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Absence of Long-Wavelength Cerenkov Radiation With Isotropic Lorentz and CPT Violation

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Abstract

Modified theories of electrodynamics that include violations of Lorentz symmetry often allow for the possibility of vacuum Cerenkov radiation. This phenomenon has previously been studied in a number of Lorentz-violating theories, but none of the methods that have previously been developed are sufficient to study a theory with a timelike Chern-Simons term k_{AF}, because such a term may generate exponentially growing solutions to the field equations. Searching for vacuum Cerenkov radiation in a theory with a purely timelike Chern-Simons term using only elementary methods, we find that, despite the presence of the runaway modes, a charge in uniform nonrelativistic motion does not radiate energy, up to second order in the velocity.

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In recent years, the closely related possibilities of Lorentz and CPT symmetry violations have gotten an increasing amount of attention. Theories that lack one or both of these symmetries, which are ordinarily considered fundamental building blocks of the laws of physics, might describe new physics related to quantum gravity. In fact, many theories that have been proposed as candidates to explain quantum gravity suggest the possibility of Lorentz symmetry breaking, at least in certain regimes. Moreover, if any violation of these symmetries were uncovered experimentally, it would be a discovery of critical importance and would open up a new window for studying fundmental physics.

Even if Lorentz and CPT violation are not realized in nature (and, as yet, there is no compelling evidence that they are), studying the kinds of theories that contain such symmetry violations may provide important new insights into the behavior of quantum fields. There is an effective quantum field theory, known as the standard model extension (SME), which has been developed to accomodate Lorentz and CPT violation in a general fashion. The SME action contains operators that may be constructed from standard model fields [1, 2]. These operators are much more general than those that appear in the action for the standard model itself, because they are not constrained by Lorentz invariance. However, some other restrictions on the operators are still necessary in order to make the theory tractable. Such restrictions typically include locality, superficial renormalizability, and gauge invariance.

In this paper, we shall be interested in the quantum electrodynamics sector of the SME. Although there may also be Lorentz-violating modifications to the behavior of matter fields, we shall be chiefly interested in a form of Lorentz and CPT violation that affects the purely electromagnetic sector. This is one of the most interesting terms that appears in the SME—the electromagnetic Chern-Simons term. This term defines a preferred spacetime direction, which may be spacelike or timelike. The Chern-Simons term screens static fields over long distances and splits the dispersion relations for right-handed and left-handed electromagnetic waves. This birefringence effect has been searched for and not seen. Polarized light that has traversed even cosmological distances shows no sign of the kind of systematic rotation of polarization that would result from the presence of the Chern-Simons term [3, 4, 5, 6], and this produces numerical constraints that are extremely strong. Yet despite the tightness of the empirical bounds, the term is still extremely interesting, and understanding the behavior of quantum electrodynamics with a Chern-Simons term can reveal new insights about the general structure of field theory. For example, the Chern-Simons term in the Lagrange density is not actually gauge invariant, although the integrated action associated with the term is. Because of this subtlety, the radiative corrections to the Chern-Simons terms are rather complicated, and for a while they were quite controversial [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18].

Another subject that is of very general interest in theories with Lorentz violation is the possibility of vacuum Cerenkov radiation. Ordinary Cerenkov radiation is emitted by charged particles moving in matter, where the equations of electrodynamics differ from their vacuum forms and Lorentz symmetry is lost. In a Lorentz-invariant vacuum, a

Cerenkov process such as $e^- \to e^- + \gamma$ is forbidden by energy-momentum conservation. However, processes that are kinematically forbidden when Lorentz symmetry is exact may become allowed when this symmetry is weakly broken. In theories in which the dispersion relations for particles differ from their conventional relativistic forms, the Cerenkov process could occur. So for any Lorentz-violating modification of electrodynamics, two very natural questions to ask are, *Under what circumstances does the theory permit vacuum Cerenkov radiation?* and, *What are the properties of the radiation emitted in the Cerenkov process?*

The possibility of vacuum Cerenkov radiation in SME electrodynamics has been studied in significant detail, but questions still remain. This paper will address the most important outstanding problem in this area, which has actually represented a glaring deficiency of our understanding of Lorentz-violating Cerenkov physics. Previous analyses of vacuum Cerenkov processes have used a variety of techniques. Some of the methods involved the use of Green's functions—either directly, in combination with a prescribed current source [19], or through the use of Feynman diagrams [20]. Other methods used a transformation to carry the Lorentz violation into the charged matter sector; then radiation occurs when the matter-sector Lorentz violation leads to faster-than-light motion. Depending on the theory involved, it may be possible to use this method globally [21] or only locally in Fourier space (by looking at whether the motion of a charged particle exceeds the phase speed of light for a particular propagation mode [22, 23]).

None of these methods, however, is really equipped to deal with what happens at very long wavelengths when the electromagnetic Lorentz violation is isotropic and odd under CPT. The methods discussed above have different levels of usefulness, depending on the nature of the Lorentz violation involved. However, they all ultimately rely on an understanding of the modified dispersion relations for electromagnetic excitations. In fact, the radiation rates in all the situations previously studied can be estimated fairly accurately simply by looking at the phase space available for the process, using the correctly modified dispersion relations.

The use of methods based on understanding the electromagnetic dispersion relations leaves a major gap in our understanding of these theories, because in certain cases, the Lorentz violation may lead to a fundamental change in the electromagnetic propagation structure at long wavelengths. Rather than traveling waves with real frequencies, there may be long-wavelength runaway modes that grow exponentially with time. The instability itself may be cured by using an acausal Green's function, but this obviously introduces other complications, and it may not be clear how exactly the physical theory should be defined. However, we shall see that it is still possible to learn quite a bit about the possibility of vacuum Cerenkov radiation in this regime, without being troubled by such potential ambiguities.

The electromagnetic Lagrange density, including all power-counting renormalizable

Lorentz-violating terms that can appear solely in the photon sector is

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}k_F^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + \frac{1}{2}k_{AF}^{\mu}\epsilon_{\mu\nu\rho\sigma}F^{\nu\rho}A^{\sigma} - j^{\mu}A_{\mu}. \tag{1}$$

The k_F term is even under CPT, while the k_{AF} term (the Lorentz-violating Chern-Simons term of interst here) is odd under CPT. The particular case of interest here is that of a timelike k_{AF}. In fact, we shall consider a strictly timelike vector $k_{AF}^{\mu} = (k, \vec{0})$. The difficulty with this theory is that the photon dispersion relation becomes $\omega_{\pm}^2 = p(p \mp 2k)$, where the $\pm$ denotes the helicities of the modes. At very long wavelengths, $p < |2k|$, one of the two helicity modes possesses an imaginary frequency. As a result, there may be runaway solutions that grow exponentially with time. In [3], a Green's function was given that alleviates this difficulty; the function provides solutions to the equations of motion that do not involve exponential growth, but the cost is that charged particles may begin to radiate before they are actually in motion.

The problem with runaway modes is also tied to the structure of the energy-momentum tensor in the k_{AF} theory. With only the Chern-Simons term present, the purely electromagnetic part of the tensor is [3]

$$\Theta^{\mu\nu} = -F^{\mu\alpha}F^{\nu}{}_{\alpha} + \frac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta} - \frac{1}{2}k_{AF}^{\nu}\epsilon^{\mu\alpha\beta\gamma}F_{\beta\gamma}A_{\alpha}. \tag{2}$$

With the strictly timelike k_{AF}, the energy density (Θ^{00}), momentum density (Θ^{0j}), and energy flux (Θ^{j0}) respectively are

$$\begin{aligned} \mathcal{E} &= \frac{1}{2}\vec{E}^2 + \frac{1}{2}\vec{B}^2 - k\vec{B}\cdot\vec{A} && (3)\\ \vec{\mathcal{P}} &= \vec{E}\times\vec{B} && (4)\\ \vec{S} &= \vec{E}\times\vec{B} - kA_0\vec{B} + k\vec{A}\times\vec{E}. && (5) \end{aligned}$$

These quantities are not gauge invariant, although the integrated electromagnetic energy and momentum are. Note, moreover, that $\mathcal{E}$ contains a term that is not bounded below; this accounts for the existence of runaway solutions.

The question we want to address is whether a charge moving with nonrelativistic speed v in this vacuum will emit radiation. It has already been established that there will be Cerenkov radiation at higher order in v. For a charge with a finite speed, there will always be a range of wavelengths for which the electromagnetic phase speeds are real and slower than v, so Cerenkov radiation will be emitted at these wavelengths.

The algorithm we previously developed for dealing with radiation into these modes [22] gives the following results. The range of photon momenta p satisfying $0 < \omega/p < v$ is $2k < p < 2\gamma^2 k$, where $\gamma = (1-v^2)^{-1/2}$ is the usual Lorentz factor. The (basically phase space) estimate for the radiated power per unit wave vector is $P(\omega) = \frac{q^2}{8\pi}(1-\omega^2/p^2v^2)\omega$.

So the total power emitted into the positive-frequency modes is

$$P=\frac{e^2}{8\pi}\int_{2k}^{2\gamma^2 k}dp\left(1-\frac{1-2k/p}{v^2}\right)p\sqrt{1-2k/p}. \tag{6}$$

Changing variables to $u=(1-2k/p)/v^2$ (the square of the ratio of the phase speed to the particle speed), this becomes,

$$\begin{aligned}
P &= \frac{k^2e^2v^3}{2\pi}\int_0^1 du\frac{\sqrt{u}(1-u)}{(1-v^2u)^3} &(7)\\
&= \frac{k^2e^2v^3}{8\pi\gamma^2}\frac{v(3-v^2)-(3+v^2)(1-v^2)\tanh^{-1}(v)}{v^5} &(8)\\
&= \frac{k^2e^2v^3}{30\pi}+\mathcal{O}(v^5). &(9)
\end{aligned}$$

[In spite of the v^5 in the denominator, the final fraction in (8) is regular at $v=0$.] The origin of the overall v^3 dependence is fairly simple to understand; the size of the p-space region covered by the integration is $\mathcal{O}(v^2k)$, and the typical energy of photon emitted in this region is $\mathcal{O}(vk)$.

This shows the character of the radiation related to the modes with real frequencies. However, this algorithm tells us nothing about what happens to the very longest wavelength modes—those corresponding to the runaway solutions. It is entirely plausible that these modes will be excited by a charged particle moving with even an infinitesimal speed.

Therefore, we shall consider the possibility of radiation from a charge moving with speed v and only look at results up to $\mathcal{O}(v^2)$. The purpose of this restriction is to disentangle the known Cerenkov radiation at shorter wavelengths from what happens in the $p<|2k|$ modes. Fortunately, once this approximation is made, the $\mathcal{O}(v^2)$ results may be determined using quite elementary methods. We shall also restrict attention to results that are $\mathcal{O}(k^2)$, which is the lowest order at which nontrivial radiation effects could occur; in fact, for dimensional reasons alone, the total energy loss rate must be proportional to k^2. At these orders, we shall be able to show, without needing to "correct" the theory with an acausal Green's function, that a charged particle in uniform nonrelativistic motion does not emit any energy as radiation.

The key to demonstrating this fact is calculating the $\mathcal{O}(k)$ contribution to the magnetic field of the moving charge. For a charge q located at the origin and moving uniformly with a nonrelativistic velocity $\vec{v}=v\hat{z}$, the conventional magnetic field is

$$\vec{B}_0(\vec{r})=\frac{qv}{4\pi}\frac{\sin\theta}{r^2}\hat{\phi}. \tag{10}$$

This is not the entirety of the magnetic field, however. The standard $\vec{B}_0$ generates a further magnetic field through the k_{AF}-modified Ampere's Law (which is the only one of

Maxwell's equations that is modified by the presence of the purely timelike k_{AF}),

$$\vec{\nabla}\times\vec{B}-\frac{\partial\vec{E}}{\partial t}=2k\vec{B}+\vec{J}. \tag{11}$$

We see that the magnetic field itself behaves like an effective current source of strength $\vec{J}_{\rm eff}=2k\vec{B}$. It turns out that when $\vec{B}$ is given by (10), the field of the corresponding $\vec{J}_{\rm eff}$ may be found exactly. This is obviously equivalent to the problem of finding the vector potential of a charge in uniform motion, but in the Coulomb gauge (since $\vec{\nabla}\cdot\vec{B}=0$), rather than the more usual Lorenz gauge.

An elementary version of the calculation proceeds as follows. Between radii r and $r+dr$ there is an effective surface current $\vec{K}_{\rm eff}=\vec{J}_{\rm eff}\,dr$, and the field of a surface current $\vec{K}=K_0\sin\theta\,\hat{\phi}$ at radius R is well known. (This $\vec{K}$ is, for example, the bound surface current on a uniformly magnetized sphere of radius R.) The field generated by just this surface current is

$$\vec{B}_K(\vec{r})=\left\{\begin{array}{ll}\frac{2}{3}K_0\hat{z}, & r<R\\ \frac{R^3}{3r^3}K_0(3\cos\theta\hat{r}-\hat{z}), & r>R\end{array}\right. \tag{12}$$

These contributions to the total magnetic field field must be integrated over all the shells from radii $R=0$ to ∞, remembering that the equatorial surface current density K_0 is also a function of R. The required integral is

$$\begin{aligned}\vec{B}(\vec{r}) &= \vec{B}_0(\vec{r})+\int_0^r dR\left(\frac{kqv}{2\pi R^2}\right)\left[\frac{R^3}{3r^3}(3\cos\theta\,\hat{r}-\hat{z})\right]+\int_r^\infty dR\left(\frac{kqv}{2\pi R^2}\right)\left(\frac{2}{3}\hat{z}\right) && (13)\\ &= \vec{B}_0(\vec{r})+\frac{kqv}{4\pi r}(\cos\theta\,\hat{r}+\hat{z}) && (14)\\ &= \vec{B}_0(\vec{r})+\frac{kqv}{4\pi r}(2\cos\theta\,\hat{r}-\sin\theta\,\hat{\theta}). && (15)\end{aligned}$$

Because of the presence of the dimensional parameter k, this field decays only as r^{-1}—less rapidly than the conventional field $\vec{B}_0$. This means that the field at $\mathcal{O}(k)$ is potentially capable of producing outgoing energy and momentum fluxes at infinity.

To see whether there really are such fluxes, we must turn our attention to the energetics of the fields at large r. We shall look specifically at the energy flux $\vec{S}$, which is a modified Poynting vector. An outward energy flux at large r will be the signature of energy losses from vacuum Cerenkov radiation.

We recall that we are only interested in effects at $\mathcal{O}(v^2)$, since the presence of Cerenkov radiation into real-frequency modes at $\mathcal{O}(v^3)$ has already been established. There are three terms in $\vec{S}$, and two of them involve cross products with $\vec{E}$. However, neither of these can contribute to a flux at infinity at this order. The standard Coulomb field of the charge points radially outward, and so a cross product containing the conventional part of $\vec{E}$ can never contribute to an outgoing flux. The magnetic quantities $\vec{A}$ and $\vec{B}$ are each proportional to v, and the inductive part of $\vec{E}$ [the part sourced by $\partial B/\partial t=v(\partial B/\partial z)$]

is $\mathcal{O}(v^2)$; so the products $(\vec{E}\times\vec{B})\cdot\hat{r}$ and $(\vec{A}\times\vec{E})\cdot\hat{r}$ are necessarily $\mathcal{O}(v^3)$. Then the only term in $\vec{S}$ that remains a concern is $-kA_0\vec{B}$. Since this term contains an explicit factor of k, we evidently do not need to consider $\vec{B}$ beyond $\mathcal{O}(k)$ in order to determine the $\mathcal{O}(k^2)$ flux. Any k-dependent contributions to A_0 would also be $\mathcal{O}(v^2)$ (as are the next k-independent contributions to A_0), and so these may be neglected, leaving the only contribution as that coming from the k-dependent $\vec{B}$ (15).

The radial flux at spatial infinity is thus

$$\vec{S}\cdot\hat{r} = -k\left(\frac{q}{4\pi r}\right)\left[\frac{kqv}{4\pi r}(2\cos\theta\,\hat{r}-\sin\theta\,\hat{\theta})\right]\cdot\hat{r} = -\frac{k^2q^2v}{8\pi^2r^2}\cos\theta. \tag{16}$$

This is not, on its own, a meaningful result. There naively appears to be a constant energy flux along the $-z$-direction. However, we must recall that local expressions for energy densities are not gauge invariant in this theory. What is gauge invariant, however, is the integral of this flux over the full surface Σ at spatial infinity, and

$$\int_{r\to\infty} d\Sigma\,(\vec{S}\cdot\hat{r}) = 0. \tag{17}$$

There is no net energy flux away from the particle—no vacuum Cerenkov radiation.

This can be immediately extended to show that there is no momentum flux at infinity at $\mathcal{O}(v)$ either. The momentum flux involves Θ^{jk}, which takes its standard form when k_{AF} is purely timelike. Because the Maxwell stress terms only involve products E_jE_k and B_jB_k, there are no terms that are linear in v that do not fall off faster than r^{-2}. So there are no energy or momentum fluxes at spatial infinity at the lowest orders in v and k.

If k_{AF} is not purely timelike, but $|\vec{k}_{AF}|$ is small compared with k_{AF}^0, then it is possible to boost into a frame in which k'_{AF} is indeed purely timelike, without giving the charged particle a relativistic velocity in that frame. The speed added to the particle is $v' = |\vec{k}_{AF}|/k_{AF}^0$; the absence of vacuum Cerenkov radiation for a particle moving with this speed in the boosted frame—a result which is valid to $\mathcal{O}[(k_{AF}^0)^2v']$—ensures that there is no radiation from a stationary particle to $\mathcal{O}(|\vec{k}_{AF}|k_{AF}^0)$. It follows from the series expansion that there is no contribution to the emission rate at this particular order, even if $|\vec{k}_{AF}|$ and k_{AF}^0 are comparable in size. This mirrors results from [19], in which it was found that there was no radiation from a stationary particle at this order in the alternative $|\vec{k}_{AF}| > |k_{AF}^0|$ spacelike case.

We should note, however, that it is by no means a triviality that a stationary particle should emit no vacuum Cerenkov radiation. In fact, in the spacelike case, such radiation does occur, albeit not at the low order just discussed. When the k_{AF} is anisotropic, it provides a preferred spatial direction along which energy and momentum may be emitted.

It is similarly non-obvious that a charge moving in an isotropic k_{AF} background should not emit radiation at $\mathcal{O}(k^2v^2)$. There is no obvious symmetry that could prevent the emission (except Lorentz symmetry, which has been explicitly broken). The k term is odd under P and CPT, but this does not preclude a P-even radiation process from occurring at $\mathcal{O}(k^2)$. The situation is not isotropic, because the axis of motion $\vec{v}$ picks out a particular direction, and it would not be surprising if the charge lost energy and momentum to electromagnetic waves emitted into a cone of angles around $\hat{v}$. This is precisely what happens at higher order in v, through the interaction with non-runaway modes.

The main result of this paper has been the demonstration that a charged particle moving slowly through a vacuum that violates Lorentz boost and CPT symmetries (but not spatial isotropy) does not emit any radiation at $\mathcal{O}(v^2)$. While there is radiation at $\mathcal{O}(v^3)$, it can be understood as occurring primarily because energy-momentum conservation allows it. When the phase speed for a given mode of the electromagnetic field is less than the speed at which a charged particle is moving, Cerenkov radiation is expected, in analogy with the emission by relativistic particles passing through material with a significant index of refraction. However, things are qualitatively different for the long-wavelength modes of the isotropic theory. These modes lead to the possibility of runaway solutions, and their contributions to vacuum Cerenkov radiation could not have been evaluated by any of the methods that had previously been introduced to study this kind of radiation process in other regimes.

So this paper has dealt with the greatest remaining puzzle that had arisen in studies of Lorentz-violating vacuum Cerenkov radiation. The central result is simple and aesthetically interesting on several levels. For instance, as a side effect of this calculation, we determined the magnetic field of a nonrelativistically moving charge in the Lorentz-violating theory, to leading order in k. This was itself an interesting result, since it turned out the field could be built up from the interior and exterior fields of a set of concentric shells.

On a deeper level, this work also addresses the direct problem of the instability in the timelike k_{AF} theory. As we noted, runaway modes can be wiped out of the theory by using a Green's function for the electromagnetic field that is acausal—the radiation at a time t depending on the behavior of the source at times $t' > t$. This cures one problem by introducing another. However, we saw in this paper that the runaway modes were not a problem in the context being studied. We showed that no acausal signaling was required to avoid exponentially growing energy-carrying solutions. Although this was obviously restricted to a very particular regime, it provides an important insight into the behavior of this fascinating Lorentz-violating theory.

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