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Wave Propagation in Metamaterial using Multiscale Resonators by Creating Local Anisotropy

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WAVE PROPAGATION IN METAMATERIAL USING MULTISCALE RESONATORS BY CREATING LOCAL ANISOTROPY

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Abstract

Directional guiding, passing or stopping of elastic waves through engineered materials have many applications to the engineering fields. Recently, such engineered composite materials received great attention by the broader research community. In elastic waves, the longitudinal and transverse motion of material particles are coupled, which exhibits richer physics and demands greater attention than electromagnetic waves and acoustic waves in fluids. Waves in periodic media exhibit the property of Bragg scattering and create frequency band gaps in which the energy propagation is prohibited. However, in addition to the Bragg scattering, it has been found that local resonance of artificially designed resonators can also play a critical role in the generation of low-frequency band gaps. It has been found that negative effective mass density and negative effective elastic modulus are created by virtue of the local resonators and are correlated with the creation of the frequency band gaps that can be artificially perturbed.

In this paper, the authors present a novel anisotropic design of metamaterial using local split-ring resonators of multiple-length scales. Unlike traditional metamaterials, multiple split rings of different dimensions are embedded in a polymer matrix. Considering the complexity of the proposed material, it is extremely difficult to find the dynamic response of the material using analytical methods. Thus, a numerical simulation was performed in order to find frequency band gaps. Simultaneously, correlation between the band gaps and negative effective mass density and negative effective elastic modulus was verified. Both unidirectional split rings and bidirectional chiral split rings were studied. The effects of discontinuity in the rings at larger scales were compared with the dynamic characteristics of full rings in the proposed metamaterial. Application of such metamaterials will be primarily for vibration isolation and impact mitigation of structures. The proposed configuration is based on unit dimension and is, thus, dimensionless. The concept can be easily commutable between macro-scale structures for low-frequency applications and micro-scale MEMS devices for high-frequency applications.

Introduction

Sonic and ultrasonic wave propagation through periodic structures has been the subject of interest for many years. Recently, this topic received greater attention in solid-state physics and acoustics. Periodic media has a unique capability of being able to manipulate wave propagation by exhibiting innumerous possibilities, e.g., creating directional guiding, frequency filtration, negative dispersion and many other novel phenomena yet to be discovered. Such properties of the periodic media have triggered many new ideas of possible applications to different engineering fields. Periodic elastic structures are capable of producing frequency band gaps, where apparently no wave energy can be transmitted through the structures; however, energy is not actually lost but is in fact absorbed by the local resonators. A significant amount of research has been conducted in recent years on the manipulation of frequency band gaps in different frequency ranges through an innovative design of the acousto-elastic metamaterials.

Acoustoelastic metamaterials and a novel arrangement of phononic crystals are used for vibration control, impact mitigation and acoustic wave manipulation. Structures (e.g., civil, mechanical and aerospace) or individual structural components are vulnerable to external vibration, especially when the external vibration frequencies are at the proximity of the natural frequencies of the structures. The natural frequencies of a system are known during the design of the structural systems. Thus, if possible, the proposed metamaterials can be placed on the structures and the design of the metamaterials can be tuned in such a way that the external vibration frequencies close to the natural frequencies of the structure will be completely damped, keeping the structure safe. Other possible applications of metamaterials are evident in naval research. Metamaterials are capable of guiding and bending acoustic waves; for example, sonar radiated waves can be twisted and guided so that they will never be reflected back to the enemies. And while a perfect acoustic clock could be built, it is admittedly a rather farfetched futuristic application. These acoustoelastic metamaterials are classified into two broader categories. One set of metamaterials is made by changing the structural geometry by processes such as grooving, cutting, etc.; in other words, perturbing the surface texture of the structures. On the other hand, metamaterials can also be made by designing novel inclusion patterns in the host matrix of the materials and considering them composite or engineered materials.

El-Bahrawy [1] and Banerjee et al. [2-4] studied wave propagation in periodic wave guides, where the surfaces of the elastic media (wave guides) were proposed to be perturbed sinusoidally without creating any inclusions inside the material (metamaterial of type 1). Thus, surface perturbations (surface etching) and volume perturbations (inclusions) are completely different genre of metamaterials. Here, the metamaterials with volume perturbation is discussed. Photonic researchers are also engaged in designing electromagnetic metamaterials [5-7] in the exploration of photonic band gaps, a range of frequencies where electromagnetic waves cannot propagate [8]. The same is true for the physical understanding of stress (elastic) wave propagation in phononic crystals and electromagnetic wave propagation in photonic crystals [9-11]. Thus, the concepts of designing newly engineered materials are commutable between electrodynamics and elastodynamics. As a rule of thumb, mass-in-mass systems are frequently proposed for elastodynamic problems to predicatively manipulate the frequency band gaps with engineered volume inclusion (metamaterials of type II). Applications of such acoustic metamaterials have been envisioned for acoustic cloaking, vibration control, sound isolation, etc. [12], [13].

Band-gap manipulation is very important in periodic structures in order to diversify their applicability. The formation of a negative bulk modulus and negative effective mass densities can result in band gaps in periodic structures [14-16]. Negative effective mass density arises from the negative momentum of the unit cell with a positive velocity field [17]. In mass-in-mass unit cells, the effective mass of the cell becomes negative at frequencies near the local resonance frequency, due to the special decay in wave amplitudes [18]. Hsu [19] showed that frequency band gaps can be formed by multiple scattering of the periodic inclusions, also known as Bragg scattering. Low-frequency sounds can be controlled by introducing locally resonant components into phononic crystals [20-22]. Generally speaking, a locally resonant medium consists of a heavy material core coated with softer material and then embedded into a host of other media. There are always multiple possibilities for opening new low-frequency forbidden gaps by setting an engineered local resonator in phononic crystals. Band gaps can be obtained or manipulated by altering the geometry of the local resonators [19].

Table 1. Material Properties

Material	Young's Modulus (GPa)	Density (kg/m^3)	Poisson's Ratio
Steel	205	7850	0.28
Epoxy	2.35	1110	0.38

Recently, researchers have attempted to open new portals of band gaps by artificially designing the metamaterial system. Huang et al. [14] proposed a multi-resonator system by introducing an additional mass in a previously proposed one -resonator mass-in-mass model [18]. Similarly a layer-in-layer system was proposed by Zhu et al. [23] to calculate the effective dynamic properties of the finite acoustic metamaterial. Low-frequency or high-frequency (high or low wavelengths, respectively) bands were already engineered by the proposed models. A model that can devise a wider band of frequency gaps (at both low- and high-frequency regions) is desired. If by any means higher numbers of closely spaced band gaps are obtained, they can eventually be merged by manipulation. Such multiple band gaps could potentially form a wider band gap. Such possibilities were not demonstrated by any previous models. In this study, then, a novel split-ring metamaterial was proposed. The proposed model not only obtains wider frequency bands but also multiple band gaps in both low- and high-frequency regions. The proposed unit cell is composed of a multiscale mass-in-mass system that forms a multiscale mass-in-mass model (MMM), which was proposed by any researchers. A two-dimensional study was performed and the full structure was considered infinite with periodic unit cells.

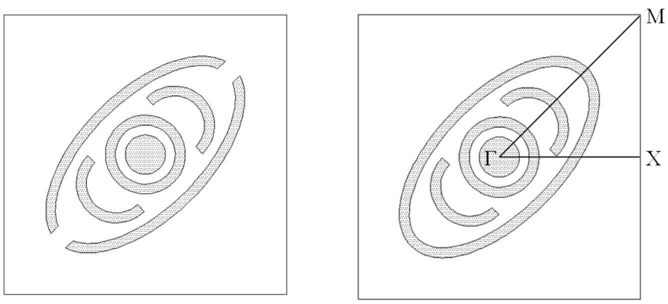
The concept of split rings was inspired by its counterpart from electromagnetic waves in photonics. Guenneau et al. [24] proposed a double 'C' resonator for wave focusing and confinement. Movchan et al. [25] used split-ring resonators to control electromagnetic bands in two-dimensional photonic structures. Many other photonic researchers found split rings useful in manipulating electromagnetic waves for specific purposes. To the best of authors' knowledge, split-ring resonators were introduced into the acoustic wave arena for the very first time in this study.

A few possible futuristic applications of the proposed metamaterials were mentioned in previous paragraphs. However, in the near term, this research will likely have a significant impact on the design of innovative microphones for directional sensitivity of sounds [26]. Speech recognition by reducing noise will improve the quality of synthesized sound for clean speech recognition in transformed domains [27]. Similar to the concept of Filtered-XRLS, an algorithm with sequential updates [28] can be used to optimize the design of appropriate acoustic metamaterial filters, which will eventually help active acoustic noise control. In this paper, a step towards the engineering design of such a metamaterial is presented.

Numerical Implementation

A two-dimensional structure with a multiple-resonator system was proposed in this study. Figure 1a illustrates the schematic of the unit cells. One unit cell is composed of a steel core with a diameter of 0.1414" embedded in a circular ring with an outside diameter of 0.2828". A softer material (epoxy) was used to seal the space between the steel ball and the circular ring. A similar mass-in-mass system was proposed by Huang et al. [18].

In this paper, an additional set of semicircular rings was placed symmetrically in order to increase the number of band gaps. To generate access to the new portal in order to manipulate further frequency bands, another pair of elliptical split rings was positioned symmetrically but orthogonally to the previously positioned semicircular rings. All of the rings were made of steel with a thickness of 0.037" and included in a softer material. For simplicity, the steel cores, elastic coating and the split rings were considered embedded in an epoxy matrix. Material properties of the unit cell are listed in Table 1. Additional studies were performed in order to analyze the geometrical dependency of elliptical split rings compared to elliptical full rings (see Figure 1b).



(a) Split Ellipse **(b) Full Ellipse**

Figure 1. Proposed Multiple Resonator Unit Cell

The proposed model was fairly complicated for performing analytical solutions for obtaining the dispersion curves. The eigenvalue analysis (dispersion) was performed using Finite Element simulation. The Floquet periodic boundary condition was enforced at all four boundaries of the unit cell. Floquet boundary conditions can be written as ($x + dx$, $y + dy$) = $F(x,y)$, where dx and dy are the period length of the periodic media along x and y directions, respectively. Floquet boundary conditions are based on the Floquet theory, which can be applied to the problem of small-amplitude vibrations of spatially periodic structures. A Bloch solution [29] for the frequency response to a small-amplitude time-periodic excitation that also possesses spatial periodicity can be sought in the form of the product of two functions. One follows the periodicity of the structure, while the other follows the periodicity of the excitation. The problem can be solved on a unit cell of periodicity by applying the corresponding periodicity conditions to each of the two components in the product. The Floquet periodicity conditions at the corresponding boundaries of the periodicity cell are expressed as

$$u_T = u_S e^{(i\mathbf{k}(x_t - x_s) - i\omega t)} \tag{1}$$

where $\boldsymbol{u}$ is the displacement vector and vector $\mathbf{k}$ represents the spatial periodicity of the excitation or the wave number. $\boldsymbol{T}$ stands for target and $\boldsymbol{S}$ stands for source element. $\mathbf{x_t} - \mathbf{x_s}$ $\mathbf{k}.$ $\mathbf{x_t} - \mathbf{x_s}$ calculates the distance between source and target and, thus, $\mathbf{k}$ provides the phase difference between them. The general linear relation between the stress and strain tensors in solid materials at x_m is expressed by Hook's law:

$$\sigma_{ij} = C_{ijkl}(x_m)\varepsilon_{kl} \tag{2}$$

Here, σ is the Cauchy's stress tensor, ε is the strain tensor and C_{ijkl} is a fourth-order elasticity tensor. For small deformations, the strain tensor is defined as

$$\varepsilon_{kl} = \frac{1}{2}(u_{k,l} + u_{l,k}) \tag{3}$$

where i, j, k and l take on the values of 1, 2 and 3; typical index notation applies. u_i represents particle displacement along i. The elastic wave equation is then expressed as

$$\frac{\rho(x_m)(\partial^2 u_i)}{\partial t^2} - \nabla \cdot (C_{ijkl}(x_m)\varepsilon_{kl} - s_i) = F_i \tag{4}$$

Here, $\rho(x_m)$ is the medium density, which is a function of space; $\boldsymbol{s}$ and $\boldsymbol{F}$ represent the source terms. If a time-harmonic wave is assumed, the displacement function can be written as

$$u(x, \mathbf{k}, t) = \boldsymbol{u}(x, k)e^{i\omega t} \tag{5}$$

where f represents the frequency and $\omega = 2\pi f$ is the angular frequency. Assuming the same time-harmonic dependency for the source terms $\boldsymbol{s_0}$ and $\boldsymbol{F}$, the wave equation for linear elastic waves reduces to an inhomogeneous equation:

$$-\omega^2\rho(x_m)\boldsymbol{u}_i - \nabla \cdot \left(C_{ijkl}(x_m)\varepsilon_{kl} - \boldsymbol{s}_i\right) = F_i \tag{6}$$

Alternatively, eigenmodes and eigenfrequencies can be solved by treating this Helmholtz equation as an eigenvalue partial differential equation. Eigenfrequency equations are derived by assuming a harmonic displacement field, similar to the frequency response formulation. The only difference is that the eigenfrequency study uses a new variable, $j\omega$, explicitly expressed in the eigenvalue $j\omega = -\lambda$. The eigenfrequency f is then derived from $j\omega$ as

$$f = \left|\frac{\mathrm{Im}(j\omega)}{2\pi}\right| \tag{7}$$

A dispersion relation was obtained by discretizing the wave number domain from zero to ka/π, where 'a' is the periodicity of the unit cell and '**k**' is the fundamental or incident wave number. Please note that, according to the Bloch-Floquet theorem, any solution of wave number $(\tilde{k})$ in periodic media can be written as

$$\tilde{k} = k + \mathbf{G} \tag{8}$$

$$\mathbf{G} = \frac{2\pi n}{a}\hat{x} + \frac{2\pi m}{b}\hat{y} \tag{9}$$

where n and m take on the values 1,2,3,…∞. a and b are the periodicity of the unit cell along the x and y directions, respectively. In this study, $a = b$. **G** is the reciprocal basis of the proposed periodic media.

Dispersion Relation and Geometric Dependency

The primary objective of this study was to obtain multiple band gaps at both low- and high-frequency regions. Frequencies up to 180 kHz were analyzed in this study. Frequencies less than 50 kHz were considered to be in the low-frequency region and 50–180 kHz frequencies were considered to be in the high-frequency region. Figure 2 shows the dispersion relation for the multi-resonant system proposed in Figure 1a. The total band structure was computed for the ΓXMΓ boundary (see Figure 1b). Three band gaps were observed in the low-frequency region from 20.83 to 22.07 kHz, 27.22 to 29.94 kHz and 36.72 to 37.47 kHz, with bandwidths of 1245 Hz, 2723 Hz and 743 Hz, respectively.

To understand the explanation of band-gap formation in gap frequencies, a frequency-domain analysis was performed. A uniformly distributed compressive load of magnitude 1 N (P in Figure 3a) was applied on both sides of the unit cell along the x direction. According to load-deformation laws, one can expect to observe compressive deformation in unit cells along the x direction. At an arbitrary frequency of 28.5 kHz (a frequency within the band gap 27.22 to 29.94 kHz) it was observed that the unit cell tends to elongate along the x direction. Such an unusual phenomenon signifies the formation of negative bulk modulus at frequencies where band gaps exist and verifies the established hypothesis on creation of resonant band gaps.

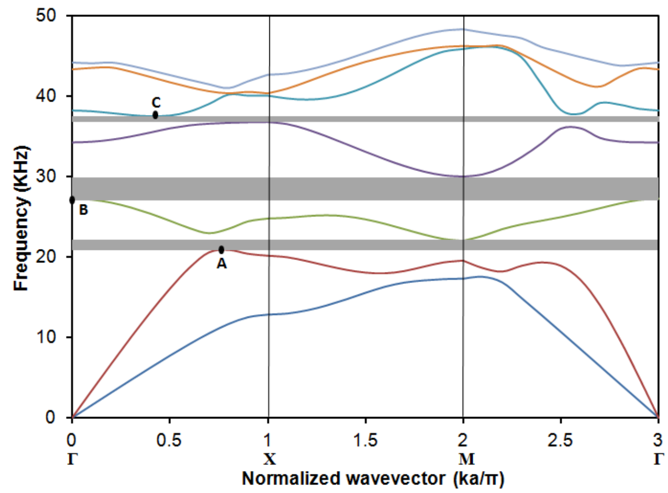


Figure 2. Dispersion Relation with Split Ellipse Resonator

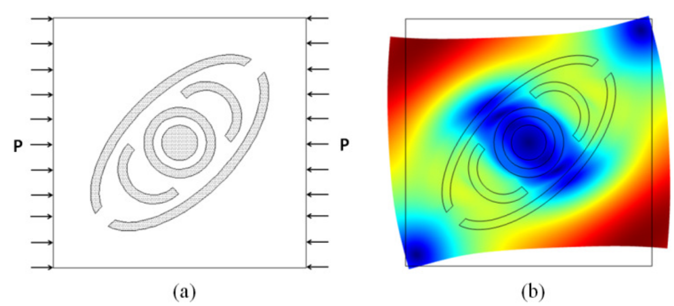


Figure 3. (a) Loading Setup in Unit Cell and (b) Displacement Mode at f = 28.5 kHz

Another objective of this study was to understand the geometric effect of elliptical resonators. Both the models with split rings (model 1) and full-ring resonators (model 2) were introduced in Figure 1. The full-ellipse resonator model also produced three band gaps from 20.59 to 22.7 kHz, 26.64 to 30.37 kHz and 37.17 to 37.9 kHz, with bandwidths of 2118 Hz, 3726 Hz and 729 Hz, respectively (see Figure 4). Both proposed models generated three low-frequency band gaps for the ΓXMΓ boundary. No high-frequency band gaps were observed. In model 2, each band gap was formed in more or less the same frequency ranges, compared to model 1. However, higher bandwidths were noticed for full-ellipse configurations for the first two band gaps, while the third band gap thickness was relatively unchanged in both models.

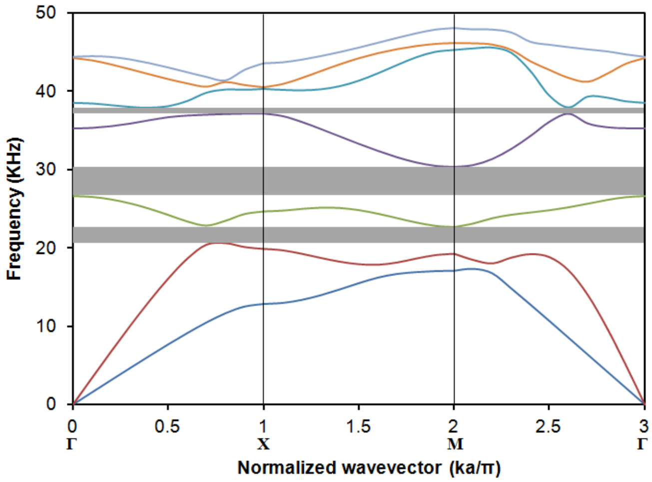
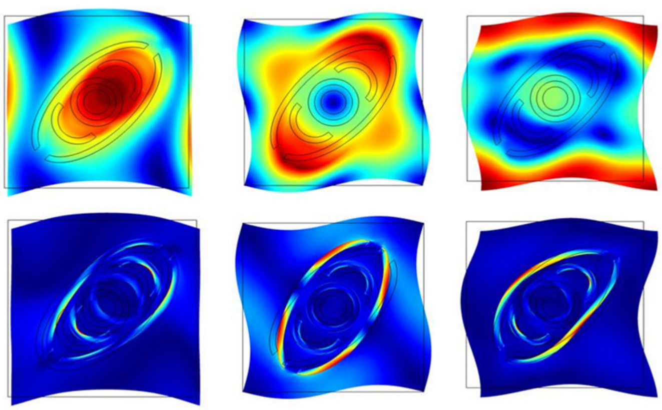


Figure 4. Dispersion Relation with Full Ellipse Resonator

Vibration in steel-core and steel resonators created the band modes in the first band gap. Steel-ball and closed steel rings were the dominating resonators in these bands and elliptical rings were vibrating with smaller amplitudes. Plate vibration was also coupled with these modes. In the second band gap, resonances at elliptical rings were more prominent than the first mode. Since elliptical rings are responsible for both first and second band mode resonances, a change in resonator geometry affected the bandwidth. Vibration modes at location A & B (see Figure 2) is shown in Figures 5a and 5b. A full ellipse can absorb more energy than a split resonator, which results in higher bandwidth at the second band gap in the full-ellipse model. In the third gap, very little vibration was observed in the elliptical rings (see Figure 5c), which signifies almost the same bandwidth for both models.



(a) (b) (c)

Figure 5. Displacement (top) and Von Mises Stress (bottom) Mode at (a) A, (b) B and (c) C Positions (see also Figure 2)

Band gaps over the full frequency range (0-180 kHz) for the ΓXMΓ boundary and their dependency on elliptical resonators are reported in Figures 2, 4 and 5. The geometric influence of elliptical resonators in the Γ-X, X-M and M-Γ directions were also explored individually.

A comparative study was conducted and the band structures obtained from the models are presented in Figures 6, 7 and 8. A remarkable number of band gaps, more than 20, were achieved in both the Γ-X and X-M directions for each model. In this paper, bandwidths higher than 700 Hz are reported. In the figures shown in this paper, the y axis represents the location of the band gaps along the frequency axis and the x axis represents the frequency bandwidth of the band gaps. Larger band gaps were observed at low frequencies in all directions. In the low-frequency region, frequency bands were generated very close to each other. Larger band modes were mostly dependent on elliptical ring resonators. Thus, geometric alteration will help to manipulate the band structure. It was also observed that the local resonances were a function of the incident wave direction.

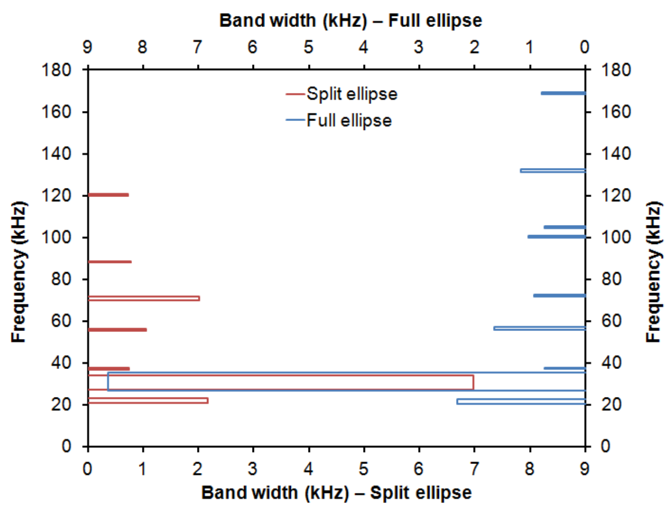


Figure 6. Band-Gap Comparison for Split and Full Ellipse Resonator in Γ-X Direction

At higher frequencies, significant numbers of smaller band gaps were observed (in both the Γ-X and X-M directions). These band gaps were strongly influenced by the cell geometry. Different frequency ranges were more pronounced for different geometries. Epoxy modes were mainly dominant in the high-frequency regions. Low-amplitude vibrations of elliptical and half-circular resonators were also coupled with those modes. Since epoxy modes were dominant at higher frequencies, little scattering can be achieved, which is another reason to get smaller band gaps. Using this understanding, a new orientation was proposed to expand the first band gap at the lower frequencies. The proposed configuration is shown in Figure 9.

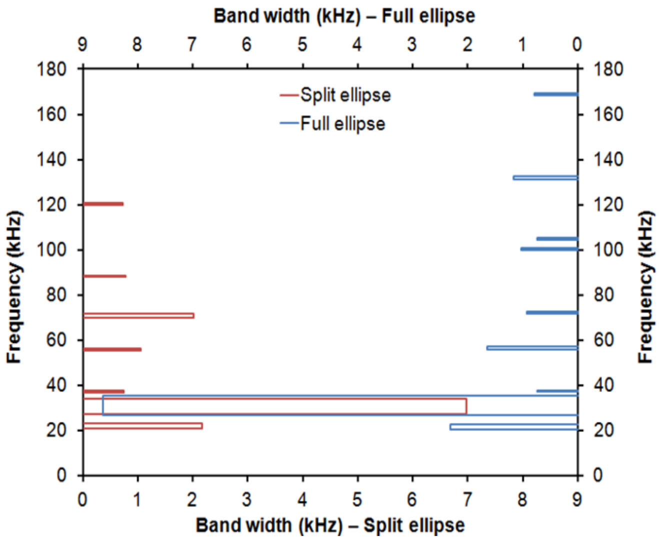


Figure 7. Band-Gap Comparison for Split and Full Ellipse Resonator in X-M Direction

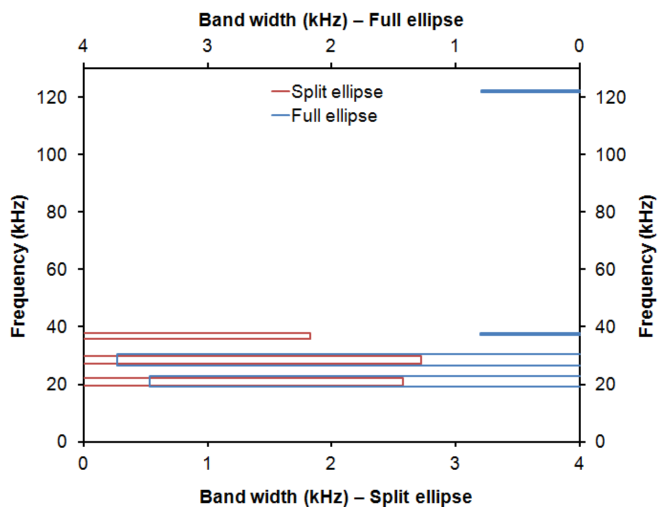


Figure 8. Band-Gap Comparison for Split and Full Ellipse Resonator in M-Γ Direction

A similar eigenvalue analysis of the system was performed and the dispersion relation between frequency and wave number for the proposed 2D mirror image pattern is shown in Figures 10 and 11. Figures 10 and 11 show the dispersion curves in the Γ-X, X-M and M-Γ directions for both the split-ellipse and the full-ellipse configurations, respectively.

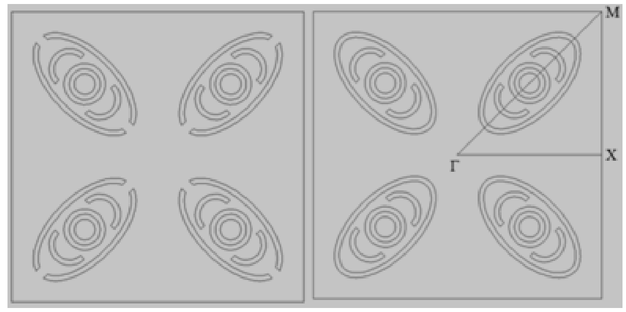


Figure 9. Proposed Multiple Diagonally Symmetric Resonator Unit Cell with Split-Ring Ellipse (left) and Full-Ring Ellipse (right)

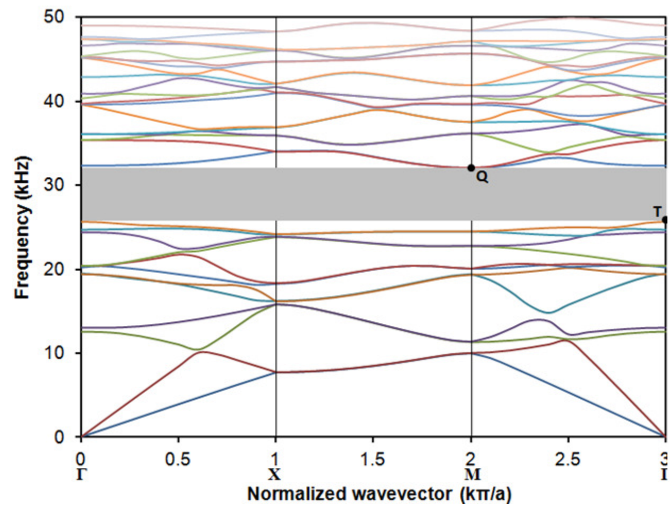


Figure 10. Dispersion Relation for the Split Ellipse Resonators

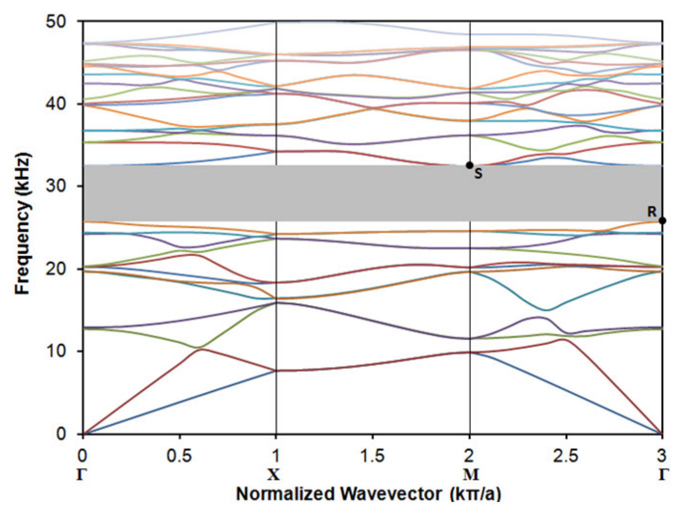


Figure 11. Dispersion Relation for the Full Ellipse Resonators

The dispersion relations show the existence of absolute band gaps below 50 kHz. These band gaps are essentially wider (between 24 kHz and 33 kHz) than the band gaps obtained in Figures 2 and 3. Figure 12 shows the displacement and Von Mises stress pattern for the modes T and Q (see Figure 10). Figure 13 shows the complete band structure of the diagonally symmetric pattern between 0–180 kHz along the X-M direction. It clearly shows that the number of band gaps is improved by the split-ring configuration if organized in a diagonally symmetric (mirror image) pattern of the unit cells, which was originally proposed in Figure 1 over a full-ring configuration.

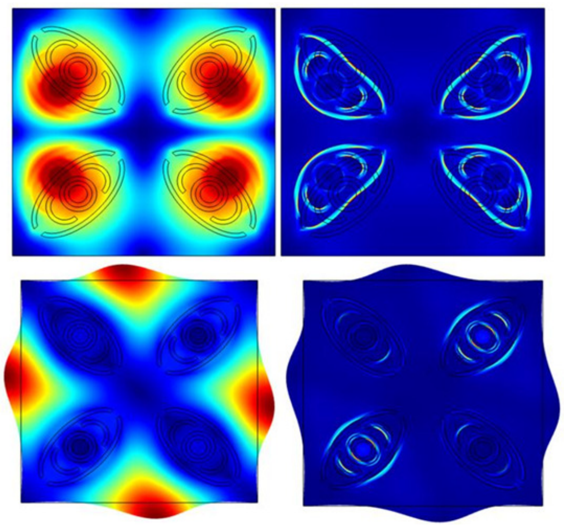


Figure 12. Displacement (left column) and Von Mises Stress (right column) for Mode T (top rows) and Mode Q (bottom rows)

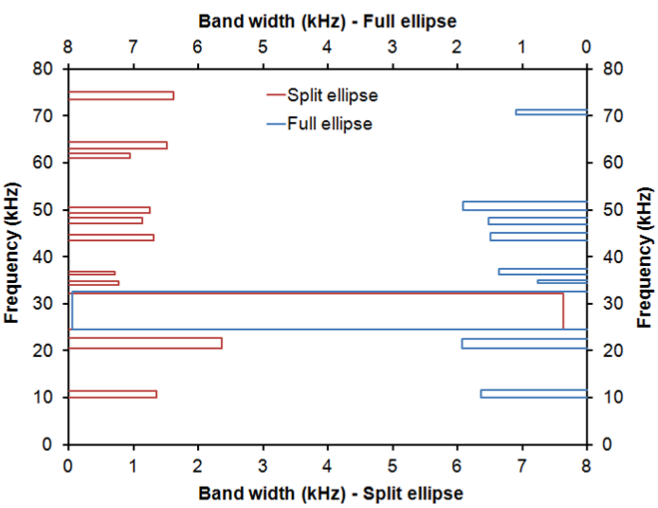


Figure 13. Band-Gap Comparison for Split and Full Ellipse Resonators in Diagonally Symmetric Configuration in X-M Direction

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Conclusion

Obtaining multiple band gaps by designing a novel configuration of metamaterials is one of the key areas of interest in recent days. Mass-in-mass and layer-in-layer models were proposed by previous researchers for finding multiple band gaps. But the number of band gaps was not significantly increased by these models. In this current study, a novel multi-scale mass-in-mass (MMM) system containing split-ring resonators for achieving several band gaps in both low- and high-frequency regions was proposed. Split-ring resonators are frequently used for guiding electromagnetic waves; however, it was initiated in order to implement this idea for guiding the acoustic waves for the first time. Three distinct band gaps were obtained (ΓXYΓ boundary) from the proposed model. However, multiple and wider band gaps were achieved for wave propagation along the direction other than the orthogonal directions. Wider band gaps were visible in the low-frequency region, while other smaller band gaps were noticed in the high-frequency region. It was also found that the frequency bands were influenced by the geometry of the elliptical resonators. Band position and width can be manipulated with the change of resonator geometry. A diagonally symmetric orientation of the unit cell was able to drastically improve the extent of the band gaps. This study can be useful to isolate or guide different directional wave and control the vibration over a wide range of frequencies, which can be a useful note for structural health monitoring.

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