6-2009

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Publication Info

Postprint version. Published in *IEEE Transactions on Communications*, Volume 57, Issue 6, 2009, pages 1628-1633.

Matolak, D. (2009). Probability density functions for SNIR in DS-CDMA. *IEEE Transactions on Communications, 57(6)*, 1628-1633.
http://dx.doi.org/10.1109/TCOMM.2009.06.040385

Probability Density Functions for SNIR in DS-CDMA

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***Abstract*—Analytical expressions for the probability density function of block-wise signal-to-noise-plus-interference ratio for both synchronous and asynchronous direct-sequence spread spectrum code-division multiple access systems are developed, for equal average energy signals on the Gaussian and Rayleigh flat fading channels. Using the standard Gaussian approximation for multi-user interference, accurate density approximations are obtained, which agree very well with computer simulation results.**

***Index Terms*—Spread spectrum, code division multiple access.**

I. Introduction

DIRECT-sequence spread spectrum (DS-SS) code-division multiple access (CDMA) systems have seen much attention in the literature, and also in application. Many such systems use packet transmission, and in this case, short-term statistics are of interest for assessing quality of service (QoS), and potentially for advanced detection approaches such as multi-user detection (MUD) [1].

Although there have been numerous publications on MUD and related techniques – see for example the extensive reference lists in [1]- [3] – and much on the statistics of multi-user interference (MUI), determination of the statistics of the actual signal-to-noise-plus-interference ratio (SNIR) has *not* been done. The history of the study of MUI statistics in DS-SS is long, so only several well-known references are cited to justify our use of the Gaussian approximation. Most investigations employ BPSK modulation and equal received signal energies for all users; extension to quaternary modulations and unequal received energies (near-far conditions) has also been performed (addressed subsequently).

One of the earliest references on MUI statistics is [4], in which accurate bounds on the second moment of MUI were obtained. This author notes that the standard Gaussian approximation (SGA) is often very good, particularly when processing gain L is not small, the number of users K is large, and the signal-to-noise ratio (SNR) is not too large. We use the term SGA in this paper to refer to both (1) the approximation of the correlator output MUI, for any bit, as a zero mean Gaussian random variable with variance dependent upon L, K, and constants that depend upon the presence or absence of carrier phase and code chip alignment; and (2) the resulting bit error probability (P_b) expression that obtains from this approximation.

In [5]- [8], the authors derived exact statistics of MUI and corresponding P_b bounds. These approaches are computationally intensive, requiring *K-2* convolutions (or *K-1* integrals [8]) of a probability density function (pdf) to obtain the pdf for the total MUI. In all these references, the SGA was cited as very good for large *K/L* and moderate SNR, and was shown to be accurate even for small *K/L* (e.g., 2/31) when P_b is larger than about 10^{-5}.

Other authors have derived accurate techniques for determining P_b with far less computation and for near-far cases. The most widely used is [10], which employs a difference approximation to derivatives in the Taylor series expansion of P_b about the SGA value, yielding a weighted sum of Gaussian tail integrals ("Q-functions"). This P_b approximation has also been termed the improved GA (IGA), or as slightly modified in [9], the simplified IGA (SIGA). Numerous references have employed these versions of the SGA – a few examples include [11]- [14] – with unanimous concurrence regarding their accuracy for the conditions previously mentioned. Although there have been exceptions, e.g., [15], wherein a Laplacian density was found to better fit simulated data for small *K/L*, the Gaussian MUI approximation is firmly established as a useful and accurate approximation under the conditions for which it was derived. To be specific, the SGA is accurate for the "system loading" ratio *K/L* larger than a value we denote q_{min}. The value of q_{min} is approximately 0.3 [10], and decreases slightly as L increases, and also decreases as thermal noise increases. Use of the SGA for loading values of q_{min} and larger translates to an (uncoded) error probability on the order of $10^{-4} \sim 10^{-3}$ or larger (for PSK modulation with coherent detection); these error probabilities are lower than those typically required of the code symbols output from the demodulator in practical systems that employ forward error correction coding. The SGA is even better for higher values of system loading where error probabilities are higher, and where practical systems may operate. When quaternary modulation is used, the SGA is even better [16], [17], and the use of raised-cosine filtering has also been shown not to invalidate the above conditions on q_{min} [18].

Given the accuracy and usefulness of the SGA, we employ it here for our investigation, which derives the SNIR distribution for packet transmission using long codes. We apply the ensemble average 2^{nd}-order statistics for MUI to each MUI sample to develop the block-wise (time-average) SNIR distribution models, for both AWGN and slow Rayleigh flat fading channels. These SNIR distributions are of interest for several reasons. First, from a theoretical perspective, since in a long

Paper approved by F. Santucci, the Editor for Wireless System Performance of the IEEE Communications Society. Manuscript received November 30, 2004; revised September 15, 2005, December 21, 2006, July 30, 2007, and September 23, 2008.

A portion of this work was presented at MILCOM 2003 [19]. This work was supported by a grant from the National Science Foundation, ITR program grant number 0113307.

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Digital Object Identifier 10.1109/TCOMM.2009.06.040385

code case the MUI changes each symbol, the SNIR, computed on a packet basis, can be modeled as a random variable whose distribution heretofore was not described in a simple form – deriving the distribution using exact MUI statistics (e.g., from [5]) would not only be prohibitively complex, but would also improve upon our results only under conditions where the SGA is inaccurate, i.e., low loading factors *K/L* and large SNR. Second, since SNIR can be rapidly and accurately estimated [19], it can be used in rapidly assessing signal quality of service (QoS), even in terms of approximate error probability performance, and the distribution of SNIR is of interest for this assessment. Third, for some advanced detection techniques such as MUD, where SNIR may be used for example instead of correlator output amplitude for successive interference cancelling (SIC) [20] or group interference cancelling (GIC) [21], the SNIR distribution provides information usable by the MUD for classifying user signals for processing.

Finally, other recent work has aimed at deriving SNIR or carrier-to-interference ratio (CIR) distributions, but almost exclusively for non-DS-SS transmission. This includes [22] and [23] for ad hoc and downlink cases, respectively. Another reference [24] describes how to use SNR (averaged over a block) for multiuser diversity. These authors assume SNIR is estimated perfectly, but do not comment on its distribution. They also assume near-constant fading over a block, and block-to-block independence of fading, as we do here, but again, for non-DS-SS transmissions. In [25], the authors note that the pdf of SNR is of use for analyzing cross-layer multiple-input/multiple-output (MIMO) schemes, without providing any SNIR pdf. Last, reference [26] describes how the SNIR distributions can be used for both power control and data rate control, in a DS-SS CDMA system, and does derive such a distribution, for a dispersive wide-sense stationary, uncorrelated scattering Rayleigh fading uplink case. This distribution is far more complex than the one derived here, as it is expressed in terms of a set of *JK* individual average powers on the *J* channel taps for each of the *K* users, and on a set of $J^2(K-1)$ ratios of the *J* desired signal channel tap powers and the *J(K-1)* interfering signal channel tap powers on all the *J(K-1)* interferer channel taps. In addition, these authors neglect thermal noise, which we do not.

II. Signal Model

An asynchronous DS-SS CDMA system with *K* users and BPSK signaling is the basic model; the conditions of carrier phase and/or chip synchronism can easily be treated as special cases of this model. We address this in the sequel. User *k*'s baseband signal is $u_k(t)=A_k\sum_n b_k(n)s_k(t-\lfloor n/a\rfloor aT)$, with $b_k(n)\in\{\pm1\}$ user *k*'s n^{th} bit, $A_k=\sqrt{E_{bk}}$ the signal amplitude, E_{bk} the bit energy, and T the bit duration. The constant a is a positive integer with $a>1$ for our long code case of interest (a=1 for short codes (periodic over T)). The function $\lfloor x\rfloor$ is the greatest integer less than or equal to x. The signature sequence (spreading) waveform, unit-energy over T, is $s_k(t)=\sum_{m=0}^{N-1}c_k(m)p(t-mT_c)/\sqrt{L}$, with binary spreading code chips $c_k(m)\in\pm1$, and chip pulse shape $p(t)$ rectangular over the chip time T_c, with amplitude$1/\sqrt{T_c}$. The processing gain is $L=T/T_c$, with code length $N=aL$ for long codes ($L=N$ for short codes). All users employ a common data rate $R_b=1/T$, and random, long spreading codes. We cite subsequently one multi-rate case to which our results apply. User *k*'s transmitted signal is $x_k(t)=\sqrt{2}u_k(t)cos(\omega_c t)$.

The received bandpass signal is $r(t)=\sqrt{2}\sum_{k=1}^{K}\alpha_k u_k(t-\tau_k)cos(\omega_c t-\theta_k)+w(t)$, where user *k*'s delay τ_k is modeled as a uniform random variable on $[0,T)$ modulo-*T*, phase $\theta_k=\omega_c\tau_k+\phi_k$ is also modeled as random, uniform on $[0,2\pi)$, and $w(t)$ is the zero-mean AWGN, with two-sided spectral density $N_0/2$. Variable α_k is the Rayleigh fading amplitude, with mean-square value of *1*, and ϕ_k is the random channel phase. Equal flat fading average energies are assumed for all users, as in [27], [28]. For slow fading, we assume the fading variables are constant over a block duration, as in [24]; the non-fading AWGN channel case is obtained by setting the fading amplitude variables to *1*. The carrier phase synchronous case is obtained by setting all $\theta_k=0$, and similarly, the chip synchronous case is obtained by setting all $\tau_k=0$.

Synchronization (acquisition) is assumed for each user signal, so after coherent downconversion, the k^{th} user's j^{th} bit correlator output is given by

$$\begin{aligned}y_k(j)=&\sqrt{2}\int_{jT+\tau_k}^{(j+1)T+\tau_k}r(t)s_k(t-jT-\tau_k)cos(\omega_c t-\theta_k)dt\\ =&A_k\alpha_k b_k(j)+M_k(j)+w_k(j)\end{aligned}\tag{1}$$

where the first term is the user-*k* desired component, $M_k(j)$ denotes the multiuser interference imposed on user *k*'s j^{th} bit, and $w_k(j)$ is the noise sample, zero mean, with variance $N_0/2$.

Using notation similar to that of [1], the MUI term in (1) is

$$\begin{aligned}M_k(j)=\quad&\sum_{i<k}\tilde{A}_i[b_i(j+1)\rho_{ki}+b_i(j)\rho_{ik}]\\ &+\sum_{i>k}\tilde{A}_i[b_i(j)\rho_{ki}+b_i(j-1)\rho_{ik}],\end{aligned}\tag{2}$$

where the first sum is over users whose delay $\tau_i<\tau_k$, and the second sum is over users whose delay $\tau_i>\tau_k$, and $\tilde{A}_i=A_i\alpha_i cos(\theta_k-\theta_i)$. The partial cross correlations are a generalized version of those defined for short codes in [1]. For long codes, the partial correlations are a function of both the delays and the bit index *j*, where for $\tau_i<\tau_k$, using $\delta_{j,a}=\lfloor j/a\rfloor$ and $\beta_{j,a}=\lfloor(j-1)/a\rfloor$, we have

$$\begin{aligned}&\rho_{ik}(\tau_i,\tau_k,j)\\ &=\int_{jT+\tau_k}^{(j+1)T+\tau_i}s_i(t-\delta_{j,a}aT-\tau_i)s_k(t-\delta_{j,a}aT-\tau_k)dt,\end{aligned}\tag{3}$$

$$\begin{aligned}&\rho_{ki}(\tau_i,\tau_k,j)\\ &=\int_{jT+\tau_i}^{jT+\tau_k}s_i(t-\delta_{j,a}aT-\tau_i)s_k(t-\beta_{j,a}aT-\tau_k)dt.\end{aligned}\tag{4}$$

III. SNIR & PDF

For the AWGN channel, the ensemble average SNIR is defined as $E_b/(N_0/2+I_0)$, where I_0 is the variance (equal to the mean-square) of the MUI term of (2). For the fading channel, the SNIR expression is identical except that the numerator is multiplied by the square of the fading amplitude, α_k^2, and I_0 also incorporates the fading variables. In the block-wise case, we compute a time-average SNIR, instead of the ensemble average as in [1]; the time-average is something an actual receiver can compute. This time average MUI+AWGN energy over a packet of size N_b bits for user k is

$$I_k(N_b) = \frac{1}{N_b}\sum_{j=1}^{N_b}(M_k(j)+w_k(j))^2. \quad (5)$$

The actual time average SNIR for user k over these N_b bits, $SNIR_{k,a}$, is then

$$SNIR_{k,a}(N_b) = \frac{A_k^2\alpha_k^2}{I_k(N_b)} = \frac{N_b E_{b,k}\alpha_k^2}{\sum_{j=1}^{N_b}(M_k(j)+w_k(j))^2}. \quad (6)$$

where $A_k^2\alpha_k^2$ is the energy of the desired signal, constant over a block. This energy is also equal to the square of the mean of the j^{th} correlator output conditioned on user k's data ($b_k(j)$) since the MUI and AWGN are zero mean, i.e., $A_k^2\alpha_k^2 = \{E[y_k(j)|b_k(j)]^2\}$. The MUI+AWGN energy $I_k(N_b)$ is the time-average conditional variance, or the time-average corresponding to $Var[y_k(j)|b_k(j)] = E\{[M_k(j)+w_k(j)]^2\}$.

We address the AWGN channel case first (all $\alpha_k = 1$). Let $H_k(j) = M_k(j) + w_k(j)$, and note that via the SGA, $H_k(j)$ is Gaussian, with zero mean, and variance given by

$$\sigma_H^2 = (K-1)E_b/(DL) + N_0/2. \quad (7)$$

This variance is obtained by averaging over the carrier phases, delays, and random code chips, and where using results from [7], the constant D is as follows: $D = 3$ for chip and phase asynchronism; $D = 1$ for chip and phase synchronism; $D = 2$ for chip synchronism, phase asynchronism; and $D = 3/2$ for chip asynchronism, phase synchronism. Asynchronism is random and uniform, as defined prior to (1). For any of these cases, for any given random delay and carrier phase, if the bits and code chips are random, $H_k(j)$ is independent of $H_k(j\pm q)$, for any q other than one [7], and for $q = 1$, the dependence is weak enough so that the independent approximation is very good, as our simulation results show. Thus, with H_k^2 a chi-squared variate, the MUI $I_k(N_b)$ of (5) is also a chi-squared variate with N_b degrees of freedom [29], and the SNIR is the ratio of a constant $E_{b,k} = A_k^2$ divided by this chi-squared variable. Via a simple transformation of random variables [30], we obtain the pdf of SNIR as follows [19]:

$$p_{SNIR}(g) = \frac{(A_k^2 N_b)^{N_b/2}}{\sigma_H^{N_b} 2^{N_b/2}\Gamma(N_b/2)} g^{-\frac{N_b}{2}-1} exp\left\{-\frac{N_b A_k^2}{2g\sigma_H^2}\right\} \quad (8)$$

where g is the average $SNIR_{k,a}$ of (6) over N_b bits, and Γ is the gamma function. In the case of a single user, with MUI exactly zero, the pdf of (8) is exact (with $\sigma_H^2 = N_0/2$). Accounting for correlations among the adjacent MUI terms does not appear to allow a closed-form expression for the pdf of $I_k(N_b)$. To find the pdf of the sum of N_b terms, one can proceed interatively, first finding the joint pdf of two correlated chi-squares and integrating as in [30] to find the pdf of the sum of two terms. The joint pdf of two correlated chi-squares is known [31]. Beyond this though, the procedure becomes intractable, since deriving the pdf of the sum of even three of the N_b terms requires the derivation (then integration) of the joint pdf of the 3^{rd} chi-square variate and the sum of the previous two (with pdf in [31]), and this joint pdf for the three correlated variates does not appear to be known.

For large values of block size N_b, the Central Limit Theorem can be applied to the MUI+AWGN term of (5), to yield an approximate pdf, obtained in a manner analogous to that used to obtain (8), where with $v_H = 2\sigma_H^4/N_b$, we have

$$p_{SNIR,a}(g) = \frac{A_k^2}{g^2\sqrt{2\pi v_H}} exp\left[-\left(\frac{A_k^2}{g} - \sigma_H^2\right)^2 / (2v_H)\right] \quad (9)$$

We also note that these pdf results apply to the multi-rate case where user signals have unequal data rates, but equal values of received power; equal power ensures the amplitudes $\{A_k\}_{k=1}^K$ are identical, and does not alter the derivation.

For the Rayleigh fading case, the procedure is analogous. We again employ the SGA for MUI, as done in [14] and [20]. As noted, fading is assumed slow for all user signals, and is modeled as constant over a packet (fading can be either independent or correlated between blocks – this will not affect the pdf). The mean and variance of the MUI term of (5) do not change (recall $E[\alpha_k^2]$) since we are using averages and assume independence, as in [1] and [20], so the MUI term $I_k(N_b)$ is modeled as the same chi-squared variate with N_b degrees of freedom. The distribution of the numerator $E_{b,k}\alpha_k^2$ is also chi-squared, with 2 degrees of freedom, so the SNIR is the ratio of these two chi-squared variates. Using the transformation of a ratio of random variables [30], after some algebra we can obtain the pdf. More directly, we can recognize the ratio of the two chi-squareds as a scaled (Fisher) F-variate [30], and get the same result. This pdf is as follows:

$$p_{SNIR,R}(g) = \frac{(N_b/2)^{N_b/2+1}(A_k^2/\sigma_H^2)^{N_b/2}}{\left[g + \frac{N_b A_k^2}{2\sigma_H^2}\right]^{N_b/2+1}}. \quad (10)$$

This pdf can also be used to approximate the case of Rayleigh-lognormal fading, in which the bit energies – E_{bk} in the numerator of (6) and the energies within the MUI terms of (2) – are multiplied by $z_k\alpha_k^2$, with z_k a lognormal random variable, constant over a block. Specifically, z_k is a normalized lognormal z_k=$10^{x/10}/\zeta$, where x is Gaussian with zero mean, variance σ_x^2, and ζ=$E[10^{x/10}]$=$exp\{-\sigma_x^2[ln(10)/10]^2/2\}$; thus yielding $E(z) = 1$. The value σ_x can represent either a shadowing standard deviation, or a Gaussian power imbalance in dB. The exact pdf for this Rayleigh-lognormal case can not be derived in closed form, but when the normalized lognormals have mean one, the pdf is well approximated by that of (10).

IV. Numerical Results

For all simulations described here, conducted in Matlab®, the number of trials N_t was such that the total number of

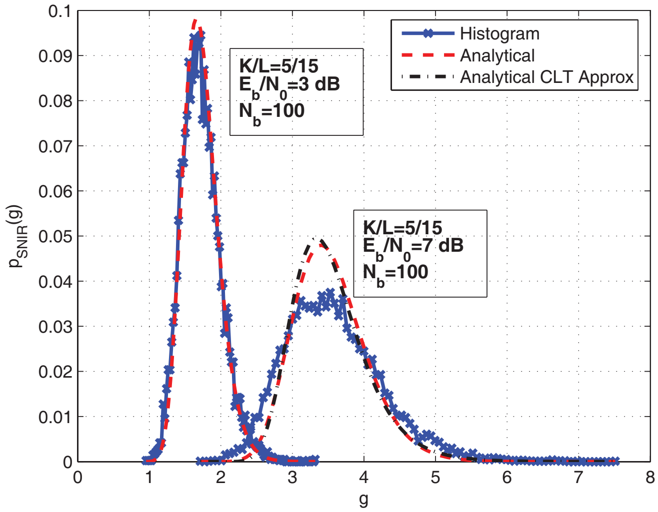


Fig. 1. Analytical pdf, analytical approximation to pdf, and histograms from simulations for SNIR, chip and phase asynchronous case, AWGN channel with N_b=100 bits, processing gain L=15, K=5 users, and two values of E_b/N_0.

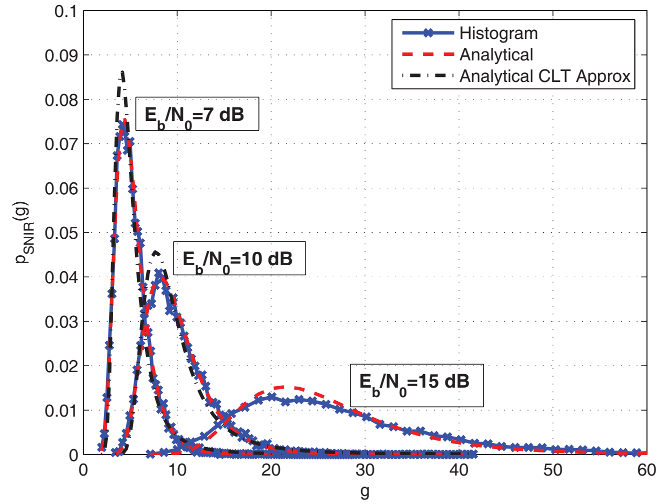


Fig. 3. Analytical pdf, analytical approximation to pdf, and histograms from simulations for SNIR, AWGN channel with N_b=20 bits, K/L=2/31, and several values of E_b/N_0 for the chip and phase asynchronous case.

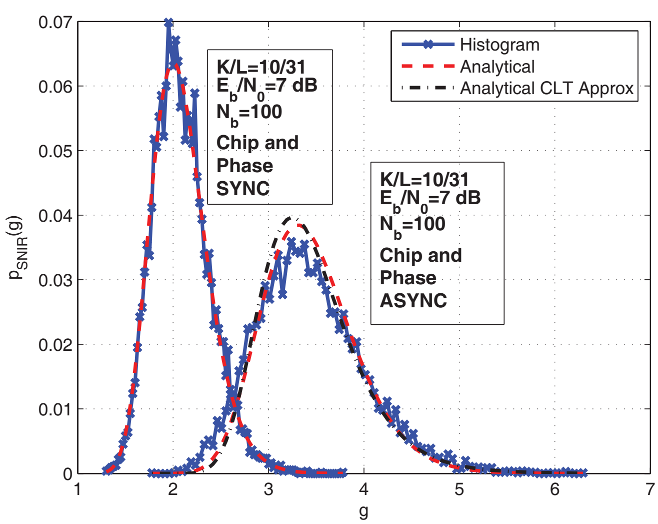


Fig. 2. Analytical pdf, analytical approximation to pdf, and histograms from simulations for SNIR, AWGN channel with N_b=100 bits, E_b/N_0=7 dB, processing gain L=31, and K=10 users, for two cases: both chip and phase asynchronous, and both chip and phase synchronous.

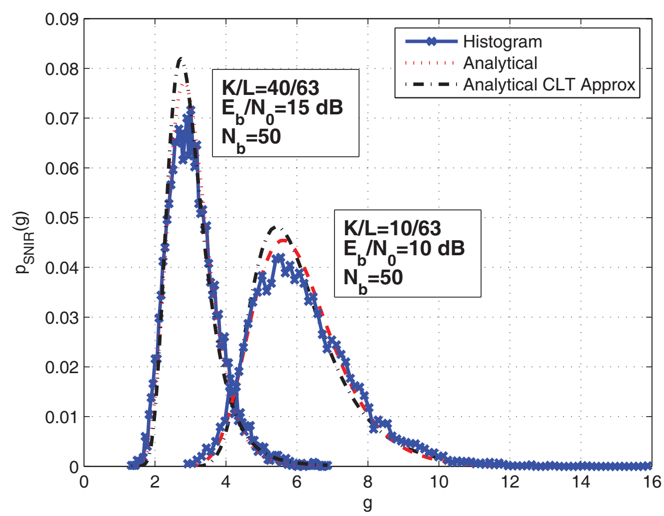
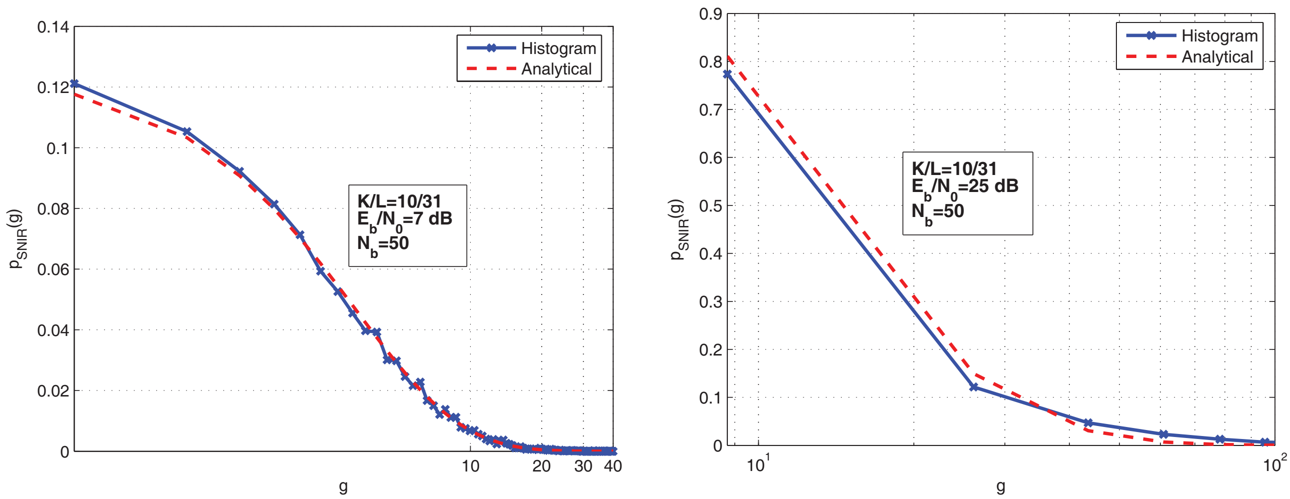


Fig. 4. Analytical pdf, analytical approximation to pdf, and histograms from simulations for SNIR, AWGN channel with N_b=50 bits; K/L=10/63 and E_b/N_0=10 dB; and K/L=40/63 and E_b/N_0=15 dB, for the chip synchronous, phase asynchronous case.

SNIRs gathered, KN_t, is equal to 10,000. In all simulations, N_s=4 samples/chip were used. For each trial of the simulation, all K users are assigned a new random long code, new random carrier phase, new random delay, new block of random data bits, and for the fading channel results, a new fading amplitude. All histograms and analytical plots use 100 points across the domain. Figure 1 plots the analytical pdf expression of (8) and histograms obtained for SNIR by computer simulation for the AWGN channel, chip and phase asynchronous case. The loading factor *K/L* is 5/15, the packet size is N_b=100 bits, and E_b/N_0=3 dB and 7 dB. The actual block-wise SNIR (histogram) is that obtained from the true amplitude and the MUI+AWGN, collected in simulation as prescribed by (6). As can be seen, the density of (8) is very well corroborated by experiment for the smaller value of E_b/N_0, but for this small value of processing gain (L=15), agreement between analysis and simulation degrades for the higher SNR value of 7 dB, since the Gaussian MUI approximation degrades as the SNR increases when the processing gain L is this small.

Figure 2 shows similar results for the same channel and asynchronism conditions, and for essentially the same loading factor but larger K and L, K/L=10/31. The block size N_b is again 100 bits, with E_b/N_0=7 dB. Similar good agreement obtains here, and on comparison with Figure 1, it can be seen that as K/L goes from 5/15 to 10/31, the validity of the SGA improves and yields better agreement between analysis and simulations. Figure 2 also shows that the analytical and simulation results are in very good agreement for the chip and phase synchronous case. Both Figures 1 and 2 also show that for N_b=100 bits, the CLT approximate pdf of (9) is in good agreement with the "exact" pdf of (8).

In Figure 3, we show AWGN channel results for the chip and phase asynchronous case again, for a very small



(a) E_b/N_0=7 dB (b) E_b/N_0=25 dB

Fig. 5. Analytical pdf and histograms from simulations for SNIR, Rayleigh channel with N_b=50 bits; K/L=10/31, and (a) E_b/N_0=7 dB and (b) E_b/N_0=25 dB, chip and phase asynchronous case.

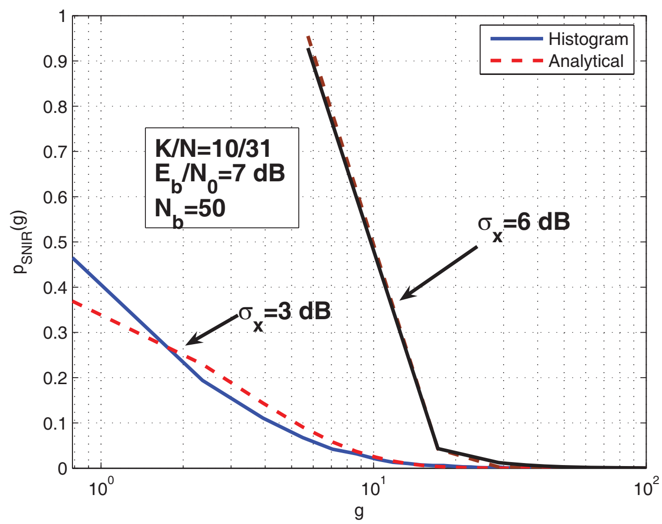


Fig. 6. Analytical pdf and histograms from simulations for SNIR, Rayleigh-lognormal channel with N_b=50 bits; K/L=10/31, $Eb/N0$=7 dB, and two values of the lognormal standard deviation, σ_x=3 and 6 dB, chip and phase asynchronous case.

loading factor, K/L=2/31, for several values of E_b/N_0, and block size N_b=20 bits. Evident is the gradual degradation of the agreement between analysis and simulations as the SNR increases, but even for the rather large SNR of 15 dB, as long as the processing gain L is large enough, the pdf of (8) is a reasonable approximation. The poorer agreement of the CLT approximation of (9) is also apparent in Figure 3, and is attributable to the smaller block size of 20 bits.

Two more AWGN channel example results are shown in Figure 4, for the chip synchronous, phase asynchronous case, with an even larger value of processing gain L=63, with two loading factors and two values of E_b/N_0. With this value of L, agreement is reasonably good even for the small loading factor of K/L=10/63, as long as the SNR is not very large. For larger loading factors, agreement is good at larger SNRs.

Two sets of Rayleigh channel results are shown in Figure 5. As with the AWGN channel case, for a loading factor K/L equal to or larger than about 1/3, analysis and simulations are in very good agreement, and agreement improves as processing gain increases. One difference in the Rayleigh case is that the SGA for MUI is good at both low and high SNR. As with the AWGN case, for given K, L, and E_b/N_0, agreement between analysis and simulations is independent of the carrier phase and code chip synchronism/asynchronism.

Finally, Figure 6 shows two sets of results for the Rayleigh-lognormal case, with K/L and N_b the same as in Figure 5, and with E_b/N_0=7 dB, and two values of lognormal standard deviation, σ_x=3 and 6 dB. Agreement between analysis and simulations is nearly as good as in the case of Rayleigh fading alone, particularly when the lognormal standard deviation increases.

V. Conclusion

Analytical expressions for the probability density function of the block-wise signal-to-noise-plus-interference ratio for DS-SS CDMA in AWGN and Rayleigh flat fading channels were derived for the case when all user energies are equal, and for all combinations of chip and carrier phase synchronism/asynchronism. The pdfs employ the standard Gaussian approximation for multi-user interference; given this approximation, and the mild approximation that MUI terms within a block are independent, the first AWGN pdf and the Rayleigh pdf are exact, and the second AWGN channel pdf employs the Central Limit Theorem to approximate the average MUI plus Gaussian noise energy over a block as another Gaussian. Simulation results agree very well with the analysis for a range of parameters. Specifically, for loading factors above approximately 0.3, because the SGA is accurate, our pdfs model very well the simulated SNIRs as long as E_b/N_0 is not too large. As with the SGA itself, it is difficult to be precise about all parameter settings, but some additional observations are as follows: for a fixed value of loading factor and E_b/N_0, as processing gain increases, our pdfs become more accurate at modeling the distribution of SNIRs. As loading factor increases, the largest value of SNR at which our pdfs apply also increases. The CLT approximate pdf (9) is generally in very good agreement with the "exact" pdf of (8) for block sizes above approximately 50. All these observations apply to all four combinations of code chip and carrier phase synchronism/asynchronism. Finally, for the case when all signals undergo flat Rayleigh fading with the same average energy, or with energies lognormally distributed, using the same model for MUI as in (9), the resulting SNIR is an F-variate [30], and the analytical pdf approximates well that obtained in simulation, even at high SNR.

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