}_{r\alpha} = \begin{pmatrix} 12.96 & 3.51 & 2.33 & -447.09 & 0 & \cdots & 0 \\ 3.35 & 3.05 & 1.89 & -93.58 & 0 & \cdots & 0 \\ -54.16 & 1.73 & -2.27 & 1831.03 & 0 & \cdots & 0 \end{pmatrix}. \tag{20}$$

where the measured correlated parameters are: $\alpha_1 \equiv \mathrm{T}_{db}$, $\alpha_2 \equiv \mathrm{T}_{dp}$, $\alpha_3 \equiv \mathrm{T}_{w,in}$, and $\alpha_4 \equiv P_{atm}$.

The a priori parameter covariance matrix, $\mathbf{C}_{\alpha\alpha}$, has also been already computed in [1], Equation (B1) (see the Appendix of PART I.), and is also reproduced below:

$$\begin{aligned} \mathbf{C}_{\alpha\alpha} &\triangleq \begin{pmatrix} Var(\alpha_1) & Cov(\alpha_1,\alpha_2) & \bullet & Cov(\alpha_1,\alpha_{52}) \\ Cov(\alpha_2,\alpha_1) & Var(\alpha_2) & \bullet & Cov(\alpha_2,\alpha_{52}) \\ \bullet & \bullet & \bullet & \bullet \\ Cov(\alpha_{52},\alpha_1) & \bullet & \bullet & Var(\alpha_{52}) \end{pmatrix} \\ &= \begin{pmatrix} 17.37 & 2.83 & 1.81 & -529.26 & 0 & \bullet & 0 \\ 2.83 & 5.56 & 2.31 & -87.16 & 0 & \bullet & 0 \\ 1.81 & 2.31 & 2.90 & -47.22 & 0 & \bullet & 0 \\ -529.26 & -87.16 & -47.22 & 160597.01 & 0 & \bullet & 0 \\ 0 & 0 & 0 & 0 & 0 & \bullet & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & 0 & 0 & \bullet & 25.81 \end{pmatrix} \end{aligned} \tag{21}$$

The a priori covariance matrix of the computed responses, $\mathbf{C}_{rr}^{comp}$, is obtained by using Equations (15) and (21) together with the sensitivity results presented in Tables 1–4; the final result is given below:

$$\begin{aligned} &\mathbf{C}_{rr}^{comp} \equiv Cov\left(T_a^{(1)}, T_w^{(50)}, RH^{(1)}\right) = \mathbf{S}_{r\alpha}\mathbf{C}_{\alpha\alpha}\mathbf{S}_{r\alpha}^{+} \\ &= \begin{pmatrix} \frac{\partial T_a^{(1)}}{\partial \alpha_1}, \ldots, \frac{\partial T_a^{(1)}}{\partial \alpha_{N\alpha}} \\ \frac{\partial T_w^{(50)}}{\partial \alpha_1}, \ldots, \frac{\partial T_w^{(50)}}{\partial \alpha_{N\alpha}} \\ \frac{\partial RH^{(1)}}{\partial \alpha_1}, \ldots, \frac{\partial RH^{(1)}}{\partial \alpha_{N\alpha}} \end{pmatrix} \begin{pmatrix} Var(\alpha_1) & Cov(\alpha_1,\alpha_2) & \bullet & Cov(\alpha_1,\alpha_{52}) \\ Cov(\alpha_2,\alpha_1) & Var(\alpha_2) & \bullet & Cov(\alpha_2,\alpha_{52}) \\ \bullet & \bullet & \bullet & \bullet \\ Cov(\alpha_{52},\alpha_1) & \bullet & \bullet & Var(\alpha_{52}) \end{pmatrix} \begin{pmatrix} \frac{\partial T_a^{(1)}}{\partial \alpha_1}, \ldots, \frac{\partial T_a^{(1)}}{\partial \alpha_{N\alpha}} \\ \frac{\partial T_w^{(50)}}{\partial \alpha_1}, \ldots, \frac{\partial T_w^{(50)}}{\partial \alpha_{N\alpha}} \\ \frac{\partial RH^{(1)}}{\partial \alpha_1}, \ldots, \frac{\partial RH^{(1)}}{\partial \alpha_{N\alpha}} \end{pmatrix}^{+} \\ &= \begin{pmatrix} 10.87 & 7.19 & -34.81 \\ 7.19 & 7.72 & -13.97 \\ -34.81 & -13.97 & 221.88 \end{pmatrix}. \end{aligned} \tag{22}$$

The a priori covariance matrix, $Cov\left(T_{a,out}^{meas}, T_{w,out}^{meas}, RH_{out}^{meas}\right) \equiv \mathbf{C}_{rr}$, of the measured responses (namely: the outlet air temperature, $T_{a,out}^{meas} \equiv \left[T_a^{(1)}\right]^{measured}$; the outlet water temperature, $T_{w,out}^{meas} \equiv \left[T_w^{(50)}\right]^{measured}$, and the outlet air relative humidity, $RH_{out}^{meas} \equiv \left[RH^{(1)}\right]^{measured}$ was also computed in [1], Equation (A4), and is reproduced below:

$$Cov\left(T_{a,out}^{meas},\ T_{w,out}^{meas},\ RH_{out}^{meas}\right) \triangleq \mathbf{C}_{rr} = \begin{pmatrix} 11.29 & 3.55 & -43.85 \\ 3.55 & 2.53 & -5.31 \\ -43.85 & -5.31 & 252.49 \end{pmatrix}. \tag{23}$$

2.2.1. Model Calibration: Predicted Best-Estimated Parameter Values with Reduced Predicted Standard Deviations

The best-estimate nominal parameter values have been computed using Equation (12) in conjunction with the a priori matrices given in Equations (20)–(23) and the sensitivities presented in Tables 1–4. The resulting best-estimate nominal values are listed in Table 5, below. The corresponding best-estimate absolute standard deviations for these parameters are also presented in this table. These values are the square-roots of the diagonal elements of the matrix $\mathbf{C}_{\alpha\alpha}^{pred}$, which is computed using Equation (16) in conjunction with the a priori matrices given in Equations (20)–(23) and the sensitivities presented in Tables 1–4. For comparison, the original nominal parameter values and original absolute standard deviations are also listed. As the results in Table 5 indicate, the predicted best-estimate standard deviations are all smaller or at most equal to (i.e., left unaffected) the original standard deviations. The parameters are affected proportionally to the magnitudes of their corresponding sensitivities: the parameters experiencing the largest reductions in their predicted standard deviations are those having the largest sensitivities.

Table 5. Best-estimated nominal parameter values and their standard deviations.

i	Independent Scalar Parameters (α_i)	Math. Notation	Original Nominal Value	Original Standard Deviation	Best-Estimated Nominal Value	Best-Estimated Standard Deviation
1	Air temperature (dry bulb), (K)	T_{db}	299.11	4.17	299.37	3.44
2	Dew point temperature (K)	T_{dp}	292.05	2.36	292.23	2.28
3	Inlet water temperature (K)	$T_{w,in}$	298.79	1.70	298.77	1.70
4	Atmospheric pressure (Pa)	P_{atm}	100,586	401	100,576	389
5	Wetted fraction of fill surface area	w_{tsa}	1	0	1	0
6	Sum of loss coefficients above fill	k_{sum}	10	5	10	5
7	Dynamic viscosity of air at T = 300 K (kg/m·s)	μ	1.983×10^{-5}	9.676×10^{-7}	1.984×10^{-5}	9.668×10^{-7}
8	Kinematic viscosity of air at T = 300 K (m^2/s)	v	1.568×10^{-5}	1.895×10^{-6}	1.564×10^{-5}	1.893×10^{-6}
9	Thermal conductivity of air at T = 300 K (W/m·K)	k_{air}	0.02624	1.584×10^{-3}	0.02625	1.583×10^{-3}
10	Heat transfer coefficient multiplier	f_{ht}	1	0.5	1.0316	0.47
11	Mass transfer coefficient multiplier	f_{mt}	1	0.5	0.882	0.41
12	Fill section frictional loss multiplier	f	4	2	4	2.00
13	P_{vs}(T) parameters	a_0	25.5943	0.01	25.5943	0.01
14		a_1	−5229.89	4.4	−5229.92	4.40
15	C_{pa}(T) parameters	$a_{0,cpa}$	1030.5	0.2940	1030.5	0.294
16		$a_{1,cpa}$	−0.19975	0.0020	−0.19975	0.0020
17		$a_{2,cpa}$	3.9734×10^{-4}	3.345×10^{-6}	3.9734×10^{-4}	3.345×10^{-6}
18	D_{av}(T) parameters	$a_{0,dav}$	7.06085×10^{-9}	0	7.06085×10^{-9}	0
19		$a_{1,dav}$	2.65322	0.003	2.65322	0.003
20		$a_{2,dav}$	-6.1681×10^{-3}	2.3×10^{-5}	-6.16806×10^{-3}	2.3×10^{-5}
21		$a_{3,dav}$	6.552659×10^{-6}	3.8×10^{-8}	6.552688×10^{-6}	3.8×10^{-8}
22	h_f(T) parameters	a_{0f}	−1,143,423.8	543	−1,143,423.7	543
23		a_{1f}	4186.50768	1.8	4186.50818	1.8
24	h_g(T) parameters	a_{0g}	2,005,743.99	1046	2,005,743.80	1046
25		a_{1g}	1815.437	3.5	1815.436	3.5
26	Nu parameters	$a_{0,Nu}$	8.235	2.059	8.235	2.059
27		$a_{1,Nu}$	0.00314987	0.001	0.0030475	0.001
28		$a_{2,Nu}$	0.9902987	0.327	0.987827	0.327
29		$a_{3,Nu}$	0.023	0.0088	0.023	0.088
30	Cooling tower deck width in x-dir (m)	W_{dkx}	8.5	0.085	8.5	0.085
31	Cooling tower deck width in y-dir (m)	W_{dky}	8.5	0.085	8.5	0.085
32	Cooling tower deck height above ground (m)	Δz_{dk}	10	0.1	10	0.1
33	Fan shroud height (m)	Δz_{fan}	3.0	0.03	3.0	0.03
34	Fan shroud inner diameter (m)	D_{fan}	4.1	0.041	4.1	0.041
35	Fill section height (m)	Δz_{fill}	2.013	0.02013	2.013	0.02013
36	Rain section height (m)	Δz_{rain}	1.633	0.01633	1.633	0.01633
37	Basin section height (m)	Δz_{bs}	1.168	0.01168	1.168	0.01168
38	Drift eliminator thickness (m)	Δz_{de}	0.1524	0.001524	0.1524	0.001524
39	Fill section equivalent diameter (m)	D_h	0.0381	0.000381	0.0381	0.000381
40	Fill section flow area (m^2)	A_{fill}	67.29	6.729	67.507	6.705
41	Fill section surface area (m^2)	A_{surf}	14,221	3555.3	13914	3463
42	Prandlt number of air at T = 80 C	P_r	0.708	0.005	0.708	0.005
43	Wind speed (m/s)	V_w	1.80	0.92	1.80	0.92
44	Exit air speed at the shroud (m/s)	V_{exit}	10.0	1.0	9.978	1.0
i	**Boundary Parameters**	**Math. Notation**	**Original Nominal Value**	**Original Standard Deviation**	**Best-Estimated Nominal Value**	**Best-Estimated Standard Deviation**
45	Inlet water mass flow rate (kg/s)	$m_{w,in}$	44.02	2.201	44.05	2.199
46	Inlet air temperature (K)	$T_{a,in}$	299.11	4.17	300.14	2.64
47	Inlet air mass flow rate (kg/s)	m_a	155.07	15.91	154.70	15.87
48	Inlet air humidity ratio	ω_{in}	0.0138	0.00206	0.0142	0.00137
i	**Special Dependent Parameters**	**Math. Notation**	**Original Nominal Value**	**Original Standard Deviation**	**Best-Estimated Nominal Value**	**Best-Estimated Standard Deviation**
49	Reynold's number	Re_d	4428	671.6	4395	666.1
50	Schmidt number	Sc	0.60	0.074	0.5986	0.0739
51	Sherwood number	Sh	14.13	4.84	13.35	4.44
52	Nusselt number	Nu	14.94	5.08	14.34	4.83

2.2.2. Predicted Best-Estimated Response Values with Reduced Predicted Standard Deviations

Using the a priori matrices given in Equations (20)–(23) together with the sensitivities presented in Tables 1–4 in Equation (18) yields the following predicted response covariance matrix, $\mathbf{C}_{rr}^{pred}$:

$$\mathbf{C}_{rr}^{pred} \equiv Cov\left(\left[T_a^{(1)}\right]^{be}, \left[T_w^{(50)}\right]^{be}, \left[RH^{(1)}\right]^{be}\right) = \begin{pmatrix} 6.71 & 2.73 & -22.80 \\ 2.73 & 2.37 & -1.79 \\ -22.80 & -1.79 & 145.19 \end{pmatrix}. \quad (24)$$

The best-estimate response-parameter correlation matrix, $\mathbf{C}_{\alpha r}^{pred}$, is obtained using Equation (19) together with the a priori matrices given in Equations (20)–(23) and the sensitivities presented in Tables 1–4. The non-zero elements with the largest magnitudes are as follows:

$$\begin{array}{lll} rel.cor.(R_1, \alpha_4) = -0.278; & rel.cor.(R_1, \alpha_{41}) = -0.070; & rel.cor.(R_1, \alpha_{49}) = -0.039; \\ rel.cor.(R_2, \alpha_4) = -0.108; & rel.cor.(R_2, \alpha_{41}) = -0.019; & \\ rel.cor.(R_3, \alpha_4) = 0.232; & rel.cor.(R_3, \alpha_{41}) = 0.127; & rel.cor.(R_3, \alpha_{49}) = 0.072. \end{array} \quad (25)$$

The notation used in Equation (25) is as follows: $R_1 \equiv T_a^{(1)}$, $R_2 \equiv T_w^{(50)}$, $R_3 \equiv RH^{(1)}$, $\alpha_4 \equiv P_{atm}$, $\alpha_{41} \equiv A_{surf}$ and $\alpha_{49} \equiv \mathrm{Re}_d$.

The best-estimate nominal values of the (model responses) outlet air temperature, $T_a^{(1)}$; outlet water temperature $T_w^{(50)}$; and outlet air relative humidity, $RH^{(1)}$, have been computed using Equation (17) together with the a priori matrices given in Equations (20)–(23) and the sensitivities presented in Tables 1–4. The resulting best-estimate predicted nominal values are summarized in Table 6. To facilitate comparison, the corresponding measured and computed nominal values are also presented in this table. Note that there are no direct measurements for the outlet water flow rate, $m_w^{(50)}$. For this response, therefore, the predicted best-estimate nominal value has been obtained by a forward re-computation using the best-estimate nominal parameter values listed in Table 5, while the predicted best estimate standard deviation for this response has been obtained by using "best-estimate" values in Equation (15), i.e.:

$$\left[\mathbf{C}_{rr}^{comp}\right]^{be} = \left[\mathbf{S}_{r\alpha}\right]^{be}\left[\mathbf{C}_{\alpha\alpha}\right]^{be}\left[\mathbf{S}_{r\alpha}^{+}\right]^{be}. \quad (26)$$

Table 6. Computed, measured, and optimal best-estimate nominal values and standard deviations for the outlet air temperature, outlet water temperature, outlet air relative humidity, and outlet water flow rate responses.

Nominal Values and Standard Deviations	$T_a^{(1)}$ (K)	$T_w^{(50)}$ (K)	$RH^{(1)}$ (%)	$m_w^{(50)}$ (kg/s)
Measured				
Nominal value	298.34	295.68	81.98	—
Standard deviation	±3.36	±1.59	±15.89	—
Computed				
Nominal value	297.46	294.58	86.12	43.60
Standard deviation	±3.30	±2.78	±14.90	±2.21
Best-estimate				
Nominal value	298.45	295.67	82.12	43.67
Standard deviation	±2.59	±1.54	±12.05	±2.20

The results presented in Table 6 indicate that the predicted standard deviations are smaller than either the computed or the experimentally measured ones. This is indeed the consequence of using the PM_CMPS methodology in conjunction with consistent (as opposed to discrepant) computational and experimental information. Often, however, the information is inconsistent, usually due to the presence of unrecognized errors. Solutions for addressing such situations have been proposed in [10].

It is also important to note that the PM_CMPS methodology has improved (i.e., reduced, albeit not by a significant amount) the predicted standard deviation for the outlet water flow rate response, for which no measurements were available. This improvement stems from the global characteristics of the PM_CMPS methodology, which combines all of the available simultaneously on phase-space, as opposed to combining it sequentially, as is the case with the current state-of-the-art data assimilation procedures [11,12].

3. Discussion

In the present work, the adjoint sensitivity model of the counter-flow cooling tower derived in the accompanying PART I [1] was used to obtain the expressions and relative numerical rankings of the sensitivities, to all model parameters, of the following responses (quantities of interest): (i) the outlet air temperature; (ii) the outlet water temperature; (iii) the outlet water mass flow rate; and (iv) the air outlet relative humidity. These sensitivities were subsequently used within the "predictive modeling for coupled multi-physics systems" (PM_CMPS) methodology [2] to obtain *explicit formulas* for the predicted optimal nominal values for the model responses and parameters, along with reduced predicted standard deviations for the predicted model parameters and responses. These explicit formulas embody the assimilation of experimental data and the "calibration" of the model's parameters.

The results presented in this work indicate that the predicted standard deviations are smaller than either the computed or the experimentally measured ones. It is also important to note that the PM_CMPS methodology has improved (i.e., reduced, albeit not by a significant amount) the predicted standard deviation for the outlet water flow rate response, for which no measurements were available. This improvement stems from the global characteristics of the PM_CMPS methodology, which combines all of the available information simultaneously in phase-space, as opposed to combining it sequentially, as is the case with the current state-of-the-art data assimilation procedures [11,12]. This is indeed the consequence of using the PM_CMPS methodology in conjunction with *consistent* (as opposed to discrepant) computational and experimental information. Often, however, the information is inconsistent, usually due to the presence of unrecognized errors. Solutions for addressing such situations have been proposed in [10].

The adjoint sensitivity analysis methodology used in PART I [1] for computing exactly and efficiently the 1^{st}-order response sensitivities to model parameters has been recently extended to computing efficiently and exactly the 2nd-order response sensitivities to parameters for linear [13] and nonlinear [14] large-scale systems. As has been shown in [15–18], the 2nd-order response sensitivities have the following major impacts on the computed moments of the response distribution: (a) they cause the "expected value of the response" to differ from the "computed nominal value of the response"; and (b) they contribute decisively to causing asymmetries in the response distribution. Indeed, neglecting the second-order sensitivities would nullify the third-order response correlations, and hence would nullify the skewness of the response. Consequently, non-Gaussian features (i.e., asymmetries, long-tails) any events occurring in a response's long and/or short tails, which are characteristic of rare but decisive events (e.g., major accidents, catastrophes), would likely be missed. Ongoing work aims at further applications and generalization of the adjoint sensitivity analysis and the PM_CMPS methodologies, to enable the computation of 3rd- and higher-order sensitivities and response distributions. The exact and efficient computation of high-order response sensitivities for large-scale systems is expected to advance significantly the areas of uncertainty quantification, model validation, reduced-order modeling, and predictive modeling/data assimilation.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A. Derivatives of Cooling Tower Model Equations with Respect To Model Parameters

For convenience, the model parameters are reproduced in Table A1 below from Appendix B of PART I [1]. The independent model parameters are used for computing various dependent model parameters and thermal material properties, as shown in Tables A2 and A3, below.

Table A1. Parameters for SRNL f-area cooling towers.

Index i of α_i	Independent Scalar Parameters	C + + String	Math. Notation	Nominal Value(s)	Absolute Standard Deviation	Relative Standard Deviation (%)
1	Air temperature (dry bulb) (K)	tdb	T_{db}	299.11	4.17	1.39
2	Dew point temperature (K)	tdp	T_{dp}	292.05	2.36	0.81
3	Inlet water temperature (K)	twin	$T_{w,in}$	298.79	1.70	0.57
4	Atmospheric pressure (Pa)	patm	P_{atm}	100586	401	0.40
5	Wetted fraction of fill surface area	wtsa	w_{tsa}	1	0	0
6	Sum of loss coefficients above fill	ksum	k_{sum}	10	5	50
7	Dynamic viscosity of air at T = 300 K (kg/m·s)	muair	μ	1.983×10^{-5}	9.676×10^{-7}	4.88
8	Kinematic viscosity of air at T = 300 K (m^2/s)	nuair	v	1.568×10^{-5}	1.895×10^{-6}	12.09
9	Thermal conductivity of air at T = 300 K (W/m·K)	tcair	k_{air}	0.02624	1.584×10^{-3}	6.04
10	Heat transfer coefficient multiplier	mlthtc	f_{ht}	1	0.5	50
11	Mass transfer coefficient multiplier	mltmtc	f_{mt}	1	0.5	50
12	Fill section frictional loss multiplier	mltfil	f	4	2	50
13	P_{vs}(T) parameters	a0	a_0	25.5943	0.01	0.04
14		a1	a_1	−5229.89	4.4	0.08
15	C_{pa}(T) parameters	A(1)	$a_{0,cpa}$	1030.5	0.2940	0.03
16		A(2)	$a_{1,cpa}$	−0.19975	0.0020	1.00
17		A(3)	$a_{2,cpa}$	3.9734×10^{-4}	3.345×10^{-6}	0.84
18	D_{av}(T) parameters	A(1)	$a_{0,dav}$	7.06085×10^{-9}	0	0
19		A(2)	$a_{1,dav}$	2.65322	0.003	0.11
20		A(3)	$a_{2,dav}$	-6.1681×10^{-3}	2.3×10^{-5}	0.37
21		A(4)	$a_{3,dav}$	6.55266×10^{-6}	3.8×10^{-8}	0.58
22	h_f(T) parameters	a0f	a_{0f}	−1,143,423.78	543.	0.05
23		a1f	a_{1f}	4186.50768	1.8	0.04
24	h_g(T) parameters	a0g	a_{0g}	2,005,743.99	1046	0.05
25		a1g	a_{1g}	1815.437	3.5	0.19
26	Nu parameters	-	$a_{0,Nu}$	8.235	2.059	25
27		-	$a_{1,Nu}$	0.00314987	0.001	31.75
28		-	$a_{2,Nu}$	0.9902987	0.327	33.02
29		-	$a_{3,Nu}$	0.023	0.0088	38.26
30	Cooling tower deck width in x-dir. (m)	dkxw	W_{dkx}	8.5	0.085	1
31	Cooling tower deck width in y-dir. (m)	dkyw	W_{dky}	8.5	0.085	1
32	Cooling tower deck height above ground (m)	dkht	Δz_{dk}	10	0.1	1
33	Fan shroud height (m)	fsht	Δz_{fan}	3.0	0.03	1
34	Fan shroud inner diameter (m)	fsid	D_{fan}	4.1	0.041	1
35	Fill section height (m)	flht	Δz_{fill}	2.013	0.02013	1
36	Rain section height (m)	rsht	Δz_{rain}	1.633	0.01633	1
37	Basin section height (m)	bsht	Δz_{bs}	1.168	0.01168	1
38	Drift eliminator thickness (m)	detk	Δz_{de}	0.1524	0.001524	1
39	Fill section equivalent diameter (m)	deqv	D_h	0.0381	0.000381	1
40	Fill section flow area (m^2)	flfa	A_{fill}	67.29	6.729	10
41	Fill section surface area (m^2)	flsa	A_{surf}	14221	3555.3	25
42	Prandlt number of air at T = 80 C	Pr	P_r	0.708	0.005	0.71
43	Wind speed (m/s)	wspd	V_w	1.80	0.92	51.1
44	Exit air speed at the shroud (m/s)	vexit	V_{exit}	10.0	1.0	10.0
Index i of α_i	**Boundary Parameters**	**C + + String**	**Math. Notation**	**Nominal Value**	**Absolute Standard Deviation**	**Relative Standard Deviation (%)**
45	Inlet water mass flow rate (kg/s)	mfwin	$m_{w,in}$	44.02	2.201	5
46	Inlet air temperature (K)	tain	$T_{a,in}$	set to T_{db}	4.17	1.39
47	Inlet air mass flow rate (kg/s)	main	m_a	155.07	15.91	10.26
48	Inlet air humidity ratio (Dependent Scalar Parameter)	hrin	ω_{in}; ω_{rain}	0.0138	0.00206	14.93
49	Reynold's number	Re; Reh	Re_d	4428	671.6	15.17
50	Schmidt number	Sc	Sc	0.60	0.074	12.41
51	Sherwood number	Sh	Sh	14.13	4.84	34.25
52	Nusselt number	Nu	Nu	14.94	5.08	34.00

Table A2. Dependent scalar model parameters.

Dependent Scalar Parameters	Math. Notation	Defining Equation or Correlation
Mass diffusivity of water vapor in air (m^2/s)	$D_{av}(T_a,\boldsymbol{\alpha})$	$\frac{a_{0,dav}T^{1.5}}{a_{1,dav}+(a_{2,dav}+a_{3,dav}T)T}$
Heat transfer coefficient ($W/m^2\cdot K$)	$h(\boldsymbol{\alpha})$	$\frac{f_{ht}N_u k_{air}}{D_h}$
Mass transfer coefficient (m/s)	$k_m(\boldsymbol{\alpha})$	$\frac{f_{mt}S_h D_{av}(T_{db},\alpha)}{D_h}$
Heat transfer term (W/K)	$H(m_a,\boldsymbol{\alpha})$	$h\,(\alpha)\,w_{tsa}A_{ff}$
Mass transfer term (m^3/s)	$M(m_a,\boldsymbol{\alpha})$	$M_{H_2O}k_m\,(\alpha)\,w_{tsa}A_{ff}$
Density of dry air (kg/m^3)	$\rho(\boldsymbol{\alpha})$	$\frac{P_{atm}}{R_{air}T_{db}}$
Air velocity in the fill section (m/s)	$v_a(m\mathrm{a},\boldsymbol{\alpha})$	$\frac{\lvert m_a\rvert}{\rho(\alpha)A_{fill}}$
Fill falling-film surface area per vertical section (m^2)	A_{ff}	$\frac{A_{surf}}{I}$
Rain section inlet flow area (m^2)	A_{in}	$W_{dkx}W_{dky}$
Height for natural convection (m)	Z	$z_{dk}+z_{fan}-z_{bs}$
Height above fill section (m)	$\Delta z_{4\text{–}2}$	$Z-z_{fill}-z_{rain}$
Fill section control volume height (m)	Δz	$\frac{z_{fill}}{I}$
Fill section length, including drift eliminator (m)	L_{fill}	$z_{fill}+z_{de}$
Fan shroud inner radius (m)	r_{fan}	$0.5D_{fan}$
Fan shroud flow area (m^2)	A_{out}	$\pi r_{fan}{}^2$

Table A3. Thermal properties (dependent scalar model parameters).

Thermal Properties (Functions of State Variables)	Math. Notation	Defining Equation or Correlation
$h_f(T_w)$ = saturated liquid enthalpy (J/kg)	$h_f(T_w,\boldsymbol{\alpha})$	$a_{0f}+a_{1f}T_w$
$H_g(T_w)$ = saturated vapor enthalpy (J/kg)	$h_{g,w}(T_w,\boldsymbol{\alpha})$	$a_{0g}+a_{1g}T_w$
$H_g(T_a)$ = saturated vapor enthalpy (J/kg)	$h_{g,a}(T_a,\boldsymbol{\alpha})$	$a_{0g}+a_{1g}T_a$
$C_p(T)$ = specific heat of dry air (J/kg·K)	$C_p(T,\boldsymbol{\alpha})$	$a_{0,cpa}+(a_{1,cpa}+a_{2,cpa}T)T$
$P_{vs}(T_w)$ = saturation pressure (Pa)	$P_{vs}(T_w,\boldsymbol{\alpha})$	$P_c\cdot e^{a_0+\frac{a_1}{T_w}}$, in which P_c = 1.0 Pa
$P_{vs}(T_a)$ = saturation pressure (Pa)	$P_{vs}(T_a,\boldsymbol{\alpha})$	$P_c\cdot e^{a_0+\frac{a_1}{T_a}}$, in which P_c = 1.0 Pa

Note: The parameters α_1 through α_4 (i.e., the dry bulb air temperature, dew point temperature, inlet water temperature, and atmospheric pressure) were measured at the SRNL site at which the F-area cooling towers are located. Among the 8079 measured benchmark data sets [8], 7688 data sets are considered to represent "unsaturated conditions", which have been used to derive the statistical properties (means, variance and covariance, skewness and kurtosis) for these model parameters, as shown in Figures B1 through B4 and Tables B4 through B7 in Appendix B of PART I [1].

Recall that the cooling tower model comprises conservation balances representing mathematically the following physical phenomena: A. liquid continuity; B. liquid energy balance; C. water vapor continuity; D. air and water vapor energy balance. For easy reference, these conservation equations are reproduced below from Section 2 of PART I [1]:

A. Liquid continuity equations:

(i) *Control Volume i = 1:*

$$N_1^{(1)}\left(\mathbf{m}_w,\mathbf{T}_w,\mathbf{T}_a,\boldsymbol{\omega};\boldsymbol{\alpha}\right)\triangleq m_w^{(2)}-m_{w,in}+\frac{M(m_a,\boldsymbol{\alpha})}{\overline{R}}\left[\frac{P_{vs}^{(2)}(T_w^{(2)},\boldsymbol{\alpha})}{T_w^{(2)}}-\frac{\omega^{(1)}P_{atm}}{T_a^{(1)}(0.622+\omega^{(1)})}\right]=0;\tag{A1}$$

(ii) *Control Volumes i = 2,..., I − 1:*

$$N_1^{(i)}\left(\mathbf{m}_w,\mathbf{T}_w,\mathbf{T}_a,\boldsymbol{\omega};\boldsymbol{\alpha}\right)\triangleq m_w^{(i+1)}-m_w^{(i)}+\frac{M(m_a,\boldsymbol{\alpha})}{\overline{R}}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{T_w^{(i+1)}}-\frac{\omega^{(i)}P_{atm}}{T_a^{(i)}(0.622+\omega^{(i)})}\right]=0;\tag{A2}$$

(iii) *Control Volume i = I:*

$$N_1^{(I)}\left(\mathbf{m}_w,\mathbf{T}_w,\mathbf{T}_a,\boldsymbol{\omega};\boldsymbol{\alpha}\right)\triangleq m_w^{(I+1)}-m_w^{(I)}+\frac{M(m_a,\boldsymbol{\alpha})}{\overline{R}}\left[\frac{P_{vs}^{(I+1)}(T_w^{(I+1)},\boldsymbol{\alpha})}{T_w^{(I+1)}}-\frac{\omega^{(I)}P_{atm}}{T_a^{(I)}(0.622+\omega^{(I)})}\right]=0;\tag{A3}$$

B. Liquid energy balance equations:

(i) *Control Volume i = 1:*

$$N_2^{(1)}\left(\mathbf{m}_w,\mathbf{T}_w,\mathbf{T}_a,\boldsymbol{\omega};\boldsymbol{\alpha}\right) \triangleq m_{w,in}h_f(T_{w,in},\boldsymbol{\alpha}) - (T_w^{(2)} - T_a^{(1)})H(m_a,\boldsymbol{\alpha}) \\ - m_w^{(2)}h_f^{(2)}(T_w^{(2)},\boldsymbol{\alpha}) - (m_{w,in} - m_w^{(2)})h_{g,w}^{(2)}(T_w^{(2)},\boldsymbol{\alpha}) = 0; \tag{A4}$$

(ii) *Control Volumes i = 2,..., I − 1:*

$$N_2^{(i)}\left(\mathbf{m}_w,\mathbf{T}_w,\mathbf{T}_a,\boldsymbol{\omega};\boldsymbol{\alpha}\right) \triangleq m_w^{(i)}h_f^{(i)}(T_w^{(i)},\boldsymbol{\alpha}) - (T_w^{(i+1)} - T_a^{(i)})H(m_a,\boldsymbol{\alpha}) \\ - m_w^{(i+1)}h_f^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha}) - (m_w^{(i)} - m_w^{(i+1)})h_{g,w}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha}) = 0; \tag{A5}$$

(iii) *Control Volume i = I:*

$$N_2^{(I)}\left(\mathbf{m}_w,\mathbf{T}_w,\mathbf{T}_a,\boldsymbol{\omega};\boldsymbol{\alpha}\right) \triangleq m_w^{(I)}h_f^{(I)}(T_w^{(I)},\boldsymbol{\alpha}) - (T_w^{(I+1)} - T_a^{(I)})H(m_a,\boldsymbol{\alpha}) \\ - m_w^{(I+1)}h_f^{(I+1)}(T_w^{(I+1)},\boldsymbol{\alpha}) - (m_w^{(I)} - m_w^{(I+1)})h_{g,w}^{(I+1)}(T_w^{(I+1)},\boldsymbol{\alpha}) = 0; \tag{A6}$$

C. Water vapor continuity equations:

(i) *Control Volume i = 1:*

$$N_3^{(1)}\left(\mathbf{m}_w,\mathbf{T}_w,\mathbf{T}_a,\boldsymbol{\omega};\boldsymbol{\alpha}\right) \triangleq \omega^{(2)} - \omega^{(1)} + \frac{m_{w.in} - m_w^{(2)}}{|m_a|} = 0; \tag{A7}$$

(ii) *Control Volumes i = 2,..., I − 1:*

$$N_3^{(i)}\left(\mathbf{m}_w,\mathbf{T}_w,\mathbf{T}_a,\boldsymbol{\omega};\boldsymbol{\alpha}\right) \triangleq \omega^{(i+1)} - \omega^{(i)} + \frac{m_w^{(i)} - m_w^{(i+1)}}{|m_a|} = 0; \tag{A8}$$

(iii) *Control Volume i = I:*

$$N_3^{(I)}\left(\mathbf{m}_w,\mathbf{T}_w,\mathbf{T}_a,\boldsymbol{\omega};\boldsymbol{\alpha}\right) \triangleq \omega_{in} - \omega^{(I)} + \frac{m_w^{(I)} - m_w^{(I+1)}}{|m_a|} = 0; \tag{A9}$$

D. The air/water vapor energy balance equations:

(i) *Control Volume i = 1:*

$$N_4^{(1)}\left(\mathbf{m}_w,\mathbf{T}_w,\mathbf{T}_a,\boldsymbol{\omega};\boldsymbol{\alpha}\right) \triangleq (T_a^{(2)} - T_a^{(1)})C_p^{(1)}\left(\tfrac{T_a^{(1)}+273.15}{2},\boldsymbol{\alpha}\right) - \omega^{(1)}h_{g,a}^{(1)}(T_a^{(1)},\boldsymbol{\alpha}) \\ + \tfrac{(T_w^{(2)}-T_a^{(1)})H(m_a,\boldsymbol{\alpha})}{|m_a|} + \tfrac{(m_{w,in}-m_w^{(2)})h_{g,w}^{(2)}(T_w^{(2)},\boldsymbol{\alpha})}{|m_a|} + \omega^{(2)}h_{g,a}^{(2)}(T_a^{(2)},\boldsymbol{\alpha}) = 0; \tag{A10}$$

(ii) *Control Volumes i = 2,..., I − 1:*

$$N_4^{(i)}\left(\mathbf{m}_w,\mathbf{T}_w,\mathbf{T}_a,\boldsymbol{\omega};\boldsymbol{\alpha}\right) \triangleq (T_a^{(i+1)} - T_a^{(i)})C_p^{(i)}\left(\tfrac{T_a^{(i)}+273.15}{2},\boldsymbol{\alpha}\right) - \omega^{(i)}h_{g,a}^{(i)}(T_a^{(i)},\boldsymbol{\alpha}) \\ + \tfrac{(T_w^{(i+1)}-T_a^{(i)})H(m_a,\boldsymbol{\alpha})}{|m_a|} + \tfrac{(m_w^{(i)}-m_w^{(i+1)})h_{g,w}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{|m_a|} + \omega^{(i+1)}h_{g,a}^{(i+1)}(T_a^{(i+1)},\boldsymbol{\alpha}) = 0; \tag{A11}$$

(iii) *Control Volume i = I:*

$$N_4^{(I)}\left(\mathbf{m}_w,\mathbf{T}_w,\mathbf{T}_a,\boldsymbol{\omega};\boldsymbol{\alpha}\right) \triangleq (T_{a,in} - T_a^{(I)})C_p{}^{(I)}\left(\tfrac{T_a^{(I)}+273.15}{2},\boldsymbol{\alpha}\right) - \omega^{(I)}h_{g,a}^{(I)}(T_a^{(I)},\boldsymbol{\alpha}) \\ + \tfrac{(T_w^{(I+1)}-T_a^{(I)})H(m_a,\boldsymbol{\alpha})}{|m_a|} + \tfrac{(m_w^{(I)}-m_w^{(I+1)})h_{g,w}^{(I+1)}(T_w^{(I+1)},\boldsymbol{\alpha})}{|m_a|} + \omega_{in}h_{g,a}(T_{a,in},\boldsymbol{\alpha}) = 0. \tag{A12}$$

The components of the vector $\boldsymbol{\alpha}$, which appears in Equations (A1)–(A12), comprise the model parameters, i.e.:

$$\boldsymbol{\alpha} \triangleq (\alpha_1, \ldots, \alpha_{N_\alpha}) \tag{A13}$$

where N_α denotes the total number of model parameters. These model parameters are described in Table A1.

The following notation will be used for the derivatives of the above equations with respect to the parameters:

$$a_\ell^{i,j} \equiv \frac{\partial N_\ell^{(i)}}{\partial \alpha^{(j)}};\ \ell = 1,\ 2,\ 3,\ 4;\ i = 1, \ldots, I;\ j = 1, \ldots, N_\alpha. \tag{A14}$$

A1. Derivatives of the Liquid Continuity Equations with Respect to the Parameters

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(1)} \equiv T_{db}$ are as follows:

$$\begin{gathered}\frac{\partial N_1^{(i)}}{\partial \alpha^{(1)}} = \frac{\partial N_1^{(i)}}{\partial T_{db}} \equiv a_1^{i,1} = \frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{T_w^{(i+1)}} - \frac{\omega^{(i)} P_{atm}}{\left(0.622+\omega^{(i)}\right)T_a^{(i)}}\right]\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial D_{av}(T_{db},\boldsymbol{\alpha})}\cdot\frac{\partial D_{av}(T_{db},\boldsymbol{\alpha})}{\partial T_{db}}; \\ \ell = 1;\ i = 1, \ldots, I;\ j = 1,\end{gathered} \tag{A15}$$

where:

$$\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial D_{av}(T_{db},\boldsymbol{\alpha})} = \frac{2}{3}\cdot\frac{M(m_a,\boldsymbol{\alpha})}{D_{av}(T_{db},\boldsymbol{\alpha})} \tag{A16}$$

$$\frac{\partial D_{av}(T_{db},\boldsymbol{\alpha})}{\partial T_{db}} = \frac{1.5\cdot a_{0dav}{T_{db}}^{0.5} - D_{av}(T_{db},\boldsymbol{\alpha})\cdot(a_{2dav} + 2\cdot a_{3dav}T_{db})}{a_{1dav} + a_{2dav}T_{db} + a_{3dav}{T_{db}}^2} \tag{A17}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(2)} \equiv T_{dp}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(2)}} = \frac{\partial N_1^{(i)}}{\partial T_{dp}} \equiv a_1^{i,2} = 0;\quad \ell = 1;\ i = 1, \ldots, I;\ j = 2. \tag{A18}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(3)} \equiv T_{w,in}$ are as follows:

$$\frac{\partial N_1^{(1)}}{\partial \alpha^{(3)}} = \frac{\partial N_1^{(1)}}{\partial T_{w,in}} \equiv a_1^{1,3} = -\frac{\partial m_{w,in}}{\partial T_{w,in}};\quad \ell = 1;\ i = 1;\ j = 3, \tag{A19}$$

where:

$$\begin{aligned}\frac{\partial m_{w,in}}{\partial T_{w,in}} &= \frac{\partial}{\partial T_{w,in}}\left[\rho\left(T_{w,in}\right)\cdot\frac{700.0}{15850.32}\right] \\ &= \left[a_{2,\rho} + 2\cdot a_{3,\rho}\left(T_{w,in} - 273.15\right) + 3\cdot a_{4,\rho}\left(T_{w,in} - 273.15\right)^2\right]\cdot\frac{700.0}{15850.32};\end{aligned} \tag{A20}$$

and where $a_{2,\rho} = -0.26847207$; $a_{3,\rho} = -1.8113691 \times 10^{-3}$; $a_{4,\rho} = -1.7041217 \times 10^{-6}$.

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(3)}} = \frac{\partial N_1^{(i)}}{\partial T_{w,in}} \equiv a_1^{i,3} = 0;\quad \ell = 1;\ i = 2, \ldots, I;\ j = 3. \tag{A21}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(4)} \equiv P_{atm}$ are as follows:

$$\begin{aligned}\frac{\partial N_1^{(i)}}{\partial \alpha^{(4)}} &= \frac{\partial N_1^{(i)}}{\partial P_{atm}} \equiv a_1^{i,4} = -\frac{M(m_a,\boldsymbol{\alpha})}{R}\frac{\omega^{(i)}}{T_a^{(i)}(0.622+\omega^{(i)})} \\ &\quad + \frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{T_w^{(i+1)}} - \frac{\omega^{(i)}P_{atm}}{\left(0.622+\omega^{(i)}\right)T_a^{(i)}}\right]\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial m_a}\frac{\partial m_a}{\partial P_{atm}}; \\ &\quad \ell = 1;\ i = 1, \ldots, I;\ j = 4,\end{aligned} \tag{A22}$$

where:

$$\frac{\partial M(\mathrm{Re}, \boldsymbol{\alpha})}{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})} = \frac{M(m_a, \boldsymbol{\alpha})}{Nu(\mathrm{Re}, \boldsymbol{\alpha})}, \tag{A23}$$

$$\frac{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})}{\partial m_a} = \begin{cases} 0 & \mathrm{Re}_d < 2300 \\ a_{1,Nu}\,\mathrm{Re}(m_a, \boldsymbol{\alpha})/m_a & 2300 \le \mathrm{Re}_d \le 10000 \\ 0.8Nu(\mathrm{Re}, \boldsymbol{\alpha})/m_a & \mathrm{Re}_d > 10000 \end{cases}, \tag{A24}$$

$$\frac{\partial m_a}{\partial P_{atm}} = \frac{1}{R_{air} T_{a,in}} \cdot V_{exit} \cdot \frac{\pi {D_{fan}}^2}{4}; \tag{A25}$$

Note: The term on the right hand side of Equation (A25) stems from the following relation:

$$m_a = \rho\,(T_a) \cdot V_{exit} \cdot \frac{\pi {D_{fan}}^2}{4} = \frac{P_{atm}}{R_{air} T_a} \cdot V_{exit} \cdot \frac{\pi {D_{fan}}^2}{4}. \tag{A26}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(5)} \equiv w_{tsa}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(5)}} = \frac{\partial N_1^{(i)}}{\partial w_{tsa}} \equiv a_1^{i,5} = \frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)}, \boldsymbol{\alpha})}{T_w^{(i+1)}} - \frac{\omega^{(i)} P_{atm}}{\left(0.622 + \omega^{(i)}\right) T_a^{(i)}}\right] \frac{\partial M(m_a, \boldsymbol{\alpha})}{\partial w_{tsa}}; \quad \ell = 1;\ i = 1, \ldots, I;\ j = 5, \tag{A27}$$

where:

$$\frac{\partial M(m_a, \boldsymbol{\alpha})}{\partial w_{tsa}} = \frac{M_{H2O} f_{mt} Nu(\mathrm{Re}, \boldsymbol{\alpha}) \left(\frac{\nu}{\mathrm{Pr}}\right)^{\frac{1}{3}} \left[D_{av}(T_{db}, \boldsymbol{\alpha})\right]^{\frac{2}{3}} A_{surf}}{\mathrm{D}_h I} \tag{A28}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(6)} \equiv k_{sum}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(6)}} = \frac{\partial N_1^{(i)}}{\partial k_{sum}} \equiv a_1^{i,6} = 0; \quad \ell = 1;\ i = 1, \ldots, I;\ j = 6. \tag{A29}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(7)} \equiv \mu$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(7)}} = \frac{\partial N_1^{(i)}}{\partial \mu} \equiv a_1^{i,7} = \frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)}, \boldsymbol{\alpha})}{T_w^{(i+1)}} - \frac{\omega^{(i)} P_{atm}}{\left(0.622 + \omega^{(i)}\right) T_a^{(i)}}\right] \frac{\partial M(m_a, \boldsymbol{\alpha})}{\partial \mu}; \quad \ell = 1;\ i = 1, \ldots, I;\ j = 7, \tag{A30}$$

where:

$$\frac{\partial M(m_a, \boldsymbol{\alpha})}{\partial \mu} = \begin{cases} 0 & \mathrm{Re}_d < 2300 \\ -\frac{a_{1,Nu} \cdot M(m_a, \boldsymbol{\alpha}) \cdot \mathrm{Re}(m_a, \boldsymbol{\alpha})}{Nu(\mathrm{Re}, \boldsymbol{\alpha}) \cdot \mu} & 2300 \le \mathrm{Re}_d \le 10000 \\ -0.8 \cdot \frac{M(m_a, \boldsymbol{\alpha})}{\mu} & \mathrm{Re}_d > 10000 \end{cases}. \tag{A31}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(8)} \equiv v$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(8)}} = \frac{\partial N_1^{(i)}}{\partial v} \equiv a_1^{i,8} = \frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)}, \boldsymbol{\alpha})}{T_w^{(i+1)}} - \frac{\omega^{(i)} P_{atm}}{\left(0.622 + \omega^{(i)}\right) T_a^{(i)}}\right] \frac{\partial M(m_a, \boldsymbol{\alpha})}{\partial v}; \quad \ell = 1;\ i = 1, \ldots, I;\ j = 8, \tag{A32}$$

where:

$$\frac{\partial M(m_a, \boldsymbol{\alpha})}{\partial v} = \frac{1}{3} \frac{M(m_a, \boldsymbol{\alpha})}{v}. \tag{A33}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(9)} \equiv k_{air}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(9)}} = \frac{\partial N_1^{(i)}}{\partial k_{air}} \equiv a_1^{i,9} = 0; \quad \ell = 1;\ i = 1, \ldots, I;\ j = 9. \tag{A34}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(10)} \equiv f_{ht}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(10)}} = \frac{\partial N_1^{(i)}}{\partial f_{ht}} \equiv a_1^{i,10} = 0; \quad \ell = 1;\ i = 1, \ldots, I;\ j = 10. \tag{A35}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(11)} \equiv f_{mt}$ are as follows:

$$\begin{gathered}\frac{\partial N_1^{(i)}}{\partial \alpha^{(11)}} = \frac{\partial N_1^{(i)}}{\partial f_{mt}} \equiv a_1^{i,11} = \frac{1}{\overline{R}}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{T_w^{(i+1)}} - \frac{\omega^{(i)} P_{atm}}{\left(0.622+\omega^{(i)}\right)T_a^{(i)}}\right]\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial f_{mt}}; \\ \ell = 1;\ i = 1, \ldots, I;\ j = 11,\end{gathered} \tag{A36}$$

where:

$$\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial f_{mt}} = \frac{M_{H2O} Nu(\mathrm{Re},\boldsymbol{\alpha}) \left(\frac{\nu}{\mathrm{Pr}}\right)^{\frac{1}{3}} \left[D_{av}(T_{db},\boldsymbol{\alpha})\right]^{\frac{2}{3}} w_{tsa} A_{surf}}{\mathrm{D}_h I}. \tag{A37}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(12)} \equiv f$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(12)}} = \frac{\partial N_1^{(i)}}{\partial f} \equiv a_1^{i,12} = 0; \quad \ell = 1;\ i = 1, \ldots, I;\ j = 12. \tag{A38}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(13)} \equiv a_0$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(13)}} = \frac{\partial N_1^{(i)}}{\partial a_0} \equiv a_1^{i,13} = \frac{M(m_a,\boldsymbol{\alpha})}{\overline{R}} \frac{1}{T_w^{(i+1)}} \frac{\partial P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{\partial a_0}; \quad \ell = 1;\ i = 1, \ldots, I;\ j = 13, \tag{A39}$$

where:

$$\frac{\partial P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{\partial a_0} = P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha}). \tag{A40}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(14)} \equiv a_1$ are as follows:

$$\begin{gathered}\frac{\partial N_1^{(i)}}{\partial \alpha^{(14)}} = \frac{\partial N_1^{(i)}}{\partial a_1} \equiv a_1^{i,14} = \frac{M(m_a,\boldsymbol{\alpha})}{\overline{R}} \frac{1}{T_w^{(i+1)}} \frac{\partial P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{\partial a_1}; \\ \ell = 1;\ i = 1, \ldots, I;\ j = 14,\end{gathered} \tag{A41}$$

where:

$$\frac{\partial P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{\partial a_1} = \frac{P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{T_w^{(i+1)}}. \tag{A42}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(15)} \equiv a_{0,cpa}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(15)}} = \frac{\partial N_1^{(i)}}{\partial a_{0,cpa}} \equiv a_1^{i,15} = 0; \quad \ell = 1;\ i = 1, \ldots, I;\ j = 15. \tag{A43}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(16)} \equiv a_{1,cpa}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(16)}} = \frac{\partial N_1^{(i)}}{\partial a_{1,cpa}} \equiv a_1^{i,16} = 0; \quad \ell = 1;\ i = 1,\ldots,I;\ j = 16. \tag{A44}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(17)} \equiv a_{2,cpa}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(17)}} = \frac{\partial N_1^{(i)}}{\partial a_{2,cpa}} \equiv a_1^{i,17} = 0; \quad \ell = 1;\ i = 1,\ldots,I;\ j = 17. \tag{A45}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\boldsymbol{\alpha^{(18)}} \equiv \boldsymbol{a_{0,dav}}$ are as follows:

$$\begin{gathered}\frac{\partial N_1^{(i)}}{\partial \alpha^{(18)}} = \frac{\partial N_1^{(i)}}{\partial a_{0,dav}} \equiv a_1^{i,18} = \frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{T_w^{(i+1)}} - \frac{\omega^{(i)}P_{atm}}{\left(0.622+\omega^{(i)}\right)T_a^{(i)}}\right]\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial D_{av}(T_{db},\boldsymbol{\alpha})}\cdot\frac{\partial D_{av}(T_{db},\boldsymbol{\alpha})}{\partial a_{0,dav}}; \\ \ell = 1;\ i = 1,\ldots,I;\ j = 18,\end{gathered} \tag{A46}$$

where $\frac{\partial M(m_a,\alpha)}{\partial D_{av}(T_{db},\alpha)}$ was defined previously in Equation (A16), and:

$$\frac{\partial D_{av}(T_{db},\boldsymbol{\alpha})}{\partial a_{0,dav}} = \frac{{T_{db}}^{1.5}}{a_{1dav} + a_{2dav}T_{db} + a_{3dav}{T_{db}}^2}. \tag{A47}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(19)} \equiv a_{1,dav}$ are as follows:

$$\begin{gathered}\frac{\partial N_1^{(i)}}{\partial \alpha^{(19)}} = \frac{\partial N_1^{(i)}}{\partial a_{1,dav}} \equiv a_1^{i,19} = \frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{T_w^{(i+1)}} - \frac{\omega^{(i)}P_{atm}}{\left(0.622+\omega^{(i)}\right)T_a^{(i)}}\right]\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial D_{av}(T_{db},\boldsymbol{\alpha})}\cdot\frac{\partial D_{av}(T_{db},\boldsymbol{\alpha})}{\partial a_{1,dav}}; \\ \ell = 1;\ i = 1,\ldots,I;\ j = 19,\end{gathered} \tag{A48}$$

where $\frac{\partial M(m_a,\alpha)}{\partial D_{av}(T_{db},\alpha)}$ was defined previously in Equation (A16), and:

$$\frac{\partial D_{av}(T_{db},\boldsymbol{\alpha})}{\partial a_{1,dav}} = -\frac{a_{0dav}{T_{db}}^{1.5}}{\left(a_{1dav} + a_{2dav}T_{db} + a_{3dav}{T_{db}}^2\right)^2}. \tag{A49}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(20)} \equiv a_{2,dav}$ are as follows:

$$\begin{gathered}\frac{\partial N_1^{(i)}}{\partial \alpha^{(20)}} = \frac{\partial N_1^{(i)}}{\partial a_{2,dav}} \equiv a_1^{i,20} = \frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{T_w^{(i+1)}} - \frac{\omega^{(i)}P_{atm}}{\left(0.622+\omega^{(i)}\right)T_a^{(i)}}\right]\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial D_{av}(T_{db},\boldsymbol{\alpha})}\cdot\frac{\partial D_{av}(T_{db},\boldsymbol{\alpha})}{\partial a_{2,dav}}; \\ \ell = 1;\ i = 1,\ldots,I;\ j = 20,\end{gathered} \tag{A50}$$

where $\frac{\partial M(m_a,\alpha)}{\partial D_{av}(T_{db},\alpha)}$ was defined previously in Equation (A16), and

$$\frac{\partial D_{av}(T_{db},\boldsymbol{\alpha})}{\partial a_{2,dav}} = -\frac{a_{0dav}{T_{db}}^{2.5}}{\left(a_{1dav} + a_{2dav}T_{db} + a_{3dav}{T_{db}}^2\right)^2}. \tag{A51}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(21)} \equiv a_{3,dav}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(21)}} = \frac{\partial N_1^{(i)}}{\partial a_{3,dav}} \equiv a_1^{i,21} = \frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{T_w^{(i+1)}} - \frac{\omega^{(i)}P_{atm}}{\left(0.622+\omega^{(i)}\right)T_a^{(i)}}\right]\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial D_{av}(T_{db},\boldsymbol{\alpha})}\cdot\frac{\partial D_{av}(T_{db},\boldsymbol{\alpha})}{\partial a_{3,dav}}; \quad \ell = 1;\ i = 1,\ldots,I;\ j = 21, \tag{A52}$$

where $\frac{\partial M(m_a,\alpha)}{\partial D_{av}(T_{db},\alpha)}$ was defined previously in Equation (A16), and

$$\frac{\partial D_{av}(T_{db},\boldsymbol{\alpha})}{\partial a_{3,dav}} = \frac{a_{0dav}{T_{db}}^{3.5}}{\left(a_{1dav} + a_{2dav}T_{db} + a_{3dav}{T_{db}}^2\right)^2}. \tag{A53}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(22)} \equiv a_{0f}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(22)}} = \frac{\partial N_1^{(i)}}{\partial a_{0f}} \equiv a_1^{i,22} = 0; \quad \ell = 1;\ i = 1,\ldots,I;\ j = 22. \tag{A54}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(23)} \equiv a_{1f}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(23)}} = \frac{\partial N_1^{(i)}}{\partial a_{1f}} \equiv a_1^{i,23} = 0; \quad \ell = 1;\ i = 1,\ldots,I;\ j = 23. \tag{A55}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(24)} \equiv a_{0g}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(24)}} = \frac{\partial N_1^{(i)}}{\partial a_{0g}} \equiv a_1^{i,24} = 0; \quad \ell = 1;\ i = 1,\ldots,I;\ j = 24. \tag{A56}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(25)} \equiv a_{1g}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(25)}} = \frac{\partial N_1^{(i)}}{\partial a_{1g}} \equiv a_1^{i,25} = 0; \quad \ell = 1;\ i = 1,\ldots,I;\ j = 25. \tag{A57}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(26)} \equiv a_{0,Nu}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(26)}} = \frac{\partial N_1^{(i)}}{\partial a_{0,Nu}} \equiv a_1^{i,26} = \frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{T_w^{(i+1)}} - \frac{\omega^{(i)}P_{atm}}{\left(0.622+\omega^{(i)}\right)T_a^{(i)}}\right]\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial a_{0,Nu}}; \quad \ell = 1;\ i = 1,\ldots,I;\ j = 26, \tag{A58}$$

where $\frac{\partial M(m_a,\alpha)}{\partial Nu(\mathrm{Re},\alpha)}$ was defined previously in Equation (A23), and

$$\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial a_{0,Nu}} = \begin{cases} 1 & \mathrm{Re}_d < 2300 \\ 0 & 2300 \le \mathrm{Re}_d \le 10000 \\ 0 & \mathrm{Re}_d > 10000 \end{cases}. \tag{A59}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(27)} \equiv a_{1,Nu}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(27)}} = \frac{\partial N_1^{(i)}}{\partial a_{1,Nu}} \equiv a_1^{i,27} = \frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{T_w^{(i+1)}} - \frac{\omega^{(i)} P_{atm}}{\left(0.622+\omega^{(i)}\right)T_a^{(i)}}\right]\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial a_{1,Nu}}; \quad \ell = 1;\ i = 1,\ldots,I;\ j = 27, \tag{A60}$$

where $\frac{\partial M(m_a,\alpha)}{\partial Nu(\mathrm{Re},\alpha)}$ was defined previously in Equation (A23), and:

$$\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial a_{1,Nu}} = \begin{cases} 0 & \mathrm{Re}_d < 2300 \\ \mathrm{Re}(m_a,\boldsymbol{\alpha}) & 2300 \le \mathrm{Re}_d \le 10000 \\ 0 & \mathrm{Re}_d > 10000 \end{cases}. \tag{A61}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(28)} \equiv a_{2,Nu}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(28)}} = \frac{\partial N_1^{(i)}}{\partial a_{2,Nu}} \equiv a_1^{i,28} = \frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{T_w^{(i+1)}} - \frac{\omega^{(i)} P_{atm}}{\left(0.622+\omega^{(i)}\right)T_a^{(i)}}\right]\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial a_{2,Nu}}; \quad \ell = 1;\ i = 1,\ldots,I;\ j = 28, \tag{A62}$$

where $\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}$ was defined previously in Equation (A23), and:

$$\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial a_{2,Nu}} = \begin{cases} 0 & \mathrm{Re}_d < 2300 \\ 1 & 2300 \le \mathrm{Re}_d \le 10000 \\ 0 & \mathrm{Re}_d > 10000 \end{cases} \tag{A63}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(29)} \equiv a_{3,Nu}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(29)}} = \frac{\partial N_1^{(i)}}{\partial a_{3,\ Nu}} \equiv a_1^{i,29} = \frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{T_w^{(i+1)}} - \frac{\omega^{(i)} P_{atm}}{\left(0.622+\omega^{(i)}\right)T_a^{(i)}}\right]\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial a_{3,Nu}}; \quad \ell = 1;\ i = 1,\ldots,I;\ j = 29, \tag{A64}$$

where $\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}$ was defined previously in Equation (A23), and:

$$\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial a_{3,Nu}} = \begin{cases} 0 & \mathrm{Re}_d < 2300 \\ 0 & 2300 \le \mathrm{Re}_d \le 10000 \\ [\mathrm{Re}(m_a,\boldsymbol{\alpha})]^{0.8} \cdot \mathrm{Pr}^{\frac{1}{3}} & \mathrm{Re}_d > 10000 \end{cases}. \tag{A65}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(30)} \equiv W_{dkx}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(30)}} = \frac{\partial N_1^{(i)}}{\partial W_{dkx}} \equiv a_1^{i,30} = 0; \quad \ell = 1;\ i = 1,\ldots,I;\ j = 30. \tag{A66}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(31)} \equiv W_{dky}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(31)}} = \frac{\partial N_1^{(i)}}{\partial W_{dky}} \equiv a_1^{i,31} = 0; \quad \ell = 1;\ i = 1,\ldots,I;\ j = 31. \tag{A67}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(32)} \equiv \Delta z_{dk}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(32)}} = \frac{\partial N_1^{(i)}}{\partial \Delta z_{dk}} \equiv a_1^{i,32} = 0; \quad \ell = 1;\ i = 1, \ldots, I;\ j = 32. \tag{A68}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(33)} \equiv \Delta z_{fan}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(33)}} = \frac{\partial N_1^{(i)}}{\partial \Delta z_{fan}} \equiv a_1^{i,33} = 0; \quad \ell = 1;\ i = 1, \ldots, I;\ j = 33. \tag{A69}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(34)} \equiv D_{fan}$ are as follows:

$$\begin{gathered}\frac{\partial N_1^{(i)}}{\partial \alpha^{(34)}} = \frac{\partial N_1^{(i)}}{\partial D_{fan}} \equiv a_1^{i,34} = \frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{T_w^{(i+1)}} - \frac{\omega^{(i)} P_{atm}}{\left(0.622+\omega^{(i)}\right)T_a^{(i)}}\right]\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial m_a}\frac{\partial m_a}{\partial D_{fan}}; \\ \ell = 1;\ i = 1, \ldots, I;\ j = 34,\end{gathered} \tag{A70}$$

where $\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}$ and $\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial m_a}$ were defined previously in Equations (A23) and (A24), respectively, and:

$$\frac{\partial m_a}{\partial D_{fan}} = \frac{2 \cdot m_a}{D_{fan}}. \tag{A71}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(35)} \equiv \Delta z_{fill}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(35)}} = \frac{\partial N_1^{(i)}}{\partial \Delta z_{fill}} \equiv a_1^{i,35} = 0; \quad \ell = 1;\ i = 1, \ldots, I;\ j = 35. \tag{A72}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(36)} \equiv \Delta z_{rain}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(36)}} = \frac{\partial N_1^{(i)}}{\partial \Delta z_{rain}} \equiv a_1^{i,36} = 0; \quad \ell = 1;\ i = 1, \ldots, I;\ j = 36. \tag{A73}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(37)} \equiv \Delta z_{bs}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(37)}} = \frac{\partial N_1^{(i)}}{\partial \Delta z_{bs}} \equiv a_1^{i,37} = 0; \quad \ell = 1;\ i = 1, \ldots, I;\ j = 37. \tag{A74}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(38)} \equiv \Delta z_{de}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(38)}} = \frac{\partial N_1^{(i)}}{\partial \Delta z_{de}} \equiv a_1^{i,38} = 0; \quad \ell = 1;\ i = 1, \ldots, I;\ j = 38. \tag{A75}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(39)} \equiv D_h$ are as follows:

$$\begin{gathered}\frac{\partial N_1^{(i)}}{\partial \alpha^{(39)}} = \frac{\partial N_1^{(i)}}{\partial D_h} \equiv a_1^{i,39} = \frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{T_w^{(i+1)}} - \frac{\omega^{(i)} P_{atm}}{\left(0.622+\omega^{(i)}\right)T_a^{(i)}}\right]\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial D_h}; \\ \ell = 1;\ i = 1, \ldots, I;\ j = 39,\end{gathered} \tag{A76}$$

where:

$$\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial D_h}=\begin{cases}-M(m_a,\boldsymbol{\alpha})/D_h & \mathrm{Re}_d<2300\\ -\frac{a_{2,Nu}M(m_a,\boldsymbol{\alpha})}{Nu(\mathrm{Re},\boldsymbol{\alpha})D_h} & 2300\le \mathrm{Re}_d\le 10000\\ -0.2\cdot M(m_a,\boldsymbol{\alpha})/D_h & \mathrm{Re}_d>10000\end{cases}. \tag{A77}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(40)}\equiv A_{fill}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial\alpha^{(40)}}=\frac{\partial N_1^{(i)}}{\partial A_{fill}}\equiv a_1^{i,40}=\frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{T_w^{(i+1)}}-\frac{\omega^{(i)}P_{atm}}{\left(0.622+\omega^{(i)}\right)T_a^{(i)}}\right]\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial A_{fill}};\\ \ell=1;\ i=1,\ldots,I;\ j=40, \tag{A78}$$

where:

$$\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial A_{fill}}=\begin{cases}0 & \mathrm{Re}_d<2300\\ -\frac{a_{1,Nu}M(m_a,\boldsymbol{\alpha})\mathrm{Re}(m_a,\boldsymbol{\alpha})}{Nu(\mathrm{Re},\boldsymbol{\alpha})A_{fill}} & 2300\le \mathrm{Re}_d\le 10000\\ -0.8\cdot M(m_a,\boldsymbol{\alpha})/A_{fill} & \mathrm{Re}_d>10000\end{cases}. \tag{A79}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(41)}\equiv A_{surf}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial\alpha^{(41)}}=\frac{\partial N_1^{(i)}}{\partial A_{surf}}\equiv a_1^{i,41}=\frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{T_w^{(i+1)}}-\frac{\omega^{(i)}P_{atm}}{\left(0.622+\omega^{(i)}\right)T_a^{(i)}}\right]\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial A_{surf}};\\ \ell=1;\ i=1,\ldots,I;\ j=41, \tag{A80}$$

where:

$$\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial A_{surf}}=\frac{M(m_a,\boldsymbol{\alpha})}{A_{surf}} \tag{A81}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(42)}\equiv \mathrm{Pr}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial\alpha^{(42)}}=\frac{\partial N_1^{(i)}}{\partial \mathrm{Pr}}\equiv a_1^{i,42}=\frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{T_w^{(i+1)}}-\frac{\omega^{(i)}P_{atm}}{\left(0.622+\omega^{(i)}\right)T_a^{(i)}}\right]\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial \mathrm{Pr}};\\ \ell=1;\ i=1,\ldots,I;\ j=42, \tag{A82}$$

where:

$$\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial \mathrm{Pr}}=\begin{cases}-M(m_a,\boldsymbol{\alpha})/(3\cdot\mathrm{Pr}) & \mathrm{Re}_d\le 10000\\ 0 & \mathrm{Re}_d>10000\end{cases}. \tag{A83}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(43)}\equiv V_w$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial\alpha^{(43)}}=\frac{\partial N_1^{(i)}}{\partial V_w}\equiv a_1^{i,43}=0;\quad \ell=1;\ i=1,\ldots,I;\ j=43. \tag{A84}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(44)}\equiv V_{exit}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial\alpha^{(44)}}=\frac{\partial N_1^{(i)}}{\partial V_{exit}}\equiv a_1^{i,44}=\frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{T_w^{(i+1)}}-\frac{\omega^{(i)}P_{atm}}{\left(0.622+\omega^{(i)}\right)T_a^{(i)}}\right]\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial m_a}\frac{\partial m_a}{\partial V_{exit}};\\ \ell=1;\ i=1,\ldots,I;\ j=44, \tag{A85}$$

where $\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}$ and $\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial m_a}$ were defined previously in Equations (A23) and (A24), respectively, and

$$\frac{\partial m_a}{\partial V_{exit}} = \frac{P_{atm}}{R_{air} T_{a,in}} \cdot \frac{\pi D_{fan}{}^2}{4} \tag{A86}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(45)} \equiv m_{w,in}$ are as follows:

$$\frac{\partial N_1^{(1)}}{\partial \alpha^{(45)}} = \frac{\partial N_1^{(1)}}{\partial m_{w,in}} \equiv a_1^{1,45} = -1; \quad \ell = 1;\ i = 1;\ j = 45, \tag{A87}$$

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(45)}} = \frac{\partial N_1^{(i)}}{\partial m_{w,in}} \equiv a_1^{i,45} = 0; \quad \ell = 1;\ i = 2, \ldots, I;\ j = 45. \tag{A88}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(46)} \equiv T_{a,in}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(46)}} = \frac{\partial N_1^{(i)}}{\partial T_{a,in}} \equiv a_1^{i,46} = 0; \quad \ell = 1;\ i = 1, \ldots, I;\ j = 46. \tag{A89}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(47)} \equiv m_a$ are as follows:

$$\begin{gathered}\frac{\partial N_1^{(i)}}{\partial \alpha^{(47)}} = \frac{\partial N_1^{(i)}}{\partial m_a} \equiv a_1^{i,47} = \frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{T_w^{(i+1)}} - \frac{\omega^{(i)} P_{atm}}{\left(0.622+\omega^{(i)}\right) T_a^{(i)}}\right] \frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})} \frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial m_a}; \\ \ell = 1;\ i = 1, \ldots, I;\ j = 47,\end{gathered} \tag{A90}$$

where $\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}$ and $\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial m_a}$ were defined previously in Equations (A23) and (A24), respectively.

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(48)} \equiv \omega_{in}$ are as follows:

$$\frac{\partial N_1^{(i)}}{\partial \alpha^{(48)}} = \frac{\partial N_1^{(i)}}{\partial \omega_{in}} \equiv a_1^{i,48} = 0; \quad \ell = 1;\ i = 1, \ldots, I;\ j = 48. \tag{A91}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(49)} \equiv \mathrm{Re}_d$ are as follows:

$$\begin{gathered}\frac{\partial N_1^{(i)}}{\partial \alpha^{(49)}} = \frac{\partial N_1^{(i)}}{\partial \mathrm{Re}_d} \equiv a_1^{i,49} = \frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{T_w^{(i+1)}} - \frac{\omega^{(i)} P_{atm}}{\left(0.622+\omega^{(i)}\right) T_a^{(i)}}\right] \frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re}_d,\boldsymbol{\alpha})} \frac{\partial Nu(\mathrm{Re}_d,\boldsymbol{\alpha})}{\partial \mathrm{Re}_d}; \\ \ell = 1;\ i = 1, \ldots, I;\ j = 49,\end{gathered} \tag{A92}$$

where $\frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}$ was defined in Equation (A23), and:

$$\frac{\partial Nu(\mathrm{Re}_d,\boldsymbol{\alpha})}{\partial \mathrm{Re}_d} = \begin{cases} 0 & \mathrm{Re}_d < 2300 \\ a_{1,Nu} & 2300 \le \mathrm{Re}_d \le 10000 \\ 0.8 \cdot a_{3,Nu} \cdot \mathrm{Re}_d{}^{-0.2} \mathrm{Pr}^{1/3} & \mathrm{Re}_d > 10000 \end{cases} \tag{A93}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(50)} \equiv Sc$ are as follows:

$$\begin{gathered}\frac{\partial N_1^{(i)}}{\partial \alpha^{(50)}} = \frac{\partial N_1^{(i)}}{\partial Sc} \equiv a_1^{i,50} = \frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{T_w^{(i+1)}} - \frac{\omega^{(i)} P_{atm}}{\left(0.622+\omega^{(i)}\right) T_a^{(i)}}\right] \frac{\partial M(m_a,\boldsymbol{\alpha})}{\partial Sc}; \\ \ell = 1;\ i = 1, \ldots, I;\ j = 50,\end{gathered} \tag{A94}$$

where:

$$\frac{\partial M(m_a, \boldsymbol{\alpha})}{\partial Sc} = \frac{1}{3}\frac{M(m_a, \boldsymbol{\alpha})}{Sc}. \tag{A95}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(51)} \equiv Sh$ are as follows:

$$\begin{gathered}\frac{\partial N_1^{(i)}}{\partial \alpha^{(51)}} = \frac{\partial N_1^{(i)}}{\partial Sh} \equiv a_1^{i,51} = \frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)}, \boldsymbol{\alpha})}{T_w^{(i+1)}} - \frac{\omega^{(i)} P_{atm}}{\left(0.622+\omega^{(i)}\right)T_a^{(i)}}\right]\frac{\partial M(m_a, \boldsymbol{\alpha})}{\partial Sh}; \\ \ell = 1;\ i = 1, \ldots, I;\ j = 51,\end{gathered} \tag{A96}$$

where:

$$\frac{\partial M(m_a, \boldsymbol{\alpha})}{\partial Sh} = \frac{M(m_a, \boldsymbol{\alpha})}{Sh}. \tag{A97}$$

The derivatives of the "liquid continuity equations" [cf. Equations (A1)–(A3)] with respect to the parameter $\alpha^{(52)} \equiv Nu$ are as follows:

$$\begin{gathered}\frac{\partial N_1^{(i)}}{\partial \alpha^{(52)}} = \frac{\partial N_1^{(i)}}{\partial Nu} \equiv a_1^{i,52} = \frac{1}{R}\left[\frac{P_{vs}^{(i+1)}(T_w^{(i+1)}, \boldsymbol{\alpha})}{T_w^{(i+1)}} - \frac{\omega^{(i)} P_{atm}}{\left(0.622+\omega^{(i)}\right)T_a^{(i)}}\right]\frac{\partial M(m_a, \boldsymbol{\alpha})}{\partial Nu}; \\ \ell = 1;\ i = 1, \ldots, I;\ j = 52,\end{gathered} \tag{A98}$$

where $\frac{\partial M(m_a, \boldsymbol{\alpha})}{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})}$ was defined in Equation (A23).

A2. Derivatives of the Liquid Energy Balance Equations with Respect to the Parameters

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(1)} \equiv T_{db}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(1)}} = \frac{\partial N_2^{(i)}}{\partial T_{db}} \equiv a_2^{i,1} = 0; \quad \ell = 2;\ i = 1, \ldots, I;\ j = 1. \tag{A99}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(2)} \equiv T_{dp}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(2)}} = \frac{\partial N_2^{(i)}}{\partial T_{dp}} \equiv a_2^{i,2} = 0; \quad \ell = 2;\ i = 1, \ldots, I;\ j = 2. \tag{A100}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(3)} \equiv T_{w,in}$ are as follows:

$$\begin{gathered}\frac{\partial N_2^{(1)}}{\partial \alpha^{(3)}} = \frac{\partial N_2^{(1)}}{\partial T_{w,in}} \equiv a_2^{1,3} = m_{w,in}\frac{\partial h_f^{(1)}(T_{w,in}, \boldsymbol{\alpha})}{\partial T_{w,in}} + h_f^{(1)}(T_{w,in}, \boldsymbol{\alpha})\frac{\partial m_{w,in}}{\partial T_{w,in}} - h_{g,w}^{(2)}(T_w^{(2)}, \boldsymbol{\alpha})\frac{\partial m_{w,in}}{\partial T_{w,in}}; \\ \ell = 2;\ i = 1;\ j = 3,\end{gathered} \tag{A101}$$

where $\frac{\partial m_{w,in}}{\partial T_{w,in}}$ was defined in Equation (A20), and:

$$\frac{\partial h_f^{(1)}(T_{w,in}, \boldsymbol{\alpha})}{\partial T_{w,in}} = a_{1f}, \tag{A102}$$

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(3)}} = \frac{\partial N_2^{(i)}}{\partial T_{w,in}} \equiv a_2^{i,3} = 0; \quad \ell = 2;\ i = 2, \ldots I;\ j = 3. \tag{A103}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(4)} \equiv P_{atm}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(4)}} = \frac{\partial N_2^{(i)}}{\partial P_{atm}} \equiv a_2^{i,4} = -\left(T_w^{(i+1)} - T_a^{(i)}\right) \frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})} \frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial m_a} \frac{\partial m_a}{\partial P_{atm}}; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 4, \tag{A104}$$

where $\frac{\partial Nu(\mathrm{Re},\alpha)}{\partial m_a}$ and $\frac{\partial m_a}{\partial P_{atm}}$ were defined in Equations (A24) and (A25), respectively, and:

$$\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})} = \frac{H(m_a,\boldsymbol{\alpha})}{Nu(\mathrm{Re},\boldsymbol{\alpha})}. \tag{A105}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(5)} \equiv w_{tas}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(5)}} = \frac{\partial N_2^{(i)}}{\partial w_{tsa}} \equiv a_2^{i,5} = -\left(T_w^{(i+1)} - T_a^{(i)}\right) \frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial w_{tsa}}; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 5, \tag{A106}$$

where:

$$\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial w_{tsa}} = \frac{f_{ht} Nu(\mathrm{Re},\boldsymbol{\alpha}) k_{air} A_{surf}}{\mathrm{D}_h I}. \tag{A107}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(6)} \equiv k_{sum}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(6)}} = \frac{\partial N_2^{(i)}}{\partial k_{sum}} \equiv a_2^{i,6} = 0; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 6. \tag{A108}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(7)} \equiv \mu$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(7)}} = \frac{\partial N_2^{(i)}}{\partial \mu} \equiv a_2^{i,7} = -\left(T_w^{(i+1)} - T_a^{(i)}\right) \frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial \mu}; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 7, \tag{A109}$$

where:

$$\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial \mu} = \begin{cases} 0 & \mathrm{Re}_d < 2300 \\ -\frac{a_{1,Nu} \cdot H(m_a,\boldsymbol{\alpha}) \cdot \mathrm{Re}(m_a,\boldsymbol{\alpha})}{Nu(\mathrm{Re},\boldsymbol{\alpha}) \cdot \mu} & 2300 \le \mathrm{Re}_d \le 10000 \\ -0.8 \cdot \frac{H(m_a,\boldsymbol{\alpha})}{\mu} & \mathrm{Re}_d > 10000 \end{cases} \tag{A110}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(8)} \equiv v$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(8)}} = \frac{\partial N_2^{(i)}}{\partial v} \equiv a_2^{i,8} = 0; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 8. \tag{A111}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(9)} \equiv k_{air}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(9)}} = \frac{\partial N_2^{(i)}}{\partial k_{air}} \equiv a_2^{i,9} = -\left(T_w^{(i+1)} - T_a^{(i)}\right) \frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial k_{air}}; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 9, \tag{A112}$$

where:

$$\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial k_{air}} = \frac{H(m_a,\boldsymbol{\alpha})}{k_{air}} = \frac{f_{ht} Nu(\mathrm{Re},\boldsymbol{\alpha}) w_{tsa} A_{surf}}{\mathrm{D}_h I}. \tag{A113}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(10)} \equiv f_{ht}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(10)}} = \frac{\partial N_2^{(i)}}{\partial f_{ht}} \equiv a_2^{i,10} = -\left(T_w^{(i+1)} - T_a^{(i)}\right)\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial f_{ht}}; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 10, \tag{A114}$$

where:

$$\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial f_{ht}} = \frac{H(m_a,\boldsymbol{\alpha})}{f_{ht}} = \frac{k_{air} Nu(\mathrm{Re},\boldsymbol{\alpha}) w_{tsa} A_{surf}}{\mathrm{D}_h I}. \tag{A115}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(11)} \equiv f_{mt}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(11)}} = \frac{\partial N_2^{(i)}}{\partial f_{mt}} \equiv a_2^{i,11} = 0; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 11. \tag{A116}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(12)} \equiv f$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(12)}} = \frac{\partial N_2^{(i)}}{\partial f} \equiv a_2^{i,12} = 0; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 12. \tag{A117}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(13)} \equiv a_0$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(13)}} = \frac{\partial N_2^{(i)}}{\partial a_0} \equiv a_2^{i,13} = 0; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 13. \tag{A118}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(14)} \equiv a_1$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(14)}} = \frac{\partial N_2^{(i)}}{\partial a_1} \equiv a_2^{i,14} = 0; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 14. \tag{A119}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(15)} \equiv a_{0,cpa}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(15)}} = \frac{\partial N_2^{(i)}}{\partial a_{0,cpa}} \equiv a_2^{i,15} = 0; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 15. \tag{A120}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(16)} \equiv a_{1,cpa}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(16)}} = \frac{\partial N_2^{(i)}}{\partial a_{1,cpa}} \equiv a_2^{i,16} = 0; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 16. \tag{A121}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(17)} \equiv a_{2,cpa}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(17)}} = \frac{\partial N_2^{(i)}}{\partial a_{2,cpa}} \equiv a_2^{i,17} = 0; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 17. \tag{A122}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(18)} \equiv a_{0,dav}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(18)}} = \frac{\partial N_2^{(i)}}{\partial a_{0,dav}} \equiv a_2^{i,18} = 0; \quad \ell = 2;\ i = 1, \ldots, I;\ j = 18. \tag{A123}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(19)} \equiv a_{1,dav}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(19)}} = \frac{\partial N_2^{(i)}}{\partial a_{1,dav}} \equiv a_2^{i,19} = 0;\ \ell = 2;\ i = 1, \ldots, I;\ j = 19. \tag{A124}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(20)} \equiv a_{2,dav}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(20)}} = \frac{\partial N_2^{(i)}}{\partial a_{2,dav}} \equiv a_2^{i,20} = 0; \quad \ell = 2;\ i = 1, \ldots, I;\ j = 20. \tag{A125}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(21)} \equiv a_{3,dav}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(21)}} = \frac{\partial N_2^{(i)}}{\partial a_{3,dav}} \equiv a_2^{i,21} = 0; \quad \ell = 2;\ i = 1, \ldots, I;\ j = 21. \tag{A126}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(22)} \equiv a_{0f}$ are as follows:

$$\begin{gathered}\frac{\partial N_2^{(i)}}{\partial \alpha^{(22)}} = \frac{\partial N_2^{(i)}}{\partial a_{0f}} \equiv a_2^{i,22} = m_w^{(i)} \frac{\partial h_f^{(i)}(T_w^{(i)},\boldsymbol{\alpha})}{\partial a_{0f}} - m_w^{(i+1)} \frac{\partial h_f^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{\partial a_{0f}} = m_w^{(i)} - m_w^{(i+1)}; \\ \ell = 2;\ i = 1, \ldots, I;\ j = 22.\end{gathered} \tag{A127}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(23)} \equiv a_{1f}$ are as follows:

$$\begin{aligned}\frac{\partial N_2^{(i)}}{\partial \alpha^{(23)}} = \frac{\partial N_2^{(i)}}{\partial a_{1f}} \equiv a_2^{i,23} = m_w^{(i)} \frac{\partial h_f^{(i)}(T_w^{(i)},\boldsymbol{\alpha})}{\partial a_{1f}} - m_w^{(i+1)} \frac{\partial h_f^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{\partial a_{1f}} \\ = T_w^{(i)} m_w^{(i)} - T_w^{(i+1)} m_w^{(i+1)}; \quad \ell = 2;\ i = 1, \ldots, I;\ j = 23.\end{aligned} \tag{A128}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(24)} \equiv a_{0g}$ are as follows:

$$\begin{gathered}\frac{\partial N_2^{(i)}}{\partial \alpha^{(24)}} = \frac{\partial N_2^{(i)}}{\partial a_{0g}} \equiv a_2^{i,24} = -(m_w^{(i)} - m_w^{(i+1)}) \frac{\partial h_{g,w}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{\partial a_{0g}} = m_w^{(i+1)} - m_w^{(i)}; \\ \ell = 2;\ i = 1, \ldots, I;\ j = 24.\end{gathered} \tag{A129}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(25)} \equiv a_{1g}$ are as follows:

$$\begin{gathered}\frac{\partial N_2^{(i)}}{\partial \alpha^{(25)}} = \frac{\partial N_2^{(i)}}{\partial a_{1g}} \equiv a_2^{i,25} = -(m_w^{(i)} - m_w^{(i+1)}) \frac{\partial h_{g,w}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{\partial a_{1g}} = -\left(m_w^{(i)} - m_w^{(i+1)}\right) T_w^{(i+1)}; \\ \ell = 2;\ i = 1, \ldots, I;\ j = 25.\end{gathered} \tag{A130}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(26)} \equiv a_{0,Nu}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(26)}} = \frac{\partial N_2^{(i)}}{\partial a_{0,Nu}} \equiv a_2^{i,26} = -\left(T_w^{(i+1)} - T_a^{(i)}\right) \frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})} \frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial a_{0,Nu}}; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 26, \tag{A131}$$

where $\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}$ was defined in Equation (A105) and $\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial a_{0,Nu}}$ was defined in Equation (A59).

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(27)} \equiv a_{1,Nu}$ are as follows:

$$\begin{gathered}\frac{\partial N_2^{(i)}}{\partial \alpha^{(27)}} = \frac{\partial N_2^{(i)}}{\partial a_{1,Nu}} \equiv a_2^{i,27} = -\left(T_w^{(i+1)} - T_a^{(i)}\right) \frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})} \frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial a_{1,Nu}}; \\ \ell = 2;\ i = 1,\ldots,I;\ j = 27,\end{gathered} \tag{A132}$$

where $\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}$ was defined in Equation (A105) and $\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial a_{1,Nu}}$ was defined in Equation (A61).

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(28)} \equiv a_{2,Nu}$ are as follows:

$$\begin{gathered}\frac{\partial N_2^{(i)}}{\partial \alpha^{(28)}} = \frac{\partial N_2^{(i)}}{\partial a_{2,Nu}} \equiv a_2^{i,28} = -\left(T_w^{(i+1)} - T_a^{(i)}\right) \frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})} \frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial a_{2,Nu}}; \\ \ell = 2;\ i = 1,\ldots,I;\ j = 28,\end{gathered} \tag{A133}$$

where $\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}$ was defined in Equation (A105) and $\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial a_{2,Nu}}$ was defined in Equation (A63).

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(29)} \equiv a_{3,Nu}$ are as follows:

$$\begin{gathered}\frac{\partial N_2^{(i)}}{\partial \alpha^{(29)}} = \frac{\partial N_2^{(i)}}{\partial a_{3,Nu}} \equiv a_2^{i,29} = -\left(T_w^{(i+1)} - T_a^{(i)}\right) \frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})} \frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial a_{3,Nu}}; \\ \ell = 2;\ i = 1,\ldots,I;\ j = 29,\end{gathered} \tag{A134}$$

where $\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}$ was defined in Equation (A105) and $\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial a_{3,Nu}}$ was defined in Equation (A65).

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(30)} \equiv W_{dkx}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(30)}} = \frac{\partial N_2^{(i)}}{\partial W_{dkx}} \equiv a_2^{i,30} = 0; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 30. \tag{A135}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(31)} \equiv W_{dky}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(31)}} = \frac{\partial N_2^{(i)}}{\partial W_{dky}} \equiv a_2^{i,31} = 0; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 31. \tag{A136}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(32)} \equiv \Delta z_{dk}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(32)}} = \frac{\partial N_2^{(i)}}{\partial \Delta z_{dk}} \equiv a_2^{i,32} = 0; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 32. \tag{A137}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(33)} \equiv \Delta z_{fan}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(33)}} = \frac{\partial N_2^{(i)}}{\partial \Delta z_{fan}} \equiv a_2^{i,33} = 0; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 33. \tag{A138}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(34)} \equiv D_{fan}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(34)}} = \frac{\partial N_2^{(i)}}{\partial D_{fan}} \equiv a_2^{i,34} = -\left(T_w^{(i+1)} - T_a^{(i)}\right) \frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})} \frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial m_a} \frac{\partial m_a}{\partial D_{fan}}; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 34, \tag{A139}$$

where $\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}$ was defined in Equation (A105), while $\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial m_a}$ and $\frac{\partial m_a}{\partial D_{fan}}$ were defined in Equations (A24) and (A71), respectively.

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(35)} \equiv \Delta z_{fill}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(35)}} = \frac{\partial N_2^{(i)}}{\partial \Delta z_{fill}} \equiv a_2^{i,35} = 0; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 35. \tag{A140}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(36)} \equiv \Delta z_{rain}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(36)}} = \frac{\partial N_2^{(i)}}{\partial \Delta z_{rain}} \equiv a_2^{i,36} = 0; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 36. \tag{A141}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(37)} \equiv \Delta z_{bs}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(37)}} = \frac{\partial N_2^{(i)}}{\partial \Delta z_{bs}} \equiv a_2^{i,37} = 0; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 37. \tag{A142}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(38)} \equiv \Delta z_{de}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(38)}} = \frac{\partial N_2^{(i)}}{\partial \Delta z_{de}} \equiv a_2^{i,38} = 0; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 38. \tag{A143}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(39)} \equiv D_h$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(39)}} = \frac{\partial N_2^{(i)}}{\partial D_h} \equiv a_2^{i,39} = -\left(T_w^{(i+1)} - T_a^{(i)}\right) \frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial D_h}; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 39, \tag{A144}$$

where

$$\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial D_h} = \begin{cases} -H(m_a,\boldsymbol{\alpha})/D_h & \mathrm{Re}_d < 2300 \\ -\frac{a_{2,Nu} H(m_a,\boldsymbol{\alpha})}{Nu(\mathrm{Re},\boldsymbol{\alpha}) D_h} & 2300 \le \mathrm{Re}_d \le 10000 \\ -0.2 \cdot H(m_a,\boldsymbol{\alpha})/D_h & \mathrm{Re}_d > 10000 \end{cases} \tag{A145}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(40)} \equiv A_{fill}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(40)}} = \frac{\partial N_2^{(i)}}{\partial A_{fill}} \equiv a_2^{i,40} = -\left(T_w^{(i+1)} - T_a^{(i)}\right) \frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial A_{fill}}; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 40, \tag{A146}$$

where:

$$\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial A_{fill}} = \begin{cases} 0 & \mathrm{Re}_d < 2300 \\ -\frac{a_{1,Nu} H(m_a,\boldsymbol{\alpha}) \mathrm{Re}(m_a,\boldsymbol{\alpha})}{Nu(\mathrm{Re},\boldsymbol{\alpha}) A_{fill}} & 2300 \le \mathrm{Re}_d \le 10000 \\ -0.8 \cdot H(m_a,\boldsymbol{\alpha})/A_{fill} & \mathrm{Re}_d > 10000 \end{cases} \tag{A147}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(41)} \equiv A_{surf}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(41)}} = \frac{\partial N_2^{(i)}}{\partial A_{surf}} \equiv a_2^{i,41} = -\left(T_w^{(i+1)} - T_a^{(i)}\right)\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial A_{surf}}; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 41, \tag{A148}$$

where:

$$\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial A_{surf}} = \frac{H(m_a,\boldsymbol{\alpha})}{A_{surf}} = \frac{f_{ht}k_{air}Nu(\mathrm{Re},\boldsymbol{\alpha})w_{tsa}}{\mathrm{D}_h I}. \tag{A149}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(42)} \equiv \mathrm{Pr}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(42)}} = \frac{\partial N_2^{(i)}}{\partial \mathrm{Pr}} \equiv a_2^{i,42} = -\left(T_w^{(i+1)} - T_a^{(i)}\right)\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial \mathrm{Pr}}; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 42, \tag{A150}$$

where:

$$\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial \mathrm{Pr}} = \begin{cases} 0 & \mathrm{Re}_d \le 10000 \\ H(m_a,\boldsymbol{\alpha})/(3\cdot \mathrm{Pr}) & \mathrm{Re}_d > 10000 \end{cases} \tag{A151}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(33)} \equiv V_w$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(43)}} = \frac{\partial N_2^{(i)}}{\partial V_w} \equiv a_2^{i,43} = 0; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 43. \tag{A152}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(44)} \equiv V_{exit}$ are as follows:

$$\begin{gathered}\frac{\partial N_2^{(i)}}{\partial \alpha^{(44)}} = \frac{\partial N_2^{(i)}}{\partial V_{exit}} \equiv a_2^{i,44} = -\left(T_w^{(i+1)} - T_a^{(i)}\right)\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial m_a}\frac{\partial m_a}{\partial V_{exit}}; \\ \ell = 2;\ i = 1,\ldots,I;\ j = 44,\end{gathered} \tag{A153}$$

where $\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}$ was defined in Equation (A105), while $\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial m_a}$ and $\frac{\partial m_a}{\partial V_{exit}}$ and were defined previously in Equations (A24) and (A86), respectively.

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(45)} \equiv m_{w,in}$ are as follows:

$$\begin{gathered}\frac{\partial N_2^{(1)}}{\partial \alpha^{(45)}} = \frac{\partial N_2^{(1)}}{\partial m_{w,in}} \equiv a_2^{1,45} = h_f^{(1)}(T_{w,in},\boldsymbol{\alpha}) - h_{g,w}^{(2)}(T_w^{(2)},\boldsymbol{\alpha}) = T_{w,in}a_{1\,f} - a_{1g}T_w^{(2)} + a_{0\,f} - a_{0\,\mathrm{g}}, \\ \ell = 2;\ i = 1;\ j = 45,\end{gathered} \tag{A154}$$

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(45)}} = \frac{\partial N_2^{(i)}}{\partial m_{w,in}} \equiv a_2^{i,45} = 0; \quad \ell = 2;\ i = 2,\ldots,I;\ j = 45. \tag{A155}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(46)} \equiv T_{a,in}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(46)}} = \frac{\partial N_2^{(i)}}{\partial T_{a,in}} \equiv a_2^{i,46} = 0; \quad \ell = 2;\ i = 1,\ldots,I;\ j = 46. \tag{A156}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(47)} \equiv m_a$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(47)}} = \frac{\partial N_2^{(i)}}{\partial m_a} \equiv a_2^{i,47} = -\left(T_w^{(i+1)} - T_a^{(i)}\right) \frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})} \frac{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})}{\partial m_a}; \quad \ell = 2;\ i = 1, \ldots, I;\ j = 47, \tag{A157}$$

where $\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}$ and $\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial m_a}$ were defined previously in Equations (A24) and (A105), respectively.

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(48)} \equiv \omega_{in}$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(48)}} = \frac{\partial N_2^{(i)}}{\partial \omega_{in}} \equiv a_2^{i,48} = 0; \quad \ell = 2;\ i = 1, \ldots, I;\ j = 48. \tag{A158}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(49)} \equiv \mathrm{Re}_d$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(49)}} = \frac{\partial N_2^{(i)}}{\partial \mathrm{Re}_d} \equiv a_2^{i,49} = -\left(T_w^{(i+1)} - T_a^{(i)}\right) \frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial Nu(\mathrm{Re}_d, \boldsymbol{\alpha})} \frac{\partial Nu(\mathrm{Re}_d, \boldsymbol{\alpha})}{\partial \mathrm{Re}_d}; \quad \ell = 2;\ i = 1, \ldots, I;\ j = 49, \tag{A159}$$

where $\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}$ was defined in Equation (A105), and $\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial m_a}$ was defined in Equation (A93).

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(50)} \equiv Sc$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(50)}} = \frac{\partial N_2^{(i)}}{\partial Sc} \equiv a_2^{i,50} = 0; \quad \ell = 2;\ i = 1, \ldots, I;\ j = 50. \tag{A160}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(51)} \equiv Sh$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(51)}} = \frac{\partial N_2^{(i)}}{\partial Sh} \equiv a_2^{i,51} = 0; \quad \ell = 2;\ i = 1, \ldots, I;\ j = 51. \tag{A161}$$

The derivatives of the liquid energy balance equations [cf. Equations (A4)–(A6)] with respect to the parameter $\alpha^{(52)} \equiv Nu$ are as follows:

$$\frac{\partial N_2^{(i)}}{\partial \alpha^{(52)}} = \frac{\partial N_2^{(i)}}{\partial Nu} \equiv a_2^{i,52} = -\left(T_w^{(i+1)} - T_a^{(i)}\right) \frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial Nu}; \quad \ell = 2;\ i = 1, \ldots, I;\ j = 52, \tag{A162}$$

where $\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}$ was defined in Equation (A105).

A3. Derivatives of the Water Vapor Continuity Equations with Respect to the Parameters

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(1)} \equiv T_{db}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(1)}} = \frac{\partial N_3^{(i)}}{\partial T_{db}} \equiv a_3^{i,1} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 1. \tag{A163}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(2)} \equiv T_{dp}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(2)}} = \frac{\partial N_3^{(i)}}{\partial T_{dp}} \equiv a_3^{i,2} = 0; \quad \ell = 3;\ i = 1, \ldots, I-1;\ j = 2, \tag{A164}$$

$$\frac{\partial N_3^{(I)}}{\partial \alpha^{(2)}} = \frac{\partial N_3^{(I)}}{\partial T_{dp}} \equiv a_3^{I,2} = \frac{\partial \omega_{in}}{\partial T_{dp}}; \quad \ell = 3;\ i = I;\ j = 2, \tag{A165}$$

where:

$$\frac{\partial \omega_{in}}{\partial T_{dp}} = -\frac{0.622 a_1 P_{atm} e^{a_0 + \frac{a_1}{T_{dp}}}}{T_{tdp}{}^2 \left(P_{atm} - e^{a_0 + \frac{a_1}{T_{dp}}} \right)^2}. \tag{A166}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(3)} \equiv T_{w,in}$ are as follows:

$$\frac{\partial N_3^{(1)}}{\partial \alpha^{(3)}} = \frac{\partial N_3^{(1)}}{\partial T_{w,in}} \equiv a_3^{1,3} = \frac{1}{m_a} \frac{\partial m_{w,in}}{\partial T_{w,in}}; \quad \ell = 3;\ i = 1;\ j = 3, \tag{A167}$$

where $\frac{\partial w,in}{\partial T_{w,in}}$ was defined in Equation (A20).

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(3)}} = \frac{\partial N_3^{(i)}}{\partial T_{w,in}} \equiv a_3^{i,3} = 0; \quad \ell = 3;\ i = 2, \ldots I;\ j = 3. \tag{A168}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(4)} \equiv P_{atm}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(4)}} = \frac{\partial N_3^{(i)}}{\partial P_{atm}} \equiv a_3^{i,4} = -\frac{m_w^{(i)} - m_w^{(i+1)}}{m_a{}^2} \frac{\partial m_a}{\partial P_{atm}}; \quad \ell = 3;\ i = 1, \ldots, I-1;\ j = 4, \tag{A169}$$

$$\frac{\partial N_3^{(I)}}{\partial \alpha^{(4)}} = \frac{\partial N_3^{(I)}}{\partial P_{atm}} \equiv a_3^{I,4} = \frac{\partial \omega_{in}}{\partial P_{atm}} - \frac{m_w^{(i)} - m_w^{(i+1)}}{m_a{}^2} \frac{\partial m_a}{\partial P_{atm}}; \quad \ell = 3;\ i = I;\ j = 4, \tag{A170}$$

where $\frac{\partial m_a}{\partial P_{atm}}$ was defined in Equation (A25) and:

$$\frac{\partial \omega_{in}}{\partial P_{atm}} = -\frac{0.622 e^{a_0 + \frac{a_1}{T_{dp}}}}{\left(P_{atm} - e^{a_0 + \frac{a_1}{T_{dp}}} \right)^2}. \tag{A171}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(5)} \equiv w_{tsa}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(5)}} = \frac{\partial N_3^{(i)}}{\partial w_{tsa}} \equiv a_3^{i,5} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 5. \tag{A172}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(6)} \equiv k_{sum}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(6)}} = \frac{\partial N_3^{(i)}}{\partial k_{sum}} \equiv a_3^{i,6} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 6. \tag{A173}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(7)} \equiv \mu$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(7)}} = \frac{\partial N_3^{(i)}}{\partial \mu} \equiv a_3^{i,7} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 7. \tag{A174}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(8)} \equiv v$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(8)}} = \frac{\partial N_3^{(i)}}{\partial v} \equiv a_3^{i,8} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 8. \tag{A175}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(9)} \equiv k_{air}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(9)}} = \frac{\partial N_3^{(i)}}{\partial k_{air}} \equiv a_3^{i,9} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 9. \tag{A176}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(10)} \equiv f_{ht}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(10)}} = \frac{\partial N_3^{(i)}}{\partial f_{ht}} \equiv a_3^{i,10} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 10. \tag{A177}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(11)} \equiv f_{mt}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(11)}} = \frac{\partial N_3^{(i)}}{\partial f_{mt}} \equiv a_3^{i,11} = 0; \ell = 3;\ i = 1, \ldots, I;\ j = 11. \tag{A178}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(12)} \equiv f$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(12)}} = \frac{\partial N_3^{(i)}}{\partial f} \equiv a_3^{i,12} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 12. \tag{A179}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(13)} \equiv a_0$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(13)}} = \frac{\partial N_3^{(i)}}{\partial a_0} \equiv a_3^{i,13} = 0; \quad \ell = 3;\ i = 1, \ldots, I-1;\ j = 13, \tag{A180}$$

$$\frac{\partial N_3^{(I)}}{\partial \alpha^{(13)}} = \frac{\partial N_3^{(I)}}{\partial a_0} \equiv a_3^{I,13} = \frac{\partial \omega_{in}}{\partial a_0}; \quad \ell = 3;\ i = I;\ j = 13, \tag{A181}$$

where:

$$\frac{\partial \omega_{in}}{\partial a_0} = \frac{0.622 P_{atm} \mathrm{e}^{a_0 + \frac{a_1}{T_{dp}}}}{\left(P_{atm} - \mathrm{e}^{a_0 + \frac{a_1}{T_{dp}}}\right)^2}; \tag{A182}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(14)} \equiv a_1$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(14)}} = \frac{\partial N_3^{(i)}}{\partial a_1} \equiv a_3^{i,14} = 0; \quad \ell = 3;\ i = 1, \ldots, I-1;\ j = 14, \tag{A183}$$

$$\frac{\partial N_3^{(I)}}{\partial \alpha^{(14)}} = \frac{\partial N_3^{(I)}}{\partial a_1} \equiv a_3^{I,14} = \frac{\partial \omega_{in}}{\partial a_1}; \quad \ell = 3;\ i = I;\ j = 14, \tag{A184}$$

where:

$$\frac{\partial \omega_{in}}{\partial a_1} = \frac{0.622 P_{atm} e^{a_0 + \frac{a_1}{T_{dp}}}}{T_{dp} \left(P_{atm} - e^{a_0 + \frac{a_1}{T_{dp}}} \right)^2}. \quad \text{(A185)}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(15)} \equiv a_{0,cpa}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(15)}} = \frac{\partial N_3^{(i)}}{\partial a_{0,cpa}} \equiv a_3^{i,15} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 15. \quad \text{(A186)}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(16)} \equiv a_{1,cpa}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(16)}} = \frac{\partial N_3^{(i)}}{\partial a_{1,cpa}} \equiv a_3^{i,16} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 16. \quad \text{(A187)}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(17)} \equiv a_{2,cpa}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(17)}} = \frac{\partial N_3^{(i)}}{\partial a_{2,cpa}} \equiv a_3^{i,17} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 17. \quad \text{(A188)}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(18)} \equiv a_{0,dav}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(18)}} = \frac{\partial N_3^{(i)}}{\partial a_{0,dav}} \equiv a_3^{i,18} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 18. \quad \text{(A189)}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(19)} \equiv a_{1,dav}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(19)}} = \frac{\partial N_3^{(i)}}{\partial a_{1,dav}} \equiv a_3^{i,19} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 19. \quad \text{(A190)}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(20)} \equiv a_{2,dav}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(20)}} = \frac{\partial N_3^{(i)}}{\partial a_{2,dav}} \equiv a_3^{i,20} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 20. \quad \text{(A191)}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(21)} \equiv a_{3,dav}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(21)}} = \frac{\partial N_3^{(i)}}{\partial a_{3,dav}} \equiv a_3^{i,21} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 21. \quad \text{(A192)}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(22)} \equiv a_{0f}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(22)}} = \frac{\partial N_3^{(i)}}{\partial a_{0f}} \equiv a_3^{i,22} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 22. \quad \text{(A193)}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(23)} \equiv a_{1f}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(23)}} = \frac{\partial N_3^{(i)}}{\partial a_{1f}} \equiv a_3^{i,23} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 23. \tag{A194}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(24)} \equiv a_{0g}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(24)}} = \frac{\partial N_3^{(i)}}{\partial a_{0g}} \equiv a_3^{i,24} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 24. \tag{A195}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(25)} \equiv a_{1g}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(25)}} = \frac{\partial N_3^{(i)}}{\partial a_{1g}} \equiv a_3^{i,25} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 25. \tag{A196}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(26)} \equiv a_{0,Nu}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(26)}} = \frac{\partial N_3^{(i)}}{\partial a_{0,Nu}} \equiv a_3^{i,26} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 26. \tag{A197}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(27)} \equiv a_{1,Nu}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(27)}} = \frac{\partial N_3^{(i)}}{\partial a_{1,Nu}} \equiv a_3^{i,27} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 27. \tag{A198}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(28)} \equiv a_{2,Nu}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(28)}} = \frac{\partial N_3^{(i)}}{\partial a_{2,Nu}} \equiv a_3^{i,28} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 28. \tag{A199}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(29)} \equiv a_{3,Nu}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(29)}} = \frac{\partial N_3^{(i)}}{\partial a_{3,Nu}} \equiv a_3^{i,29} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 29. \tag{A200}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(30)} \equiv W_{dkx}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(30)}} = \frac{\partial N_3^{(i)}}{\partial W_{dkx}} \equiv a_3^{i,30} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 30. \tag{A201}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(31)} \equiv W_{dky}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(31)}} = \frac{\partial N_3^{(i)}}{\partial W_{dky}} \equiv a_3^{i,31} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 31. \tag{A202}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(32)} \equiv \Delta z_{dk}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(32)}} = \frac{\partial N_3^{(i)}}{\partial \Delta z_{dk}} \equiv a_3^{i,32} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 32. \tag{A203}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(33)} \equiv \Delta z_{fan}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(33)}} = \frac{\partial N_3^{(i)}}{\partial \Delta z_{fan}} \equiv a_3^{i,33} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 33. \tag{A204}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(34)} \equiv D_{fan}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(34)}} = \frac{\partial N_3^{(i)}}{\partial D_{fan}} \equiv a_3^{i,34} = -\frac{m_w^{(i)} - m_w^{(i+1)}}{{m_a}^2} \frac{\partial m_a}{\partial D_{fan}}; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 34, \tag{A205}$$

where $\frac{\partial m_a}{\partial D_{fan}}$ was defined in Equation (A71).

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(35)} \equiv \Delta z_{fill}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(35)}} = \frac{\partial N_3^{(i)}}{\partial \Delta z_{fill}} \equiv a_3^{i,35} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 35. \tag{A206}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(36)} \equiv \Delta z_{rain}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(36)}} = \frac{\partial N_3^{(i)}}{\partial \Delta z_{rain}} \equiv a_3^{i,36} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 36. \tag{A207}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(37)} \equiv \Delta z_{bs}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(37)}} = \frac{\partial N_3^{(i)}}{\partial \Delta z_{bs}} \equiv a_3^{i,37} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 37. \tag{A208}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(38)} \equiv \Delta z_{de}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(38)}} = \frac{\partial N_3^{(i)}}{\partial \Delta z_{de}} \equiv a_3^{i,38} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 38. \tag{A209}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(39)} \equiv D_h$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(39)}} = \frac{\partial N_3^{(i)}}{\partial D_h} \equiv a_3^{i,39} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 39. \tag{A210}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(40)} \equiv A_{fill}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(40)}} = \frac{\partial N_3^{(i)}}{\partial A_{fill}} \equiv a_3^{i,40} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 40. \tag{A211}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(41)} \equiv A_{surf}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(41)}} = \frac{\partial N_3^{(i)}}{\partial A_{surf}} \equiv a_3^{i,41} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 41. \tag{A212}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(42)} \equiv \mathrm{Pr}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(42)}} = \frac{\partial N_3^{(i)}}{\partial \mathrm{Pr}} \equiv a_3^{i,42} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 42. \tag{A213}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(43)} \equiv V_w$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(43)}} = \frac{\partial N_3^{(i)}}{\partial V_w} \equiv a_3^{i,43} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 43. \tag{A214}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(44)} \equiv V_{exit}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(44)}} = \frac{\partial N_3^{(i)}}{\partial V_{exit}} \equiv a_3^{i,44} = -\frac{m_w^{(i)} - m_w^{(i+1)}}{{m_a}^2} \frac{\partial m_a}{\partial V_{exit}}; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 44, \tag{A215}$$

where $\frac{\partial m_a}{\partial V_{exit}}$ was defined in Equation (A86).

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(45)} \equiv m_{w,in}$ are as follows:

$$\frac{\partial N_3^{(1)}}{\partial \alpha^{(45)}} = \frac{\partial N_3^{(1)}}{\partial m_{w,in}} \equiv a_3^{1,45} = \frac{1}{m_a}; \quad \ell = 3;\ i = 1;\ j = 45, \tag{A216}$$

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(45)}} = \frac{\partial N_3^{(i)}}{\partial m_{w,in}} \equiv a_3^{i,45} = 0; \quad \ell = 3;\ i = 2, \ldots, I;\ j = 45. \tag{A217}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(46)} \equiv T_{a,in}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(46)}} = \frac{\partial N_3^{(i)}}{\partial T_{a,in}} \equiv a_3^{i,46} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 46. \tag{A218}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(47)} \equiv m_a$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(47)}} = \frac{\partial N_3^{(i)}}{\partial m_a} \equiv a_3^{i,47} = -\frac{m_w^{(i)} - m_w^{(i+1)}}{{m_a}^2}; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 47. \tag{A219}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(48)} \equiv \omega_{in}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(48)}} = \frac{\partial N_3^{(i)}}{\partial \omega_{in}} \equiv a_3^{i,48} = 0; \quad \ell = 3;\ i = 1, \ldots, I-1;\ j = 48, \tag{A220}$$

$$\frac{\partial N_3^{(I)}}{\partial \alpha^{(48)}} = \frac{\partial N_3^{(I)}}{\partial \omega_{in}} \equiv a_3^{I,48} = 1; \quad \ell = 3;\ i = I;\ j = 48. \tag{A221}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(49)} \equiv \mathrm{Re}_d$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(49)}} = \frac{\partial N_3^{(i)}}{\partial \mathrm{Re}_d} \equiv a_3^{i,49} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 49. \tag{A222}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(50)} \equiv \mathrm{Sc}$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(50)}} = \frac{\partial N_3^{(i)}}{\partial Sc} \equiv a_3^{i,50} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 50. \tag{A223}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(51)} \equiv Sh$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(51)}} = \frac{\partial N_3^{(i)}}{\partial Sh} \equiv a_3^{i,51} = 0; \ell = 3;\ i = 1, \ldots, I;\ j = 51. \tag{A224}$$

The derivatives of the water vapor continuity equations [cf. Equations (A7)–(A9)] with respect to the parameter $\alpha^{(52)} \equiv Nu$ are as follows:

$$\frac{\partial N_3^{(i)}}{\partial \alpha^{(52)}} = \frac{\partial N_3^{(i)}}{\partial Nu} \equiv a_3^{i,52} = 0; \quad \ell = 3;\ i = 1, \ldots, I;\ j = 52. \tag{A225}$$

A4. Derivatives of the Air and Water Vapor Energy Balance Equations with Respect to the Parameters

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(1)} \equiv T_{db}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(1)}} = \frac{\partial N_4^{(i)}}{\partial T_{db}} \equiv a_4^{i,1} = 0; \quad \ell = 4;\ i = 1, \ldots, I-1;\ j = 1, \tag{A226}$$

$$\begin{aligned} \frac{\partial N_4^{(I)}}{\partial \alpha^{(1)}} &= \frac{\partial N_4^{(I)}}{\partial T_{db}} \equiv a_4^{I,1} = C_p^{(I)}\left(\frac{T_a^{(I)}+273.15}{2}, \boldsymbol{\alpha}\right) + \omega_{in}\frac{\partial h_{g,a}^{(I+1)}(T_{a,in}, \boldsymbol{\alpha})}{\partial T_{a,in}} \\ &= C_p^{(I)}\left(\frac{T_a^{(I)}+273.15}{2}, \boldsymbol{\alpha}\right) + \omega_{in} a_{1g}; \quad \ell = 4;\ i = I;\ j = 1. \end{aligned} \tag{A227}$$

Note: The value of the inlet air temperature is set equal to dry-bulb temperature, although these quantities are treated as two different parameters in the model. The dry-bulb temperature is used in mass diffusivity calculations. The relation between the two parameters, i.e., $T_{a,in}$ = T_{db}, needs to be accounted for when computing the respective derivatives: the derivative of Equation (A12) with respect to the dry-bulb temperature must be the same as the derivative of Equation (A12) with respect to the inlet air temperature.

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(2)} \equiv T_{dp}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(2)}} = \frac{\partial N_4^{(i)}}{\partial T_{dp}} \equiv a_4^{i,2} = 0; \quad \ell = 4;\ i = 1, \ldots, I-1;\ j = 2, \tag{A228}$$

$$\frac{\partial N_4^{(I)}}{\partial \alpha^{(2)}} = \frac{\partial N_4^{(I)}}{\partial T_{dp}} \equiv a_4^{I,2} = \frac{\partial \omega_{in}}{\partial T_{dp}} \left(a_{1g} T_{a,in} + a_{0g} \right); \quad \ell = 4;\ i = I;\ j = 2, \tag{A229}$$

where $\frac{\partial \omega_{in}}{\partial T_{dp}}$ was defined in Equation (A166).

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(3)} \equiv T_{w,in}$ are as follows:

$$\begin{gathered} \frac{\partial N_4^{(1)}}{\partial \alpha^{(3)}} = \frac{\partial N_4^{(1)}}{\partial T_{w,in}} \equiv a_4^{1,3} = \frac{h_{g,w}^{(2)}(T_w^{(2)}, \boldsymbol{\alpha})}{m_a} \frac{\partial m_{w,in}}{\partial T_{w,in}} = \frac{a_{1g} T_w^{(2)} + a_{0g}}{m_a} \frac{\partial m_{w,in}}{\partial T_{w,in}}; \\ \ell = 4;\ i = 1;\ j = 3, \end{gathered} \tag{A230}$$

where $\frac{\partial \omega_{w,in}}{\partial T_{w,in}}$ was defined in Equation (A20), and

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(3)}} = \frac{\partial N_4^{(i)}}{\partial T_{w,in}} \equiv a_4^{i,3} = 0; \quad \ell = 4;\ i = 2, \ldots, I;\ j = 3. \tag{A231}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(4)} \equiv P_{atm}$ are as follows:

$$\begin{gathered} \frac{\partial N_4^{(i)}}{\partial \alpha^{(4)}} = \frac{\partial N_4^{(i)}}{\partial P_{atm}} \equiv a_4^{i,4} = \left(T_w^{(i+1)} - T_a^{(i)} \right) \left[\frac{1}{m_a} \frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})} \frac{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})}{\partial m_a} \frac{\partial m_a}{\partial P_{atm}} - \frac{H(m_a, \boldsymbol{\alpha})}{m_a{}^2} \frac{\partial m_a}{\partial P_{atm}} \right] \\ - \frac{(m_w^{(i)} - m_w^{(i+1)}) h_{g,w}^{(i+1)}(T_w^{(i+1)}, \boldsymbol{\alpha})}{m_a{}^2} \frac{\partial m_a}{\partial P_{atm}}; \quad \ell = 4;\ i = 1, \ldots, I-1;\ j = 4, \end{gathered} \tag{A232}$$

$$\begin{gathered} \frac{\partial N_4^{(I)}}{\partial \alpha^{(4)}} = \frac{\partial N_4^{(I)}}{\partial P_{atm}} \equiv a_4^{I,4} = \left(T_w^{(i+1)} - T_a^{(i)} \right) \left[\frac{1}{m_a} \frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})} \frac{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})}{\partial m_a} \frac{\partial m_a}{\partial P_{atm}} - \frac{H(m_a, \boldsymbol{\alpha})}{m_a{}^2} \frac{\partial m_a}{\partial P_{atm}} \right] \\ - \frac{(m_w^{(i)} - m_w^{(i+1)}) h_{g,w}^{(i+1)}(T_w^{(i+1)}, \boldsymbol{\alpha})}{m_a{}^2} \frac{\partial m_a}{\partial P_{atm}} + \frac{\partial \omega_{in}}{\partial P_{atm}} \left(a_{1g} T_{a,in} + a_{0g} \right); \quad \ell = 4;\ i = I;\ j = 4, \end{gathered} \tag{A233}$$

where $\frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})}$ and $\frac{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})}{\partial m_a}$ were defined in Equations (A24) and (A105), respectively, while $\frac{\partial m_a}{\partial P_{atm}}$ and $\frac{\partial \omega_{in}}{\partial P_{atm}}$ were defined in Equations (A25) and (A171), respectively.

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(5)} \equiv w_{tsa}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(5)}} = \frac{\partial N_4^{(i)}}{\partial w_{tsa}} \equiv a_4^{i,5} = \frac{(T_w^{(i+1)} - T_a^{(i)})}{m_a} \frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial w_{tsa}}; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 5, \tag{A234}$$

where $\frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial w_{tsa}}$ was defined in Equation (A107).

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(6)} \equiv k_{sum}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(6)}} = \frac{\partial N_4^{(i)}}{\partial k_{sum}} \equiv a_4^{i,6} = 0; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 6. \tag{A235}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(7)} \equiv \mu$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(7)}} = \frac{\partial N_4^{(i)}}{\partial \mu} \equiv a_4^{i,7} = \frac{(T_w^{(i+1)} - T_a^{(i)})}{m_a} \frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial \mu}; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 7, \tag{A236}$$

where $\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial}$ was defined in Equation (A110).

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(8)} \equiv v$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(8)}} = \frac{\partial N_4^{(i)}}{\partial v} \equiv a_4^{i,8} = 0; \quad \ell = 4;\ i = 1,\ldots,I;\ j = 8. \tag{A237}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(9)} \equiv k_{air}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(9)}} = \frac{\partial N_4^{(i)}}{\partial k_{air}} \equiv a_4^{i,9} = \frac{(T_w^{(i+1)} - T_a^{(i)})}{m_a} \frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial k_{air}}; \quad \ell = 4;\ i = 1,\ldots,I;\ j = 9, \tag{A238}$$

where $\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial k_{air}}$ was defined in Equation (A113).

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(10)} \equiv f_{ht}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(10)}} = \frac{\partial N_4^{(i)}}{\partial f_{ht}} \equiv a_4^{i,10} = \frac{(T_w^{(i+1)} - T_a^{(i)})}{m_a} \frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial f_{ht}}; \quad \ell = 4;\ i = 1,\ldots,I;\ j = 10, \tag{A239}$$

where $\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial f_{ht}}$ was defined in Equation (A115).

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(11)} \equiv f_{mt}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(11)}} = \frac{\partial N_4^{(i)}}{\partial f_{mt}} \equiv a_4^{i,11} = 0; \quad \ell = 4;\ i = 1,\ldots,I;\ j = 11. \tag{A240}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(12)} \equiv f$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(12)}} = \frac{\partial N_4^{(i)}}{\partial f} \equiv a_4^{i,12} = 0; \quad \ell = 4;\ i = 1,\ldots,I;\ j = 12. \tag{A241}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(13)} \equiv a_0$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(13)}} = \frac{\partial N_4^{(i)}}{\partial a_0} \equiv a_4^{i,13} = 0; \quad \ell = 4;\ i = 1,\ldots,I-1;\ j = 13, \tag{A242}$$

$$\frac{\partial N_4^{(I)}}{\partial \alpha^{(13)}} = \frac{\partial N_4^{(I)}}{\partial a_0} \equiv a_4^{I,13} = \frac{\partial \omega_{in}}{\partial a_0}\left(a_{1g}T_{a,in} + a_{0g}\right); \quad \ell = 4;\ i = I;\ j = 13, \tag{A243}$$

where $\frac{\partial \omega_{in}}{\partial a_0}$ was defined in Equation (A182).

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(14)} \equiv a_1$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(14)}} = \frac{\partial N_4^{(i)}}{\partial a_1} \equiv a_4^{i,14} = 0; \quad \ell = 4;\ i = 1,\ldots,I-1;\ j = 14, \tag{A244}$$

$$\frac{\partial N_4^{(I)}}{\partial \alpha^{(14)}} = \frac{\partial N_4^{(I)}}{\partial a_1} \equiv a_4^{I,14} = \frac{\partial \omega_{in}}{\partial a_1}\left(a_{1g}T_{a,in} + a_{0g}\right); \quad \ell = 4;\ i = I;\ j = 14, \tag{A245}$$

where $\frac{\partial \omega_{in}}{\partial a_1}$ was defined in Equation (A185).

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(15)} \equiv a_{0,cpa}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(15)}} = \frac{\partial N_4^{(i)}}{\partial a_{0,cpa}} \equiv a_4^{i,15} = (T_a^{(i+1)} - T_a^{(i)}) \frac{\partial C_p^{(i)}\left(\frac{T_a^{(i)}+273.15}{2}, \boldsymbol{\alpha}\right)}{\partial a_{0,cpa}} = T_a^{(i+1)} - T_a^{(i)}; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 15. \tag{A246}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(16)} \equiv a_{1,cpa}$ are as follows:

$$\begin{aligned}\frac{\partial N_4^{(i)}}{\partial \alpha^{(16)}} &= \frac{\partial N_4^{(i)}}{\partial a_{1,cpa}} \equiv a_4^{i,16} = (T_a^{(i+1)} - T_a^{(i)}) \frac{\partial C_p^{(i)}\left(\frac{T_a^{(i)}+273.15}{2}, \boldsymbol{\alpha}\right)}{\partial a_{1,cpa}} \\ &= 0.5\left(T_a^{(i+1)} - T_a^{(i)}\right)(T_a^{(i)} + 273.15); \quad \ell = 4;\ i = 1, \ldots, I;\ j = 16.\end{aligned} \tag{A247}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(17)} \equiv a_{2,cpa}$ are as follows:

$$\begin{aligned}\frac{\partial N_4^{(i)}}{\partial \alpha^{(17)}} &= \frac{\partial N_4^{(i)}}{\partial a_{2,cpa}} \equiv a_4^{i,17} = (T_a^{(i+1)} - T_a^{(i)}) \frac{\partial C_p^{(i)}\left(\frac{T_a^{(i)}+273.15}{2}, \boldsymbol{\alpha}\right)}{\partial a_{2,cpa}} \\ &= 0.25\left(T_a^{(i+1)} - T_a^{(i)}\right)\left[T_a^{(i)} + 273.15\right]^2; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 17.\end{aligned} \tag{A248}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(18)} \equiv a_{0,dav}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(18)}} = \frac{\partial N_4^{(i)}}{\partial a_{0,dav}} \equiv a_4^{i,18} = 0; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 18. \tag{A249}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(19)} \equiv a_{1,dav}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(19)}} = \frac{\partial N_4^{(i)}}{\partial a_{1,dav}} \equiv a_4^{i,19} = 0; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 19. \tag{A250}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(20)} \equiv a_{2,dav}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(20)}} = \frac{\partial N_4^{(i)}}{\partial a_{2,dav}} \equiv a_4^{i,20} = 0; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 20. \tag{A251}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(21)} \equiv a_{3,dav}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(21)}} = \frac{\partial N_4^{(i)}}{\partial a_{3,dav}} \equiv a_4^{i,21} = 0; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 21. \tag{A252}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(22)} \equiv a_{0f}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(22)}} = \frac{\partial N_4^{(i)}}{\partial a_{0f}} \equiv a_4^{i,22} = 0; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 22. \tag{A253}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(23)} \equiv a_{1f}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(23)}} = \frac{\partial N_4^{(i)}}{\partial a_{1f}} \equiv a_4^{i,23} = 0; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 23. \tag{A254}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(24)} \equiv a_{0g}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(24)}} = \frac{\partial N_4^{(i)}}{\partial a_{0g}} \equiv a_4^{i,24} = \omega^{(i+1)} - \omega^{(i)} + \frac{m_w^{(i)} - m_w^{(i+1)}}{m_a}; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 24. \tag{A255}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(25)} \equiv a_{1g}$ are as follows:

$$\begin{gathered}\frac{\partial N_4^{(i)}}{\partial \alpha^{(25)}} = \frac{\partial N_4^{(i)}}{\partial a_{1g}} \equiv a_4^{i,25} = \omega^{(i+1)} T_a^{(i+1)} - \omega^{(i)} T_a^{(i)} + \frac{\left(m_w^{(i)} - m_w^{(i+1)}\right) T_w^{(i+1)}}{m_a}; \\ \ell = 4;\ i = 1, \ldots, I;\ j = 25.\end{gathered} \tag{A256}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(26)} \equiv a_{0,Nu}$ are as follows:

$$\begin{gathered}\frac{\partial N_4^{(i)}}{\partial \alpha^{(26)}} = \frac{\partial N_4^{(i)}}{\partial a_{0,Nu}} \equiv a_4^{i,26} = \frac{(T_w^{(i+1)} - T_a^{(i)})}{m_a} \frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})} \frac{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})}{\partial a_{0,Nu}}; \\ \ell = 4;\ i = 1, \ldots, I;\ j = 26,\end{gathered} \tag{A257}$$

where $\frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})}$ and $\frac{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})}{\partial a_{0,Nu}}$ were defined previously in Equations (A59) and (A105), respectively.

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(27)} \equiv a_{1,Nu}$ are as follows:

$$\begin{gathered}\frac{\partial N_4^{(i)}}{\partial \alpha^{(27)}} = \frac{\partial N_4^{(i)}}{\partial a_{1,Nu}} \equiv a_4^{i,27} = \frac{(T_w^{(i+1)} - T_a^{(i)})}{m_a} \frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})} \frac{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})}{\partial a_{1,Nu}}; \\ \ell = 4;\ i = 1, \ldots, I;\ j = 27,\end{gathered} \tag{A258}$$

where $\frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})}$ and $\frac{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})}{\partial a_{1,Nu}}$ were defined previously in Equations (A61) and (A105), respectively.

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(28)} \equiv a_{2,Nu}$ are as follows:

$$\begin{gathered}\frac{\partial N_4^{(i)}}{\partial \alpha^{(28)}} = \frac{\partial N_4^{(i)}}{\partial a_{2,Nu}} \equiv a_4^{i,28} = \frac{(T_w^{(i+1)} - T_a^{(i)})}{m_a} \frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})} \frac{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})}{\partial a_{2,Nu}}; \\ \ell = 4;\ i = 1, \ldots, I;\ j = 28,\end{gathered} \tag{A259}$$

where $\frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})}$ and $\frac{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})}{\partial a_{2,Nu}}$ were defined previously in Equations (A63) and (A105), respectively.

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(29)} \equiv a_{3,Nu}$ are as follows:

$$\begin{gathered}\frac{\partial N_4^{(i)}}{\partial \alpha^{(29)}} = \frac{\partial N_4^{(i)}}{\partial a_{3,Nu}} \equiv a_4^{i,29} = \frac{(T_w^{(i+1)} - T_a^{(i)})}{m_a} \frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})} \frac{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})}{\partial a_{3,Nu}}; \\ \ell = 4;\ i = 1, \ldots, I;\ j = 29,\end{gathered} \tag{A260}$$

where $\frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})}$ and $\frac{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})}{\partial a_{3,Nu}}$ were defined previously in Equations (A65) and (A105), respectively.

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(30)} \equiv W_{dkx}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(30)}} = \frac{\partial N_4^{(i)}}{\partial W_{dkx}} \equiv a_4^{i,30} = 0; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 30. \tag{A261}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(31)} \equiv W_{dky}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(31)}} = \frac{\partial N_4^{(i)}}{\partial W_{dky}} \equiv a_4^{i,31} = 0; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 31. \tag{A262}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(32)} \equiv \Delta z_{dk}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(32)}} = \frac{\partial N_4^{(i)}}{\partial \Delta z_{dk}} \equiv a_4^{i,32} = 0; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 32. \tag{A263}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(33)} \equiv \Delta z_{fan}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(33)}} = \frac{\partial N_4^{(i)}}{\partial \Delta z_{fan}} \equiv a_4^{i,33} = 0; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 33. \tag{A264}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(34)} \equiv D_{fan}$ are as follows:

$$\begin{aligned} \frac{\partial N_4^{(i)}}{\partial \alpha^{(34)}} = \frac{\partial N_4^{(i)}}{\partial D_{fan}} \equiv a_4^{i,34} = \left(T_w^{(i+1)} - T_a^{(i)}\right) \left[\frac{1}{m_a} \frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})} \frac{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})}{\partial m_a} \frac{\partial m_a}{\partial D_{fan}} - \frac{H(m_a, \boldsymbol{\alpha})}{m_a{}^2} \frac{\partial m_a}{\partial D_{fan}} \right] \\ - \frac{(m_w^{(i)} - m_w^{(i+1)}) h_{g,w}^{(i+1)}(T_w^{(i+1)}, \mathbf{a})}{m_a{}^2} \frac{\partial m_a}{\partial D_{fan}}; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 34, \end{aligned} \tag{A265}$$

where $\frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})}$ and $\frac{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})}{\partial m_a}$ $\frac{\partial Nu(\mathrm{Re}, \boldsymbol{\alpha})}{\partial m_a}$ were defined previously in Equations (A105) and (A24), respectively, while $\frac{\partial m_a}{\partial D_{fan}}$ was defined in Equation (A71).

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(35)} \equiv \Delta z_{fill}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(35)}} = \frac{\partial N_4^{(i)}}{\partial \Delta z_{fill}} \equiv a_4^{i,35} = 0; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 35. \tag{A266}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(36)} \equiv \Delta z_{rain}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(36)}} = \frac{\partial N_4^{(i)}}{\partial \Delta z_{rain}} \equiv a_4^{i,36} = 0; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 36. \tag{A267}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(37)} \equiv \Delta z_{bs}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(37)}} = \frac{\partial N_4^{(i)}}{\partial \Delta z_{bs}} \equiv a_4^{i,37} = 0; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 37. \tag{A268}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(38)} \equiv \Delta z_{de}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(38)}} = \frac{\partial N_4^{(i)}}{\partial \Delta z_{de}} \equiv a_4^{i,38} = 0; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 38. \tag{A269}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(39)} \equiv D_h$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(39)}} = \frac{\partial N_4^{(i)}}{\partial D_h} \equiv a_4^{i,39} = \frac{(T_w^{(i+1)} - T_a^{(i)})}{m_a} \frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial D_h}; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 39, \tag{A270}$$

where $\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial D_h}$ was defined in Equation (A145).

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(40)} \equiv A_{fill}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(40)}} = \frac{\partial N_4^{(i)}}{\partial A_{fill}} \equiv a_4^{i,40} = \frac{(T_w^{(i+1)} - T_a^{(i)})}{m_a} \frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial A_{fill}}; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 40, \tag{A271}$$

where $\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial A_{fill}}$ was defined in Equation (A147).

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(41)} \equiv A_{surf}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(41)}} = \frac{\partial N_4^{(i)}}{\partial A_{surf}} \equiv a_4^{i,41} = \frac{(T_w^{(i+1)} - T_a^{(i)})}{m_a} \frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial A_{surf}}; \ell = 4;\ i = 1, \ldots, I;\ j = 41, \tag{A272}$$

where $\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial A_{surf}}$ was defined in Equation (A149).

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(42)} \equiv \mathrm{Pr}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(42)}} = \frac{\partial N_4^{(i)}}{\partial \mathrm{Pr}} \equiv a_4^{i,42} = \frac{(T_w^{(i+1)} - T_a^{(i)})}{m_a} \frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial \mathrm{Pr}}; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 42, \tag{A273}$$

where $\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial \mathrm{Pr}}$ was defined in Equation (A151).

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(43)} \equiv V_w$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(43)}} = \frac{\partial N_4^{(i)}}{\partial V_w} \equiv a_4^{i,43} = 0; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 43. \tag{A274}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(44)} \equiv V_{exit}$ are as follows:

$$\begin{aligned} \frac{\partial N_4^{(i)}}{\partial \alpha^{(44)}} = \frac{\partial N_4^{(i)}}{\partial V_{exit}} \equiv a_4^{i,44} = (T_w^{(i+1)} - T_a^{(i)}) \left[\frac{1}{m_a} \frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})} \frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial m_a} \frac{\partial m_a}{\partial V_{exit}} - \frac{H(m_a,\boldsymbol{\alpha})}{{m_a}^2} \frac{\partial m_a}{\partial V_{exit}} \right] \\ - \frac{(m_w^{(i)} - m_w^{(i+1)}) h_{g,w}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{{m_a}^2} \frac{\partial m_a}{\partial V_{exit}}; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 44, \end{aligned} \tag{A275}$$

where $\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}$ and $\frac{\partial Nu(\mathrm{Re},\alpha)}{\partial m_a}$ were defined previously in Equations (A105) and (A24), respectively, while $\frac{\partial m_a}{\partial V_{exit}}$ was defined in Equation (A86).

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(45)} \equiv m_{w,in}$ are as follows:

$$\frac{\partial N_4^{(1)}}{\partial \alpha^{(45)}} = \frac{\partial N_4^{(1)}}{\partial m_{w,in}} \equiv a_4^{1,45} = \frac{h_{g,w}^{(2)}(T_w^{(2)}, \boldsymbol{\alpha})}{m_a} = \frac{a_{1g}T_w^{(2)} + a_{0g}}{m_a}; \quad \ell = 4;\ i = 1;\ j = 45, \tag{A276}$$

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(45)}} = \frac{\partial N_4^{(i)}}{\partial m_{w,in}} \equiv a_4^{i,45} = 0; \quad \ell = 4;\ i = 2, \ldots, I;\ j = 45. \tag{A277}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(46)} \equiv T_{a,in}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(46)}} = \frac{\partial N_4^{(i)}}{\partial T_{a,in}} \equiv a_4^{i,46} = 0; \quad \ell = 4;\ i = 1, \ldots, I-1;\ j = 46, \tag{A278}$$

$$\begin{aligned} \frac{\partial N_4^{(I)}}{\partial \alpha^{(46)}} = \frac{\partial N_4^{(I)}}{\partial T_{a,in}} \equiv a_4^{I,46} = C_p^{(I)}\left(\frac{T_a^{(I)}+273.15}{2}, \boldsymbol{\alpha}\right) + \omega_{in}\frac{\partial h_{g,a}^{(I+1)}(T_{a,in},\boldsymbol{\alpha})}{\partial T_{a,in}} \\ = C_p^{(I)}\left(\frac{T_a^{(I)}+273.15}{2}, \boldsymbol{\alpha}\right) + \omega_{in}a_{1g}; \quad \ell = 4;\ i = I;\ j = 46. \end{aligned} \tag{A279}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(47)} \equiv m_a$ are as follows:

$$\begin{aligned} \frac{\partial N_4^{(i)}}{\partial \alpha^{(47)}} = \frac{\partial N_4^{(i)}}{\partial m_a} \equiv a_4^{i,47} = -\left(m_w^{(i)} - m_w^{(i+1)}\right)\frac{h_{g,w}^{(i+1)}(T_w^{(i+1)},\boldsymbol{\alpha})}{{m_a}^2} \\ + \left(T_w^{(i+1)} - T_a^{(i)}\right)\left[\frac{1}{m_a}\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial m_a} - \frac{H(m_a,\boldsymbol{\alpha})}{{m_a}^2}\right]; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 47, \end{aligned} \tag{A280}$$

where $\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}$ and $\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial m_a}$ were defined previously in Equations (A24) and (A105), respectively.

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(48)} \equiv \omega_{in}$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(48)}} = \frac{\partial N_4^{(i)}}{\partial \omega_{in}} \equiv a_4^{i,48} = 0; \quad \ell = 4;\ i = 1, \ldots, I-1;\ j = 48, \tag{A281}$$

$$\frac{\partial N_4^{(I)}}{\partial \alpha^{(48)}} = \frac{\partial N_4^{(I)}}{\partial \omega_{in}} \equiv a_4^{I,48} = h_{g,a}^{(I+1)}(T_{a,in}, \boldsymbol{\alpha}); \quad \ell = 4;\ i = I;\ j = 48. \tag{A282}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(49)} \equiv \mathrm{Re}_d$ are as follows:

$$\begin{aligned} \frac{\partial N_4^{(i)}}{\partial \alpha^{(49)}} = \frac{\partial N_4^{(i)}}{\partial \mathrm{Re}_d} \equiv a_4^{i,49} = \frac{(T_w^{(i+1)} - T_a^{(i)})}{m_a}\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re}_d,\boldsymbol{\alpha})}\frac{\partial Nu(\mathrm{Re}_d,\boldsymbol{\alpha})}{\partial \mathrm{Re}_d}; \\ \ell = 4;\ i = 1, \ldots, I;\ j = 49, \end{aligned} \tag{A283}$$

where $\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}$ was defined in Equation (A105) and $\frac{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}{\partial \mathrm{Re}_d}$ was defined in Equation (A93).

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(50)} \equiv Sc$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(50)}} = \frac{\partial N_4^{(i)}}{\partial Sc} \equiv a_4^{i,50} = 0; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 50. \tag{A284}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(51)} \equiv Sh$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(51)}} = \frac{\partial N_4^{(i)}}{\partial Sh} \equiv a_4^{i,51} = 0; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 51. \tag{A285}$$

The derivatives of the air/water vapor energy balance equations [cf. Equations (A10)–(A12)] with respect to the parameter $\alpha^{(52)} \equiv Nu$ are as follows:

$$\frac{\partial N_4^{(i)}}{\partial \alpha^{(52)}} = \frac{\partial N_4^{(i)}}{\partial Nu} \equiv a_4^{i,52} = \frac{\left(T_w^{(i+1)} - T_a^{(i)}\right)}{m_a} \frac{\partial H(m_a, \boldsymbol{\alpha})}{\partial Nu}; \quad \ell = 4;\ i = 1, \ldots, I;\ j = 52, \tag{A286}$$

where $\frac{\partial H(m_a,\boldsymbol{\alpha})}{\partial Nu(\mathrm{Re},\boldsymbol{\alpha})}$ was defined in Equation (A105).

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