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- [11] J. Foerster, "The effects of multipath interference on the performance of UWB systems in an indoor wireless channel," in *VTC 2001-Spring*, vol. 2, Rhodes, Greece, May 2001, pp. 1176–1180.
- [12] L. Zhao and A. M. Haimovich, "Performance of ultra-wideband communications in the presence of interference," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 9, pp. 1684–1691, Dec. 2002.
- [13] S. Kim and S. Im, "The effects of PRF and slot interval on the data throughput of PPM-based ultra wide-band systems in multi-path channels," in *Proc. Int. Conf. Wireless Netw. (ICWN 2003)*, vol. 1, Las Vegas, NV, Jun. 2003, pp. 19–23.
- [14] J. G. Proakis, *Digital Communications*. New York: McGraw-Hill, 1995.
- [15] C. Daniel, *Fitting Equations to Data*. New York: Wiley, 1980.

Asynchronous DS-SS CDMA Random Spreading Code Correlation Statistics in the Presence of Timing Error

David W. Matolak

Abstract—We quantify the effect of timing tracking errors upon 2nd order correlation statistics of random binary spreading codes and, in so doing, fill a gap in the literature. Using a Gaussian model for timing tracking error, new expressions for autocorrelation statistics are derived. For crosscorrelations, we show that a zero mean Gaussian timing error has no effect upon 2nd order crosscorrelation statistics.

Index Terms—Correlation, pseudonoise coded communication, spread spectrum communication.

I. INTRODUCTION

For analyses of performance of asynchronous direct sequence spread spectrum (DS-SS) code division multiple access (CDMA), spreading code properties are needed. In particular, the statistics of spreading code correlations—both auto and crosscorrelations—are essential to quantify the effective signal and multiuser interference (MUI) energies. In addition, the correlation properties of random sequences alone, in the presence of random synchronization errors, are of interest.

The random code model for the spreading codes is a common one; e.g., [1], [2], in particular for systems that use "long codes," or codes whose period is a large number of symbols. The correlation properties of random spreading codes, and the resulting effect on system performance, are well known in the case of perfect timing estimation (and rectangular chip pulse shapes) [3]–[6]. The effect of imperfect timing estimates has been given only modest attention, e.g., [7]–[10], and somewhat surprisingly, the explicit effect of imperfect timing upon correlation statistics has, to our knowledge, not been reported. Also, in no case have we found results for a nonzero mean Gaussian timing error, which we provide here. We first review some of the pertinent literature.

In [7], the authors derive an approximation to the performance degradation of (single user) DS-SS in the presence of carrier phase and timing errors, in terms of the required symbol energy increase, over that of perfect synchronization, for a given bit error ratio (BER). Their results are applicable only for very small energy in-

creases (~ 0.2 dB). Also, their expressions must be evaluated numerically, either iteratively, or over the distributions of the carrier and timing errors, and so are somewhat computationally intensive. In [8], the authors determine CDMA system error probability via a power series approximation to the Gaussian probability density function (pdf) tail integral and characteristic functions for the MUI and chip timing error. Their results are conditioned upon a *specific* value of timing error, and include several computationally intensive special functions.

The results of [9] include the effect of Gaussian distributed phase and timing errors on correlator reception of DS-SS CDMA. The authors derive the pdf for the (zero mean) timing error, but the resulting effect on the correlation is only an approximation. As in [8], the authors of [10] also assume a fixed value of timing error, and apply the result to a parallel interference cancelling (PIC) multiuser detector.

In none of the works we have found are the actual effects of imperfect timing estimates quantified fully in terms of 2nd order statistics. In no case are the effects cast in terms of modifications to code correlation statistics. Most sources ([7], [8], [10]) estimate the resulting effect of imperfect timing/phase upon system performance (e.g., BER)—which is, of course, ultimately paramount—yet the quantification of the effect upon correlations is more portable in that it can be widely and easily applied to single or multiple user systems and multiuser detectors, and it can also be applied to random spreading waveforms (sequences) alone, without data modulation. In this paper, we employ a Gaussian model for the error in the estimate of the chip timing, and derive, either in closed form or by bounds and simulations, the effect of imperfect estimates upon the auto and cross correlations of random spreading codes. Section II contains our analysis, Section III contains numerical results, and Section IV contains conclusions.

II. ANALYSIS

We consider an asynchronous DS-SS CDMA system, but as noted, our treatment can also be viewed as pertaining to the random sequences alone, without regard to modulation. In most CDMA works, such as [3], [4], modulation is binary phase modulation, with coherent detection. We analyze a baseband case, applicable with slight modification to any phase modulation scheme. The spreading waveform for user k is

$$s_k(t) = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} c_k(m)p(t - mT_c) \quad (1)$$

where the spreading code elements $c_k(m)$ are equiprobable binary variates in $\{\pm 1\}$, and the unit energy chip pulse shape $p(t)$ is rectangular over the chip time T_c , with amplitude $1/(T_c)^{0.5}$. The energy of $s_k(t)$ over the bit period T is also unity. The processing gain is $N = T/T_c$. If $s_k(t)$ repeats each bit period, (1) describes a "short code" (with period T); otherwise, (1) represents a single length- N subsequence of a long code.

A correlator receiver is common for single user detection, or for the first stage of most multiuser detection schemes. The correlator is actually applied to $s_k(t - \tau_k)$, where τ_k is the delay induced by the channel, and which user k 's tracking loop must estimate and follow. The k th user's tracking loop estimate is $\hat{\tau}_k = \tau_k + e_{\tau k}$, where $e_{\tau k}$ is the tracking error, which we model as Gaussian with mean $\mu_{e k}$, and variance $\sigma_{e k}^2$. Generally the mean $\mu_{e k} = 0$ [9]. For reference, Fig. 1 shows a diagram of the baseband portion of the correlator receiver, with an input signal consisting of K CDMA signals and additive white Gaussian noise (AWGN).

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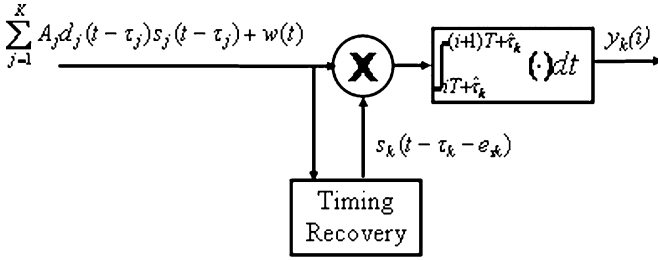


Fig. 1. Baseband portion of coherent DS-SS BPSK receiver for k th user in CDMA system with K active users. For j th user, A_j = signal amplitude, $d_j(t)$ is binary data signal, $s_j(t)$ is spreading signal of (1), and τ_j is delay. Signal $w(t)$ is AWGN, and timing error is $e_{\tau k}$.

For CDMA systems, the importance of the correlation statistics we analyze is as follows. The bit error probability P_b for any user can be approximated as $Q(\sqrt{\text{SNIR}})$ [6], where SNIR is the signal-to-noise plus interference ratio, and $Q(x) = \int_x^\infty e^{-t^2/2} dt / \sqrt{2\pi}$. For binary phase modulation, the SNIR is given by $E_{b_{k,e}} / (N_0/2 + I_{0k})$, where $E_{b_{k,e}}$ is user k 's effective bit energy, $N_0/2$ is the two-sided white Gaussian noise density, and I_{0k} is the total MUI energy imposed upon user k . The MUI energy I_{0k} is assumed Gaussian for the Q -function approximation, but even in the absence of the Gaussian approximation, the SNIR is as given above. The MUI energy is composed of the contributions of the $K - 1$ other active users.

For perfect timing tracking, the bit energy for user k is $E_{b_{k,e}} = E_{b_{kk}} \rho_{kk}^2(0)$, where $E_{b_{kk}}$ is user k 's actual received bit energy, and $\rho_{kk}(0) = 1$ is the code autocorrelation for user k at timing offset zero. If a tracking error $e_{\tau k}$ exists, the resulting bit energy $E_{b_{k,e}}$ is reduced to less than $E_{b_{kk}}$. In fact, for random codes the bit energy can be upper bounded, via conventional analysis (e.g., [8], [11]), by $E_{b_{kk}} \{E[\rho_{kk}(e_{\tau k})]\}^2$, where "E" is the expectation operator. The autocorrelation $|\rho_{kk}(e_{\tau k})| \leq 1$ has a maximum value of one at $e_{\tau k} = 0$. Specifically, when the k th user's i th data bit is $d_k(i)$ and only user k 's signal is present, the correlator output of Fig. 1. is given by $y_k(i) = \sqrt{2E_{b_{kk}}} d_k(i) [\rho_{kk}(e_{\tau k}) - 2r_{kk}(e_{\tau k})\delta_{dk}] + w(i)$, with $w(i)$ the zero-mean AWGN noise sample with variance N_0 , the partial correlation function $r_{kk}(x) = \int_T^{T+x} s_k(t)s_k(t-x) dt$, and δ_{dk} is equal to one for $d_k(i-1) \neq d_k(i)$ and zero for $d_k(i-1) = d_k(i)$. The function $r_{kk}(x)$ has magnitude $|r_{kk}(x)| \ll |\rho_{kk}(x)|$ for $x < T_c$. This correlator output would be employed as usual to obtain P_b conditioned upon the timing offset. Similarly, the MUI contribution to I_{0k} from any user i is proportional to $E_{b_{i}} \rho_{ik}^2(\tau_{ik})$, where τ_{ik} is the relative delay between the received user i and user k 's signals, and ρ_{ik} is the cross-correlation between the spreading signals $s_i(t)$ and $s_k(t)$ at this delay. Thus, auto and crosscorrelations both directly affect the SNIR, which directly affects P_b .

A. Autocorrelation

The autocorrelation of (1) is expressed as follows [5], where without loss of generality we assume $\tau_k = 0$:

$$\rho_{kk}(e_{\tau k}) = \int_{e_{\tau k}}^{T+e_{\tau k}} s_k(t)s_k(t-e_{\tau k}) dt. \quad (2)$$

Strictly, this autocorrelation is conditioned upon τ_k , since we assume that acquisition has taken place, and the code tracking loop is in operation. The statistics of (2) can also be viewed as conditioned upon τ_k . Both the mean and mean square value of autocorrelation are of interest in the analysis of tracking loop performance [11]. We compute the first two moments of (2) for a Gaussian timing error.

Using notation similar to that in [5], we can express (2) as

$$\rho_{kk}(e_{\tau k}) = \frac{1}{N} \sum_{m=0}^{N-1} \sum_{j=0}^{N-1} c_k(m)c_k(j)R_p(e_{\tau k} + (j-m)T_c) \quad (3)$$

where $R_p(x)$, the autocorrelation function of the rectangular pulse $p(t)$, is equal to $1 - |x|/T_c$ for $|x| \leq T_c$, and zero elsewhere. To find the mean of (3), we take expectation with respect to both the code chips, and the timing error $e_{\tau k}$. Taking the expectation of (3) with respect to the random codes, we obtain $R_p(e_{\tau k})$, which must be averaged over the pdf of the timing error to obtain the mean of (3), μ_{kk} :

$$\mu_{kk} = E_{c,e_{\tau k}}[\rho_{kk}] = \int_{-T_c}^{T_c} p_{e_{\tau}}(x)(1 - |x|/T_c) dx \quad (4)$$

where $p_{e_{\tau}}(x)$ is the Gaussian pdf of $e_{\tau k}$. The limits of integration of (4) arise from the finite support of $R_p(e_{\tau k})$. Appealing to integral tables [12], and after some algebraic manipulation we obtain

$$\begin{aligned} \mu_{kk} &= \frac{\mu_{ek}}{T_c} \left[2Q\left(\frac{\mu_{ek}}{\sigma_{ek}}\right) - Q\left(\frac{\mu_{ek} - T_c}{\sigma_{ek}}\right) - Q\left(\frac{\mu_{ek} + T_c}{\sigma_{ek}}\right) \right] \\ &+ Q\left(\frac{\mu_{ek} - T_c}{\sigma_{ek}}\right) - Q\left(\frac{\mu_{ek} + T_c}{\sigma_{ek}}\right) \\ &+ \frac{\sigma_{ek} \exp(-\mu_{ek}^2 / (2\sigma_{ek}^2))}{T_c \sqrt{\pi}} \left[\exp\left(\frac{-T_c^2}{2\sigma_{ek}^2}\right) \cosh\left(\frac{\mu_{ek} T_c}{\sigma_{ek}^2}\right) - 1 \right] \end{aligned} \quad (5)$$

which, for a zero mean Gaussian timing error ($\mu_{ek} = 0$), simplifies to

$$\mu_{kk} = 1 - 2Q\left(\frac{T_c}{\sigma_{ek}}\right) + \frac{2\sigma_{ek}}{T_c \sqrt{2\pi}} \left(\exp\left(\frac{-T_c^2}{2\sigma_{ek}^2}\right) - 1 \right). \quad (6)$$

For the mean square value of (3), denoted m_{kk} , the random code chip independence yields again a single integral expression:

$$m_{kk} = E_{c,e_{\tau k}}[\rho_{kk}^2] = \int_{-T_c}^{T_c} p_{e_{\tau}}(x)(1 - |x|/T_c)^2 dx. \quad (7)$$

This can similarly be solved using integral tables and standard functions, with the result given by

$$\begin{aligned} m_{kk} &= \frac{T_c^2 + \mu_{ek}^2 + \sigma_{ek}^2}{T_c^2} \left[1 - Q\left(\frac{T_c - \mu_{ek}}{\sigma_{ek}}\right) - Q\left(\frac{T_c + \mu_{ek}}{\sigma_{ek}}\right) \right] \\ &+ \frac{2\mu_{ek}}{T_c} \left[Q\left(\frac{T_c - \mu_{ek}}{\sigma_{ek}}\right) - Q\left(\frac{T_c + \mu_{ek}}{\sigma_{ek}}\right) + 2Q\left(\frac{\mu_{ek}}{\sigma_{ek}}\right) - 1 \right] \\ &+ \frac{\sigma_{ek} \exp(-\mu_{ek}^2 / (2\sigma_{ek}^2))}{T_c^2 \sqrt{2\pi}} \\ &\times \left[(T_c + \mu_{ek}) \exp\left[\frac{-(T_c^2 + 2\mu_{ek} T_c)}{2\sigma_{ek}^2}\right] \right] \\ &+ \frac{\sigma_{ek} \exp(-\mu_{ek}^2 / (2\sigma_{ek}^2))}{T_c^2 \sqrt{2\pi}} \\ &\times \left[(T_c - \mu_{ek}) \exp\left[\frac{-(T_c^2 - 2\mu_{ek} T_c)}{2\sigma_{ek}^2}\right] - 4T_c \right] \end{aligned} \quad (8)$$

and the result for the zero mean case simplifies to

$$m_{kk} = (1 - 2Q(T_c/\sigma_{ek})) \left(1 + \sigma_{ek}^2/T_c^2\right) + \frac{2\sigma_{ek}}{T_c\sqrt{2\pi}} \left(\exp[-T_c^2/(2\sigma_{ek}^2)] - 2\right). \quad (9)$$

We corroborate these results via simulation.

B. Crosscorrelations

For the case of crosscorrelations, in an asynchronous setting, we have *partial* correlations—correlations computed over only a portion of a bit period. Thus, two partial correlations affect any given bit. When averaged over the actual range of possible delays of the interfering users, the statistics of these two partial correlations are identical; hence it is sufficient to compute statistics for one of them. We make the common assumption that the delays $\{\tau_i\}_{i=1, i \neq k}^K$, modulo the bit period T , are uniform on $[0, T)$.

Let the partial correlation between the received i th signal $s_i(t - \tau_i)$ and user k 's locally generated spreading signal $s_k(t - \hat{\tau}_k)$ be expressed as

$$\rho_{ik}(\tau_i, \hat{\tau}_k) = \int_{\hat{\tau}_k}^{\tau_i + T} s_i(t - \tau_i) s_k(t - \hat{\tau}_k) dt \quad (10)$$

which applies to the first part of user k 's bit, for the case $\tau_i < \tau_k$. With $\hat{\tau}_k = \tau_k + e_{\tau k}$, and a change of variables, we can rewrite (10) in the equivalent simpler form

$$\rho_{ik}(\tau_i, \hat{\tau}_k) = \int_0^{T+v_{ik}} s_i(t) s_k(t - v_{ik}) dt \quad (11)$$

where $v_{ik} = \tau_i - e_{\tau k}$. From the perspective of user k 's correlator, τ_i modulo- T is uniform on $[0, T)$, and given $e_{\tau k}$ is Gaussian with mean μ_{ek} , and variance σ_{ek}^2 , we can easily find the pdf of v_{ik} via convolution of these pdfs: in (12), shown at the bottom of the page.

As with the autocorrelation, we compute statistics of (11) by averaging over both the random code chips and the random delay variable, here the delay and error “composite” variable v_{ik} . Expressing (11) in summation form as in [3], we have

$$\rho_{ik}(v_{ik}) = \frac{1}{N} \sum_{m=0}^{N-1} \sum_{j=0}^{N-1} c_k(m) c_i(j) R_p(v_{ik} + (j - m)T_c) \quad (13)$$

where we have abbreviated the argument using the composite variable v_{ik} . For random (independent) codes for signals i and k , the mean of (13) is zero. Via independence of the code chips for different user signals, we obtain for the mean square value m_{ik}

$$m_{ik} = E \left[\rho_{ik}^2(v_{ik}) \right] = E_{v_{ik}} \left[\frac{1}{N^2} \sum_{m=0}^{N-1} \sum_{j=0}^{N-1} R_p^2(v_{ik} + (j - m)T_c) \right] \quad (14)$$

and so to complete the computation we take expectation with respect to v_{ik} , by integrating the product of the double summation in (14) and the pdf of v_{ik} , given in (12). Using the fact that $R_p(x) = 0$ outside $|x| < T_c$, and letting $r = j - m$, after algebraic simplification we can express the average in (14) as

$$m_{ik} = \frac{1}{N^2} \sum_{r=1-N}^{N-1} (N - |r|) \int_{-T_c}^{T_c} R_p^2(x) p_{v_{ik}}(x - rT_c) dx. \quad (15)$$

The mean-square value m_{ik} in (15) can not be obtained in closed form. For perfect chip timing, $m_{ik} = 1/(3N)$ [5]. For the case when the timing error mean value μ_{ek} is zero, a simple upper bound can be obtained by upper bounding the pdf $p_{v_{ik}}$ by $1/T$ for summation index $r = 1 - N$ to 0, and by $1/(2T)$ for index $r = 1$ to $N - 1$: $m_{ik} < 1/(2N) - 1/(6N^2) + 1/(3N^3)$. Similarly, we can lower bound the pdf by $1/(2T)$ for the summation index $r = 1 - N$ to -1 , and truncate the sum afterward, yielding $m_{ik} > 1/(6N) - 1/(6N^2)$. Neither bound depends upon the timing error variance.

Finally, in some situations, such as interference cancelling, we may have need of the crosscorrelation between two signals, both of which are subject to timing error. This correlation is the same as (10) with both delays given by estimates, and is expressed as in (11) as follows:

$$\rho_{ik}(\hat{\tau}_i, \hat{\tau}_k) = \int_0^{T+y_{ik}} s_k(t) s_i(t - \zeta_{ik}) dt \quad (16)$$

where $\zeta_{ik} = v_{ik} + e_{\tau i} = \tau_i + e_{\tau i} - e_{\tau k}$. As with (11), the mean value of (16) is zero. Assuming independent Gaussian timing errors for the i th and k th tracking loops, the pdf of ζ_{ik} is the same as that of v_{ik} , namely, (12), but with μ_{ek} replaced by $\mu_{ek} + \mu_{ei}$, and σ_{ek} replaced by $\sqrt{\sigma_{ek}^2 + \sigma_{ei}^2}$, and hence the same bounds that apply to (15) then apply to the meansquare value of (16), again independent of the timing error.

III. NUMERICAL RESULTS

In Fig. 2, we plot versus normalized timing error standard deviation σ_{ek}/T_c , both analytical and simulation results for mean and mean square autocorrelation ((6) and (9)), and simulation results for mean square crosscorrelation (15), for $N = 31$, and a timing error mean value $\mu_{ek} = 0$. Simulations were conducted in MATLAB, averaged over 1000 trials using 1000 samples/chip. Agreement between analytical and simulation results for autocorrelation statistics is excellent. Simulated crosscorrelation values are within a few percent of $1/(3N)$ —identical to the result for perfect chip timing.

Fig. 3 shows similar results for $N = 63$, for autocorrelation only ((5) and (8)), where the timing error mean value $\mu_{ek} = 0.1T_c$. Agreement between analytical and simulation results is again very good. Results for different values of processing gain, and different timing error means and standard deviations, are similar.

IV. CONCLUSION

In this paper, we have quantified the effect of timing errors upon the 2nd order correlation statistics of random spreading codes. New

$$p_{v_{ik}}(x) = \frac{1}{T} \begin{cases} Q\left(\frac{-(x+\mu_{ek})}{\sigma_{ek}}\right) - Q\left(\frac{-(x+\mu_{ek}-T)}{\sigma_{ek}}\right); & x < -\mu_{ek} \\ 1 - Q\left(\frac{x+\mu_{ek}}{\sigma_{ek}}\right) - Q\left(\frac{-(x+\mu_{ek}-T)}{\sigma_{ek}}\right); & -\mu_{ek} \leq x \leq -\mu_{ek} + T \\ Q\left(\frac{x+\mu_{ek}-T}{\sigma_{ek}}\right) - Q\left(\frac{x+\mu_{ek}}{\sigma_{ek}}\right); & x > -\mu_{ek} + T \end{cases} \quad (12)$$

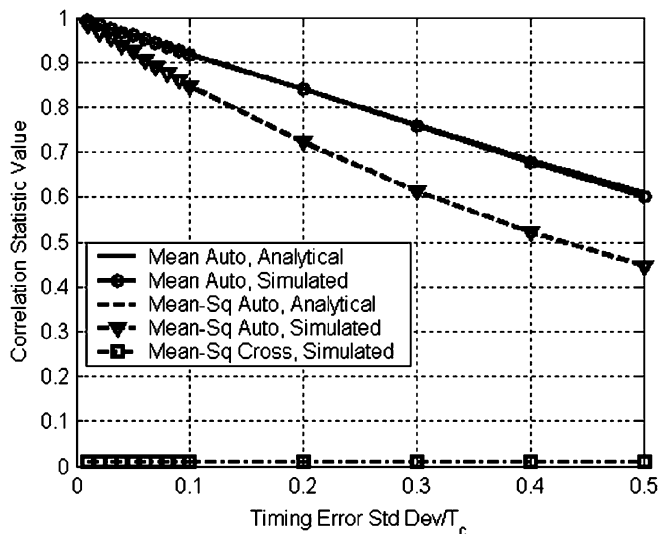


Fig. 2. Auto and crosscorrelation statistics vs. normalized timing error standard deviation. Processing gain $N = 31$, timing error mean $\mu_{ek} = 0$.

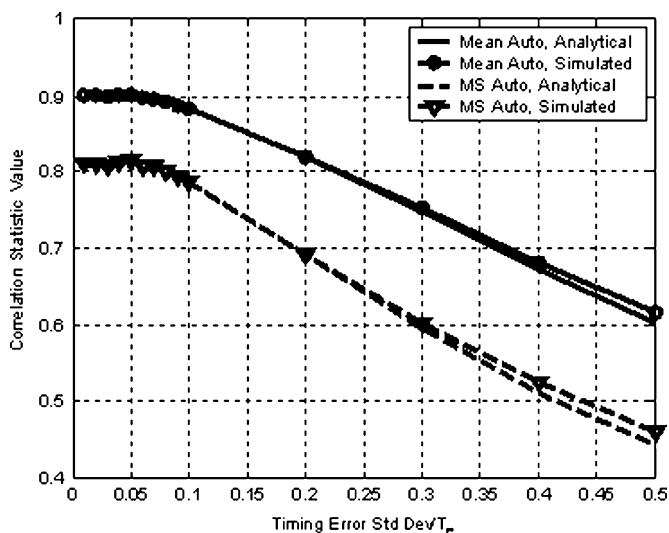


Fig. 3. Autocorrelation statistics vs. normalized timing error standard deviation. Processing gain $N = 63$, timing error mean $\mu_{ek} = 0.1T_c$.

expressions for code autocorrelation statistics were derived, and it was shown that a zero mean Gaussian timing error has no effect upon average crosscorrelation statistics.

REFERENCES

- [1] "Mobile station-base station compatibility standard for dual-mode wide-band spread spectrum cellular system," TIA/EIA/IS-95-A, 1995.
- [2] "Technical specification group radio access network spreading and modulation (FDD), Release 1999," Third Generation Partnership Project, 3G TS 25.213, version 3.2.0, Mar. 2000.
- [3] M. B. Pursley, "Performance evaluation for phase-coded spread-spectrum multiple-access communication—Part I: System analysis," *IEEE Trans. Commun.*, vol. COM-25, no. 8, pp. 795–799, Aug. 1977.
- [4] M. B. Pursley and D. V. Sarwate, "Performance evaluation for phase-coded spread-spectrum multiple-access communication—Part II: Code sequence analysis," *IEEE Trans. Commun.*, vol. COM-25, no. 8, pp. 800–803, Aug. 1977.
- [5] S. Verdú, *Multiuser Detection*, Cambridge, U.K.: Cambridge Univ. Press, 1998.
- [6] R. K. Morrow and J. S. Lehnert, "Bit-to-bit error dependence in slotted DS/SSMA packet systems with random signature sequences," *IEEE Trans. Commun.*, vol. 37, no. 10, pp. 1052–1061, Oct. 1989.
- [7] K. Buckley and M. Moeneclaey, "Effect of carrier phase and timing errors on the detection of narrowband M-PSK and bandlimited DS/SS M-PSK signals," *IEEE Trans. Commun.*, vol. 43, no. 2/3/4, pp. 1260–1263, Feb./Mar./Apr. 1995.
- [8] M. O. Sunay and P. J. McLane, "Sensitivity of a DS CDMA system with long PN sequences to synchronization errors," in *Proc. of ICC'95*, Seattle, WA, Jun. 1995, pp. 1029–1035.
- [9] J. Panicker and S. Kumar, "Effect of system imperfections on BER performance of CDMA correlator receiver," in *Proc. of ICC'95*, Seattle, WA, Jun. 1995, pp. 894–899.
- [10] R. M. Buehrer, A. Kaul, S. Striglis, and B. D. Woerner, "Analysis of DS-SS CDMA parallel interference cancellation with phase and timing errors," in *IEEE J. Sel. Areas Commun.*, vol. 14, no. 8, Oct. 1996, pp. 1522–1535.
- [11] R. L. Peterson, R. E. Ziemer, and D. E. Borth, *Introduction to Spread Spectrum Commun.*, Upper Saddle River, NJ: Prentice-Hall, 1995.
- [12] W. H. Beyer (ed.), *Standard Mathematical Tables*, 24th ed., CRC Press, Palm Beach, FL, 1976.