


10-23-2019

## **Lorentz- and C P T -Violating Standard Model Extension in Chiral Perturbation Theory**

Brett Altschul

Matthias R. Schindler

Follow this and additional works at: [https://scholarcommons.sc.edu/phys\\_facpub](https://scholarcommons.sc.edu/phys_facpub)


 Part of the **Physics Commons**

---

# Lorentz- and *CPT*-violating standard model extension in chiral perturbation theory

Brett Altschul<sup>\*</sup> and Matthias R. Schindler<sup>†</sup>

*Department of Physics and Astronomy, University of South Carolina,  
Columbia, South Carolina 29208, USA*

 (Received 4 July 2019; published 23 October 2019)

Lorentz and *CPT* violation in hadronic physics must be tied to symmetry violations at the underlying quark and gluon level. Chiral perturbation theory provides a method for translating novel operators that may appear in the Lagrange density for color-charged parton fields into equivalent forms for effective theories at the meson and baryon levels. We extend the application of this technique to the study of Lorentz-violating and potentially *CPT*-violating operators from the minimal standard model extension. For dimension-4 operators, there are nontrivial relations between the coefficients of baryon-level operators related to underlying quark and gluon operators with the same Lorentz structures. Moreover, in the mapping of the dimension-3 operators from the quark and gluon level to the hadron level (considered here for the first time), many of the hadronic observables contain no new low-energy coupling constants at all, which makes it possible to make direct translations of bounds derived using experiments on one kind of hadron into bounds in a completely different corner of the hadronic sector. A notable consequence of this is bounds (at  $10^{-15} - 10^{-20}$  GeV levels) on differences  $a_B^\mu - a_{B'}^\mu$  of Lorentz and *CPT* violation coefficients for  $SU(3)_f$  octet baryons that differ in their structure by the replacement of a single valance  $d$  quark by a  $s$  quark. Never before has there been any proposal for how these kinds of differences could be constrained.

DOI: [10.1103/PhysRevD.100.075031](https://doi.org/10.1103/PhysRevD.100.075031)

## I. INTRODUCTION

Recent developments in our understanding of fundamental symmetry principles have led to a great deal of interest in testing how well the symmetries that seem to underlie the fundamental physics we currently understand—the standard model of particle physics and the general theory of relativity for gravitation—have really been verified experimentally. Particular attention has been paid to Lorentz symmetry and *CPT*, because these symmetries can be given up without needing to abandon the general structure of field theory. Other exotic possibilities (such as violations of the spin-statistics relation) are even less well behaved, and it may not even be possible to formulate completely self-consistent test theories for such possibilities.

The experimental discovery of any kind of really exotic new fundamental phenomena would obviously be of singular importance, on par with the development of renormalizable quantum field theories, which provided a

comprehensive framework for the study of interacting elementary particles. If Lorentz or *CPT* violation is ever found experimentally, the new result can immediately be analyzed in the context of effective field theory (EFT), since an effective field theory framework capable of incorporating these symmetry violations into the description of standard model quanta has already been developed [1,2]. This EFT, known as the standard model extension (SME), describes Lorentz violation, and then is automatically capable of describing *CPT* violation as well—because a quantum field theory (QFT) with a well-defined *S*-matrix that is not invariant under *CPT* cannot be invariant under Lorentz symmetry either [3].

Thanks to its generality, the SME has become the standard formalism used for parametrizing the results of experimental Lorentz and *CPT* tests. Most reasonable test theories previously proposed for use in explorations of how these symmetries might be broken have turned out to be special cases of the SME. As an EFT, the SME really contains a potentially infinite hierarchy of Lorentz-violating operators that can be constructed out of standard model fields. However, in many cases, attention is restricted to the minimal SME (mSME), which contains only gauge invariant and renormalizable operators in its action. The mSME is expected to describe most low-energy Lorentz- and *CPT*-violating phenomena, and experimental verifications of these symmetries can usually be expressed most usefully as

<sup>\*</sup>baltschu@physics.sc.edu

<sup>†</sup>mschindl@mailbox.sc.edu

*Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP<sup>3</sup>.*

bounds on the coupling constants of the mSME (of which there is a large but finite number). The Lagrange density for the mSME looks qualitatively similar to the Lagrange density for the standard model itself. The key difference is that the mSME operators do not need to be Lorentz scalars; each Lorentz-violating term will have one or more free Lorentz indices, which is contracted with a constant background vector that describes a preferred directional structure in spacetime. These background constants are the parameters that can be bounded experimentally, and the current state of the art for such bounds may be found summarized in [4]. The best bounds on different strains of Lorentz-violating operators come from many different areas of experimentation—including astronomy, atomic physics, and collider physics.

There are still significant challenges for the interpretation of experimental results in terms of SME parameters. One of the most important ones is a challenge that is also present in analyses in a conventional standard model context. Although there are additional subtleties when Lorentz and  $CPT$  are potentially broken, there is a common basic issue that the fundamental parameters of the theory are the coefficients of operators that are formed from the elementary fields, which do not necessarily represent the quanta that are physically accessible at low energies. In particular, it is not so easy to take the results of measurements made on hadrons—particles with residual strong interactions mediated largely by the exchange of mesons and mesonlike resonance states—and relate those to the fundamental description in terms of color-charged fields that are capable of exchanging gluons. The purpose of chiral perturbation theory ( $\chi$ PT) [5–7] (and see [8] for a pedagogical introduction to the subject) is to bridge this gap between the descriptions at the hadron level and the quark and gluon level.

Previous work has introduced a number of SME operators for quarks [9,10] and gluons [11,12] and used  $\chi$ PT methods to translate them into equivalent formulations for mesons and baryons. The author of Ref. [12] also considered certain radiative corrections and meson-exchange potentials. However, there has not previously been a complete treatment of all the mSME operators for strongly interacting fields that are amenable to  $\chi$ PT methods simultaneously. Such a treatment is our goal in this paper. This is actually a slightly less onerous undertaking than it might initially appear, since any Lorentz violation in nature is known to be a very small effect. That means that it is a pretty much universally valid approximation to work only to first order in the SME parameters; we shall not consider any operators or phenomena that involve products of multiple SME coefficients. However, even at linear order, there are some interesting relationships to be found between the coefficients.

The outline of this paper is as follows. In Sec. II, we introduce mSME Lorentz violation for the fields at the level

of two-flavor quantum chromodynamics (QCD)—the quarks and gluons. The methodology of  $\chi$ PT is discussed in Sec. III. Then, in Secs. IV and V, we construct the leading order (LO) effective actions for the pion and nucleon sectors, respectively. Experimental consequences, including some involving kaons and other strange particles, are discussed in Sec. VI. Finally, Sec. VII summarizes our conclusions and areas for future study.

## II. LORENTZ VIOLATION WITH QCD FIELDS

### A. Quark operators

The starting point for our analysis will be the mSME Lagrange density, expressed in terms of the QCD fields. The mSME action is built out of gauge-invariant operators of dimensions 2, 3, and 4, which are constructed out of the standard model’s quantum fields. This is the same basic approach taken in the usual standard model, except that the new operators specific to the mSME will have free Lorentz indices. These indices are contracted with constant background tensors; if the Lorentz violation arises through spontaneous symmetry breaking, then the background tensors are determined from the vacuum expectation values of tensor-valued bosonic fields. In the presence of such background tensors, otherwise identical experiments done in different coordinate reference frames may yield different outcomes. By comparing the results of experiments done with the apparatus at different orientations, or moving with different velocities, it is possible to place bounds on the symmetry-breaking backgrounds.

The Lagrange density for the QCD sector of the mSME has operators that can be constructed out of quark field bilinears and the gluon field. Our focus will primarily be on Lorentz violation in two-flavor QCD. However, when it is straightforward to do so, we will present generalizations to the theory containing a strange ( $s$ ) quark field, in addition to up ( $u$ ) and down ( $d$ ), with an approximate  $SU(3)_f$  flavor symmetry. However, the inclusion of a heavier quark does significantly increase the complexity of the theory, because there are no gauge symmetries to prevent there existing a large number of Lorentz-violating mixing terms between the  $d$  and  $s$  fields. The situation is analogous to having not just a single Cabibbo angle to describe the difference between the mass eigenstates and electroweak eigenstates of the quarks, but a potentially different mixing angle for every single component of the Lorentz-violating background tensors.

Moreover, although the focus of our analyses will always be the strongly interacting sector of the mSME, we will also make use of results from other sectors of the theory. In addition to chiral symmetry and the  $SU(3)_c$  gauge symmetry of QCD, there are additional symmetry requirements that the hadronic Lagrangians must respect. Some of these are simply the additional electroweak gauge symmetries of the standard model. However, there

are also other conditions that will need to be satisfied if the mSME (which is a QFT) is to be embedded into a larger geometric theory that also encompasses gravitation. We will employ these additional consistency conditions freely, whenever they can be used to simplify the analysis.

We may further subdivide the various forms of Lorentz violation into those which are odd under *CPT*, versus those that are *CPT* invariant. In the mSME, the *CPT*-violating operators are those with odd numbers of Lorentz indices to be contracted with the external background tensors. The *CPT*-even operators are then those with even numbers of free indices; these include, naturally enough, the regular standard model operators, which possess zero free Lorentz indices. (This rule—that whether an operator is *CPT* violating can be determined simply by counting its indices—holds for most operators in the full SME. However, there is an important exception [13,14]—the *f*-type operators, which do not violate *CPT*, in spite of having odd numbers of indices.) In the mSME, the only quark and gluon operators that can exist at mass dimension 4 are even under *CPT*. There are *CPT*-odd dimension-4 operators that can exist in a SME version of pure quantum electrodynamics (QED), but all such operators involve Dirac matrix structures that mix left- and right-chiral fermion fields in a way that is not consistent with the  $SU(2)_L$  electroweak gauge symmetry of the full standard model. Since these terms are not gauge invariant (and are correspondingly not expected to be renormalizable), they are not truly part of the mSME. However, similar terms that break the electroweak gauge symmetry actually can exist as dimension-3 operators, where they may arise as vacuum expectation values of dimension-4 operators involving the Higgs field. This is the same way that the Dirac fermion mass terms arise in the conventional standard model; when the Higgs acquires a vacuum expectation, certain Yukawa-like dimension-4 operators are converted into dimension-3 mass terms.

We shall first look at the dimension-4 operators, beginning with those for the quarks. The *CPT*-even terms of this dimension that can exist in the quark sector are [2]

$$\begin{aligned} \mathcal{L}_{\text{quark}}^{d=4, \text{CPT-even}} = & i(c_Q)_{\mu\nu AB} \bar{Q}_A \gamma^\mu D^\nu Q_B \\ & + i(c_U)_{\mu\nu AB} \bar{U}_A \gamma^\mu D^\nu U_B \\ & + i(c_D)_{\mu\nu AB} \bar{D}_A \gamma^\mu D^\nu D_B. \end{aligned} \quad (1)$$

The covariant derivatives contain all the standard model gauge fields, and in curved spacetime, any derivatives must be taken as 50-50 linear combinations of derivative operators acting to the right and left. The left- and right-handed quark multiplets are denoted by

$$Q_A = \begin{bmatrix} u_A \\ d_A \end{bmatrix}_L, U_A = [u_A]_R, D_A = [d_A]_R. \quad (2)$$

where the left and right multiplets are of different dimensionalities because they transform differently under the  $SU(2)_L$  electroweak gauge symmetry.

The labels  $A, B = 1, 2, 3$  denote the quark generations. Terms that are off diagonal in the  $(A, B)$  basis correspond to mixing between the generations due to Lorentz violation. It is familiar from the standard model that there is generally not a single natural basis for the quark fields. The standard model is formulated so that the mass terms in the quark Lagrangian are diagonal, so that there is no flavor mixing during free quark propagation. However, the electroweak interactions are not diagonalized in the quark mass basis, leading to flavor-changing interactions. In general, the Lorentz violation coefficients will also not be diagonal in the mass basis. If all the heavier quarks are integrated out of the theory via the renormalization group, leaving just the  $u$  and  $d$  fields, then the mixing issue becomes moot. However, if the  $s$  field is retained, then for each Lorentz component of the  $(c_Q)_{\mu\nu AB}$  and  $(c_D)_{\mu\nu AB}$ , there are coefficients for unmixed  $d$  and  $s$  propagation, as well as a mixing angle between them, analogous to the Cabibbo angle. As a result, the full parameter space of Lorentz-violating flavor physics may be extremely difficult to probe, even with just three flavors.

The predominant effects of the dimension-4 Lorentz-violating operators are expected to come from terms that are symmetric in the indices  $(\mu, \nu)$ . In particular, the antisymmetric parts cannot modify the dimension-4 kinetic terms for baryons at leading order in the Lorentz violation, and they cannot affect the dimension-4 kinetic operators for mesons at all. The generic mSME Lagrange density for a single species of fermion is

$$\mathcal{L}_{\text{spin-}\frac{1}{2}} = \bar{\psi}(i\Gamma^\mu \partial_\mu - M)\psi, \quad (3)$$

$$\Gamma^\mu = \gamma^\mu + c^{\mu\nu} \gamma_\nu + d^{\mu\nu} \gamma_5 \gamma_\nu, \quad (4)$$

$$M = m + im_5 \gamma_5 + a^\mu \gamma_\mu + b^\mu \gamma_5 \gamma_\mu + \frac{1}{2} H^{\mu\nu} \sigma_{\mu\nu}. \quad (5)$$

With the only potential form of Lorentz violation coming from an antisymmetric tensor  $c^{\mu\nu} = -c^{\nu\mu}$ , it is clear that the effect of  $c^{\mu\nu}$  is, at leading order, just a change in the basis of the Dirac matrices. A complementary transformation of the fermion field removes the antisymmetric  $c^{\mu\nu}$  from the Lagrange density at leading order [13]. So the antisymmetric term cannot have any observable consequences at leading order. The same fact can be seen manifested in the exact energy-moment relation for a fermion described by  $\mathcal{L}_{\text{spin-}\frac{1}{2}}$  with just  $c_{jk} \neq 0$ ,

$$E = \sqrt{m^2 + p_j p_j - 2c_{jk} p_j p_k + c_{jl} c_{kl} p_j p_k}. \quad (6)$$

In fact, it has been demonstrated that there is an exact supersymmetry transformation between  $\mathcal{L}_{\text{spin-}\frac{1}{2}}$  with just a

$c^{\mu\nu}$  coefficient and the general Lagrange density for a complex scalar field

$$\mathcal{L}_{\text{spin-0}} = (\partial^\mu - ia_\phi^\mu)\phi^*(\partial_\mu + ia_\phi^\mu)\phi + k^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi - m^2|\phi|^2, \quad (7)$$

so long as  $a_\phi^\mu = 0$  and  $k^{\mu\nu} = c^{\mu\nu} + c^{\nu\mu} + c^{\mu\rho}c^\nu{}_\rho$  [15]. [Note that it is not even possible for the bosonic  $k^{\mu\nu}$  to have an antisymmetric part without additionally breaking the charge conjugation ( $C$ ) symmetry of the Lagrange density.]

In the two-flavor QCD limit, the Lagrange density simplifies quite a bit. Each of the  $c_{\mu\nu}$  parameters in (1) is a dimensionless coupling constant, and they form matrices which are Hermitian in the  $(A, B)$  flavor space. Restricting the Lagrange density of (1) to one with just  $u$  and  $d$  fields, it reduces to

$$\mathcal{L}_{\text{light quarks}}^{d=4, CPT\text{-even}} = i\bar{Q}_L C_{L\mu\nu}\gamma^\mu D^\nu Q_L + i\bar{Q}_R C_{R\mu\nu}\gamma^\mu D^\nu Q_R, \quad (8)$$

where the quark fields are now  $Q_{L/R} = [u_{L/R}, d_{L/R}]^T$ , and the Lorentz-violation coefficients can be collected in the matrices

$$C_{L/R}^{\mu\nu} = \begin{bmatrix} c_{u_{L/R}}^{\mu\nu} & 0 \\ 0 & c_{d_{L/R}}^{\mu\nu} \end{bmatrix}. \quad (9)$$

Note that there is no mixing between the  $u$  and  $d$  quarks; that is forbidden by the standard model's unbroken electromagnetic gauge invariance. This formalism actually allows for there to be different coefficients  $c_{u_L}^{\mu\nu}$  and  $c_{d_L}^{\mu\nu}$ , whereas in actuality,  $SU(2)_L$  gauge invariance requires these to be equal,  $c_{u_L}^{\mu\nu} = c_{d_L}^{\mu\nu} = c_{q_L}^{\mu\nu}$ . However, this is somewhat modified when the  $s$  quarks are included, and we shall generally consider the  $c_{u_L}^{\mu\nu}$  and  $c_{d_L}^{\mu\nu}$  separately.

Because the coefficients in (8) are given in the chiral basis, they multiply operators that are not simply even or odd under parity ( $P$ ) and  $C$ . Since most precision experiments will measure effects that are unambiguously odd or even under  $P$ , the resulting bounds are usually quoted on the linear combination  $c^{\mu\nu} = \frac{1}{2}(c_L^{\mu\nu} + c_R^{\mu\nu})$  and  $d^{\mu\nu} = \frac{1}{2}(c_L^{\mu\nu} - c_R^{\mu\nu})$ . When dealing with hadrons and chiral symmetry, it is often convenient to use different linear combinations of coefficients, broken up by their transformation properties under isospin. The isosinglet is  ${}^1C_{L/R}^{\mu\nu} = \frac{1}{2}\text{Tr}(C_{L/R}^{\mu\nu})$ , and the isotriplet is  ${}^3C_{L/R}^{\mu\nu} = C_{L/R}^{\mu\nu} - \mathbb{1}{}^1C_{L/R}^{\mu\nu}$ , where  $\mathbb{1}$  is the identity in isospin space.

There are also dimension-3 quark operators. Note that in the generic  $\mathcal{L}_{\text{spin-}\frac{1}{2}}$ , the dimension-3 terms from (5) exhaust all the possible Dirac matrix structures; each dimension-3 Lorentz-violating operator is composed of a fermion bilinear  $\bar{\psi}A\psi$ , multiplied by a matching background tensor. At dimension 4, some of the Dirac bilinear quantities

$\bar{\psi}B\partial\psi$  were forbidden by electroweak gauge invariance. However, at dimension 3, terms that mix left- and right-chiral fields can arise as vacuum expectations; in the standard model, this is precisely how the mass  $m$  appears. Among the allowed dimension-3 fermion bilinears in (3), there are two mass terms, parametrized by  $m$  and  $m_5$ . We shall operate under the assumption that the  $m_5$  has already been transformed away, so there are only pure Dirac mass terms  $m_u$  and  $m_d$  in the two-flavor QCD Lagrange density. The way these masses (which break chiral symmetry) are encoded in the hadronic sector will provide us with a guide for how to include additional Lorentz-violating terms that may also softly break chiral invariance.

The softest breaking is by terms that are  $CPT$  odd,

$$\mathcal{L}_{\text{light quarks}}^{d=3, CPT\text{-odd}} = -\bar{Q}_L A_{L\mu}\gamma^\mu Q_L - \bar{Q}_R A_{R\mu}\gamma^\mu Q_R, \quad (10)$$

where the  $A_{L/R}^\mu$  have a flavor-space matrix structure analogous to the  $C_{L/R}^{\mu\nu}$ :

$$A_{L/R}^\mu = \begin{bmatrix} a_{u_{L/R}}^\mu & 0 \\ 0 & a_{d_{L/R}}^\mu \end{bmatrix}. \quad (11)$$

Bounds on mSME coupling constants are usually expressed in terms of the vector  $a^\mu$  and axial vector  $b^\mu$  linear combinations,

$$a_{u/d}^\mu = \frac{1}{2}(a_{u_L/d_L}^\mu + a_{u_R/d_R}^\mu), \quad b_{u/d}^\mu = \frac{1}{2}(a_{u_L/d_L}^\mu - a_{u_R/d_R}^\mu). \quad (12)$$

These also have isosinglet and isotriplet linear combinations analogous to  ${}^1C_{L/R}^{\mu\nu}$  and  ${}^3C_{L/R}^{\mu\nu}$ . In terms of these combinations, (10) can be rewritten as

$$\begin{aligned} \mathcal{L}_{\text{light quarks}}^{d=3, CPT\text{-odd}} &= -\bar{Q}_L \left[ {}^3A_{L\mu} + \frac{1}{2}({}^1A_{R\mu} + {}^1A_{L\mu})\mathbb{1} \right] \gamma^\mu Q_L \\ &\quad - \bar{Q}_R \left[ {}^3A_{R\mu} + \frac{1}{2}({}^1A_{R\mu} + {}^1A_{L\mu})\mathbb{1} \right] \gamma^\mu Q_R \\ &\quad - \bar{Q} \frac{1}{2}({}^1A_{L\mu} - {}^1A_{R\mu})\gamma_5\gamma^\mu Q, \end{aligned} \quad (13)$$

which shows that this term includes an isosinglet axial vector current. This form of the Lagrange density is particularly convenient when mapping to  $\chi$ PT.

Following the pattern of (5), there is one remaining possibility for  $d = 3$  operators—those of the  $H^{\mu\nu}$  type. Like the mass terms  $m$  and  $m_5$ , the  $H^{\mu\nu}$  Lorentz violation mixes the left- and right-chiral fields directly, so the  $H^{\mu\nu}$  do not need to have the kind of natural chiral decomposition that the other SME terms possess. In fact, the antisymmetry of  $H^{\mu\nu}$  terms essentially preclude them making contributions to the LO  $\chi$ PT Lagrange density, and so we will have little to say about these operators here.

### B. Gluon operators

There are also mSME operators in the purely gluonic sector. As in the quark sector, the dimension-4 gluon operators are even under  $CPT$ . In a strictly Minkowski spacetime, there is also an  $CPT$ -odd operator with mass dimension 3, but this runs into difficulties when the EFT is embedded in a gravitometric theory such as general relativity. This will ultimately mean that the  $CPT$ -even terms are the only ones that will need to be considered.

Those  $CPT$ -even terms are collected in the form

$$\mathcal{L}_{\text{gluon}}^{d=4, CPT\text{-even}} = -\frac{1}{2} k_G^{\mu\nu\rho\sigma} \text{Tr}(G_{\mu\nu} G_{\rho\sigma}), \quad (14)$$

with two powers of the gluon field strength tensor  $G^{\mu\nu a}$ . This allows for essentially arbitrary bilinear products composed of spatial components of the chromoelectric and chromomagnetic fields, summed symmetrically over the three colors. The four-index tensor  $k_G^{\mu\nu\rho\sigma}$  has the symmetries of the Riemann tensor and is double traceless. (A nonzero double trace would just provide a rescaling of the usual QCD gluon Lagrange density.) Like the Riemann tensor, which can be broken into Ricci and Weyl parts, the  $k_G^{\mu\nu\rho\sigma}$  background can be split into two pieces with different characteristic behaviors,

$$k_G^{\mu\nu\rho\sigma} = \frac{1}{2} (\eta^{\mu\rho} \tilde{k}_G^{\nu\sigma} - \eta^{\mu\sigma} \tilde{k}_G^{\nu\rho} - \eta^{\nu\rho} \tilde{k}_G^{\mu\sigma} + \eta^{\nu\sigma} \tilde{k}_G^{\mu\rho}) + \hat{k}_G^{\mu\nu\rho\sigma}, \quad (15)$$

where  $\tilde{k}_G^{\mu\nu} = k_{G\alpha}^{\mu\alpha\nu}$  is symmetric, traceless in  $(\mu, \nu)$ , and invariant under both  $C$  and  $PT$ .  $\tilde{k}_G^{\mu\nu}$  is the gauge analogue of the  $c_{L/R}^{\mu\nu}$  terms for the chiral fermions. These terms represent there being different “natural” coordinates, which are oblique to the usual Cartesian coordinates, for the affected species.

While  $\tilde{k}_G^{\mu\nu}$  is the “Ricci-like” part of the  $k_G^{\mu\nu\rho\sigma}$  tensor, the “Weyl-like” part is  $\hat{k}_G^{\mu\nu\rho\sigma}$ . The two parts of the tensor have qualitatively different features, and, in general  $\hat{k}_G^{\mu\nu\rho\sigma}$  is expected to be less important in  $\chi$ PT. There are two separate reasons for this. The first reason is that, because it has four free Lorentz indices, any terms in the hadronic Lagrange density will need to involve either multiparticle interactions or additional derivatives. In the mesonic sector, this immediately corresponds to terms that are higher order in the chiral power counting. In the baryon sector, completely symmetrized combinations of the covariant baryon derivatives can be included without a power counting penalty; however, the antisymmetry of the  $\hat{k}_G^{\mu\nu\rho\sigma}$  ensures that these terms vanish.

The second reason is that the electromagnetic analog of  $\hat{k}_G^{\mu\nu\rho\sigma}$  is extremely tightly constrained. The most important qualitative difference between the Ricci-like and Weyl-like tensors in the QED sector of the mSME is that the ten Weyl-like terms generate photon birefringence, while the nine

Ricci-like components do not. The birefringent terms can be bounded extremely well, by looking at photons that have traveled cosmological distances—from radio galaxies,  $\gamma$ -ray bursts, and the cosmic microwave background. Some specific linear combinations of these terms in the photon sector are constrained at the  $10^{-37}$  level, and all the birefringent terms are bounded at the  $10^{-32}$  level, at least. This means that, in many contexts, it is reasonable to neglect the birefringent electromagnetic terms entirely. The bounds on the Weyl-like gluonic terms are not as strong as those for their electromagnetic equivalents. However, there will necessarily be mixing between the different gauge sectors due to radiative corrections. A nonzero  $\hat{k}_G^{\mu\nu\rho\sigma}$  will contribute to the renormalization of the birefringent photon terms; the mixing will be suppressed by powers of the standard model coupling constants, but even with this modest suppression, the  $\hat{k}_G^{\mu\nu\rho\sigma}$  would need to be exceedingly small to be consistent with the existing electromagnetic bounds.

The  $CPT$ -odd operator of dimension 3 has the form

$$\mathcal{L}_{\text{gluon}}^{d=3, CPT\text{-odd}} = k_3^\mu \epsilon_{\mu\nu\rho\sigma} \text{Tr} \left( G^\sigma G^{\nu\rho} + \frac{2}{3} i g_s G^\sigma G^\nu G^\rho \right). \quad (16)$$

The electromagnetic analog of this term will always generate birefringence, so it would also be justifiable to neglect this term in any context in which  $\hat{k}_G^{\mu\nu\rho\sigma}$  could be similarly neglected.

However, there is actually an even stronger reason to drop this term. The Lagrange density in (16) is not gauge invariant on its own. Instead, it changes by a total derivative under a gauge transformation, provided the background tensor  $k_3^\mu$  is a constant. This means that the integrated action is gauge invariant, which is sufficient to ensure the equations of motion are similarly gauge invariant. This is entirely satisfactory in a pure EFT approach in flat spacetime. However, the physical mSME, if it is to represent the Lorentz and  $CPT$  violation that are possible for real-world particles, must be embedded in a dynamical theory of gravitation. Explicit breaking of Lorentz invariance by constant vacuum tensors such as  $k_3^\mu$  is inconsistent with a metric theory of gravitation [16]. Lorentz violation in a Riemannian theory of gravity is only possible if the background tensors are themselves dynamical, with  $k_3^\mu$  being determined by the vacuum expectation value of a dynamical axial vector field; without this, the geometrical Bianchi identities cannot be satisfied. Once there are nontrivial dynamics associated with  $k_3^\mu$ ,  $\mathcal{L}_{\text{gluon}}^{d=3, CPT\text{-odd}}$  no longer changes by a total derivative under a gauge transformation, meaning that the term is not allowed, even in an asymptotically flat spacetime [17]. We shall not, therefore, consider this term any further, although if it were included in the  $\chi$ PT Lagrangian, it would be coupled to hadrons in the same way as a quark  $b^\mu$  term.

### III. ELEMENTS OF $\chi$ PT

With the full quark and gluon Lagrange density set down, we now find ourselves in a position to construct a new, effective Lagrange density for the hadrons. Our analysis of how the Lorentz-violating mSME operators are to be embedded in  $\chi$ PT will begin with a treatment of the purely mesonic Lagrangian. (Some qualitative results for pions can even be extended to their octet partners with nonzero strangeness, especially to  $K$  mesons.) There can be a basically self-contained description of the pions in  $\chi$ PT, without needing to simultaneously introduce nucleons. In contrast, a low-energy  $\chi$ PT treatment of baryons automatically includes, in addition to a description of the free propagation of the baryons, a set of meson-baryon interaction vertices.

Whichever baryon sector is under consideration, using  $\chi$ PT means considering all possible terms that are permitted by the symmetries of the underlying theory [5–7]. Normally, in Lorentz-invariant QCD, this suite of symmetries includes rotations, boosts, and the discrete transformations of  $C$ ,  $P$ , and time reversal ( $T$ ). There is also an accidental chiral symmetry to QCD. This symmetry is exact when the quarks are massless,  $m_u = m_d = 0$ , and even when the masses are nonvanishing, the chiral transformations generate an approximate symmetry that has many useful consequences at energy scales well below the symmetry breaking scale of  $\sim 4\pi F \approx 1$  GeV, where  $F \approx 92.4$  MeV is the pion decay constant. The strongly interacting QCD dynamics break the full chiral symmetry group  $SU(2)_L \times SU(2)_R$  down to its diagonal subgroup  $SU(2)_V$ .<sup>1</sup> The pions are the associated pseudo-Goldstone bosons; in the  $m_u = m_d = 0$  limit, in which the original chiral symmetry is exact, the pions are precisely massless.

The massless, two-flavor QCD Lagrange density

$$\mathcal{L}_{\text{QCD}}^0 = \bar{Q}_L i \not{D} Q_L + \bar{Q}_R i \not{D} Q_R - \frac{1}{2} \text{Tr}(G_{\mu\nu} G^{\mu\nu}) \quad (17)$$

will be the starting point for  $\chi$ PT. (We are continuing to follow our previous convention [9,10] of using the letter variants  $\mathcal{L}$  for Lorentz-invariant Lagrange densities and  $\mathcal{L}$  for Lorentz-violating ones.) In (17),  $Q_{L/R} = [u_{L/R}, d_{L/R}]^T$  are the doublets of left- and right-chiral quark fields; and  $D_\mu q = (\partial_\mu + igG_\mu)q$  is the QCD covariant derivative, with  $G_\mu$  the gluon fields,  $g$  the strong coupling constant, and  $G_{\mu\nu}$  the gluon field strength tensor. If (17) is the entire Lagrange density (that is, if the  $u$  and  $d$  masses, along with any other sources of explicit chiral symmetry breaking, are vanishing), then there are global symmetry transformations,

<sup>1</sup>While it is common to refer to  $SU(2)_L \times SU(2)_R$  as the chiral symmetry group, the QCD Lagrange density for massless  $u$  and  $d$  quarks has a  $U(2)_L \times U(2)_R$  symmetry. Because of the axial anomaly, this reduces to a  $SU(2)_L \times SU(2)_R \times U(1)_V$  symmetry, with the  $U(1)_V$  symmetry related to baryon number. This will be relevant for the baryonic sector.

$$Q_L \rightarrow L Q_L, \quad Q_R \rightarrow R Q_R, \quad (18)$$

where  $(L, R)$  are a pair of matrices in  $SU(2)_L \times SU(2)_R$ .

However, since this chiral symmetry is broken down to  $SU(2)_V$ , there are Goldstone modes. The Goldstone boson fields carry the quantum numbers of the broken symmetry generators. This means that pion fields can be encoded in the  $SU(2)$  matrix [18]

$$U(x) = \exp \left[ i \frac{\phi(x)}{F} \right]. \quad (19)$$

Here,  $\phi = \sum \phi_a \tau_a$  [so that the  $\phi$  contains the three  $SU(2)$  generators], and  $F$  is the pion decay constant in the  $SU(2)$  chiral limit. Global chiral transformations act on  $U(x)$  as

$$U(x) \rightarrow U'(x) = R U(x) L^\dagger, \quad (20)$$

for  $(L, R) \in SU(2)_L \times SU(2)_R$ .

The effective action for the pure pion EFT (the lowest-energy limit of QCD) can be constructed from the matrix  $U(x)$  and its derivatives. The power counting scheme used in  $\chi$ PT dictates that each additional derivative acting on a pion field indicates an additional power of a small parameter; this applies to both spatial and temporal derivatives, because the pion mass is small in the chiral limit. The lowest-order chirally invariant term that can be constructed out of  $U(x)$  contains the meson kinetic terms. The standard LO pion Lagrange density thus has a term of the form

$$\mathcal{L}_\pi^{\text{LO}} \supset \frac{F^2}{4} \text{Tr}(\partial_\mu U \partial^\mu U^\dagger), \quad (21)$$

where the trace  $\text{Tr}$  is taken over flavor space.

However, in real-world QCD, the masses of the light quarks cannot usually be so glibly neglected. Moreover, in addition to gluon interactions, there are also interactions between the quarks and the electroweak gauge fields. Both of these facts can be included in the  $\chi$ PT in a unified way, by treating the quark masses and the electroweak gauge boson fields as external fields. These external fields are included in the QCD Lagrange density in the form

$$\mathcal{L} = \mathcal{L}_{\text{QCD}}^0 + \mathcal{L}_{\text{external}}, \quad (22)$$

in which the coupling to the external fields is described by [6,7]

$$\begin{aligned} \mathcal{L}_{\text{external}} = & \bar{Q}_L \gamma^\mu \left( l_\mu + \frac{1}{3} v_\mu^{(s)} \mathbb{1} \right) Q_L + \bar{Q}_R \gamma^\mu \left( r_\mu + \frac{1}{3} v_\mu^{(s)} \mathbb{1} \right) Q_R \\ & + \bar{Q}_L (s - ip) Q_R + \bar{Q}_R (s + ip) Q_L. \end{aligned} \quad (23)$$

The external fields  $l_\mu$ ,  $r_\mu$ ,  $s$ , and  $p$  can have nontrivial structures in flavor space. As chiral fields,  $l_\mu$  and  $r_\mu$  may be taken to be traceless [the trace part of the Lagrange density

being taken care of through the isosinglet term  $v_\mu^{(s)}$ ; no axial vector singlet term is needed because the diagonal chiral symmetry is broken at a higher energy scale by the chiral anomaly] and thus represented in terms of the generators

$$l^\mu = \frac{1}{2} \sum \tau_a l_a^\mu, \quad r^\mu = \frac{1}{2} \sum \tau_a r_a^\mu. \quad (24)$$

With appropriate choices, these can give the couplings of the quarks to the electroweak gauge bosons. For instance, setting just  $l_3^\mu = r_3^\mu = v^{(s)\mu} = -\frac{1}{2}eA^\mu$  to be nonzero gives the vector couplings of the  $u$  and  $d$  quarks to the electromagnetic four-vector potential  $A^\mu$ . [The combinations including  $v_\mu^{(s)}$  as they appear in (23), which are also frequently useful, can be denoted  $\tilde{l}_\mu = l_\mu + \frac{1}{3}v_\mu^{(s)}\mathbb{1}$  and  $\tilde{r}_\mu = r_\mu + \frac{1}{3}v_\mu^{(s)}\mathbb{1}$ .]

The Dirac mass terms for the  $u$  and  $d$  fields can be introduced similarly, through the scalar external field  $s = \mathcal{M} = \text{diag}(m_u, m_d)$ . [The pseudoscalar  $p$  could be used for Majorana masses like  $m_5$  in (5).] All of the external fields break the chiral symmetry, so the form that this symmetry breaking takes must be mirrored between the Lagrange densities at the QCD level and hadron level. To match the symmetry breaking patterns it is necessary to determine how the external fields would need to transform if (22) were actually to remain chirally invariant. In fact, the Lagrange density (22) is invariant under not just a global chiral transformation, but a local  $(V_L, V_R, \mathcal{U}) \in SU(2)_L \times SU(2)_R \times U(1)_V$ ,

$$\begin{aligned} Q_L &\rightarrow \exp\left[-\frac{i\Theta(x)}{3}\right] V_L(x) Q_L, \\ Q_R &\rightarrow \exp\left[-\frac{i\Theta(x)}{3}\right] V_R(x) Q_R, \end{aligned} \quad (25)$$

so long as the external fields transform as

$$\begin{aligned} l_\mu &\rightarrow V_L l_\mu V_L^\dagger + iV_L \partial_\mu V_L^\dagger, \\ r_\mu &\rightarrow V_R r_\mu V_R^\dagger + iV_R \partial_\mu V_R^\dagger, \\ v_\mu^{(s)} &\rightarrow v_\mu^{(s)} - \partial_\mu \Theta, \\ s + ip &\rightarrow V_R (s + ip) V_L^\dagger, \\ s - ip &\rightarrow V_L (s - ip) V_R^\dagger. \end{aligned} \quad (26)$$

The  $\Theta(x)$  is associated with the  $U(1)_V$  baryon number symmetry, which is separate from the chiral  $SU(2)_L \times SU(2)_R$ . The invariance under *local* chiral transformations ensures that the chiral Ward identities are satisfied [6,19]. With the quark mass terms transforming as  $s$ , (26) implies the transformation behavior  $\mathcal{M} \rightarrow V_R \mathcal{M} V_L^\dagger$ .

At the hadronic level, the particle excitations may also have nonminimal couplings to external fields. To get the

minimal couplings, we ensure invariance under local chiral transformations by replacing the derivative  $\partial_\mu U$  of  $U(x)$  by a covariant derivative with a chiral connection,

$$D_\mu U = \partial_\mu U + iU l_\mu - i r_\mu U. \quad (27)$$

This transforms under local transformations according to  $D_\mu U \rightarrow V_R D_\mu U V_L^\dagger$ . Then the possible nonminimal couplings can be constructed from the ‘‘field strengths’’ formed out of the chiral connection fields  $l^\mu$  and  $r^\mu$ ,

$$f_L^{\mu\nu} = \partial^\mu l^\nu - \partial^\nu l^\mu - i[l^\mu, l^\nu], \quad (28)$$

$$f_R^{\mu\nu} = \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu]. \quad (29)$$

These transform covariantly under the local transformations,

$$f_L^{\mu\nu} \rightarrow V_L f_L^{\mu\nu} V_L^\dagger, \quad f_R^{\mu\nu} \rightarrow V_R f_R^{\mu\nu} V_R^\dagger. \quad (30)$$

The mass enters in a similar fashion, via the external field

$$\chi = 2B(s + ip), \quad (31)$$

transforming as  $\chi \rightarrow V_R \chi V_L^\dagger$ . The constant  $B$  is numerically determined by the nontrivial dynamics of strong-field QCD. However, it can be directly related to the chiral condensate density,  $B = -\frac{1}{2}\langle \bar{Q}Q \rangle$ . Thus the full LO pion Lagrangian, including nonzero quark masses and the couplings to external fields, is given by [7]

$$\mathcal{L}_\pi^{\text{LO}} = \frac{F^2}{4} \text{Tr}(D_\mu U D^\mu U^\dagger) + \frac{F^2}{4} \text{Tr}(\chi U^\dagger + U \chi^\dagger). \quad (32)$$

This provides a relationship,  $M_\pi^2 = -\frac{1}{2}\langle \bar{Q}Q \rangle (m_u + m_d)/F^2$ , between the pion mass and the underlying quark masses. (Although the quark masses are real,  $\chi^\dagger$  is still formally distinguished from  $\chi$  in this situation.)

For the various quantities that can be used to assemble the mesonic Lagrange densities, the power counting scheme is

$$\begin{aligned} U &= \mathcal{O}(q^0), & D^\mu U &= \mathcal{O}(q), \\ \chi &= \mathcal{O}(q^2), & f_{L/R}^{\mu\nu} &= \mathcal{O}(q^2), \end{aligned} \quad (33)$$

where  $q$  is a small momentum expansion parameter.

For the baryonic sector, which resides at a slightly higher natural momentum scale than the minimal meson theory, there are additional quantities that can be invoked in the construction of chirally invariant Lagrange densities. The starting point is the nucleon doublet  $\Psi = [p, n]^T$ , which transforms as [18,20,21]

$$\Psi \rightarrow K(V_L, V_R, U)\Psi, \quad (34)$$



with the matrix  $K(V_L, V_R, U)$  determined in terms of the transformation rules for the square root  $u(x)$  of  $U(x)$ . If  $[u(x)]^2 = U(x)$ , then in order to have  $u(x) \rightarrow \sqrt{V_R U V_L^\dagger}$ , the matrix  $u(x)$  itself must transform according to

$$u(x) \rightarrow V_R u K^\dagger = K u V_L^\dagger. \quad (35)$$

For the baryon field  $\Psi$ , the chiral covariant derivative is more complicated than the one (27) for the pions. Probably most notably, the covariant derivative that acts on the fermions includes not just the external fields, but also the meson fields themselves, which enter through combinations of  $u(x)$  and  $u^\dagger(x)$ ,

$$\Gamma_\mu = \frac{1}{2} [u^\dagger (\partial_\mu - i r_\mu) u + u (\partial_\mu - i l_\mu) u^\dagger], \quad (36)$$

so that

$$D_\mu \Psi = [\partial_\mu + \Gamma_\mu - i v_\mu^{(s)} \mathbb{1}] \Psi. \quad (37)$$

This covariant derivative is constructed so that  $D_\mu \Psi$  transforms in the same way as  $\Psi$  itself,  $D_\mu \Psi \rightarrow K D_\mu \Psi$ .

In addition to a kinetic coupling term involving  $D_\mu \Psi$ , it is well known that the nucleon also has an axial vector coupling term. With this term included, the Lorentz-invariant LO pion-nucleon Lagrangian has the form [22]

$$\mathcal{L}_{\pi N}^{\text{LO}} = \bar{\Psi} \left( i \mathcal{D} - m + \frac{g_A}{2} \gamma^\mu \gamma_5 u_\mu \right) \Psi. \quad (38)$$

In this equation,  $m$  is the nucleon mass and  $g_A$  the axial coupling, both in the chiral limit. At LO, these may be replaced by their physical values of  $m_N \approx 939$  MeV and  $g_A \approx 1.27$ , although there are further corrections to the physical values at higher chiral orders. The chiral vielbein  $u_\mu$  is defined as

$$u_\mu = i [u^\dagger (\partial_\mu - i r_\mu) u - u (\partial_\mu - i l_\mu) u^\dagger], \quad (39)$$

which transforms according to  $u_\mu \rightarrow K u_\mu K^\dagger$ .

Because the nucleon mass  $m_N$  does not vanish in the chiral limit, a timelike derivative acting on the nucleon field will not be suppressed, even at low energies. This affects the chiral  $q$ -counting scheme. The additional building blocks defined in the nucleon sector are counted as

$$\Psi = \mathcal{O}(q^0), \quad D_\mu \Psi = \mathcal{O}(q^0), \quad u_\mu = \mathcal{O}(q). \quad (40)$$

However, because  $\Psi$  must obey a field equation, the particular combination  $(i \mathcal{D} - m_N) \Psi$  is counted as  $\mathcal{O}(q)$ . This means, for instance, that  $\mathcal{D} \Psi$  may be exchanged for  $-i m_N \Psi$  if terms of higher chiral orders are being neglected [23].

## IV. LORENTZ-VIOLATING MESONIC LAGRANGE DENSITY

### A. CPT-even operators

The Lagrange density (32) can be generalized in a straightforward way to include Lorentz violation coming from the quark and gluon sectors. We shall begin with generalizations to the kinetic Lagrange density (21). The results with just the dimension-4 quark terms have already been given [9]. The argument that led to these terms was based on matching the transformation properties of the QCD-level Lagrange density (8) onto the equivalent meson-scale Lagrange density. Under a chiral transformation with matrices  $(L, R)$ , the doublets of  $u$  and  $d$  quark fields transform as  $Q_R \rightarrow R Q_R$  and  $Q_L \rightarrow L Q_L$ . This takes (8)

$$\begin{aligned} \mathcal{L}_{\text{light quarks}}^{d=4, \text{CPT-even}} &\rightarrow i \bar{Q}_L L^\dagger C_{L\mu\nu} L \gamma^\mu D^\nu Q_L \\ &+ i \bar{Q}_R R^\dagger C_{R\mu\nu} R \gamma^\mu D^\nu Q_R. \end{aligned} \quad (41)$$

With constant matrices  $C_{L/R}^{\mu\nu}$  that do not transform under  $SU(2)_L \times SU(2)_R$ , the presence of the Lorentz-violating term (8) would break the chiral symmetry. However, if the  $C_{L/R}^{\mu\nu}$  were also to transform,

$$C_L^{\mu\nu} \rightarrow L C_L^{\mu\nu} L^\dagger, \quad C_R^{\mu\nu} \rightarrow R C_R^{\mu\nu} R^\dagger, \quad (42)$$

the chiral symmetry would be restored. Since the transformation properties (42) would keep the quark-level Lagrange density chirally invariant, applying those same transformation prescriptions must also maintain the chiral symmetry at the hadron level. This rule allows us to identify what kinds of operators the  $C_{L/R}^{\mu\nu}$  can be associated with in the pion Lagrange density.

The transformation properties (42) are more usefully expressed in terms of the isospin singlet and triplet components of the  $C_{L/R}^{\mu\nu}$ . The isosinglet is useful because it does not transform at all under chiral rotations, while the isotriplet retains the transformation properties of the underlying  $C_{L/R}^{\mu\nu}$ .

Moreover, along with the  $C_{L/R}^{\mu\nu}$ , which modify the kinetic terms in the quark Lagrange density, there is also the gluon  $\tilde{k}_G^{\mu\nu}$ , which—since it appears in a term (14) involving only the gauge fields—also does not transform at all under the action of the chiral  $SU(2)_L \times SU(2)_R$ . So the transformation rules for the coefficients of the dimension-4 operators are

$$\begin{aligned} {}^1 C_L &\rightarrow {}^1 C_L, & {}^3 C_L &\rightarrow L^3 C_L L^\dagger, \\ {}^1 C_R &\rightarrow {}^1 C_R, & {}^3 C_R &\rightarrow R^3 C_R R^\dagger, \\ \tilde{k}_G^{\mu\nu} &\rightarrow \tilde{k}_G^{\mu\nu}. \end{aligned} \quad (43)$$

These transformation rules—(42) or (43), along with the discrete transformation properties of the SME terms—are

sufficient for us to determine the qualitative forms of the operators these coefficients are associated with in the LO mesonic Lagrangian. The process begins with writing down all the possible operator forms that are consistent with the chiral symmetry. However, the Lorentz-violating terms in the quark-level Lagrange density are also the only potential sources of *C*, *P*, and *T* violations in the theory. So at LO, any terms in the hadronic Lagrange densities need to have the same discrete symmetries as the terms in the underlying quark density that are multiplied by the same SME coefficients. This means

$$\mathcal{L}_\pi^{d=4,\text{LO}} = [\beta^{(1)}({}^1C_{R\mu\nu} + {}^1C_{L\mu\nu}) + \beta^{(3)}\tilde{k}_{G\mu\nu}] \frac{F^2}{4} \text{Tr}[(D^\mu U)^\dagger D^\nu U] + \beta^{(2)} \frac{F^2}{4} \text{Tr}[(D^\mu U)^\dagger {}^3C_{R\mu\nu} D^\nu U + D^\mu U {}^3C_{L\mu\nu} (D^\nu U)^\dagger] \quad (44)$$

where the  $\beta^{(n)}$  are dimensionless LECs. (The “ $d = 4$ ” superscript denotes the mass dimension of the operators in the underlying QCD Lagrange density that give rise to this mesonic expression, rather than the dimension of the  $\mathcal{L}_\pi^{d=4,\text{LO}}$  operators themselves.) The factor of  $F^2/4$  in (44) is present to mirror the form of the standard pion Lagrange density and is also chosen such that based on naive dimensional analysis [24], the  $\beta^{(n)}$  are expected to have a natural size that is  $\mathcal{O}(1)$ . Actually, the  $\beta^{(2)}$  term does not contribute at all at leading order. It was shown in [9] that with symmetric tensors  ${}^3C_{L/R}^{\mu\nu}$ , the  $\beta^{(2)}$  reduces to a total derivative. As we shall see below, this actually holds for antisymmetric  ${}^3C_{L/R}^{\mu\nu}$  as well.

The short-distance QCD physics is entirely encapsulated in the LECs. A complete determination of their values would entail the use of nonperturbative QCD, and to our knowledge, no numerical computation of these values has thus far been undertaken. Relative to the formulation given in [9,10], the portion of (44) that is symmetric in  $(\mu, \nu)$  contains one additional term, since in addition to the four quark tensors  $c_{u_L}^{\mu\nu}$ ,  $c_{d_L}^{\mu\nu}$ ,  $c_{u_R}^{\mu\nu}$ , and  $c_{d_R}^{\mu\nu}$ , (44) also includes the contribution from the gluon tensor  $\tilde{k}_G^{\mu\nu}$  [12]. However, it turns out that, when all five of these tensors from the mSME are included, there is actually a nontrivial relation between the LECs, which will allow us to express  $\beta^{(3)}$  in terms of  $\beta^{(1)}$ .

What the  $c^{\mu\nu}$  and  $\tilde{k}_G^{\mu\nu}$  tensors represent is a form of Lorentz violation in which the natural spacetime coordinates for different standard model fields are actually different. Having solely a nonzero  $c_{u_L}^{\mu\nu}$ , for example, indicates that the left-chiral *u* quarks propagate according to normal relativistic rules in a coordinate system that is oblique to the usual coordinates. If we change to the oblique coordinates, which are given (at leading order) by  $x'^\mu = x^\mu - \frac{1}{2}(c_{u_L})^\mu{}_\nu x^\nu$ , the dynamics for the *u* quark field are standard, but all the other fields will have Lorentz-violating behavior, dictated by  $c'^{\mu\nu} = \frac{1}{2}\tilde{k}'^{\mu\nu} = -c_{u_L}^{\mu\nu}$  for the remaining species. The fact

that the coefficients for left- and right-handed quark fields must always enter the pion Lagrange density multiplied by the same low-energy couplings (LECs). In this way, imposing the discrete symmetries drastically reduces the number of independent terms in the Lagrangian. Moreover, a number of the remaining terms turn out to be linearly dependent (or at least linearly dependent at LO). Using integration by parts, the additional redundant terms may also be eliminated from the description of the theory.

The LO minimal mesonic Lagrange density is given by

that *c*-type Lorentz violation can be moved from one sector to another by coordinate redefinitions like these means that any physical measurement of a *c*-type coefficient really has to be a measurement of a difference of the coefficients for different particle types.

Expanding  $U(x)$  to second order in the pion fields, the Lagrange density (44) gives the Lorentz-violating kinetic terms for the pions. [Expanding  $U(x)$  to higher orders in the pion fields produces Lorentz-violating meson interaction vertices.] The two-pion portion of the Lagrange density is

$$\mathcal{L}_\pi^{\text{LO},2\phi} = \left[ \frac{\beta^{(1)}}{4} (c_{u_L}^{\mu\nu} + c_{d_L}^{\mu\nu} + c_{u_R}^{\mu\nu} + c_{d_R}^{\mu\nu}) + \frac{\beta^{(3)}}{2} \tilde{k}_G^{\mu\nu} \right] \partial_\mu \phi_a \partial_\nu \phi_a \quad (45)$$

$$= \left[ \frac{\beta^{(1)}}{4} (c_{u_L}^{\mu\nu} + c_{d_L}^{\mu\nu} + c_{u_R}^{\mu\nu} + c_{d_R}^{\mu\nu}) + \frac{\beta^{(3)}}{2} \tilde{k}_G^{\mu\nu} \right] \times (\partial_\mu \pi^+ \partial_\nu \pi^- + \partial_\mu \pi^- \partial_\nu \pi^+ + \partial_\mu \pi^0 \partial_\nu \pi^0). \quad (46)$$

[ $\pi^0 = \phi_3$ ,  $\pi^+ = \frac{1}{\sqrt{2}}(\phi_1 - i\phi_2)$ , and  $\pi^- = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$  are the physical pion fields.] This  $\mathcal{L}_\pi^{\text{LO},2\phi}$  has the form of the Lorentz-violating *k* term from (7). There are three species of pions, but in the chiral limit, there is a just single

$$k_\pi^{\mu\nu} = \frac{\beta^{(1)}}{2} (c_{u_L}^{\mu\nu} + c_{u_R}^{\mu\nu} + c_{d_L}^{\mu\nu} + c_{d_R}^{\mu\nu}) + \beta^{(3)} \tilde{k}_G^{\mu\nu} \quad (47)$$

tensor common to all three of the physical pion fields. Note that since the pion wave functions are all equal mixtures of left- and right-chiral, *u* and *d* quarks, the quark portion of  $k_\pi^{\mu\nu}$  receives equal contributions from each of the four quark types.

The nontrivial relation between  $\beta^{(1)}$  and  $\beta^{(3)}$  arises from the fact that, by making a change of coordinates in the usual two-flavor QCD Lagrange density  $x^\mu \rightarrow x'^\mu = x^\mu + \kappa^\mu{}_\nu x^\nu$  (for some arbitrary symmetric tensor  $\kappa^{\mu\nu}$ ), we can turn the conventional QCD expression into a Lorentz-violating

Lagrange density with  $c_{u_L}^{\mu\nu} = c_{d_L}^{\mu\nu} = c_{u_R}^{\mu\nu} = c_{d_R}^{\mu\nu} = \frac{1}{2}\tilde{k}_G^{\mu\nu} = \kappa^{\mu\nu}$ . Since the theory this describes is really just the standard, Lorentz-invariant one, merely viewed in unconventional coordinates, the pion sector must also be the usual one, expressed in the same oblique coordinates. This means that  $\frac{1}{2}k_\pi^{\mu\nu} = \kappa^{\mu\nu}$  also.

Taken together with (46), this relation indicates that  $\beta^{(3)} = 1 - \beta^{(1)}$ . The same kind of relation for the  $c$ -type Lorentz-violation coefficients for composite particles was found in [25], with the coefficient for a composite being a sum of the constituents' coefficients, each one weighted by the fraction of the total momentum carried by a particular constituent. In this case,  $\beta^{(1)}$  represents the fraction of the pion momentum carried by all the constituent quarks, with the remainder carried by the gluons. The values of these weights still cannot be determined without recourse to nonperturbative QCD, but (47) does simplify to

$$k_\pi^{\mu\nu} = \frac{\beta^{(1)}}{2}(c_{u_L}^{\mu\nu} + c_{u_R}^{\mu\nu} + c_{d_L}^{\mu\nu} + c_{d_R}^{\mu\nu}) + [1 - \beta^{(1)}]\tilde{k}_G^{\mu\nu}. \quad (48)$$

This specific results also supports the general presumption that each of the LECs should be  $\mathcal{O}(1)$ .

The  $k_\pi^{\mu\nu}$  coefficients are the easiest ones to observe directly for pions. They affect the energy-momentum relations for ultrarelativistic pions, which in turn can lead to new thresholds (including upper energy thresholds) for reactions involving extremely energetic mesons. There are also pion vertices, which are in some cases straightforward Lorentz-violating generalizations of the usual pion vertex operators, involving even numbers of fields. The form of (46) involves the insertion of a Lorentz-violating symmetric tensor between the  $(\mu, \nu)$  indices of the derivatives  $\partial_\mu\phi_a\partial_\nu\phi_a$ . At higher orders in the fields  $\phi_a$ , there are homologous expressions, such as

$$\mathcal{L}_\pi^{\text{LO},4\phi} = \frac{k_\pi^{\mu\nu}}{6F^2}(\phi_a\phi_b\partial_\mu\phi_a\partial_\nu\phi_b - \phi_b\phi_b\partial_\mu\phi_a\partial_\nu\phi_a) \quad (49)$$

at fourth order. Note that all these higher-order terms depend on the same linear combination of quark and gluon SME coefficients.

Naively it looks like there might be other terms, associated with the antisymmetric parts of  ${}^1C_{L/R}^{\mu\nu}$  and  ${}^3C_{L/R}^{\mu\nu}$  or with the  $H^{\mu\nu}$ , which would be qualitatively different in structure. (Note that, by virtue of its structure,  $\tilde{k}_G^{\mu\nu}$  cannot have an antisymmetric part, so that the antisymmetric terms can only involve quark parameters.) For example, if the  $C_{L/R}^{\mu\nu}$  are all antisymmetric, then direct expansion of the Lagrange density gives

$$\mathcal{L}_\pi^{\text{LO}} \supset \frac{\beta^{(2)}}{4}(c_{u_L}^{\mu\nu} + c_{u_R}^{\mu\nu} + c_{d_L}^{\mu\nu} + c_{d_R}^{\mu\nu})(\partial_\mu\phi_1\partial_\nu\phi_2 - \partial_\nu\phi_1\partial_\mu\phi_2). \quad (50)$$

However, (50) is actually a total derivative (both with respect to  $\partial_\mu$  and  $\partial_\nu$ ), which makes no contribution to the physics.

We might also anticipate a three- $\phi$  term involving  $\partial_\mu\phi_3\partial_\nu\phi_a\phi_a - \partial_\nu\phi_3\partial_\mu\phi_a\phi_a$ , or equivalently,  $\partial_\mu\pi^0(\pi^-\partial_\nu\pi^+ + \pi^+\partial_\nu\pi^-) - \partial_\nu\pi^0(\pi^-\partial_\mu\pi^+ + \pi^+\partial_\mu\pi^-)$ . However, not only would this term be another total derivative, but the three-pion form gives an operator that is manifestly odd under  $C$ , which does not match the symmetry of the SME coefficients multiplying the term; this  $C$ -odd behavior is a general feature of antisymmetric tensor SME coefficients in scalar field theories [26]. In fact, there appears to be no term that can be written down in the pion sector at LO that involves an antisymmetric tensor structure. This observation was already prefigured by the fact that there was no antisymmetric tensor among the external fields (26) that could be coupled to the hadrons at leading order. This also justifies the absence of any LO terms involving  $\hat{k}_G$ , which is separately antisymmetric in two sets of Lorentz indices.

## B. CPT-odd operators

For the  $d = 3$ ,  $CPT$ -odd operators coming from the quark sector, finding their couplings to pions is actually quite straightforward. These terms can simply be inserted as external fields of the left- and right-chiral vector forms, through  $-\tilde{\gamma}^\mu$  and  $-\tilde{\gamma}^\mu$ . The correct signs and magnitudes for these terms can be read off directly from the SME coupling (10) [or equivalently (13)] to the quarks. The pion term is then

$$\begin{aligned} \mathcal{L}_\pi^{d=3,\text{LO}} &= \frac{F^2}{4}\text{Tr}[(\partial^\mu U + iUA_L^\mu - iA_R^\mu U) \\ &\quad \times (\partial^\mu U + iUA_L^\mu - iA_R^\mu U)^\dagger] - \frac{F^2}{4}\text{Tr}(\partial_\mu U \partial^\mu U^\dagger). \end{aligned} \quad (51)$$

The scalar part with  $v_\mu^{(s)}$  cancels between the left- and right-chiral terms, which ensures that the expression has the correct behavior under  $C$  and  $P$  transformations. Moreover, (51) is structured to contain only Lorentz-violating terms, since the usual LO meson kinetic term has been explicitly subtracted away. In Lorentz-invariant  $\chi$ PT, the singlet axial-vector current is not considered, and even in the SME, it is not possible to construct an axial vector current operator entirely out of pseudoscalar meson fields.

Simplifying (51), and noting that  $A_L^\mu + A_R^\mu = (a_u^\mu + a_d^\mu)\mathbb{1} + (a_u^\mu - a_d^\mu)\tau_3$ , the  $CPT$ -odd expression reduces to

$$\mathcal{L}_\pi^{d=3,\text{LO}} \supset \frac{i}{4}\text{Tr}[(A_L^\mu + A_R^\mu)(\phi_a\partial_\mu\phi_b - \partial_\mu\phi_a\phi_b)\tau_a\tau_b] \quad (52)$$

$$= -\frac{i}{2}(a_{u_L}^\mu + a_{u_R}^\mu - a_{d_L}^\mu - a_{d_R}^\mu)(\pi^+\partial_\mu\pi^- - \pi^-\partial_\mu\pi^+), \quad (53)$$

up to a total derivative. The form of (53) is essentially what is expected for a charged spin-0 field. Note that this kind of term cannot exist for a single real scalar field, so the  $CPT$ -odd term does not affect the  $\pi^0$  part of the Lagrange density. As far back as [27], it was argued that the net  $a_\phi^\mu$  term for a meson should be a difference of the  $a$ -type coefficients for the constituent quark fields, times a dimensionless factor not too different from unity. This calculation grounds that conclusion firmly in  $\chi$ PT. In fact, since the  $a$ -type terms are odd under  $C$ , but independent of spin and momentum, it makes sense that the expectation value of the contribution from virtual quark-antiquark pairs to the net meson  $a_\phi^\mu$  should vanish.

Like the  $c$ -type coefficients, the  $a^\mu$  coefficients for fermions can only be observed as differences between different species, not in isolation. Moreover, the difference must be between the coefficients for species that can interconvert. For example, in a theory with multiple species of massless, noninteracting fermions, none of the  $a_{L/R}^\mu$  can be observed by propagation effects. The free propagation of a particle with SME coefficient  $a^\mu$  and momentum  $p^\mu$  is indistinguishable from the motion of a particle with  $a^\mu = 0$  and momentum  $p'^\mu = p^\mu - a^\mu$ , and without the ability to create or annihilate particles, it is impossible to make an absolute measurement of the momentum carried by an excitation. Introducing a Dirac mass term generates a coupling between the left- and right-chiral fermion modes, which makes differences of  $a_L^\mu$  and  $a_R^\mu$  physically observable; these are precisely the fermion  $b^\mu$  terms, which affect the energy-momentum relations of massive particles in a directly observable fashion.

The reason that only differences between  $a^\mu$  values are observable is tied to the observation that  $a^\mu$  effectively represents a translation of the momentum space for a single species. That translation can be undone by applying a field redefinition [13] that changes the phase of the fermion field

by  $e^{-ia \cdot x}$ . For uncoupled species, the phases of their fields may be varied independently. However, if two types of fermions are coupled by an interaction term of the form  $\bar{\psi}_a C \psi_b$ , then the phases of  $\psi_a$  and  $\psi_b$  cannot be set separately; trying to define away both  $a_a^\mu$  and  $a_b^\mu$  will leave behind a residual term in the Lagrange density, proportional to  $a_a^\mu - a_b^\mu$ .

The combination  $a_{\pi^+}^\mu = a_u^\mu - a_d^\mu = \frac{1}{2}(a_{u_L}^\mu + a_{u_R}^\mu - a_{d_L}^\mu - a_{d_R}^\mu)$  that appears in (53) is thus not actually yet an observable, since it is a difference of  $a$ -type parameters for two species ( $u$  and  $d$  quarks) which do not have the same charge and thus cannot interconvert. In fact, to form a physical observable, we must construct a difference of two  $a$ -type parameters for like-charged meson species. (There are possible exceptions to this rule if the  $a$ -type coefficients are to be measured in a gravitational experiment; however, even there, nonminimal gravitational couplings are required, placing this scenario outside the mSME framework.) We shall return to this topic in Sec. VI, when we discuss experimental bounds on  $CPT$  violation for mesons.

## V. LORENTZ-VIOLATING BARYONIC LAGRANGE DENSITY

### A. $CPT$ -even operators

The analysis of the contributions from dimension-4 mSME operators in the nucleon sector proceeds along similar lines to the treatment in the pion sector. Again, there is a straightforward generalization of earlier results [9,10] to include the additional contributions from a gluon  $\tilde{k}_G^{\mu\nu}$  term. Because of the presence of chirally covariant derivatives, the form of the free baryon Lagrange density also determines the LO meson-baryon couplings.

The LO baryonic Lagrange density for the nucleon doublet field  $\Psi$  is

$$\begin{aligned} \mathcal{L}_{\pi N}^{d=4, \text{LO}} = & \alpha^{(1)} \bar{\Psi} [(u^\dagger)^3 C_R^{\mu\nu} u + u^3 C_L^{\mu\nu} u^\dagger] (\gamma_\nu i D_\mu + \gamma_\mu i D_\nu) \Psi + \alpha^{(2)} ({}^1 C_R^{\mu\nu} + {}^1 C_L^{\mu\nu}) \bar{\Psi} (\gamma_\nu i D_\mu + \gamma_\mu i D_\nu) \Psi \\ & + \alpha^{(3)} \bar{\Psi} [(u^\dagger)^3 C_R^{\mu\nu} u - u^3 C_L^{\mu\nu} u^\dagger] (\gamma_\nu \gamma^5 i D_\mu + \gamma_\mu \gamma^5 i D_\nu) \Psi + \alpha^{(4)} ({}^1 C_R^{\mu\nu} - {}^1 C_L^{\mu\nu}) \bar{\Psi} (\gamma_\nu \gamma^5 i D_\mu + \gamma_\mu \gamma^5 i D_\nu) \Psi \\ & + \alpha^{(5)} \tilde{k}_G^{\mu\nu} \bar{\Psi} (\gamma_\nu i D_\mu + \gamma_\mu i D_\nu) \Psi, \end{aligned} \quad (54)$$

where the  $\alpha^{(n)}$ 's are the dimensionless LECs for this sector of the theory. By naive dimensional analysis, these are again anticipated to be  $\mathcal{O}(1)$ . The structural properties of these various terms are discussed in detail in [9].

As there was for the pions, there is a nontrivial constraint coming from the fact that, when all the quark  $c_{u_L}^{\mu\nu} = c_{d_L}^{\mu\nu} = c_{u_R}^{\mu\nu} = c_{d_R}^{\mu\nu}$  and gluon  $\frac{1}{2} \tilde{k}_G^{\mu\nu}$  are equal to  $\kappa^{\mu\nu}$ , the theory is really just conventional QCD written in skewed coordinates. From the expression for the proton coefficient

$$\begin{aligned} c_p^{\mu\nu} = & \alpha^{(1)} (c_{u_L}^{\mu\nu} + c_{u_R}^{\mu\nu} - c_{d_L}^{\mu\nu} - c_{d_R}^{\mu\nu}) \\ & + \alpha^{(2)} (c_{u_L}^{\mu\nu} + c_{u_R}^{\mu\nu} + c_{d_L}^{\mu\nu} + c_{d_R}^{\mu\nu}) + 2\alpha^{(5)} \tilde{k}_G^{\mu\nu}, \end{aligned} \quad (55)$$

it again follows, from  $c_p^{\mu\nu} = \kappa^{\mu\nu}$ , that  $\alpha^{(5)} = \frac{1}{4} - \alpha^{(2)}$ . [Precisely the same result could be obtained from the neutron coefficient  $c_n^{\mu\nu}$ , because the  $\alpha^{(1)}$  term, which changes sign between protons and neutrons, vanishes when all the quark coefficients are equal.] So in spite of the inclusion of the additional gluonic SME coefficients

relative to [9], the number of independent LECs corresponding to the  $d = 4$  QCD operators has not increased.

### B. *CPT*-odd operators

The LO contributions from the *CPT*-violating vector and axial vector operators enter through their couplings to the chiral connection (36). Here, in order to get the correct *C* and *P* transformation properties, we must set the chiral sources  $l_\mu = -{}^3A_{L\mu}$  and  $r_\mu = -{}^3A_{R\mu}$ . In addition, from comparing (13) and (23), we see that

$$v_\mu^{(s)} = -\frac{3}{2}({}^1A_L^\mu + {}^1A_R^\mu) = -\frac{3}{4}(a_{u_L}^\mu + a_{u_R}^\mu + a_{d_L}^\mu + a_{d_R}^\mu). \quad (56)$$

Inserting these into the chiral covariant derivative gives

$$(D_\mu - \partial_\mu)\Psi = \frac{1}{2} \left\{ u^\dagger (\partial_\mu + i{}^3A_{R\mu})u + u(\partial_\mu + i{}^3A_{L\mu})u^\dagger + i\frac{3}{2}[{}^1A_{L\mu} + {}^1A_{R\mu}]\mathbb{1} \right\} \Psi. \quad (57)$$

There is also the axial coupling term, which likewise depends on  $l_\mu$  and  $r_\mu$ ,

$$\frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu = i\frac{g_A}{2}\gamma^\mu\gamma_5 [u^\dagger(\partial_\mu + i{}^3A_{R\mu})u - u(\partial_\mu + i{}^3A_{L\mu})u^\dagger]. \quad (58)$$

In addition, we need to include the singlet axial vector contribution from the quark-level Lagrange density. While chiral symmetry does not constrain this piece of the interaction and thus provides no relationships between various terms with different numbers of pion fields, only the contribution without pions will be relevant for the following discussion. The corresponding baryonic operator takes the form

$$\mathcal{L}_N^{d=3} \supset -\alpha^{(6)}\bar{\Psi}\gamma_5\gamma^\mu({}^1A_{L\mu} - {}^1A_{R\mu})\mathbb{1}\Psi, \quad (59)$$

where  $\alpha^{(6)}$  is a new LEC. (If we had considered hadronic terms arising from the *CPT*-odd gluon operator with coefficient  $k_3^\mu$ , they would also have entered here, through yet another  $\bar{\Psi}\gamma_5\gamma^\mu\Psi$  operator with another new LEC.)

So, with the neglect of the pion coupling terms [setting  $u(x) = 1$ ] the *CPT*-violating part of the purely baryonic action reads

$$\mathcal{L}_N^{d=3} = \bar{\Psi} \left\{ \gamma_\mu \frac{1}{2} [-({}^3A_L^\mu + {}^3A_R^\mu) - 3({}^1A_L^\mu + {}^1A_R^\mu)\mathbb{1}] - \frac{g_A}{2}\gamma_5\gamma_\mu({}^3A_L^\mu - {}^3A_R^\mu) - \alpha^{(6)}\gamma_5\gamma_\mu({}^1A_L^\mu - {}^1A_R^\mu)\mathbb{1} \right\} \Psi. \quad (60)$$

From this, coefficients such as the proton  $a^\mu$  and  $b^\mu$  can be read off,

$$a_p^\mu = (a_{u_L}^\mu + a_{u_R}^\mu) + \frac{1}{2}(a_{d_L}^\mu + a_{d_R}^\mu) = 2a_u^\mu + a_d^\mu, \quad (61)$$

$$\begin{aligned} b_p^\mu &= \frac{g_A}{4}(a_{u_L}^\mu - a_{u_R}^\mu) + \frac{\alpha^{(6)}}{2}(a_{u_L}^\mu - a_{u_R}^\mu + a_{d_L}^\mu - a_{d_R}^\mu) \\ &= \frac{g_A}{2}b_u^\mu + \alpha^{(6)}(b_u^\mu + b_d^\mu). \end{aligned} \quad (62)$$

Since  $b_p^\mu$  is directly observable, it is a sum of direct differences between the  $a$ -type coefficients for pairs of equally charged chiral species. Moreover, while  $a_p^\mu$  is not an independent physical observable, it has a very natural form—the sum of the (spin-averaged)  $a$ -type coefficients for the proton's three valance quarks. It is actually quite remarkable that, at LO, there is only a single undetermined LEC (which only affects the baryons'  $b$ -type coefficients, not any of the  $a$ -type coefficients) that appears in the dimension-3 Lagrange densities for both the pions and the nucleons.

## VI. EXPERIMENTAL CONSTRAINTS

We shall now turn to an exploration of how the various LECs for mesons and baryons can be constrained using existing and future experimental data. In purely phenomenological analyses, it has been commonplace to assign a separate set of SME coefficients to each observable hadron species. However, this will end up significantly overcounting the number of independent parameters, because the true number of mSME coefficients for strongly interacting particles is determined by the structure of the quark and gluon sectors. The coefficients for different types of hadrons are not independent, and this makes it possible to carry bounds over from one part of the strongly interacting sector to another. There will be modest uncertainties, due to the presence of unknown LECs; however, it will be possible to set constraints on the SME parameters for baryons using measurements made on mesons, and vice versa. This is one of the things that makes  $\chi$ PT such a powerful technique.

We have previously discussed [9] how bounds on pion Lorentz violation could be improved by making reference to atomic clock experiments that measured Lorentz violation for nucleons, and [12] took a similar approach to constraining the gluon coefficients  $\tilde{k}_G^\mu$ .  $\chi$ PT methods can also be used to help isolate Lorentz-violating observables in the weak sector [10]. All these approaches have dealt with the dimension-4, *CPT*-even coefficients. Since this paper has, for the first time, given a  $\chi$ PT description of dimension-3, *CPT*-odd operators for quarks, gluons, and hadrons, we shall primarily concentrate our attention on how new bounds may be placed on these dimension-3 operators.

However, we should first point out that the specific bounds derived in [9] were set under the simplifying assumption that there was no dimension-4 Lorentz violation in the gluon Lagrange density. In that case, particular sums of proton and neutron observables ended up probing the exact same linear combinations  $c_{u_L}^{\mu\nu} + c_{u_R}^{\mu\nu} + c_{d_L}^{\mu\nu} + c_{d_R}^{\mu\nu}$  as a separate set of pion observables (in the chiral limit). Meanwhile, [12] adopted a complementary approach, effectively assuming that there was Lorentz violation in the gluon sector, and none for the quarks. If, as discussed here, all the phenomenologically viable dimension-4 QCD operators are included, the actual effective coefficients for mesons and baryons are linear combinations of elements from the quark and gluon Lagrange densities, and the relative weights for the two kinds of coefficients are not known. As a result, bounds such as those derived in [9,12] should be considered order of magnitude estimates for the sizes of the underlying quark and gluon SME coefficients; the bounds (at the  $10^{-19} - 10^{-27}$  levels) represent the largest those coefficients could be without there being unnatural fine tuning in the form of a nearly exact cancellation between the quark and gluon parameters.

We now turn to the experimental status of the dimension-3 hadronic terms. In many cases, the  $b$ -type coefficients for nucleons are extremely well bounded. The reason is that the  $b^\mu$  coefficients alter the energies of spin states, meaning that these coefficients can be measured in extremely sensitive spin flip and spin precession experiments. Except for the proton time component  $b_p^T$ , all the components of  $b_p^\mu$  and  $b_n^\mu$  have been bounded at the  $10^{-25}$  GeV level or better [4]. Bounds on forms of Lorentz violation are by convention expressed in a system of Sun-centered celestial equatorial coordinates  $(T, X, Y, Z)$ , with the  $Z$ -axis coinciding with the Earth's rotation axis. The  $X$ - and  $Y$ -components of a vector such as  $b_p^\mu$  are relatively easy to constrain, because they affect observables that oscillate as the Earth rotates; bounds on a  $Z$ -component are trickier, since while such a component does give rise to anisotropic phenomena, they are not of a type that can be observed just by looking for sidereal variations in some observable; and measuring a time component is the hardest, as it requires a direct test of either boost invariance or a discrete symmetry. This explains why  $b_p^T$  has, thus far, only been bounded at the  $3 \times 10^{-8}$  level [28].

However, before we delve into questions about the  $b$ -type coefficients for quarks, we shall consider a much less well studied area of the SME—the  $a$ -type coefficients for baryons. As pointed out in Sec. IV B, the  $a^\mu$  are only observable as differences between the coefficients for like-charged particles that can be interconverted. This immediately means that to set any experimental bounds, it is necessary to go beyond two-flavor QCD;  $a_p^\mu - a_n^\mu$  is not a QCD observable, even in principle. We shall therefore extend our analysis to three-flavor QCD, with a  $s$  quark and assuming that there is a fairly robust  $SU(3)_f$  symmetry.

TABLE I. Strengths of the existing constraints on the *CPT*-violating differences between the  $a$ -type coefficients for  $d$  and  $s$  quarks. The values are taken from [4], based on experimental kaon results reported in [29,30].

Coefficient	Bound
$\Delta a^X = \frac{1}{2}(a_{d_L}^X + a_{d_R}^X - a_{s_L}^X - a_{s_R}^X)$	$10^{-21}$ GeV
$\Delta a^Y$	$10^{-21}$ GeV
$\Delta a^Z$	$10^{-17}$ GeV
$\Delta a^T$	$10^{-16}$ GeV

With this assumption, the  $a$ -type coefficients for kaons as well as pions can be inferred from our formulas [as the kaons are also pseudo-Goldstone bosons for the spontaneously broken  $SU(3)_L \times SU(3)_R$ , together with the pions and the  $\eta_8$ , they form a flavor octet; we briefly discuss the extension of our  $\chi$ PT methods to the  $SU(3)_f$  sector in the Appendix]. Specifically, the kaon coefficient is  $a_{K^0}^\mu = a_d^\mu - a_s^\mu$  with no  $s$ - $d$  mixing. Since the  $K^0$  can oscillate into a  $\bar{K}^0$ , it is possible to measure the difference of  $a_{K^0}$  and  $a_{\bar{K}^0} = -a_{K^0}$ . A number of strong bounds on the difference in quark coefficients, as measured in kaon oscillations experiments, have been reported in the literature. The orders of magnitude of the best current constraints are listed in Table I.

What is remarkable is that, in the  $SU(3)_f$  limit, the difference  $a_d^\mu - a_s^\mu$  is the basis of another observable: the difference between the  $a$ -type coefficients for octet baryons that differ in their valence quark content by the replacement of a  $d$  quark with a  $s$  quark. This means a difference such as  $a_p^\mu - a_{\Sigma^+}^\mu$ , or the even more exotic  $a_{\Sigma^-}^\mu - a_{\Xi^-}^\mu$ . The key relations follow from (61) and its analogs for other species; these yield, for example,

$$a_p^\mu - a_{\Sigma^+}^\mu = \frac{1}{2}(a_{d_L}^\mu + a_{d_R}^\mu - a_{s_L}^\mu - a_{s_R}^\mu). \quad (63)$$

Conservative bounds [leaving at least an order of magnitude buffer to account for possible deviations from  $SU(3)_f$  symmetry] on such quantities are listed in Table II.

One thing that is notable about these bounds is that no method for constraining these baryon coefficient

TABLE II. Order of magnitude bounds for differences between the  $a$ -type coefficients for  $SU(3)_f$  octet baryons  $B$  and  $B'$  that differ in quark content by one  $d \leftrightarrow s$  replacement.

Coefficient	Bound
$a_B^X - a_{B'}^X$	$10^{-20}$ GeV
$a_B^Y - a_{B'}^Y$	$10^{-20}$ GeV
$a_B^Z - a_{B'}^Z$	$10^{-16}$ GeV
$a_B^T - a_{B'}^T$	$10^{-15}$ GeV

differences has ever been proposed before. They would, in fact, be exceedingly difficult to measure directly. (This is different from the situation with  $a_p^\mu - a_n^\mu$  which is not directly observable, even in principle—at least not without nonminimal couplings to gravity.) Although baryons such as the proton and the  $\Sigma^+$  can, in theory, interconvert (there being no conserved quantity that differentiates them), the fact that there are (in the standard model) no flavor-changing neutral currents means that there can be no direct transitions between these species. What makes the  $K^0$ - $\bar{K}^0$  system special is that the oscillation process is mediated by a box diagram that exchanges both a  $W^+$  and  $W^-$ , so that the net charges of the initial and final particles are the same. There is no similar process for the baryons, so methods utilizing comparisons between different hadron types represent essentially the only practicable way to constrain these differences.

The relations derived here from  $\chi$ PT can be used not just to place bounds on new combinations of hadron SME parameters, but also on the underlying quark coefficients. This can be illustrated by considering differences of nucleon  $b$ -type coefficients. According to (62)—as well as the homologous formula for neutrons—

$$b_p^\mu - b_n^\mu = \frac{g_A}{2} (b_u^\mu - b_d^\mu), \quad (64)$$

which contains no unknown LECs at LO in  $\chi$ PT.

There are bounds (coming from precision magnetometer experiments) on linear combinations of mSME coefficients that include all the proton and neutron spatial components  $b_p^J$  and  $b_n^J$  ( $J = X, Y, Z$ ), at  $10^{-28} - 10^{-33}$  GeV levels. With direct bounds on the proton and neutron  $b$ -type terms, we could construct similarly precise bounds on the fundamental quark parameters in (64). Unfortunately however, the extant bounds are actually on somewhat complicated linear combinations of proton and neutron coefficients, including both dimension-3 and dimension-4 terms. These mixtures of coefficients for operators of different mass dimensions are unavoidable in purely nonrelativistic experiments, although it is possible to disentangle the effects of, for instance,  $b^J$  and  $d^{JT}$  at higher energies. In fact, this disentanglement can actually be accomplished by using relativistic corrections related to nuclear binding and the internal motions of constituent nucleons [31], although separating the operators of different dimensions does come with a significant cost in precision. The disentangled bounds will be worse than the raw experimental ones by a sizable factor of  $\sim m_N/\Delta e$ , where  $\Delta e$  is the difference in the binding energies of the nucleons that are being probed in different nuclei.

However, to distinguish proton and neutron contributions, as well as to separate dimension-3 and dimension-4 operators, would require measurements of  $b$ -type Lorentz violation for at least four different nuclear systems. At present, the best bounds on  $b$ -type coefficients are

dominated by measurements made on just two nuclei:  $^3\text{He}$  and  $^{129}\text{Xe}$  [32,33], which are very convenient to use in atomic magnetometers, because they are spin- $\frac{1}{2}$  noble gasses. There is only one other nucleus,  $^{199}\text{Hg}$ , for which comparably precise measurements have been made [34], which means there are not enough independent measurements to extract complete and robust bounds on the quark sector coefficients. However, naturalness does still suggest that the  $b_u^\mu$  and  $b_d^\mu$  should probably not be much larger than the best inferred bounds on  $b_p^\mu$  and  $b_n^\mu$ .

## VII. CONCLUSIONS AND OUTLOOK

In this paper, we have given the first explorations of simultaneous quark and gluon SME operators of dimension 4 in  $\chi$ PT, finding nontrivial relationships between the LECs that characterize their effects at the hadron level. We have also presented the first  $\chi$ PT analysis of dimension-3 SME operators. The results for the dimension-3  $CPT$ -violating terms have allowed us to place new bounds on certain combinations of octet hadron  $a$ -type coefficients, based on comparisons to the octet meson sector. This provides a novel avenue for constraining certain mSME parameters that are, in principle, observable, but which would be extremely difficult to investigate directly.

In the course of our analyses, we have also made some additional observations about the character of Lorentz-violating operators in  $\chi$ PT. There is a notable difference between the structure that  $\chi$ PT dictates for the  $CPT$ -even SME operators (of dimension 4 and higher) and the  $CPT$ -odd ones (which begin at dimension 3). The dimension-4 terms behave as modifications of the kinetic terms for the hadrons, and their sizes depend on the amount of momentum carried by the individual quarks and gluons. There are nontrivial relations between the coefficients for the  $PT$ -even quark-derived and gluon-derived terms. The relations are tied to the physical fact that all the momentum of a given hadron must ultimately be carried by its constituent partons (although those parton components generally include sea quarks as well as valance quarks and gluons). However, there are still a number of undetermined coefficients in the effective Lagrange densities for the hadrons. These parametrize, for instance, the relative contributions from the isosinglet and isotriplet Lorentz violation tensors, and they are ultimately determined by the interior wave functions of the nucleons. Determination of the  $\alpha^{(n)}$  and  $\beta^{(n)}$  LECs, using nonperturbative methods such as lattice QCD, would be a welcome development.

The situation is quite different for the dimension-3 operators, whose coefficients are, in the chiral limit, completely determined by the transformation behavior of the quarks. The Lorentz violation enters through external fields that couple to the quarks, which means that the  $l_\mu$ ,  $r_\mu$ , and  $v_\mu^{(s)}$  terms contribute unambiguously to the pion and baryon effective actions. This also makes sense, since, for

example, the net  $a$ -type coefficient for a baryon will just be the sum of expectation values of the  $a$ -type coefficients of its constituent quark fields. The contributions from the three valence quarks in an  $SU(3)_f$  octet baryon simply add up, while the contribution from the virtual sea of quark-antiquark pairs cancels out.

There is, however, a subtlety to the  $SU(3)_f$  analysis. For bounds that are based on kinematical considerations—such as direction- and boost-dependent differences between the effective masses of  $K^0$  and  $\bar{K}^0$  mesons—it is correct to phrase those bounds in terms of the mSME coefficients (such as  $a_d^\mu$  and  $a_s^\mu$ ) for well-defined quark species. However, if the experimental results are to be interpreted in terms of “direct”  $CPT$  violation—involving  $CPT$ -violating decays with strangeness change  $\Delta S = \pm 1$ , rather than asymmetric  $K^0$ - $\bar{K}^0$  oscillations involving  $\Delta S = \pm 2$ —it would also be necessary to include in the analysis terms such as  $a_{ds}^\mu$ , which parametrizes an operator

$$\mathcal{L}_{s-d\text{mixing}}^{d=3,CPT\text{-odd}} = -\frac{i}{2}\bar{s}a_{ds}^\mu\gamma_\mu d + \text{H.c.}, \quad (65)$$

where “H.c.” indicates the Hermitian conjugate. A term like (65), which is off diagonal in flavor space, would contribute directly to the kaon decay process, in an intrinsically Lorentz- and  $CPT$ -violating fashion. Whereas the Cabibbo angle describes the mixing between the  $s$  and  $d$  species in the matrix of the standard model’s fermion-Higgs Yukawa couplings, the  $a_{ds}^\mu$  play analogous roles in the Lorentz-violating sector. Further exploration of how neutral meson experiments could be used to place constraints on  $a_{ds}^\mu$  (as well as the other analogous mixing parameters that appear when more than three flavors are taken into account) would be quite interesting.

In fact, it would also be useful to have systematic methods for determining the effective SME coefficients for heavier hadron species. Using techniques for the study of hadrons containing heavy quarks ( $c$  or  $b$  flavors), it should be possible to generalize the  $\chi$ PT results to answer questions about heavier mesons and the related spin- $\frac{1}{2}$  baryons. The differences between the  $a$ -type coefficients for the constituents of  $D^0$  and  $B^0$  mesons have already been measured, at roughly  $10^{-15}$  GeV levels of precision. These limits can presumably be translated into bounds on the differences of  $a$ -type coefficients for baryons with the same heavy valence quarks.

It may also be possible to extend our analysis to mesons with spin. There has been some recent work on higher-dimensional forms of Lorentz violation for spin-1 bosons [35]. Lorentz violation for a massive spin-1 particle is similar to that for a photon, although without the restriction of gauge invariance there are additional allowed operators. The general features of a Lorentz-violating mass term have been explored and appear to be qualitatively understood [36–38]. If the mass-squared matrix  $M^\mu{}_\nu$  for the vector

boson field has an eigenvalue  $m_0^2$  corresponding to a timelike direction and a larger eigenvalue  $m_1^2$  corresponding to a spacelike eigenvector, then there may be propagation with signal and group velocities as large as  $\frac{m_1}{m_0} > 1$  for the approximately longitudinal mode. However, in spite of these interesting results, there has been no systematic survey of all possible Lorentz-violating operators of dimensions 3 and 4.

Existing work on Lorentz-invariant applications of  $\chi$ PT to spin-1 octet mesons, such as in [39–44], has often focused on the forms taken by interaction vertices involving vector particles like the  $\rho^0$ , rather than on the behavior of the vector propagator. This focus is partially motivated by the vector meson dominance (VMD) phenomenon, in which the interactions of hadrons with deeply virtual photons can be dominated by diagrams in which the photon makes a virtual transition into a neutral vector meson such as the  $\rho^0$  before interacting with real hadrons. Because of the existence of VMD, understanding the role of the vector meson sector of the SME may actually be quite important for the interpretation of some high-energy collider tests of Lorentz and  $CPT$  symmetries.

Moreover, there are other heavy particles for which a different suite of techniques might be needed. The  $\chi$ PT methodology has been useful for determining the effective Lorentz violation coefficients for nucleons and pions. In terms of flavor  $SU(3)_f$ , these are the lightest representatives of the meson and baryon octets. A natural additional question is how to determine the coefficients for decuplet baryons as well. In fact, the mSME structure for a spin- $\frac{3}{2}$  field operator has not yet been worked out, so even the general forms of the possible operators (much less their relationships to the underlying quark and gluon operator structures) are unknown. The chief complication with a spin- $\frac{3}{2}$  field is that the Rarita-Schwinger equation [45] describes the behavior of a field with both a Dirac index and a Lorentz index—and thus 16 apparent components. However, an actual spin- $\frac{3}{2}$  quantum has only eight possible states (four helicity projections, along with a binary choice for particle versus antiparticle identity). Therefore only a certain subspace of solutions of the Rarita-Schwinger equation actually represents the propagation of spin- $\frac{3}{2}$  particles. This significantly complicates the construction of any EFT theory for such particles; many of the operators that might be constructed in generalizations of the Rarita-Schwinger Lagrange density will turn out to be spurious (because they only affect the behavior of the unphysical part of the solution space) or pathological (because they induce transitions between the physical subspace and the unphysical one, thus destroying unitarity). This is a serious problem even for Lorentz-invariant Rarita-Schwinger theories with nonminimal couplings [46,47], and it is likely to be an even greater challenge when the most general Lorentz-violating couplings are included. The inclusion of



the  $\Delta$  resonance in  $\chi$ PT in the Lorentz-invariant sector has been treated extensively in the literature, addressing issues of power counting as well as the treatment of the unphysical degrees of freedom, in such works as [48–54]. Extensions of these methods to the Lorentz-violating sector might be feasible.

In any event, understanding Lorentz violation for spin- $\frac{3}{2}$  composite particles such as  $\Delta^+$  baryons would be very interesting, because of the importance of such particles to the Greisen-Zatsepin-Kuzmin (GZK) cutoff [55,56]. Primary cosmic ray protons of sufficient energy interact with cosmic microwave background photons according to

$$p^+ + \gamma \rightarrow \Delta^+ \rightarrow \begin{cases} p^+ + \pi^0 \\ n^0 + \pi^- \end{cases}, \quad (66)$$

and the threshold energy depends sensitively on the relevant  $c$ -type coefficient for the  $\Delta^+$ . The process must be allowed for at least one  $\Delta^+$  helicity state that is accessible from each proton helicity state, in order for all the protons above the  $\sim 5 \times 10^{10}$  GeV GZK threshold to have their energies drained away over intergalactic distances, as is observed experimentally.

However, it is not even known how many different parameters actually govern the ultrarelativistic dispersion relations for the  $\Delta^+$  modes under the mSME. The propagation of a field with spin- $\frac{3}{2}$  excitations may be controlled by up to four  $c$ -type symmetric tensors, one for each helicity state. Alternatively, it may be that there are only two independent tensors involved, with the  $c$ -type coefficients for a  $\Delta^+$  taking the form  $c_{\Delta^+}^{\mu\nu} + 2hd_{\Delta^+}^{\mu\nu}$ , with  $h$  being the helicity component of the particle's angular momentum.

Either type of Lorentz-violating spin structure would be at least partially analogous to the Lorentz-violating behavior of relativistic spin- $\frac{1}{2}$  fermions, which have two helicity states and whose dispersion relations are set by  $c_L^{\mu\nu} = c^{\mu\nu} + d^{\mu\nu}$  and  $c_R^{\mu\nu} = c^{\mu\nu} - d^{\mu\nu}$ . Note, however, that in spite of the Dirac spinor having four components—allowing for the presence of two particle and two antiparticle excitation modes for each momentum eigenvalue—there are not four separate  $c$ -type tensors, only the two. When the  $C$ -parity of  $\gamma_5$  is taken into account, the behavior of antiparticle modes is governed by the same tensors as the particle modes. Something similar is expected for the spin- $\frac{3}{2}$  modes as well, although the details of which Lorentz-violating terms actually change signs under the action of  $C$  are unknown. (For relativistic fermion fields, regardless of their total spins, the *zitterbewegung* process ensures that only helicity eigenstates are eigenstates of propagation. This ensures that the even more complicated spin structure that is possible for Lorentz-violating integer-spin fields such as photons—which is represented by the birefringent part of their bosonic Lagrange densities—cannot be replicated for higher-spin fermions.)

Ultimately, although progress is being made in understanding the relationships between Lorentz violation at the quark and gluon level and at the hadronic level, there are still important unanswered questions. As  $\chi$ PT and other methods are used to further elucidate the connections between the SME coefficients for different strongly interacting particles, we expect there to be many strong new bounds based on the understanding of these connections.

## ACKNOWLEDGMENTS

This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics, under Award No. DE-SC0019647 (M. R. S.).

## APPENDIX: $SU(3)_f$ FORMALISM

The extension of  $\chi$ PT methods to  $SU(3)_f$  in the meson sector is straightforward. As in the  $SU(2)$  case, the Goldstone bosons are encoded in the matrix  $U(x)$  of (19), which still transforms as in (20). However, the matrix  $\phi$  in the exponential now takes the form

$$\phi = \sum_{a=1}^8 \phi_a \lambda_a = \begin{bmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta_8 & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta_8 & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 \end{bmatrix}, \quad (A1)$$

and the constant  $F$  is now the pseudoscalar decay constant in the  $SU(3)$  chiral limit—that is, with the strange quark mass also set to zero. Because the transformation properties are unchanged compared to the  $SU(2)_f$  case, the LO Lagrange densities for both the Lorentz-invariant and Lorentz-violating sectors still take the same forms as in (32), (44), and (51), respectively. Differences between the two- and three-flavor cases appear in the values of the low-energy constants, as well as possibly in the forms of higher-order Lagrange densities, as some techniques used in reducing the number of independent terms at a given order (such as the Caley-Hamilton formalism) may differ.

The extension to  $SU(3)_f$  in the baryon sector is more complicated. Instead of the nucleon doublet  $\Psi$ , the baryon octet is encoded in a traceless  $3 \times 3$  matrix

$$B = \sum_{a=1}^8 B_a \frac{\lambda_a}{\sqrt{2}} = \begin{bmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{bmatrix}, \quad (A2)$$

with the chiral transformation property

$$B \rightarrow KBK^\dagger. \quad (A3)$$

The corresponding covariant derivative is naively given by

$$D_\mu B = \partial_\mu B + [\Gamma_\mu, B]. \quad (\text{A4})$$

The Lagrangian is constructed by forming products of terms  $X$  that each transform as  $KXK^\dagger$  and then taking a trace. For example, the LO Lorentz-conserving meson-baryon Lagrange density is

$$\begin{aligned} \mathcal{L}_{MB}^{\text{LO}} = & \text{Tr}[\bar{B}(i\mathcal{D} - m_0)B] - \frac{D}{2} \text{Tr}[\bar{B}\gamma^\mu\gamma_5\{u_\mu, B\}] \\ & - \frac{F}{2} \text{Tr}[\bar{B}\gamma^\mu\gamma_5[u_\mu, B]]. \end{aligned} \quad (\text{A5})$$

Here,  $m_0$  is the octet baryon mass in the chiral limit, while  $D$  and  $F$  are LECs that can be related to semileptonic decays. Note that there are three parameters, compared to two in the  $SU(2)_f$  case.

Analogously, we expect the form of the Lorentz-violating Lagrange density in the  $SU(3)_f$  sector to be more complex. However, for the discussion in Sec. VI, we

are only interested in the baryon octet  $a$ -type coefficients. At LO, these enter through the covariant derivative term in (A5); the terms proportional to  $D$  and  $F$  contribute to  $b$ -type terms, since they are proportional to  $u_\mu$ . However, to properly include the Lorentz-violating interactions, the baryon covariant derivative has to be modified to

$$D_\mu B = \partial_\mu B + [\Gamma_\mu, B] - i v_\mu^{(s)} B. \quad (\text{A6})$$

In standard  $\chi$ PT, coupling to the vector current describes electromagnetic interactions, which at the quark level are proportional to the quark charge matrix. Since this matrix is traceless, the singlet vector current is identically zero. For the *CPT*-odd terms considered here, this is no longer the case, and the  $v_\mu^{(s)}$  contribution has to be considered. The  $a$ -type terms for the baryon octet can then be determined from the first term in (A5). In addition to reproducing the  $SU(2)_f$  results of Sec. VB, we find, for example,

$$a_{\Sigma^+}^\mu = 2a_u^\mu + a_s^\mu. \quad (\text{A7})$$

- 
- [1] D. Colladay and V. A. Kostelecký, *Phys. Rev. D* **55**, 6760 (1997).
- [2] D. Colladay and V. A. Kostelecký, *Phys. Rev. D* **58**, 116002 (1998).
- [3] O. W. Greenberg, *Phys. Rev. Lett.* **89**, 231602 (2002).
- [4] V. A. Kostelecký and N. Russell, *Rev. Mod. Phys.* **83**, 11 (2011); updated as arXiv:0801.0287v12.
- [5] S. Weinberg, *Physica (Amsterdam)* **96A**, 327 (1979).
- [6] J. Gasser and H. Leutwyler, *Ann. Phys. (N.Y.)* **158**, 142 (1984).
- [7] J. Gasser and H. Leutwyler, *Nucl. Phys.* **B250**, 465 (1985).
- [8] S. Scherer and M. R. Schindler, *A Primer for Chiral Perturbation Theory* (Springer, New York, 2012).
- [9] R. Kamand, B. Altschul, and M. R. Schindler, *Phys. Rev. D* **95**, 056005 (2017).
- [10] R. Kamand, B. Altschul, and M. R. Schindler, *Phys. Rev. D* **97**, 095027 (2018).
- [11] J. P. Noordmans, J. de Vries, and R. G. E. Timmermans, *Phys. Rev. C* **94**, 025502 (2016).
- [12] J. P. Noordmans, *Phys. Rev. D* **95**, 075030 (2017).
- [13] D. Colladay and P. McDonald, *J. Math. Phys. (N.Y.)* **43**, 3554 (2002).
- [14] B. Altschul, *J. Phys. A* **39**, 13757 (2006).
- [15] M. S. Berger and V. A. Kostelecký, *Phys. Rev. D* **65**, 091701 (R) (2002).
- [16] V. A. Kostelecký, *Phys. Rev. D* **69**, 105009 (2004).
- [17] B. Altschul, *Phys. Rev. D* **99**, 125009 (2019).
- [18] S. Coleman, J. Wess, and B. Zumino, *Phys. Rev.* **177**, 2239 (1969).
- [19] H. Leutwyler, *Ann. Phys. (N.Y.)* **235**, 165 (1994).
- [20] C. G. Callan, Jr., S. R. Coleman, J. Wess, and B. Zumino, *Phys. Rev.* **177**, 2247 (1969).
- [21] H. Georgi, *Weak Interactions and Modern Particle Theory* (Benjamin/Cummings, Menlo Park, CA, 1984).
- [22] J. Gasser, M. E. Sainio, and A. Švarc, *Nucl. Phys.* **B307**, 779 (1988).
- [23] N. Fettes, U. G. Meißner, M. Mojžiš, and S. teininger, *Ann. Phys. (N.Y.)* **283**, 273 (2000).
- [24] A. Manohar and H. Georgi, *Nucl. Phys.* **B234**, 189 (1984).
- [25] B. Altschul, *Phys. Rev. D* **79**, 016004 (2009).
- [26] B. Altschul, *Phys. Rev. D* **86**, 045008 (2012).
- [27] V. A. Kostelecký and R. Potting, *Phys. Rev. D* **51**, 3923 (1995).
- [28] B. M. Roberts, Y. V. Stadnik, V. A. Dzuba, V. V. Flambaum, N. Leefer, and D. Budker, *Phys. Rev. Lett.* **113**, 081601 (2014).
- [29] H. Nguyen (KTeV Collaboration) in *Proceedings of the Fourth Meeting on CPT and Lorentz Symmetry*, edited by V. A. Kostelecký (World Scientific, Singapore, 2008).
- [30] D. Babusci *et al.* (KLOE Collaboration), *Phys. Lett. B* **730**, 89 (2014).
- [31] B. Altschul, *Phys. Rev. D* **79**, 061702(R) (2009).
- [32] J. M. Brown, S. J. Smullin, T. W. Kornack, and M. V. Romalis, *Phys. Rev. Lett.* **105**, 151604 (2010).
- [33] F. Allmendinger, W. Heil, S. Karpuk, W. Kilian, A. Scharth, U. Schmidt, A. Schnabel, Y. Sobolev, and K. Tullney, *Phys. Rev. Lett.* **112**, 110801 (2014).
- [34] S. K. Peck, D. K. Kim, D. Stein, D. Orbaker, A. Foss, M. T. Hummon, and L. R. Hunter, *Phys. Rev. A* **86**, 012109 (2012).

- [35] V. E. Mouchrek-Santos, M. M. Ferreira, Jr., and C. Miller, *Nucl. Phys.* **B945**, 114677 (2019).
- [36] G. Gabadadze and L. Grisa, *Phys. Lett. B* **617**, 124 (2005).
- [37] G. Dvali, M. Papucci, and M. D. Schwartz, *Phys. Rev. Lett.* **94**, 191602 (2005).
- [38] B. Altschul, *Phys. Rev. D* **73**, 036005 (2006).
- [39] E. Jenkins, A. V. Manohar, and M. B. Wise, *Phys. Rev. Lett.* **75**, 2272 (1995).
- [40] M. F. M. Lutz and E. E. Kolomeitsev, *Nucl. Phys.* **A730**, 392 (2004).
- [41] M. F. M. Lutz and S. Leupold, *Nucl. Phys.* **A813**, 96 (2008).
- [42] C. Terschläusen, S. Leupold, and M. F. M. Lutz, *Eur. Phys. J. A* **48**, 190 (2012).
- [43] P. C. Bruns, M. Emmerich, L. Greil, and A. Schäfer, *Phys. Rev. D* **88**, 114503 (2013).
- [44] C. Terschläusen and S. Leupold, *Phys. Rev. D* **94**, 014021 (2016).
- [45] W. Rarita and J. Schwinger, *Phys. Rev.* **60**, 61 (1941).
- [46] G. Velo and D. Zwanziger, *Phys. Rev.* **186**, 1337 (1969).
- [47] M. Porrati and R. Rahman, *Phys. Rev. D* **80**, 025009 (2009).
- [48] E. E. Jenkins and A. V. Manohar, *Phys. Lett. B* **259**, 353 (1991).
- [49] M. N. Butler, M. J. Savage, and R. P. Springer, *Nucl. Phys.* **B399**, 69 (1993).
- [50] T. R. Hemmert, B. R. Holstein, and J. Kambor, *J. Phys. G* **24**, 1831 (1998).
- [51] V. Pascalutsa and D. R. Phillips, *Phys. Rev. C* **67**, 055202 (2003).
- [52] V. Bernard, T. R. Hemmert, and U. G. Meißner, *Phys. Lett. B* **565**, 137 (2003).
- [53] C. Hacker, N. Wies, J. Gegelia, and S. Scherer, *Phys. Rev. C* **72**, 055203 (2005).
- [54] N. Wies, J. Gegelia, and S. Scherer, *Phys. Rev. D* **73**, 094012 (2006).
- [55] K. Greisen, *Phys. Rev. Lett.* **16**, 748 (1966).
- [56] G. T. Zatsepin and V. A. Kuz'min, *Pis'ma Zh. Eksp. Teor. Fiz.* **4**, 114 (1966) [*JETP Lett.* **4**, 78 (1966)].