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Modeling-Free Bounds on Nonrenormalizable Isotropic Lorentz and CPT Violation in QED

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Abstract

The strongest bounds on some forms of Lorentz and CPT violation come from astrophysical data, and placing such bounds may require understanding and modeling distant sources of radiation. However, it is also desirable to have bounds that do not rely on these kinds of detailed models. Bounds that do not rely on any modeling of astrophysical objects may be derived both from laboratory experiments and certain kinds of astrophysical observations. The strongest such bounds on isotropic modifications of electron, positron, and photon dispersion relations of the form $E^2 = p^2 + m^2 + \epsilon p^3$ come from data on cosmological birefringence, the absence of photon decay, and radiation from lepton beams. The bounds range in strength from the 4×10^{-13} to 6×10^{-33} (GeV)$^{-1}$ levels.

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In recent years, there has grown to be a great deal of interest in the possibilities of Lorentz and CPT symmetries not being exact in nature. There is no compelling evidence that calls these symmetries into question, but if such evidence were uncovered, that would be a discovery of profound importance and an indication that there are new fundamental physical laws still to be discovered. Responding to this possibility, there has been an explosion of experimental work testing Lorentz and CPT invariances. Different types of experiments, done at many different energy scales, are sensitive to different possible types of Lorentz violation.

Forms of Lorentz violation whose effects grow more important with increasing energy present a peculiar case. Obviously, the best bounds on these forms of Lorentz violation are likely to come from observations of extremely energetic quanta. The most energetic phenomena we can study are astrophysical. If we possess an accurate understanding of these phenomena, we may be able to place very strong constraints. However, it is not always entirely certain precisely what astrophysical interactions are responsible for the observations we make here on Earth, and this is a particular problem at extremely high energies. For example, there is controversy about the relative importances of inverse Compton scattering and π^0 decay in the production of observed TeV γ-rays.

When dealing with the possibility of especially exotic phenomena like Lorentz violation, it is desirable to have bounds that are as clean as possible. It is particularly useful to have bounds that require no inferences about the nature of distant, high-energy emission sources, since the modeling of such sources could introduce additional uncertainties. There are, however, many situations in which it is possible to place bounds without any need for such modeling. For example, bounds may be derived from observations of known particles in terrestrial laboratory environments.

Of course, the key distinction is not simply between experiments performed in a laboratory and observations of astrophysical phenomena. A more important distinction is whether inferences about the behavior of distant objects are required. Pulsar wind nebulae like the Crab nebula are probably teeming with extremely energetic particles, but drawing inferences about precisely what is happening inside them—especially at the very highest energies—is tricky. Experiments done in the laboratory are much less prone to this kind of uncertainty, but some astrophysical results are also strongly model independent. For example, the survival of photons traveling over long distances is not controversial or subject to modeling ambiguities. Bounds based simply on the observation of such photons are extremely clean, and also taking into account their polarizations will allow us to draw additional conclusions, with only very simple assumptions about the polarization at the source.

For those dimension-4, renormalizable forms of Lorentz violation that are most influential in their effects at high energies (meaning those forms of Lorentz violation that add a term $\Delta \propto p^2$ to the conventional dispersion relation $E^2 = p^2 + m^2$), there has already been a significant amount of work on both astrophysical [1, 2, 3, 4, 5] and laboratory [6, 7, 8, 9, 10, 11, 12, 13] bounds. (See [14] for a summary of the results.) The bounds based on astrophysical data are often stronger, but the laboratory bounds may be more secure. However, there has been little corresponding analysis of how higher-dimensional forms of Lorentz violation may be constrained without using astrophysical models. In this paper, we shall perform such an analysis, placing constraints on dimension-5 Lorentz violations (which add a $\Delta \propto p^3$ to the dispersion relation). Since these nonrenormalizable forms of Lorentz violation grow very rapidly in importance at high energies, the best bounds will generally come from observations of very high-energy quanta—the large energies serving to enhance ordinarily small effects.

Lorentz violation includes violations of both rotation and boost invariances. However, we shall concentrate here on forms of Lorentz violation that are purely isotropic; only boost symmetry is violated. Obviously, perfect isotropy can only exist in one preferred frame, but if the preferred frame is moving nonrelativistically relative to the Earth (as is the rest frame of the cosmic microwave background), anisotropic effects will be suppressed. We shall also only consider Lorentz violations for electrons, positrons, and photons.

However, even with these simplifying assumptions, there are multiple forms the Lorentz violation can take. In the modified dispersion relation, $E^2 = m^2 + p^2 + \epsilon p^3$, the coefficient ϵ (which parameterizes the isotropic, dimension-5 Lorentz violation) may depend on helicity and particle versus antiparticle identity. Yet not all such dependences are consistent with the Lorentz violation appearing as part of a local quantum field theory (QFT) [15], in which the unperturbed Lagrange density is $\mathcal{L}_0 = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}(i\not{\partial} - m)\psi$. For photons, a spin-independent ϵ is not possible in QFT; the only possible isotropic, dimension-5 electromagnetic bilinears are equivalent to $\Delta\mathcal{L}_\gamma = -\frac{1}{4}(\epsilon_\gamma^+ - \epsilon_\gamma^-)F^{0\mu}\partial_0\tilde{F}_{0\mu}$, where $\tilde{F}$ is the dual of F. For fermions, either a spin-independent $\Delta\mathcal{L}_{\psi1} = \frac{1}{8}(\epsilon_e^+ + \epsilon_e^-)\bar{\psi}\gamma_0\partial_0^2\psi$ or a spin-dependent $\Delta\mathcal{L}_{\psi2} = \frac{1}{8}(\epsilon_e^+ - \epsilon_e^-)\bar{\psi}\gamma_0\gamma_5\partial_0^2\psi$ is possible. Note that, in addition to having an obvious dependence on particle chirality, $\Delta\mathcal{L}_{\psi2}$ produces different dispersion relations for particles and antiparticles. All the ϵ coefficients that can exist in local QFT are odd under CPT.

However, we shall not restrict our attention solely to operators that exist in QFT. We shall consider more general modified dispersion relations determined by six parameters, $\epsilon_w^\pm$, where w denotes the species (γ, e, or $\bar{e}$) and the sign label indicates the helicity. If there is an underlying local field theory, the number of parameters is only three, since $\epsilon_\gamma^+ = -\epsilon_\gamma^-$, $\epsilon_{\bar{e}}^+ - \epsilon_{\bar{e}}^- = \epsilon_e^- - \epsilon_e^+$, and $\epsilon_{\bar{e}}^+ + \epsilon_{\bar{e}}^- = \epsilon_e^+ + \epsilon_e^-$. The particular parameters that can be nonzero in local QFT are much more interesting than those that cannot, since they descend from a fully dynamical theory. All of the bounds we shall describe will apply equally to electron and positron coefficients, and we shall denote the coefficient for a generic charged particle by ϵ_q^s.

Many of the techniques used to place bounds on dimension-4 violations of boost invariance can be adapted to constrain dimension-5 effects as well. However, some tests of boost symmetry only constrain possible anisotropies in boost behavior, and so they cannot be used to bound the ϵ coefficients. The physical effects we shall consider in this paper are (in order of decreasing sensitivity): photon birefringence, photon decay, synchrotron radiation, and vacuum Cerenkov (VC) radiation. The impact of dimension-5 Lorentz violation on certain aspects of these processes has already been studied in expressly astrophysical contexts [16, 17, 18, 19, 20, 21, 22]. However, there has been significant confusion about the rate and spin structure of the photon decay process, $\gamma \rightarrow e^- + e^+$, which is important to a number of these studies. The decay rate turns out to be closely tied to the spins of the decay products; the details of their relationship—which are also relevant to studies of other kinds of Lorentz violation—will be discussed for the first time below.

Strong bounds on dimension-5 Lorentz violation in QFT have been derived from models of the Crab nebula [22]. While these bounds are stronger than those derived here, the models involved do not do a perfect job of describing the Crab's emission spectrum; there is some tension between the populations of high-energy electrons inferred from the synchrotron and inverse Compton parts of the spectrum. Consequently, it is useful to have bounds that do not require any significant degree of astrophysical modeling. Moreover, the strength of the bounds depends a great deal on what forms of spin dependence in the Lorentz violation are considered [18]. Strong bounds on some of these dispersion relation modifications have also been derived from assumptions of naturalness [15]—either assuming that Lorentz violations in different sectors should be comparable in their orders of magnitude, because of renormalization group mixing; or assuming that isotropy should hold only in the rest frame of the cosmic microwave background.

This paper contains a number of new results and places some previously published results in context. The new elements include: an analysis of the rate of photon decay in terms of the spin states of the decay products (a topic that has previously caused some confusion, with different calculations producing different results [21]); analysis of synchrotron and vacuum Cerenkov processes for accelerator electrons and positrons; and simultaneous consideration of all possible isotropic, dimension-5 modifications of particle dispersion relations, whether or not such modifications are allowed in QFT.

The tightest constraint on any ϵ comes from the polarization of radiation from γ-ray bursts. If $\epsilon_\gamma^+ \neq \epsilon_\gamma^-$, right- and left-handed photons have different phase speeds. This leads to birefringence, a rotation of the polarization over distance. Since the rate of rotation depends on frequency, an initially linearly polarized polychromatic source would be depolarized by if the birefringence were strong enough.

There are already several bounds on $\epsilon_\gamma^+ - \epsilon_\gamma^-$ derived from γ-ray burst data. We shall quote the best of these bounds below, but they are not new to this analysis. However, they are the best limits on $\epsilon_\gamma^+ - \epsilon_\gamma^-$ and are included here for two reasons: for completeness, and because they will be used (along with the genuinely new bounds described later in this paper) to obtain the final, two-sided bounds on some of the other coefficients.

The birefringence effect has been considered in several papers. In [20], the reported polarization $\Pi_{021206} = (80 \pm 20)\%$ [23] of the γ-ray burst GRB 021206 (Π denoting the polarization fraction of the γ-rays, averaged over the observations' energy range) for photons covering a 15–2000 keV range was used as the basis for a bound on $\epsilon_\gamma^+ - \epsilon_\gamma^-$. However, the accuracy of the polarization measurement has been called strongly into question [24, 25]. In [26] two more polarized γ-ray bursts, GRB 930131 and GRB 960924, with observed polarizations of $\Pi_{930131} > 35\%$ and $\Pi_{960924} > 50\%$ were considered. These three sources are fairly distant, each with redshift $z \gtrsim 0.1$. Most recently, the polarization of GRB 041219a (a source with a pseudo-redshift $z \gtrsim 0.2$) over an energy range of 100–350 keV [27], has been used to derive another bound [28].

With the simplest assumption about the initial polarization of a γ-ray burst, that the radiation began completely linearly polarized, the survival of the observed degrees of polarization over cosmological distances indicates that $(\epsilon_\gamma^+ - \epsilon_\gamma^-) \lesssim [d(\omega_2^2 - \omega_1^2)]^{-1}$, where d is the distance to the source, while ω_1 and ω_2 are the lower and upper limits of the observed γ-ray frequency range. That the inequality is satisfied ensures that the plane of polarization does not change by an $\mathcal{O}(1)$ angle over the energy range involved. The best published bound resulting from this kind of analysis comes from [26]; in the notation of that paper, the bound is

$$\left|k^{(5)}_{(V)00}\right| = \left|-\sqrt{\pi}(\epsilon_\gamma^+ - \epsilon_\gamma^-)\right| < 10^{-32}\,(\mathrm{GeV})^{-1}. \tag{1}$$

The details of the analysis are found in [20, 26]; we omit further discussion of how they were derived, because no modifications to the analysis are necessary for the present purposes.

The order of magnitude of the bound (1) is fairly robust, because the only assumptions it requires are related to the sources' distances (for which lower limits are available) and the initial polarization of the radiation. Barring some exceedingly unlikely conspiracy of circumstance, in which the radiation began with a frequency-dependent plane of polarization, such that, after birefringence, the polarizations all lined up at the time of observation, a strong initial linear polarization represents the most natural hypothesis.

Moreover, even if there were reason to doubt the accuracy of the bound (1) derived in [26], the constraint is so strong that, even if the constraint on $(\epsilon_\gamma^+ - \epsilon_\gamma^-)$ were actually determined to be 10^{11} times weaker, none of the later results in this paper would be affected. In fact, if the GRB 021206 results were accurate, the bound would actually be about a factor of 7 stronger. Moreover, the observed polarization of synchrotron x-rays from the Crab nebula indicates a bound on $|\epsilon_\gamma^+ - \epsilon_\gamma^-|$ at the 10^{-27} $(\mathrm{GeV})^{-1}$ level or better [29]. Henceforth, we shall simply neglect photon birefringence, because it is so much more strongly constrained than any of the other effects to be considered. Including the possibility of birefringence at the allowed level (1) would not affect any of our other numerical results. So we shall only consider the possibility of a single $\epsilon_\gamma = \epsilon_\gamma^+ = \epsilon_\gamma^-$. (Note that in the QFT framework, in which $\epsilon_\gamma^- = -\epsilon_\gamma^+$, this means neglecting Lorentz violation in the photon sector altogether; there can be no dimension-5 photon Lorentz violation in QFT without birefringence.)

There are no other existing bounds that can so simply be incorporated into this analysis without modification. The next process to consider is photon decay, $\gamma \rightarrow e^- + e^+$, which is ordinarily forbidden by boost invariance and momentum conservation. However, it has already been noted that the existence of Lorentz violation may make this decay allowed for sufficiently energetic photons. Most previous discussions have focused primarily on the threshold for the process; however, understanding the process's rate—especially how the rate depends on the spin states of the decay products—is also crucial to understanding the bounds that follow from the observed absence of the decay. Some correct bounds have already been published [20], but there has been significant confusion about the spin structure of the decay.

It is not necessary that the photon decay threshold correspond to a symmetric configuration in which the daughter particles are emitted with equal energies. In fact, even if $\epsilon_e^s = \epsilon_{\bar{e}}^{s'}$, the threshold (at which all the particles involved are moving collinearly) may be asymmetric; but nonetheless we shall focus on the symmetric threshold, which is simpler. In some regimes, including the possibility of an asymmetric threshold produces a modest improvement in the numerical bounds that can be set. However, there is no such improvement possible in the final constraints to be derived in this paper.

To get the most comprehensive bounds possible from a study of photon decay, we must look at the spin structure of the reaction and look at energies above the threshold, so that the particles involved need not be perfectly collinear. We shall discuss configurations near the symmetric threshold, but the asymmetric case is similar. If the e^- and e^+ each make an angle θ with the original photon's momentum $\vec{p}$, energy-momentum conservation requires

$$\epsilon_\gamma - \frac{1}{4}(\epsilon_e^s + \epsilon_{\bar{e}}^{s'}) \sec^3\theta = \frac{4m^2 + p^2 \tan^2\theta}{p^3}. \tag{2}$$

If $\theta \lesssim \frac{m}{p} \ll 1$, the right-hand side of (2) is only modestly greater than $\frac{4m^2}{p^3}$. The characteristic range of angles θ for decays only slightly above threshold is therefore $\theta \lesssim \frac{m}{p}$. For these angles, $\sec\theta \approx 1$, and so the left-hand side of (2)—which is the combination of Lorentz violation coefficients upon which the decay rate depends—reduces to the θ-independent sum $\epsilon_\gamma - \frac{1}{4}\epsilon_e^s - \frac{1}{4}\epsilon_{\bar{e}}^{s'}$.

Since the initial photon is a $J = 1$ state, the final spins must be parallel, oriented the same way along the photon's propagation axis. At threshold, where all the particles are collinear, this means the e^+ and e^- must have the same helicity as the photon. However, if the particles each veer off at an angle θ, each may be found in a helicity state opposite that of the initial photon with probability $\sin^2\frac{\theta}{2}$. The rate for the decay process in which both final particles have the same helicity as the photon is $\Gamma_0 \sim e^2 p \cos^4\frac{\theta}{2}$. For each particle with helicity opposite that of the photon, one factor of $\cos^2\frac{\theta}{2}$ changes to $\sin^2\frac{\theta}{2}$. These results are consistent with the very different energy dependences inferred for the decay rates by Jacobson, Liberati, and Mattingly in [19] and [21]. They identified $\Gamma \sim e^2 p$ if the final spin states are arbitrary and the e^- and e^+ dispersion relations are independent of spin; contrariwise, if the daughter particles have opposite spins but still the same dispersion relation, $\Gamma \sim e^2 \epsilon p^2$. The difference derives from the trigonometric factors in the decay rates. If the photon has helicity s, each daughter particle with helicity $-s$ slows the decay by a factor of $\tan^2\frac{\theta}{2}$; and near threshold, $\tan^2\frac{\theta}{2} \sim \frac{m^2}{p^2} \sim \epsilon p$.

If the slowest possible decay rate, $\Gamma_0 \tan^4 \frac{\theta}{2}$, is large compared with the reciprocal of a photon's time of flight, the reaction $\gamma \rightarrow e^- + e^+$ has time to occur for any of the four possible outgoing helicity states. For extremely energetic γ-rays $\Gamma_0 \sim 10^{25} \left(\frac{p}{10\,\mathrm{TeV}}\right)$. For a 50 TeV photon, $\frac{m}{p} = 10^{-8}$; this makes the typical angle for a near-threshold decay $\theta \sim 10^{-8}$. The decay lifetime of such a 50 TeV photon is therefore $16\Gamma_0^{-1}\theta^{-4} \sim 10^8$ s. For a source such as the Crab nebula, lying approximately 2 kpc away and emitting photons above 50 TeV, the time of flight is 2×10^{11} s; such photons have ample time to decay, provided the condition, $4\epsilon_\gamma - \epsilon_e^s - \epsilon_{\bar{e}}^{s'} > \frac{20m^2}{p^3} = 4 \times 10^{-20}\ (\mathrm{GeV})^{-1}$ is met. Conversely, the stability of the 50 TeV photons indicates that

$$4\epsilon_\gamma - \epsilon_e^s - \epsilon_{\bar{e}}^{s'} < 4 \times 10^{-20}\,(\mathrm{GeV})^{-1}; \tag{3}$$

our analysis of the decay products' spin behavior enables us to set this bound for all combinations of s and s'.

The derivation of the result (3) required no inferences about how the TeV γ-rays were produced at their source; it only matters that the photons traversed the full distance from the Crab nebula to Earth. This independence of source characteristics is a general feature of bounds inferred from the absence of photon decay. The bounds (3) are the strongest ones available on the fermionic ϵ_q^s coefficients that are independent of a model of the source, but they have the disadvantage of being strictly one-sided.

It is possible to place two-sided bounds using observations of the synchrotron losses at the Large Electron-Positron Collider (LEP). This has already been done for the dimension-4 coefficients in the standard model extension [10, 30]. Yet in a theory that lacks Lorentz invariance and is not necessarily based on QFT, it may not be immediately obvious what form the electromagnetic coupling to relativistic charged matter should take. We shall assume that the modified theory has a gauge-invariant single-particle Hamiltonian H for each charge, and this H is (almost) minimally coupled to the electromagnetic field. Then H is a function of the electromagnetic field (almost) solely via the combination $\vec{p} - e\vec{A}$. It follows that the electromagnetic field couples principally to the four-velocity v^μ. If the electromagnetic sector were completely conventional, then given a known particle trajectory, we could determine the emitted radiation by standard methods. Moreover, even if the photon dispersion relation is also Lorentz violating, ϵ_γ has minimal impact on the synchrotron process, because the individual photons that are emitted have energies much smaller than do the orbiting charges.

So neglecting ϵ_γ, the power radiated by a synchrotron electron is $P = \frac{e^2a^2}{6\pi m^2}\gamma^4$, where a is the magnitude of the acceleration and $\gamma = (1 - v^2)^{-1/2}$ is the usual Lorentz factor. For ultrarelativistic particles, γ is a very rapidly increasing function of v. A charged particle's velocity in the presence of the Lorentz violation is

$$v = \frac{\partial E}{\partial p} \approx 1 - \frac{m^2}{2p^2} + \epsilon_q^s p \tag{4}$$

at high energies. This makes the radiated power

$$P = P_0(1 + 4\gamma^2\epsilon_q^s p), \tag{5}$$

where P_0 is the radiation rate in the conventional case, and ϵ_q^s is whichever coefficient is relevant for the charged particle in question.

A precise determination of the beam energy E_b at LEP was important, since the machine was used to make precision W and Z boson mass measurements. The primary method of determining E_b involved measuring the magnetic field profile with nuclear magnetic resonance and studying the beam trajectory. The field strength and the trajectory through the bending magnets were known to high precision, and together those quantities determined E_b.

The synchrotron radiation rate was used as part of a redundant measurement of the beam energies. This redundant measurement looked at the synchrotron tune, Q_s, which is the ratio of the synchrotron oscillation frequency to the orbital frequency [31]. Synchrotron oscillations occur when beam particles' energies differ from the nominal value E_b. Less energetic particles revolve around smaller orbits and thus travel between the accelerating radio frequency (RF) cavities more quickly. They arrive at the cavities earlier in the RF cycle and receive larger than nominal energy boosts, pushing their energies back towards E_b. The opposite effect occurs for particles with greater than the nominal energy.

Q_s is proportional to $(P/E_b)^2$, and since E_b is known from other measurements, synchrotron tune measurements can be turned around to give measurements of the e^- and e^+ radiation rates (and hence velocities). The extent to which the synchrotron tune energy measurement agreed with the primary energy calibration determines how small ϵ must be. A conservative 2σ bound on the fractional deviation of P from its conventionally expected value is $\left|\frac{\Delta P}{P}\right| < 6 \times 10^{-4}$, for measurements performed at the Z pole energy of $E_b = 91$ GeV.

The LEP beams were maintained in transverse polarization states, except sometimes near interaction points. (An initially pure helicity state would precess many times during a single orbit.) Since the average beam helicity was zero, the synchrotron radiation rate would depend only on the average $\frac{1}{2}(\epsilon_q^+ + \epsilon_q^-)$ for each beam. Moreover, the synchrotron oscillations could not be observed separately for the counterpropagating e^- and e^+ beams; the oscillations were only observed for the beam system as a whole. With nominally equal e^- and e^+ beam currents, any effect to be observed would have been averaged over the particle types. (The existence of differing Q_s values for e^- and e^+ is another phenomenon that does not occur in QFT.) The agreement of the synchrotron tune measurement with expectations therefore yields the constraint

$$|\epsilon_e^+ + \epsilon_e^- + \epsilon_{\bar{e}}^+ + \epsilon_{\bar{e}}^-| < \frac{|\Delta P/P|}{\gamma^2 p} = 2.2 \times 10^{-16}\,(\text{GeV})^{-1}, \tag{6}$$

using $p \approx E_b$. This is weaker than the photon survival bound, but it is two sided.

In order to have two-sided constraints on all the coefficients, one more process needs to be considered. Photon decay would occur if photon energies grew more rapidly with momentum than electron and positron energies; conversely, if the fermion energies were more rapidly increasing functions of p, the VC radiation process $e^{\pm} \rightarrow e^{\pm} + \gamma$ could occur. However, the data from LEP showed no evidence of this process up to $E_b = 104.5$ GeV. Above threshold, such radiation would be quite rapid, so any combination of Lorentz violation coefficients that would allow the radiation to occur can be reliably ruled out.

There are two type of VC radiation: hard and soft. The threshold for emission of a soft photon is basically insensitive to ϵ_γ, since the photon energy is so small. The threshold in this case is $p_T = (m^2/2\epsilon_q^s)^{1/3}$ (which is the momentum at which a charge's speed v exceeds 1), and the absence of soft emission by a charged particle of momentum $p \approx E_b$ implies

$$\epsilon_q^s < \frac{m^2}{2p^3} = 1.5 \times 10^{-13}. \tag{7}$$

This bound applies to both helicity states of both electrons and positrons.

In a hard VC process, in which a charge of momentum p retains a portion xp of its momentum while emitting a collinear photon of momentum $(1-x)p$, the energy-momentum conservation condition reduces to $\epsilon_\gamma - \left(\frac{1+x}{1-x}\right)\epsilon_q^s = -\frac{1}{x(1-x)}\frac{m^2}{p^3}$. (This assumes there is no change in the charge's helicity state, but including helicity-changing reactions only produces additional, more complicated inequalities that do not improve the ultimate numerical results.)

The absence of VC radiation from charges of momentum p implies the threshold condition cannot be satisfied for any relevant x, and thus

$$\epsilon_\gamma - \left(\frac{1+x}{1-x}\right)\epsilon_q^s > -\frac{1}{x(1-x)}\frac{m^2}{p^3} \tag{8}$$

for all $\frac{m}{p} \ll x < 1$. The lower limit on x comes from the requirement that the charge remain ultrarelativistic after radiating, or $xp \gg m$. The inequality (8) is needed to place a lower bound on ϵ_γ. The bound becomes more sensitive to ϵ_γ (as opposed to ϵ_q^s) for smaller x, but the magnitude of the right-hand side of (8) grows rapidly as $x \rightarrow 0$. (Note that the soft threshold corresponds to $x \approx 1$.) Although (8) represents a continuous set of inequalities, the key features are captured by the $x = \frac{1}{2}$ version,

$$\epsilon_\gamma - 3\epsilon_q^s > -\frac{4m^2}{p^3} = -3 \times 10^{-13}\,(\mathrm{GeV})^{-1}, \tag{9}$$

again using $p \approx E_b$.

Separate bounds on important combinations of ϵ coefficients can be extracted from (1), (3), (6), (7) and (8) by linear programming. The results are given in table 1, which contains entries with each of of the characteristic strengths seen in the raw bounds. (A bound at a given level typically depends on a raw bound at the same level, as well as the

Coefficient	Maximum	Minimum	QFT Maximum	QFT Minimum
$\epsilon_{\gamma}^{+}-\epsilon_{\gamma}^{-}$	6×10^{-33}	-6×10^{-33}	6×10^{-33}	-6×10^{-33}
$\epsilon_{\gamma}^{+}+\epsilon_{\gamma}^{-}$	6×10^{-17}	-6×10^{-13}	–	–
ϵ_{e}^{+}, ϵ_{e}^{-}, $\epsilon_{\bar{e}}^{+}$, and $\epsilon_{\bar{e}}^{-}$	10^{-13}	-3×10^{-13}	1.1×10^{-16}	-2×10^{-20}
$\epsilon_{e}^{+}+\epsilon_{e}^{-}$	2×10^{-13}	-2×10^{-13}	1.1×10^{-16}	-4×10^{-20}
$\epsilon_{e}^{+}-\epsilon_{e}^{-}$	4×10^{-13}	-4×10^{-13}	1.1×10^{-16}	-1.1×10^{-16}

Table 1: Maximum and minimum allowed values of electron, positron, and photon ϵ coefficients, in units of $(\mathrm{GeV})^{-1}$. The fourth and fifth columns give the bounds if only the coefficients that can exist in local QFT are considered. The dashes in those columns denote combinations that are identically zero in the QFT framework.

stronger raw bounds discussed previous to it in this paper. A result at the 10^{-16} level, for example, relies on the birefringence, photon decay, and synchrotron data.) Note that, since all the raw results treat electrons and positrons, of positive and negative helicity, equivalently, the constraints on ϵ_{e}^{+}, ϵ_{e}^{-}, $\epsilon_{\bar{e}}^{+}$, and $\epsilon_{\bar{e}}^{-}$ are all identical. Table 1 also includes bounds for the QFT case, in which there are only three independent coefficients. In that case, the bounds are generally better, and the weakest raw constraints (coming from the absence of VC radiation at LEP) are not needed.

For several of the ϵ coefficients, the results given in Table 1 are the first modeling-free constraints to be placed on them: none of these bounds require any understanding of the behavior of high-energy astrophysical sources, as opposed to many earlier bounds, which relied on our having an accurate picture of how the photons we see coming from particular sources are produced. So far, these results are limited to isotropic forms of Lorentz violation, but many of the same techniques discussed here could also be used to study anisotropic, dimension-5 Lorentz- and CPT-violating operators (which are extremely numerous). Moreover, the results are not limited to those forms of Lorentz violation that can exist in QFT; purely phenomenalistic modifications of particle dispersion relations—although they are less plausible—were also found to be subject to strong bounds.

References

[1] F. W. Stecker, S. L. Glashow, Astropart. Phys. **16**, 97 (2001).

[2] V. A. Kostelecký, M. Mewes, Phys. Rev. Lett. **87**, 251304 (2001).

[3] V. A. Kostelecký, M. Mewes, Phys. Rev. Lett. **97**, 140401 (2006).

[4] B. Altschul, Phys. Rev. D **74**, 083003 (2006).

[5] B. Altschul, Phys. Rev. D **75**, 041301 (R) (2007).

[6] H. Müller, *et al.*, Phys. Rev. Lett. **99**, 050401 (2007).

[7] B. R. Heckel, E. G. Adelberger, C. E. Cramer, T. S. Cook, S. Schlamminger, U. Schmidt, Phys. Rev. D **78**, 092006 (2008).

[8] M. A. Hohensee, R. Lehnert, D. F. Phillips, R. L. Walsworth, Phys. Rev. Lett. **102**, 170402 (2009).

[9] Ch. Eisele, A. Yu. Nevsky, S. Schiller, Phys. Rev. Lett. **103**, 090401 (2009).

[10] B. Altschul, Phys. Rev. D **80**, 091901 (R) (2009).

[11] S. Herrmann, *et al.*, Phys. Rev. D 80, 105011 (2009).

[12] B. Altschul, Phys. Rev. D **81**, 041701 (R) (2010).

[13] J.-P. Bocquet, *et al.*, Phys. Rev. Lett. **104**, 241601 (2010).

[14] V. A. Kostelecký, N. Russell, arXiv:0801.0287.

[15] R. C. Myers, M. Pospelov, Phys. Rev. Lett. **90**, 211601 (2003).

[16] G. Amelino-Camelia, J. Ellis, N. E. Mavromatos, D. V. Nanopoulos, S. Sarka, Nature **393**, 763 (1998).

[17] B. E. Schaefer, Phys. Rev. Lett. **82**, 4964 (1999).

[18] T. Jacobson, S. Liberati, D. Mattingly, Nature **424**, 1019 (2003).

[19] T. Jacobson, S. Liberati, D. Mattingly, Phys. Rev. D **67**, 124011 (2003).

[20] T. Jacobson, S. Liberati, D. Mattingly, F. W. Stecker, Phys. Rev. Lett. **93**, 021101 (2004).

[21] T. Jacobson, S. Liberati, D. Mattingly, Ann. Phys. **321**, 150 (2006).

[22] L. Maccione, S. Liberati, A. Celotti, J. G. Kirk, JCAP **10**, 013 (2007).

[23] W. Coburn, S. E. Boggs, Nature **423**, 415 (2003).

[24] R. E. Rutledge, D. B. Fox, Mon. Not. Roy. Astr. Soc. **350**, 1288 (2004).

[25] C. Wigger, W. Hajdas, K. Arzner, M. Güdel, A. Zehnder, Astrophys. J. **613**, 1088 (2004).

[26] V. A. Kostelecký, M. Mewes, Phys. Rev. D **80**, 015020 (2009).

[27] S. McGlynn, *et al.*, Atstron. Astrophys. **466**, 895 (2007).

[28] F. W. Stecker, arXiv:1102.2784.

[29] L. Maccione, S. Liberati, A. Celotti, J. G. Kirk, P. Ubertini, Phys. Rev. D **78**, 103003 (2008).

[30] B. Altschul, Phys. Rev. D **72**, 085003 (2005).

[31] R. Assmann, *et al.*, Eur. Phys. J. C **39**, 253 (2005).