2-13-2013

Lorentz and CPT Violation in Scalar-Mediated Potentials

Brett David Altschul
altschul@mailbox.sc.edu

Publication Info

Published in *Physical Review D*, Volume 87, Issue 4, 2013.

arXiv:1211.6614

Lorentz and CPT Violation in Scalar-Mediated Potentials

Brett Altschul[1]

Department of Physics and Astronomy
University of South Carolina
Columbia, SC 29208

Abstract

In Lorentz- and CPT-violating effective field theories involving scalar and spinor fields, there exist forms of Lorentz violation that modify only the scalar-spinor Yukawa interaction vertices. These affect low-energy fermion and antifermion scattering processes through modifications to the nonrelativistic Yukawa potentials. The modified potentials involve novel combinations of momentum, spin, and Lorentz-violating background tensors.

[1]baltschu@physics.sc.edu

1 Introduction

In recent years, a significant amount of attention has been paid to the possibility that the laws of physics at the most fundamental level may not respect Lorentz and CPT symmetries exactly. While there is thus far no compelling experimental reason to believe that Lorentz or CPT invariances are not exact, many candidate theories of quantum gravity suggest the possibility of such symmetry violations, at least in certain regimes. The possibilities for symmetry breaking include spontaneous breaking in string theory [1, 2] and elsewhere [3], mechanisms in loop quantum gravity [4, 5] and non-commutative geometry [6, 7], Lorentz violation through spacetime-varying couplings [8, 9], and anomalous breaking of Lorentz and CPT symmetries [10].

Because any confirmed discovery of Lorentz violation would be a sure sign of new physics—with a fundamentally different structure from anything previously observed—this subject remains quite interesting and an active area of both experimental and theoretical research. Most theoretical work is performed within the context of effective quantum field theory. There is an effective field theory known as the standard model extension (SME) that contains all possible translation-invariant but Lorentz-violating operators that may be constructed out of the fields of the standard model. (Generalizations to include additional fields are straightforward.) Each Lorentz-violating operator that appears in the SME Lagrangian is parameterized by a small background tensor [11, 12]. If the Lorentz violation arises from spontaneous symmetry breaking, these background tensors are essentially the vacuum expectation values of tensor-valued fields. Moreover, because the existence of CPT violation in a stable, unitary quantum field theory implies that there must also be Lorentz violation [13], the SME is also the most general effective field theory describing CPT violation.

The most frequently considered subset of the SME is the minimal SME, which contains only gauge-invariant, local, superficially renormalizable forms of Lorentz violation. The minimal SME has become the standard framework used for parameterizing the results of experimental Lorentz tests. Recent searches for Lorentz violation have included studies of matter-antimatter asymmetries for trapped charged particles [14, 15, 16] and bound state systems [17, 18], measurements of muon properties [19, 20], analyses of the behavior of spin-polarized matter [21], frequency standard comparisons [22, 23, 24, 25], Michelson-Morley experiments with cryogenic resonators [26, 27, 28, 29, 30], Doppler effect measurements [31, 32], measurements of neutral meson oscillations [33, 34, 35, 36, 37, 38], polarization measurements on the light from cosmological sources [39, 40, 41, 42], high-energy astrophysical tests [43, 44, 45, 46, 47], precision tests of gravity [48, 49], and others. The results of these experiments set constraints on the various SME coefficients, and up-to-date information about most of these constraints may be found in [50].

The least studied areas of the SME are those that involve scalar fields. There has been a good deal of theoretical investigation into the behavior of scalars in the (Lorentz-invariant) standard model—both the fundamental Higgs and composite pseudoscalar mesons in the hadronic sector. However, little attention has been paid to scalars by theorists studying Lorentz violation. For example, the one-loop renormalization of the SME Higgs sector has not yet been studied systematically. Although the one-loop renormalization of the Abelian [51], non-Abelian [52], and chiral [53] gauge theories with spinor matter that make up parts of the SME were completed some time ago, only recently have the corresponding scalar field theories including Yukawa interactions [54] received similar treatments. Yet the study of the renormalization of the scalar sector is still not complete; gauge theories with scalar matter and theories with spontaneous gauge symmetry breaking have not been adequately examined.

This paper discusses how low-energy scalar-mediated interactions between fermions and antifermions may be affected by Lorentz violation. At nonrelativistic energies, these interactions are described by modified Yukawa potentials. The changes to the Yukawa potential induced by Lorentz violation in the pure scalar propagation sector were previously considered in [55]. However, as emphasized in [54], the Yukawa sector contains Lorentz-violating operators that modify the scalar-spinor interactions. These operators can have a much more intricate structure than those considered in [55].

The nonrelativistic limit is most relevant in low-energy hadronic physics. Understanding symmetry breaking in few-baryon systems is an important topic. Much of the research in this area has focused on parity (P) violation, since this is the most strongly broken spacetime symmetry in the standard model. Weak P violation in processes that are normally dominated by the strong interaction has become a very interesting area of research, with experimental data coming from a number of different nuclear systems [56]. The same kinds of hadronic experiments, particularly those involving precision measurements of neutron spin rotations, may be useful as tests of Lorentz and CPT invariances.

P-violating observables in systems of interacting nucleons have traditionally been analyzed in terms of interparticle potentials, as in the Desplanques-Donoghue-Holstein (DDH) model [57]. The DDH model uses a collection of one-boson exchange potentials, generated by the exchange of both scalar and vector mesons. More recent analyses have used effective field theories, particularly a pionless [58] effective theory that should be reliable at energies less than m_π^2/M_N.

The state of the art for P violation in field theory has moved beyond two-nucleon systems. Three-nucleon observables, such as spin rotations in neutron-deuteron scattering, have been analyzed using the pionless effective field theory [59, 60], as well as hybrid methods that use effective field theory to describe the symmetry violation along with modified wave functions derived from the P-invariant portions of the interparticle potentials [61, 62]. These field theory methods have also been extended to deal with time reversal invariance violation [63], which is much weaker than P violation in the standard model.

However, this paper will only consider two-body potentials. The rich additional structure that arises when violations of isotropy and boost invariance are allowed may make three-body physics more complicated (in contrast to the Lorentz-invariant pionless theory, which has no leading-order three-body interactions [64]). The potential theory provides a starting point for the analysis of observables that break Lorentz invariance. Our approach to deriving the relevant potentials will be somewhat similar to the one used to determine the low-energy forms of P violation in [65]—starting with the possible relativistic forms of symmetry violation, then reducing to a nonrelativistic theory and observing any redundancies.

The outline of this paper is as follows. Section 2 describes the structure of the Lorentz-violating operators that appear in the scalar sector of a Lorentz-violating effective field theory. Then section 3 examines how Lorentz-violating modifications to scalar-spinor couplings affect the Yukawa potentials between spin-$\frac{1}{2}$ particles. The results are summarized in section 4, along with some additional remarks placing this work in context.

2 Scalar Sector Lorentz Violation

The Lagrange density for the minimal SME contains local operators that can be constructed out of the standard model's scalar, spinor, and gauge fields. To maintain superficial renormalizability, only gauge invariant operators with mass dimensions of up to 4 are included. In all cases that have been checked explicitly, these conditions are indeed sufficient to make the theories renormalizable at one-loop order. (As already noted, the Yukawa theory discussed in this paper is among the cases for which such a check has been made.)

For a single species of Dirac fermion, the minimal SME Lagrange density is

$$\mathcal{L}_f = \bar{\psi}(i\Gamma^\mu\partial_\mu - M)\psi, \tag{1}$$

where

$$\Gamma^\mu = \gamma^\mu + \Gamma_1^\mu = \gamma^\mu + c^{\nu\mu}\gamma_\nu + d^{\nu\mu}\gamma_5\gamma_\nu + e^\mu + if^\mu\gamma_5 + \frac{1}{2}g^{\lambda\nu\mu}\sigma_{\lambda\nu} \tag{2}$$

and

$$M = m + im_5\gamma_5 + M_1 = m + im_5\gamma_5 + a^\mu\gamma_\mu + b^\mu\gamma_5\gamma_\mu + \frac{1}{2}H^{\mu\nu}\sigma_{\mu\nu}. \tag{3}$$

These are the only operators satisfying the listed conditions that can exist in a purely fermionic theory. The Γ coefficients are dimensionless, while the M coefficients have dimension (mass)^1. However, some of the coefficients appearing in Γ and M are more interesting than others. Several, such as m_5, a, and f, may be eliminated from the theory by a redefinition of the fermion field [66, 67].

The species of interest in low-energy hadronic physics are composites formed from the fundamental quark and gluon fields. However, there are still Lorentz violation coefficients for the composite species; they are linear combinations of the coefficients for the fundamental fields. Many of the more precise bounds on SME coefficients are actually constraints on the composite coefficients for protons and neutrons.

Many of the terms present in (1–3) violate CPT as well as Lorentz symmetry. Of the coefficients that cannot be eliminated by field redefinitions, those with odd numbers of Lorentz indices are also odd under CPT. However, the coefficients that have even numbers of indices are CPT invariant. It is thus possible to break Lorentz symmetry but leave CPT intact (although not vice versa). The full discrete symmetry properties of the minimal SME operators are discussed in [51, 67].

To the fermionic theory may be appended one or more boson fields—of either scalar or gauged vector types. This also introduces new possible forms of Lorentz violation. However, there is a fundamental difference between the possibilities in scalar- and gauge-mediated interactions. In a gauge theory, whatever renormalizable Lorentz violation exists in the free fermion sector completely determines the Lorentz violation present at the boson-fermion vertex. The same quantity Γ^μ appears in both the fermion propagator and the vertex, because of gauge invariance. However, the situation is quite different in a Yukawa theory. With no additional condition analogous to gauge invariance, there is a completely independent set of Lorentz-violating operators that can appear in the fermion-scalar vertex.

With the addition of a scalar field ϕ, the most general Lorentz-violating Lagrange density that does not lead to spontaneous symmetry breaking becomes

$$\mathcal{L} = \bar{\psi}(i\Gamma^\mu\partial_\mu - M)\psi + \frac{1}{2}(\partial^\mu\phi)(\partial_\mu\phi) + \frac{1}{2}K^{\mu\nu}(\partial_\nu\phi)(\partial_\mu\phi) - \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4 - \bar{\psi}G\psi\phi. \quad (4)$$

The symmetric tensor $K^{\mu\nu} = K^{\nu\mu}$ represents the only kind of Lorentz violation that can be introduced in the pure bosonic sector with a real scalar field. Much more intricate in structure is the operator G appearing in the Yukawa vertex term. G has essentially the same structure as the M term in the pure scalar sector,

$$G = g + ig'\gamma_5 + G_1 = g + ig'\gamma_5 + I^\mu\gamma_\mu + J^\mu\gamma_5\gamma_\mu + \frac{1}{2}L^{\mu\nu}\sigma_{\mu\nu}. \quad (5)$$

The terms g and g' are the usual scalar and pseudoscalar Yukawa couplings, while the other terms are Lorentz violating. All the coefficients contained in G are dimensionless. The tensor term $L^{\mu\nu}$ is naturally antisymmetric. The discrete symmetries of the operators that make up G are similar to the symmetries of the corresponding operators contained in M. If the ϕ field is a true scalar, the symmetries are exactly the same; for a pseudoscalar field, the parity and time reversal behaviors are opposite between M and G, while the charge conjugation properties are still the same. Ultimately, the I and J terms violate CPT as well as Lorentz symmetry, while L is CPT invariant.

While the fact that this rich structure of operators could exist in the spinor-scalar coupling term was noted as part of the original formulation of the SME, very little attention has been paid to the G terms. There has been essentially no calculations of their phenomenalistic effects, and only recently [54] have the effects of these terms on the one-loop renormalization of the SME been considered.

3 Modified Yukawa Potentials

Both the K and G terms will affect the Yukawa potentials for interacting fermions and antifermions, because both of them appear in the four-point correlation functions that describe two-particle scattering. The purpose of this section will be to evaluate the inter-particle potentials that are associated with this scattering. Since the K term was already discussed in [55], the focus here will be on the effects of the I, J, and L terms that, together with the Lorentz-invariant g and g' terms, comprise G.

In discussing the modified Yukawa potential, we shall only consider Lorentz-violating effects that are linear in the SME coefficients. Since Lorentz violation is known to be a very small phenomenon physically, higher-order effects should be minuscule. A similar leading-order analysis of electromagnetic potentials was conducted in [68].

Lorentz violation in the pure fermion sector (and in the fermion-gauge interaction sector) is relatively well constrained, at least for the first generation fermions that make up the stable constituents of everyday matter. For this reason, we shall neglect the Γ_1 and M_1 terms (even though some small nonzero Γ_1 and M_1 terms could be generated from G_1 by radiative corrections [54]).

However, if the coefficients of the operators involved were not too small, the forms of Lorentz violation described by Γ_1 and M_1 would affect fermionic scattering in a significant way. These pure fermion sector terms would affect both the amputated matrix element for the one-boson exchange process that dominates low-energy scattering and the dispersion relations for the external particles, which would in turn affect the kinematics of a reaction. In fact, the changes to scattering and decay rates due to changes in particle velocities and available phase space may be as large as or larger than the changes arising from Lorentz violation in the invariant matrix element itself [69, 70, 71].

3.1 Spinor Products

When Γ_1 and M_1 are neglected, it is possible to use standard external fermion and antifermion spinor states for the calculation of a matrix element. Since the behaviors of the G operators depend in nontrivial ways on the spins of the external particles, it is simplest to perform the matrix element calculation using explicit spinor eigenstates. Using the Dirac-Pauli basis for the Dirac matrices and a relativistic normalization convention, the Dirac spinor $u_s(p)$ (corresponding to momentum p and spin s) is

$$u_s(p) = \sqrt{\frac{2E(E+m)}{2m}} \left[\begin{array}{c} \xi_s \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\xi_s \end{array} \right], \tag{6}$$

where, ξ_s is a two-component spinor. In the nonrelativistic limit, this becomes

$$u_s(p) = \sqrt{2m} \left[\begin{array}{c} \xi_s \\ \frac{\vec{\sigma}\cdot\vec{p}}{2m}\xi_s \end{array} \right]. \tag{7}$$

Using the explicit spinors, it is possible to calculate the fermion bilinears that appear in a scattering amplitude. In particular, if the external particles are nonrelativistic, so that terms with more than a single power of p/m may be neglected,

$$\begin{aligned}
\bar{u}_{s'}(p')Gu_s(p) &= 2m\left[\xi_{s'}^{\dagger}, -\xi_{s'}^{\dagger}\frac{\vec{\sigma}\cdot\vec{p}\,'}{2m}\right](g+i\gamma_5 g'+G_1)\left[\begin{array}{c}\xi_s\\ \frac{\vec{\sigma}\cdot\vec{p}}{2m}\xi_s\end{array}\right] && (8)\\
&= 2m(g+I_0)\xi_{s'}^{\dagger}\xi_s + ig'(p_j-p_j')\xi_{s'}^{\dagger}\sigma_j\xi_s + 2m(J_j+\epsilon_{jkl}L_{kl})\xi_{s'}^{\dagger}\sigma_j\xi_s \\
&\quad -I_j\left[(p_j+p_j')\xi_{s'}^{\dagger}\xi_s - i\epsilon_{jkl}(p_k-p_k')\xi_{s'}^{\dagger}\sigma_l\xi_s\right] - J_0(p_j+p_j')\xi_{s'}^{\dagger}\sigma_j\xi_s \\
&\quad -L_{0j}\left[i(p_j-p_j')\xi_{s'}^{\dagger}\xi_s - \epsilon_{jkl}(p_k+p_k')\xi_{s'}^{\dagger}\sigma_l\xi_s\right]. && (9)
\end{aligned}$$

If the two fermions involved are of different species, direct scattering is the only possible channel. We shall henceforth assume that the particles involved in a scattering event are indeed distinguishable. However, in the scattering of identical particles, the usual method of replacing the scattering amplitude $f(\theta,\phi)$ with $f(\theta,\phi)-f(\pi-\theta,\pi+\phi)$ will give the correct result.

For antiparticle scattering, we shall similarly assume that the annihilation scattering channel is not available, and only a single diagram contributes to the potential. We may also take advantage of the fact that $v_s(p)=\gamma_5 u_{-s}(p)$, where the subscript $-s$ on the spinor $u(p)$ indicates a spinor with spin projections that are opposite those of $u_s(p)$. Then we have that

$$\begin{aligned}
\bar{v}_s(p)Gv_{s'}(p') &= -\bar{u}_{-s}(p)\gamma_5 G\gamma_5 u_{-s'}(p') && (10)\\
&= -2m(g-I_0)\xi_{-s}^{\dagger}\xi_{-s'} + ig'(p_j-p_j')\xi_{-s}^{\dagger}\sigma_j\xi_{-s'} + 2m(J_j-\epsilon_{jkl}L_{kl})\xi_{-s}^{\dagger}\sigma_j\xi_{-s'} \\
&\quad -I_j\left[(p_j+p_j')\xi_{-s}^{\dagger}\xi_{-s'} + i\epsilon_{jkl}(p_k-p_k')\xi_{-s}^{\dagger}\sigma_l\xi_{-s'}\right] - J_0(p_j+p_j')\xi_{-s}^{\dagger}\sigma_j\xi_{-s'} \\
&\quad -L_{0j}\left[i(p_j-p_j')\xi_{-s}^{\dagger}\xi_{-s'} + \epsilon_{jkl}(p_k+p_k')\xi_{-s}^{\dagger}\sigma_l\xi_{-s'}\right]. && (11)
\end{aligned}$$

The remaining products of Pauli spinors can be simplified further. The inner product $\xi_{-s}^{\dagger}\xi_{-s'}$ simply equals $\xi_{s'}^{\dagger}\xi_s=\delta_{ss'}$. However, because of the spin reversal present in ξ_{-s}, the matrix element $\xi_{-s}^{\dagger}\sigma_j\xi_{-s'}$ is equal to $-\xi_{s'}^{\dagger}\sigma_j\xi_s$.

Note that present in (9) [and (11)] are almost all the possible vector structures that can be constructed at first order in the momenta. There are three independent three-vector operators that may be constructed: the total momenta along the incoming and outgoing lines from a vertex ($\vec{p}+\vec{p}\,'$), the momentum transfer ($\vec{q}=\vec{p}-\vec{p}\,'$), and the spin operator ($\vec{\sigma}$). Each of these three may form a dot product with an isotropy-breaking background vector; either of the momentum observables may form a dot product with the spin; or there may be a triple product with a background vector, the spin, and one of the momenta. The only possible structures that are missing are contractions of the momenta and spin with symmetric, traceless three-tensors; however, these structures cannot exist because there is no such symmetric, traceless tensor that can be constructed at first order in G.

The terms involving background three-vectors manifestly break isotropy. Moreover, Lorentz boost invariance normally prevents the appearance of the $\vec{p}+\vec{p}\,'$ terms. The average velocity $\vec{v}_{av}=\frac{1}{2}(\vec{v}+\vec{v}\,')$ (which is, of course, the passing velocity $\vec{v}_{av}\approx\vec{v}\approx\vec{v}\,'$ in a glancing collision) is measured in the specific laboratory frame in which the calculation has been performed. The dot product of this velocity with the spin is not invariant under nonrelativistic Galilean transformations, and this signifies the failure of Lorentz boost symmetry. In contrast, the difference $\vec{v}-\vec{v}\,'$, which appears in the Lorentz-invariant theory in conjunction with the pseudoscalar g' term, transforms covariantly under Galielan transformations

3.2 Fermion Potential

For the scattering of two nonidentical fermions, with the exchange of a single scalar boson between them, the matrix element is

$$i\mathcal{M}=\bar{u}^{a}_{s'_a}(p'_a)G^{a}u^{a}_{s_a}(p_a)\frac{-i}{q^2-\mu^2+i\epsilon}\bar{u}^{b}_{s'_b}(p'_b)G^{b}u^{b}_{s_b}(p_b). \tag{12}$$

The indices a and b denote the identities of the species involved. However, for simplicity, we shall assume that there is only Lorentz violation for one species of particle (the one labeled a). Including Lorentz violation for both is a straightforward generalization. We shall consider both scalar and pseudoscalar couplings for the second particle, however.

For nonrelativistic scattering, the interaction may be described using a potential, such that

$$V(\vec{r})=\int\frac{d^3q}{(2\pi)^3}e^{i\vec{q}\cdot\vec{r}}i\mathcal{M}(\vec{q}), \tag{13}$$

in the limit where $q_0=0$. The integrand involves the usual scalar Yukawa amplitude proportional to $1/(\vec{q}^{\,2}+\mu^2)$, as well as terms with additional factors of q_j. The spatial extent of the interactions is therefore determined by the Yukawa potential function and its derivative,

$$\begin{aligned} f(\vec{r}) &= -\frac{e^{-\mu r}}{4\pi r} && (14)\\ g_j(\vec{r}) &= \partial_j f(\vec{r})=\frac{e^{-\mu r}}{4\pi r^2}\left(\mu+\frac{1}{r}\right)x_j. && (15)\end{aligned}$$

Although higher powers of $\vec{q}$ (and thus additional spatial derivatives) will be largely neglected in this paper, we notice that the next order term is

$$h_{jk}(\vec{r})=\partial_j\partial_k f(r)=\frac{e^{-\mu r}}{4\pi r^2}\left[\left(\mu+\frac{1}{r}\right)\delta_{jk}-\left(\frac{\mu^2}{r}+\frac{3\mu}{r^2}+\frac{3}{r^3}\right)x_jx_k\right]+\frac{1}{3}\delta^3(\vec{r})\delta_{jk}. \tag{16}$$

Spatial potentials with this shape already appear in the Lorentz-invariant theory, coming from terms with pseudoscalar g' couplings at both vertices. It is also possible to have a first-order Lorentz-violating potential with this shape, if one vertex involves a G_1 term and the other vertex a g'.

The function f contributes to that part of the potential that comes from vertex terms in which q does not appear. These generate a potential

$$V_f(\vec{r}) = [\tilde{g} - I_j(v^a_{av})_j - J_0(v^a_{av})_j\sigma^a_j + \tilde{J}_j\sigma^a_j + \epsilon_{jkl}\tilde{L}_j(v^a_{av})_k\sigma^a_l]g_b f(\vec{r}). \tag{17}$$

The terms in square in brackets in (17) comes from the vertex with the a species, while the g_b term comes from the b vertex. The species labels are omitted on the Lorentz violation coefficients, since there is assumed to be no Lorentz violation in the vertex with the b species. The scalar $\tilde{g}$ and three-vectors $\tilde{J}_j$ and $\tilde{L}_j$ marked with tildes denote the combinations

$$\begin{aligned} \tilde{g} &= g + I_0 && (18)\\ \tilde{J}_j &= J_j + \epsilon_{jkl}L_{kl} && (19)\\ \tilde{L}_j &= L_{0j} = -L_{j0}, && (20) \end{aligned}$$

which are the combinations that are observable in experiments with nonrelativistic fermions conducted in a single Lorentz frame. They are analogous to the tilde-marked coefficients defined in other sectors of the SME, although the ones defined in (18–20) are defined to be dimensionless, unlike the tilde coefficients in other sectors, which most typically have units of mass. These combinations of indistinguishable terms exist because the leading order fermionic matrix elements of even Dirac operators (which involve only the large components of the Dirac spinors) are unchanged when multiplied by γ_0. So if $\mathcal{E}$ is an even operator, the nonrelativistic matrix elements of $\mathcal{E}$ and $\gamma_0\mathcal{E}$ are identical. Note however, that different combinations exist for antifermions, because γ_0 is equivalent to -1 in the corresponding matrix elements for antiparticles.

Despite the existence of these degenerate combinations, the structure of the physical matrix elements of G is still quite a bit richer than the corresponding structure for matrix elements of M. Because M appears in the bilinear propagation Lagrangian for fermion species, its physical matrix elements always involved particles with identical incoming and outgoing momenta. So a matrix element such as $\bar{u}_{s'}(p)Mu_s(p)$ lacks any terms that depend on $\vec{p}-\vec{p}\,'$.

The part of the interaction potential with the more complicated $g_j(\vec{r})$ spatial dependence arises from those terms in the scattering amplitude with just a single factor of q_j. This factor can appear at either vertex. At the a vertex, either a Lorentz-violating term or the g'_a coupling may be responsible; or at the b vertex, there may be a coupling g'_b. Taken together, these terms generate a potential

$$V_g(\vec{r}) \;=\; \left\{\frac{1}{2m_a}\left[g'_a\sigma^a_j - \tilde{L}_j - \epsilon_{jkl}I_k\sigma^a_l\right]g_b - \frac{1}{2m_b}\left[\tilde{g} - I_k(v^a_{av})_k\right.\right.$$

$$-J_0(v^a_{av})_k\sigma^a_k + \tilde{J}_k\sigma^a_k + \epsilon_{kln}\tilde{L}_k(v^a_{av})_l\sigma^a_n\Big]\, g'_b\sigma^b_j\Big\}\, g_j(\vec{r}\,). \tag{21}$$

For completeness, we may mention the potential term that arises when a factor of q appears at both vertices. This potential has the spatial shape $h_{jk}(\vec{r}\,)$, so that

$$V_h(\vec{r}\,) = \frac{1}{4m_a m_b}\left[g'_a\sigma^a_j - \tilde{L}_j - \epsilon_{jln}I_l\sigma^a_n\right]\left(g'_b\sigma^b_k\right) h_{jk}(\vec{r}\,). \tag{22}$$

However, this is not a complete description of the potential at this order. Other terms with multiple factors of p/m have also been neglected (for example, in the normalization of the Dirac spinors).

3.3 Antifermion Potential

The one-boson scalar exchange also generates potentials between antifermions and other particles. We shall now look explicitly at the case where the species-a particle is an antifermion, while the species-b particle remains a fermion. The two species are still different, so there is no annihilation scattering. In this case, the potential is derived from a matrix element similar to (12):

$$i\mathcal{M} = -\bar{v}^a_{s_a}(p_a)G^a v^a_{s'_a}(p'_a)\frac{-i}{q^2-\mu^2+i\epsilon}\bar{u}^b_{s'_b}(p'_b)G^b u^b_{s_b}(p_b), \tag{23}$$

with the usual overall minus sign for antiparticle scattering, coming from the fields' anticommutation. This cancels the overall minus sign from (10).

There are additional minus signs associated with the I and L terms. These can be read off directly from the explicit expression (11). However, they can be derived most straightforwardly simply by noting that exactly those operators associated with I and L are odd under fermion-antifermion charge conjugation.

Therefore, the potentials, up to linear order in p/m, are

$$V^*_f(\vec{r}\,) = [\tilde{g}^* + I_j(v^a_{av})_j - J_0(v^a_{av})_j\sigma^a_j + \tilde{J}^*_j\sigma^a_j - \epsilon_{jkl}\tilde{L}_j(v^a_{av})_k\sigma^a_l]g_b f(\vec{r}\,) \tag{24}$$

$$\begin{aligned} V^*_g(\vec{r}\,) = &\left\{\frac{1}{2m_a}\left[g'_a\sigma^a_j + \tilde{L}_j + \epsilon_{jkl}I_k\sigma^a_l\right] g_b - \frac{1}{2m_b}\Big[\tilde{g}^* + I_k(v^a_{av})_k\right. \\ &\left. -J_0(v^a_{av})_k\sigma^a_k + \tilde{J}^*_k\sigma^a_k - \epsilon_{kln}\tilde{L}_k(v^a_{av})_l\sigma^a_n\Big]\, g'_b\sigma^b_j\right\} g_j(\vec{r}\,). \end{aligned} \tag{25}$$

Because of the sign changes associated with charge conjugation, two additional linear combinations of coefficients are relevant for nonrelativistic experiments with antiparticles:

$$\tilde{g}^* = g - I_0 \tag{26}$$

$$\tilde{J}^*_j = J_j - \epsilon_{jkl}L_{kl}, \tag{27}$$

where the star superscript notation continues to follow [50].

4 Conclusions

The potential $V_f + V_g$ provides a description of the $\mathcal{O}(p/m)$ nonrelativistic interactions between two fermion species—one with Lorentz violation present in the scalar-spinor vertex and one without. For a Lorentz-invariant fermion and Lorentz-violating antifermion, the equivalent potential is $V_f^* + V_g^*$, which is related by charge conjugation in the Lorentz-violating a sector. The Lorentz-violating structure of these terms is evident in several ways. Dependences on specific projections of the spin and momenta break spatial isotropy, and structures such as $\vec{v}_{av} \cdot \vec{\sigma}$ break boost invariance.

This study has already demonstrated several important properties of the Lorentz-violating operators that form G_1. In most sectors of the SME, there are operators that are not physically observable. However, all the terms that compose G appear in the nonrelativistic potentials, making observable contributions to the energy. While it may not be surprising that terms from G, which only affect interactions, not free particle propagation, cannot be eliminated from observables in the same fashion as m_5, a, or f, neither is it obvious that such is the case.

The potentials derived in this paper provide a fairly general formalism for studying violations of fundamental symmetries in low-energy, potential-dominated interaction processes. As noted in section 3.1, most of the observables than can be constructed at $\mathcal{O}(p/m)$ are included in the potentials, and those that are not included cannot descend from a renormalizable relativistic quantum field theory. The study of symmetry violation in low-energy processes is an active area of hadronic research, and it may be possible to place constraints on completely new SME parameters through studies of meson-mediated interactions between baryons.

Naturally, further generalizations of the results discussed here are also possible. There may be Lorentz violation at both vertices, and accounting for this possibility is entirely straightforward, as are accounting for the additional diagrams that appear when the external particles are associated with the same species. However, it is possible that the V_f and V_g potentials may not actually include the predominant effects, even when the momentum transfer in a collision is very low. The $g'_a g'_b$ term in V_h is $\mathcal{O}(p^2/m^2)$, but it is Lorentz invariant; so it would be no surprise if that term were substantially larger than Lorentz-violating terms that are nominally lower order in p/m. The $g'_a g'_b$ term is in fact the dominant term in standard model interactions involving pseudoscalar mesons when P violation is small.

Lorentz violation for the external fermion states has also been neglected, although if such Lorentz violation exists, it will modify the interparticle potentials further. The purely fermionic Γ_1 and M_1 terms in the SME Lagrangian were neglected because such terms, which would affect freely propagating fermions, are rather well constrained for first-generation species. However, Lorentz violation in the scalar sector is a separate matter, and the effects of Lorentz violation in the scalar sector on the Yukawa potential have already been studied [55]. The effect of a (CPT-even) tensor $K_{\mu\nu}$ is to modify $f(r)$ to

$$f^K(\vec{r}) = -\frac{e^{-\mu r}}{4\pi r}\left[1+\frac{1}{2}K_{jj}-\frac{1}{2}K_{jk}\left(\frac{\mu}{r}+\frac{1}{r^2}\right)x_jx_k\right]. \tag{28}$$

When the vertex interactions involve the Lorentz-invariant g' term, K leads to a further modified version of the derivative

$$\begin{aligned} g_j^K(\vec{r}) &= \partial_j f^K(\vec{r}) = g_j(\vec{r})\left[1+\frac{1}{2}K_{kk}-\frac{1}{2}K_{kl}\left(\frac{\mu}{r}+\frac{1}{r^2}\right)x_kx_l\right] \\ &+\frac{1}{2}f(\vec{r})K_{kl}\left[\left(\frac{\mu}{r^3}+\frac{2}{r^4}\right)x_jx_kx_l-\left(\frac{\mu}{r}+\frac{1}{r^2}\right)(\delta_{jk}x_l+\delta_{jl}x_k)\right]. \end{aligned} \tag{29}$$

The modified $g_j^K(\vec{r})$ is not needed in the $\vec{q}$-dependent terms that are themselves Lorentz violating; any resulting changes to the potentials calculated in section 3 would be higher order in the small Lorentz violation coefficients.

Finally, this work provides another stepping stone in the general analysis of Lorentz violation in scalar theories. With the advent of the Large Hadron Collider, it appears that it is finally possible to see direct evidence of the Higgs boson [72, 73]. This naturally opens up the possibility of studying the Lorentz symmetry behavior of fundamental scalar fields. The era of direct experimental studies of the Higgs particle is just beginning, and the theoretical foundation for understanding such studies needs to be prepared. While spontaneous gauge symmetry breaking is one of the most important features of the standard model, its complexity has limited studies of SME scalar fields to particular sub-topics. In addition to the renormalization of the Yukawa and pure scalar sectors, the tree-level quantization of the theory in the spontaneously broken phase [74, 75] has already been studied. Certain specific quantum corrections, originating in the Faddeev-Popov ghost sector [76], or involving higher powers of the SME coefficients [77], have also been examined. This paper presents some new results that increase our understanding of the scalar sector of the SME. While particles interacting via direct Higgs exchange are generally not at low nonrelativistic energies, this work nonetheless gives a new window into the dynamics of Higgs interactions.

References

[1] V. A. Kostelecký, S. Samuel, Phys. Rev. D **39**, 683 (1989).

[2] V. A. Kostelecký, R. Potting, Nucl. Phys. B **359**, 545 (1991).

[3] B. Altschul, V. A. Kostelecký, Phys. Lett. B **628**, 106 (2005).

[4] R. Gambini, J. Pullin, Phys. Rev. D **59**, 124021 (1999).

[5] J. Alfaro, H. A. Morales-Técotl, L. F. Urrutia, Phys. Rev. D **65**, 103509 (2002).

[6] I. Mocioiu, M. Pospelov, R. Roiban, Phys. Lett. B **489**, 390 (2000).

[7] S. M. Carroll, J. A. Harvey, V. A. Kostelecký, C. D. Lane, T. Okamoto, Phys. Rev. Lett. **87**, 141601 (2001).

[8] V. A. Kostelecký, R. Lehnert, M. J. Perry, Phys. Rev. D **68**, 123511 (2003).

[9] A. Ferrero, B. Altschul, Phys. Rev. D **80**, 125010 (2009).

[10] F. R. Klinkhamer, C. Rupp, Phys. Rev. D **70**, 045020 (2004).

[11] D. Colladay, V. A. Kostelecký, Phys. Rev. D **55**, 6760 (1997).

[12] D. Colladay, V. A. Kostelecký, Phys. Rev. D **58**, 116002 (1998).

[13] O. W. Greenberg, Phys. Rev. Lett. **89**, 231602 (2002).

[14] R. Bluhm, V. A. Kostelecký, N. Russell, Phys. Rev. Lett. **79**, 1432 (1997).

[15] G. Gabrielse, A. Khabbaz, D. S. Hall, C. Heimann, H. Kalinowsky, W. Jhe, Phys. Rev. Lett. **82**, 3198 (1999).

[16] H. Dehmelt, R. Mittleman, R. S. Van Dyck, Jr., P. Schwinberg, Phys. Rev. Lett. **83**, 4694 (1999).

[17] R. Bluhm, V. A. Kostelecký, N. Russell , Phys. Rev. Lett. **82**, 2254 (1999).

[18] D. F. Phillips, M. A. Humphrey, E. M. Mattison, R. E. Stoner, R. F. C. Vessot, R. L. Walsworth , Phys. Rev. D **63**, 111101(R) (2001).

[19] R. Bluhm, V. A. Kostelecký, C. D. Lane, Phys. Rev. Lett. **84**, 1098 (2000).

[20] V. W. Hughes, *et al.*, Phys. Rev. Lett. **87**, 111804 (2001).

[21] B. R. Heckel, E. G. Adelberger, C. E. Cramer, T. S. Cook, S. Schlamminger, U. Schmidt, Phys. Rev. D **78**, 092006 (2008).

[22] C. J. Berglund, L. R. Hunter, D. Krause, Jr., E. O. Prigge, M. S. Ronfeldt, S. K. Lamoreaux, Phys. Rev. Lett. **75**, 1879 (1995).

[23] V. A. Kostelecký, C. D. Lane, Phys. Rev. D **60**, 116010 (1999).

[24] D. Bear, R. E. Stoner, R. L. Walsworth, V. A. Kostelecký, C. D. Lane, Phys. Rev. Lett. **85**, 5038 (2000).

[25] P. Wolf, F. Chapelet, S. Bize, A. Clairon, Phys. Rev. Lett. **96**, 060801 (2006).

[26] H. Müller, *et al.*, Phys. Rev. Lett. **99**, 050401 (2007).

[27] S. Herrmann, A. Senger, K. Möhle, E. V. Kovalchuk, A. Peters, in *CPT and Lorentz Symmetry IV*, edited by V. A. Kostelecký (World Scientific, Singapore, 2008), p. 9.

[28] S. Herrmann, *et al.*, Phys. Rev. D 80, 105011 (2009).

[29] Ch. Eisele, A. Yu. Nevsky, S. Schiller, Phys. Rev. Lett. 103, 090401 (2009).

[30] H. Müller, Phys. Rev. D **71**, 045004 (2005).

[31] G. Saathoff, S. Karpuk, U. Eisenbarth, G. Huber, S. Krohn, R. Muñoz Horta, S. Reinhardt, D. Schwalm, A. Wolf, G. Gwinner, Phys. Rev. Lett. **91**, 190403 (2003).

[32] C. D. Lane, Phys. Rev. D **72**, 016005 (2005).

[33] V. A. Kostelecký, Phys. Rev. Lett. **80**, 1818 (1998).

[34] V. A. Kostelecký, Phys. Rev. D **61**, 016002 (1999).

[35] Y. B. Hsiung, Nucl. Phys. Proc. Suppl. **86**, 312 (2000).

[36] K. Abe *et al.*, Phys. Rev. Lett. **86**, 3228 (2001).

[37] J. M. Link *et al.*, Phys. Lett. B **556**, 7 (2003).

[38] B. Aubert *et al.*, Phys. Rev. Lett. **96**, 251802 (2006).

[39] S. M. Carroll, G. B. Field, Phys. Rev. Lett. **79**, 2394 (1997).

[40] V. A. Kostelecký, M. Mewes, Phys. Rev. Lett. **87**, 251304 (2001).

[41] V. A. Kostelecký, M. Mewes, Phys. Rev. Lett. **97**, 140401 (2006).

[42] V. A. Kostelecký, M. Mewes, Phys. Rev. Lett. **99**, 011601 (2007).

[43] F. W. Stecker, S. L. Glashow, Astropart. Phys. **16**, 97 (2001).

[44] T. Jacobson, S. Liberati, D. Mattingly, Nature **424**, 1019 (2003).

[45] B. Altschul, Phys. Rev. Lett. **96**, 201101 (2006).

[46] B. Altschul, Phys. Rev. D **74**, 083003 (2006).

[47] F. R. Klinkhamer, M. Risse, Phys. Rev. D **77**, 016002 (2008); addendum Phys. Rev. D **77**, 117901 (2008).

[48] J. B. R. Battat, J. F. Chandler, C. W. Stubbs, Phys. Rev. Lett. **99**, 241103 (2007).

[49] H. Müller, S. W. Chiow, S. Herrmann, S. Chu, K.-Y. Chung, Phys. Rev. Lett. **100**, 031101 (2008).

[50] V. A. Kostelecký, N. Russell, arXiv:0801.0287v5.

[51] V. A. Kostelecký, C. D. Lane, A. G. M. Pickering, Phys. Rev. D **65**, 056006 (2002).

[52] D. Colladay, P. McDonald, Phys. Rev. D, **75**, 105002 (2007).

[53] D. Colladay, P. McDonald, Phys. Rev. D, **79**, 125019 (2009).

[54] A. Ferrero, B. Altschul, Phys. Rev. D **84**, 065030 (2011).

[55] B. Altschul, Phys. Lett. B **639**, 679 (2006).

[56] B. R. Holstein, Nucl. Phys. A **844**, 160c (2010).

[57] B. Desplanques, J. F. Donoghue, B. R. Holstein, Ann. Phys. (NY) **124**, 499 (1980).

[58] S.-L. Zhu, C. M. Maekawa, B. R. Holstein, M. J. Ramsey-Musolf, U. van Kolck, Nucl. Phys. A **748**, 435 (2005).

[59] H. W. Greisshammer, M. R. Schindler, R. P. Springer, Eur. Phys. J. A **48**, 7 (2012).

[60] J. Vanasse, Phys. Rev. C **86**, 014001 (2012).

[61] R. Schiavilla, M. Viviani, L. Girlanda, A. Kievsky, L. E. Marcucci, Phys. Rev. C **78**, 014002 (2008); **83**, 029902 (E) (2011).

[62] Y.-H. Song, R. Lazauskas, V. Gudkov, Phys. Rev. C **83**, 015501 (2011).

[63] Y.-H. Song, R. Lazauskas, V. Gudkov, Phys. Rev. C **83**, 065503 (2011).

[64] H. W. Greisshammer, M. R. Schindler, Eur. Phys. J. A **46**, 73 (2010)

[65] L. Girlanda, Phys. Rev. C **77**, 067001 (2008).

[66] D. Colladay, P. McDonald, J. Math. Phys. **43**, 3554 (2002).

[67] B. Altschul, J. Phys. A **39**, 13757 (2006).

[68] Q. G. Bailey, V. A. Kostelecký, Phys. Rev. D **70**, 076006 (2004).

[69] D. Colladay, V. A. Kostelecký, Phys. Lett. B **511** 209 (2001).

[70] B. Altschul, Phys. Rev. D **70**, 056005 (2004).

[71] B. Altschul, Phys. Rev. D **84**, 091902(R) (2011).

[72] G. Aad, *et al.* (ATLAS Collaboration), Phys. Lett. B **716**, 1 (2012).

[73] S. Chatrchyan, *et al.* (CMS Collaboration), Phys. Lett. B **716**, 30 (2012).

[74] D. L. Anderson, M. Sher, I. Turan, Phys. Rev. D **70**, 016001 (2004).

[75] B. Altschul, Phys. Rev. D **86**, 045008 (2012).

[76] B. Altschul, Phys. Rev. D **73**, 045004 (2006).

[77] M. Gomes, J. R. Nascimento, A. Yu. Petrov, A. J. da Silva, Phys. Rev. D **81**, 045018 (2010).