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# Measurement of the branching fraction and time-dependent $CP$ asymmetry for $B^0 \rightarrow J/\psi\pi^0$ decays

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We measure the branching fraction and time-dependent  $CP$ -violating asymmetry for  $B^0 \rightarrow J/\psi\pi^0$  decays using a data sample of  $711 \text{ fb}^{-1}$  collected on the  $\Upsilon(4S)$  resonance by the Belle experiment running at the KEKB  $e^+e^-$  collider. The branching fraction is measured to be  $\mathcal{B}(B^0 \rightarrow J/\psi\pi^0) = [1.62 \pm 0.11(\text{stat}) \pm 0.06(\text{syst})] \times 10^{-5}$ , which is the most precise measurement to date. The measured  $CP$  asymmetry parameters are  $\mathcal{S} = -0.59 \pm 0.19(\text{stat}) \pm 0.03(\text{syst})$  and  $\mathcal{A} = -0.15 \pm 0.14(\text{stat})_{-0.03}^{+0.04}(\text{syst})$ . The mixing-induced  $CP$  asymmetry ( $\mathcal{S}$ ) differs from the case of no  $CP$  violation by 3.0 standard deviations, and the direct  $CP$  asymmetry ( $\mathcal{A}$ ) is consistent with zero.

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At the quark level, the decay  $B^0 \rightarrow J/\psi\pi^0$  proceeds via  $b \rightarrow c\bar{c}d$  “tree” and “penguin” amplitudes, as shown in Fig. 1. Both amplitudes are suppressed in the Standard Model (the first one is color and Cabibbo suppressed), and thus the branching fraction is small. The tree-level amplitude has the same weak phase as that of the  $b \rightarrow c\bar{c}s$  amplitude governing, e.g.,  $B^0 \rightarrow J/\psi K_S^0$  decays, while the penguin amplitude has a different weak phase. The former dominates mixing-induced  $CP$  violation, while the addition of the latter gives rise to direct  $CP$  violation.

In the process  $\Upsilon(4S) \rightarrow B^0\bar{B}^0$ , one of the two  $B$  mesons can decay into a  $CP$  eigenstate  $f_{CP}$  at time  $t_{CP}$ , while the other can decay into a flavor-specific state  $f_{\text{tag}}$  at time  $t_{\text{tag}}$ . The decay time evolution for the  $B \rightarrow f_{CP}$  is [1]

$$\mathcal{P}(\Delta t, q) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} \times (1 + q[\mathcal{S} \sin(\Delta m_d \Delta t) + \mathcal{A} \cos(\Delta m_d \Delta t)]), \quad (1)$$

where  $\Delta t = t_{CP} - t_{\text{tag}}$  is the difference in proper decay times between the two  $B$  mesons;  $q = +1(-1)$  for signal  $\bar{B}^0(B^0)$  decays;  $\Delta m_d$  is the mass difference between the two mass eigenstates of the  $B^0 - \bar{B}^0$  system; and  $\tau_{B^0}$  is the  $B^0$  lifetime. The parameters  $\mathcal{S}$  and  $\mathcal{A}$  are  $CP$  violating and characterize mixing-induced and direct  $CP$  violation, respectively. In the absence of the penguin amplitude,  $\mathcal{A} = 0$  and  $\mathcal{S} = -\sin(2\phi_1)$ , where  $\phi_1 = \arg[-(V_{cb}^* V_{cd})/(V_{tb}^* V_{td})]$ . However, this amplitude and any new physics (NP) process having a different weak phase will shift  $\mathcal{S}$  and  $\mathcal{A}$  from these values. Thus, measuring these parameters provides a way to search for NP. The values of  $\mathcal{S}$  and  $\mathcal{A}$  measured in

$B^0 \rightarrow J/\psi\pi^0$  decays can also be used to constrain the small penguin contribution to  $B^0 \rightarrow J/\psi K_S^0$  decays [2–7]. This small contribution is important as the decay  $B^0 \rightarrow J/\psi K_S^0$  provides the most precise determination of  $\phi_1$ .

The parameter  $\mathcal{S}$  for  $B^0 \rightarrow J/\psi\pi^0$  has previously been measured by Belle [8] and BABAR [9], but the results are not in good agreement. The BABAR result lies outside the physically allowed region, but the uncertainties are large. The previous result from Belle was based on  $535 \times 10^6 B\bar{B}$  pairs [8]. Here we update that measurement using the final Belle data set of  $772 \times 10^6 B\bar{B}$  pairs. We also update the  $B^0 \rightarrow J/\psi\pi^0$  branching fraction, for which our previous measurement used only  $32 \times 10^6 B\bar{B}$  pairs [10]. In addition to more data, the analysis presented here also uses improved tracking and photon reconstruction.

The Belle detector is a large-solid-angle magnetic spectrometer consisting of a silicon vertex detector (SVD), a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters, a barrel-like arrangement of time-of-flight scintillation counters, and an electromagnetic calorimeter (ECL) comprising CsI(Tl) crystals. These detector components are located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux-return (KLM) located outside the coil is instrumented to detect  $K_L^0$  mesons and to identify muons. Two inner detector configurations were used: a 2.0 cm radius beampipe and a three-layer SVD were used for the first  $152 \times 10^6 B\bar{B}$  pairs of data, while a 1.5 cm radius beampipe, a four-layer SVD, and a small-cell inner drift chamber were used for the remaining  $620 \times 10^6 B\bar{B}$

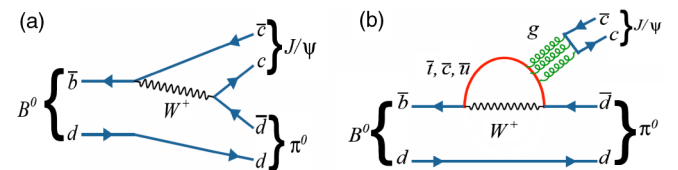


FIG. 1. (a) Tree and (b) penguin amplitudes for the decay  $B^0 \rightarrow J/\psi\pi^0$ .



pairs of data. The detector is described in detail in Ref. [11]. Event selection requirements are optimized using Monte Carlo (MC) simulation. MC events are generated using EVTGEN [12], and the detector response is modeled using GEANT3 [13]. Final-state radiation is taken into account using the PHOTOS package [14].

The  $\Upsilon(4S)$  is produced with a Lorentz boost of  $\beta\gamma = 0.425$  along the  $+z$  axis, which is defined as antiparallel to the  $e^+$  beam direction. Since the  $B^0$  and  $\bar{B}^0$  mesons are approximately at rest in the  $\Upsilon(4S)$  center-of-mass (CM) system,  $\Delta t$  is determined from the displacement in  $z$  between the two  $B$  decay vertices:  $\Delta t \approx \Delta z / c\beta\gamma$ .

The reconstruction of  $B^0 \rightarrow J/\psi\pi^0$  proceeds by first reconstructing  $\pi^0 \rightarrow \gamma\gamma$  candidates. An ECL cluster not matched to any track is identified as a photon candidate. Such candidates are required to have an energy greater than 50 MeV in the barrel region and greater than 100 MeV in the end-cap regions, where the barrel region covers the polar angle  $32^\circ < \theta < 130^\circ$  and the end-cap regions cover the ranges  $12^\circ < \theta < 32^\circ$  and  $130^\circ < \theta < 157^\circ$ . We require that the  $\gamma\gamma$  invariant mass be within 20 MeV/ $c^2$  (about  $3.5\sigma$  in resolution) of the  $\pi^0$  mass [15]. To improve the  $\pi^0$  momentum resolution, we perform a mass-constrained fit and require that the resulting  $\chi^2$  be less than 30. This requirement is relatively loose, retaining more than 99% of events.

We subsequently combine  $\pi^0$  candidates with  $J/\psi$  candidates, which are reconstructed in the  $e^+e^-$  and  $\mu^+\mu^-$  decay channels. All charged tracks are required to have a minimum number of SVD hits:  $\geq 2$  in the beam direction, and  $\geq 1$  in the transverse direction. Electron identification is based on the ratio of the ECL cluster energy to the particle momentum as measured in the CDC, as well as the position and shape of the electromagnetic shower in the ECL. In order to account for radiative energy loss in  $e^+e^-$  decays, we include up to two bremsstrahlung photons that lie within 50 mrad of each of the reconstructed tracks when calculating the  $e^+$  and  $e^-$  four-momenta. Muons are identified by corresponding hit positions and the track penetration depth in the KLM. The reconstructed  $J/\psi$  invariant masses  $M_{ee(\gamma)}$  and  $M_{\mu\mu}$  are required to satisfy  $-150 \text{ MeV}/c^2 < M_{ee(\gamma)} - m_{J/\psi} < +36 \text{ MeV}/c^2$  and  $-60 \text{ MeV}/c^2 < M_{\mu\mu} - m_{J/\psi} < +36 \text{ MeV}/c^2$ , where  $m_{J/\psi}$  is the nominal  $J/\psi$  mass [15]. The asymmetric mass ranges account for the radiative tail, which biases the reconstructed mass towards lower values. For selected  $J/\psi$  candidates, vertex- and mass-constrained fits are performed to improve the momentum resolution.

Candidate  $B^0$  mesons are identified using the beam-energy-constrained mass  $M_{bc} = (\sqrt{E_{\text{beam}}^2 - |\vec{p}_B|^2 c^2})/c^2$ , and the energy difference  $\Delta E = E_B - E_{\text{beam}}$ , where  $E_{\text{beam}}$  is the beam energy, and  $E_B$  and  $\vec{p}_B$  are the reconstructed energy and momentum, respectively, of the  $B^0$  candidate. All quantities are evaluated in the CM frame. Events

satisfying  $M_{bc} > 5.24 \text{ GeV}/c^2$  and  $-0.20 \text{ GeV} < \Delta E < 0.10 \text{ GeV}$  are retained for further analysis. To calculate the signal yield, we define a smaller signal region:  $5.27 \text{ GeV}/c^2 < M_{bc} < 5.29 \text{ GeV}/c^2$  and  $-0.10 \text{ GeV} < \Delta E < 0.05 \text{ GeV}$ . In order to suppress “continuum” background arising from light quark production ( $e^+e^- \rightarrow q\bar{q}$ ,  $q = u, d, s, c$ ), we require that the event shape variable  $R_2$ , which is the ratio of the second to zeroth Fox-Wolfgram moments [16], satisfies  $R_2 < 0.4$ .

After applying all selection criteria, 2.9% of events have multiple  $B^0$  candidates in the signal region. For these events, we retain the candidate having the smallest sum of  $\chi^2$  values obtained from the  $\pi^0 \rightarrow \gamma\gamma$  mass-constrained fit and the  $J/\psi \rightarrow \ell^+\ell^-$  vertex- and mass-constrained fit. According to MC simulations, this criterion selects the correct  $B^0$  candidate in 74% of multiple-candidate events.

We tag (identify) the flavor of the accompanying  $B$  meson using inclusive properties of particles not associated with the signal  $B^0 \rightarrow J/\psi\pi^0$  decay. The algorithm for flavor tagging is described in Ref. [17]. Two parameters,  $q$  and  $r$ , are used to represent the tagging information. The former is the implied flavor of the signal  $B$  decays as used in Eq. (1). The latter is an event-by-event MC-determined quality factor that ranges from  $r = 0$  for no flavor discrimination to  $r = 1$  for unambiguous flavor assignment. It is used for sorting candidate events into seven  $r$  ranges. For events having  $r > 0.10$ , we determine the wrong-tag fractions  $\omega_l$  ( $l = 1, 7$ ) and their differences  $\Delta\omega_l$  between  $B^0$  and  $\bar{B}^0$  decays from a control sample of self-tagged semileptonic and hadronic  $b \rightarrow c$  decays [18,19]. If  $r < 0.10$ , the wrong tag fraction is set to 0.5.

The vertex position for the  $B^0 \rightarrow J/\psi\pi^0$  decay is reconstructed using lepton tracks from the  $J/\psi$  decays. We perform a vertex fit with a constraint to the interaction point (IP) profile. A vertex position for  $f_{\text{tag}}$  is obtained using tracks that are not assigned to the  $B^0 \rightarrow J/\psi\pi^0$  candidate, plus the IP constraint. This constraint allows for reconstruction of an  $f_{\text{tag}}$  vertex even in cases when only one track candidate satisfies the requirement on SVD hits. The fraction of single-track vertices for  $f_{\text{tag}}$  is approximately 12%, estimated from MC. To reject events with poorly reconstructed vertices, we require  $\sigma_z < 200 \mu\text{m}$  and  $h < 50$  for multitrack vertices, and  $\sigma_z < 500 \mu\text{m}$  for single-track vertices, where  $\sigma_z$  is the error on the vertex  $z$  coordinate, and  $h$  is the  $\chi^2$  value calculated in three-dimensional space without using the IP constraint [19]. We retain events in which both the  $J/\psi$  and  $f_{\text{tag}}$  vertices satisfy  $|\Delta t| < 70 \text{ ps}$ .

To extract the signal yield, we perform a two-dimensional unbinned maximum likelihood fit to the variables  $M_{bc}$  and  $\Delta E$ . The probability density function (PDF) of signal events consists of two parts: one for candidates that are correctly reconstructed, and one for those incorrectly reconstructed, i.e., at least one daughter

originates from the other (tag-side)  $B$ . For the former case, both the  $M_{bc}$  and  $\Delta E$  distributions are modeled with Crystal Ball (CB) functions [20]. For the latter case, the correlated two-dimensional  $M_{bc} - \Delta E$  distribution is modeled with a nonparametric PDF [21]. The fraction of incorrectly reconstructed decays ( $\sim 10\%$  in the signal region) is taken from MC simulation. The CB parameters that describe the lower tail of the  $M_{bc}$  and  $\Delta E$  distributions are also fixed to MC values.

The remaining background is small and dominated by  $B\bar{B}$  events in which one of the  $B$  mesons decays into a final state containing a  $J/\psi$ . We divide this background into three categories: (a)  $B^0 \rightarrow J/\psi K_S^0$ , (b)  $B^0 \rightarrow J/\psi K_L^0$ , and (c)  $B \rightarrow J/\psi X$  other than  $B^0 \rightarrow J/\psi K^0$ . We use two-dimensional nonparametric PDFs [21] to model the  $M_{bc} - \Delta E$  distributions for all three categories. We fix the background yields to those expected based on MC simulation: 10.8  $J/\psi K_S^0$  events, 10.0  $J/\psi K_L^0$  events, and 17.5 other  $J/\psi X$  events in the  $M_{bc} - \Delta E$  signal region. The remaining background comes from continuum  $q\bar{q}$  events. We model the  $M_{bc}$  and  $\Delta E$  distributions of continuum background with an ARGUS [22] function having its end point fixed to 5.29  $\text{GeV}/c^2$ , and a first-order polynomial, respectively. Background coming from  $B\bar{B}$  not containing a real  $J/\psi$  is negligible. From the fit we obtain  $330.2 \pm 22.1$  signal events and  $16.3 \pm 3.5$  continuum events. The purity of the signal is 86% in the signal region. Projections of the fit are shown in Fig. 2.

The branching fraction is calculated from the formula

$$\mathcal{B}(B^0 \rightarrow J/\psi \pi^0) = \frac{Y_{\text{sig}}}{\epsilon \times N_{B\bar{B}} \times \mathcal{B}_{J/\psi} \times \mathcal{B}_{\pi^0}}, \quad (2)$$

where  $Y_{\text{sig}}$  is the fitted signal yield;  $N_{B\bar{B}} = (772 \pm 11) \times 10^6$  is the number of  $B\bar{B}$  events;  $\epsilon = (22.3 \pm 0.1)\%$  is the signal efficiency for  $e^+e^-$  and  $\mu^+\mu^-$  combined as obtained from MC simulation;  $\mathcal{B}_{J/\psi}$  is the sum of  $\mathcal{B}(J/\psi \rightarrow \mu^+\mu^-)$  and  $\mathcal{B}(J/\psi \rightarrow e^+e^-)$  [15]; and  $\mathcal{B}_{\pi^0}$  is the branching fraction of  $\pi^0 \rightarrow \gamma\gamma$  [15]. In Eq. (2) we assume equal production of  $B^0\bar{B}^0$  and  $B^+B^-$  pairs at the  $\Upsilon(4S)$  resonance. The result is

$$\mathcal{B}(B^0 \rightarrow J/\psi \pi^0) = (1.62 \pm 0.11 \pm 0.06) \times 10^{-5},$$

where the first uncertainty is statistical and the second is systematic.

The systematic uncertainty on  $\mathcal{B}(B^0 \rightarrow J/\psi \pi^0)$  arises from several sources, as listed in Table I. The uncertainty due to the fixed parameters in the PDF is estimated by varying each parameter individually according to its statistical uncertainty. The resulting changes in the branching fraction are added in quadrature and the result is taken as the systematic uncertainty. The nonparametric shapes are also varied by changing their smoothing, and the associated systematic uncertainty is found to be negligible. We assign a 1.5% systematic uncertainty due to  $\pi^0$  reconstruction, as

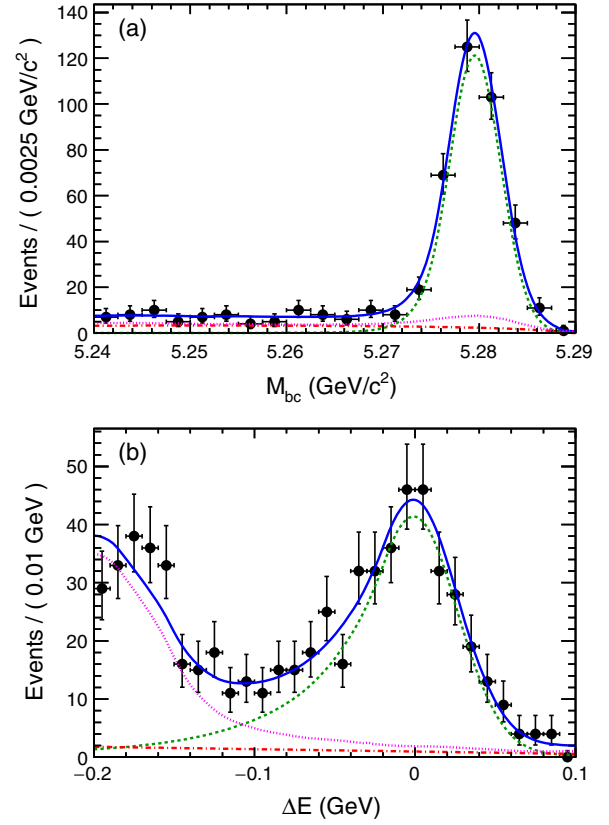


FIG. 2. Projections of the two-dimensional fit: (a)  $M_{bc}$  in the  $\Delta E$  signal region, and (b)  $\Delta E$  in the  $M_{bc}$  signal region. The points are data, the (green) dashed curves show the signal, the (red) dotted-dashed curves show the  $q\bar{q}$  background, the (magenta) dotted curves show the  $B\bar{B}$  background, and the (blue) solid curves show the total PDF.

determined from a study of  $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$  decays [23]. The uncertainty due to charged track reconstruction is 0.35% per track, as determined from a study of partially reconstructed  $D^{*+} \rightarrow D^0 \pi^+$ ,  $D^0 \rightarrow K_S^0 \pi^+ \pi^-$  decays. We assign a 2.1% uncertainty due to lepton identification, as obtained from a study of two-photon  $\gamma\gamma \rightarrow \ell^+ \ell^-$  production events.

TABLE I. Fractional systematic uncertainties for  $\mathcal{B}(B^0 \rightarrow J/\psi \pi^0)$ .

Source	Uncertainty (%)
PDF parametrization	0.1
$\pi^0$ reconstruction	1.5
Tracking	0.7
Lepton identification selection	2.1
Incorrectly reconstructed signal events	0.8
$B \rightarrow J/\psi(K_S^0, K_L^0, X)$ background	+1.8 -2.0
MC statistics	0.4
Secondary branching fractions	0.8
Number of $B\bar{B}$ pairs	1.4
Total	+3.7 -3.9

The uncertainty due to the estimated fraction of incorrectly reconstructed signal events is obtained by varying this fraction by  $\pm 100\%$ . As  $B \rightarrow J/\psi(K_S^0, K_L^0, X)$  decays are well measured, we evaluate the uncertainty due to their estimated amounts by varying them by  $\pm 20\%$ . The uncertainty due to the number of  $B\bar{B}$  pairs is 1.4%, and the uncertainty on the reconstruction efficiency  $\varepsilon$  due to the MC sample size is 0.4%. The total systematic uncertainty is obtained by summing all individual contributions in quadrature.

We determine  $\mathcal{S}$  and  $\mathcal{A}$  by performing an unbinned maximum likelihood fit to the  $\Delta t$  distribution of candidate events in the signal region. The PDF for the signal component,  $\mathcal{P}_{\text{sig}}(\Delta t; \mathcal{S}, \mathcal{A}, q, \omega_l, \Delta\omega_l)$ , is given by Eq. (1) with the parameters  $\tau_{B^0}$  and  $\Delta m_d$  fixed to the world-average values [24]. We modify this expression to take into account the effect of incorrect flavor assignment, which is parametrized by  $\omega_l$  and  $\Delta\omega_l$ . This PDF is then convolved with the decay-time resolution function  $R_{\text{sig}}(\Delta t)$ . The resolution function is itself a convolution of four components: the detector resolutions for  $z_{J/\psi\pi^0}$  and  $z_{\text{tag}}$ ; the shift of the  $z_{\text{tag}}$  vertex position due to secondary tracks from charmed particle decays; and the kinematic approximation that the  $B$  mesons are at rest in the CM frame [19]. The PDFs for the  $B^0 \rightarrow J/\psi K_S^0$  and  $B^0 \rightarrow J/\psi K_L^0$  backgrounds are the same as  $\mathcal{P}_{\text{sig}}$  but with  $CP$  parameters  $\mathcal{A}$  and  $\mathcal{S}$  fixed to the recent Belle results [19]. The PDF for the  $B \rightarrow J/\psi X$  background is taken to have the same form as  $\mathcal{P}_{\text{sig}}$  but with  $\mathcal{A}$  and  $\mathcal{S}$  set to zero, and with an effective lifetime  $\tau_{\text{eff}}$  determined from MC simulation. The PDF for continuum background is taken to be the sum of two Gaussian functions whose parameters are obtained by fitting events in the sideband region  $5.20 \text{ GeV}/c^2 < M_{\text{bc}} < 5.26 \text{ GeV}/c^2$  and  $0.10 \text{ GeV} < \Delta E < 0.50 \text{ GeV}$ .

We assign the following likelihood to the  $i$ th event:

$$\begin{aligned} \mathcal{P}_i(\Delta t) = & (1 - f_{\text{ol}}) \int d(\Delta t') [R_{\text{sig}}(\Delta t_i - \Delta t') \\ & \times (f_{\text{sig}} \mathcal{P}_{\text{sig}}(\Delta t') + f_{J/\psi K_S^0} \mathcal{P}_{J/\psi K_S^0}(\Delta t') \\ & + f_{J/\psi K_L^0} \mathcal{P}_{J/\psi K_L^0}(\Delta t') + f_{J/\psi X} \mathcal{P}_{J/\psi X}(\Delta t')) \\ & + f_{q\bar{q}} \mathcal{P}_{q\bar{q}}(\Delta t_i)] + f_{\text{ol}} \mathcal{P}_{\text{ol}}(\Delta t_i), \end{aligned} \quad (3)$$

where  $f_{\text{sig}}$ ,  $f_{J/\psi K_S^0}$ ,  $f_{J/\psi K_L^0}$ ,  $f_{J/\psi X}$ , and  $f_{q\bar{q}}$  are the fractions of the signal,  $B^0 \rightarrow J/\psi K_S^0$ ,  $B^0 \rightarrow J/\psi K_L^0$ ,  $B \rightarrow J/\psi X$ , and  $q\bar{q}$  continuum background, respectively. All fractions depend on the flavor tagging quality  $r$  and are functions of  $\Delta E$  and  $M_{\text{bc}}$ . The term  $\mathcal{P}_{\text{ol}}(\Delta t)$  is a broad Gaussian function that represents an outlier component with a small fraction  $f_{\text{ol}} \approx 0.5\%$ . The only free parameters in the fit are  $\mathcal{S}$  and  $\mathcal{A}$ ; these are determined by maximizing the likelihood  $\mathcal{L}(\mathcal{S}, \mathcal{A}) = \prod_i \mathcal{P}_i(\Delta t_i; \mathcal{S}, \mathcal{A})$ . Figure 3 shows the fitted  $\Delta t$  distribution and the time-dependent decay

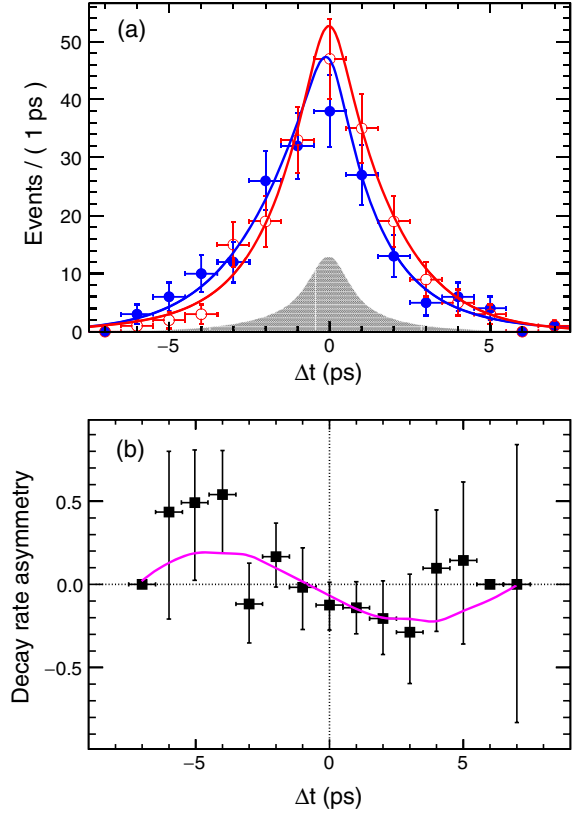


FIG. 3. (a) Distributions of  $\Delta t$ . The (blue) solid and (red) open points represent the  $q = +1$  and  $q = -1$  events, respectively, and the solid curves show the corresponding fit projections. The gray shaded region represents the sum of all backgrounds. (b) Time-dependent  $CP$  asymmetry  $\mathcal{A}_{CP}$  (see text).

rate asymmetry  $\mathcal{A}_{CP}$ , where  $\mathcal{A}_{CP} = (Y_{\text{sig}}^{(q=+1)} - Y_{\text{sig}}^{(q=-1)}) / (Y_{\text{sig}}^{(q=+1)} + Y_{\text{sig}}^{(q=-1)})$ , and  $Y_{\text{sig}}^{(q=\pm 1)}$  is the signal yield with  $q = \pm 1$ . The results of the fit are

$$\begin{aligned} \mathcal{S} &= -0.59 \pm 0.19 \pm 0.03 \\ \mathcal{A} &= -0.15 \pm 0.14^{+0.04}_{-0.03}, \end{aligned}$$

where the first uncertainty is statistical and the second is systematic. The correlation between  $\mathcal{A}$  and  $\mathcal{S}$  is  $-0.005$ .

The systematic uncertainties for  $\mathcal{S}$  and  $\mathcal{A}$  are listed in Table II. They are small compared to the corresponding statistical uncertainties. The largest contributions to  $\mathcal{S}$  arise from vertex reconstruction and the resolution function. The uncertainty due to the former includes uncertainties in the IP profile, charged track selection, vertex quality selection, and SVD misalignment. We vary each parameter of the resolution function by one standard deviation ( $\pm 1\sigma$ ) and compare the resulting fit result with that of the nominal fit; the difference between the two is taken as the systematic uncertainty. Each physics parameter that is fixed to its world average value [24], e.g.,  $\tau_{B^0}$  and  $\Delta m_d$ , is varied by the corresponding error; the uncertainty is taken to be the



TABLE II. Absolute systematic uncertainties for  $\mathcal{S}$  and  $\mathcal{A}$ .

Source	$\sigma_{\mathcal{S}}$ (%)	$\sigma_{\mathcal{A}}$ (%)
Vertex reconstruction	+2.36 -1.75	+1.40 -2.22
Resolution function	+1.43 -2.37	+1.00 -0.91
Physics parameters	+0.04 -0.03	$\pm 0.04$
Fit bias	$\pm 0.68$	$\pm 0.27$
Wrong tag fraction	+0.41 -0.20	+0.43 -0.17
$M_{bc}$ , $\Delta E$ shapes	+0.52 -0.45	+0.50 -0.48
Signal and background fraction	+0.71 -0.62	+0.49 -0.72
Background $\Delta t$ shape	+0.20 -0.12	$\pm 0.10$
Tag-side interference	$\pm 0.20$	+3.80 -0.00
Total	+3.02 -3.14	+4.26 -2.57

resulting difference with the nominal fit result. The uncertainty due to possible fit bias is evaluated using large ensembles of MC signal events; the differences of the fit results with the MC inputs are assigned as systematic uncertainties. The uncertainties due to  $\omega_l$  and  $\Delta\omega_l$  are estimated by varying these parameters individually by  $\pm 1\sigma$ . The  $M_{bc}$  and  $\Delta E$  shape parameters, and the fractions of signal and background, are varied to estimate their contributions to the systematic uncertainty. We vary each parameter in  $\mathcal{P}_{q\bar{q}}(\Delta t)$  and  $\mathcal{P}_{J/\psi X}(\Delta t)$  by  $\pm 1\sigma$ . For  $\mathcal{P}_{J/\psi K_S^0}(\Delta t)$  and  $\mathcal{P}_{J/\psi K_L^0}(\Delta t)$ , we vary the  $CP$  asymmetry parameters by their statistical errors [19]. We include the effect of tag-side interference [25], which introduces a significant contribution to the systematic uncertainty for  $\mathcal{A}$ . Tag-side interference is caused by interference between the two tree-level amplitudes contributing to  $B \rightarrow DX$  decays.

In summary, we have measured the branching fraction and time-dependent  $CP$  asymmetry for  $B^0 \rightarrow J/\psi\pi^0$  decays using the full Belle  $\Upsilon(4S)$  data set. The results are

$$\mathcal{B} = (1.62 \pm 0.11 \pm 0.06) \times 10^{-5}$$

$$\mathcal{S} = -0.59 \pm 0.19 \pm 0.03$$

$$\mathcal{A} = -0.15 \pm 0.14^{+0.04}_{-0.03},$$

where the first uncertainty is statistical and the second is systematic. The measured value for the branching fraction is the most precise value to date and supersedes the previous measurement [10]. It is consistent with measurements made by other experiments [9,26]. The measured  $CP$  asymmetries are consistent with, and supersede, our previous results [8]. The direct  $CP$  asymmetry  $\mathcal{A}$  is consistent with zero. The mixing-induced  $CP$  asymmetry  $\mathcal{S}$  differs from zero (i.e., no  $CP$  violation) by  $3.0\sigma$ , and it differs from the *BABAR* result [9] (which is outside the physical region) by  $3.2\sigma$ . The value is consistent with the

value of  $\sin 2\phi_1$  measured using  $b \rightarrow c\bar{c}s$  decays [15]. These results indicate that the penguin and any NP contribution to  $B^0 \rightarrow J/\psi\pi^0$  are small.

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