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# Radiatively Induced Lorentz and Gauge Symmetry Violation in Electrodynamics with Varying $\alpha$

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## Abstract

A time-varying fine structure constant  $\alpha(t)$  could give rise to Lorentz- and *CPT*-violating changes to the vacuum polarization, which would affect photon propagation. Such changes to the effective action can violate gauge invariance, but they are otherwise permitted. However, in the minimal theory of varying  $\alpha$ , no such terms are generated at lowest order. At second order, vacuum polarization can generate an instability—a Lorentz-violating analogue of a negative photon mass squared  $-m_\gamma^2 \propto \alpha \left(\frac{\dot{\alpha}}{2\alpha}\right)^2 \log(\Lambda^2)$ , where  $\Lambda$  is the cutoff for the low-energy effective theory.

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Two exotic forms of physics beyond the standard model that have recently gotten a lot of attention are Lorentz symmetry violations [1] and time-dependent fundamental constants. If these two phenomena exist, they are likely to be closely related. For example, if the fine structure constant  $\alpha = \frac{e^2}{4\pi}$  is actually a function of time  $\alpha(t)$ , there is naturally a preferred spacetime direction,  $\partial_\mu\alpha$ , which violates boost invariance. We shall investigate this potential connection, which may also be related to violations of electro-dynamical gauge invariance.

The most common theory with varying constants that has been studied is one with a time-dependent  $\alpha$ . Other possibilities have included changes in the quantum chromodynamics scale  $\Lambda_{QCD}$  and therefore the electron-proton mass ratio. However, these possibilities are usually approached phenomenologically, without reference to a full underlying quantum field theory. Lorentz violation has been treated somewhat differently in recent years. Although there is a long history of searches for deviations from special relativity—also frequently handled in purely phenomenological fashion—the standard approach now is to use effective field theory. The effective field theory containing all possible local, Lorentz-violating operators built from standard model fields is called the standard model extension (SME) [2]. The SME Lagrange density contains new operators, involving tensor objects constructed from quantum fields, contracted with constant background tensors. An example of such an operator is  $-a^\mu\bar{\psi}\gamma_\mu\psi$ . For parameterizing the results of experimental tests, a restricted subset of the theory, the minimal SME, is typically used. The minimal SME contains only operators that are gauge invariant and power counting renormalizable.

Lorentz violation and varying constants have both been tightly constrained experimentally, and many of the most stringent tests are in quantum electrodynamics (QED). In most cases, it is conventional to consider only the leading order effects of Lorentz violation or a varying  $\alpha$ ; since  $\dot{\alpha}$  must be small,  $\mathcal{O}(\dot{\alpha}^2)$  terms may be of negligible importance. In this paper, we shall begin by following this convention; however, the higher order terms are interesting when they can produce qualitatively different effects than those possible at leading order. For this reason, after discussing  $\mathcal{O}(\dot{\alpha})$  radiative corrections, we shall consider additional effects that are  $\mathcal{O}(\dot{\alpha}^2)$  and  $\mathcal{O}(\ddot{\alpha})$ .

Measurements of  $\frac{\dot{\alpha}}{\alpha}$  may be made in a number of different ways: with pairs of precision spectroscopy experiments done years apart [3], by determination of the production rates for certain isotopes in natural reactors [4], and by observing spectra from cosmologically distant sources [5]. The resulting bounds are typically at the  $\frac{\dot{\alpha}}{\alpha} < 10^{-14} \text{ yr}^{-1}$  level for measurements of the present rate of change and a comparable  $\frac{\Delta\alpha}{\alpha} < 10^{-5}$  level over cosmological time scales.

Observations of very old photons can be an excellent way to probe exotic physics. If  $\alpha$  was different at the time of their emission, these photons could reveal different characteristic atomic spectra than are seen today. The photons' extremely long travel times could magnify the effects of a very small  $\dot{\alpha}$ . If there are any changes in the propagation characteristics of the photons, this could also be magnified by the extremely long line of sight to

cosmological sources. This is how many of the best constraints on Lorentz-violating effects have been set. For example, many SME operators give rise to a polarization dependence in the phase speed of photons; this in turn leads to birefringence and a change in the polarization of radiation as it propagates—an effect which has not been seen [6]. One can also look for energy dependence in photon arrival times, indicative of a nontrivial photon dispersion relation [7], deflection of photon trajectories [8], or phase differences [9]. All these can be used to probe exotic new physics possibilities, and the long photon propagation times give tiny effects a long time to build up, leading to many of the best bounds on various novel effects.

It is therefore worthwhile to ask whether a changing  $\alpha$  would leave an imprint on pure photon propagation. The problem is theoretically interesting, and it could potentially lead to new ways of constraining  $\dot{\alpha}$ . Obviously, for a time-dependent  $\alpha$  to affect photons' propagation, the photons must interact with some charged matter. However, it is well known that, even in vacuum, the electromagnetic field is constantly interacting with virtual particle-antiparticle pairs. The polarizability of the QED vacuum could give rise to  $\dot{\alpha}$ -dependent effects in the pure photon sector. And since a theory with varying  $\alpha$  supports a preferred direction  $\partial_\mu\alpha$ , it is natural to wonder whether the pure photon sector will exhibit timelike Lorentz violation as a result of  $\dot{\alpha}$ .

Lorentz symmetry is closely related to *CPT* symmetry. Breaking of *CPT* symmetry requires either a breaking of Lorentz invariance or something even more exotic [10]. The phenomenon of time-varying fundamental constants is odd under time reversal, even under parity and charge conjugation—hence odd under *CPT*. The behavior under *T* is obvious, and the behavior under *P* is dictated by isotropy. One might wonder if a different behavior under *C* is possible, but to have *C*-odd, time-dependent charges would violate the equality of particle and antiparticle charge.

Gauge invariance is another crucial property of electrodynamics, and one which has also been subjected to stringent experimental tests. Ordinarily, gauge symmetry is responsible for ensuring charge conservation, but charge is not conserved if  $\dot{\alpha} \neq 0$ . So it would not be unexpected that radiative corrections in the photon sector might break gauge symmetry. The most common form of gauge symmetry breaking to be considered is a photon mass, which leads to a screening of electromagnetic fields. Measurements of the solar wind and the persistence of the sun's magnetic field out to Pluto's orbit give bounds at the  $m_\gamma < 10^{-27}$  GeV level [11]; less secure limits based on galaxy-scale fields are many orders of magnitude better [12]. Discrete symmetries rule out the generation of a conventional photon mass (which would be even under *C*, *P*, and *T*) by  $\mathcal{O}(\dot{\alpha})$  effects, but qualitatively similar Lorentz- and gauge-symmetry-violating effects might be possible. A simple screening of static sources would not arise, since such screening is time reversal invariant, but changes in the magnitude of the electromagnetic field with time are not ruled out.

Moreover, any Lorentz-violating radiative correction that is linear in  $\dot{\alpha}$  must break gauge symmetry or otherwise lie outside the minimal SME. The minimal SME contains

no operator with the same discrete symmetries as  $\dot{\alpha}$ . There are operators [ $b_j$ ,  $(k_{AF})_j$ ,  $g_{jk0}$ , and  $g_{j0k}$ ] that are  $C$ -even,  $P$ -even, and  $T$ -odd. However, the behavior of Lorentz-violating effects under parity can be further subcategorized. The parity operator is defined as inverting all three spatial coordinates,  $\vec{x} \rightarrow -\vec{x}$ , but  $P$  may be broken down into the product of three separate reflections,  $P = R_1 R_2 R_3$ , where  $R_j$  takes  $x_j \rightarrow -x_j$  and leaves the other two coordinates unchanged. A changing  $\alpha$  is an isotropic effect, and it must behave the same way under all three  $R_j$  reflections. However, the  $b_j$ ,  $(k_{AF})_j$ ,  $g_{jk0}$ , and  $g_{j0k}$  coefficients are actually odd under two reflections and even under the third; the overall behavior under parity is the same, but the different symmetries still prevent  $\dot{\alpha}$  from generating any of these operators through radiation corrections.

We may actually hope that gauge symmetry breaking might lead to an enhancement of some  $\dot{\alpha}$ -dependent radiative corrections. New contributions to the self-energy tensor  $\Pi^{\mu\nu}$  that do not satisfy the Ward identity,  $p_\mu \Pi^{\mu\nu}(p) = 0$  (which ensures that radiative corrections do not generate a quadratically divergent photon mass), might naively be expected to exhibit power law divergences. At lowest order, we would expect the divergence to be  $\mathcal{O}(\dot{\alpha}\Lambda)$ , where  $\Lambda$  is the cutoff for the theory. A radiative correction like this could have profound experimental implications; if the appropriate form of gauge symmetry violation could be constrained at a scale comparable to the best bounds on  $m_\gamma$ , the corresponding bound on  $\frac{\dot{\alpha}}{\alpha}$  would be at the  $\alpha^{-1} \left(\frac{m_\gamma}{\Lambda}\right) m_\gamma$  level, with the factor in parentheses providing a huge boost in sensitivity. Unfortunately however,  $\mathcal{O}(\dot{\alpha}\Lambda)$  divergences are actually forbidden; we shall see that the radiative corrections must depend on a positive power of  $p$ , which means that a linear divergence is ruled out by power counting. But for  $\mathcal{O}(\dot{\alpha}^2)$  corrections the situation is different, and radiative corrections that violate gauge invariance may indeed be proportional to a (logarithmically) divergent function of the cutoff.

The form  $\dot{\alpha}$ -dependent effects might take may be model dependent, and even if the value of  $\alpha$  has changed significantly over the lifetime of the universe, it is not really clear what the course of the time dependence might have been. For definiteness, we shall assume a particularly simple function  $\alpha(t)$  for our discussion of  $\mathcal{O}(\dot{\alpha})$  effects. We shall assume that the change is smooth and uniform in time, even on very short time scales, less than the lifetime of a virtual electron-positron pair,  $\tau \sim (2m)^{-1}$ .

If the electromagnetic coupling is changing on very short time scales, then we can envision a scenario in which electromagnetic fields might grow or decay with time. An incoming photon produces a virtual electron-positron pair, which annihilate at a slightly later time, producing an outgoing photon. During the lifetime of the virtual pair, the coupling constant changes slightly, and so the final field may be weaker or stronger than the initial field was. There will, however, be cancellation between this process and one in which the outgoing photon is produced simultaneously with the electron and positron (which are annihilated at a later time, along with the incoming photon). In the second process, the change in  $\alpha$  between the photon absorption and emission has the opposite sign relative to the first. However, the cancellation between the two should be incomplete, since the second process has a larger energy defect in the intermediate state.

The standard vacuum polarization Feynman diagram encompasses both virtual electron-positron processes. Normally, the Feynman rules are derived from the action using functional derivatives. It is not automatically clear, however, whether a theory with varying constants should be derived from a Lagrangian. In fact, there are ambiguities in the Lagrangian formalism. The standard electromagnetic Lagrange density may be written as

$$\mathcal{L}_{EM} = -\frac{1}{4e^{2n}}F^{\mu\nu}F_{\mu\nu} + e^{1-n}j^\mu A_\mu \quad (1)$$

for any  $n$ . When  $e$  is constant, the value of  $n$  is irrelevant. However, when the coupling varies with time, only the Lagrangian with  $n = 1$  is invariant under the usual  $U(1)$  gauge transformation. The equation of motion becomes

$$\frac{1}{e^{2n}}\partial_\mu F^{\mu\nu} - \frac{4\pi n}{e^{2n+2}}(\partial_\mu\alpha)F^{\mu\nu} = e^{1-n}j^\nu. \quad (2)$$

This includes a Lorentz-violating term with an undetermined coefficient. Theories of this sort are discussed in [13]. The Lorentz-violating term in (2) mirrors the effects of a gauge-noninvariant  $\vec{A}\cdot\vec{E}$  term in  $\mathcal{L}$ . However, since the undetermined dependent coefficient can be set to zero (using the very the common choice of  $n = 0$ ), it is possible to have a theory with varying  $\alpha(t)$  and no Lorentz violation of this sort. Moreover, the differences between the theories with different values of  $n$  are not generally apparent in experiments that simply compare the instantaneous value of  $\alpha$  at two different times.

Rather than dealing with the Lagrangian, we shall simply take the theory to be defined by its Feynman rules. This allows us to consider the minimal modification to QED that accounts for a time-varying  $\alpha$ . However, this approach is essentially equivalent to using the particular electromagnetic Lagrangian with  $n = 1$ .

Because the coupling constant, and thus the Feynman rules, will be time dependent, we shall set up the rules in configuration space. The key change to the QED Feynman rules is in the vertex; a vertex at position  $x$  corresponds to a factor  $-\int d^4x ie\left(1 + \frac{\dot{\alpha}}{2\alpha}x^0\right)\gamma^\mu$ . Any theory with varying  $\alpha$  essentially must include this modified vertex. We shall take the factors corresponding to other elements of Feynman graphs to be unchanged. A fully consistent theory might include additional modifications to the Feynman rules, but we want to consider only those effects which are necessary consequences of having a time-dependent  $\alpha$ . The remaining rules that will be needed for the calculation of the modified photon self-energy are the factors of  $e^{-iq\cdot x}\epsilon_\mu$  ( $e^{iq\cdot x}\epsilon_\mu^*$ ) for incoming (outgoing) photon lines and the fermion propagator

$$S_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i(\not{p} + m)}{p^2 - m^2 - i\epsilon} e^{-ip\cdot(x-y)}, \quad (3)$$

with a  $-1$  for a closed fermion loop.

Novel effects in the pure photon sector would be generated by the vacuum polarization diagram. Excising the external photon lines and the corresponding polarization vectors,

the amplitude for this diagram is

$$\begin{aligned}
i\mathcal{M}^{\mu\nu}(q, q') &= \int d^4x \int d^4y \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} (-ie)^2 \left(1 + \frac{\dot{\alpha}}{2\alpha} x^0\right) \left(1 + \frac{\dot{\alpha}}{2\alpha} y^0\right) \\
&\quad \times (-1) \text{tr} \left[ \gamma^\mu \frac{i(\not{k}_1 + m)}{k_1^2 - m^2} \gamma^\nu \frac{i(\not{k}_2 + m)}{k_2^2 - m^2} \right] e^{-i(k_1 - k_2) \cdot (x - y)} e^{-iq \cdot x} e^{iq' \cdot y}.
\end{aligned} \tag{4}$$

The integrals may be rewritten in terms of  $k = k_1$  and  $p = k_2 - k_1$ . Symmetries dictate that, when we calculate only the fermion-antifermion loop, the  $x^0$  and  $y^0$  contributions are equal apart from surface terms. For the  $\mathcal{O}(\dot{\alpha})$  part of the self-energy, we therefore have

$$\begin{aligned}
i\mathcal{M}_{\dot{\alpha}}^{\mu\nu}(q, q') &= e^2 \left(\frac{\dot{\alpha}}{\alpha}\right) \int d^4x \int d^4y y^0 \int \frac{d^4p}{(2\pi)^4} e^{ip \cdot (x - y)} e^{-iq \cdot x} e^{iq' \cdot y} \\
&\quad \times \int \frac{d^4k}{(2\pi)^4} \text{tr} \left[ \gamma^\mu \frac{i(\not{k} + m)}{k^2 - m^2} \gamma^\nu \frac{i(\not{k} + \not{p} + m)}{(k + p)^2 - m^2} \right] \\
&= \left(\frac{\dot{\alpha}}{\alpha}\right) \int d^4x \int d^4y y^0 \int \frac{d^4p}{(2\pi)^4} e^{ip \cdot (x - y)} e^{-iq \cdot x} e^{iq' \cdot y} [i\Pi_2^{\mu\nu}(p)],
\end{aligned} \tag{5}$$

$$\tag{6}$$

where  $\Pi_2^{\mu\nu}(p)$  is the usual one-loop photon self-energy.

The  $y$ -integration may be eliminated via  $\int d^4y y^0 e^{-ip \cdot y} e^{iq' \cdot y} = i(2\pi)^4 \delta^3(\vec{p} - \vec{q}') \delta'(p^0 - q'^0)$ . This reduces the self-energy to

$$\begin{aligned}
i\mathcal{M}_{\dot{\alpha}}^{\mu\nu}(q, q') &= \left(\frac{\dot{\alpha}}{\alpha}\right) \int d^4x \int d^4p i e^{-i(\vec{p} - \vec{q}') \cdot \vec{x}} e^{i(p^0 - q'^0)x^0} \delta^3(\vec{p} - \vec{q}') \delta'(p^0 - q'^0) [i\Pi_2^{\mu\nu}(p)] \\
&= \left(\frac{\dot{\alpha}}{\alpha}\right) (2\pi)^3 \delta^3(\vec{q} - \vec{q}') \int dx^0 \int dp^0 i e^{i(p^0 - q'^0)x^0} \delta'(p^0 - q'^0) [i\Pi_2^{\mu\nu}(p^0, \vec{q})]
\end{aligned} \tag{7}$$

$$\begin{aligned}
&= \left(\frac{\dot{\alpha}}{\alpha}\right) (2\pi)^3 \delta^3(\vec{q} - \vec{q}') \int dx^0 \left\{ x^0 e^{-i(q^0 - q'^0)x^0} [i\Pi_2^{\mu\nu}(q^0, \vec{q})] \right. \\
&\quad \left. + e^{-i(q^0 - q'^0)x^0} \left[ \frac{\partial \Pi_2^{\mu\nu}(q)}{\partial q^0} \right] \right\}
\end{aligned} \tag{8}$$

It is not possible to factor out an energy-conserving  $\delta$ -function, because the theory is not time translation invariant. However, the first term in French brackets in (8) has a straightforward interpretation. Even though it appears to violate energy conservation, it is actually a natural extension of the usual photon self-energy. Combined with the  $\dot{\alpha}$ -independent contribution, it gives a contribution to the effective Lagrangian  $\Delta\mathcal{L} \supset \frac{1}{2} \left(1 + \frac{\dot{\alpha}}{\alpha} x^0\right) A_\mu(x) [\Pi_2^{\mu\nu}(i\partial)] A_\nu(x)$ . This just represents the usual quantum correction, but with the full, time-dependent  $\alpha(t)$ . A crucial feature of  $\Pi_2^{\mu\nu}$  is that it obeys the Ward identity. It is this property [and the fact that the self-energy is regular at  $p^2 = 0$ ] that ensures photons are massless. The first term in (8) also obeys the Ward identity.

If we had retained the two external photon propagators in the preceding calculation and determined the amplitude for a photon to propagate from  $z_1$  to  $z_2$ , splitting into an electron-positron pair one time along the way, the result would be just the usual expression, but with the coupling constant  $\alpha$  evaluated at the average time  $(z_1^0 + z_2^0)/2$ .

The fact that the effective Lagrangian depends explicitly on time suggests an interesting possibility. The effective Hamiltonian for the electromagnetic field also appears to be time dependent, which could lead to energy nonconservation. However, this apparent effect is actually unphysical. The time dependence of the effective Hamiltonian density  $\mathcal{H} + \Delta\mathcal{H}$  arises from the time dependence of  $\Delta\mathcal{L}$ , which is exactly canceled by a time-dependent field strength renormalization constant  $Z(t)$ ; the instantaneous value of  $Z(t)$  sets the scale on which field strengths are measured. That such a cancellation must occur is actually clear from the fact that the momentum density of the field appears to vary in time in exactly the same way that the Hamiltonian density does. However, the theory is invariant under spatial translations, and so momentum must be conserved; any apparent time variation is canceled by  $Z(t)$ .

The second term on the right-hand side of (8) is superficially Lorentz-, *CPT*-, and gauge-symmetry-violating, but it is actually a total derivative. It gives rise to terms such as  $A^\mu \partial^0 A_\mu = \frac{1}{2} \partial^0 A^2$  and  $A^0(\partial_\mu A^\mu) + A^\mu(\partial_\mu A^0) = \partial_\mu(A^0 A^\mu)$ , which certainly cannot generate a novel time dependence for electromagnetic fields, nor any other new effects. In fact, had we performed the various integrations in a different order, this term need not have appeared at all.

There are several reasons why the radiative corrections take these kinds of forms. In the physical picture outlined earlier, there was a partial cancellation between processes in which the incoming field was absorbed before the outgoing one was emitted and processes with the time ordering inverted. If the incoming field carried no energy, the intermediate states in the two processes would be equally off shell, and the cancellation would be exact; therefore, any nonvanishing contribution must be proportional to the photon energy  $p^0$ . For a radiative correction to produce the notional time dependence (with a growing or decaying field), it must lead to an equation of motion that may be written schematically as  $\dot{A} - \xi \dot{\alpha}(\partial)^{2n+1} A = 0$ . The  $\dot{\alpha}$  term must have an odd number of derivatives, so that changing the sign of  $\dot{\alpha}$  will reverse the decay or enhancement effect. Alternatively, having the radiative correction depend on an odd power of the momentum is the only way to preserve the *C*-even, *P*-even, *T*-odd behavior characteristic of  $\mathcal{O}(\dot{\alpha})$  effects.

There are terms in the SME with odd powers of  $p$  that are not total derivatives—for example, the purely timelike Chern-Simons term  $\frac{1}{2} k_{AF}^0 \vec{A} \cdot \vec{B}$ . However, this term relies on the presence of the Levi-Civita  $\epsilon$ -tensor in the definition of the magnetic field and is consequently odd under parity; it affects right- and left-circularly polarized photons in opposite fashions. Moreover, the Chern-Simons term is gauge invariant (up to a total divergence). However, a gauge symmetry breaking term such as  $A^0(\partial^\mu A_\mu)$  could make equally real contributions to photon behavior, if it appeared singly (and not in combination as a total derivative). In that case, the term would duplicate the effects of the



Lorentz-violating term in (2).

Since one of our major results is that the variation of  $\alpha$  does not give rise to a Lorentz-violating two-photon operator at lowest order, it is natural to wonder what was wrong with the scenario we described earlier, with the change in the coupling constant leading to temporal enhancement or suppression of electromagnetic fields. In that scenario, we would expect the amplitude of the field to track the value of  $\alpha(t)$ ; as a photon is continually being absorbed and re-emitted by virtual electron-positron pairs, the field strength should increase (or decrease) as the charge does. However, this is problematic when considered in the context of quantum mechanics. When a single photon is propagating, a continuous change in the magnitude of the field strength is not possible; such a change would violate the quantization of energy and hence the uncertainty principle. Rather than the field strength, what tracks the changing value of  $\alpha$  is the amount of mixing between photon and fermion-antifermion states. This argument also suggests that there ought to be no Lorentz-violating radiative corrections at  $\mathcal{O}(\dot{\alpha})$  even when diagrams with more loops are considered.

We shall now delve into the question of what happens at higher orders, either  $\mathcal{O}(\dot{\alpha}^2)$  or  $\mathcal{O}(\ddot{\alpha})$ . In a systematic expansion of the photon self-energy, these two orders should be considered simultaneously. We shall take the time-dependent electric charge  $e(t)$  as simply proportional to the square root of  $\alpha(t)$ :

$$\frac{e(t)}{\sqrt{4\pi\alpha_0}} = 1 + \left(\frac{\dot{\alpha}}{2\alpha_0}\right)t + \frac{1}{2} \left[ \left(\frac{\ddot{\alpha}}{2\alpha_0}\right) - \left(\frac{\dot{\alpha}}{2\alpha_0}\right)^2 \right] t^2, \quad (9)$$

where  $\alpha_0$  is the fine structure constant at the reference time  $t = 0$ . However, calculations that include these higher order terms do not enjoy the model independence of the  $\mathcal{O}(\dot{\alpha})$  results; it would be entirely natural to have other changes to the Feynman rules at this order. It is also important to note that the  $\mathcal{O}(\dot{\alpha}^2)$  radiative corrections may be smaller than corrections that are only  $\mathcal{O}(\dot{\alpha})$ , but which arise from diagrams with more than one loop—although the terms at the different orders can always be distinguished by their behavior under  $T$ .

With the higher order corrections, we encounter a new subtlety. To determine the one-loop effective action in this theory, it is not generally sufficient to evaluate an isolated fermion-antifermion loop, amputated of external legs. This complication arises because of the explicit time dependence in the Feynman rules. The factors of  $x^0$  lead to derivatives of  $\delta$ -functions; these produce  $\partial/\partial q_0$  derivative operators which act not only on  $\Pi^{\mu\nu}(q)$ , but also on the external propagators in the photon self-energy diagram. This subtlety does not become a problem at  $\mathcal{O}(\dot{\alpha})$ , where the full results may be inferred from just the fermion-antifermion part of the self-energy. However, at higher orders, we must calculate the full two-point correlation function  $\langle A^\mu(z_1)A^\nu(z_2) \rangle$ .

The calculation of this two-point function is outlined in the Appendix. The result,

including all terms up to  $\mathcal{O}(\dot{\alpha}^2)$  and  $\mathcal{O}(\ddot{\alpha})$  is

$$\begin{aligned} \langle A^\mu(z_2)A^\nu(z_1) \rangle &= \frac{\bar{\alpha}}{\alpha_0} \int \frac{d^4q}{(2\pi)^4} \frac{-i-i}{q^2} e^{-iq \cdot (z_2 - z_1)} [i\Pi_2^{\mu\nu}(q)] \\ &+ \frac{1}{8} \left( \frac{\dot{\alpha}}{\alpha_0} \right)^2 \int \frac{d^4q}{(2\pi)^4} \frac{-i-i}{q^2} e^{-iq \cdot (z_2 - z_1)} \left[ i \frac{\partial^2}{\partial q_0^2} \Pi_2^{\mu\nu}(q) \right] \\ &+ \frac{1}{2} \left( \frac{\ddot{\alpha}}{\alpha_0} \right) \int \frac{d^4q}{(2\pi)^4} \frac{-i-i}{q^2} e^{-iq \cdot (z_2 - z_1)} \left\{ i q_0 \frac{\partial}{\partial q_0} \left[ \frac{\Pi_2^{\mu\nu}(q)}{q^2} \right] \right\}, \end{aligned} \quad (10)$$

where  $\bar{\alpha}$  is a suitably averaged value of the coupling between the times  $z_1^0$  and  $z_2^0$ :

$$\frac{\bar{\alpha}}{\alpha_0} = 1 + \frac{\dot{\alpha}}{2\alpha_0} (z_1^0 + z_2^0) + \frac{\ddot{\alpha}}{4\alpha_0} \left[ (z_1^0)^2 + (z_2^0)^2 \right]. \quad (11)$$

Equation (10) factorizes the two-point function into approximately the form usually seen. It includes the usual self-energy, modified by an average of the time-dependent coupling constant, and also new, Lorentz-violating self-energy terms. We can almost just read off the effective potential, as we would conventionally. However, when multiple fermion-antifermion loops are inserted into the photon propagator, the loops do not quite produce an exactly resumable geometric series. The existence of the  $\partial/\partial q_0$  derivatives acting on the several parts of the diagram is responsible for this. However, the terms do approximately resum; the errors associated with the resummation are  $\mathcal{O}(\dot{\alpha}^2)$ , equivalent to other multiple-loop corrections that have already been neglected. So at the order under consideration, the  $\mathcal{O}(\dot{\alpha}^2)$  and  $\mathcal{O}(\ddot{\alpha})$  terms can be treated like any other Lorentz-violating contribution to the effective Lagrangian.

The  $\mathcal{O}(\dot{\alpha}^2)$  part of the self-energy is

$$\Pi_{m_\gamma}^{\mu\nu}(q) = \frac{1}{8} \left( \frac{\dot{\alpha}}{\alpha_0} \right)^2 \left[ \frac{\partial^2}{\partial q_0^2} \Pi_2^{\mu\nu}(q) \right], \quad (12)$$

which has the general structure of a photon mass term. The discrete symmetries that prevented the generation of a mass term at first order do not come into play here. However, this mass-like contribution differs from the usual Proca mass term [14] and clearly breaks Lorentz boost symmetry.

The Lorentz structure of  $\Pi_2^{\mu\nu}(q)$  is  $\Pi_2^{\mu\nu}(q) = (q^2 g^{\mu\nu} - q^\mu q^\nu) \Pi_2(q^2)$ . The scalar factor is  $\Pi_2(q^2) = \Pi_2(0) + \frac{2\alpha}{\pi} \int_0^1 dz z(1-z) \log[1 - z(1-z)q^2/m^2]$ . The constant term in  $\Pi_2(q^2)$  is formally divergent and in any case should be much larger than the momentum-dependent term when  $q^2 \ll m^2$ . So the dominant part of the radiative correction is captured in the approximation  $\Pi_2(q) \approx \Pi_2(0)$ . The second derivative then becomes  $\frac{\partial^2}{\partial q_0^2} \Pi_2^{\mu\nu}(q) \approx 2(g^{\mu\nu} - g^{0\mu}g^{0\nu})\Pi_2(0)$ , and the  $\mathcal{O}(\dot{\alpha}^2)$  contribution to the effective Lagrange density is

$$\Delta\mathcal{L} \supset -\frac{1}{8} \left( \frac{\dot{\alpha}}{\alpha_0} \right)^2 \Pi_2(0) \vec{A}^2. \quad (13)$$

Of course, this expression is problematic, because  $\Pi_2(0)$  is formally infinite. The divergence itself is not surprising, since the term involved is not gauge invariant; however, we must understand what the physical interpretation of this divergence could be. The mass-like term represents a new renormalizable operator, which is consistent with the broken gauge and Lorentz symmetries of the theory. The term will be generated by radiative corrections even if its bare value vanishes, and naturalness suggests that the physically measurable coefficient of the term should not be substantially smaller in magnitude than the radiative corrections it receives. With a single species of fermion, the divergence associated with  $\Pi_2(0)$  takes the form  $\Pi_2(0) = -\frac{\alpha_0}{3\pi} \log \frac{\Lambda^2}{m^2}$ , where  $\Lambda$  is the ultraviolet cutoff. What value of  $\Lambda$  is appropriate depends on the scale at which the new physics ultimately responsible for the varying  $\alpha$  enters, but the cutoff should presumably lie somewhere between the electroweak scale and the Planck scale. Taking  $\Lambda = 200$  GeV as a conservative estimate and including all species of charged fermions gives  $|\Pi_2(0)| > 10\alpha_0$ .

A mass term of the form  $-m_\gamma^2 \vec{A}^2$  has interesting properties [15, 16]. While many theories that invoke Lorentz violation envision it occurring primarily as a high-energy phenomenon, this form of Lorentz violation is most important far in the infrared (just like the changing  $\alpha$  that generates it). With this mass term, the magnetic field of a source takes the same form as in Proca electrodynamics, while the electric field exhibits a mixture of Proca and conventional behavior. There is an instantaneous Coulomb's Law, while propagating waves have a massive dispersion relation. The signal speed for the theory is infinite, but electromagnetic disturbances caused by distant sources are slow to reach full strength. Constraints on the photon mass based on observations of static magnetic fields would remain valid even if the mass has this Lorentz-violating form, but constraints based on the behavior of static electric fields would not.

Of course, the radiative correction is not actually a photon mass term, because  $m_\gamma^2$  is negative. There are also the derivatives acting on  $\Pi_2(q^2) - \Pi_2(0)$ . These give additional smaller corrections—some of which are gauge invariant, some of which are not. However, these terms will not change the overall sign of the mass-like term. Because of its dependence on  $\dot{\alpha}^2$ , this term cannot represent a genuine mass, whether the coupling strength is increasing or decreasing. Instead, it should correspond to some kind of instability. The existence of either a mass term or an instability at  $\mathcal{O}(\dot{\alpha}^2)$  would suggest the possibility of constraining  $\dot{\alpha}$  indirectly, using large scale electromagnetic field measurements. However, the best limits on the time dependence of  $\alpha$  are already many orders of magnitude better than the best current limits on  $m_\gamma$  and related quantities, so direct measurements cannot provide any useful new constraints on  $\dot{\alpha}$ . In fact, the bounds on the variation of  $\alpha$  are so tight that it is not clear that the usual interpretation assigned to an instability is still meaningful. An instability on a time scale longer than the lifetime of the universe would not be directly measurable (and with a positive  $m_\gamma^2$  of the same size, there would be a similar problem; the energy uncertainty of any observed photon cannot be less than the reciprocal lifetime of the universe). So the presence of this mass-like term is theoretically interesting but probably not experimentally significant.

The Lorentz-violating term that arises at  $\mathcal{O}(\ddot{\alpha})$  has a different structure. Again neglecting  $\Pi_2(q^2) - \Pi_2(0)$ , this term is equivalent to a self-energy contribution

$$\Pi_{\ddot{\alpha}}^{\mu\nu}(q) = \frac{\ddot{\alpha}}{2\alpha_0} \left[ -\frac{g^{\mu 0} q_0 q^\nu + g^{\nu 0} q_0 q^\mu}{q^2} + \frac{2q_0^2 q^\mu q^\nu}{(q^2)^2} \right] \Pi_2(0). \quad (14)$$

This term violates gauge invariance— $q_\mu \Pi_{\ddot{\alpha}}^{\mu\nu}(q) \propto g^{\nu 0} q_0 - q_0^2 q^\nu / q^2$ . However, it is not a photon mass term. At  $\vec{q} = 0$ , it vanishes; it does not endow long-wavelength excitations with a nonzero energy. Nor, at leading order, does it affect the dispersion relation for standard photons; contracted with a transverse spatial polarization vector, it also gives 0. Indeed, if this term is inserted (as part of the photon propagator) between two conserved currents, it will always give a vanishing result. The term is not completely trivial in a theory that incorporates charge nonconservation, but any physical effects will be additionally suppressed by the smallness of  $\partial^\mu j_\mu$ —which, in this theory, is  $\mathcal{O}(\dot{\alpha})$ . So this term only has effects at higher order.

At  $\mathcal{O}(\ddot{\alpha})$ , the additional terms arising from derivatives of  $\Pi_2(q^2)$  are all transverse and thus gauge invariant. The largest correction of this type comes from

$$\frac{q_0}{q^2} \frac{\partial \Pi_2(q^2)}{\partial q_0} \Big|_{q^2 \rightarrow 0} = -\frac{2\alpha_0 q_0^2}{15\pi m^2 q^2}. \quad (15)$$

On its own, this term would modify the dispersion relation in the denominator of the photon propagator to  $q^2 + \left(\frac{\ddot{\alpha}}{\alpha_0}\right) \frac{\alpha_0}{15\pi m^2} q_0^2$ . This represents an isotropic change in the propagation speed of photons and is a radiative contribution to the SME parameter  $\tilde{\kappa}_{\text{tr}}$ , with  $\Delta\tilde{\kappa}_{\text{tr}} = \left(\frac{\ddot{\alpha}}{\alpha_0}\right) \frac{\alpha_0}{30\pi m^2}$ . This shows that a varying  $\alpha(t)$  does generate quantum corrections to minimal SME operators. However, given the experimentally allowed values of  $\ddot{\alpha}$  and the size of the electron mass, this correction must be incredibly minuscule and not observable directly.

In summary, we have studied the radiative corrections to the photon sector caused by a time varying fine structure constant  $\alpha$ . If  $\dot{\alpha} \neq 0$ , the theory is not invariant under Lorentz or  $CPT$  symmetry, nor is charge conserved. So there is expected to be no symmetry preventing the appearance of Lorentz- and gauge-symmetry-violating terms in the effective action. However, in a theory that has been minimally modified to include a varying  $\alpha(t)$ , the one-loop vacuum polarization does not generate any Lorentz-violation in the photon sector at  $\mathcal{O}(\dot{\alpha})$ . In other words, it is possible to have a varying  $\alpha$  that is not accompanied by any electromagnetic Lorentz violation at lowest order. At  $\mathcal{O}(\dot{\alpha}^2)$ , a  $CPT$ -preserving but Lorentz-violating photon-mass-like term is possible, and in the minimal varying  $\alpha$  theory, such a term is duly generated.

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## Appendix: Self-Energy at Second Order

The amplitude for the electromagnetic field to propagate from  $z_1$  to  $z_2$  with one fermion-antifermion loop insertion is

$$\begin{aligned} \langle A^\mu(z_2)A^\nu(z_1) \rangle &= \int \frac{d^4q}{(2\pi)^4} \int \frac{d^4q'}{(2\pi)^4} \int \frac{d^4p}{(2\pi)^4} \int d^4x \int d^4y \frac{-ig^\mu{}_\alpha}{q^2} \frac{-ig_\beta{}^\nu}{q'^2} \\ &\quad e^{-iq \cdot (x-z_1)} e^{ip \cdot (y-x)} e^{-iq' \cdot (z_2-y)} \left[ 1 + \frac{\dot{\alpha}}{2\alpha_0} (x_0 + y_0) \right. \\ &\quad \left. + \frac{\dot{\alpha}^2}{4\alpha_0^2} x_0 y_0 + \frac{1}{2} \left( \frac{1}{2} \frac{\ddot{\alpha}}{\alpha_0} - \frac{1}{4} \frac{\dot{\alpha}^2}{\alpha_0^2} \right) (x_0^2 + y_0^2) \right] [i\Pi_2^{\alpha\beta}(p)]. \end{aligned} \quad (16)$$

This is correct to second order in the variation of the coupling constant and represents a straightforward generalization of the first-order expression (6). Defining

$$\begin{aligned} M^{\mu\nu}(\xi) &= \int \frac{d^4q}{(2\pi)^4} \int \frac{d^4q'}{(2\pi)^4} \int \frac{d^4p}{(2\pi)^4} \int d^4x \int d^4y \xi \\ &\quad \frac{-i}{q^2} \frac{-i}{q'^2} e^{iq \cdot z_1} e^{i(p-q) \cdot x} e^{-i(p-q') \cdot y} e^{-iq' \cdot z_2} [i\Pi_2^{\alpha\beta}(p)], \end{aligned} \quad (17)$$

the  $\mathcal{O}(\dot{\alpha})$  calculation becomes nothing more than the evaluation of  $M^{\mu\nu}(x_0 + y_0)$ .

The  $\mathcal{O}(\dot{\alpha}^2)$  and  $\mathcal{O}(\ddot{\alpha})$  terms are trickier. In order to obtain the fewest extraneous surface terms and otherwise simplify the calculation, we can perform the necessary integrations in the most symmetric fashion possible. We begin with

$$M^{\mu\nu}(x_0 y_0) = \int \frac{d^4q}{(2\pi)^4} \int \frac{d^4q'}{(2\pi)^4} \int \frac{d^4p}{(2\pi)^4} \int d^4x \int d^4y x_0 y_0 f(q_0, q'_0) e^{i(p-q) \cdot x} e^{-i(p-q') \cdot y} [i\Pi_2^{\mu\nu}(p)], \quad (18)$$

where

$$f(q_0, q'_0) = \frac{-i}{q_0^2 - \vec{q}^2} \frac{-i}{q_0'^2 - \vec{q}'^2} e^{iq_0 z_1^0} e^{-iq_0' z_2^0} e^{i\vec{q} \cdot (\vec{z}_2 - \vec{z}_1)}. \quad (19)$$

If we perform the integral over  $x$  first, it will produce a factor of  $(2\pi)^4 i\delta^3(\vec{q} - \vec{p}) \frac{1}{2} (\partial_{q_0} - \partial_{p_0}) \delta(q_0 - p_0)$ . If the integral over  $y$  is done first, the contribution would instead be  $(2\pi)^4 i\delta^3(\vec{p} - \vec{q}') \frac{1}{2} (\partial_{p_0} - \partial_{q_0'}) \delta(p_0 - q'_0)$ . Averaging the contributions from the two orders of integration, we find

$$\begin{aligned} M^{\mu\nu}(x_0 y_0) &= \frac{1}{4} \int \frac{d^4q}{(2\pi)^4} \int \frac{d^4q'}{(2\pi)^4} \left\{ \int d^4y e^{-i(q-q') \cdot y} [i y_0^2 \Pi_2^{\mu\nu}(q) f(q_0, q'_0) \right. \\ &\quad \left. - y_0 f(q_0, q'_0) \partial_{q_0} \Pi_2^{\mu\nu}(q) + y_0 \Pi_2^{\mu\nu}(q) \partial_{q_0} f(q_0, q'_0)] + \int d^4x e^{-i(q-q') \cdot x} \right. \\ &\quad \left. [i x_0^2 \Pi_2^{\mu\nu}(q') f(q_0, q'_0) + x_0 f(q_0, q'_0) \partial_{q_0'} \Pi_2^{\mu\nu}(q') - x_0 \Pi_2^{\mu\nu}(q') \partial_{q_0'} f(q_0, q'_0)] \right\}. \end{aligned} \quad (20)$$

This can be written as  $M^{\mu\nu}(x_0 y_0) = \frac{1}{4} M^{\mu\nu}(x_0^2 + y_0^2) + \frac{1}{4} \bar{M}^{\mu\nu}(x_0 y_0)$ , where

$$\begin{aligned}\bar{M}^{\mu\nu}(x_0 y_0) &= \int \frac{d^4 q}{(2\pi)^4} \int \frac{d^4 q'}{(2\pi)^4} \int d^4 y y_0 e^{-i(q_0 - q'_0) y_0} e^{i(\vec{q} - \vec{q}') \cdot \vec{y}} \{ \Pi_2^{\mu\nu}(q) \partial_{q_0} f(q_0, q'_0) \\ &\quad - \Pi_2^{\mu\nu}(q') \partial_{q'_0} f(q_0, q'_0) + f(q_0, q'_0) [\partial_{q'_0} \Pi_2^{\mu\nu}(q') - \partial_{q_0} \Pi_2^{\mu\nu}(q)] \} \end{aligned} \quad (21)$$

$$\begin{aligned} &= \frac{i}{2} \int \frac{d^4 q}{(2\pi)^4} \left\{ 2f(q_0, q_0) \partial_{q_0}^2 \Pi_2^{\mu\nu}(q) - [\partial_{q_0} f(q_0, q_0)] [\partial_{q_0} \Pi_2^{\mu\nu}(q)] \right. \\ &\quad \left. - \Pi_2^{\mu\nu}(q) \partial_{q_0}^2 f(q_0, q_0) + 4\Pi_2^{\mu\nu}(q) [\partial_{q_0} \partial_{q'_0} f(q_0, q'_0)] \Big|_{q'_0=q_0} \right\}. \end{aligned} \quad (22)$$

Similarly,

$$\begin{aligned} M^{\mu\nu}(x_0^2 + y_0^2) &= -i \int \frac{d^4 q}{(2\pi)^4} \left\{ f(q_0, q_0) \partial_{q_0}^2 \Pi_2^{\mu\nu}(q) + [\partial_{q_0} f(q_0, q_0)] [\partial_{q_0} \Pi_2^{\mu\nu}(q)] \right. \\ &\quad \left. + \Pi_2^{\mu\nu}(q) \partial_{q_0}^2 f(q_0, q_0) - 2\Pi_2^{\mu\nu}(q) [\partial_{q_0} \partial_{q'_0} f(q_0, q'_0)] \Big|_{q'_0=q_0} \right\}. \end{aligned} \quad (23)$$

Tabulating the derivatives of  $f$  that appear, we have

$$f(q_0, q_0) = \frac{-i-i}{q^2 q^2} e^{-iq \cdot (z_2 - z_1)} \quad (24)$$

$$\partial_{q_0} f(q_0, q_0) = f(q_0, q_0) \left[ -i(z_2^0 - z_1^0) - \frac{4q_0}{q^2} \right] \quad (25)$$

$$\partial_{q_0}^2 f(q_0, q_0) = f(q_0, q_0) \left[ \left( iz_2^0 - iz_1^0 + \frac{4q_0}{q^2} \right)^2 - \frac{4}{q^2} + \frac{8q_0^2}{(q^2)^2} \right] \quad (26)$$

$$[\partial_{q_0} \partial_{q'_0} f(q_0, q'_0)] \Big|_{q'_0=q_0} = f(q_0, q_0) \left( -iz_2^0 - \frac{2q_0}{q^2} \right) \left( iz_1^0 - \frac{2q_0}{q^2} \right). \quad (27)$$

Combining these gives expressions for  $M^{\mu\nu}(x_0^2 + y_0^2)$  and  $\bar{M}^{\mu\nu}(x_0 y_0)$ . By adding the total derivatives  $\partial_{q_0}^2 [\Pi_2^{\mu\nu}(q) f(q_0, q_0)]$  and  $\partial_{q_0} [f(q_0, q_0) \partial_{q_0} \Pi_2^{\mu\nu}(q)]$ , we may eliminate the  $iz^0$  terms. When this is done, the  $\mathcal{O}(\dot{\alpha}^2)$  term is more straightforward to evaluate; it takes the value

$$\langle A^\mu(z_2) A^\nu(z_1) \rangle_{\dot{\alpha}^2} = \frac{1}{8} \left( \frac{\dot{\alpha}}{\alpha_0} \right)^2 \int \frac{d^4 q}{(2\pi)^4} e^{-iq \cdot (z_2 - z_1)} \frac{-i-i}{q^2 q^2} \left[ i \frac{\partial^2}{\partial q_0^2} \Pi_2^{\mu\nu}(q) \right]. \quad (28)$$

The  $\mathcal{O}(\ddot{\alpha})$  term is more complicated. It involves both derivatives of  $\Pi_2^{\mu\nu}(q)$  and the external coordinates  $z_1$  and  $z_2$ . This kind of dependence of the external coordinates is also present in the expression for  $M(x_0 + y_0)$ ; in that case, the external coordinates just give the average time between a photon's emission at  $z_1$  and its absorption at  $z_2$ . For the second-order terms, it is not so clear what form the corresponding average should take,

except that at  $z_1^0 = z_2^0$ , the self-energy should be proportional to  $\alpha(z^0)$ , the value of the coupling constant at that instant. The ultimate expression for the  $\mathcal{O}(\ddot{\alpha})$  term is

$$\begin{aligned} \langle A^\mu(z_2)A^\nu(z_1) \rangle_{\ddot{\alpha}} &= \frac{\ddot{\alpha}}{4\alpha_0} \left[ (z_1^0)^2 + (z_2^0)^2 \right] \int \frac{d^4q}{(2\pi)^4} \frac{-i}{q^2} \frac{-i}{q^2} e^{-iq \cdot (z_2 - z_1)} [i\Pi_2^{\mu\nu}(q)] \\ &+ \frac{\ddot{\alpha}}{2\alpha_0} \int \frac{d^4q}{(2\pi)^4} \frac{-i}{q^2} \frac{-i}{q^2} e^{-iq \cdot (z_2 - z_1)} \left\{ iq_0 \frac{\partial}{\partial q_0} \left[ \frac{\Pi_2^{\mu\nu}(q)}{q^2} \right] \right\}. \end{aligned} \quad (29)$$

Together with the lower-order terms, (28) and (29) give the complete photon propagation amplitude (10).

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