10-2011

Consequences of Neutrino Lorentz Violation For Leptonic Meson Decays

Brett David Altschul
altschul@mailbox.sc.edu

Publication Info
Volume 84, Issue 9, 2011.

arXiv:1110.2123

Consequences of Neutrino Lorentz Violation For Leptonic Meson Decays

Brett Altschul[1]

Department of Physics and Astronomy
University of South Carolina
Columbia, SC 29208

Abstract

If the observation by OPERA of apparently superluminal neutrinos is correct, the Lagrangian for second-generation leptons must break Lorentz invariance. We calculate the effects of an energy-independent change in the neutrino speed on another observable, the charged pion decay rate. The rate decreases by a factor $\left[1-\frac{3}{1-m_\mu^2/m_\pi^2}\left(\langle v_\nu\rangle-1\right)\right]$, where $\langle v_\nu\rangle$ is the (directionally averaged) neutrino speed in the pion's rest frame. This provides a completely independent experimental observable that is sensitive to the same forms of Lorentz violation as a neutrino time of flight measurement.

[1]baltschu@physics.sc.edu

The recent announcement by the OPERA experiment that muon neutrinos may be traveling faster than light with speeds of $\sim 1 + (2.5 \times 10^{-5})$ [1] has attracted a tremendous amount of attention. Whether this experimental conclusion will withstand further scrutiny remains to be seen. However, the mere possibility that superluminal travel might occur has already produced an explosion of interest in theories that do not precisely obey Lorentz symmetry.

Before the intriguing neutrino results were reported, there was already a thriving community of physicists working on such Lorentz-violating theories. This community includes both experimenters and theorists. One of the most important theoretical developments in the field was an effective quantum field theory capable of describing all possible forms of Lorentz violation that might occur at low energies with standard model fields [2, 3]. This theory is known as the standard model extension (SME). Both the renormalizability [4, 5, 6] and stability [7] of the SME have been studied. The SME is now the standard framework for parameterizing the results of experimental Lorentz tests. Many different systems and phenomena may be used for such tests, and up-to-date information about most of the resulting constraints may be found in [8].

In this paper, we shall use the SME formalism to examine one very important consequence that the existence of superluminal neutrinos could have. If the neutrino energy-momentum relation is modified, the rates of scattering and decay processes involving these neutrinos will necessarily be modified. Of particular interest are meson decays. The phase space available to the decay products in a process such as $\pi^+ \rightarrow \mu^+ + \nu_\mu$ depends crucially on the neutrino velocity. This provides a completely new observable, which is nonetheless sensitive to some of the same parameters as the OPERA experiment. Precision decay measurements can thus be directly compared with the results of a neutrino time of flight experiment.

We shall calculate how an energy-independent change in the muon neutrino speed affects the decay rate of a charged pion. The choice of the pion is partially just for definiteness. However, we shall also find that the Lorentz-violating modifications to the decay rate are most pronounced for the lightest parent particles. While this makes pions the ideal species to study, our calculations naturally apply to any particle that decays via a W boson primarily to a muon and its associated neutrino.

Moreover, we expect that similar phenomena would occur in more complicated decay processes, such as $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$. In fact, calculations of Cerenkov-like decay processes such as $\nu_\mu \rightarrow \nu_\mu + e^+ + e^-$ have already raised serious questions about the OPERA result. Such processes, which could be allowed for superluminal neutrinos, would have major effects on the neutrino beam in transit—effects which have seemingly not been observed [9, 10].

In our calculations, we shall assume that the only Lorentz violation is in the lepton sector. The structures and interactions of the pion itself and the W that mediates the decay are exactly as in the standard model. This means that the entire calculation may be performed, as usual, in the rest frame of the pion. The kinematic factors associated with the motion of the parent particle are unchanged from the conventional case.

However, it is not possible in the SME formalism to have Lorentz violation for muon neutrinos but simultaneously to have a muon sector that is free of Lorentz violation. The lepton Lagrange density that contains a spin- and energy-independent modification to the neutrino velocity is

$$\mathcal{L} = i\bar{L}(\gamma^\mu + c^{\nu\mu}\gamma_\nu)D_\mu L + i\bar{R}(\gamma^\mu + c^{\nu\mu}\gamma_\nu)D_\mu R + \text{mass terms}, \tag{1}$$

where L and R are the left- and right-chiral lepton multiplets,

$$L = \left[\begin{array}{c} \nu_L \\ \ell_L \end{array} \right], \quad R = \left[\ell_R\right]. \tag{2}$$

and D_μ is the covariant derivative, containing the electromagnetic and weak gauge interactions. $SU(2)_L$ gauge invariance requires that the Lorentz violation coefficients $c^{\nu\mu}$ be the same for the charged lepton and its neutrino. (Actually, it is possible to have a different set of coefficients for the right-chiral charged lepton field, but those coefficients are irrelevant in reactions involving charged current weak interactions.) The c coefficients together form a traceless background tensor, which imbues spacetime with a preferred frame structure. While the c coefficients control violations both of isotropy and of boost invariance, we shall mostly be concerned here with the isotropic element of the tensor c_{00}. Because Lorentz violation is a small effect, we may neglect all terms beyond first order in c.

The choice of an energy-independent modification of the lepton velocities is motivated by two important factors. First, the OPERA data does not show evidence of any energy dependence in the neutrino speed. Second, the observed effect is rather large, suggesting that the modification of the velocity is not an effect suppressed by any powers of a large energy scale. This requires that the Lorentz-violating operator responsible for the effect be renormalizable, leading to the choice of c.

There are very few direct constraints on the c coefficients for second-generation leptons. The absence of the photon decay process $\gamma \rightarrow \mu^+ + \mu^-$ up to TeV photon energies constrains the possibility that the leptons might have limiting velocities less than the speed of light [11], but there are few constraints on the possibility that muons and their corresponding neutrinos might move faster than light. Superluminal muons would emit vacuum Cerenkov radiation [12], but to constrain the muon c coefficients at the level seen by OPERA would require careful observation of ~ 15 GeV muons.

The strongest bounds on other forms of muon Lorentz violation are based on $g_\mu - 2$ measurements [13]. This data could also be used to constrain the muon c, potentially at the 10^{-6} level. The behavior of a fermion in an external field, with a form of photon-sector Lorentz violation that is essentially equivalent to c, has been calculated. The result is that the spin couples to both the electric and magnetic fields in a potentially anisotropic fashion [14]. A full analysis of this phenomenon would need to include the effects of the substantial $\vec{E}$ fields present in muon $g_\mu - 2$ experiments as well as the possible influences of other SME parameters.

On the other hand, there are some constraints on c-type Lorentz violation in the neutrino sector. If the c coefficients are nondiagonal in flavor space, they will lead to neutrino oscillations, with a different characteristic energy dependence from the oscillations generated by nondiagonal neutrino masses [15]. This possibility has been constrained, and Lorentz violation as the principal cause of observed neutrino oscillations has already been ruled out [16]. Oscillation measurements can also constrain the differences between the flavor-diagonal c parameters for different species, since if the different neutrino flavors had substantially different speeds, the wave packet overlap necessary for oscillations could be destroyed. With only isotropic c coefficients, the oscillation data require a very high degree of flavor independence (at the 10^{-19} level), which would make the OPERA data incompatible with numerous measurements of electron-sector Lorentz violation [17, 18]. However, the number of SME parameters that can influence neutrino oscillations is very large, and it is not clear whether this conclusion applies in more general models. Finally, there is the famous time of flight measurement made on neutrinos from SN 1987A [19, 20], which (assuming the interpretation of the data was correct) constrains the c coefficients for electron neutrinos at the 10^{-9} level.

Constraints on the SME parameters are conventionally expressed in a sun-centered reference frame, with coordinates (X, Y, Z, T), with the Z-direction parallel to the Earth's rotation axis [21]. The OPERA neutrino timing data was collected over a long period of time, effectively averaging over the range of neutrino directions $\hat{v}$ that are possible as the Earth rotates. As shown below in (11), the speed of a massless particle in the presence of the c parameters is $1 - c_{00} - c_{(0j)}\hat{v}_j - c_{jk}\hat{v}_j\hat{v}_k$, where $c_{(0j)}$ is the symmetrized combination $c_{0j} + c_{j0}$. This makes the average speed of the OPERA neutrinos $1 - c_{00} - c_{(0Z)}\cos\theta - \frac{1}{2}(c_{XX} + c_{YY})\sin^2\theta - c_{ZZ}\cos^2\theta$, where θ is the angle between the CERN-to-Gran Sasso direction and the Z-axis, for which $\cos\theta \approx -0.41$.

We now turn to the calculation of the pion decay rate. This rate is not directly sensitive to all the c parameters that might have affected the average neutrino speed. Because, in the absence of Lorentz violation, the distribution of the decay products is isotropic, the modifications to the total decay rate can depend only on the isotropic $c_{00} = c_{jj}$, as measured in the decaying particle's rest frame. We may therefore treat c as if it were a diagonal tensor, with elements $[c_{00}, \frac{1}{3}c_{00}, \frac{1}{3}c_{00}, \frac{1}{3}c_{00}]$ in the center of mass frame. If this isotropic c is the sole contributor to the neutrinos' faster-than-light travel, $v_{\nu_\mu} = 1 - \frac{4}{3}c_{00}$. However, the value of c_{00} will depend on the pion momentum, and other components of c will affect the differences in the decay rates of pions with different velocities.

Although the ultimate form taken by the modified decay rate is quite simple, the calculations leading up to it are somewhat tricky. Methods for calculating cross sections and decay rates in the SME context have been worked out [22]. The formulas for physical observables can be put in forms quite similar to those that appear conventionally. However, in these formulas, both the matrix element $\mathcal{M}$ for the decay and the kinematics are modified from their conventional forms. The kinematic modifications, which are related to changes in the phase space available to the external particles, are often more important than the changes to the matrix element.

The matrix element $\mathcal{M}$ can be determined from the same tree-level Feynman diagram that governs conventional pion decay. Since there is assumed to be no Lorentz violation in the meson or W boson sectors, only those parts of the diagram involving the final muon and neutrino need to be considered. This means that $\mathcal{M}$ may be modified in two ways. The first is via the replacement of γ^μ at the muon-neutrino-gauge boson vertex with $\gamma^\mu + c^{\mu\nu}\gamma_\nu$. The free index μ is ultimately to be contracted with the total momentum vector p_μ, which has only a time component. This amounts to a rescaling of the matrix element simply by a factor of $(1 + c_{00})$.

The second modification of the matrix element comes from the modified spinors, which obey the modified Dirac equation

$$\left[(1 + c_{00})\gamma_0 E - \left(1 - \frac{1}{3}c_{00}\right)\gamma_j p_j - m\right] u(p) = 0. \tag{3}$$

The spinor solutions are standard, except for an evident rescaling of E and p_j. Therefore, if they are normalized to obey $\bar{u}^{s'}(p)u^s(p) = 2E\delta_{ss'}$ [and analogously for $v(p)$], the closure relations needed for calculating cross sections are $\sum_s u^s(p)\bar{u}^s(p) = \gamma^0 E + \left(1 - \frac{4}{3}c_{00}\right)\gamma_j p_j + m$ and $\sum_s v^s(p)\bar{v}^s(p) = \gamma^0 E + \left(1 - \frac{4}{3}c_{00}\right)\gamma_j p_j - m$. This makes the matrix element squared proportional to

$$\begin{aligned} |\mathcal{M}|^2 &\propto (1 + c_{00})^2 \sum_{s,s'} \operatorname{tr}\left\{ v_\nu^{s'}(-\vec{p}_\mu)\gamma_0(1-\gamma_5)u_\mu^s(\vec{p}_\mu)\bar{u}_\mu^s(\vec{p}_\mu)(1+\gamma_5)\gamma_0 v_\nu^s(-\vec{p}_\mu)\right\} && (4) \\ &= (1 + c_{00})^2 \operatorname{tr}\left\{\left[\gamma_0 E_\nu - \left(1 - \frac{4}{3}c_{00}\right)\gamma_j(-p_\mu)_j\right]\gamma_0(1-\gamma_5)\right. \\ &\quad \left.\times\left[\gamma_0 E_\mu - \left(1 - \frac{4}{3}c_{00}\right)\gamma_j(p_\mu)_j\right](1+\gamma_5)\gamma_0\right\} && (5) \\ &= 2\left(1 + \frac{2}{3}c_{00}\right)p_\mu\left[E_\mu - \left(1 - \frac{4}{3}c_{00}\right)p_\mu\right]. && (6) \end{aligned}$$

In these formulas, p_μ and p_ν denote the magnitudes of the muon and neutrino three-momenta. (Despite the indices, these symbols do not represent four-vectors.) E_μ and E_ν are the corresponding energies. The derivation of (6) used the facts that the neutrino momentum $\vec{p}_\nu$ is opposite to the muon momentum $\vec{p}_\mu$ and that $E_\nu = \left(1 - \frac{4}{3}c_{00}\right)p_\nu$, which is an obvious consequence of the massless Dirac equation.

Further simplification requires analysis of the more complicated dispersion relation for the muon. The energy of a fermion, including the leading order effects of c, is

$$E = \sqrt{(m^2 + p_j p_j)(1 - 2c_{00}) - 2c_{jk}p_j p_k - 2c_{0j}p_j\sqrt{m^2 + p_j p_j}}, \tag{7}$$

which reduces to $E=\sqrt{(1-2c_{00})m^2+(1-\frac{8}{3}c_{00})p_jp_j}$ when only the isotropic c_{00} is considered. Energy and momentum conservation then dictate that the muon momentum is $p_\mu=\frac{1-\xi^2}{2}m_\pi\left[1+\frac{2(2+\xi^2)}{3(1-\xi^2)}c_{00}\right]$, where $\xi=\frac{m_\mu}{m_\pi}$ is the muon-pion mass ratio—the main dimensionless parameter determining the kinematics of the Lorentz-invariant case. The energies of the decay products are $E_\nu=\frac{1-\xi^2}{2}m_\pi\left(1+\frac{2\xi^2}{1-\xi^2}c_{00}\right)$ and $E_\mu=\frac{1+\xi^2}{2}m_\pi\left(1-\frac{2\xi^2}{1+\xi^2}c_{00}\right)$. The quantity $E_\mu-\left(1-\frac{4}{3}c_{00}\right)p_\mu$ is then $\xi^2m_\pi(1-2c_{00})$, so the ratio of the matrix element squared in the presence of c_{00} to its value at $c_{00}=0$ is

$$\left|\frac{\mathcal{M}}{\mathcal{M}_0}\right|^2=\left(1-\frac{4}{3}c_{00}\right)\left(\frac{p_\mu}{p_{\mu0}}\right), \tag{8}$$

with the subscript 0 referring to the Lorentz-invariant values of particular quantities.

As mentioned, the Lorentz violation changes not just the matrix element for this process, but also the phase space factors in the decay rate formula, since c_{00} affects the free propagation of the daughter particles. The phase space integral appearing in the decay rate is

$$\int d\Pi = \int\frac{d^3p_\mu}{(2\pi)^3}\frac{1}{2E_\mu}\frac{d^3p_\nu}{(2\pi)^3}\frac{1}{2E_\nu}(2\pi)^4\delta^4(p_\pi-p_\mu-p_\nu) \tag{9}$$

$$= \frac{1}{16\pi^2}\int d^3p_\mu\frac{1}{E_\mu(p_\mu)E_\nu(p_\mu)}\delta(m_\pi-E_\mu-E_\nu). \tag{10}$$

After eliminating p_ν, it remains to evaluate the integral over the muon momentum, $d^3p_\mu=p_\mu^2\,dp_\mu\,d\Omega_\mu$. The angular integration can be done by symmetry, yielding 4π; then the δ-function eliminates the dp_μ integration, fixing p_μ at its physical value. The energy δ-function also introduces a factor $\left|\frac{d}{dp_\mu}(m_\pi-E_\mu-E_\nu)\right|^{-1}=|v_\mu+v_\nu|^{-1}$, where the velocities are evaluated at the fixed momentum p_μ.

The neutrino velocity is simple, $v_\nu=1-\frac{4}{3}c_{00}$, but the muon case is trickier. The general expression for a massive particle's group velocity is

$$v_j=\frac{\pi_j}{\sqrt{m^2+\vec{\pi}^2}}-c_{00}\frac{\pi_j}{\sqrt{m^2+\vec{\pi}^2}}-2c_{jk}\frac{\pi_k}{\sqrt{m^2+\vec{\pi}^2}}+c_{kl}\frac{\pi_j\pi_k\pi_l}{(m^2+\vec{\pi}^2)^{3/2}}-2c_{0j}, \tag{11}$$

which, in the isotropic limit, reduces to

$$v_\mu=\frac{p_\mu}{\sqrt{m_\mu^2+p_\mu^2}}\left\{1+\left[-\frac{5}{3}+\frac{p_\mu^3}{3(m_\mu^2+p_\mu^2)^{3/2}}\right]c_{00}\right\}. \tag{12}$$

Evaluated at the real muon momentum, this becomes

$$v_\mu=\frac{1-\xi^2}{1+\xi^2}\left\{1+\left[\frac{2(2+\xi^2)}{3(1+\xi^2)}\left(\frac{1+\xi^2}{1-\xi^2}-\frac{1-\xi^2}{1+\xi^2}\right)-\frac{5}{3}+\frac{1}{3}\left(\frac{1-\xi^2}{1+\xi^2}\right)^2\right]c_{00}\right\}, \tag{13}$$

which makes $v_\mu + v_\nu$ the considerably simpler

$$v_\mu + v_\nu = \frac{2}{1+\xi^2}\left[1 - \frac{2(2-\xi^2)}{3(1+\xi^2)}c_{00}\right]. \qquad (14)$$

It is now possible to assemble all the pieces of the cross section. Since there is no Lorentz violation in the meson sector, the kinematic factors associated with the initial state are not modified. The remaining effects give

$$\begin{aligned} \frac{\Gamma}{\Gamma_0} &= \left(\frac{p_\mu}{p_{\mu 0}}\right)^2 \left(\frac{E_{\mu 0}}{E_\mu}\right)\left(\frac{E_{\nu 0}}{E_\nu}\right)\left(\frac{v_{\mu 0}+v_{\nu 0}}{v_\mu + v_\nu}\right)\left|\frac{\mathcal{M}}{\mathcal{M}_0}\right|^2 & (15) \\ &= 1 + \frac{4}{1-\xi^2}c_{00}. & (16) \end{aligned}$$

The first factor on the right-hand-side of (15) arises from the p_μ^2 in d^3p_μ.

The final formula (16) is quite simple, and it depends on the same single quantity $1-\xi^2$ that typically characterizes leptonic decay rates. [The simplicity of the final result also suggests that there may be a more convenient way of organizing this calculation, in which complicated rational functions of ξ^2, such as those in (13), need not appear.] Using the physical value of $\xi = 0.757$ for pion decay, $\Gamma = \Gamma_0(1+9.4c_{00})$. For heavier mesons, the dependence on c_{00} will be smaller. Note that c_{00} can also be expressed as $-\frac{3}{4}(\langle v_\nu\rangle - 1)$, where $\langle v_\nu\rangle$ is the neutrino speed (averaged over all directions $\hat{v}$) in the rest frame of the decaying pion.

The modification to Γ given in (16) depends on the mass of the parent particle. Most of the c dependence in Γ comes from the kinematic factors rather than from the dynamical matrix element. The fact that the kinematics are more strongly affected when ξ is close to 1 is relatively straightforward to understand. At small momenta, a small addition to the velocity produces a much larger fractional change in the available phase space than would a similar term at large momenta. This tends to make the decay rate more sensitive to c when the energy available to the decay products is small.

Since superluminal neutrinos correspond to negative values of c_{00}, superluminal behavior will tend to reduce the rate at which a pion will decay. This is natural, since the decay products (both the neutrinos and the corresponding muons) have dispersion relations for which the energy increases more rapidly than usual as a function of the three-momentum. Since the energy available in the decay is fixed to be m_π, the negative value of c_{00} decreases the momentum that the daughter particles can carry and consequently also decreases the available phase space.

There are two straightforward kinds of comparisons that might be made, in order to test the hypothesis of a nonzero muon neutrino c. The first involves a comparison of the decay rates for pions (or other mesons) moving with different velocities. The c_{00} in (16) is the coefficient in the decaying particle's rest frame. In terms of the coefficients c_{TT} and c_{TJ} in the sun-centered frame, the c_{00} is

$$c_{00} = \gamma_\pi^2\left[c_{TT} + c_{(TJ)}(v_\pi)_J + c_{JK}(v_\pi)_J(v_\pi)_K\right], \qquad (17)$$

where $\vec{v}_\pi$ is the meson velocity in the sun-centered frame, and γ_π is the corresponding Lorentz factor $\gamma = (1 - v_\pi^2)^{-1/2}$.

Generally, the size of the Lorentz violation effect grows as the square of the meson energy. (This novel phenomenon is, of course, in addition to the conventional time dilation effect that lengthens the apparent lifetime of the meson, as measured by a stationary observer, by γ_π.) Although the absolute magnitude of the pion decay rate depends on difficult-to-compute hadronic effects, comparisons of the decay rates for mesons with different boosts can be used to search for a nonzero c_{TT}. Most obviously, measurements of highly boosted decays may be compared with the reference value of the decay lifetime for a meson at rest. Moreover, studying the decays of mesons moving in different directions adds potential sensitivity to the other coefficients $c_{(TJ)}$ and c_{JK}. Even if, at a given source facility, the mesons produced are traveling in relatively tight beam, oriented along a fixed compass direction, the rotation of the Earth will make it possible to sample multiple directions $\hat{v}$. The current best value of the charged pion lifetime is accurate at the 2×10^{-4} level in measurements with nonrelativistic pions. Experiments of comparable precision with relativistic pions with $\gamma_\pi \gtrsim 3$ would be sensitive to anomalies in the neutrino dispersion relation at the level reported by OPERA. At higher energies, experiments can be less precise by a factor of γ_π^2 and still be capable of confirming the novel effects.

The other way in which this effect might be straightforwardly tested involves comparisons of multiple meson decay modes. Light charged mesons are generally much more likely to decay into muon and neutrino pairs than electron and neutrino pairs. However, the latter decay does occur, and the ratio of the two decay rates can be predicted quite precisely in the standard model. Tests of lepton universality, which measure the relative sizes of the branching ratios for decays such as $\pi^+ \to \mu^+ + \nu_\mu$ and $\pi^+ \to e^+ + \nu_e$, are therefore sensitive to the muon c coefficients.

Of course, the $\pi^+ \to e^+ + \nu_e$ decay rate is sensitive to the c coefficients in the electron sector. However, the coefficients for electrons (and, therefore, their associated neutrinos) are much more tightly constrained than the coefficients for the second-generation leptons. The reason is that electrons are stable and extremely numerous. They play a crucial role in many high-energy phenomena, and their energy-momentum relation has been mapped out very carefully, even up to PeV energies. This can be done by studying the radiation emitted by highly boosted electrons; such electromagnetic emissions are sensitive probes of both electron velocities and energies. By looking at the ways various processes would be modified by the inclusion of electron c coefficients and comparing these predictions to observations of energetic electrons—both in astrophysical sources [23] and at particle accelerators [24]—all the relevant coefficients may be constrained at the 10^{-14} level or better.

Finally, it is worth noting that for sufficiently large boosts, the decay $\pi^+ \to \mu^+ + \nu_\mu$ may be impossible. If $E_\pi > \sqrt{(m_\pi^2 - m_\mu^2)/2[-c_{TT} - c_{(TJ)}(v_\pi)_J - c_{JK}(v_\pi)_J(v_\pi)_K]}$, the greater-than-normal growth of the muon and neutrino energies makes the decay energetically impossible [9, 25]. The pion energy $\gamma_\pi m_\pi$ is insufficient to produce the highly boosted decay products. In contrast, in a purely phenomenalistic model, which violates gauge invariance by giving only the neutrino a modified dispersion relation, there is no threshold above which the pion is stable [26].

If neutrinos do move faster than light, this will have many consequences in particle physics. Some of these consequences are rather counter-intuitive. For example, without also violating electroweak gauge invariance, it is impossible to have an energy-independent form of Lorentz violation for neutrinos without having it for the charged leptons as well. A complete dynamical calculation of the pion decay rate shows that it is fairly sensitive to this kind of Lorentz violation, and this provides a completely independent avenue for testing the fascinating OPERA result.

References

[1] T. Adam, *et al.* (OPERA collaboration), arXiv:1109.4897.

[2] D. Colladay, V. A. Kostelecký, Phys. Rev. D **55**, 6760 (1997).

[3] D. Colladay, V. A. Kostelecký, Phys. Rev. D **58**, 116002 (1998).

[4] V. A. Kostelecký, C. D. Lane, A. G. M. Pickering, Phys. Rev. D **65**, 056006 (2002).

[5] D. Colladay, P. McDonald, Phys. Rev. D **75**, 105002 (2007).

[6] A. Ferrero, B. Altschul, Phys. Rev. D, **84**, 065030 (2011).

[7] V. A. Kostelecký, R. Lehnert, Phys. Rev. D **63**, 065008 (2001).

[8] V. A. Kostelecký, N. Russell, Rev. Mod. Phys. **83**, 11 (2011).

[9] X.-J. Bi, P.-F. Yin, Z.-H. Yu, Q. Yuan, arXiv:1109.6667.

[10] A. G. Cohen, S. L. Glashow, Phys. Rev. Lett **107**, 181803 (2011).

[11] B. Altschul, Astropart. Phys. **28**, 380 (2007).

[12] B. Altschul, Phys. Rev. Lett. **98**, 041603 (2007).

[13] G. W. Bennett, *et al.* (Muon $g-2$ collaboration), Phys. Rev. Lett. **100**, 091602 (2008).

[14] C. D. Carone, M. Sher, M. Vanderhaeghen, Phys. Rev. D **74**, 077901 (2006).

[15] V. A. Kostelecký, M. Mewes, Phys. Rev. D **69**, 016005 (2004).

[16] V. Barger, J. Liao, D. Marfatia, K. Whisnant, Phys. Rev. D **84**, 056014 (2011).

[17] G. F. Giudice, S. Sibiryakov, A. Strumia, arXiv:1109.5682.

[18] A. Drago, I. Masina, G. Pagliara, R. Tripiccione, arXiv:1109.5917.

[19] M. J. Longo, Phys. Rev. D **36**, 3276 (1987).

[20] L. Stodolsky, Phys. Lett. B **201**, 353 (1988).

[21] R. Bluhm, V. A. Kostelecký, C. D. Lane, N. Russell, Phys. Rev. D **68**, 125008 (2003).

[22] D. Colladay, V. A. Kostelecký, Phys. Lett. B **511** 209 (2001).

[23] B. Altschul, Phys. Rev. Lett. **96**, 201101 (2006).

[24] B. Altschul, Phys. Rev. D **82**, 016002 (2010).

[25] L. Gonzalez-Mestres, arXiv:1109.6630.

[26] R. Cowsik, S. Nussinov, U. Sarkar, arXiv:1110.0241.