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## Experiments on Dynamic Parallel Magnetism in Superfluid $^3\text{He}$

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temperature. Near  $T_\lambda$ , the healing length becomes large, and the apparent porosity will be reduced. For  $^4\text{He}$  in 200-Å powders below 1.6 K, the effect is negligible. But in  $^3\text{He}$  the correlation length is believed to be at least  $10^3$  Å and possibly even  $10^4$  Å. With grain sizes  $\sim 30$   $\mu\text{m}$ , this can lead to a correction of from 1 to 10%.

<sup>10</sup>Although we have derived Eq. (4) from Eq. (3), it is

obvious that (4) is really the more general relation—it expresses the proportionality of  $F_d$  to  $\dot{v}_r$ , and in particular the obvious requirement that there shall be no drag if  $\dot{v}_r = 0$  (even if  $\dot{v}_n \approx \dot{v}_s \neq 0$ ).

<sup>11</sup>We wish to thank Professor I. Rudnick for providing us with preliminary data for a range of porosities in the region  $\mathcal{P} \sim 0.7$ . These results strongly suggest a form close to Eq. (12).

## Experiments on Dynamic Parallel Magnetism in Superfluid $^3\text{He}\dagger$

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Observations are reported of the ringing of parallel magnetization in superfluid  $^3\text{He}$  when an incremental magnetic field parallel to a steady field is suddenly turned off.

We report novel experiments on the dynamic response of the nuclear magnetization of superfluid  $^3\text{He}$  *A* and  $^3\text{He}$  *B* parallel to the axis of a magnetic field which is suddenly changed by an incremental amount parallel to itself. These experiments, motivated by the theoretical work of Leggett<sup>1</sup> and the experimental static magnetization measurements of Paulson, Johnson, and Wheatley,<sup>2</sup> both illuminate the theoretical concepts and provide new insights into the nature of the superfluid state. The present experiments, performed over wide ranges of temperature, pressure, and magnetic field, reflect and support the remarkable parallel-resonance experiments on liquid  $^3\text{He}$  at melting pressure of Osheroff and Brinkman<sup>3</sup> and of Bozler *et al.*<sup>4</sup> which were reported while our measurements were in progress. Our experiments also allow important nonlinear phenomena to be studied. Moreover, they may be interpreted, at least in the case of  $^3\text{He}$  *A* and following both Leggett<sup>1</sup> and Maki and Tsuneto,<sup>5</sup> as a manifestation of an internal Josephson effect in the spin-triplet superfluid.

The  $^3\text{He}$  measured was contained in a 3-mm-i.d. tower above a main cell nearly filled with powdered cerium magnesium nitrate (CMN). A steady field  $H_0$  parallel to the axis of the cylinder was trapped in a 6.5-mm-i.d. Nb tube. An incremental field  $\Delta H$  parallel to  $H_0$  was provided by a single-layer Nb solenoid of 3.25-mm i.d. The  $^3\text{He}$  magnetization was sensed by a 4-mm-i.d. by 4-mm-long Nb pickup coil with one end 3.5 mm from the end of the solenoid. The pickup coil was connected with Nb leads to an rf-biased superconducting quantum interference device oper-

ated as a nonoverloading, wide-band flux sensor of low dynamic range. Temperatures were sensed both in CMN in a second magnetically shielded tower above the cell and in the main CMN, although the latter could not be used while small fields in the magnetizing solenoid were controlling the temperature. Liquid  $^3\text{He}$  provided thermal contact between all parts of the cell. Provisional Kelvin temperatures based on noise thermometry and the attenuation of zero sound were assigned to magnetic temperatures by comparison with the line of second-order transitions ( $P$ ,  $T_c$ ) as in our earlier work.<sup>6</sup>

Concepts regarding the nature of superfluid  $^3\text{He}$  which we used to design our experiments are illustrated in Fig. 1. The superfluid is assumed to be characterized by a vector order parameter  $\vec{d}$  in spin space which is a function of orbital variables described by the direction  $\vec{n}$  of BCS-like triplet pairs.<sup>3,1</sup> To be specific, consider the case of  $^3\text{He}$  *A* where, following Anderson and Brinkman<sup>7</sup> and in the absence of boundary or other orienting effects, the vector  $\vec{d}$  is thought to lie in a plane perpendicular to  $H_0$ , as in Fig. 1(a). For a fixed orbital configuration the temperature-dependent average nuclear dipolar interaction energy  $E_D$  varies periodically with angle  $\theta$  as shown in Fig. 1(b) leading to a torque  $R_D$  on the nuclear spins [Fig. 1(c)] which is also periodic in  $\theta$ . At equilibrium, in this example,  $\theta$  is either 0 or  $\pi$ . In our experiment a steady field  $(H_0 + \Delta H)\hat{z}$  is applied for a sufficiently long time, and then at time  $t_0$  it is suddenly reduced to  $H_0\hat{z}$  as in Fig. 1(d). Following Leggett<sup>1</sup> and neglecting relaxation, the equation for the change of the  $z$  compo-

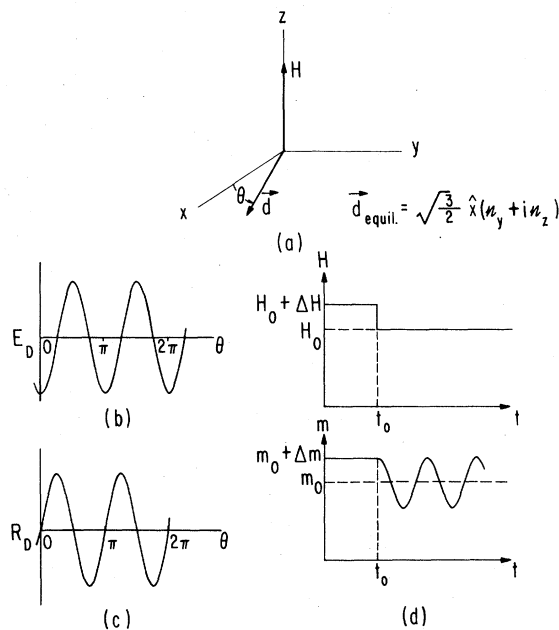


FIG. 1. (a) Configuration of field  $H_0$  and order parameter  $\vec{d}$  for the nonequilibrium ABM state (Ref. 7). (b) Dipolar energy versus  $\theta$  for fixed orbital variables. (c) Dipolar torque versus  $\theta$  for fixed orbital variables. (d) Schematic dependence of magnetic field  $\vec{H}$  and parallel magnetization  $\vec{m}$  on time for the conditions of our experiment. The ringing frequency of  $\vec{m}$  depends on temperature.

ment of spin angular momentum per unit volume is

$$d \Delta S_z / dt = R_D \cong \lambda \theta, \quad (1)$$

where in approximating  $R_D$  we assume that  $\theta$  is small. Leggett showed that  $\vec{d}$  precesses in the field  $\vec{H} - \gamma \vec{S} / \chi$ , where  $\gamma$  is the gyromagnetic ratio and  $\chi$  is the susceptibility, so in the present case we have

$$\dot{\theta} = \gamma (\Delta H - \gamma \Delta S_z / \chi). \quad (2)$$

It is assumed that the orbital variables do not change during this precession. Combining the above two equations, we find that

$$d^2 \Delta S_z / dt^2 + \gamma^2 \lambda \Delta S_z / \chi = \gamma \lambda \Delta H. \quad (3)$$

The magnetization  $\gamma S_z$  thus rings at the temperature-dependent frequency  $(\gamma/2\pi)(\lambda/\chi)^{1/2}$  with amplitude  $\chi \Delta H$  as shown in Fig. 1(d). A similar result has recently been derived for  $^3\text{He A}$  by Maki and Tsuneto<sup>5</sup> who observe that the angle  $2\theta$  is the phase difference between the order parameters  $d_{\uparrow\uparrow}$  and  $d_{\downarrow\downarrow}$  of the two equal-spin-pairing states which are coupled by the phase-dependent cou-

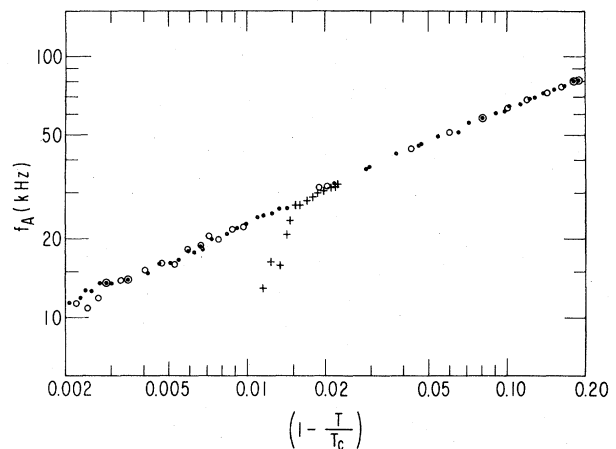


FIG. 2. Ringing frequency in  $^3\text{He A}$  versus reduced temperature difference for 29.4 bar and  $H_0 = 50$  and 310 G (closed and open circles, respectively). The plus symbols denote response to a nonlinear effect; see text.

pling energy  $E_D$  of Fig. 1(b) in an internal Josephson effect.

The ringing phenomenon has been observed in  $^3\text{He A}$  over a pressure range from 8.5 to 33 bar and an  $H_0$  range from 1 to 310 G, consistent with the effect of field on the phase diagram.<sup>9</sup> Typical ringing-frequency results as a function of  $1 - T/T_c$  are shown in Fig. 2 for two values of  $H_0$  at a pressure of 29.4 bar. The ringing frequency  $f_A$  is field independent. However,  $f_A$  can depend on  $\Delta H$ , as shown in Fig. 2 by the plus symbols, for which  $\gamma \Delta H / 2\pi = 25$  kHz. If  $\Delta H$  is too large the angle  $\theta$  can become too large and the response is nonlinear. We will discuss this interesting effect elsewhere; for the present results we maintained  $\gamma \Delta H / 2\pi \lesssim \frac{2}{3} f_A(T)$ , where  $f_A(T)$  is the  $\Delta H$ -independent ringing frequency. A plot of  $f_A^2(T)$  versus  $T$  was linear near  $T_c$  although the extrapolation to zero did not give perfect agreement either with the center of the second-order specific-heat transition<sup>6</sup> in the main cell or with the center of the  $A \rightarrow N$  magnetic feature.<sup>10</sup> The data in Fig. 2 use  $T_c$  from the specific-heat transition. The ringing frequency at  $1 - T/T_c = 0.01$  varied from about 24.5 kHz at 33 bar to about 19 kHz at 21 bar. At 33 bar our values of  $f_A$  agreed within 1 or 2% over the full range of comparison possible with the resonance frequencies and shifts given by Osheroff and Brinkman<sup>3</sup> at melting pressure, but were lower by 5 to 10% than those of Bozler *et al.*<sup>4</sup> At very low values of  $H_0$  the signal was degraded: For  $H_0 = 0$ , degradation was observed for  $1 - T/T_c < 0.01$  with the region of degradation progressively narrowing for 0.3 and 1 G. In all

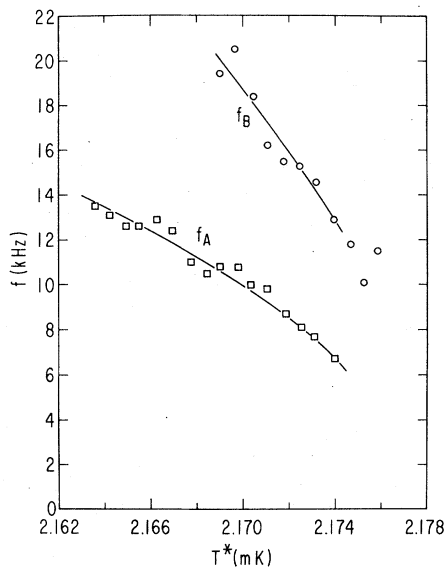


FIG. 3. Comparison of ringing frequencies  $f_B$  and  $f_A$  in  $^3\text{He } B$  and  $^3\text{He } A$  at 21.2 bar and 30 G. The slope of the line through the  $f_B$  data points is 1.9 times the slope of the line through the  $f_A$  points. Note that  $f_B$  can be observed at higher temperature than  $f_A$ .  $T^*$  is a magnetic temperature scale.

fields decay of signals in  $^3\text{He } A$  was not reproducible and was characterized more by a "beating" phenomenon than by exponential decay. Good quality ringing was observed typically for 0.5 – 1 msec.

Ringing has been observed in  $^3\text{He } B$  in the pressure range from 8.5 to 21.5 bar in  $H_0$  from 0 to 310 G, consistent with the phase diagram and the observation that for  $H_0 > 1$  to 5 G no persistently good quality ringing is observed unless  $1 - T/T_c$  is less than 0.1 to 0.2%. Within this temperature range and for fields from 5 to 310 G we observed frequencies like those shown in Fig. 3 for 21.2 bar and 30 G. The  $A$ -phase frequencies  $f_A$  were observed as functions of magnetic temperature  $T^*$  on cooling below  $T_c$ , while the  $B$ -phase frequencies  $f_B$  were observed on warming. The fortunate temperature overlap is a consequence of supercooling-superheating effects. Except below ca. 10 G, we found  $f_B(T)/f_A(T) \approx 1.9 \pm 0.1$ , independent of magnetic field to 310 G. The region of good  $B$ -phase signals did increase somewhat as  $H_0$  increased up to about 30 G, but observations at 100 and 300 G showed no further change. In this region the signal had acceptable quality for 0.3–0.5 msec.

At fields of 0, 0.3, and 1 G we observed an entirely different behavior in  $^3\text{He } B$ . Very near  $T_c$

some typical  $B$ -phase ringing might be observed. Then for  $1 - T/T_c \gtrsim 0.01$  we observed a ringing phenomenon in which the frequency depended only weakly, if at all, on temperature and pressure. (Signal quality did decrease at lower temperatures, but this may have reflected only the decreasing susceptibility.) Rather the frequency depended both on  $\Delta H$  and time. For  $\Delta H = 2$  G the frequency damping was not perceptible, but for  $\Delta H = 10$  G the frequency could be observed to drop to  $1/e$  of its initial value in about 2 msec. The initial ringing frequency  $f$  depended approximately linearly on  $\Delta H$ , for  $\Delta H$  in the range 2 to 10 G, with  $2\pi f \approx \alpha(\gamma\Delta H)$ . The coefficient  $\alpha$  is about  $\frac{1}{2}$  at  $H_0 = 0$  and somewhat larger at  $H_0 = 1$  G.

Owing to the presence of a significant magnetic background it was not possible to study the non-oscillatory decay of the  $^3\text{He}$  magnetization to its new equilibrium value following an incremental field change.

The existence of ringing in  $^3\text{He } B$  suggests that it, like  $^3\text{He } A$ , may be a spin-triplet superfluid.<sup>1</sup> Observation of ringing would also agree with the parallel absorption reported by Osheroff and Brinkman.<sup>3</sup> The ratio of  $1.9 \pm 0.1$  for  $f_B/f_A$  is greater than the  $(\frac{5}{2})^{1/2}$  expected near  $T_c$  and the polycritical point from Leggett's calculations<sup>1</sup> if  $^3\text{He } A$  were the Anderson-Brinkman-Morel (ABM) state<sup>7</sup> and  $^3\text{He } B$  the Balian-Werthamer (BW) state.<sup>8,11</sup> However, Maki<sup>11</sup> has suggested a variety of nearly degenerate BW-like states, each giving a different possible  $f_B/f_A$  ratio. Osheroff and Brinkman<sup>3</sup> and Brinkman *et al.*<sup>12</sup> have interpreted perpendicular NMR in  $^3\text{He } B$  at melting pressure and for  $T/T_c \lesssim 0.8$  in a 6-mm-diam container as being profoundly affected by boundaries in the entire field range of our experiments. Our parallel-ringing measurements, carried out at lower pressures, in a 3-mm-diam tube and at temperatures down to about  $0.8T_c$ , show low- and high-field regions with a transition between them in the range 1 to 5 G for both  $^3\text{He } A$  and  $^3\text{He } B$ . We presently suspect that all these different measurements represent truth and that  $^3\text{He } B$  is more complex than currently believed. Experiments have been performed under sufficiently different conditions as to make intercomparison questionable, particularly regarding magnetic properties. We cite in this connection Ahonen, Haikala, and Krusius<sup>13</sup> who find in  $^3\text{He } B$  mixed with platinum powder a profoundly altered magnetic state but little change in thermal properties such as  $T_c$  and  $T_{AB}$ .

We suspect that the easily observed ringing in

$^3\text{He A}$  and the rather sudden appearance of good quality ringing in  $^3\text{He B}$  only very near to  $T_c$  reflect the known flow properties of the two phases.<sup>14</sup> The chemical potential differences in space produced by  $\Delta H$  in our rather inhomogeneous geometry are comparable to those due to  $\Delta T$  in heat-flow experiments which drive  $^3\text{He A}$  supercritical over a wide temperature range in  $^3\text{He A}$ , and  $^3\text{He B}$  supercritical only very near  $T_c$ . The tendency to "stir" the system by the flow of magnetization supercurrents<sup>15,5</sup> is thus inhibited in  $^3\text{He A}$  but not inhibited in  $^3\text{He B}$  except very near  $T_c$ . Experiments to test this possibility are now being considered.

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## Coupled Nonlinear Electron-Plasma and Ion-Acoustic Waves

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We obtain solutions describing stationary one-dimensional propagation of a coupled nonlinear electron-plasma wave and a nonlinear ion-acoustic wave. These waves have amplitudes linearly proportional to one another, and propagate with approximately the ion-acoustic velocity in the form of periodic wave trains, including solitary waves as a special case.

Nonlinear stationary propagation of plasma waves has been investigated extensively in recent years.<sup>1-4</sup> One-dimensional propagation of small- but finite-amplitude ion-acoustic waves in a collisionless cold-ion plasma is described by a Korteweg-deVries equation,<sup>5</sup> and the theoretical prediction of steepening and soliton formation has been confirmed by experiments.<sup>6</sup> A long-wavelength electron-plasma wave obeys a nonlinear Schrödinger equation.<sup>7</sup> Its stationary solu-

tions in the one-dimensional case include envelope soliton, periodic wave train, and finite-amplitude plane wave. The latter is subject to a modulational instability under certain conditions.

In this paper, we present some special solutions which describe coupled, stationary, one-dimensional propagation of a nonlinear electron wave and a nonlinear ion wave. The basic equations are the Schrödinger equation for the electron wave, with a potential proportional to the