The Pósa Method: Students’ Perceptions of Hungarian Mathematics Pedagogy

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THE PÓSA METHOD:
STUDENTS' PERCEPTIONS OF HUNGARIAN MATHEMATICS PEDAGOGY

By

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Abstract

This study examines the reactions and experiences of students who are educated under the Pósa method, a Hungarian mathematics pedagogy previously employed in mathematics camps designed for gifted students. The focus of this research was to analyze the experiences of students in a Pósa method class in Budapest, Hungary. The holistic approach of gathering historical, qualitative, and analytical data pursued answers to the subsequent three research questions that guided the study: What do the students at Petrik School think about the Pósa method? What do the students in the Pósa method class think about math? What do the students’ and teachers’ comments reveal about the structure of the method? This qualitative phenomenological study inspects the beliefs students hold towards the Pósa method. Their perceptions support the implementation of additional Pósa method classes in Hungary as well as around the world. Moreover, implications of the study recommend further discussions on the execution of gifted pedagogy being applied in all types of classrooms, particularly those with disadvantaged students.

*Keywords:* Pósa method, discovery learning, student perceptions, gifted mathematics pedagogy
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Introduction

Hungary is well known for its excellence in mathematics and the sciences. The country prides itself in its ability to build mathematical talent, particularly through special math programs designed for gifted students. The Hungarian gifted math programs have been around for decades and have proven successful in furthering students in mathematics and building problem-solving skills. These programs provide hope and inspiration for mathematics education in Hungary: by implementing the effective pedagogy from these programs into all classrooms, students of all backgrounds may learn to thrive in mathematics. The idea that all students should be educated under effective pedagogy is simple and inspiring; the implementation is more challenging.

The Pósa method, named after creator Lajos Pósa the Hungarian mathematician and advocate for math education, is one such effective pedagogy for teaching mathematics. It has recently been implemented and is currently being researched in three regular\(^1\) education math classrooms in Hungary. The foundation of the Pósa method is discovery learning, where a “learner draws upon past knowledge and experience to infer underlying strategies and gain an understanding of concepts” (Honomichl & Chen, 2012, pg. 615). Students of the method are expected to be active participants in the learning process. As it has been used in gifted math camps since the 1980s, the Pósa method has strong evidence for turning out successful math students, yet it has never been used in general education. The method was confined to math camps for gifted math students until its latest introduction and experimentation in three math classes in Hungary, led by a group of Hungarian teachers and researchers. These researchers of the MTA-Rényi Research Group on Discovery Learning in Mathematics (MTA-Rényi research

\(^1\) The terms “regular” or “general” education students will be used interchangeably in this thesis. Both terms indicate students who are enrolled in non-gifted classes. The terms do not necessarily mean the student is not gifted, simply that they are not placed on gifted tracks.
group) have been analyzing whether the Pósa method could be integrated successfully in regular education classes for disadvantaged students\(^2\). Having started in 2016, this project is part of a Content Pedagogy Research Program, which has been specifically looking analyzing if discovery learning techniques in Pósa method are feasible and applicable for students in public education. The purpose of the study conducted by the MTA-Rényi research group is to “make mathematics teaching more interesting, exciting, and effective” in school “by using innovative methods” (Juhász, 2016, pg. 8). The researchers have hopes that their study will play “an important role in efforts to create equal chances for all” in education (Juhász, 2016, pg. 13).

I learned of the Pósa method through a study abroad program - Budapest Semester in Mathematics Education - which I attended in the spring of 2020. Several researchers in the MTA-Rényi research group were my professors in the study abroad program and introduced me to their research on the Pósa method. Their research included a case study of a Pósa method classroom at Petrik Lajos Bilingual Vocational School of Chemistry, Environmental Protection, and Information Technology (*Petrik-Lajos Két Tanítási Nyelvű Vegyipari, Környezetvédelmi és Informatikai Szakközépiskola*) referred to as “Petrik School” throughout the thesis. Petrik School is a Hungarian-English bilingual high school in Budapest, Hungary. As an aspiring high school mathematics teacher, the incorporation of the Pósa method at Petrik School appealed to my interests as a future secondary mathematics educator interested in mathematics pedagogy. Seeing as the Pósa method has not been widely studied in academic research due to its novelty, it was the perfect new topic to introduce myself to the world of mathematics education research.

\(^2\) The term “disadvantaged student” comes from the research proposal written by Juhász (2016) to the Hungarian Government, requesting permission to introduce the Pósa method in public education. “Disadvantaged students”, in this case, references students who are not members of the gifted math classes nor camps. These students are participants in general education, disadvantaged by their lack of access to higher-quality education that is often associated with gifted classes.
Consequently, I developed a study, delving into the opinions that students hold about the Pósa method and the implications of their reactions. The aim of my thesis is to explore the student understanding and appreciation of the Pósa method through student narratives. The hope is that this research will draw questions, considerations, and curiosity about the Pósa method, Hungary, and the use of gifted pedagogy with all types of students.

In this study, I address the student experiences as a result of the application of gifted pedagogy, specifically the Pósa method, in regular education. Through conversations had with the students at Petrik School in Hungary, the benefits and consequences of this gifted pedagogy are brought to light. Seeing and hearing students voice their enjoyment of mathematics as well as the joy of discovery brought to light why the Pósa method could be considered a viable method of teaching mathematics to secondary students in Hungary. While the research focused on one classroom in Hungary, we can begin to see how the concept of gifted pedagogy may be generalized for all students. As a review of the literature shows, the concept of using gifted pedagogy among “typical” students is not new. Given the parallel conversations and implementations of student-centered pedagogies in education, like inquiry-based learning and problem-based learning, this study of the Pósa method in a regular education classroom is one from which all educators could learn from.

Statement of the Problem

Throughout its history in gifted math camps, the Pósa method has proven to be effective in its ability to generate and mold intelligent mathematicians across Hungary. Specifically, alumni of the camps have been known to be successful in international math competitions. While the method has not undergone scholarly review, data from these international competitions as well as verbal praises from the creators of the method have portrayed the Pósa method in a
positive light, making it a striking method to discuss among educators. Given its decades-long success, the Pósa method is a prime method to test among students who are not labeled ‘gifted’. Testing the Pósa method with regular education students aligns with the educational conversations in the United States and worldwide about gifted education for all. Discovery learning, as a part of the Pósa method, has parallel ideas with inquiry-based learning and problem-based learning strategies, especially the former. An analysis of the Pósa method, particularly through the eyes of the students who use it on a daily basis, provides insight into why the method has positive results and why further research may prove that it could be applied in the general Hungarian curriculum.

**Research Questions**

I set out to gather and assess the opinions that students hold towards a Hungarian mathematics pedagogy called the Pósa method. An examination of the history of mathematics education in Hungary, and an analysis of classroom observations, interviews, and a questionnaire were essential aspects of the research in order to embody the students’ experiences in a Pósa method class. To examine and understand the Petrik School and Pósa Method students’ perspectives, the study focuses on the following research questions:

1. What do the students at Petrik School think about the Pósa method?
2. What do the students in the Pósa method class think about math?
3. What do the students’ and teacher’s comments reveal about the structure of the method?

**Theoretical Framework**

Informed by a phenomenological framework, this thesis explored how students made sense of their experiences using the Pósa method in their mathematics class. The dictations made by the students helped fill the lack of scholarly research on the Pósa method. Given that this
study is qualitative, a phenomenological framework fits the aims and objectives set out at the implementation of the study. The focus of the study was on student perceptions of the Pósa method to ensure the voices of the students were embodied adequately. An analysis of learner motivation, perceived beliefs about mathematics, and development of mathematical talent in the classroom were involved in the interpretation of their experiences using the Pósa method.

**Methodology**

I engaged in a qualitative phenomenological case study to craft a narrative of the opinions that the students held in the Pósa method classroom. The focus of this research was to analyze the experiences of the students in the Pósa method class, and explore student insights about the method, specifically, and mathematics education, generally. Successful and agreeable attributes of the method, as reflected in students’ perceptions, were considered as well as negative perceptions of the Pósa method. There were multiple methods I employed to collect data for the study. Through studying the history of mathematics education in Hungary, I was informed of the practices and changes that the country has undergone, especially in the last century. Data collection consisted of various formats: observations of the classroom for a week, journals from the students while they worked on a series of prepared problems, a questionnaire given at the end of the observational week, a test based on Pósa method problems, and a series of five interviews with the students as well as the teacher. Lastly, a thorough data analysis through coding of interviews, the questionnaire, and journals was conducted, examining the commonalities and differences among student responses.

**Participants**

The participants of the study were the teacher and students enrolled at Petrik Lajos Bilingual Vocational School of Chemistry, Environmental Protection, and Information
Technology, located in Budapest, Hungary. The participants were eleventh grade students on a vocational pathway for specialization in chemistry. At Petrik School, all students are immersed in the English language for one year and then spend four years in specialized STEM classes, taught in both English and Hungarian. Thus, the participants of the study were bilingual, though many students preferred to participate in this study in their native tongue Hungarian. The class observed had a total of 16 students, 6 of whom were female. Their teacher is a doctoral student at Eötvös Loránd University (Eötvös Loránd Tudományegyetem). Her current dissertation research is focused on gamification in a secondary school setting.

**Purpose of the Study**

Through exchanges with Pósa method students, the study aims to explore the student experiences in a Pósa method class. The experience of general education students in gifted education pedagogy is important to consider and document. This research provides a glance into the educational lives of Hungarian students in regular education as they navigate the Pósa method. Through their opinions, evidence for why the Pósa method is currently being discussed, studied, and implemented by the Hungarian education system in general classrooms is brought to light. One goal of this educational research is to contribute to the body of literature that addresses gifted education for all students. Furthermore, my hope is that educators in Hungary and worldwide, specifically in mathematics, can see the value in discovery learning through the Pósa method. As well, I hope teachers will begin to learn about, advocate for, and implement this or other similar methods in their classrooms.

**Significance of the Study**

Mathematics education in the United States was founded in part on the works of famous Hungarians and their methods for teaching mathematics. In American elementary classrooms,
one can find Zoltán Pál Dienes (1916-2014) blocks, inspired by Dienes’ “new mathematics” theory which focused on learning through play and games. These blocks are typically not called “Zoltán Pál Dienes blocks”; American students commonly know them as “Base Ten Blocks” and “Place Value Blocks.” Another notable figure is George Plóya (1887-1985), who simultaneously influenced mathematics education in Hungary and the United States. He was the father of emphasizing problem solving in math education. His name has gone down in history for writing numerous books still in use today on mathematical discovery and teaching students to think. All his books are filled with examples of math problems. Not only do his books include instructional problems, but also one can find educational strategies directed towards teachers. For example, in one of his most noteworthy novels, “Mathematical Discovery”, a list of commandments are included for teachers to prescribe by while in the classroom. His devotion to math education specifically, while not as strong as Dienes’, was and continues to be influential in the American mathematics education system.

This study turns to the latest developments in mathematics education in Hungary in an effort to pick up this thread of Hungarian mathematics development and help introduce it to the United States, just as Dienes and Pólya did with their math theories for mathematics education. This thesis’ focused look at the Pósa method holds important ideas and concepts about international methodology in mathematics education that should be considered and discussed among American educators.

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3 A few examples of his commandments include: 1) Be interested in your subject, 2) Know your subject, 3) Know about the ways of learning: the best way to learn anything is to discover it by yourself, 4) Try to read the faces of your students, try to see their expectations and difficulties, put yourself in their place. For a full list of his commandments, see Chapter 14 of George Polya’s novel “Mathematical Discovery”.
Key Terms (In Hungarian)

**Pósa method (Pósa módszer):** a method of mathematics instruction used in Hungarian gifted math camps which emphasizes discovery learning. The aim of this method of teaching is to make learning enjoyable for the students. The application of the method is through problem threads.

**Problem threads (Problémacsomag):** “a set of connected problems, with a not completely fixed order” that may last for years and “foster the development of specific ways of mathematical thinking, or methods, called the ‘kernels of the threads’” (Juhász & Katona, 2019, pg. 273).

**Discovery learning (Felfedezéses tanulás):** “is a technique for helping learners create and organize knowledge. Involving mindful participation and active inquiry, it typically takes place during problem-solving situations. The learner draws upon past knowledge and experience to infer underlying strategies and gain an understanding of concepts” (Honomichl & Chen, 2012, pg. 615).

**Literature Review**

**Introduction**

The literature review presents three factors to consider regarding the Pósa method: gifted curriculum and pedagogy, major educational trends in learning strategies, and student and teacher perceptions of educational strategies, specifically mathematics. This analysis of the literature includes an assessment of research studies written in Hungarian, largely from Hungarian researchers. The findings from scholarly Hungarian articles both differed and related to findings from the research written in English, which encompasses the other half of the literature review. Thus, the following section establishes the foundation and highlights the
underpinnings for why research on the Pósa method is simultaneously unique and fitting in the current Hungarian and English conversations on mathematics education.

**Gifted Curriculum and Pedagogy in Regular Education**

The gifted education research field, while primarily focused on advanced students, occasionally touches upon the needs of disadvantaged, under-served, and non-traditional learners. Seeing as the strategies in gifted education are grounded in theory, especially cognitive psychology, the techniques employed in gifted education has been shown to work for all types of students (Tomlinson & Callahan, 1992). When looking at gifted education studies in English, they show that the introduction of new math teaching methods with a focus on problem solving has proven to increase academic success among students in general mathematics education settings (András, 2015). Thus, these gifted education studies bring to light that curriculum should be changed to ensure problem solving is at the forefront of mathematics education.

In Hungary, there is no one rule for how to teach, but changing the curriculum, especially so that it emphasizes students, has shown to be best for Hungarian students. The current approach to gifted education in Hungary is built on two concurrent pillars: individual student support and discrete establishments for gifted students (Gyarmathy, 2013). Gifted students are identified by Renzulli’s model of giftedness, which has been simplified to the following: “individuals can be regarded as gifted if their excellent abilities – as a combination of four components – enable them to be capable of a high-level achievement in some area of life” (Gyarmathy, 2013, pg. 22). Additionally, gifted education in Hungary is influenced by Czeizel’s 4*2+1 factor model of giftedness, which is based on nine factors: four genetic, four environmental, and one factor of fate (Gyarmathy, 2013). However, these pillars of gifted education are not implemented uniformly, and students are left out of the gifted identification
process. For the general education students in Hungary, the instructional strategies in regular classrooms have proven not to be adequate. There are remarkable teaching styles that work for exceptionally good students where the teacher speaks less, uses more multimedia tools, and manages the class time well, yet students who do not fit into the “gifted” category are left out of these best practices.

**Gifted Strategies in Mathematics Education**

One strategy used for teaching gifted students is acceleration. Students are given the opportunity to advance to higher level classes in one or more content areas. A hindrance to acceleration is ‘tracking’, where students are grouped by their abilities in various classrooms. Typically, students are broken into two groups: the top students of the class take higher-level courses and the rest of the students are in general education classes (Venkatakrishnan & William, 2003). Often, the tracked placements that students are given affect their mood and attitudes toward learning in ways that may not have been present if the students had not been tracked (Chambers, 2009; Venkatakrishnan & William, 2003). As various studies have shown, tracking has not been proven to be a more impactful educational method than simply keeping mixed ability groups of students (Burris, Heubert, & Levin, 2004; Chambers, 2009; Venkatakrishnan & William, 2003). Thus, this supports the idea that when accelerating students in math, one should consider the benefits of accelerating all students, rather than only ‘high-ability’ students.

Burris, Heubert, & Levin (2004) and Dougherty, Goodman, Hill, Litke, & Page (2015) advocate for equitable mathematics education for all students and suggest algebra should be taught in seventh grade. Their ideas are not new; since the 1980s, researchers like Zalman Usiskin have been pushing for younger and younger students to be taught algebra. In Usiskin’s 1987 article, “Why Elementary Algebra Can, Should, and Must Be an Eighth-Grade Course for
Average Students”, she noted that only 9% of eighth graders had been taught algebra. Of recent, the enrollment rates of algebra have greatly increased, with students, both gifted and regular, enrolling in algebra in eighth grade rather than later grades. With this example, it can be seen how the gifted strategy of acceleration has filtered its way into regular education.

One particular research article further promotes that math acceleration should be used with students of all abilities (Burris, Heubert, & Levin, 2004). Positive effects such as closing an achievement gap, increased performance of low achievers, and accessibility to quality curriculum were cited as reasons to educate all students with accelerated standards (Burris, Heubert, & Levin, 2004). Strikingly, the authors discussed that the “high achievers [did] better, and more students [became] high achievers” when accelerated with all of their peers in middle school algebra classes (Burris, Heubert, & Levin, 2004, pg. 71). The findings of the studies are clear: tracking can be less beneficial than the idea suggests, and acceleration among heterogeneous groups can significantly increase overall classroom performance. The findings from the research relate to mathematics education as a whole, showing how the application of gifted education strategies, like acceleration in mathematics education, can be beneficial among regular education students, especially those labeled as low achievers.

**Access and Inclusiveness to Educational Opportunities**

Many students in the general education classes, especially minorities, are often left out of gifted classrooms, as indicated by trends in studies conducted by English researchers (Baldwin, 2005). Selection of students for gifted education can be a faulty process in schools, oftentimes leading to programs overlooking gifted students (Peters, Cater, & Plucker, 2020). Public education has long held the ideals of preparing all students for life beyond school, making sure students meet at least the minimum standard for education. Minimum, however, is not enough.
Gifted students are given the opportunities to excel through accelerated and enriched programs whereas their peers may not be afforded the same quality of education. If schools began to focus on developing the needs and skills of all children, rather than only those of gifted-labeled children, the education system could become more equitable, particularly by not excluding the overlooked students who have the potential for high achievement (Peters, Cater, & Plucker, 2020). Inclusivity in gifted programs may not always happen, thus implementing gifted strategies in non-gifted classes is one step towards equity in the quality of education for all students.

Equity in education, however, cannot come without an understanding of students as well as the intellect they bring into the classroom. As Howard Gardner (1983) first postulated, there does not exist one type of intelligence, which is typically related to IQ, but rather the following multiple intelligences: linguistic, logical-mathematical, spatial, musical, naturalist, bodily-kinesthetic, interpersonal, and intrapersonal. Taking this into consideration, the theory of multiple intelligences explains why gifted students thrive when their various multiple intelligences are taken into consideration. On the other hand, general education pedagogies tend to forgo acknowledging the multiple intelligences in regular students, thus further emphasizing the inequity in education between gifted and regular students.

Research has shown that gifted strategies in regular education classrooms can help recognize students who go unidentified by nominators, gifted assessments, or their performance in school as “gifted” (Reis & Renzulli, 1982; Renzulli, 1978; Sternberg, 1991). Therefore, not only are more gifted students identified but also general education students are given the fruitful education they deserve. Incorporating gifted strategies into general education is not only possible among general learners, but also encouraged for better identification of gifted students. Students who are typically left out of gifted programs can benefit from some, if not all, pedagogies that
are usually employed in these gifted education programs (Reis, Gentry, & Park, 1995). Training teachers, as well as pre-service teachers, to acknowledge the multiple theories of intelligence, differentiate curriculum, and attend to under-served students gives regular education students a chance to thrive in an environment where they are too often excluded or overlooked. This approach only seeks to benefit all regular education students by providing them the opportunity to develop their talents.

Similar to the conversations in the English literature, the Hungarian education system has had trouble identifying unapparent giftedness in students, as well as providing opportunities for disadvantaged students (Gyarmathy, 2013). In Hungary, parents compete to make sure their kids go to the best schools. However, there is inequality in the system. Due to differences in upbringing, some students may have issues in getting into the higher quality schools. Since selection starts early (e.g., age 10), it is inevitable that students from a disadvantaged background will not make it to higher quality education, especially by the time the students hit their teens (Radó, 2007). While there are efforts in Hungary to reach disadvantaged students, like the Arany János Programme, most programs do not reach the students until their teens, if at all. As Gyarmathy (2013) states, by this time in the students’ lives, “disadvantage turns into considerable deficit” (Gyarmathy, 2013, pg. 27).

The Hungarian literature states that Hungary has an “unfair” selection, particularly among the Roma populations, the largest minorities in Hungary (Radó, 2007). Discrimination of the Roma is not unique to Hungary; examples of systemic racism against the Roma can be found throughout middle Europe and, in general, systemic racism against minority groups exists worldwide (Radó, 2007). The early selection of gifted students restricts students, especially those from a Roma background, from experiencing the diverse opportunities that should be available to
them (Gyarmathy, 2013). Without examining and putting an end to the reasons of discrimination and selection, there may not be any success in providing quality education for all students, especially overlooked gifted students.

When it comes to educational inequalities, there is not always an issue of access to educational tools such as schools or classroom manipulatives. The inequity occurs in the quality of education for different groups of students. Social background, the place of residence, ethnicity, language barriers, and gender can positively or negatively influence the performance of students in education, as can be seen and discovered across the globe. The leading factor on whether a student is set up for success is familial status; thus, where a child comes from impacts their educational opportunities the most (Radó, 2007). These findings from the literature suggest that governments should provide more opportunities for families of all backgrounds in order to provide equitable education. Increasing the quality of education, especially through gifted education strategies, would be one step to take towards equality.

**Major Educational Trends in Learning Strategies**

As the conversation on education expands, there has been an increased focus on topics instructional strategies, educational policy, and curriculum. The trends in the literature suggest new methods of instruction have gained preference and traction by individuals in educational settings (Luera, Killu, & O’Hagan, 2003). One such instructional strategy is inquiry-based learning (IBL). This learning strategy focuses on the student in the classroom, emphasizing their position in the learning process. Teachers are aids to learning, generally guiding and designing lessons to fit the students’ interests. The difference between IBL and the Pósa method’s ‘discovery learning’ lies in how students tackle problems. Through a comparative analysis, one can see how the discovery learning aspect of the Pósa method is similar to the IBL method. It is
important to note that there are other similar methods worldwide, but the IBL method is more widespread, especially in countries like the U.S., and is the most similar to discovery learning.

**Discovery and Inquiry Based Learning**

The main distinction between discovery learning and IBL is a matter of how students tackle problem solving: either through the use of previous knowledge, as in discovery learning, or experimentation, in IBL. IBL is a method that is intended to be used with all students, since it cultivates learning in a way that appeals to the individual student’s needs and interests. Philosophers from the early 20th century advocated for this concept, suggesting that by integrating all students in IBL classrooms, students would learn “to play an active role in the development of societies” (Artigue & Blomhøj, 2013, pg. 798; Luera, Killu, & O’Hagan, 2003).

Given the newness of discovery learning, there is little stated in the literature about how discovery learning works with all students. It can only be speculated by the definition of the instructional strategy. Discovery learning has been described to be “a technique for helping learners create and organize knowledge. Involving mindful participation and active inquiry, it typically takes place during problem-solving situations. The learner draws upon past knowledge and experience to infer underlying strategies and gain an understanding of concepts” (Honomichl & Chen, 2012, pg. 615). This suggests that like IBL, discovery learning could appeal to the needs of all students. It is worth noting that in both IBL and discovery learning, “pupils become participants in creativity” and generate meaningful ideas which “engender a sense of self-worth” (Steed, 2009, pg. 471). These two methods invoke observation skills, as students are expected to develop their intellect through keen examinations of their environment (Artigue & Blomhøj, 2013). For example, when the techniques are used in math classrooms, students are intended to think like a mathematician would when examining mathematical structures and problems.
The positive effects of IBL and discovery learning are worthy of consideration in mathematics education, since their methodology is student-centered and focused on the intellectual growth of the students. As Stonewater (2005) found in their research on student perspectives in a math class that used IBL, over two thirds of the students “described in one way or another how their inquiry experience awakened them— for perhaps the first time—to the importance of developing reasoning skills in mathematics and, as such, expanded their view of the role of student” (Stonewater, 2005, pg. 42). Students became more inquisitive and social in their group work. Given that IBL and discovery learning are similar in their structure, one would expect parallel reactions to discovery learning in the Pósa method.

**Student and Teacher Perceptions in Education**

Individuals possess unique perspectives which may offer insight into the structure, perceptions, and consequences of their experiences. In educational research, students can provide a layer of comprehension that can only add to theories previously established by researchers and former studies (Amitay & Rahav, 2018). As well, students’ experiences can be a segue into understanding the underpinnings of an unfamiliar topic in education. Thus, the student experience provides fruitful data about methods, curriculum, and pedagogies employed in the learning environment. Student responses during studies can reveal relevant pedagogical practices which may go unnoticed in studies that use research methodologies that do not emphasize the student experience. As the literature suggests, “to promote adaptive learning, we should take into account students’ self-views” (Luo et al., 2014, pg. 193). While focusing on student perceptions is not entirely objective, one should not denounce the importance of their feelings and reactions to educational phenomenon (Amitay & Rahav, 2018).
**Student Self-Concept**

Through participant-focused research, researchers can uncover not only descriptions of the subject the participants experience, but also the perceptions that participants hold about themselves. In relation to an educational topic, student-focused studies can help divulge students’ self-concepts, enjoyment of a particular content area, reactions to a phenomenon, and motivations, both internal and external (Luo et al., 2014). The literature has shown how students’ internal and external experiences can impact academic outcomes. High student performance in school as well as prosperous future ambitions are strongly related to positive cognitive and affective reactions that students have towards a particular topic (Pinxten et al., 2014). On the opposite end, negative self-concepts, and emotions, such as anxiety, have been known to lead to poor performance by students in school (Luo et al., 2014).

Motivation and emotion have been found to be linked to cultural standards and thus international studies have differed on conclusions for self-mastery in students. Internalized values can have a large influence on academic achievement. Particularly in math, students do not often discuss having intrinsic motivation in their studies (Rowan-Kenyon, Swan, & Creager, 2010). Students, however, can be trained to develop higher motivation. By doing so, this higher self-concept enforces the idea of content mastery and knowledge retention (Luo et al., 2014).

Turning to the conversation in Hungary, research on Hungarian students’ perceptions of education in general has been primarily focused on responses to large-scale studies like the PISA and TIMSS. It has been shown that Hungarian students’ opinions about themselves when it comes to learning is quite negative (Németh & Habók, 2006). The PISA 200 survey showed that the majority of Hungarian students do not believe in their own skills and qualities in the classroom (Németh & Habók, 2006). While the literature does provide some context for how
students perceive education in general, typically, student perceptions are analyzed in a specific content area, such as math.

**Student Perceptions in Mathematics**

A large quantity of qualitative research studies have focused on academic self-concept in mathematical settings, especially in math classrooms. The push and intrigue for a deeper understanding of student perceptions through qualitative research is justified. As Rowan-Kenyon, Swan, and Creager (2010, pg. 3) state in their article on social cognitive factors among math students, “given that the development of math interest is complex and highly phenomenological, based on each person’s experiences and meaning making, qualitative inquiry is the most appropriate approach for examining these perceptions.” While math is a highly quantitative subject, the experiences that students face in mathematics classes are very much qualitative. The findings in the current body of literature have shown interesting concepts which need more research. For example, mathematics’ self-efficacy as well as a positive outlook on achievement among high school students has been strongly linked to pursuing a career in a STEM field, although women are less likely to act upon their interest (Rowan-Kenyon, Swan, & Creager, 2010).

Students’ perceptions of their competency and enjoyment of mathematics strongly correlate to achievement, with competency being more influential over time (Pinxten et al., 2014). However, enjoyment of mathematics, specifically, has often been overlooked in the current research. Enjoyment is strongly linked to a positive judgement of the quality of the instruction and thus is difficult to examine without looking at other student perceptions (Frenzel, Pekrun, & Goetz, 2007; Vandecandelaere, Speybroeck, Vanlaar, De Fraine, & Van Damme, 2012). Self-concept and general interest in mathematics are also intertwined with enjoyment, so
the measurement of enjoyment is not a simple task. The exposure to mathematical related
activities can also influence the enjoyment of mathematics, ultimately influencing a career in the
subject of mathematics. By analyzing students’ interests beyond the classroom and perceptions
of these activities, the literature suggests researchers can uncover a cyclical feedback loop where
interests lead to career which in turn leads further interests (Rowan-Kenyon, Swan, & Creager,
2010). In addition, when students believe their teachers are supportive in the classroom, the
students are more likely to be engaged and motivated in the subject area.

Hungarian research sheds light on the importance of confidence in mathematics,
especially since there is a country-wide emphasis on excellence in mathematics yet many
students report having little confidence in the classroom. In comparison, students will rate their
verbal skills much higher than their math skills. Students doubt whether they can successfully
complete math exercises which results in a negative perception of their self-belief in math
(Németh & Habók, 2006). Students who have strict and negative opinions of math education are
more likely to be passive about learning in general, thus adversely affecting their performance in
the classroom (Pehkonen & Tompa, 1993).

One reason why math is more difficult for some Hungarian students to grasp is due in
part to the opinions that students hold about mathematics in general. Hungarian students are
taught advanced level mathematics from an early age. In class, it is important for all the math
problems to be solved, to be as close to the correct answer as possible, to be precise, and to be
proven completely. What students put on the table is crucial to their success and the grade they
achieve is very important (Pehkonen & Tompa, 1993). As a result, students perceive math to be
tiring, boring, and often a cognitive burden (András, 2015; Pintér, 1994.). Even the way students
think about the math problems affects their success in the subject. Application of their
knowledge has shown to be weak, emphasized by low analytical skills. Simply changing their order of thinking can positively influence their perceptions and performance in math (András, 2015). Focusing on educational strategies backed by cognitive psychology has the potential to positively influence education, as supported by international research.

When it comes to increasing motivation for learning math, students in Hungary believe emphasizing their emotional, cognitive, and moral well-being to be an effective educational strategy (Orosz, 1975). The most important thing to students, however, is whether the math lessons are interesting. Successful learning of mathematics heavily depends on how the students feel about the question. If they do not find it interesting, then they will simply memorize formulas, forgoing the enjoyment of exploring mathematics (Pehkonen & Tompa, 1993; Pintér, 1994) As well, students will note that a close bond with their math teacher improves their perception of the subject. Student-teacher relationships enhance enjoyment, decrease anxiety, and influence achievement (Orosz, 1975). In conclusion, the primary condition for successful math education in Hungary is to teach math with an understanding of how the human mind and psyche work. This involves finding enjoyable instructional strategies, which in turn lessens cognitive burden, and encouraging teacher rapport with students.

**Teacher Perceptions:**

Through narrative research, Andrews (2007) discovered how a teacher’s voice can reveal their perceptions of and relationship with the content area they teach. His research on teacher’s exposed beliefs is not new to the field of educational research. A teacher’s voice holds heavy weight, particularly since they are perceived to be the head of the classroom, the central learning place in education. As previous studies have shown, Andrews’ research focused on how cultures can influence teacher perspectives on teaching and learning. He analyzed teacher beliefs on
curriculum from English and Hungarian teachers. English teachers believed in the importance of curriculum as well as emphasizing the competency of numbers, believing they are an essential prerequisite for mathematics. These teachers indicated that the importance of numbers was due to their prominence in everyday life, suggesting a world view of mathematics. Bearing these in mind, it is obvious that teachers have a strong foothold in the education in their classrooms, which in turn affects their students. Their perceptions have the ability to contribute to the success of their students (Stonewater, 2005).

While my study focuses on student perceptions, the teacher’s perspective in the classroom is one to consider, particularly as it affects the ideas students hold about the method of instruction, content, and overall satisfaction with the class. Thus, it was necessary to look into the experience of Hungarian teachers. Currently, enthusiastic teachers in Hungary have been working on spreading a new method of teaching math with more or less success. It is not an easy task because current teachers who were not taught with a successful and researched method may think that learning math is tiring and boring. It is hard for them to see the enjoyment of discovery learning or the beauty of mathematics (Pintér, 1994). A few universities have made math camps for math educators to encourage enthusiasm about problem solving. Once these math educators found an interesting topic for themselves, they became enthusiastic about it (Pintér, 1994). In a survey conducted among math educators, it was found that most Hungarian teachers think the education system is centralized and that this governmental control is not good for education (Dudok, & Dudok, 2020). Teachers believe that they cannot make their voice heard or validate their opinions since schools are tightly controlled by the government. Most teachers believe they should have more say in what happens in math education. The largest finding of this study was that many teachers discuss international results from math education studies, hoping to
implement new practices in their classrooms (Dudok, & Dudok, 2020). These teacher perceptions shed light on the current educational situation in Hungary. The research has shown that there needs to be more leeway in how teachers run the classroom. Additionally, studies advocate implementing more interesting problems since these can influence the enthusiasm teachers hold towards math. The incorporation of these ideas and new teaching techniques can provide excellent environments in which students may thrive in math.

**Summary**

First, seeing as the Pósa method has been used with gifted students in the past, it is beneficial to review studies that have analyzed the experiences and outcomes that students face in gifted classrooms. Additionally, the Pósa method employs techniques similar to current trends in education, especially learning strategies that are not geared purely towards gifted students. Comparing the discovery learning aspect of the Pósa method and inquiry in inquiry-based learning (IBL), one can see why these new methods are being discussed in scholarly conversations on mathematics education. Finally, previous studies have emphasized the importance of student and teacher perspectives and perceptions in educational research. Seeing as the current study was strongly impacted by student narratives, a thorough dive into the literature on student perceptions gives a clear understanding behind the methodology of this work.

Through analysis of literature in written Hungarian, one can see that Hungarian research on mathematics education engages in the same conversations as American research. This is in part because Hungarian researchers will cite English authors in their research. However, the same cannot be said about English authors with Hungarian research. Hungarian research findings provide a unique perspective that is often lost in American research, which can be seen in this
exploration of the topics of student perceptions, gifted curriculum, and teacher perspectives. It is important to delve into what is said specifically in Hungary, especially since this thesis is focused on Hungarian mathematics education. In this literature review, the Hungarian perspective on their education system was examined for similarities with and scrutinized for differences from American studies. In the following sections, important issues in Hungary are brought to light that would otherwise have been left untouched had a literature search of articles written in Hungarian been left out of the thesis.

Context

History of Mathematics Education in Hungary

The Hungarian education system is renowned for its effective strategies in pedagogy and curriculum, especially in physics and mathematics. Hungarian excellence in mathematics education is a distinction that has been around for nearly a century, having begun after the conclusion of World War II (Győri, Fried, Köves, Oláh, & Pálfalvi, 2020). During the country’s transition into socialism, Hungarian education underwent significant changes. Notably, the ‘Great Reform Era for Hungarian mathematics education’ in the 1960s to 1970s saw the largest rise of major curricular changes in the field of mathematics, particularly in gifted mathematics education (Gosztonyi, 2016a; Gosztonyi, 2016b; Győri et al., 2020, pg. 82). While it seems unusual to promote individual talent in a socialist society, it is, in fact, common for socialist regimes to encourage and promote giftedness for the betterment of their people; thus, the push for gifted mathematics education in Hungary was typical of the time and socialist regime (Gosztonyi, 2016a). The Great Reform Era for Hungarian Mathematics (The Great Reform) aligned with the international “New Math” period, when the United States and Western Europe
were ambitious to present mathematics as an interesting and necessary subject in schools. Additionally, the 1957 “Sputnik shock” worldwide aided in the large reform in Hungarian math.

The Great Reform was not a simple endeavor but rather one that included changes in curriculum for students, as well as pre-service teachers. Additionally, educators within classrooms were presented with new resources, textbooks, professional development, and instructional strategies (Gosztonyi, 2016b). The textbooks introduced at the time were regulated by the Hungarian government and obligatory, which is still the case today (Győri et al., 2020).

The new books introduced concepts that had not previously been touched on in mathematics classes: set theory, logic, topology, and discrete mathematics (Gosztonyi, 2016b). Other topics became more well-rounded and expounded upon, such as geometry and probability. The way textbooks addressed mathematics was altered as well; the dialogue shifted to one of “guided discovery” (Gosztonyi, 2016b) signaling a pedagogic shift. Students and teachers began engaging in conversations about problems, rather than simply the teacher instructing students from the front of the class. The Hungarian math curriculum shifted to include five main topics, presented each school year: sets and logic; arithmetic and algebra; relations, functions, and series; geometry and measure; and combinatorics, probability, and statistics (Gosztonyi, 2016b; Győri et al., 2020). A new method of teaching mathematics was instated and called the “spiral structure”, where students are re-exposed to topics and expand upon them each year, as if their knowledge of a concept is spiraling upwards (Győri et al., 2020). Concepts in the “spiral structure” are generally taught at similar times each school year. As a result of the Great Reform, the education system became one where students were “guided through a series of problems, while continuing a dialogue between each other and with the teacher about the problems.
Intuition, visuality and experiences played an important role in this discovery process.” (Gosztonyi, 2016b).

In 1962, the first special math class at Fazekas Gimnázium, a public high school in Budapest, was established, which set the stage for further developments in gifted education (Stockton, 2010). In special math classes, Hungarian students from all the over the country can join a cohort of peers gifted in mathematics. The classes teach advanced math topics by some of the best math teachers in Hungary, making them the top classes for learning math in the country. Understandably, the first students of Fazekas, as well as later alumni, have become some of the most well-known mathematicians in Hungary, let alone the world. The concept of appealing to gifted math students was not a new thing in Hungary, however. The country has been known to provide unique opportunities for its students to extend their mathematical knowledge. In 1893, gifted math education and talent search began with the first publication of Középiskolai Matematikai Lapok (KöMaL; translation: Mathematical Journal for Secondary Schools), the national mathematics journal written specifically for students (Győri et al., 2020). Since 1893, mathematicians have developed and published interesting math problems in the journal to be solved by students. Gifted students are encouraged to attempt the problems and submit solutions to win prizes. Since its development, the journal has been succeeded by the creation of Hungarian math competitions, gifted camps, and more special math classes (Győri et al., 2020). The country now has hundreds of math competitions for students to compete in. As one can see, with the start of Fazekas and the pre-existing methods to appeal to gifted students, Hungary began to make a prominent name for itself in the area of gifted mathematics education towards the end of the 20th century.
The end of communist rule in Hungary in 1989 had the most lasting influence on Hungarian education in general, not just in the field of mathematics, most of which is still in effect today (Győri et al., 2020). The late eighties brought about another large reform in education, largely through determining which instructional strategies from the socialist era were effective. As well, researchers began to seek new strategies for education. During this reform era, the National Core Curriculum (NCC) was established and made compulsory for all schools. The lasting effects of this change in education came from the established curriculum developed by Tamás Varga.

Roles of Individual Scholars

Hungary has a strong history of one individual or small groups of individuals influencing its education, especially in mathematics. Prior to the major changes in the 60s, the notable leaders in mathematics were László Kalmár, Rózsa Péter, Alfréd Rényi, János Surányi, George Pólya and Imre Lakatos (Gosztonyi, 2016b). They saw math as a playful rather than formal subject, one in which the teacher and the student collaborate on the “rediscovery of mathematics”, rather than the rote memorization of formulas (Gosztonyi, 2016b). Through the efforts of Kalmár, Péter, Rényi, Surányi, Pólya and Lakatos, the subject itself began to be seen as playful, simultaneously based on instinct and discovery. These conceptions of math became the groundwork for further changes in mathematics education that Hungary stands on today.

Notable Figures

The ideals held by the mathematicians of the early 1900s were soon changed during the mathematics education reformation of the 1960s. Tamás Varga, a mathematics teacher and researcher, became a force in mathematics pedagogy. Beginning with his longitudinal studies in 1963, he set out to bring changes to the national mathematics curriculum of Hungary. His project
to improve education concluded fifteen years after it had started, in 1978, at which time Hungary adopted Varga’s ideas to design new curriculum for public education (Gosztonyi, 2016b; Győri et al., 2020). The curriculum brought about a new way of seeing content in mathematics. Rather than focusing on arithmetic and geometry primarily, Varga’s curriculum introduced a spiral structure, such that multiple topics in math were taught each and every year. As Győri et al (2020) observed, “It began from the very beginning of school with a system for modern mathematics teaching.” (pg. 83). Tamás Varga’s national curriculum is the current method used in Hungary today, as no big leaps in mathematics pedagogy has been made since his work in the 1970s, other than those by Lajos Pósa.

**Lajos Pósa and the Pósa Method**

When discussing mathematics pedagogy in Hungary, one cannot go without discussing its current leader, Lajos Pósa, especially since his work in gifted education has been largely monumental. In the early 1980s, Lajos Pósa developed a method for gifted mathematics teaching, based on observations of Hungarian classrooms. The Pósa Method, as he has called it, was designed to be a system of creative thinking, aimed at making mathematics enjoyable for students, particularly through stimulating and inquisitive problems (Juhász & Katona, 2019). Lajos Pósa is known for using his method in special math camps, which started in 1988, where the top mathematics students from grades 7 to 12 are invited 2-3 times a year to think about and work on hard problems in order to develop problem solving skills (Győri & Juhász, 2018; Juhász, 2016). The structure of the camps encourages individualized and group learning. The typical structure involves “Autonomous Thinking, Plenary Discussion, Individual Thinking, Team Contest, and Homework” (Juhász & Katona, 2019). Each category of the camp is designed to facilitate a different type of mathematical learning. For example, during ‘Plenary Discussion’,
students are encouraged to ask new and interesting questions based on previously solved problems from the camps. The Pósa Method aims to mold students into thoughtful inquisitors and often, student-posed questions and problem threads become incorporated into the curriculum of the camps. Currently, the camps are run by former students of the Pósa method as well as the colleagues of Pósa. The camps are free for every student; thus, economic status is not a barrier for attending. Having been designed to accommodate gifted students, the camps have led students to considerable triumphs, which in turn emphasizes the success of the camps themselves. Of the six participants that may represent Hungary each year in the International Mathematical Olympiad and Middle European Mathematical Olympiad (both highly prestigious and selective math competitions), nearly all individuals have been students of Pósa’s math camps, with minimal exceptions since 2009 (Juhász & Katona, 2019).

The Pósa method is widely different from the way Americans educate their students in mathematics; in fact, it is unique to Hungary. Even so, Pósa’s method holds the traditional principles of Hungarian mathematics education: teaching students how to ask “good” questions, teachers acting as aids instead of sources of knowledge for students, eschewed formal language, playfulness, mistakes are not discouraged, and the vital objective is building problem solving skills (Gosztonyi, 2016). The Pósa method strays away from the process of memorizing formulas and focuses on the autonomy of the student in the classroom. It has been described as “relaxed and cheerful”, based on logic and discovery (Juhász & Katona, 2019, pg. 270).

The defining feature that sets the Pósa method apart from the current pedagogy in Hungary is its use and dependance on problem threads. Problem threads are “a set of connected problems, with a not completely fixed order … that foster the development of specific ways of mathematical thinking, or methods, called the ‘kernels of the threads’” (Juhász & Katona, 2019,
These threads of problems build upon one another, leading students to major mathematics topics in the kernels. Some examples of kernels include the Pythagorean theorem, the law of sines, and the quadratic formula. An example of a problem thread is as follows:

“Problem 1. Does there exist an arithmetic progression (AP) with six terms so that each pair of terms is relatively prime? (The terms of the AP are distinct positive integers.)

After the students solved this problem, we ask them to pose good questions. The following questions are the most typical ones:

Problem 2. What is the maximum length of an AP if each pair of terms is relatively prime?

Problem 3. Does there exist an infinite AP so that each pair of terms is relatively prime?

Some students recommended the following questions:

Follow-up Q1. Does there exist an infinite sequence so that the differences of the consecutive terms form an AP, and each pair of terms is relatively prime?

Follow-up Q2. If the terms of the AP are primes, then it implies that each pair of terms is relatively prime. Therefore, a naturally arising question is the following: Does there exist an arbitrarily long AP whose terms are primes?

The answer for this question is very hard. It is yes, and it was proved by Ben Green and Terrence Tao in 2004.” (Juhász & Katona, 2019, pg. 274)

At the beginning of the problem threads, students are not explicitly told what method they are working towards learning; that is a key feature of discovery learning in the Pósa Method. Through solving the problems, students are led to the final kernel, at which time they will be informed of which mathematical concept was involved in the threads. Since math is a subject that tends to overlap in topics, the threads are not necessarily separate; they can last for years, span different concepts, and crisscross with other threads to make a ‘web of problem threads’, a term coined by Katona and Szűcs (as cited in Juhász & Katona, 2019, pg. 273). Due to the longevity of problem threads, students begin to grasp that math is not a stagnant subject but rather one that is an accumulation of closely related topics.
As stated above, the Pósa method was originally tailored for gifted students, particularly in gifted math camps. Recently, there has been a demand to see whether more students, other than those qualified as ‘gifted’, can be taught mathematics with the Pósa Method. While one could argue that the Pósa Method is a tried-and-true model based on competition data, the method is presently lacking in scholarly and investigative proof or empirical support. Little published research has been done on the method itself, and there exists only one soon-to-be-published research project that is currently analyzing the Pósa method’s effectiveness as a teaching practice. Having begun in 2016, that study is a four-year experiment specifically designed to determine whether the Pósa Method’s theoretical model is applicable in general education, not just with gifted students. Three public high school classrooms have incorporated the method into their curriculum, with the hypothesis that this method can be used in a variety of settings, beyond the well-established camps and among regular education students. The research aims to contribute to the international conversation of inquiry-based mathematics pedagogy. Results of the study are due to be published in the fall of 2021.

Research Methodology

Overview of the Study

The qualitative research I conducted consisted of three major parts: a historical search, a qualitative data collection, and an analytical component. This holistic approach made it possible to give a well-rounded view of Hungarian mathematics and the Pósa method, neither of which I was previously an expert in. Since the Pósa method has been newly introduced in Hungarian public education, it was important for me to understand the history of Hungarian mathematics education, what the curriculum looks like currently, and why the Pósa method has not been previously utilized among regular education students. Observations in a Pósa method classroom,
interviews with students and teachers, a questionnaire administered after lessons, daily journals, and a test provided fruitful data on the students’ experiences and understanding of the Pósa method. A copy of each data collection measure can be found in the appendices at the end of the thesis document. Analysis of the data through qualitative coding revealed common attributes of the method, as expressed by the students. Every component of the methodology was chosen to ensure the Pósa method was thoroughly examined from the students’ perspectives.

**Research Questions**

I set out to gather and assess the opinions that students hold towards a Hungarian mathematics pedagogy called the Pósa method. An examination of the history of mathematics education in Hungary, and an analysis of classroom observations, interviews, and a questionnaire were essential aspects of the research in order to embody the students’ experiences in a Pósa method class. To examine and understand the Petrik School and Pósa Method students’ perspectives, the study focuses on the following research questions:

1. What do the students at Petrik School think about the Pósa method?
2. What do the students in the Pósa method class think about math?
3. What do the students’ and teacher’s comments reveal about the structure of the method?

**Setting and Population**

Petrik-Lajos Két Tanítási Nyelvű Végyipari, Környezetvédelmi és Informatikai Szakközépiskola (Petrik Lajos Bilingual Vocational School of Chemistry, Environmental Protection, and Information Technology) is a public bilingual high school in Budapest, Hungary in which students must apply to attend. At Petrik School, students are immersed in the English language for one year, while their remaining four years consist of special science, technology, engineering, and math (STEM) classes, taught in both English and Hungarian. These classes are
geared towards training students for careers in the STEM field, bridging the gap between school and work. As a result, students graduate with technical training in one of three areas: information technology and telecommunications, environment and water, or the chemical industry.

The classroom I observed was a vocational math class for general education chemistry students. As a consequence of the 2019 novel coronavirus (COVID-19), the setting of the research shifted from in-person to online, specifically over Google Classroom and Zoom. Google Classroom was used for daily observations and lesson recordings while Zoom was primarily restricted for interviews. I was unable to stay in Budapest, Hungary due to the COVID-19 pandemic and my status as a foreigner in Hungary. The schools were not allowed to stay open, so students were required to engage in virtual learning.

The Pósa method class itself is one of three classrooms in Hungary that uses the Pósa method as its pedagogy in daily math lessons. The classroom observed at Petrik School was classified as ‘middle level’ whereas the other two Pósa method classes are considered ‘elite’. The teacher at Petrik School, Márta Barbarics, works with the two other Pósa method teachers to design the special curriculum and problems used in the Pósa method classes. Each teacher tailors the problem threads of the Pósa method to their students’ needs and abilities. These three teachers have been conducting a four-year ongoing experiment through the Subject Pedagogical Research Program to develop a theoretical model of the Pósa method in public education, determining whether Pósa’s method could be a viable pedagogical tool for students in general education. The study is expected to finish this year (2021) when the Pósa students graduate and take the required the Matura, the required Hungarian high school leaving exams\(^4\).

\(^4\) These exam results will be used as evidence of the quantitative strength of the Pósa method. The researchers expect to publish the findings in the fall of 2021.
The participants of the study were the students of the Pósa method class at Petrik School. These students had agreed upon entering high school to be participants in the larger study of the Pósa method in general education. At the time of data collection for this study, they were grade 11 bilingual students in the chemistry pathway for chemical training at Petrik School. There were 16 students in the class, with 6 females. Their teacher, who participated in the study as well as in the analysis, is a graduate student at Eötvös Loránd University in Budapest Hungary, where she is conducting research on gamification in secondary classrooms. The students were not chosen randomly, but rather with purpose. It was essential that they were in general education mathematics class, not special math classes.

**Ethical Considerations**

As this study was conducted with human subjects, there are ethical considerations to address. The participants of the study gave consent to undergo the research and were given the option to decline if they did not feel comfortable at any moment. The anonymity of the participants has been maintained throughout the study and in the discussion of the findings. Each student was given a letter of the alphabet, from A to Q, to preserve confidentiality in interview transcripts and observation data. Due note, to prevent confusion in the presentation of findings, no student was labelled with the letter “I”.

The Pósa method problems that students were given did not differ largely from their regular assignments, thus undue mental stress nor harm were issues in this study. Due to interviews being conducted in an online format over Zoom, there was the potential for a breach in security, as instances of hacking were frequent in spring 2020. These online sessions could have been hacked, despite the best efforts to ensure security through waiting rooms. However, no such instances occurred. Given that the University of South Carolina does not require approval
from the Institutional Review Board for senior theses, this study was done without institutional review. Other than the oversight from my thesis director, the study was not assessed by a review board for adequate treatment of human subjects. That said, there were no known instances of ethical problems as no rights of the participants were violated.

**Positionality**

During the research process, it is important to continually acknowledge the issue of bias to maintain trustworthiness for the data. I, the researcher, had to take steps so my own personal perceptions would not alter the information collected from the students. I was an outsider and foreigner in the observed Hungarian classroom. I was unaccustomed to common Hungarian traditions, the nature of Hungarian mathematics pedagogy, and the spoken language, Hungarian, itself. I had to ensure the American social context I was raised in did not filter into my data or analysis. I understand it is impossible to guarantee there was no obtrusiveness caused by the methods and interventions used. I found myself having to change my perceptions while I was collecting data. After preliminary observations and interviews, I realized how unique mathematics education was in Hungary in comparison to the United States. This required me to re-evaluate the types of questions I was asking the students during interviews, as well as dismiss my preconceived notions in order to better understand Hungarian math education. I asked questions to further dig into what it meant to have a typical educational experience in Hungarian math classrooms; I caught my assumptions and inquired for elaborations.

I am a white, female, fourth-year college student, from a university based in the southeastern part of the United States. I attend university with the expectation of graduating with a Bachelor of Science in mathematics and minors in secondary education and psychology. I am a future educator with professional experiences in grading, tutoring, researching, and mentoring.
Thus, I have opinions about, ideas in, and aspirations for the educational field, particularly in field of mathematics. Throughout the research process, I had to set aside my beliefs and focus on the content being studied. Maintaining neutrality towards mathematics education in Hungary was simultaneously challenging and necessary.

Given that I was a researcher, foreigner, and college student, the participants recognized a hierarchy of power, despite my attempts to remove such a concept through rapport and respect. The students at Petrik School were hesitant to be completely open with their responses, at times not due to reluctance but due to a language barrier. During the interviews, their teacher was available to provide translation from English to Hungarian and vice versa since the questions were asked in English. Students were allowed to respond in Hungarian, but most chose to practice their English in their responses. As a native speaker of English, I was afforded ease in speaking native tongue in the interviews whereas the students attempted, and often struggled, to respond in this foreign tongue. My status as a native speaker contributed to the dynamic of power and thus was a barrier to fully building rapport with the students.

My foreign perspective is unique to the project; the other researchers studying the Pósa method currently are Hungarians, specifically math teachers. This fact did not truly hit me until I gave a short presentation on my research in the spring of 2020. Primarily Hungarians, both researchers and teachers, came with questions and were surprised by my findings, not because they were ‘out of this world,’ but because my position as a foreigner led me to discover pieces of the Pósa method which had previously been overlooked. My research design dove deep into the basis of the Pósa method, focusing on the student perspectives and crafting a narrative from their experiences. Whereas the Hungarian researchers are searching for proof of effectiveness of the Pósa method, I sought after the stories woven by the students. Doing this research changed my
perspective on math education, especially from students’ observations. Not only have I come to value the journeys students undergo in mathematics education but I am also more aware of the importance of student-centered classrooms.

Limitations

The results of the study indicated an appreciation for the Pósa method, while also exposing the students’ concerns about the method as a teaching technique. While there was sufficient evidence to back this study’s findings, stronger conclusions could be made about the effectiveness of the Pósa method with more research, particularly quantitative data. Furthermore, there was a language barrier of English and Hungarian between me and the participants. I speak basic Hungarian and the students’ English was often limited so during the interviews there may have been some key information lost in translation. As well, having a native Hungarian who spoke English translate my questions and the participants’ responses directly interrupted the flow of the conversation. While it was not often a problem, on the spot interpretation can be distracting and lead to brief pauses while each individual tries to understand the question or answer. To minimize the information lost, a Hungarian translator was hired after data collection to transcribe the original Hungarian used in the interviews. Despite the attempt to ensure the Hungarian language was maintained during the process, I am an outsider, and thus bits of the interviews might have contained nuances I was unable to pick up on.

This thesis looked at the perspectives that Petrik School students held towards their Pósa method mathematics class. While the sample size provided an adequate and thorough amount of data, this study could have benefitted from more interviews with students as well as in-person observations of the Pósa method lessons. Given the COVID-19 pandemic, in-person data collection was not possible for the entirety of the study. Moreover, focus groups were not
employed in this study. Focus groups can ease students’ reluctance for responding. Furthermore, peer responses in focus group sessions can inspire additional thoughts, conversations, and opinions that may not appear during individual interviews.

**Research Design**

I spent the spring of 2020 in Budapest, Hungary, participating in a study abroad program called Budapest Semester in Mathematics Education (BSME). As a student of BSME, I enrolled in a research course on discovery learning in secondary schools, focusing primarily on the Pósa method. During this time abroad, I designed a qualitative study with a fellow classmate to analyze the effectiveness of the Pósa method and its embodiment of discovery learning. The project covered multiple topics: student pace, perceived difficulty of Pósa problems, interest in the Pósa method problems, collaboration between students, and the processes students took to solve the Pósa problems. As a consequence of the COVID-19 pandemic, the study looked at how students experienced the Pósa method in an online setting.

However, I redesigned the project to focus on the students’ experiences and opinions about the Pósa method. I felt it more important to design the study around students since they are an integral part of the educational process in the Pósa method. The learning is done by the students, rather than primarily through frontal teaching. I began to question, “How do students feel about the Pósa method?” Thus, the design was altered in order to collect additional information from the students, which included further interviews, and a questionnaire. Throughout the process, I began to highlight distinctive examples of students in the class as well their reflections on the method. The project, therefore, was devised to embody the student experience in order to best present the Pósa method through their perspective. The resulting project, in essence, became a more comprehensive qualitative project than the first one my
partner and I designed in Budapest, focusing on student experience and judgements of the Pósa method specifically.

**Historical Analysis**

The first part of the research design consisted of documenting the history of mathematics education in Hungary and the origination of the Pósa method. The historical context not only provided the basic background information, but also fully elucidated the value Hungary places on high-level mathematics pedagogy. A search of academic journals and texts on the math education reforms in Hungary developed a clear picture as to why there is a current push in Hungary to include the Pósa method in its public education. These academic sources were written in English and were found through searches using the key terms such as ‘Hungary’, ‘mathematics education’, ‘history’, ‘eastern Europe’, and ‘math reformation’.

A parallel document search of Hungarian resources was carried out to ensure the conversations in Hungarian were considered as well as the conversations in English. Given that the articles used from this document search were all written in Hungarian, a Hungarian graduate student translated the articles for use in this study. The articles were compared to the English literature for similarities and differences in the topics discussed, the conclusions, and references cited.

Furthermore, contact was made with leading researchers in the field of Hungarian mathematics education: János Győri, Márta Barbarics, and Péter Juhász. Through their guidance and suggestions, the foundation of Hungarian mathematics education was studied and the Pósa method was well analyzed. Given that Márta Barbarics, and Péter Juhász are two of the three leading teacher-researchers studying the Pósa method currently, their advice and expertise was invaluable. The information collected through the various sources mentioned in this section was
incorporated in the previous ‘context’ section of this thesis, which elaborated on the history of mathematics education, the role of individual scholars, and the key figures who have shaped the current curriculum and pedagogy.

**Data Collection Measures (Observations)**

Observations provide fruitful data in qualitative research, as it allows for heightened information on the content of the study. Data collection for this study included four observations of class lessons as well as two observations of review sessions of a Pósa method classroom at Petrik School. The first observation was conducted to simply see the Pósa method in action. Following the observation, a compilation of questions, themes, and remarks were created to serve as guidelines for designing the research project. Well-defined research questions were established as a result of the preliminary observation. Any questions deemed outside the context of the study were not considered as the research continued.

The first observation was done in February of 2020, prior to the COVID-19 pandemic, just weeks before in-person observations were no longer allowed due to the pandemic. Two video cameras were set up to see all areas of the classroom clearly. Participants were given their usual task for the day, as instructed by the teacher. This task was to use the Pósa method to complete a set of 5 problems given to them earlier in the week. Some students entered into class having already solved a few of the problems from the worksheet.

The next three observations were conducted asynchronously over Google Meet. Students were given a problem thread of four questions designed by myself, my research partner, and the teacher, Márta Barbarics. During the week of observations, students were tasked with completing as many problems as possible. A copy of the Pósa method questions can be found in the
appendices at the end of this thesis. Participants worked at their own pace and spoke up if they needed more help from their teacher.

The two review sessions provided a place for students to come to the teacher and ask for further help on the problems. In one review session, the teacher went over the answers to two of the problems, problems 248 and 249, from the assigned work. The other review session was a walkthrough of the Pósa method problems, where the teacher explained one method of solving the Pósa method problems. During the review, the teacher introduced the students to the relevant topics that were used in the problem threads, or the kernel of the problem thread. The walkthrough doubled as a place for the students to ask questions and check their understanding on the problems they had been working on.

The goal of the observations was to expose myself to the typical structure of a Pósa method classroom. Participants’ engagements with their teacher, their peers, and the questions were documented. As well, the reactions students had in class, such as comments on the problems or questions for clarity were noted for later analysis.

**Data Collection Measures (Problem Thread)**

A structured list of questions for students to solve provides external validity this study, given its small sample size. Distributing a problem thread to students emphasized the importance of looking at how students solved the problems, which students were more successful in the discovery learning, and how students excelled in different content areas of math under the Pósa method. Hence, the questions assigned to the participants were chosen to emphasize the importance of discovery learning. The decision to look at different fields of mathematics in the problems was to make it so that students were not solving the same problem multiple times, but rather employing their various mindsets in the Pósa method classroom.
Students were tasked with solving four Pósa method questions by the end of the data collection week, labeled 246 to 249. Note, since the questions were part of a larger problem thread, they were labeled with the order of appearance in the curriculum. This implies that the students had been given 245 Pósa method problems over the three years leading up to this study. The Pósa method questions were carefully chosen with varying difficulty to embody different branches of mathematics: systems of equations with logarithms, quadratic trigonometry, plane geometry, and logic. The system of equations question was listed first as it was a typical calculation problem. For quadratic trigonometry, students had to apply the Pythagorean’s theorem to solve a trigonometric identity. The need to use the Pythagorean’s Theorem was not explicitly stated by the teacher. Rather students had to discover it for themselves, an aspect of discovery learning. In plane geometry, students were given a question with multiple parts, resulting in creating a general formula for features of lines and vectors. Lastly, the logic questions tested their ability to write formal proofs, which is essential in higher level mathematics. These four questions were designed by the teacher and written in such a way that they fit into the curriculum. A copy of the questions given to the students can be found in the Appendix A at the end of this document.

Data Collection Measures (Journals and Questionnaire)

Questionnaires allocate primary information for qualitative research since the researcher structures the questions to fit the context of the study. It is important to ask well thought out questions with intentional wording. Journals, where participants record their daily activity and experiences over a given period of time, are useful in longitudinal studies, in this case over the course of a week. Journals allow researchers to see changes in behaviors and notations on general activity from the participants. Invaluable information can come from using both
questionnaires and journals as components of qualitative methodology. However, if participants neglect to engage in the questionnaires or journals, the researcher may be left with less data than anticipated. Thus, it is necessary to make both questionnaires and journals well-designed so that any information collected is invaluable.

Participants were assigned a journal through Word documents, to be completed at the end of every day of the data collection week. The journal was accompanied by an introduction text, stating the objective of the daily journals and reiterating the purpose of the study. The journal itself contained questions about the time students spent on problems and their comments on the discovery learning process. While the journals originally fit the first project design, they were analyzed through a qualitative content analysis, specifically looking at student comments, the length of time that students worked on the problems, and the hours during that day that the students devoted to mathematics. Prior to filling out the journals, students were informed that it was not forbidden to ask for help or use help from outside sources to solve the Pósa method problems. That said, the final answers were to be written by the students themselves. Regular reminders were made by the teacher to the participants to fill out the journals and submit them daily. Despite the reminders, only fifteen of the seventeen students over the course of the week submitted at least one journal entry. During data collection, some days had fewer journal submissions than others, ranging from four to nine submissions on any given day.

The questionnaire was given to students as a Google Form at the end of the data collection week. Participants were tasked with filling out the questionnaire over the course of a weekend. The questionnaire asked the students about their perceived difficulty of the math problems and problems they had solved on their own, with help, and a combination of the two. At the end of the questionnaire, students were asked to reflect on whether they felt like they
developed their discovery learning over the week as a result of the Pósa Method. The questionnaire allowed the students to transcribe their opinions about the Pósa method problems they had been working on as well as the Pósa method process. Questions on the questionnaire were intentionally chosen and concerns by the teacher were addressed. One such concern related to the rating scales on the questionnaire. Hungarian teachers grade on a one to five scale, so students have associated those numbers with the grading system, much like Americans with As to Fs. To prevent skewed results due to students’ familiarity with the one to five scale, the rating scales on the questionnaire were from one to ten. Another concern was related to the language that the questionnaire was written in. Given that the participants were more well-versed in Hungarian, the short answer questions were provided in both English and Hungarian. A copy of the questionnaire, introduction text to students, and journal can be found in Appendix C, D, and E, respectively, at the end of this document.

**Data Collection Measures (Test)**

Testing participants not only gains insight into their knowledge of specific content but also whether they developed skills during and over the pre-testing process. Tests validate participant understanding and evaluate abilities in a structured manner. The test administered in this study determined whether student’s perceptions of discovery learning aligned with the number of problems they answered correctly on the test. Thus, it was an examination of perceived discovery learning by the students.

After the week of observation, students were assigned a test of 21 questions, each question relating to one of the four Pósa Method problems assigned earlier during the week. The test in particular focused on the major theorems and formulas embodied by the problem threads and the application of these mathematics topics. Seeing as the content of the test did not differ
widely from the problem threads, students were evaluated on their ability to utilize the discovery learning that they had developed over the course of the week. Participants were given three days to complete the test. A copy of the test questions can be found in Appendix B at the end of this document.

**Data Collection Measures (Interviews)**

By questioning knowledgeable individuals or participants in a given lived experience, interviews provide insights into the phenomena at hand, an in-depth understanding of influential factors, and, often, inspiration for further questions in a study. Interviews can be beneficial in narrowing data and focusing on specific research questions. Members of the study typically bring up topics not previously recognized by the researcher, which oftentimes can be useful for the study. Due to the in-depth information collected, interviews are typically the most valuable data collection strategy in qualitative research. The interviews in this study sought to illustrate the students’ experiences in the Pósa method class. The interview questions gave an understanding of participant perceptions of themselves and their previous math education classes. By looking at participants background in mathematics, it made the different student experiences in the Pósa method class more evident. The responses in the interviews helped show the motivations that students held, both in general pedagogy and in Pósa’s pedagogy. There was a strong focus in the interviews on students’ descriptions of the Pósa method and what those revealed about the method itself.

Five interviews, each about an hour long, were conducted with carefully chosen participants. Personalities, academic success, and students with unique backgrounds were considered for the interviews. The students were first chosen based on their score in the class, which ranged from one to five. After doing so, seven interesting examples of students were
discussed with the teacher. The teacher spoke of their personalities and recommended individuals who may provide the most rewarding discussion on the Pósa method. In an attempt to interview a diverse set of students, it was decided that two students with a grade of 1, two with a grade of 3, one with a grade of 4, and two with a grade of 5 would be chosen for the individual interviews. Upon initially reaching out to these seven students, two declined to participate, leaving the study with five interviewees. Each interview consisted of questions on the participant’s background, experience with online school, memories of math education from kindergarten onward, and opinions of mathematics in general as well as the Pósa method in particular. A copy of the interview questions can be found in Appendix F at the end of this document.

The interviews were conducted in English, with the teacher on the online meeting to provide translation when necessary. After the interviews, a follow-up questionnaire in both English and Hungarian was distributed to the students, based on further questions from the researchers that arose after all the interviews were conducted. The follow-up interview questions can be found at the end of this document in Appendix G. A small interview was conducted with the teacher prior to the student interviews as well as after the interviews were conducted. The interview with the teacher was to provide a deeper understanding of how she perceived the personalities of the students as well as to give any information about the students that she deemed necessary or beneficial for the nature of the study.

Data Analysis Strategy

Data analysis is an intensive and typically ongoing part of the research. Throughout the data collection process, data is examined and evaluated to be either important or less monumental for the study. Analysis can lead to further questions, which accounts for and
justifies supplementary research. In the current study, data analysis came in a variety of methods: qualitative content analysis, cross tabulation, and interpretative phenomenological analysis. Data analysis followed the three-step qualitative strategy of description, analysis, and interpretation. Thus, the method of analysis was chosen, the data was analyzed, and the interpretation was documented. Table 1 breaks down the alignment of the research questions, the data sources, and the method of analysis used to generate the overall themes of the research.

Table 1 – Data Analysis Plan:

<table>
<thead>
<tr>
<th>Research Questions</th>
<th>Data Sources</th>
<th>Method of Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>What do the students at Petrik School think about the Pósa method?</td>
<td>Student Interviews, Interview Follow-Up Questions, Questionnaire, Journals</td>
<td>Qualitative Content Analysis, Cross Tabulation, Interpretive Phenomenological Analysis</td>
</tr>
<tr>
<td>What do the students in the Pósa method class think about math?</td>
<td>Student Interviews, Interview Follow-Up Questions</td>
<td>Interpretive Phenomenological Analysis</td>
</tr>
<tr>
<td>What do the students and teacher’s comments reveal about the structure of the method?</td>
<td>Student Interviews, Teacher Interview, Interview Follow-Up Questions, Questionnaire, Daily Journals, Classroom Observations</td>
<td>Qualitative Content Analysis, Cross Tabulation, Interpretive Phenomenological Analysis</td>
</tr>
</tbody>
</table>

Not only were the English responses considered during data analysis but also the Hungarian responses in the data. In conducting the data analysis, a Hungarian translator was hired to transcribe the Hungarian spoken during the interviews. Meaning can be lost when a native language is considered, thus it was necessary to have the Hungarian responses as a part of the data interpretation. Additionally, the translator conducted a literature search for articles written in Hungarian and subsequently translated the articles. These articles and translations were used in the literature review and were compared with articles written in English to
determine if the Hungarian and English literature were engaging in parallel conversations on mathematics education, citing similar sources, and concluding related findings.

Coding and Data Collection

Qualitative studies should focus on including numerous levels of coding and analysis to ensure the findings are unbiased, valid, and truly representative of the data collected. Coding is an essential part of qualitative analysis, since it increases validity, prevents excessive bias, and precisely embodies the participants’ narratives, given that their words are neither modified nor taken out of context. This section focuses on the various levels of coding that informed this study and the major themes that resulted from the analysis of the codes. Prior to qualitative coding, interviews and observations were transcribed verbatim, and results from the questionnaire, test, problem threads, and journals were compiled into spreadsheets. Transcriptions and observations were printed so that coding could be done by hand. Each method of data collection required a variety of meticulously selected coding strategies to ensure proper links between data and interpretation were formed. Throughout the process, the anonymity of the participants was kept, known only to the individuals who conducted the study, which includes me, the Pósa method teacher, and a fellow researcher. To ensure anonymity, the students were labeled with letters of the alphabet from A to Q, excluding the letter “I” to eliminate confusion in the presentation of the findings. The following, Table 2, contains a brief breakdown of the demographics of the students according to their alphabetical identifier. Included in the table is the grade the student received after the week of data collection, which ranged from one to five.
Table 2 – Breakdown of Student Demographics and Grades

<table>
<thead>
<tr>
<th>Student</th>
<th>Gender</th>
<th>Final Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Male</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>Female</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>Male</td>
<td>4</td>
</tr>
<tr>
<td>D</td>
<td>Male</td>
<td>3</td>
</tr>
<tr>
<td>E</td>
<td>Male</td>
<td>5</td>
</tr>
<tr>
<td>F</td>
<td>Female</td>
<td>5</td>
</tr>
<tr>
<td>G</td>
<td>Male</td>
<td>1</td>
</tr>
<tr>
<td>H</td>
<td>Male</td>
<td>1</td>
</tr>
<tr>
<td>J</td>
<td>Female</td>
<td>5</td>
</tr>
<tr>
<td>K</td>
<td>Male</td>
<td>5</td>
</tr>
<tr>
<td>L</td>
<td>Male</td>
<td>3</td>
</tr>
<tr>
<td>M</td>
<td>Female</td>
<td>5</td>
</tr>
<tr>
<td>N</td>
<td>Female</td>
<td>5</td>
</tr>
<tr>
<td>O</td>
<td>Male</td>
<td>4</td>
</tr>
<tr>
<td>P</td>
<td>Female</td>
<td>5</td>
</tr>
<tr>
<td>Q</td>
<td>Male</td>
<td>4</td>
</tr>
</tbody>
</table>

The first step in coding was to categorize responses to interview questions, reflections in journals, and responses to the questionnaire. This method of coding is called structural coding, where data is partitioned into smaller increments of code based on clear divisions, in this case, the questions in the interviews, journals, and questionnaire. A sample of structural coding and two responses to an interview question can be found in the first column of Table 3. From the structural coding, direct quotes from participants were compiled and grouped, without modification to the quotes nor an analysis of the meaning behind the quotes. This process requires the researcher to be cautious with the in vivo codes to ensure the words of the participants are not taken out of context. An example of in vivo coding can be seen below in the second column of Table 3.
Table 3 - Examples of Structural and In Vivo Coding from Student Interviews

<table>
<thead>
<tr>
<th>Structural Coding</th>
<th>In Vivo Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>I enjoy math now way more than earlier,</td>
<td>“I enjoy math now way more than earlier”</td>
</tr>
<tr>
<td>because now it’s not just going through the book, but we have problems that</td>
<td>“…now it’s not just going through the book”</td>
</tr>
<tr>
<td>we can think of, and I feel that they are closer to real life. They make me think</td>
<td>“…we have problems that we think of, and I feel that they are</td>
</tr>
<tr>
<td>So, I have to think when I’m solving them</td>
<td>closer to real life.”</td>
</tr>
<tr>
<td></td>
<td>“[the problems] make me think. So, I have to think when I’m solving them.”</td>
</tr>
<tr>
<td>When I didn’t learning the Pósa method, I just memorized things and I barely</td>
<td>“When I didn’t [learn by] the Pósa method, I just memorized</td>
</tr>
<tr>
<td>figured out the way I have to solve the problems. It was dry. That’s the only</td>
<td>things and I barely figured out the way to solve the problems.”</td>
</tr>
<tr>
<td>word I can use to describe. So, it’s dry. But with the Pósa method, I think I</td>
<td>“It was dry. That’s the only word I can use to describe.”</td>
</tr>
<tr>
<td>use my head to figure out the things and I don’t …I don’t really have to</td>
<td>“but with the Pósa method, I think I use my head to figure out things”</td>
</tr>
<tr>
<td>memorize formulas and things like that because I can figure out them through the</td>
<td>“I don’t really have to memorize formulas”</td>
</tr>
<tr>
<td>problems. So yeah.</td>
<td>“I can figure out [the formulas] through the problems”</td>
</tr>
</tbody>
</table>

The categories of in vivo and structural codes were then broken into looser codes through open coding, which is a round of tentative codes. The majority of these temporary codes were deductive in nature and were predetermined codes, decided prior to data analysis and based on previous knowledge of the Pósa method and Hungarian mathematics. The tentative codes evolved as data from this study was examined, which allowed for codes to develop and emerge from the raw data. The categorized codes after the first level of coding were: (1) everybody is different and learning needs, (2) homework, time outside of class, and practice, (3) enjoy, interested, and motivated, (4) freedom and no pressure, (5) definitions and memorization (6) discovery learning, mistakes, and pride in discovery, (7) self-pacing, time management, and mood, (8) wish different method and dislike, (9) real world problems, (10) thinking about the problems, (11) points system and gamification, (12) differences from previous math, (13) textbooks, and (14) skills learned.
After further rounds of focused coding, the following list was developed to be the final group of focused codes: (1) interest and enjoyment as motivators, (2) irritations and difficulties, (3) Pósa method versus previous math methods, (4) suggested alterations, (5) discourses on learning needs, (6) free and autonomous thinking, (7) the non-necessity of homework, (8) self-pacing based on mood, and (9) real-world problems and discovery learning. Through careful examination and reflection, the larger three qualitative themes emerged during thematic coding from the focused codes: (1) Mixed Perceptions and Experiences in the Pósa Method Class, (2) Comparisons and Proposed Improvements, and (3) Structural Revelations of the Pósa Method. The themes and the focused codes which enhance them will be referenced throughout the presentation of findings.

Summary

This qualitative research study, through a combination of mixed methods, analyzed student perceptions of the Pósa method and their experiences in a classroom which utilizes the Pósa method. The research design explained the importance and process of administering the data collection strategies. Throughout the process, anonymity was maintained by assigning unique letters to students, not linked intentionally to their names. Numerous data analysis strategies were employed, through coding specifically, in order to ensure validity and present unbiased data in this study. The data was carefully examined under a scrutinious lens to ensure my positionality did not alter the data as much as possible. The results and emergent themes from the data collection and data analysis strategies will be expanded upon in the following section of the paper.
Presentation and Analysis of Data and Findings

Overview of the Study

The presentation of findings is broken down into three main sections, partitioned by the following main research themes: 1) Mixed Perceptions and Experiences in the Pósa Method Class, (2) Comparisons and Proposed Improvements, and (3) Structural Revelations of the Pósa Method. Due to the nature of the themes, there is overlap between the themes in relation to the focused codes that will be discussed as well as the direct quotations drawn from the students. As a result, common topics and excerpts will appear throughout the presentation of findings. In the following sections, each focused code will be reviewed and supported by examples from the various methods of data collection in the study. For clarity, and due to simple spelling errors, some edits were made to student responses, indicated by brackets. That said, the content of the response was not altered in any manner.

The purpose of this holistic study was to examine the perceptions of bilingual Hungarian students in a classroom that utilizes the Pósa method. Phenomenological studies, such as this current study, give a unique perspective to the experiences that individuals undergo. Not only does this thesis provide insight into the thoughts of students, but also it sheds light on an underrepresented topic in educational research: Hungarian mathematics education, more specifically extending the Pósa method, a pedagogy aimed at educating gifted students, into general education classrooms. Through the eyes of students, we are drawn into the world of the Pósa method, its foundational attributes, and the academic lives of Hungarian students. The data collection measures of classroom observations, journals, a questionnaire, a test, interviews, and a follow-up interview questionnaire were utilized to add fruitful discussion to the narrative of
studies on mathematics education. In this section, I simultaneously present analysis strategies and findings to the following research questions that were the groundwork of this thesis:

1. What do the students at Petrik School think about the Pósa method?
2. What do the students in the Pósa method class think about math?
3. What do the students’ and teacher’s comments reveal about the structure of the method?

As a future mathematics educator, I sought to understand why the Pósa method is advocated by a few dedicated researchers conducting a larger study on the method. Through a focus on the students, I was able to craft a narrative and piece together defining features of the method, based primarily on student reflections. The following sections focus on these defining features, looking at the major themes of the study.

Theme 1: Mixed Perceptions and Experiences in the Pósa Method Class

In analyzing student interviews, questionnaires, and journals, a common theme began to present itself: the concept of enjoyment as well as its contrasting counterpart of frustration with the Pósa method. In total, students spoke more often about their high regard for the method rather than their dislike of the structure. At times, students would suggest improvements to the method to benefit themselves while at the same time presenting ways they saw the method working for different kinds of students and their peers. Many participants of the study detailed their personal history in mathematics classes prior to learning with the Pósa method and their current experiences in the Pósa method class. Provided that, in general, every person offers a unique perspective about any given circumstance, the variety of views of the Pósa method is not entirely surprising. That said, as mentioned earlier, participants voiced more strongly their interest in the method, especially in regard to its influence on their motivation in mathematics. The focused codes ‘interest and enjoyment as motivators’ and ‘irritations and difficulties’
highlight the contrasting and mixed feelings that the participants held towards the Pósa method, both of which will be analyzed in the following two subsections.

**Interest and Enjoyment as Motivators**

The second most frequent focused code after data analysis was ‘interest and enjoyment as motivators’, with the most mentioned code being ‘self-pacing based on mood’. Simply understanding that students spoke frequently of their enjoyment of the method shows how influential the method was among the Pósa method students. How often do high school students acknowledge their appreciation for mathematics, specifically the method in which it is being taught? Even the participants of the study were aware that realistically, their academic peers frequently do not take pleasure in learning mathematics. As student N said in their questionnaire:

> I came [to] like the mathematics in this way. And with this [method] I can understand what I'm learning. …If you learn this way you might have more success in this subject, which is really important because most of the students don't like mathematics at all.

The word “enjoyable” was in fact referenced quite often by the students in several forms of data collection such as the interviews, questionnaires, and journals. The repeated reference of “enjoyment” was not in part due to simplification of language as a result of the translation barrier. Rather, “enjoyment” was commonly brought up when speaking or writing in both English and Hungarian. The Hungarian root for “enjoy” is élvez [commonly paired with roots for “I enjoy” (élvezem) and “I enjoy it” (élveztem)]. Élvez and its roots can be seen across the data, indicating that when students think of the Pósa method one of the first feeling that comes to mind is “enjoyment”. It goes without question that the students in the Pósa method class do have an appreciation for mathematics, especially as it is taught with the Pósa method. Their attitude towards the method is in line with one of the main mentalities that Pósa himself hoped to instill in students when creating the method: mathematics is, and should be considered, enjoyable.
Paired with enjoyment, students spoke regularly about their interest (érdek), in the method. Often, students were rather blunt and there was no hesitation in discussing why they find the Pósa method interesting. To exemplify this statement, the following is an excerpt from an interview with Student D:

Researcher: And I’m curious about why you decided to join the Pósa method class. Why did you choose this pathway for learning math?

Student D: Because I found this method interesting.

Researcher: What is interesting about it?

Student D: What I like is that we don’t just go through problems in a book, practicing a good old method, but it’s more fresh or up-to-date or new problems that we have to work on.

Student P echoed the same notion of enjoyment during an interview, stating that they found the tasks given in class, or problem threads, to be interesting which in turn made it more enjoyable to do the problems and attend math class. In addition to interest, another student labeled the method as “exciting”, especially when solving a problem. The most excitement, as indicated by the student, comes from being “the first to get the solution. And like ‘aHHa!’ I got the solution!” As seen in the students’ response, there is joy and intrigue in discovering mathematics through the Pósa method.

Since enjoyment, interest, and motivation are positively related, they influence each other simultaneously. In the data, students expressed motivation, often an increased motivation, to complete their mathematics tasks as a result of engaging in Pósa method pedagogy. Motivation (motiváció) was another common root word among the focused codes. One participant, student F, spoke repeatedly of increased motiváció as a consequence of joining the Pósa method class,
declaring “I became really really, really motivated in math,” further claiming that this surge in motivation led to their enjoyment of the math class overall.

The students of the Pósa method are given a set period of time to solve the Pósa method problems. With this in mind, they are allowed to work on the problems on their own time, which may or may not include at home. If they do not finish their tasks at school, the problems do become homework. Student P asserted that they enjoyed this aspect of the Pósa method. To student P, the Pósa method is a “positive experience, that if I work hard on the lesson, I won’t get any homework to do. So, it’s inspiring.” As indicated by their response, student P finds their motivation through negative reinforcement, or the removal of an unpleasant condition. In this case, the unpleasant condition of ‘homework’ was removed, which encouraged better performance and attitudes towards their mathematics class.

While motiváció was spoken often, other instances of motivation were seen during classroom observations and highlighted in the evaluation of the week-long daily journals. For example, in their daily journal, student A noted at the start of the week a problem “wasn’t easy or hard, I felt like I just have to solve it.” Towards the end of the week, we see the progression and improvement in their motivation as the student reveals none of the problems were easy but “after the third or fourth try, I was unstoppable.” Almost as if giving themselves a mantra, one student documented their motivational thinking in their daily journal as “I have to practice these exercises. I have to learn the theory. I have to learn how to do this.” Despite the challenge of the problem threads, these two students increased their effort, stayed motivated, and solved all the problems. Motivation was linked with students’ moods as well, which will be discussed further in the ‘structural revelations of the Pósa method’ section. When students were in the mood to
work on the problem threads, they would “work a lot and …think a lot about the problems”, as voiced by student D.

Enjoyment, interest, and motivation led to the students ultimately, and simply, liking the method. It does not come to be much clearer than student F frankly stating, “I like [the Pósa method]” as the first phrase of their questionnaire. By far, my favorite quote from the follow-up questionnaire was the response a student gave when asked, “Are there aspects of the Pósa method that you DON’T like? If so, please describe them.” The student responded with, “Not really, but if there are, I hope I will find them 😊.” And yes, student A included the smiley face.

In response to whether there are any disadvantages to the method, the same student continued with, “I cannot say, but I think it’s because there isn’t any.” Student D’s response to their follow-up questionnaire resonated with student A’s: “There is no aspect that I would dislike.” In fact, some students advocated for future use of the method, saying “I hope more people will start using this kind of method” (Student N) since “the method is useful and helpful for [other students].” (Student B)

**Irritations and Difficulties**

While many students voiced their admiration for the Pósa method, there are a select few who spoke of the negative reactions they have towards the method. Irritation, anger, and the notion of being “pissed off” were presented in the questionnaire, journals, and the interviews.

Student A voiced their concerns by referencing competition in the class, stating, “you have to [accept] when you do not get the exact solution and you're pissed off and the others got to the end and some days later you come to [find] that it was a really…easy, you just [did] not [try] it.”

Frustration in the Pósa method class was strongly related to whether students were able to solve the problems. However, in the following example from student O, the frustration can be short
“Sometimes I [get] angry [when] I can't solve a problem but when I find the answer, it causes a lot of happiness, so this little 'suffering' is okay :D.” Grades were also brought up by students, with the idea that on occasion, the participant will “need only 1 or 2 points to get the better grade so it can be irritating. But these are not huge problems.” In each case, the student made sure to acknowledge that their negative emotions towards the Pósa method were only temporary: “did not [try] it,” “little suffering,” and “not huge problems.” Though the adverse feelings are fleeting, they are important to address.

During the student interviews, student O brought to light ways that he believed the Pósa method could be changed. Student O expressed the desire to combine the Pósa method techniques with the practices of typical Hungarian mathematics instruction, in which students are given mathematical formulas, followed by problems to practice the concepts. The student discusses the difficulty of not being told the method for solving systems of equations. The main point to address is that they wanted to change the method due to it being difficult, thus making it hard to solve some problems. The following is the exchange with the student:

Another example that we had … is the system of equations that we had to figure out how to solve them before we learned the methods of how you can solve a system of equations… And I think this is what makes it harder, this is the point why it is more difficult. It would be easier if we were told the methods first and then practice solving problems with those methods… I would say a few of my classmates think the same way I do. Most classmates chose this group because they thought it would be easier.

Student H transcribed a different desire in their questionnaire, wishing to have more direction when solving problems. The student revealed that they “find [the Pósa method] to be better [than] the usual teaching method, although I wish we (I) could get more guidance during the lesson.” The Pósa method, as indicated by participants, is not perfect for every student since
there are a select few who do struggle with the method. Student P stated it most clearly: “I think that because everyone is different…there is no method which is 100% efficient for everybody.”

**Theme 2: Comparisons and Proposed Improvements**

In this section, the second research question on students’ thoughts about mathematics is discussed. During the analysis, the common focused codes that presented in the data were a comparison of ‘Pósa method versus previous math methods’ and ‘suggested alterations’ for the method. Students highlighted the stark contrasts between their previous experiences in mathematics classes and their current experience with the Pósa method. At times, some students gave their opinions on how the Pósa method could be changed to accommodate their mathematical and learning needs. The following subsections reference notable student responses and opinions in regard to their experience in the Pósa method class, previous mathematics classes in general, and their current science, technology, engineering and math (STEM) classes.

**Pósa Method versus Previous Math Methods**

One of the main topics that students brought up was how different the Pósa method is from their previous math classes. The students claimed that the Pósa method was qualitatively better, due to its lack of ‘tedious’ problems from a textbook. Rather, the students found the Pósa method problems to be interesting and well-suited. Quite often, there was a positive reaction from the students when discussing the Pósa method. “It’s better”, said student A about the Pósa method, after stating that their math classes in elementary and middle school were monotonous. The student continued by saying, “you're not punished for not being able to do something because you know that the point is to understand and develop through learning.” Failure is encouraged in the classroom since it is through mistakes that the students truly learn. Students are not expected to answer the problems correctly the first time.
The Pósa method includes exciting problems that are not “so dry”, a phrase used by student F to describe their previous math experiences. The subsequent quote comes from a response that student F gave about their experiences in mathematics education, prior to and during the Pósa method:

Before the…Pósa method, I just memorized things and I barely figured out the way I have to solve the problems. It was dry. That’s the only word I can use to describe. So, it’s dry. But with the Pósa method, I think I use my head to figure out the things and I don’t ...I don’t really have to memorize formulas and things like that because I can figure out them through the problems. So yeah.

The Pósa method teacher began to show the student mathematics problems on their level, which led to an increased motivation in mathematics. The student felt like they were given the chance to work through problems on their own for once, using their “head to figure out things” rather than depending on formulas. Previous classes did not provide the same accommodations for this student. Similarly, student P discussed how moving to a Pósa style class increased her grades and made her less afraid of mathematics. She had previously looked at math “as the worst subject but it became easier for me [after starting the Pósa method] …because people [were] almost on my level.” The students’ responses reveal how the Pósa method allows them to work at their own level in a way that previous mathematics classes did not.

One unique fact about the Pósa method classroom in this study is that the students were engaged in a gamified classroom. In other words, students collected points over the course of a “point collecting period”, where a certain number of points determined their final grade for the period, on a range of one to five. Previous math classes did not employ gamification in the classroom. Students were able to gain partial credit and were not penalized for missing points due to lack of understanding in a lesson, problem, or problem thread. As one student put it, “the point collecting system shows our work, time, and energy invested in the subject much better.”
Due to the nature of the thesis, gamification cannot be discussed fully but it is important to note that the students brought up the notion of how collecting points and getting good grades was different from their previous experiences in mathematics. In the midst of a hardship, one student “got a lot of help in math - extra chances, extra tasks, more problems to solve.” The teacher continued to give them a “chance to get a better grade.” The Pósa method accommodates unforeseen complications in students’ lives; they are able to work on the problems on their own time and can be given more tasks and points to make up for missed assignments.

Some students take advantage of the gamified system in the Pósa method classroom. “They just want to get good grades,” student O rants, “they are just doing as many exercises and get good grades as required.” Their peer, student P, was one of the students who found a way to make the gamified system work for them, as seen in the following description of how they achieve good grades: “I work hard then, of course, and I realize that if I show all of the task, it will be easier to get points, because if I didn’t finish the task, I didn’t show it to the teacher.” Student P understood that the purpose of the Pósa method was to show their thinking, not necessarily arrive at the correct answer every time. As can be seen in their reflection, students can still do decently in the class without fully completing the assignments or math problems. As the Pósa method teacher said during an interview, points can be given lesson by lesson to secure the highest grade of a five, even if the student is slightly struggling. This combination of point collecting gamified system and the Pósa method remains quite different from the typical method of instruction in Hungary.

Interestingly enough, students often compared the use of textbooks and notebooks in their previous math classes and the Pósa method. Since the Pósa method is new to public education, there is no prescribed textbook. As well, problem threads are constantly changing and open to
adjustments in the classroom. The teacher of the Pósa method collaborates with the other Pósa method teachers to create all the problems in the problem threads. Student D talked about how it was a positive thing, declaring:

I enjoy math now way more than earlier, because now it’s not just going through the book, but we have problems that we can think of, and I feel that they are closer to real life….What I like is that we don’t just go through problems in a book, practicing a good old method, but it’s more fresh or up to date or new problems that we have to work on.

As can be seen in student D’s description of the Pósa method, the lack of textbooks has led to new problems being developed for the students to tackle. In this way, the students are not left solving problems from an outdated textbook, which is frequently the case in public education. Student A voiced how the Pósa method is “not just like any other class. [In other classes,] you just sit in your desk and write in your notebook and just look at your teacher and go ‘Okay. mhmm mhmm’.” Rather than making students write in their notebook, following along as their teacher teaches at the front of the class, the Pósa method requires students to be active participants in the learning process. The method, while it follows some structure, does not exist to be bound to a set course of study. It is a method of discovery, which in turn means it is under constant adjustment like all mathematics. The lack of textbooks involved seems to be a positive aspect of the method, as noted by the students.

**Suggested Alterations**

As mentioned in the section on ‘irritations and difficulties’, some students were keen to suggest ways to make the Pósa method fit their needs in learning mathematics. Student O was very vocal about expressing their feelings on this matter. Their belief was that the Pósa method should be combined with some aspects of typical mathematics instruction, which involves introducing students to a math topic followed by giving problems to practice the math method. In his words, he said, “in my opinion, it would be sometimes better if we mixed the two approaches
and have the methods sometimes told us first and then practice solving problems using those methods.”

In relation to typical curriculum, student D brought up a thought-provoking point about how the Pósa method relates to his other STEM classes. He mentioned that “the other learnt sciences rely on math and if we don't learn math the same order as the others, we could be having difficulties in understanding them.” The Pósa method does not follow a set curriculum and topics are brought up at random intervals. This caused issues and difficulties for the student in their STEM classes, given that there were moments when they had not already learned the math involved, unlike their peers in the typical Hungarian math classes. Thus, it is evident that in some cases, students have wishes to alter the method for the sake of their academic desires, whether it be for ease in their mathematics or STEM classes. Their responses demonstrate how math is influential in multiple content areas as well as how the Pósa method is still a work in progress in general education.

**Theme 3: Structural Revelations of the Pósa Method**

Seeing that the Pósa method has only recently been introduced to public education, there is a lot about it that is still unknown to math education scholars both outside and inside Hungary. The structure of the Pósa method, especially how students understand it, is an important topic. Lajos Pósa designed the method to emphasize problem solving, discovery learning, playfulness, and autonomous learning. As described earlier, the Pósa method has been called “relaxed and cheerful” by the individuals who implement the method in the gifted math camps (Juhász & Katona, 2019, pg. 270). Considering what Lajos Pósa and researchers of the Pósa method have described as the foundations of the method, this section focuses on what the Petrik School students and their teacher reveal about the structure of the Pósa method. The following focused
codes will be discussed and supported by examples from various methods of data collection: discourses on learning needs, free and autonomous thinking, the non-necessity of homework, self-pacing based on mood, and real-world problems and discovery learning.

**Discourses on Learning Needs**

Participants of the study showed a heightened awareness of their and their peers’ learning needs in the Pósa method class. They discussed how it was important to realize the Pósa method may not fit every student’s academic needs. Additionally, the students mentioned how the method does require a level of interest in order to succeed. In other words, if a student of the Pósa method class does not enjoy thinking about the problems and are not motivated, then they are less likely to excel in a Pósa method classroom. The following is an example of two separate quotes written in a questionnaire and follow-up questionnaire by student F that bring up the concepts of academic strengths of students and interest affecting a student’s engagement with in a Pósa method classroom:

Not all people are good in the way of thinking that the Pósa method requires. I mean the students that are really good in literature and history usually aren't that good in maths and other sciences and also don't have the required way of thinking.

For students who are less interested in maths it's not the best method. If they don’t care, they won’t think about the problems, so the method in their case is useless.

In referencing difficulties for learning, this student brought up how the strengths of a student in “literature and history” can affect their performance in mathematics and the sciences. Through striking diction, the student even labels the method as “useless” for students who are not as invested in learning mathematics. Their peer, student J, echoed this of “uselessness” by stating “this method works only if the person [likes] math and wants [to think] about math problems.” The Pósa method does involve quite a bit of thinking about problems, given that it is
a student-centered method. Therefore, the motivation to think about the math problems must come almost entirely from the student.

Furthermore, it takes time to understand the Pósa method format. Student P mentioned how it took about a month to get used to the techniques used in the Pósa method. In contrast to what their classmates said, student P stated that “maybe [the Pósa method] is easier for the children who are not the best with [mathematics]. Because if you are hard-working, you can get good grades too.” This statement from student P suggests that they believed it is possible for any student to learn through the Pósa method, as long as the student practices and perseveres.

There were many students who did express how the method fit their learning style, claiming that the Pósa method is a “useful method.” Their teacher brought up how the method has helped bring success to some students in learning mathematics. If the student had been placed in a different class without the Pósa method, the teacher hypothesized that the student “would experience only failures and would hate math because [they] wouldn’t understand anything.” In discussing another student in the Pósa method class, the teacher called the method “perfect” for them since the student has not been pressured by a time limit, which allowed the student to flourish in the class. Hence, as seen in the reflections from the students and their teacher, there were some conflicting opinions about the Pósa method’s ability to meet and appeal to each student’s learning needs.

**Free and Autonomous Thinking**

As with ‘motivation’ and ‘enjoyment’, a common Hungarian word that was spoken by the students was *szabadabb*, or “freedom.” To the students, the concept of “freedom” resulted from a pressure-free environment that encouraged independent thinking. Additionally, the students felt “free” because they did not have to stick to one mathematical method of solving a
problem nor memorize facts and formulas. Rather, they were able to process the problems on their own time and make sense of them in the way they’d like. Student P asserted that “the main reason why I like this method is that we are not stressed at all.” Student A reverberated student P’s sentiment by including a comparison, stating that “you’re free from that pressure you get when you are in a basic math lesson.” The following is an exchange during an interview which provides another example of a student referencing the freedom they feel as a result of learning in a Pósa method class:

Researcher: Okay and what has this class been like with the Pósa method compared to your classes before?

Student O: What [are] the differences between them, that’s what you are curious about now? We have a kind of freedom of thinking because we can think of the exercises, we would like. We don’t have to study the definitions by heart, at least most of the time. We are not given the exact solving method. We have to find out by ourselves… I think I spoke [about] everything I [wanted to]. Yeah. The freedom. The [not] learning by heart.

As student O brings up, the students did not have to “[learn] by heart”, meaning that through discovery learning, the students developed their problem-solving skills. Through this method of learning, definitions and steps for solving a math problem are not memorized. Rather students learn the logic behind solving the problems. As follows, they are prepared to tackle problems that may follow the same logic but not necessarily the exact same procedure of a similar problem given in class. This is a standout strategy of the Pósa method: the aspect of building logic through discovery learning rather than memorization skills.

The students commented positively on the freedom from pressure as well as the freedom to think on their own in the Pósa method class. This freedom of thinking can also be termed “autonomous thinking”, which is one of the main pillars that Lajos Pósa included when creating the Pósa method. By instilling autonomous thinking, students are aware that they are “not
punished for not being able to do something because [they] know that the point is to understand and develop through learning.” (Student A). The students are given “the chance to work on the problems the way [they] would like to.” (Student D). The Pósa method thus gives the students the opportunity to tackle math problems in the ways that the students prefer rather than through a predetermined system constructed by the teacher or the Hungarian government.

As previously discussed, student O mentioned in their interview that the students do not have to learn the definitions by heart. Later in the interview, student O revealed that the reason they chose to join the Pósa method class was due to the fact that they would not have to memorize a load of mathematical definitions. Student F brought up the same concept, mentioning how in “the Pósa method, I think I use my head to figure out the things and I don't...I don't really have to memorize formulas and things like that because I can figure out them through the problems.” Since students are meant to use problem solving to find mathematical methods, formulas, and topics, there are few definitions to memorize. As a result, the students develop tremendous problem-solving skills.

Applying autonomous thinking and freedom in the classroom has its benefits. Students commented how they tend to work on their own and think for themselves, which, as student L argued, “teaches me to rely on myself, so it has long term benefits.” Student A echoed a similar statement, during an interview, by listing the following abilities they’ve learned and developed over the years in the Pósa method class: how to ask mathematical questions, how to work by oneself, how to solve math questions, and how to handle new problems. As student K disclosed, “[the Pósa method] helps me think outside the box when I face some sort of problem.” Student K’s statement further suggests that higher-level autonomous thinking is promoted and cultivated in the Pósa method classroom.
The Pósa method gives students the opportunity to develop problem solving skills, given that every task is new and requires the ability to think beyond the problem itself. Again, these qualities have been internalized by the students as positive skills developed as a consequence of learning through the Pósa method. Student Q said quite directly “I think it's a good method because if you figure out something for yourself, you will remember it much better.” Discovery, including the failure that may arise during the process, can be one of the best methods for learning. As can be seen throughout this presentation of findings, the students brought up how freedom, autonomy, and the benefits that are associated with both are important attributes of learning in a Pósa method classroom.

**The Non-Necessity of Homework**

One aspect of the Pósa method that students would comment on, and enjoyed, is the fact that homework is not a requirement. As student F put it, “I only have homework if I want to do. So, like um, we don't get homework, we just get tasks. So, if we [want to, we] can do [the tasks] at home. If I want [to], I just do [them] at home.” Problems are assigned to students and they are given a set amount of time to solve them, whether it be a week or longer. By the end of the working period, the students are given the answers to the problems. In this way, if they did not find the solution, they’ll know the answer to help they solve future problems. Due to the nature of the Pósa method, students have ‘homework’ only if they choose to. In other words, they work on tasks on their own time, whether it be in class or outside of class; what they do at home is up to each individual student.

Demands on students after school can affect whether the student chooses to work on the tasks at home. One student voiced how they often “don’t have enough time and energy after school”, making it so they “would rather deal with other things” than homework. Once the
pandemic hit, students began to attend their classes online. Technical issues and family problems threw the students’ academic lives off balance. Student J described their experience with the Pósa method during the pandemic in the following entries of their journal:

Day Two: Today my problem was that my internet (and my phone as well) didn't want to work, so I haven't really had a chance to look at the math problems, and I'll be honest, I didn't want to do anything with it while I had bigger problems. (Yesterday I couldn't write into the journal, because I couldn't find this journal, it took me long to finally find it).

Day Three: Today I read the tasks, and I was thinking on the last one, but I haven't written my thoughts down yet.

Day Four: I used some help in the 246 problem [Side notes from the researcher: 246 is one of the tasks given to the student. In their journal on day four, the student had shown their work and answers for problems 246 and 249, half of the assigned tasks for the week.]

Day Five: Today I didn’t do any tasks, mostly because I had a headache.

The student went into detail about the important technical and medical factors that affected their ability to work on the tasks. Despite the issues the student faced, by the end of the point collecting period, they had acquired enough points to get the second highest possible grade, a four. Given the flexibility of the Pósa method, students are able to take the time they need to get the problems done in a way that other pedagogies do not allow. In essence, it allows students to work at their own pace like student J did, which will be explained further in the next subsection. As teachers, it is important to understand that students have other requirements beyond school, especially at home. The structure of the Pósa method gives students flexibility with their schedules, both inside and outside the classroom, which in turn makes assigning or doing homework a non-requirement.
Self-Pacing Based on Mood

The structure of the Pósa method allows students to choose how they spend their time on the problems given to them. Sometimes, the time is not spent in the way the student would like. Student N gave an excellent example of how time management can be a struggle by writing in her journal that “today I didn't work on the exercises. I didn't manage my time well, so I almost [ran] out [of time] for some other lesson's assignments.” While the student did not spend time on the Pósa method tasks, the lenient deadlines of the Pósa method class did give her the chance to work on her other schoolwork.

An analysis of the journals showed how the students chose to spend their time on the problems as well as when they worked on the math problems. Given that this study was done partially during a pandemic, students were able to work on the problem sets at time when they might have attempted them prior to the pandemic. The subsequent quotes are entries made by student K in their journal about his process working on the problems from this study:

Day 1: exercise 246 - I am unsure whether I am doing the right calculations without the teacher.
(20 minutes later) I am clueless, and I have come to the conclusion that I need help.
Update, after I got help from the teacher, I was able to do it

Exercise 247 - started doing exercise 247 after giving up on the previous one.
Update, I decided to do it tomorrow

(after some time) This exercise is starting to irritate me because I don't know what to do.
Update, I have made some progress however I am unsure so I need guidance and that will happen tomorrow.

Day 3: 247 - It is Friday and after getting some help I was able to do it.

Notice how student K included updates in his journal, transcribing his discovery learning process. With the help of their teacher, he was able to solve the problems but not without
attempting them first. Additionally, observe how they mentioned they did “late night maths” when it was most convenient for them. Student K was not the only student to do their exercises late at night. “I do the math task in the evening because I can think better then because nobody interrupts me”, said student P after being asked how their work habits have changed due to the pandemic. They mentioned why the late-night math appeals to them: they’re the most fresh then, they feel in the mood, and nobody interrupts them while they’re working. No one was telling the students when or how they should work, so each student chose times that fit their schedules. Some students took advantage of the class time, where, in student N’s words, the students “have an exact time period for working and thinking, and this can be [helpful].” Student F also chose to work during the lessons though not for very long; the Pósa method teacher mentioned that student F “solves the problems in only a few minutes”, spending the rest of the time preparing for math competitions.

The teacher of the Pósa method class brought up during the interviews how over the years student B has alternated between loving and hating the Pósa method, with the switch between the two feelings strongly depending on the student’s “personal situation and how [they] feel” at the time. Her reflection on student B is not a unique case in the Pósa method classroom. Whether a student enjoyed the method or worked on the problems depended on their mood and feelings, as indicated by the data. In Hungarian, the students used the phrase “kedvem”, or “I feel like it”, which was loosely translated to “mood” and “my inclination” by a Hungarian translator. The concept of ‘being in the mood’ to work was touched on repeatedly in the interviews, journals, and the questionnaire. For example, student P noted that “sometimes I don't feel well, I'm tired, I'm in a bad mood, I can't do my best.” Student D echoed the same response, by claiming “sometimes, when I’m in the mood, and I feel like doing it, then I work a lot, and I
think a lot about the problems. But other times it’s just not a priority in my life.” Using mood as an indicator to work or not work is not necessarily a bad thing for the students, however, since they know that if they did poorly in one unit, they’ll work harder the next time and “still won't get a bad grade because…I can make up for it.”

**Real-World Problems and Discovery Learning**

The participants of the study highlighted the importance of having real-world problems in their math homework, acknowledging that by doing so, the Pósa method class provided unique problem sets that previous math classes have not incorporated. As previously mentioned in the section on ‘Comparisons and Proposed Improvements’, student D liked that they “don’t just go through problems in a book, practicing a good old method, but it’s more fresh or up to date or new problems that [the students] have to work on.” These “up to date” problems require lots of review and practice, which student N claimed, “would be a great help later on” in their studies and personal life. The tasks of the Pósa method are similar to puzzles and thought problems. The problems are written to fit the learning needs of the students. Student F explained that the interesting real-life problems of the Pósa method are part of the reasons why they became interested in math:

> [The teacher] showed me math problems that were sort of on my level and just interesting math problems that weren’t just dry. So, it was more. I could connect the math problems more to real life than before. Yeah, so that’s why I get so motivated, and I enjoy her classes.

Many students in math classes struggle with appreciating math class since they don’t see how the content can be applied outside of class. Notice how student F appreciates the applicability of the math problems to real life and how she finds the problems to be written on her level. On the flip-side, the teacher of the Pósa method class mentioned that she felt there were some issues with implementing real-world problems for student F. She worked too quickly
since the questions were not hard enough for her, given her academic talent. The teacher mentioned this idea during a brief interview, bringing up how she is ahead of her peers, making it difficult to write enough tasks at her level. Generally, the math problems are written to be slightly difficult so students can practice discovery learning and be prideful when they discover the answer. Other than student F as an outlier, most students in the class have to work hard and do not solve the problems very quickly. That is not to say they don’t want to. As student K admitted, “I would feel really proud if I could solve it easily on my own hastily. :D.”

One student remarked how discovery learning brings about a “feeling when you solve a system of equations correctly”, further stating that this feeling is “a good feeling” especially “when I figure out the solution of a more difficult problem by myself.” Feelings, of course, can fluctuate. Student A gave a prime example of how their emotions shifted during the problem-solving process by asserting, “at first my feelings were a little bit under the ground, but after I finally find out that my solutions were right, I was happier than ever before.” Solving the problems by oneself is ultimately what discovery learning entails; the students are given the set of problems and are tasked with using their previous knowledge to solve the questions. Student O brought up an example of this method of thinking by explaining their process of solving a logic problem that was given to them prior to data collection. In solving the problem, they said, “my first reaction was to look at the Mr. and Mrs. Smith exercise again, then I realized I have to use a [technique] that is similar in some ways to solve it.” This is a prime example of how students lean on techniques learned from previous questions in the problem thread to answer later questions.

Discovery learning, while primarily meant to be done mostly by the student, can involve outside help. The teacher of the class does carry out review sessions where students learn the
name of the math method they were solving, as well as the correct way to solve the problem, particularly if the student did not get it right during the unit. By explaining the math method and teaching them how to logically think through the problems, the teacher prepares students for the following units, since the problems in the problem thread build on one another. Student N expressed their appreciation for this type of teaching in their journal by transcribing, “I've got some help from this journal where the teacher sent some hint. I also got help on the lessons too. So, I think I managed to solve this task. :D.” Note how the student indicated their happiness and pride in discovery through the smile at the end of their quote. The students know they are “not punished for not being able to do something because [they] know that the point is to understand and develop through learning.” Thus, the students are generally not distraught if they have to use their teacher or outside sources to solve the problem. That said, the students were aware that solving the problems for themselves is the best method for retention.

Summary

In conclusion, one can see across this presentation of findings that students have fruitful descriptions, comparisons, and feelings about the Pósa method. Their words, actions, and reactions are beneficial strides towards understanding whether the Pósa method is an adequate method for mathematical instruction. While the students’ thoughts can only describe their opinions of the method, their unique position as participants in this new method for general education helps emphasize the importance of their perceptions. As mentioned previously, through careful analysis of their perceptions, there appeared the following three important themes: (1) Mixed Perceptions and Experiences in the Pósa Method Class, (2) Comparisons and Improvements, and (3) Structural Revelations of the Pósa Method. These three themes suggest
that the students have diverse attitudes towards the Pósa method, as well as exclusive narratives about the compositions of the Pósa method.

**Discussion**

Student perspectives and the data collected on their thoughts revealed many valuable and underlying components of the Pósa method. The following four components of the Pósa method will be discussed in this section: that the method is enjoyable, it does not always align with the content in other STEM-related classes, it accommodates students’ learning needs especially through pacing and less homework, and is comprised of interesting real-world problems, which are solved through discovery learning. In the following section, the results of the study will be related to the conversations held in the literature. Through synthesizing the literature with the findings, links between what is known and what has emerged from the study will be more clearly defined, creating a better understanding of the findings that came forward during the study.

**Results Related to Existing Literature**

As a result of the data analysis and findings, new concepts were brought up in the data which enhance existing scholarship on balancing problem solving and instruction, the incorporation of textbooks in math, and the effects of emotions and enjoyment in math. Largely, student comments during interviews and their questionnaire responses brought to the surface distinct and unexpected topics. These topics show the need for emphasizing student perspectives in educational research. By focusing on student perceptions in research, several unforeseen and beneficial findings may appear, as did in this research project.

Scrutiny of a method, to discover the negatives as well as the positives of its structure, can prove to be valuable in creating the most efficient and enjoyable system, in this case, the best
design and implementation of the Pósa method. The generative insights of students in the research project provided modifications to the method in order to fit their learning needs as well as how to shift the method into non-gifted spaces. Student O mentioned that they would prefer if there was a mix between the Pósa method and the typical method of instruction so that math concepts are sometimes introduced prior to using them on Pósa method problems. Fyfe, DeCaro, and Rittle-Johnson’s (2014) study supports student O’s recommendation. They found that students embodied a greater conceptual knowledge, as well as improved problem-solving skills, when conceptual instruction, a method of teaching the principles of a field, is done prior to problem solving (Fyfe, DeCaro, Rittle-Johnson, 2014). Instructing students on the procedure for solving certain problems gives them the chance to process the method without the struggle of discovery. Furthermore, conceptual instruction can increase the likelihood that students are following the correct sequence of steps, lines of logic, and levels of thinking when problem solving. Fyfe, DeCaro, and Rittle-Johnson’s (2014) study and the findings of this thesis together emphasize the importance of the conceptual instruction timing in relation to problem solving.

On the flipside, other researchers have shown that problem solving prior to instruction can encourage “productive failure,” a concept that implies learning can come from preliminary struggles (Kapur, 2012). Through productive failure, participants without prior instruction had higher cognitive abilities, given that students were required to activate prior knowledge in order to find the solution. Due to the conflicting nature of the discussions on productive failure and conceptual instruction mentioned within Kapur (2012) and Fyfe, DeCaro, and Rittle-Johnson’s (2014) studies, there should be further analysis on these topics in relation to the Pósa method.

In light of the findings of this study, the notion of not using textbooks in the Pósa method class was indicated to be a positive aspect of the method by the students, since the problems were
generated by the teacher. As Juhász (2016) noted in their proposal to the Hungarian Academy of Sciences to study the Pósa method in general education, the Pósa method will one day have a working textbook or workbook “if the experiment shows that this is the right way forward” (Juhász, 2016 pg. 12). To detail their process, they (Juhász, 2016) said,

> During the entire grant period, we would create detailed documentation of the process. We would keep a record of what happens in each lesson. After class, we would provide the students with the complete text of every problem discussed. If this goes according to plan, these documents could create the basis of a textbook and workbook. (pg. 12).

The teachers and the students would work together to design the questions, just as they do in the current Pósa method system. An intriguing study could look at how the method works with a textbook in the classroom once it is complete, especially given that students preferred not to have a structured book for discovery learning.

Education can contain an affective facet where students’ emotions can influence the way they interact with the subject. Among the students of the Pósa method class, the emotion of ‘enjoyment’ was one of the most prominent codes about the Pósa method in the data analysis. As the literature review suggested, “a higher judgement of the quality of math instruction by students was linked with increased levels of math enjoyment” (Frenzel, Pekrun, & Goetz, 2007; Vandecandelaere, Speybroeck, Vanlaar, De Fraine, & Van Damme, 2012). Students in the Pósa method class spoke highly and valued the structure of the method, giving a possible reason for why the students enjoyed learning through Pósa’s method. As a consequence, “math enjoyment is positively associated with pupils’ later math self-concept” (Pinxten et al., 2014, pg. 170). During the interviews, students brought up how their appreciation of discovery learning led to greater confidence in their mathematical abilities.
Recommendations and Implications for Practice

In attending a study abroad program in spring 2020 in Budapest, Hungary, I began to understand the nature of Hungarian education as well as the Pósa method. I witnessed the Pósa method first as a student of the method itself in classes taught by Hungarians, one of whom was a former student of Pósa’s in the Pósa gifted math camps. Through a research course I took in Hungary, I analyzed scholarly articles and preliminary research about the Pósa method. After leaving Budapest, the data began to shed light on the ways my emergent findings were nested within a broader scholarship about student perceptions, gifted education, and non-traditional learning strategies, all of which helped make this thesis well-rounded and in line with the larger conversations in mathematics education research.

From the student opinions I have acquired over the course of this research project, I have found that the students expressed high regard and enjoyment for the Pósa method. In my years as a student in mathematics, as well as the years I have spent in teacher and classroom-related settings, few math methods have been so highly viewed as the Pósa method. To listen as students shared their satisfaction with the method along with their successes, was a rare experience to witness. Thus, it is without hesitation that I recommend the Pósa method be introduced in more classrooms in Hungary. Moreover, as data continues to be collected on the method, I believe the Pósa method would make an adequate method beyond Hungary’s borders. Persons implementing the method, however, should do well to remember that the curriculum does not follow the same sequence as other STEM-related subjects, which may lead to difficulties in teaching with this pedagogy. Furthermore, the process takes time and dedication, on both the students’ and teacher’s part, particularly at the beginning of the implementation of the method.
The benefits of this research project lie in how current and future educators process the information I have disseminated. The preferable next steps involve advocating for implementing gifted education techniques in general education, changing the narrative in schools to be more student-focused, and discovering more about the Pósa method and similar pedagogies. As the main researcher and recommender for this practice, I will engage in the same steps towards encouraging best teaching practices for mathematics education. It is with high hopes that I can publish this thesis in both an English and Hungarian journal to further the conversation on what makes the Pósa method unique and valuable.

Recommendations for Future Research

As student O declared during a follow-up interview, “It is not a tested method, so we do not know if this can work, and it is also not known if this method is good enough to prepare us for the mature exam.” What the student is referencing is the Hungarian secondary school leaving exams (érettségi or Matura), which each high school senior must take across the country, at the same time. These exams are the entry exams that students need to enter higher education at a university or college. At the end of this current school year (2020 – 2021), all students of the ongoing study conducted by the Content Pedagogy Research Program will take the leaving exams and their results will be compared to those of students across Hungary. Further research on the Pósa method is needed, not just on its effectiveness in general education, but also on its success in the Pósa camps. Research on these topics would provide a deeper discussion on the structure of the method and, most importantly, why it seems to work. Beyond studying the Pósa method in Hungary, studies could analyze whether the method should be considered an applicable method in classrooms around the globe. Thus, additional data on the usefulness of the method is essential to determine if it could become part of standard math pedagogy.
Additionally, further research should engage in longitudinal studies that follow the academic experiences of groups of students prior to entering the Pósa method class as well as their experience during Pósa method instruction. The MTA-Rényi research group have conducted a longitudinal research study on three Pósa method classes and their study should reveal extensively whether the Pósa method works with its students. However, that study does not follow the students prior to entering the Pósa method class. Therefore, it would be useful to compare the progress that students make over time to see if the Pósa method is quantitatively better than typical mathematics instruction, through an analysis of grades, especially on the Hungarian high school leaving exams.

The Pósa method, as mentioned during the literature review, has some similarities to other educational techniques that have been common in the research like inquiry-based learning (IBL), problem-based learning (PBL), and personalized learning. All three of these techniques are not necessarily for gifted students, whereas the Pósa method was designed for gifted students. Valuable research could compare the Pósa method and its method of discovery learning to one of the other prominent non-traditional learning styles like IBL and PBL. Furthermore, effective training for educators with the Pósa method, like IBL and PBL, is needed. Finally, in general, more research needs to be done on the pedagogical decisions of teachers based on the track of students in math. The Pósa method touches on the concept of using gifted methods of teaching mathematics for all types of students. Future researchers should look into how to incorporate mathematics educational techniques across different tracks of students.

Conclusion

Could gifted programs be used for the masses? Why don’t we use the curriculum that works with the best math students for all students? How could we take successful gifted
mathematics pedagogies and apply them in regular education classrooms? Schools are becoming more academically diverse as students bring unique experiences and ideas. There is a need to analyze gifted programs globally among students who are labeled ‘non-gifted’ in general education classes.

In this thesis, the perceptions of Pósa method students at Petrik School were analyzed to see what their reflections revealed about the Pósa method, a method of mathematical instruction that heavily relies on discovery learning. Given that the Pósa method has primarily been used with gifted students in the past, this study sheds light on the Pósa method’s gifted pedagogy being used in general education and the student revelations of effectiveness and disadvantages that ensued. The Pósa method was shown to be enjoyable, accommodating to pace and needs, and successful in the manner of discovery learning, yet it also did not teach content at the time needed for other courses, leaving Pósa method students behind in their other STEM classes. Given that the study was done on a small-scale basis, the findings can only be generalized to the classroom in which the study was conducted. Future research should analyze the method more extensively by examining student success and failures in Pósa method classrooms to determine whether the method should be used in more classrooms with general education students.

Postscript

Given the COVID-19 pandemic, the in-person data collection was cut short, much to my frustration. Had I had more time in Budapest, I imagine I would have had more observational data from the classroom, where I could see how students interacted with each other and the math problems. Furthermore, more data could have come from the interviews, like subtle movements and physical reactions that are simply lost during online interviews. Body language can add an
extra layer to the responses, whether that be through evidence of comfort levels, enthusiasm, frustration, or otherwise.

Given that my project is one of few that have studied the Pósa method, it has qualities that are different from the current studies. My research project has an outsider to insider lens, with a focus on the student perceptions, whereas the ongoing MTA--Rényi research group’s study will be focused more on whether the Pósa method works through quantitative data. The MTA--Rényi research group will publish their findings this summer (2021), which will provide the most important and productive discussions on the Pósa method to date. Do note, that study has been anticipating the results of the Hungarian high school leaving exams (spring 2021) before publishing. Crucially, one must consider that the COVID-19 pandemic has the potential to alter the results, given that the exam will not be administered in the typical format.

This thesis changed my perspective on mathematics curriculum, especially in relation to methodology typically geared towards gifted students in regular education classrooms. Throughout the study, I became intertwined in the personal and academic stories of students in the Pósa method classroom. The positive aspects of the method shone brightly in the research, while the negative aspects were more sparse yet important to uncover. Not only did doing this research lead to my high regard for the Pósa method, but it also inspired in me an intrigue in Hungarians who have impacted mathematics’ education in the United States. In general, I have also come to question who designs the curriculum for American math education and why those names are not as prominently discussed in the United States as they are in Hungary. I hope to engage in research on Hungarians and American math education, potentially to pursue a doctorate in educational studies.
References


https://doi.org/10.1111/cogs.12107


https://doi.org/10.1111/ajsp.12058


Appendices

Appendix A: Pósa Method Questions

246. Solve the following system of equations.

\[3^x - 9^y : 27^y = 0\]
\[\log_3 xy = 2\]

247. Solve the following equation. \(\cos^2 x + 3 \sin x + 3 = 0\)

248. Points \(A(-2; 7)\) and \(B(1; 1)\) are given. Find the

(a) equation of the line that goes through \(A\) and \(B\),
(b) slope of this line (*and how you can see it in the equation),
(c) y-intercept (*and how you can see it in the equation),
(d) x-intercept (*and how you can see it in the equation),
(e) direction vector (*and how you can see it in the equation),
(f) normal vector (*and how you can see it in the equation),
(g) *Create general formulas for the equation of a line if the following are given:
   - slope \((m)\) and y-intercept \((b)\)
   - normal vector \((n_1; n_2)\) and a point on the line \((p_1; p_2)\)
   - direction vector \((d_1; d_2)\) and a point on the line \((p_1; p_2)\)
   - two points on the line \((a_1; a_2), (b_1; b_2)\)
   - Can you think of any other ways to define a line in the coordinate system? If yes, create a general formula using that information.

249. Within a group of 6 students we know the following information: Anna has 5; Betti has 4; Csaba, Dani, and Eszter have 3; and Fanni has 2 friends from this group on Facebook.

(a) Are Betti and Fanni friends on Facebook?
(b) Can Fanni have only one friend in this group (if we don’t change the other numbers)? *Why?
Appendix B: Test

1. Choose the one that is equivalent to the expression in the picture.

\[
\frac{7^a}{49^b \cdot 343}
\]

- [ ] \(7^a\)
- [ ] \(7^a(3-2b)\)
- [ ] \(7^a(2b^3)\)
- [ ] \(7^a(2b^3)\)
- [ ] \(7^a(64b^3-343)\)
- [ ] \(7^a(7a^6b^3)\)

2. Choose the one that is equivalent to the expression in the picture.

\[
\frac{4}{3\sqrt[3]{16^c}}
\]

- [ ] \(4^{-2(3.5)}\)
- [ ] \(2^{-2(4,5/3)}\)
- [ ] \(4^{-1(3,2/5)}\)
- [ ] \(2^{-2(3,2/4)}\)
- [ ] \(4^{-2(3,2/3)}\)
- [ ] \(2^{-2(3,3/4)}\)

3. Choose the one that is equivalent to the expression in the picture.

\[
125^y - 5^x \cdot \frac{1}{25} = 0
\]

- [ ] \(5^{3y-x/2}=10\)
- [ ] \(5^y=5^{x/2}\)
- [ ] \(3y-x=2\)
- [ ] \(3y-x=2\)
- [ ] \(3y-x=2\)
- [ ] \(5^{3y-x+2}=10\)

4. Choose the one that is equivalent to the expression in the picture.

\[
\log_2 y + 1 = 3
\]

- [ ] \(2^{y+1}=3\)
- [ ] \(2^y=2\)
- [ ] \(y=4\)
- [ ] \(y=5\)
- [ ] \(6=y+1\)
- [ ] \(y=7\)

5. Choose the one that is equivalent to the expression in the picture.

\[
\log_2 (y + 1) = 3
\]

- [ ] \(2^{y+1}=3\)
- [ ] \(2^y=2\)
- [ ] \(y=4\)
- [ ] \(y=5\)
- [ ] \(6=y+1\)
- [ ] \(y=7\)

6. Choose the one that is equivalent to the expression in the picture.

If \(y = \sin x\) then \(\sin^2 x = 3\sin x + 2\)

- [ ] no solution
- [ ] \(y^2-3y-2=0\)
- [ ] \(y^2+y+2\)
- [ ] \(y=3y+2\)
- [ ] \(\sin x^2=3y+2\)
- [ ] \(0=y^2+3y+2\)
7. Choose the one that is equivalent to the expression in the picture.

If \( y = \cos x \) then \( \cos^2 x = \sin^2 x + 2 \cos x - 1 \)
- no solution
- \( y^2 = y^2 + 2y - 1 \)
- \( 2y^2 - 2y + 1 = 0 \)
- \( y^2 - 2y = 0 \)
- \( 0 = 2y - 1 \)
- \( 0 = -y^2 + y + 2 \)

8. Choose the one that is true for every angle \( \beta \).

- \( (\sin \beta)^2 + (\cos \beta)^2 = 1 \)
- \( \cos \beta + \sin \beta = 1 \)
- \( \sin \beta = \cos(\beta + 90^\circ) \)
- \( \sin \beta = \sin(-\beta) \)

9. Choose the correct equation of the line in the picture.

- \( y = 5x/3 + 7/3 \)
- \( y = 5x/3 + 7/3 \)
- \( y = 3x/5 + 7/5 \)
- \( y = -1.6x + 2.3 \)

10. Find the slope of the line in the picture.

- \( 5/3 \)
- \( -5/3 \)
- \( 3/5 \)
- \( -3/5 \)
- \( 3x/5 \)
- \( 5x/3 \)

11. Find the zero point of the line in the picture.

- \((-7/5, 0)\)
- \((-7/3, 0)\)
- \((7/5, 0)\)
- \((7/3, 0)\)
12. Choose all the correct direction vectors of the line in the picture.

- (3; 5)
- (3; 5)
- (1.5; 2.5)
- (3; 5)
- (-5; 3)
- (4; 6)

13. Choose all the correct normal vectors of the line in the picture.

- (3; 5)
- (2; 5; 1.5)
- (3; 5)
- (-5; 3)
- (6; 10)
- (15; 9)

14. Choose the one that contains all the correct solutions of \( \cos x = -0.5 \) in the interval \([-480^\circ; 360^\circ]\)

- \( x = \{-150^\circ, -30^\circ, 210^\circ, 330^\circ\} \)
- \( x = \{-300^\circ, -240^\circ, -120^\circ, -60^\circ, 60^\circ, 120^\circ, 240^\circ, 300^\circ\} \)
- \( x = \{-480^\circ, -240^\circ, -120^\circ, 120^\circ, 240^\circ\} \) \( \times \)
- \( x = \{-240^\circ, -120^\circ, 120^\circ, 240^\circ\} \) \( \checkmark \)
- \( x = 120^\circ + n360^\circ, n \in \mathbb{Z} \)
- \( x = -120^\circ + n360^\circ, n \in \mathbb{Z} \)

15. Choose the one that contains all the correct solutions of \( \sin x = 0.5 \) in the interval \([-2\pi; 2\pi]\)

- \( x = \{-5\pi/3, -\pi/3, \pi/3, 5\pi/3\} \)
- \( x = \{-5\pi/6, -\pi/6, \pi/6, 5\pi/6\} \)
- \( x = \{-11\pi/6, -7\pi/6, \pi/6, 5\pi/6\} \) \( \checkmark \)
- \( x = \{-2\pi, \pi/6, 5\pi/6, 2\pi\} \)
- \( x = \{(\pi/6 + n2\pi), 5\pi/6 + n2\pi, n \in \mathbb{Z}\} \)
- \( x = \{-300^\circ, -210^\circ, 30^\circ, 150^\circ, 390^\circ\} \)
16. Choose all the correct statements.

- (cosθ)^2 + (sinθ)^2 = 1 ✓
- sinθ + cosθ = 1
- sinθ = sin(180° - θ) ✓
- cosθ = cos(-θ) ✓

17. What is the degree of vertex e in the graph in the picture?

- no solution
- 60°
- 120°
- 3
- 4 ✓
- 8
- 14
- 28

18. How many edges does the graph in the picture have?

- no solution
- 4
- 5
- 8
- 9 ✓
- 10
- 11
- 18
19. How many vertices does the graph in the picture have?

- no solution
- 1
- 3
- 4
- 5
- 8
- 9
- 18

20. Choose the text that describes the graph in the picture.

At a meeting 8 people were present. 3 people shook hands
with 4 people, 4 people shook hands with 2 people, and one
person didn’t shake any hands.

At a party 8 people had previously known each other. Ann, Bea,
Clay, and Dan knew 2 people; Eve, Frida, Gee, and Hall knew 4
people.

- There are 7 towns connected by 11 roads to each other.
- The degrees of the vertices are either 2 or 4.

21. Choose all the correct statements about the function in
the picture.

- its range is \([2; +\infty[\]
- \(y = \log_{0.5}(x+2) - 2\)
- its zero point is at \(-2\)
- it’s domain is \(x > -2\)
- it’s strictly monotonously increasing
- it has no extrema
Appendix C: Questionnaire

1. Name: *

Problem 246

246. Solve the following system of equations.
\[ 3^x - 9^3 \cdot 27^y = 0 \]
\[ \log_3 xy = 2 \]

Choose the number that best describes how you feel about problem 246.

2. Problem 246 *

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3. Problem 246 *

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4. Problem 246 – In your opinion, it’s... *

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5. Did you manage to solve problem 246? *

Mark only one oval.

☐ Yes
☐ Partly
☐ No
☐ Other:
6. How have you worked on problem 246? Check all the boxes that apply. *

Check all that apply.

☐ I didn't work on it at all
☐ Thinking on my own
☐ Using my own notes
☐ Using online resources/applications
☐ Brainstorming with others
☐ Receiving help from someone
☐ Giving help to someone
Other: ☐

7. How much of your solution to problem 246 do you feel is your own discovery? *

Mark only one oval.

☐ 0%
☐ 1-20%
☐ 21-40%
☐ 41-60%
☐ 61-80%
☐ 81-99%
☐ 100%

8. If you have any other comments about problem 246, please, write them here.

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

Problem 247

247. Solve the following equation, \( \cos^2 x + 3 \sin x + 3 = 0 \)

Choose the number that best describes how you feel about problem 247.

9. Problem 247 *

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Easy for Me  Difficult for Me

11. Problem 247 - In your opinion, it's... *

Mark only one oval.

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Easy for Others  Difficult for Others

12. Did you manage to solve problem 247? *

Mark only one oval.

☐ Yes
☐ Partly
☐ No
☐ Other: __________________________

13. How have you worked on problem 247? Check all the boxes that apply. *

Check all that apply.

☐ I didn't work on it at all
☐ Thinking on my own
☐ Using my own notes
☐ Using online resources/applications
☐ Brainstorming with others
☐ Receiving help from someone
☐ Giving help to someone

Other: ☐ __________________________
14. How much of your solution to problem 247 do you feel is your own discovery? *

*Mark only one oval.

☐ 0%
☐ 1-20%
☐ 21-40%
☐ 41-60%
☐ 61-80%
☐ 81-99%
☐ 100%

15. If you have any other comments about problem 247, please, write them here.

________________________________________________________________________
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Problem 248

248. Points A(−2; 7) and B(1; 1) are given. Find the

(a) equation of the line that goes through A and B,
(b) slope of this line (*and how you can see it in the equation),
(c) y-intercept (*and how you can see it in the equation),
(d) x-intercept (*and how you can see it in the equation),
(e) direction vector (*and how you can see it in the equation),
(f) normal vector (*and how you can see it in the equation),
(g) *Create general formulas for the equation of a line if the following are given:
   - slope (m) and y-intercept (b)
   - normal vector \((n_1; n_2)\) and a point on the line \((r_1; p_1)\)
   - direction vector \((d_1; d_2)\) and a point on the line \((p_1; p_2)\)
   - two points on the line \((a_1; a_2), (b_1; b_2)\)
   - Can you think of any other ways to define a line in the coordinate system? If yes, create a general
     formula using that information.

Choose the number that best describes how you feel about problem 248.

16. Problem 248 *

*Mark only one oval.

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17. Problem 248 *

*Mark only one oval.*

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18. Problem 248 - In your opinion, it’s... *

*Mark only one oval.*

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19. Did you manage to solve problem 248? *

*Mark only one oval.*

- [ ] Yes
- [ ] Partly
- [ ] No
- [ ] Other: __________________________

20. How have you worked on problem 248? Check all the boxes that apply. *

*Check all that apply.*

- [ ] I didn’t work on it at all
- [ ] Thinking on my own
- [ ] Using my own notes
- [ ] Using online resources/applications
- [ ] Brainstorming with others
- [ ] Receiving help from someone
- [ ] Giving help to someone

Other: [ ]

Other: __________________________
21. How much of your solution to problem 248 do you feel is your own discovery? *

Mark only one oval.

- 0%
- 1-20%
- 21-40%
- 41-60%
- 61-80%
- 81-99%
- 100%

22. If you have any other comments about problem 248, please, write them here.

____________________________________________________________________________________________________________________________________

249. Within a group of 6 students we know the following information: Anna has 5; Betti has 4; Csaba, Dani, and Eszter have 3; and Fanni has 2 friends from this group on Facebook.

(a) Are Betti and Fanni friends on Facebook?
(b) Can Fanni have only one friend in this group (if we don’t change the other numbers)? *Why?

Choose the number that best describes how you feel about problem 249.

23. Problem 249 *

Mark only one oval.

1  2  3  4  5  6  7  8  9  10
Boring  ○  ○  ○  ○  ○  ○  ○  ○  ○  Interesting

24. Problem 249 *

Mark only one oval.

1  2  3  4  5  6  7  8  9  10
Easy for Me  ○  ○  ○  ○  ○  ○  ○  ○  ○  Difficult for Me
25. Problem 249 - In your opinion, it's...

*Mark only one oval.

1 2 3 4 5 6 7 8 9 10

Easy for Others Difficult for Others

26. Did you manage to solve problem 249? *

*Mark only one oval.

☐ Yes
☐ Partly
☐ No
☐ Other:

27. How have you worked on problem 249? Check all the boxes that apply. *

*Check all that apply.

☐ I didn't work on it at all
☐ Thinking on my own
☐ Using my own notes
☐ Using online resources/applications
☐ Brainstorming with others
☐ Receiving help from someone
☐ Giving help to someone

Other: ☐

28. How much of your solution to problem 249 do you feel is your own discovery? *

*Mark only one oval.

☐ 0%
☐ 1-20%
☐ 21-40%
☐ 41-60%
☐ 61-80%
☐ 81-99%
☐ 100%
29. If you have any other comments about problem 249, please, write them here.


Reflection

30. Describe your experiences with the Pósa method (in general). / Írd le, hogy mik a tapasztalataid a Pósa módszerről (általában). *


31. What are the similarities of learning through the Pósa method both in the classroom and online? / Miben egyezik meg a Pósa módszerrel történő tanulás az osztályteremben és online? *


32. What are the differences of learning through the Pósa method in the classroom versus online? / Miben különbözik a Pósa módszerrel történő tanulás az osztályteremben illetve online? *


33. If you have any comments on your above responses or anything else you would like to share, please write them here. / Ha van még hozzáfűznivalód az előzőekhez vagy bármiféle egyéb, amit meg szeretnél osztani, akkor azt ide írhatod.
Appendix D: Introduction Text to Students

Sziasztok! We are Adam Hanford (pseudonym) and Lauren Seidl, two university students from the United States. For the next week, we will be collecting data and researching your math class. The aim of our study is to analyze the Pósa method in an online setting, specifically the process of discovery learning. We kindly ask you to follow a few simple steps, so we can get the best data possible:

- We have assigned you all four problems to work on. For the next week, your task is to complete them as you normally would in class/online and upload your work into your Homework folder in page “Journal for problems 246-249”
  - Do note: It is not forbidden to ask for help / use help, but please do try to solve them on your own to the best of your ability
  - Keep track if you do receive help from outside resources (e.g., an online tool, application, or a person etc.)
- Every day for the next week, please keep track of how long you work on each problem. This includes if you take a break and come back to a problem. Write down the time of day for each time you work. For example: If you begin at noon, take a 7-minute break after 23 minutes of working and resume for the next hour, your time log should say: 12:00-12:23 and 12:30-13:30
- At the end of every day, we have a journal for you to fill out. In the journal, you will be asked to reflect on the process of solving the problems. As well, we ask about the time you spent on the problems. This is why we need you to write down the times; it is an essential part of the journal
- We have a questionnaire that Ms. Barbarics will distribute to you. We ask that you fill this out as soon as you have completed all the problems. If you do not manage to finish all the problems, simply fill out the questionnaire on Friday
- Your online lessons according to your timetable (Monday 9:00-10:30, Wednesday 13:30-14:30, Friday 8:00-9:00) are available for live online consultations with Ms. Barbarics, where you can ask your questions. If you consent, they will be recorded and analyzed anonymously only for research purposes together with your written data.

To summarize:
1. Keep track of the time you spend working on the problems every day
2. Upload your work and fill out your journals every night for a week
3. Complete the questionnaire when you finish the problems, or at the end of the week, whichever is sooner

Be sure to contact Ms. Barbarics if you have any questions about the study, and she will reach out to us if necessary. We thank you so much for participating in our study and look forward to reading about your experiences this week!
Appendix E: Week-Long Journal

Keep track of each instance that you worked on a problem using the template given. Make a
copy for each separate instance that you work on a problem. Write the problem number and the
start and end time(s). Write a summary of what you did during each instance in the comments
section. Tell us if you used any resources (your own notes, online resources, help from other
people, etc.) If it was an online resource, give the link or the name of the app. If it was a person,
who and how did they help you? Did you do anything besides working on the problem? If there
is anything else you would like to mention, you can also write it here. Finally, show your work
(write it here or insert a photo of your notebook).

Date:
Problem number:
Start time:
End time:
Comments:
Your work:
Appendix F: Student Interview Questions

- We will start with some background questions first.
  - How old are you?
  - In this next question, we do not want to know private information but are interested in what affects your online classes. Who do you live with currently? Is it different now than it was before the pandemic?
  - Have you faced any issues during the pandemic with online classes?
- From your earliest memory in a linear order, what was your math education like?
- In comparison, what are your experiences with the Pósa method?
- How difficult has math been for you, again from your earliest memories?
  - compared to others?
- Are you satisfied with your performance in math?
  - Why?
- When it comes to mathematics, what do you think you are good at?
- What kind of math student would you say you are?
  - Describe a general week when working on math, both in general and in the online setting
- How would you describe your personality?
- What are your interests in general?
  - Do you have interests related to math?
- What are your plans for your future and career after high school?
  - After high school, are you interested in a career in a STEM field?
  - Do you have any role models?
- In thinking about our interview, is there anything else you would like to add?

*During the interviews, if a student brought up a topic that seemed interesting or conducive for the interview, the interviewee would ask questions not from the list about the topic*
Appendix G: Follow-Up Interview Questions

- Why did you choose to attend Petrik?
- Are there aspects of the Pósa method that you DON’T like? If so, please describe them.
- In general, what do you think are DISADVANTAGES of using the Pósa method?

This work was supported in part by the South Carolina Honors College Senior Thesis/Project Grant (~LS)