10-13-1975

Nonlinear Parallel Ringing of Magnetization in Superfluid 3He

Richard A. Webb
University of South Carolina - Columbia, webbra@mailbox.sc.edu

R. E. Sager

J. C. Wheatley

Publication Info
Published in *Physical Review Letters*, ed. Gene D. Sprouse, Volume 35, Issue 15, 1975, pages 1010-1013.
Webb, R. A., Sager, R. E., & Wheatley, J. C. (1975). Nonlinear parallel ringing of magnetization in superfluid ^{3}He. *Physical Review Letters, 35*(15), 1010-1013. DOI: 10.1103/PhysRevLett.35.1010

$\leq\omega\leq 1.8$ MHz and 2.2 MHz $\leq\omega\leq 3.0$ MHz precisely where structure[12] is observed in the data of Richards *et al.* We take this agreement to confirm our belief that the $(^3\mathrm{He})_2$ molecules move two dimensionally as $x_3\to 0$ and to confirm the choice $|J_{34}|=0.6$ MHz.

An interesting feature of the experimental data[9] is that 50% of the spectral density is at low frequency, $\omega\lesssim 10^{+4}$ Hz, and 50% is in the large-ω structure discussed in this paper. $L_2(\omega)$ as shown in Fig. 2 accounts for just 50% of the total spectral density at $\omega\neq 0$. Thus the planar calculation of $T_1(\omega)$ is in good quantitative agreement with experiment. The planar calculation has an additional 33% of the spectral density in a δ function at $\omega=0$. The remaining 17% of the total would be found by considering neighbors other than nearest and is expected to contribute to the $\omega\approx 0$ region. The δ function is expected to be broadened to about 10^4 MHz by complex and slow motions we have not considered here.

A complete description of the $T_1(\omega)$ data requires a combination of two dimensional motion, hindered rotations, and three-dimensional motion with the latter two providing a bit more background density than the two-dimensional motion in $L_2(\omega)$ seems to account for by itself at the concentrations studied by Richards *et al.*

†Work supported in part by the National Science Foundation.

[1]A. F. Andreev and I. M. Lifshitz, Zh. Eksp. Teor. Fiz. 56, 2054 (1969) [Sov. Phys. JETP 29, 1107 (1969)].

[2]R. A. Guyer and L. I. Zane, Phys. Rev. Lett. 24, 660 (1970).

[3]D. S. Miyoshi, R. N. Cotts, A. S. Greenberg, and R. C. Richardson, Phys. Rev. A 2, 870 (1970); A. S. Greenberg, W. C. Thomlinson, and R. C. Richardson, Phys. Rev. Lett. 27, 179 (1971).

[4]M. G. Richards, J. Pope, and A. Widom, Phys. Rev. Lett. 29, 708 (1972).

[5]V. N. Grigor'ev, B. N. Essel'son, and V. A. Mikheev, Pis'ma Zh. Eksp. Teor. Fiz. 18, 289 (1973) [JETP Lett. 18, 169 (1973)].

[6]A. Landesman and J. M. Winter, in *Low Temperature Physics, LT-13,* edited by W. J. O'Sullivan, K. D. Timmerhaus, and E. F. Hammel (Plenum, New York, 1973), Vol. 2, p. 73.

[7]W. Huang, H. A. Goldberg, and R. A. Guyer, Phys. Rev. B 11, 3384 (1975); W. Huang, H. A. Goldberg, M. T. Takemori, and R. A. Guyer, Phys. Rev. Lett. 33, 283 (1974).

[8]Y. Yamashita, J. Phys. Soc. Jpn. 37, 1210 (1974).

[9]M. G. Richards, J. H. Smith, P. S. Tofts, and W. J. Mullin, Phys. Rev. Lett. 34, 1545 (1975).

[10]A. Abragam, *Principles of Nuclear Magnetism* (Clarendon Press, Oxford, England, 1961).

[11]The exact frequency dependence of $T_1(\omega)$ at large ω seen in experiment (Ref. 3) agrees quite well with the theory of Abragam at concentrations above 10^{-3}; at lower concentrations the frequency dependence becomes somewhat stronger.

[12]The two-dimensional motions lead to a single peak of transitions. In $T_1(\omega)^{-1}$ there will then be two peaks, one corresponding to $q=1$ in Eq. (2) and the other to $q=2$. See Ref. 10.

Nonlinear Parallel Ringing of Magnetization in Superfluid ^{3}He†

R. A. Webb, R. E. Sager, and J. C. Wheatley
University of California at San Diego, La Jolla, California 92037

(Received 21 July 1975)

Experiments based on an analogy to the ac Josephson effect have shown in both ^{3}He-A and ^{3}He-B that a pairing theory of the superfluidity of ^{3}He is essentially correct. Additional observations of parallel ringing are not in agreement with the simple "pendulum models" used to describe nonlinear dynamic magnetic effects.

According to current theoretical concepts[1] as supported by experiment,[2] superfluid ^{3}He consists of one or more interpenetrating and weakly coupled supefluids as well as a normal fluid. A BCS-like pairing theory is used to describe the superfluidity with ^{3}He-A having both $\uparrow\uparrow$ and $\downarrow\downarrow$ triplet-paired superfluids and ^{3}He-B both of these plus an $\uparrow\downarrow$ paired superfluid as well. The weak coupling between superfluids is provided by a coherent dipolar interaction. In the present work we have attempted to test the pairing theory by performing an experiment analogous to the one described by Josephson[3] for two weakly coupled superconductors with a potential difference V between them and a resulting ac supercurrent of frequency $2eV/h$. In the case of ^{3}He the sudden application of a field ΔH decreases the chemical potential of $\uparrow\uparrow$ pairs by $\gamma\hbar\Delta H$, increases the chemical potential of $\downarrow\downarrow$ pairs by $\gamma\hbar\Delta H$, and leaves that of $\uparrow\downarrow$ pairs unchanged. Since the analog of the barrier supercurrent in superconductivity is a transfer of correlated pairs from one superfluid to another or a change of magnetization, we might expect that a sudden field change ΔH would lead to a ringing of the magnetization at angular frequency $2\gamma\Delta H$ in ^{3}He-A and a ringing at both $\gamma\Delta H$ and $2\gamma\Delta H$ in ^{3}He-B. These are actually upper limits since transfer of pairs reduces the chemical potential differences between the different superfluids. But since the pair transfer rate occurs at the temperature-dependent parallel ringing frequency[4] while $\gamma\Delta H$ may in principle be as large as we please it should be possible as $T \to T_c$ to observe the $2\gamma\Delta H$ ringing frequency. Detailed theoretical calculations of the ringing frequencies f_R for such experiments for arbitrary ΔH have been worked out by Maki and collaborators.[5-7] Comparison with their theory will also be possible.

Because of the demonstrated importance of geometry[1,2,8] we made measurements both in a quasi-ideal geometry for ^{3}He-A: a long rectangular cavity 1 mm×10 mm in section with both steady field H_0 and ΔH in the long direction and parallel to the walls; and in a quasi-ideal geometry for ^{3}He-B: a stack of parallel plates 5 mm in diameter with 0.46 mm separation and a 1.3-mm-diam thermal-contact hole in the center and with both H_0 and ΔH normal to the plates. For each geometry both H_0 and ΔH were provided by the same long Nb solenoid, sensing of ^{3}He magnetization was by an astatic pair of Nb coils connected to a 160-MHz rf-biased superconducting quantum-interference device (SQUID), and shielding was provided by a Nb shield. The ratio of field to current for each solenoid was measured by a Bi probe. Thermometry was by a small cerium magnesium nitrate (CMN) thermometer at a distance from the ringing areas but thermally coupled as tightly to them as possible with provisional absolute temperatures obtained as in Ref. 2. Refrigeration of the ^{3}He was achieved using powdered CMN. Observations of ringing *periods* were made using a calibrated oscilloscope time delay with visual signal averaging to obtain the times of extrema in the ringing pattern. Thus, we looked for any possible sign of relaxation in the form of a changing period with time. Finally, where necessary the temperature was stabilized to ca. 0.1 μK for adequately long-time intervals.

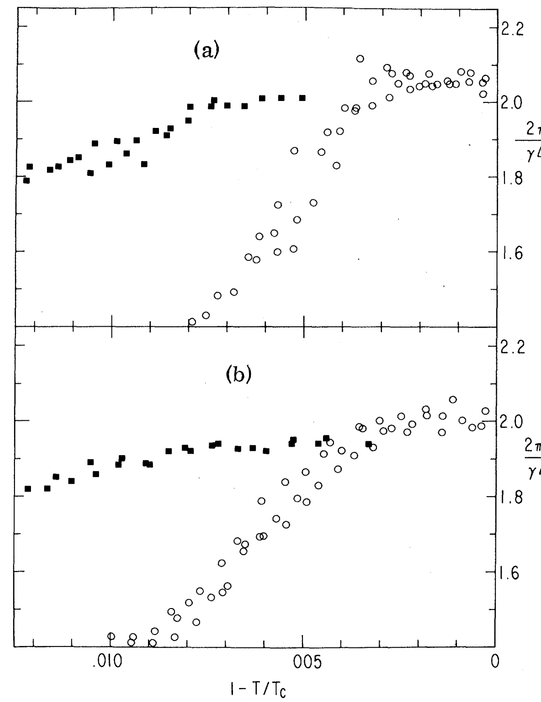

FIG. 1. Frequencies of the driven parallel ringing mode near T_c for two geometries. (a) Parallel plates: (A, 22.0 bar), ■; (B, 20.7 bar), ○; (b) rectangular cavity: (A, 22.0 bar), ■; (B, 20.7 bar), ○. f_R near T_c is ca. 80 khz. $f_A^2 \approx 3.8\times10^4\,(1-T/T_c)$ kHz^2; $f_B^2 \approx 9.0\times10^4\,(1-T/T_c)$ kHz^2.

The results of our experiments on the driven mode, the ringing which results when $\gamma\Delta H/2\pi$ is large compared to the linear or temperature-dependent frequencies f_A or f_B observed when ΔH is sufficiently small, are shown in Fig. 1. For all these data H_0 was 15 to 20 G. Measurements on the A phase were made at 22.0 bar, somewhat above the polycritical point (PCP), and on the B phase at 20.7 bar, somewhat below the PCP, so that both A and B phases could be compared in the same geometry at nearly the same pressure. The decrease of ringing frequency f_R with increasing $1-T/T_c$ presumably reflects the increasing effect of the actual transport of correlated particles from one superfluid to another although the expected[5-7] linear decrease is not demonstrated. The limiting value of $2\pi f_R/\gamma\Delta H$ is equal to 2 within our experimental accuracy and precision for both A and B fluids in both geometries. The observation of a factor of 2 is strong evidence for the pairing hypothesis used in BCS-like theories of the superfluid states of ^{3}He.

Typical results for nonlinear ringing in ^{3}He-A for both geometries are shown in Fig. 2. The the-

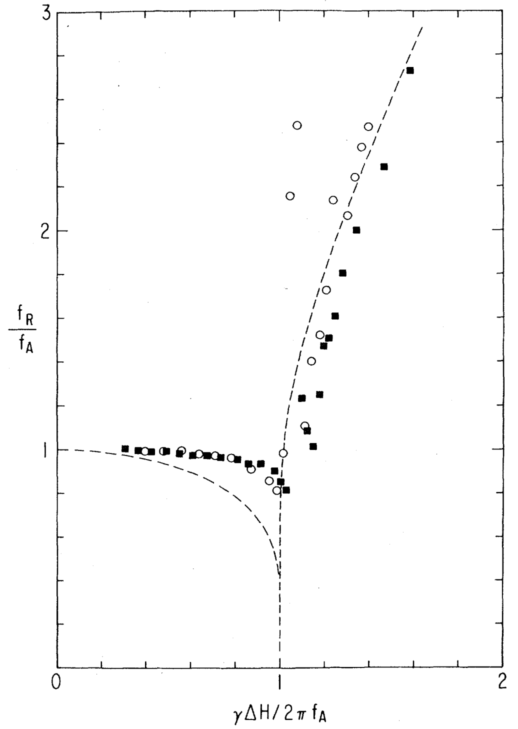

FIG. 2. Nonlinear ringing in ^{3}He-A at 22 bar for the rectangular cavity, $1-T/T_c=0.010$, $f_A=19.6$ kHz, ○; and for parallel plates, $1-T/T_c=0.013$, $f_A=22.2$ kHz, ■. Simple theory gives the dashed line. Ringing up and down occurred observably in the range $\gamma\Delta H/2\pi f_A$ of, respectively, from 0.92 to 1.0 and from 1.0 to 1.1. Points plotted are for the earliest observations after turn-off.

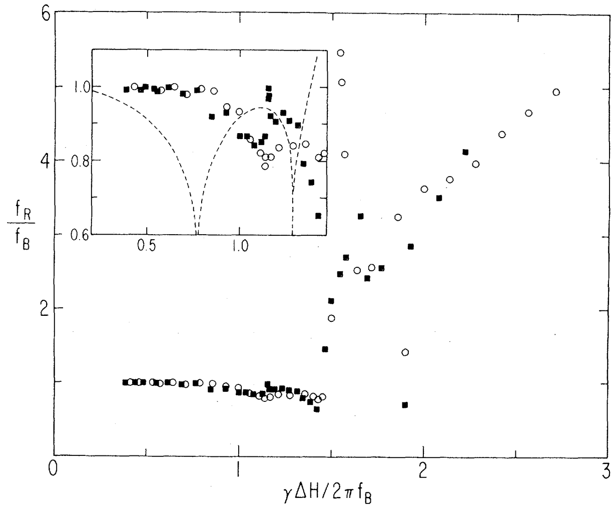

FIG. 3. Nonlinear ringing in ^{3}He-B at 20.7 bar for parallel plates, $1-T/T_c=0.0040$, $f_B=19.8$ kHz, ○; and for rectangular cavity, $1-T/T_c=0.0045$, $f_B=20.1$ kHz, ■. The inset shows the oscillatory mode with a 5× vertical scale and the *same* horizontal scale. Simple theory gives the dashed line. Ringing up and down, respectively, below and above the second minimum of the oscillatory modes was observable only in the range of $\gamma\Delta H/2\pi f_B$ of from 1.1 to 2.4 for the plates and from 0.8 to 1.8 for the rectangular cavity. Within these ranges the earliest observation after turn-off is plotted.

ory, shown by the dashed lines, predicts that $f_R \to 0$ when $\gamma\Delta H=2\pi f_A$, where f_A is the linear ringing frequency. Indeed just below $\gamma\Delta H/2\pi f_A=1$ the frequency has decreased and increases (relaxes) with time. Somewhat above $\gamma\Delta H/2\pi f_A=1$, excluding the peak, the frequency decreases with time. A quantitative analysis of this relaxation has been done by Ambegaokar and Levy,[9] who accounted for its principal features. For $\gamma\Delta H/2\pi f_A\lesssim 0.9$ we observe no relaxation while f_R/f_A is considerably larger than theory for both geometries. Just above $\gamma\Delta H/2\pi f_A=1$ we observe in the rectangular cavity *weak* high-frequency signals, probably from a nonuniform mode. These appear only in a small range of ΔH. Attempts to make quantitative observations have been frustrated by irreproducibilities, conceivably due to very slight temperature changes, indicating an uncontrolled variable. Only a vestige of the high-frequency mode is observable in the parallel-plate geometry. For higher drive ΔH we move onto the driven mode. Except in the vicinity of $\gamma\Delta H/2\pi f_A=1$ the discrepancy between theory and experiment is not caused by relaxation.

Nonlinear ringing in ^{3}He-B for both geometries is shown in Fig. 3 with the simple theory for ideal geometry shown as a dashed line. Although two minima are observed they are closer together and at a higher ΔH than predicted by theory. Again the frequency remains near the linear value up to high values of ΔH with no relaxation being observable for $\gamma\Delta H\lesssim 2\pi f_B$. A weak high-frequency, probably nonuniform, geometry-dependent mode is again observed for a small range of ΔH above the second minimum. The characteristics of such modes are again irreproducible. The observation that geometry does not grossly affect the oscillatory modes, those for $f_R/f_B\leq 1$, suggests that an uncertain texture is not the source of the discrepancy between theory and experiment.

The concept of the experiments presented here is indeed very simple.[2] The dipolar energy coupling the superfluids is analogous for ^{3}He-A to the gravitational energy of a simple pendulum and for ^{3}He-B to that of two simple pendula coupled so that the angle of rotation of one is twice that of the other (Ref. 2, Fig. 34). The action of suddenly turning the field off is analogous to giving the mechanical system an initial angular impulse or an initial incremental energy. The limiting driven modes correspond to such a large initial impulse that the angular velocities scarcely depend on potential energy; the minima in f_R correspond to the impulse being just large enough to put the system on a potential energy maximum [there are two of different heights for the Balian-Werthamer (BW) state]; and the oscillatory modes result when the initial energy increment is not large enough to overcome the maximum potential energy difference. The linear ringing and resonance frequencies (small ΔH) depend on the curvature of the dipolar energy E_D with respect to phase at minima of E_D. Although there still exist discrepancies,[2,10] we have shown[11] in the present plate geometry that the ratio of B to A linear frequencies is in reasonable agreement with the Anderson-Brinkman-Morel (ABM) and BW assignments for the A and B phases in agreement with Osheroff's conclusions[12] from his work at melting pressure. Thus, in at least one of our present geometries the apparent relative curvatures of E_D for the A and B phases are consistent with the simple picture. It is thus significant that the nonlinear response is not consistent with the simple model and further that this conclusion is only weakly affected by geometry and not affected by relaxation[9] except near the minima of f_R. On the other hand, the spin-tipping experiments of Osheroff and Corruccini[13] also sense the full dependence of dipolar energy on angle and, as interpreted by Brinkman and Smith,[14] appear to give good agreement with the simple theory. Their experiments, performed at melting pressure, are however different from the present ones in that the magnetization and hence a property of the fluid is suddenly changed rather than the magnetic field, as in the present work. Other substantial discrepancies exist: (1) The frequency of the wall-pinned mode extrapolated to zero time[15] is in quantitative disagreement with the simple theory while all other characteristics fit current ideas. (2) There is a significant difference between $[\chi_N - \chi_B(T)]/\chi_N$, the susceptibility change in the B phase relative to that of the normal liquid, as determined by spin resonance[10,12] and statically[2]; this discrepancy has been observed again in the ideal geometry of parallel plates in the present apparatus. All these results suggest that there may be more effects coming into play, possibly in orbital or collective degrees of freedom, than have been considered in the simple theory.

We are indebted to Professor V. Ambegaokar, Professor J. Bardeen, Professor A. J. Leggett, and Professor K. Maki for helpful discussions on various parts of this work.

†Work supported by the U. S. Energy Research and Development Administration under Contract No. E(04-3)-34, P. A. 143.

[1]A. J. Leggett, Rev. Mod. Phys. 47, 331 (1975). See this article for further references.

[2]J. C. Wheatley, Rev. Mod. Phys. 47, 415 (1975). See this article for further references.

[3]B. D. Josephson, Adv. Phys. 14, 419 (1965).

[4]R. A. Webb, R. L. Kleinberg, and J. C. Wheatley, Phys. Rev. Lett. 33, 145 (1974).

[5]K. Maki and T. Tsuneto, Prog. Theor. Phys. 52, 773 (1974).

[6]K. Maki and C.-R. Hu, J. Low Temp. Phys. 19, 259 (1975).

[7]K. Maki and H. Ebisawa, "Exact Magnetic Ringing Solutions in Superfluid ^{3}He-B" (to be published).

[8]W. F. Brinkman, H. Smith, D. D. Osheroff, and E. I. Blount, Phys. Rev. Lett. 33, 624 (1974).

[9]V. Ambegaokar and M. Levy, to be published.

[10]A. I. Ahonen, T. A. Alvesalo, M. T. Haikala, M. Krusius, and M. A. Paalanen, Phys. Lett. 51A, 279 (1975).

[11]R. A. Webb, R. E. Sager, and J. C. Wheatley, to be published.

[12]D. D. Osheroff, Phys. Rev. Lett. 33, 1009 (1974).

[13]D. D. Osheroff and L. R. Corruccini, Phys. Lett. 51A, 447 (1975).

[14]W. F. Brinkman and H. Smith, Phys. Lett. 51A, 449 (1975), and "Large Angle Tipping Frequency Shifts in Pulsed NMR for ^{3}He-B" (to be published).

[15]R. A. Webb, R. E. Sager, and J. C. Wheatley, to be published.