

10-8-2004

## Mohanty and Webb Reply

P. Mohanty

Richard A. Webb

University of South Carolina - Columbia, webbra@mailbox.sc.edu

Follow this and additional works at: [https://scholarcommons.sc.edu/phys\\_facpub](https://scholarcommons.sc.edu/phys_facpub)

 Part of the [Physics Commons](#)

---

### Publication Info

Published in *Physical Review Letters*, Volume 93, Issue 15, 2004, pages 159702-1-159702-1.

Mohanty, P. and Webb, R.A. (2004). Mohanty and Webb Reply. *Physical Review Letters*, 93(15), 159702-1. doi: 10.1103/PhysRevLett.93.159702

© 2004 The American Physical Society.

This Article is brought to you by the Physics and Astronomy, Department of at Scholar Commons. It has been accepted for inclusion in Faculty Publications by an authorized administrator of Scholar Commons. For more information, please contact [dillarda@mailbox.sc.edu](mailto:dillarda@mailbox.sc.edu).

**Mohanty and Webb Reply:** In our Letter [1], we reported experimental data, showing asymmetry in the conductance distribution of quasi-1D *metallic* wires at low temperatures. We argued that the observed asymmetry or deviation from the expected Gaussian shape [2,3], irrespective of its magnitude, signals a possible violation of the one-parameter scaling theory of Anderson localization [2,3]. Fal'ko, Lerner, Tsypliyatyev, and Aleiner (FLTA) [4] agree that there is a finite asymmetry in the data, and they argue that the deviation can be explained by the “limited applicability of the ergodic hypothesis.” Furthermore, they claim, there is an orders-of-magnitude difference in the values of cumulants in their analysis of the published curves [1]. In the following, we show that they obtain a huge discrepancy because of their use of a different definition of the third cumulant. We contend that both the nature and the size of the asymmetry in the data cannot be explained by the limitations of the ergodic hypothesis.

(i) *On the comparison of data with theory.*— The essential point is that the cumulants are calculated *about the mean* [5]. This is done for two specific reasons: (i) to enable a meaningful comparison of numbers (values of cumulants) between the high ( $g \sim 350$ ) and low ( $g \sim 8$ ) conductance samples, and (ii) to ascribe a meaningful definition to the asymmetry, which is defined *about the mean* conductance  $\langle g \rangle$  and not about  $g = 0$ . The textbook definitions of the first four cumulants [5] *about an arbitrary point* are  $\kappa_1 = \mu'_1$ ,  $\kappa_2 = \mu'_2 - \mu'^2_1$ ,  $\kappa_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1$ , and  $\kappa_4 = \mu'_4 - 4\mu'_3\mu'_1 - 3\mu'^2_2 + 12\mu'_2\mu'^2_1 - 6\mu'^4_1$ , where  $\kappa$  and  $\mu$  denote cumulants and moments, respectively, and the prime index represents definitions about an arbitrary point. For moments about the mean ( $\mu'_1 = 0$ ), these definitions reduce to  $\kappa_2 = \mu_2$ ,  $\kappa_3 = \mu_3$ , and  $\kappa_4 = \mu_4 - 3\mu^2_2$ . Note that the third cumulant is equal to the third moment about the mean. (Not surprisingly, the third cumulant about the mean  $\langle g^3 \rangle_\kappa$  in Ref. [1] is found to be close to the skewness, as determined by FLTA [4].) We emphasize that the numbers in Table I of Ref. [1] were evaluated using *these* definitions.

Another major reason for evaluating the cumulants *about the mean* is to recognize the fact that at least the first two moments—and therefore, the first two cumulants—have distinct temperature dependences. In quasi-1D,  $\langle g \rangle$  is dominated by electron interaction with a dependence  $T^{-1/2}$ , and  $\langle g^2 \rangle$  varies as  $(L_\phi/L)^{3/2}(L_T/L_\phi) \sim L^{1/2}_\phi L_T \sim T^{-2/3}$  (assuming  $L_\phi \sim T^{-1/3}$ ). Furthermore, from the data at 38 mK and 300 mK, it is apparent that even the third cumulant  $\langle g^3 \rangle_\kappa$  has a strong temperature

dependence. Therefore, it is not unusual to expect  $\langle g^2 \rangle_\kappa$  to be smaller than  $\langle g^3 \rangle_\kappa$ .

(ii) *On the status of reported statistical data.*— The samples studied in the experiments with length  $L = 20 \mu\text{m}$  contain  $N = L/L_\phi = 5$  independent phase-coherent segments. As pointed out in Ref. [6], the third cumulant for a phase-coherent conductor will be further increased by a factor  $N^{2n-1}$ . For  $n = 3$ , this factor is  $5^5 = 3125$ . It is important to point out that the third cumulant can be much larger than 1 for typical distributions [5].

(iii) *On the limited applicability of the ergodic hypothesis.*— The statistical uncertainty in  $\langle g^3 \rangle_\kappa$  calculated by FLTA is  $\pm(\alpha_n B_c/B_0)\langle g^2 \rangle_\kappa^{3/2}$ , which has neither the asymmetry nor the temperature dependence observed in the data. In all the samples, the excess conductance appears only on the high- $g$  side of the curve, vanishing progressively with increasing temperature (see Fig. 3 in Ref. [1]). The statistical uncertainty calculated by FLTA does not explain this asymmetry. Second, it does not explain the temperature dependence, as one would expect the exact same uncertainty at higher temperatures as well, contrary to the data.

Considering that the effects of interaction (on  $\langle g \rangle$ ) and temperature (on  $\langle g \rangle$ ,  $\langle gg \rangle$ , and  $\langle ggg \rangle$ ) are visually apparent, it is not surprising that a zero-interaction, zero-temperature theory [2] fails to explain the data.

For partial support of this work, P. M. thanks NSF and Sloan Foundation, and R. A. W. thanks NSF and NSA.

P. Mohanty<sup>1</sup> and R. A. Webb<sup>2</sup>

<sup>1</sup>Department of Physics  
Boston University  
590 Commonwealth Avenue  
Boston, Massachusetts 02215, USA

<sup>2</sup>Department of Physics and USC Nanocenter,  
University of South Carolina  
Columbia, South Carolina 29208, USA

Received 20 October 2003; published 8 October 2004

DOI: 10.1103/PhysRevLett.93.159702

PACS numbers: 72.15.-v

- [1] P. Mohanty and R. A. Webb, Phys. Rev. Lett. **88**, 146601 (2002).
- [2] E. Abrahams *et al.*, Phys. Rev. Lett. **42**, 673 (1979).
- [3] B. L. Altshuler *et al.*, Zh. Eksp. Teor. Fiz. **91**, 2276 (1986).
- [4] V. I. Fal'ko *et al.*, preceding Comment, Phys. Rev. Lett. **93**, 159701 (2004).
- [5] A. Stuart and J. K. Ord, *Kendall's Advanced Theory of Statistics* (Griffin and Co., London, 1987), 5th ed.
- [6] M. C. W. van Rossum *et al.*, Phys. Rev. B **55**, 4710 (1997).