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## The Effect of Mortgage Timeline on the Investor's Portfolio

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# THE EFFECT OF MORTGAGE TIMELINE ON THE INVESTOR'S PORTFOLIO

By

Grace Wylie

Submitted in Partial Fulfillment  
of the Requirements for  
Graduation with Honors from the  
South Carolina Honors College

May 2016

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# **Thesis Summary**

## **Overview & Purpose**

This thesis project creates a hypothetical situation in which individuals choose either a thirty-year or fifteen-year fixed-rate mortgage on the same house at the same time and uses historical data to simulate the stock portfolios resulting from investment strategies based on this choice. Both strategies have the same time horizon and total monthly payment, so investors should be indifferent between the two. Using the fifteen-year strategy, the investor pays a higher mortgage payment every month for fifteen years then invests the entire amount of the payment in the stock market monthly for the next fifteen years. Using the thirty-year strategy, the investor pays a lower mortgage payment for thirty years and invests the savings monthly throughout the mortgage's life. This project compares the simulated final values of portfolios beginning every month starting in 1971 to determine if historical data supports the use of one strategy over the other. The purpose of this thesis is to provide future homebuyers with knowledge about two possible investment strategies and to assist them in choosing one that will be beneficial in the long run.

## **Process & Methodology**

The first step in this project is to calculate the fifteen-year and thirty-year mortgage payments. To do this, the price of the average single family home in the United States was calculated for each month starting in 1971 using the current average home price published by the United States Census Bureau and the Case-Shiller Home Price Index, an index that measures the changes in home values in the United States. Once the home prices were determined, the monthly payments for both strategies were calculated

using historical mortgage rates. The future values of the investment portfolios were then calculated using historical returns of Standard and Poor's 500 Index (the S&P 500), an index of five hundred large American companies. This index is considered to be representative of the majority of the market capitalization of the American stock market. After the results of this simulation were calculated and analyzed, the future values of portfolios beginning within the past five years were projected using the geometric average return of the S&P 500 over the thirty years prior to each portfolio's start date.

## **Results & Significance**

The historical portion of this project shows that one strategy is not always a better choice than the other. In periods of low interest rates, the portfolios created using the thirty-year strategy outperformed those using the fifteen-year strategy in 96.67% of the trials. This is likely due to the advantage of compounding returns afforded to the thirty-year strategy portfolios. However, in periods of high interest rates, the fifteen-year strategy portfolios consistently outperformed their thirty-year counterparts. During these times, the monthly investment amount using the thirty-year strategy is much lower than that of the fifteen-year strategy, so even with the advantage of compounding over a longer period of time, the ending values of the thirty-year portfolios were lower than those of the fifteen-year portfolios. The forward-looking portion of the project used these findings to recommend strategies to future homebuyers. Now, while interest rates are at record lows, the thirty-year strategy is likely to be more beneficial, and this is supported by the projected future values of portfolios beginning within the past five years. However, rates are predicted to rise in coming years, so individuals planning to purchase homes several years from now should consider the fifteen-year strategy instead.

## **Section I: Introduction & Purpose**

### **Introduction**

For decades, financial planners have been divided into two major camps over how to advise their clients on the issue of mortgage timeline. The larger group recommends that, as long as interest rates are acceptably low, homebuyers opt for a mortgage with a longer timeline and lower monthly payment then use the excess cash from that lower payment to build an investment portfolio during the life of the mortgage. This strategy gives the individual's investment portfolio the advantage of compounding interest over time. However, this is not the only option, and it may not even be the most profitable one for the homeowners in the end.

Advisors in the second camp suggest the alternative strategy of choosing the shortest home mortgage timeline whose monthly payment the individual can afford. With a shorter mortgage timeline, homeowners pay less to the bank in interest and repay their debt sooner, which frees up cash to invest in financial securities when the mortgage is paid off. The purpose of this thesis is to use historical data to compare the hypothetical investment portfolios resulting from both of these options to answer the following questions. Does one option always yield a higher portfolio value at the end of the investment horizon? If not, are there certain factors, like the timing of periods of high and low stock prices, which cause one portfolio to unexpectedly outperform the other?

This project is significant in the field of personal finance because, over the course of their lives, the majority of people will enter into mortgages and participate in the financial securities market. The aim of the project is to provide the average investor with more information about the merits of two common investment strategies and to determine

whether historical data supports the use of one over the other. It is anticipated that the results of this thesis will assist future homeowners in choosing an investment strategy that will be beneficial to them in the long run.

## Methodology

This project is designed to isolate two variables: investment timeline and monthly amount invested. It creates a hypothetical situation in which two investors, A and B, purchase identical homes on the same day and choose 15-year and 30-year mortgages, respectively. Investor A will pay the mortgage payment every month for 15 years, then, once the mortgage is paid off, invests the full amount of the payment in the stock market. Investor B pays the lower mortgage payment and invests the excess (the difference between A's payment and B's payment) in the stock market monthly for the full 30 years. Both investors will invest in the same portfolio of US stocks on the same day each month. The final value of the two portfolios will be calculated for each investment period so that the results of the two strategies can be compared and long-term trends can be analyzed. The process for calculating the final values of the portfolios is shown in Exhibit 1 below.

### Exhibit 1: Calculation Process





## **Section II: Data & Calculations**

### **Mortgages**

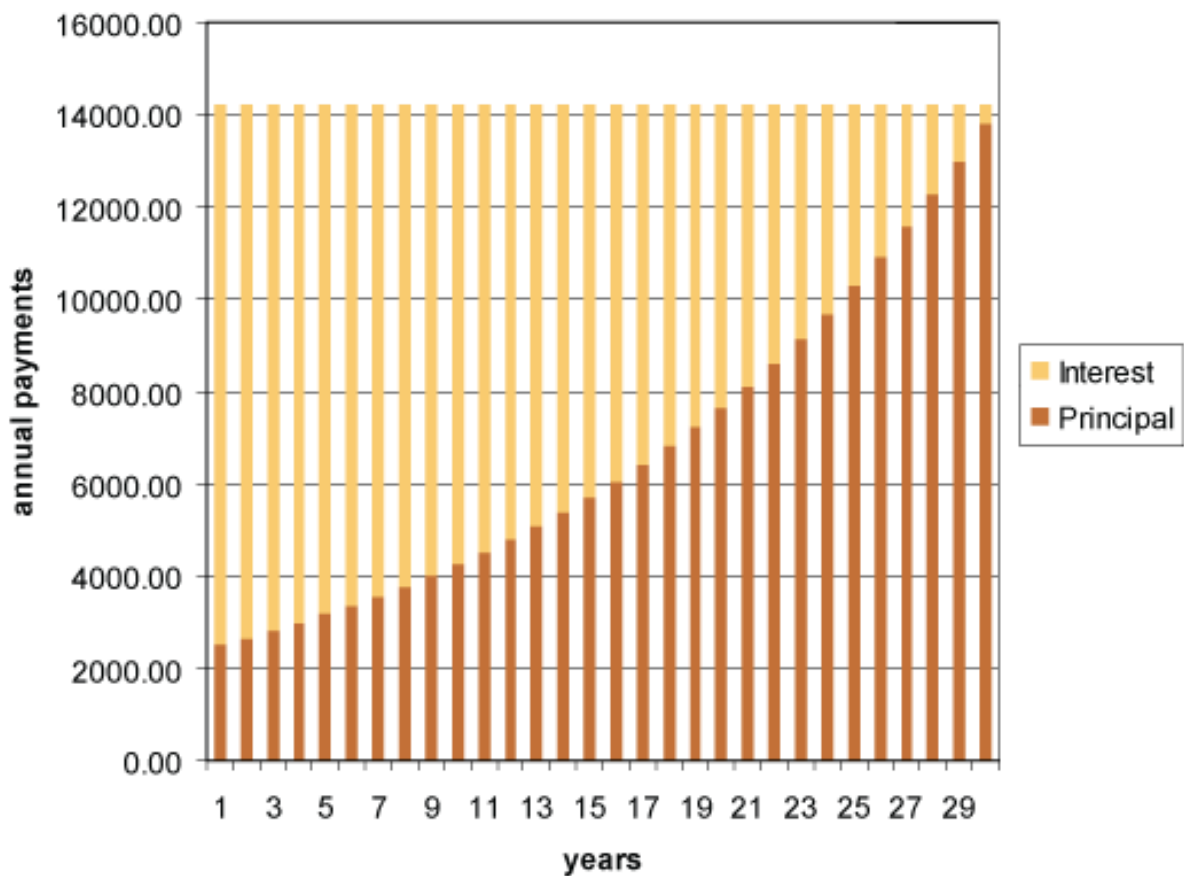
#### **How Do Mortgages Work?**

According to Investopedia, a mortgage is “a debt instrument, secured by the collateral of specified real estate property, that the borrower is obliged to pay back with a predetermined set of payments.” (Mortgage Definition: Investopedia, 2003). In other words, a mortgage is a type of real estate loan in which the lender can seize the real estate if the borrower ceases to make payments. Most individuals cannot afford to pay the entire purchase price of a home at once; by using a mortgage, the borrower can make a down payment on the home then take out a mortgage on the remaining amount. He or she will then make monthly payments over a period of several years to pay off the loan until he or she completely owns the home. The typical ratio of down payment to mortgage loan is twenty percent to eighty percent, meaning the typical borrower puts twenty percent of the property’s value down up front and borrows the other eighty percent. By paying twenty percent at the start, the borrower avoids having to purchase mortgage insurance, which lenders generally require for mortgages with lower down payments.

The majority of mortgages are fixed-rate mortgages. In a fixed-rate mortgage, both the interest rate on the loan and the scheduled monthly payments of interest and principal remain constant over the life of the loan. Most fixed-rate mortgages have a term of either fifteen years or thirty years. A key feature of mortgages is called amortization, a type of payment schedule in which the borrower pays down the principal of the loan over its life instead of paying the entire principal amount back at the end of the loan term. This way, the principal amount of the loan decreases, or amortizes, down to zero after the last

payment is made. Although the payments of a fixed-rate mortgage are the same each month over the loan's life, the fraction of the payment that goes toward interest and principal changes each month. The borrower pays the predetermined interest rate on the entire amount of principal outstanding, so the monthly payments at the beginning of the loan term mostly go toward paying interest. However, as the principal is paid down, the amount of interest due each month decreases, and more of the payment goes toward paying down the principal. Exhibit 2 below illustrates this concept.

**Exhibit 2: Amortization of a 30-Year Fixed-Rate Mortgage**



(The Federal Reserve Board, 2008)

Another important feature of mortgages is the prepayment option. At any time over the life of the mortgage, the borrower has the option to pay more than the scheduled

monthly interest and principal payment. The excess amount goes toward the outstanding principal, shortening the term of the mortgage and decreasing the amount of interest the borrower will eventually owe. For the purposes of this project, it will be assumed that the borrowers choose a fixed-rate mortgage with a term of either fifteen or thirty years and do not exercise the option to make prepayments on the principal.

## **Calculating a Mortgage Payment**

### **Home Prices**

The first step in calculating the value of a monthly mortgage payment is to determine the price of the home to be mortgaged. Because this project isolates the variable of the mortgage term, the home price for each possible investment period will be the same. In other words, the project will simulate the investment portfolios resulting from either a fifteen-year or thirty-year mortgage on the same house. To accomplish this, the average price of a home in the United States was approximated for each month using the United States Census Bureau's value for the current average home price and the Case-Shiller US National Home Price Index. The Case-Shiller Index uses residential property resale values to measure changes in home prices in various geographical areas as well as in the United States as a whole. Formula 1 below was used to calculate the average home price for each previous period. The calculated home price values for each period can be found on the "Home Price" sheet in the accompanying Excel document.

#### **Formula 1: Previous Period Price**

$$Price_{n-1} = \frac{Price_n}{1 + i_n}$$

where  $n$  = time period and  $i$  = the percent change in home prices from period  $n-1$  to  $n$

## Interest Rates

The second factor necessary to calculate a monthly mortgage payment is the mortgage's interest rate. While there are historical records of thirty-year mortgage interest rates starting in the 1890s, the American housing market changed when government-sponsored enterprises Fannie-Mae and Freddie Mac began to subsidize housing. To make this project more relevant for current investors, the portfolio simulations will be calculated beginning in April 1971, when Freddie Mac began to intervene in the housing market by purchasing and securitizing thirty-year fixed-rate mortgages.

Although fifteen-year fixed rate mortgages have been available to homebuyers for decades, Freddie Mac did not begin to purchase these loans until September 1991, so there is very little information available on their interest rates prior to that point. However, the interest rates on thirty-year and fifteen-year mortgages are highly positively correlated, meaning they move in sync with each other, so the fifteen-year mortgage interest rates prior to 1991 can be extrapolated from the post-1991 data using the percent changes in the thirty-year rates. Percent change is calculated using Formula 2 below.

$$\textbf{Formula 2: Percent Change}$$
$$\textbf{Percent Change} = \frac{X_n - X_{n-1}}{X_{n-1}}$$

where  $X_n$  = interest rate from period n and  $X_{n-1}$  = interest rate from the previous period

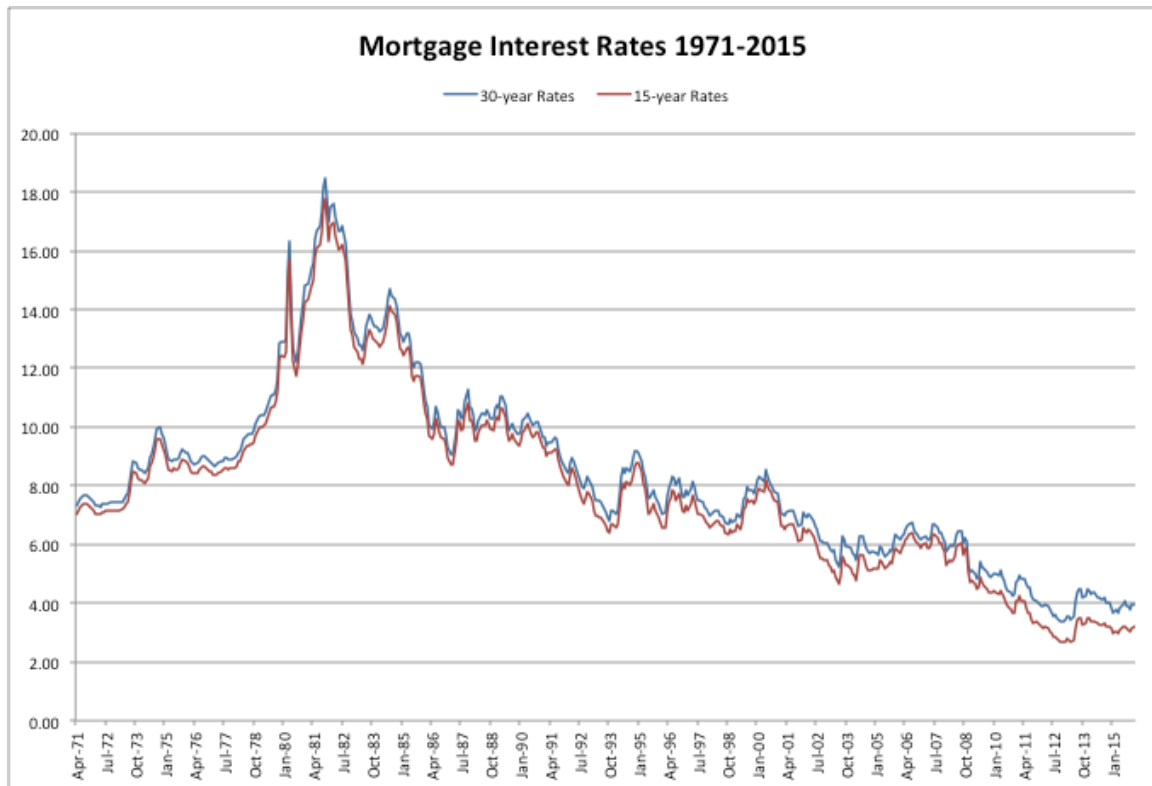
The percent change can be applied to the fifteen-year interest rates using Formula 1 on the previous page, substituting interest rates for home prices and the percent change in thirty-year rates for  $i$  as shown below.

$$Rate_{n-1} = \frac{Rate_n}{1 + i_n}$$

where  $i_n$  = percent change in thirty-year rates from period n-1 to n

The fifteen-year and thirty-year mortgage rates from 1971 to 2015 are shown in Figure 3 below, using the process above to extrapolate the fifteen-year mortgage rates from 1971 to 1991. The numerical interest rate values can be found on the “Mortgage Interest Rates” sheet in the accompanying Excel document.

**Figure 3: Mortgage Interest Rates Chart 1971-2015**



### **The Payment Calculation**

Once the purchase price of the property and the interest rate on the mortgage are known, the loan’s required monthly payments can be calculated. For the purpose of this project, it will be assumed that the homebuyers pay the typical twenty percent of the home’s value down and borrow the remaining eighty percent. Given the loan amount, interest rate, and term of the loan, the monthly payment required to completely repay the

loan in the requisite amount of time can be calculated by using Formula 3 below and solving for the payment ( $P$ ). This formula can easily be applied to data in Microsoft Excel using the payment (PMT) function. The required monthly payments for fifteen-year and thirty-year mortgages on an average house purchased in each month beginning in April 1971 can be found on the “Mortgage Payments” sheet in the accompanying Excel document.

**Formula 3: Monthly Payment Amount**

$$\text{Loan Amount} = P \frac{(1 + i)^n - 1}{i(1 + i)^n}$$

where  $P$  = monthly payment,  $i$  = monthly interest rate, and  $n$  = loan term in months

The total cost of the mortgage including principal and interest can be calculated by multiplying the monthly payment by the number of months in the loan term; the total interest cost of the loan is simply the difference between the total cost of the loan and the principal amount. These values are also included on the “Mortgage Payments” sheet. It should be noted that, due to the shorter term, a fifteen-year mortgage has a higher monthly payment than a thirty-year mortgage. However, because fifteen-year mortgages have lower interest rates, the total interest cost over the life of these loans is significantly lower than those of thirty-year mortgages.

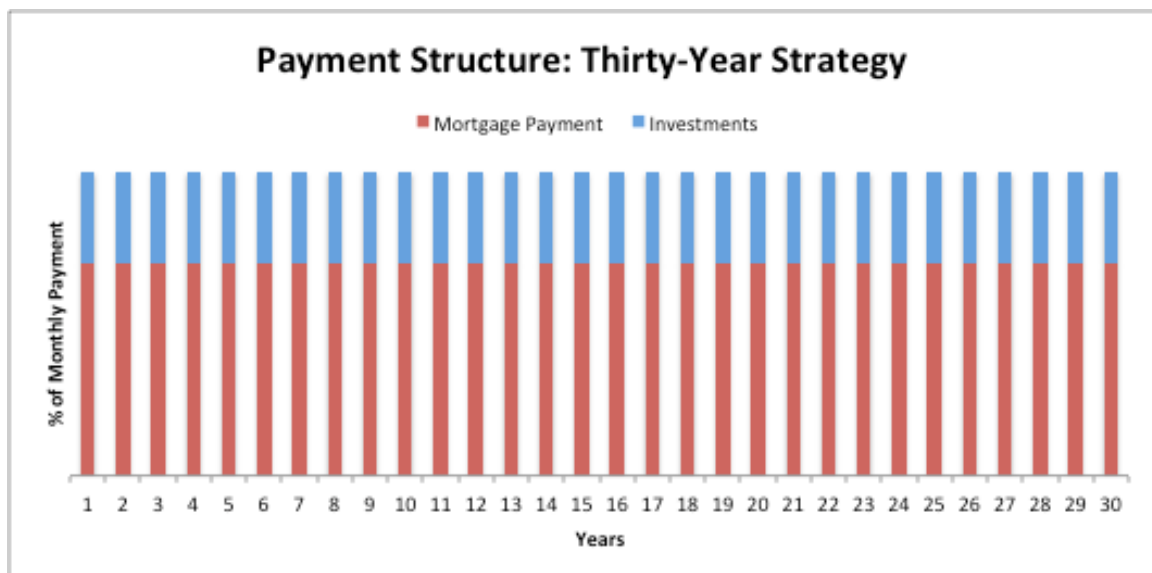
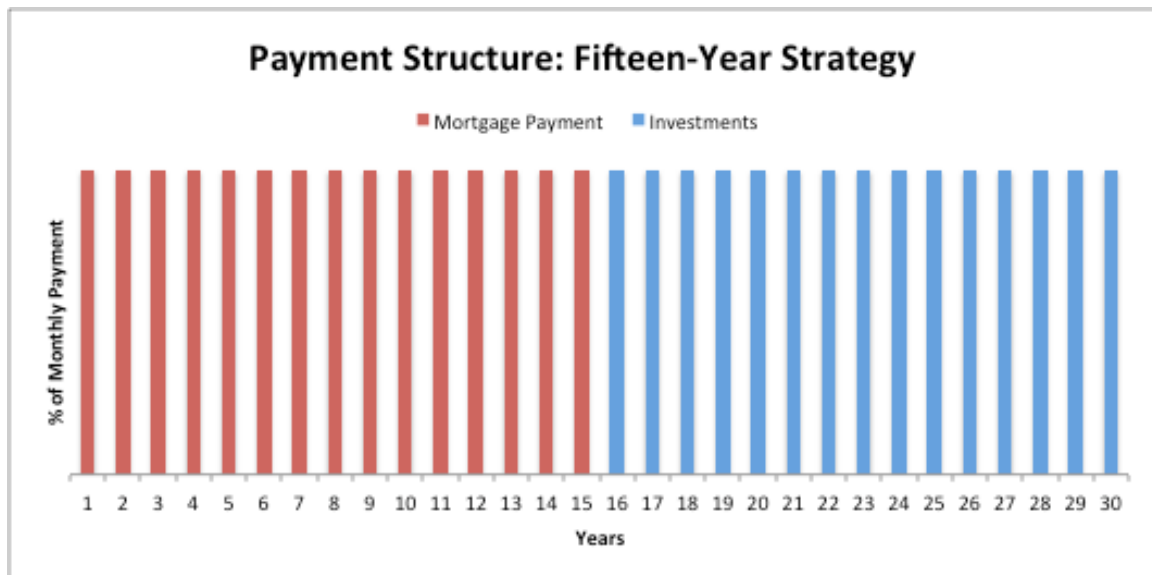
## **Investing**

### **Comparing Investment Strategies**

This project uses the monthly mortgage payments calculated in the previous section to simulate the stock portfolio resulting from two investment strategies, denoted hereafter as the fifteen-year strategy and thirty-year strategy. Both strategies have a thirty-year investment term and the total monthly payment goes toward the mortgage and/or investments. In the fifteen-year strategy, the investor pays the mortgage payment

monthly for the fifteen-year loan term then invests the amount of the payment in a stock portfolio every month for the remaining fifteen years. In the thirty-year strategy, the investor pays the lower mortgage payment each month for thirty years and invests the excess in the stock market monthly for the same amount of time. Exhibit 4 shows a diagram comparing the payment structures of the two strategies.

**Exhibit 4: Monthly Payment Structures**



## **Assumptions**

Several key assumptions are required to prevent excess variables from affecting the results of the stock portfolio projections. The primary assumption is that the individuals do not exercise their prepayment option. This means that the borrower pays exactly the required mortgage payment every month without paying extra to decrease the outstanding principal more quickly. This also includes the assumptions that the borrowers never refinance their mortgage or sell the home before the mortgage has been repaid. Another assumption is that the borrower does not make late payments, miss payments, or default on the loan.

## **Returns and the Portfolio's Value**

### **Stock Market Returns**

Investors today have a seemingly endless pool of investment opportunities from which to choose based on factors like individual risk tolerance and investment horizon. However, historically, an investor's options were much more limited. This project will assume that the individuals choose to invest only in the domestic stock market, both to facilitate comparison over time and to expose the portfolios to returns that tend to reflect the state of the American economy. To achieve this, the portfolios will track the returns of Standard & Poor's 500 Index (the S&P 500), an index of five hundred large American companies listed on the New York Stock Exchange or NASDAQ. The S&P 500 index has been reported since 1957 and is widely considered to be representative of the majority of the market capitalization of the American stock market, making it appropriate for the scope and time frame of this project.



The S&P 500's opening and closing values as well as other information like the intra-day high and low are reported daily on financial reporting websites like Yahoo Finance. The percent return of the S&P 500 for each month is calculated from the reported adjusted close using Formula 2 from the previous section with  $X$  as the S&P adjusted close on the first day of each month, as shown below. The monthly S&P returns from 1971-2015 are listed on the "S&P Returns" sheet in the accompanying Excel document.

$$\text{Percent Return} = \frac{X_n - X_{n-1}}{X_{n-1}}$$

where  $X_n$  = S&P adj. close on day 1 of month  $n$  and  $X_{n-1}$  = S&P adj. close on day 1 of the previous month

### **Calculating the Portfolios' Final Values**

The project's final step is to simulate the ending portfolio values resulting from the fifteen-year and thirty-year strategies using the S&P 500 returns calculated above and the monthly mortgage payments calculated in Section I. The investment term for the thirty-year strategy is thirty years or 360 months and the amount invested is the difference between the monthly payments of the fifteen-year and thirty-year strategies. The final value of the stock portfolio for this strategy can be calculated by multiplying the monthly investment amount by the S&P 500 return for that month plus one, adding the investment amount for the next month, multiplying the sum by the S&P return for the second month, and so on for all 360 months. Exhibit 5 on the next page shows an example of the first few iterations of this process. When the process is continued to month 360, the ending value is the final value of the stock portfolio after investing monthly for thirty years.

### Exhibit 5: Portfolio Calculation Process

|                 | Month 1     | Month 2     | Month 3     |
|-----------------|-------------|-------------|-------------|
| Beginning Value | $X$         | $X + X_1$   | $X + X_2$   |
| S&P Return      | $(1 + i_1)$ | $(1 + i_2)$ | $(1 + i_3)$ |
| Ending Value    | $X_1$       | $X_2$       | $X_3$       |

$X = (\text{15-year mortgage payment} - \text{30-year mortgage payment})$ ,  $i_n = \text{S\&P return in month } n$

Using the process described above to calculate the future value of the stock portfolio is mathematically equivalent to summing the future values of each monthly investment, as in Exhibit 6 below. This process can be further consolidated into Formula 4, a formula created specifically for this thesis project.

### Exhibit 6: Alternate Portfolio Calculation Method

$$\begin{aligned}
 FV_p = & X(1 + i_1)(1 + i_2)(1 + i_3) \dots (1 + i_{360}) \\
 & + X(1 + i_2)(1 + i_3)(1 + i_4) \dots (1 + i_{360}) \\
 & + X(1 + i_3)(1 + i_4)(1 + i_5) \dots (1 + i_{360}) \\
 & + X(1 + i_4)(1 + i_5)(1 + i_6) \dots (1 + i_{360}) \\
 & \dots + X(1 + i_{360})
 \end{aligned}$$

where  $X = (\text{15-year payment} - \text{30-year payment})$ ,  $i_n = \text{S\&P return in month } n$

### Formula 4: Future Value of Portfolio<sup>1</sup>

$$FV_p = \sum_{j=1}^{360} X \prod_{n=1}^j (1 + i_n)$$

The final values of the portfolios using the fifteen-year strategy can be calculated using the processes above with the monthly investment amount equal to the fifteen-year monthly mortgage payment and an investment term of fifteen years or 180 months. The final values for the portfolios using both strategies are listed in the “Portfolio Final Values” sheet in the Excel document.

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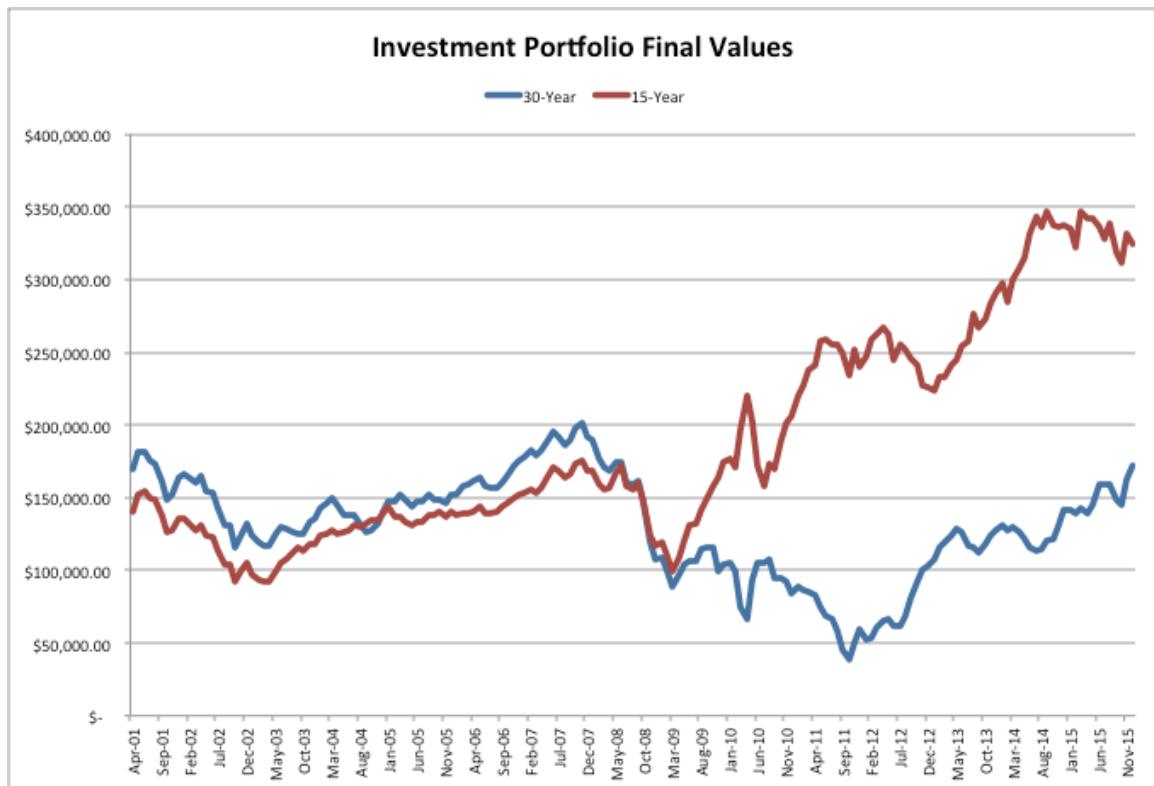
<sup>1</sup> Formula 4 was created with the assistance of Ms. Nichole Witten, a Mathematics major and member of the South Carolina Honors College Class of 2016.

## Section III: Results and Conclusions

### Comparison of Final Values

Exhibit 7 compares the final values of the stock portfolios using either the fifteen-year or thirty-year strategy for thirty-year investment periods starting in monthly increments beginning with the April 1971 – April 2001 investment period and ending with the December 2000 – December 2015 period.

**Exhibit 7: Investment Portfolio Final Values**



### Analysis of Results

The chart above shows that the stock portfolios exhibit different behavior in three time windows: April 2001 to September 2008, October 2008 to March 2009, and April 2009 to December 2015. These dates refer to the final month of the investment period, so the thirty-year strategy and fifteen-year strategy portfolio values listed for April 2001

began paying mortgages in April 1971 and investing in the stock market in April 1971 and April 1986, respectively.

#### **April 2001 – September 2008**

The final values using the two strategies during this period are fairly similar and tend to move in sync. Predictably, the thirty-year strategy portfolio ended with a higher value than the fifteen-year strategy portfolio in 96.67% of the scenarios. This is likely due to the fact that the individual using the thirty-year strategy invests for twice as long as the individual using the fifteen-year strategy, so the thirty-year portfolio has the advantage of compounding returns.

#### **October 2008 – March 2009**

The portfolios during this time period show a significant decrease in value that is likely due to the sharp decline in the performance of the American stock market during the financial crisis that took place over the same time period. The stock portfolios with investment periods ending between October 2008 and March 2009 were affected most significantly by the negative returns because the majority of the portfolios' value had already accumulated. For example, in October 2008, the S&P 500 experienced a 16.94% decline. The thirty-year strategy portfolio that began in October 1985 had a value of \$78,523.26 at the beginning of October 2008 then lost 16.94% of its value over that month, a loss of \$13,193.05. However, this portfolio's investment period continued for another seven years, and positive returns over most of that time caused the portfolio to recover from this loss and end with a value of \$145,939.97. On the other hand, the thirty-year strategy portfolio that began in November 1978 had a value of \$143,138.68 at the beginning of October 2008, its final month of investing. When it experienced the same

16.94% decline in value, the portfolio lost \$24,161.93 and had no opportunity to recover. Although the two portfolios experienced the same percentage loss in value, the portfolio ending sooner had a higher dollar value loss and no time to recover, making this investor much worse off.

#### **April 2009 – December 2015**

During this time period, the final portfolio values diverge with fifteen-year strategy portfolios far outperforming those with the thirty-year strategy. While it is easy to assume that this behavior is caused by the 2008 financial crisis or subsequent economic recovery, it is likely that the true cause of the divergence occurred several decades prior. The early 1980s saw an enormous spike in interest rates, so individuals who took out mortgages during this time would have very high monthly payments. This means that the thirty-year strategy would invest relatively little each month for the thirty-year period while the fifteen-year would have a large amount freed up to invest once the mortgage was repaid. Also, the individuals using the fifteen-year strategy beginning in the early to mid-eighties would have begun investing in the late 1990s or early 2000s during the dot-com bubble, when stock market returns were extremely high. Furthermore, investing a constant monthly amount during the subsequent market downturns in 2001 and 2002, when share prices were low, would have allowed these investors to greatly increase their share balance, the quantity of shares in the portfolio. When share values began to increase again, positive returns would have been realized by a greater number of shares, causing the portfolio's total value to increase by a greater amount. These effects would result in the fifteen-year portfolios ending with much higher values than their thirty-year counterparts.

However, it should be noted that, once interest rates fell, rational borrowers would have refinanced their mortgages at these lower rates, reducing their monthly payment and decreasing the size of the gap between the performances of the two strategies' stock portfolios. However, because this project assumes that borrowers do not refinance, the results are likely misleading for these portfolios. In reality, the thirty-year strategy portfolios would have been higher because the individuals would have had more money to invest each month after refinancing their mortgage. Conversely, the fifteen-year strategy portfolios would have had lower values because the individuals would have had lower monthly payments after refinancing, resulting in lower amounts invested in the stock market.

## **Conclusions and Implications for Future Homeowners**

The results of the historically based portion of this project show that each of the two investment strategies provides a better outcome for investors in a different scenario. One portfolio does not always outperform the other. The pivotal question arising from this conclusion is this: which strategy is a better choice for investors today? This is a difficult question to answer empirically due to finance's inherent inability to predict future interest rates or stock market returns, but historical data can be used to project the performance of future portfolios.

The future values of the portfolios of investors who began a mortgage over the past five years, January 2011 to December 2015, were projected using the monthly investment amount, calculated in the same way as those in the historical portion of the project, assuming the portfolio earns a geometric average return equal to that of the prior thirty-year investment period. For example, it is assumed that the portfolios of investors

with mortgages beginning in January 2011 grow at a rate equal to that of the geometric average S&P return from January 1981 to December 2010. Because the return is assumed to be constant, the future values of these portfolios can be calculated like annuities, using Formula 5 below.

**Formula 5: Future Value of an Annuity**

$$FV_{Annuity} = X * \frac{(1 + i)^n - 1}{i}$$

X = amount invested monthly, i = geometric average return, n=number of periods

Calculating the geometric average return over multiple periods gives the average compounded return per period and is calculated using Formula 6. Because it includes the effects of compounding interest, the geometric average return is a more appropriate measure for stock portfolios than the arithmetic, or traditional, average.

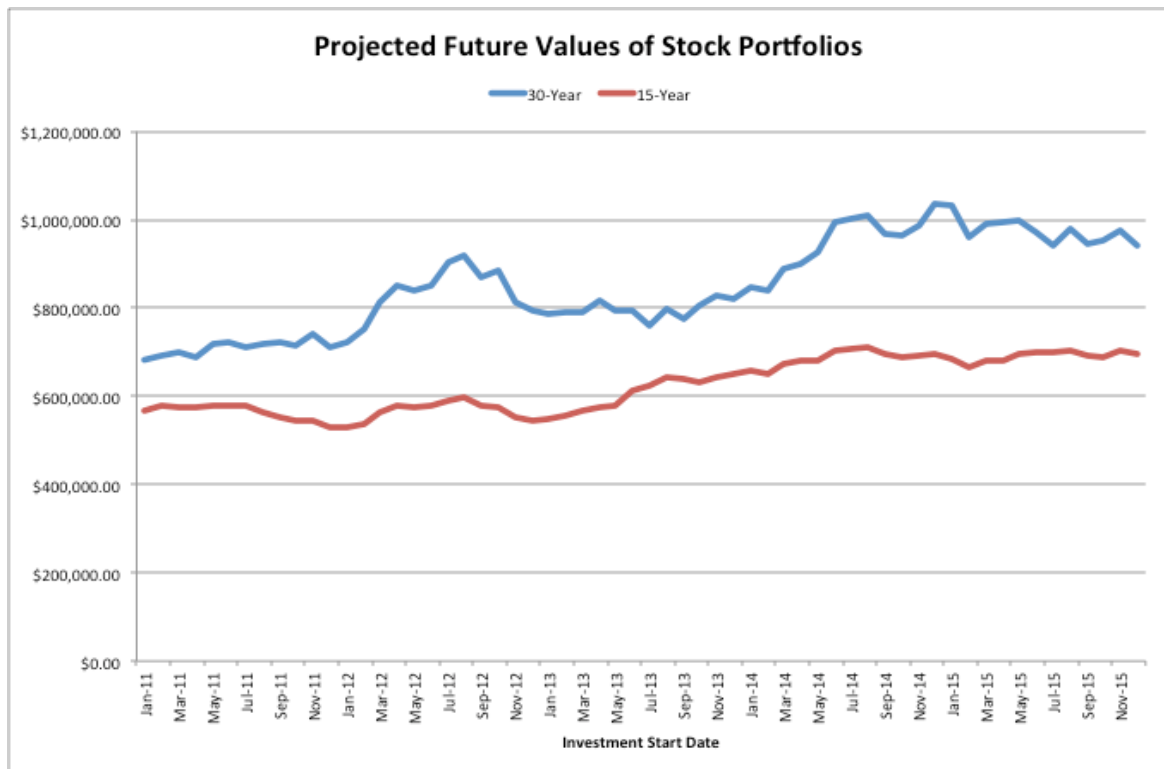
**Formula 6: Geometric Average Return**

$$Geometric\ Return = ((1 + i_1) \times (1 + i_2) \times \dots \times (1 + i_n))^{1/n}$$

where  $i_n$  = S&P return in period n, n = number of periods

The numerical results of the projection can be found in the “Projected Future Values” sheet in the Excel document and are summarized in the chart on the following page. The portfolios resulting from the thirty-year strategy are predicted to consistently outperform the portfolios resulting from the fifteen-year strategy. Interest rates are currently at record lows, and monthly payments under both strategies have decreased accordingly. During this time, although the monthly investment amount of the fifteen-year strategy is still higher than that of the thirty-year, it is not enough to overcome the advantage of compounding returns possessed by the thirty-year portfolio. The same result occurred during a previous period of low interest rates, from 1971 to 1978.

## Exhibit 8: Projection of Future Values



It is likely that the Federal Reserve will continue to increase the federal funds rate in coming years, which may cause mortgage interest rates to increase as well. In December 2015, the Federal Reserve increased the target range for the federal funds rate from near zero to between .25 and .50 percent, and according to a Reuters article from December 2015, “Fed policymakers' median projected target interest rate for 2016 remain[s] 1.375 percent, implying four quarter-point hikes next year” (Schneider, H. & Lange, J., 2015).

An increase in interest rates would likely result in the fifteen-year strategy portfolios outperforming the thirty-year strategy portfolios. This result occurred when investors began mortgages during the interest rate spike in the early 1980s. During



periods of high interest rates, both strategies have very high monthly payments, meaning the individual using the thirty-year strategy can invest relatively little each month while the individual using the fifteen-year strategy is free to invest much larger amounts once the mortgage is repaid. It is unlikely that refinancing would lessen this effect until interest rates decrease again. It will probably be several years before rates begin to trend downward since they are only starting to rise after being held down during and after the 2008 financial crisis. With all of these factors considered, it seems the optimal choice of strategy for future homebuyers depends on the time period in which they purchase a home. While individuals who intend to purchase a home now or in the near future while interest rates are low will likely benefit from the thirty-year strategy, individuals expecting to purchase a home at a later date during periods of higher interest rates should consider using the fifteen-year strategy.

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