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Effects of Dissipation and Temperature on Macroscopic Quantum Tunneling

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Measurements of the tunneling rate Γ out of the zero-voltage state for several Nb edge junctions with differing shunt capacitances are described. At zero temperature, increasing the shunt capacitance lowers Γ in agreement with dissipative calculations of the macroscopic-quantum-tunneling rate. As temperature increases, $\ln[\Gamma(T)/\Gamma(0)] \propto T^2$ as recently predicted.

Quantum mechanics must hold for the large systems as well as for small ones. Most measured attributes of large systems result from incoherent averages of quantum-mechanical processes. In macroscopic systems having sufficient internal correlation, however, a single dynamical variable describes a large number of particles.[1] Following the introductory theoretical work of Leggett[1] and Kurkijärvi,[2] several experiments[3-6] have been performed to observe the quantum behavior of such a macroscopic dynamical variable. Most have concentrated on macroscopic quantum tunneling (MQT) of either the magnetic flux confined by a SQUID ring[3] or the superconducting phase is an isolated Josephson junction.[4-6] In the latter experiment, the junction is prepared in the metastable zero-voltage state, and the escape rate into the voltage state is measured as a function of temperature and bias current. At higher temperatures, escape proceeds by classical thermal activation (TA) over the effective barrier. This process is exponentially reduced as the temperature decreases, and below some temperature, quantum-mechanical tunneling through the barrier must dominate the transition rate. Caldeira and Leggett[7] showed that the dissipation inherent in macroscopic systems reduces the quantum escape rate below the predictions for the frictionless system. It was this theory, including dissipation, that was qualitatively vindicated by the early experiments.[5,6] Using the same formalism, Chang and Chakravarty[8] have recently performed numerical calculations which provide theoretical predictions for the ($T=0$) MQT rate in the presence of arbitrary dissipation. Of more interest is the recent theoretical attention to finite-temperature corrections to MQT. Several groups[9-12] have predicted that finite temperature enhances the MQT rate in the presence of dissipation. In the following we will describe experiments designed to test the above predictions.

The collective phase difference ϕ of superconducting pairs across a narrow barrier (a Josephson junction) is analogous to the position coordinate of a microscopic particle in the tilted periodic potential

$$U(\phi) = -(I_0\Phi_0/2\pi)(x\phi + \cos\phi).$$

I_0 is the junction critical current, $\Phi_0 = h/2e$, and $I = xI_0$ is the current bias. For $0 < x < 1$, the potential consists of a series of wells, and for underdamped junctions, there are two allowed dynamical states. These correspond to a particle either trapped in one of the wells (the zero-voltage state) or sliding down the potential (the voltage state). The oscillation frequency of the particle about the well minimum is $\omega_0 = \omega_J(1-x^2)^{1/4}$, where $\omega_J^2 = 2\pi I_0/C\phi_0$, and C is the effective parallel capacitance across the junction. Because of the available, lower-energy voltage state, the zero-voltage state is metastable. For a system prepared in the zero-voltage state, the thermal-activation rate out of the minimum is $\Gamma_T = A_T \exp(-U_0/kT)$. The prefactor A_T is roughly $\omega_0/2\pi$, the rate of escape attempts, and U_0 is the height of the barrier. The effective shunt resistance R leads to dissipative corrections[13,14] to A_T. As $T \to 0$, escape from the zero-voltage state can only occur via quantum-mechanical tunneling at a rate $\Gamma_Q = A_Q e^{-B}$. For a microscopic particle, $B \simeq U_0/\hbar\omega_0$, and $A_Q \simeq \omega_0/2\pi$. For a macroscopic system, however, the quantum-mechanical-tunneling rate is suppressed by dissipation[7]; both A_Q and B depend upon the dimensionless dissipation $\alpha = (2RC\omega_0)^{-1}$. Extensions of the theory to finite temperatures[9-12] suggest that B is also a function of T. Specifically, the temperature of the thermal bath *enhances* the quantum tunneling rate in the form $\ln[\Gamma_Q(T)/\Gamma_Q(0)] = s(\alpha)T^2$. The simple sum of a constant Γ_Q and TA would, in contrast, be exponentially small at low T.

To test these predictions several nominally identical Nb edge junctions[15] were fabricated as shown in Fig. 1. Each junction was 1.2 by 0.13 μm with an oxide thickness of 20–25 Å. In order to systematically vary the dissipation α, shunt capacitors of different sizes (oxide thickness 400 Å) were formed in parallel with the junctions. (C was chosen over R as a variable to avoid parasitic inductances.) Stray capacitance plus the intrinsic capacitance of the junction was calculated from the device geometry and the critical current density[16] as $C_0 = 0.02 \pm 0.005$ pF. The shunt capacitances ranged from 0.025 to 12 pF. The larger dimensions and oxide thickness of the shunting capacitors allowed accurate determination (within $\leq 6\%$) of C from geometric measurements. At sufficiently small
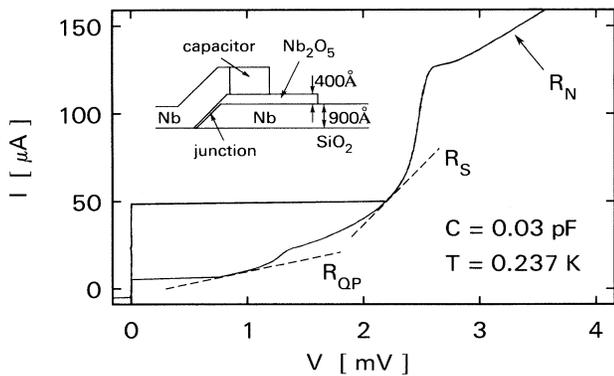

FIG. 1. Typical low-temperature I-V characteristic illustrating the quasiparticle shunting resistance $R = \partial V / \partial I$. The inset is a schematic side view of the Nb edge junction with parallel capacitance.
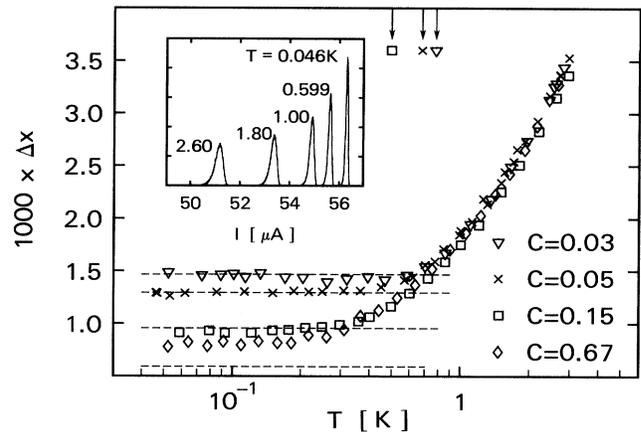

FIG. 2. The widths of the transition distributions for several junction capacitances. The various dashed lines are the theoretical predictions (Ref. 8) for MQT at $T = 0$. The inset contains representative switching distributions for $C = 0.15$ pF. The arrows at the top are the predicted crossover temperatures for the first three junctions.

plasmon frequencies ω_0 and spatial sizes, we expect that the only effect of the shunt capacitance is to increase the total junction capacitance in the widely accepted lumped circuit [resistively shunted junction (RSJ)] model.

Figure 1 contains a typical I-V curve. Almost identical characteristics were obtained for all of the junctions; the critical currents were all within 10% of 55 μA, and quasiparticle resistances varied by no more than 10%. The most noticeable effect of the shunt capacitors on the I-V curves was in the amount of hysteresis in the return to the zero-voltage state. Slight changes in the subgap resonance structure[17] were also observed with changing capacitance.

The junctions were cooled by contact with the dilute solution in a dilution refrigerator. With use of a technique described previously,[4,5] measurements of the transition rate out of the zero-voltage state were performed as a function of temperature and current bias for all of the junctions. Starting in the zero-voltage state, the bias was increased slowly until the junction switched to the voltage state. The current at which it switched was recorded, and the junction was reset. This process was repeated $\sim 10^5$ times so that a histogram, the switching distribution $P(I)$, was obtained. Extreme care was taken to isolate the junctions from noise. Figure 2 contains distribution widths Δx for several capacitances and representative raw data $P(I)$ for one junction. As T decreased Δx decreased, and the center of the distribution shifted toward $x = 1$. As expected for MQT,[7,8] below some temperature, the width and position became nearly independent of T, and for larger C, the limiting ($T = 0$) Δx was smaller.

The tunneling rate Γ is obtained from the distribution by[4]

$$\Gamma(x) = \frac{(dx/dt)P(x)}{1 - \int_0^x P(x)\,dx}.$$

In Fig. 3, some of the experimental rates Γ are compared with thermal-activation and MQT theories. The capacitance used in the theoretical curves was calculated from micrographs of the junctions and from material parameters used during growth of the devices.[16] For high-current-density junctions, the effective capacitance is enhanced by quasiparticle tunneling.[18] In our case the increase is $\delta C \simeq 0.006$ pF. The resistance was measured directly from the slope of the I-V characteristic above the gap (R_N in Fig. 1). The only remaining parameter is I_0 which is obtained from fits to thermal activation[13,14] above the crossover tempera-
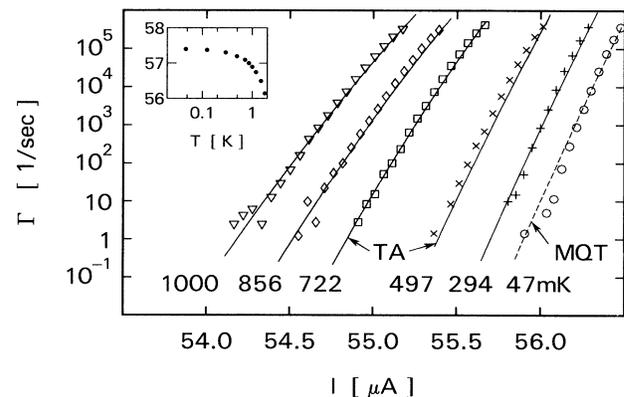

FIG. 3. Comparison of the experimental transition rate with the theory for thermal activation (solid lines) at high temperatures and for MQT (dashed line) at low temperature for $C = 0.15$ pF and $R = 23$ Ω. The inset contains the fitted I_0 (in microamperes) vs T.

ture[9-11]

$$T_0 = (\hbar\omega_0/2\pi k_B)[(1+\alpha^2)^{1/2} - \alpha].$$

Quantum tunneling enhances the escape rate even in the regime where thermal activation dominates. (This can be seen in the experiment as a rounding of the crossover from MQT to TA in Fig. 2.) The rate enhancement may be written phenomenologically as a reduction in the barrier height[10] $U_0 \rightarrow U_0 - \hbar^2\omega_0^2/12k_BT$. This approximation is valid far enough above T_0. For $T_0 < T \simeq 2T_0$, the true correction is slightly larger than the approximation, and is available only in numerical tables.[9] The escapes rates were fitted with use of these corrections to capacitance and barrier height, and the resulting I_0 are displayed in the inset. (There is a small variation in I_0 above $T = 1$ K which remains unexplained. It may be related to the temperature renormalization of junction impedances predicted recently.[19]) Below $T \simeq T_0$, I_0 is nearly constant and agrees with the $T = 0$ value. The remaining, small ($\leq 0.2\%$) variation may be an artifact of the inaccuracy of the above approximation to the quantum correction to U_0. For the three lowest-capacitance junctions, the experimental Γ and Δx (Fig. 2) are in excellent agreement with the magnitude and the functional form predicted by the two theories in their regions of validity. These data are the first which confirm the moderate-dissipation-regime calculations of MQT at $T = 0$. The $T = 0$ rate was independent of the patterned capacitance between $0.67 < C < 12$ pF. For $C \geq 0.6$ pF, where the agreement with theory is not as good, calculations indicate that the lumped circuit model may break down because the patterned shunt capacitance is spatially so large that it behaves as a distributed capacitance and inductance. A new model for a junction attached to such a transmission line is necessary.[20]

Theoretical work[7-11] has relied heavily on the RSJ model. (Calculations that do not follow the RSJ model[21] have offered no prescription for measuring the effective R from the I-V curve.) Since the I-V curve in Fig. 1 differs from the RSJ prediction, it is not obvious that R_N measures the damping which appears in the theory. In fact, since the relevant dissipation is that for small oscillations about the well minimum ($V = 0$), it is surprising that R_N (the dissipation above the gap: $V > 2\Delta/e$) fits the theory. We can only say that the internal consistency of the data implies that R_N is the best measure of the damping in this experiment (as it was in the earlier experiment[5]). Not only does R_N correctly predict the magnitude and the slope of the MQT rate, but it also correctly gives the crossover temperatures for the three junctions. For completeness, we note that theoretical fits of slightly lower quality can be obtained by using the unrenormalized capacitance and the phenomenological

quasiparticle resistance R_S measured near the switch out of the zero-voltage state (see Fig. 1). No self-consistent choice of junction parameters would fit if R_{QP} (measured near the return to the zero-voltage state) was used as the resistance.

Even at very low temperatures where TA is negligible, coupling of the metastable quantum-mechanical system to a thermal bath should increase the tunneling rate[9-12] as $\exp(T^2)$ for small T. According to Refs. 9 and 10, the T^2 dependence should be observed regardless of the amount of damping or the form of the potential. The only criterion is that the dissipation be Ohmic, i.e., that RC approaches a constant as $\omega \rightarrow 0$, and as long as this is satisfied, the T^2 dependence of the rate is universal.[9] To test the dependence of Γ_Q on temperature, it is necessary to study the behavior at fixed bias $I = xI_0$. Figure 4 displays $\ln\Gamma(T)$ vs T^2 for the three junctions at $x = 0.97$. At low T^2, the data fall on straight lines as predicted. The region of the T^2 dependence is at low temperature where the fitted I_0 is constant: The data in Fig. 4 are *not* an artifact of the variations in I_0. This is the first experimental observation of this behavior. The region of T^2 behavior was largest for the lowest-capacitance junction which had the highest T_0, and the region shrinks as T_0 decreases. This also is consistent with the theory. Although the T^2 dependence and its variation with C are unmistakable in Fig. 4, we do not find quantitative agreement with theoretical prediction[9] for the slope $s(\alpha)$. We do find, however, that $s(\alpha)$ varies approximately as predicted when C changes. Although the T^2 enhancement is universal for Ohmic damping, the slope $s(\alpha)$ depends upon the form of the potential in the voltage state. That the experimental slopes are larger than the theoretical predictions is partly the result of the junction potential being larger than the approximate model
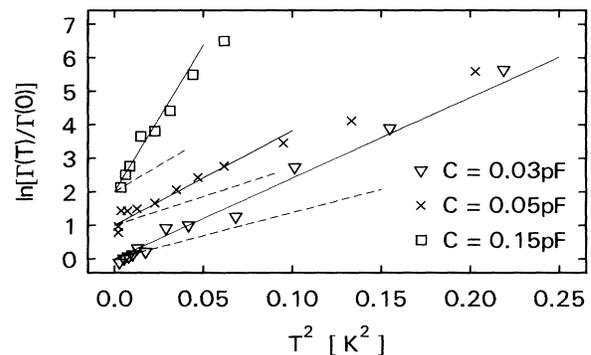

FIG. 4. The low-temperature transition rate for three junctions as a function of T^2 at $x = 0.97$. The straight lines illustrate the region in temperature where the data are linear (Crosses and squares have been offset for clarity of display.) The dashed lines are the theoretical predictions for $s(\alpha)$; they are about $\frac{1}{2}$ of the experimental values.

potential $[U(\phi) \sim \omega_0^2\phi^2 - \frac{2}{3}\omega_J^2 x\phi^3]$ used in the calculation. Predictions for the real junction potentials would have been slightly higher.[10]

In summary, experiments on a series of capacitively shunted Nb edge junctions have qualitatively verified theoretical predictions of the $T = 0$ macroscopic-quantum-tunneling rate with variations in capacitance and dissipation in the range $0.2 < \alpha < 0.7$. This agreement was obtained with use of the renormalized capacitance and the normal-state resistance. Evidence has also been found that indicates the importance of quantum corrections to the escape process at temperatures where thermal activation dominates the escape rate. Moreover, for the first time, experimental evidence has been found to confirm the T^2 enhancement of the MQT rate from coupling to the environment.

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