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In-medium behavior of the QCD θ term and the value of *CP* violation in nuclei

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The expectation value of the θ term in QCD for nuclear matter is estimated in the nucleon gas approximation. There is no significant renormalization (to an accuracy $\sim 10\%$) of the *CP* violation in nuclei due to the similar behaviors for the in-medium values of the θ term and quark condensates.

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I. INTRODUCTION

The problem of searching for *CP* violation in neutron scattering has attracted attention during the last few years due to the large enhancement factors ($\sim 10^6$) connected with nuclear structure and reaction mechanisms (see, e.g., Ref. [1] and references within). One of the advantages of this reaction is the possibility of testing different models of *CP* violation in a way independent of nuclear models [2].

To remind ourselves of the main ideas, let us consider the *T*-odd and *P*-odd correlation $(\sigma[\mathbf{k}\times\mathbf{I}])$, where σ and $\mathbf{I}$ are neutron and target spins, and $\mathbf{k}$ is the neutron momentum. This correlation leads [3,4] to the difference of the total cross sections for the transmission of neutrons polarized parallel and antiparallel to the axis $[\mathbf{k}\times\mathbf{I}]$ through the polarized target

$$\Delta\sigma_{CP}=\frac{4\pi}{k}\mathrm{Im}(f_{\uparrow}-f_{\downarrow}) \ . \tag{1.1}$$

Here $f_{\uparrow,\downarrow}$ are the zero-angle scattering amplitudes for neutrons polarized parallel and antiparallel to the $[\mathbf{k}\times\mathbf{I}]$ axis, respectively. The parameter $\Delta\sigma_{CP}$ looks like the *P*-violating $\Delta\sigma_P$ caused by the *T*-even *P*-odd correlation $(\sigma\mathbf{k})$ and the relation between these values is [2]

$$\Delta\sigma_{CP}\sim\lambda\Delta\sigma_P \ , \tag{1.2}$$

where $\lambda=g_{CP}/g_P$ is the ratio of the *CP*-odd g_{CP} to *P*-odd g_P nucleon coupling constants. Therefore, the magnitude of *CP*-violating effects in neutron scattering is related to the *CP*-odd nucleon coupling constants which have been calculated for some models of *CP* violation [5,6,2]. In these calculations the vacuum values of *P*-odd and *CP*-odd coupling constants were used. For almost all models of *CP*-violation this choice for the coupling constants provides the correct result because the *CP*-odd values in nuclei are proportional to the parameter λ [the ratio of the *CP*-odd and *P*-odd coupling constants for light meson (π,ρ,ω) interactions with a nucleon]. Therefore, if the origin of *CP* violation is not related to the strong interaction, this ratio of the coupling constants must be the same for nuclear matter.

From this point of view, the model of *CP* violation due to the θ term in the QCD Lagrangian is a special case: this mechanism of *CP* violation is related to the properties of the strong interaction. Therefore, the relative value of *CP*-odd effects in nuclear matter may be drastically changed compared to the vacuum (free particle interaction) case. However, it seems impossible to calculate such a renormalization for the *CP*-odd operator in QCD since it is necessary to account for the strong interaction. Fortunately, the recent results for the calculation of the quark and gluon condensate renormalization in nuclear matter [7–9] give an opportunity to estimate the renormalization factor for the θ-term in nuclear matter to the first power in nuclear density (in the noninteracting nucleon gas approximation). Moreover, various model-dependent calculations of higher density contributions to the quark condensate provide the hope that the renormalization factor of the θ term may have a good accuracy ($\sim 10\%$) up to the saturation nuclear density [7].

The organization of the paper is as follows. In Sec. II the simple nucleon gas model approximation is used to reproduce the main results for the values of quark and gluon condensates in nuclear matter. In Sec. III this approximation is used to estimate the renormalization factor of the θ term in nuclear matter. The consequences of this renormalization for *CP* violation in neutron scattering are discussed in Sec. IV.

II. QUARK AND GLUON CONDENSATES

We will present the computation of quark and gluon condensates in a simple model for a noninteracting nucleon gas to reproduce the same parameters calculated in paper [7] to the first power in nuclear matter density. Firstly, let us recall that quark and gluon condensates in nuclear matter have been calculated in a model-independent way to first order in nucleon density using the Hellmann-Feynman theorem as [7]

$$\langle\bar{q}q\rangle_\rho\simeq\langle\bar{q}q\rangle_{\mathrm{vac}}+\rho\frac{\sigma_N}{m_u+m_d} \ , \tag{2.1}$$

$$\left\langle\frac{\alpha_s}{\pi}GG\right\rangle_\rho\simeq\left\langle\frac{\alpha_s}{\pi}GG\right\rangle_{\mathrm{vac}}-\rho\frac{8}{9}(M-\sigma_N-S) \ . \tag{2.2}$$

Here $\langle\ \rangle_{\mathrm{vac}}$ and $\langle\ \rangle_\rho$ are vacuum and in-medium conden-

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sates, ρ is the medium (nuclear matter) density, σ_N is the nucleon σ term, $m_{u,d}$ are current masses of u,d quarks, M is the nucleon mass, and S is the strangeness content of the nucleon. As has been shown [7], the deviation from the linear ρ dependence in Eq. (2.1) is small ($\simeq 10\%$) up to the nuclear saturation density, and Eqs. (2.1) and (2.2) lead to a reduction in the quark and gluon condensates at the nuclear saturation density by about 25%–50% and 5%, respectively.

In this paper we will consider only nuclear matter with a density less than or equal to the saturation density $\rho_{\rm sat}$. To describe this matter, the free nucleon gas approximation will be used. Therefore, the matter wave function is a sum of the vacuum wave function $|0\rangle$ and the wave functions of free nucleons $|N\rangle$ (we will not distinguish between protons and neutrons)

$$|\rho\rangle = |0\rangle + \sum_N \frac{1}{\sqrt{V}} |N\rangle \ , \tag{2.3}$$

where V is the volume. Using this wave function, we easily obtain the following results for the quark and gluon condensates:

$$\langle \bar{q}q \rangle_\rho \simeq \langle \bar{q}q \rangle_{\rm vac} + \rho \langle N|\bar{q}q|N\rangle \ , \tag{2.4}$$

$$\left\langle \frac{\alpha_s}{\pi} GG \right\rangle_\rho \simeq \left\langle \frac{\alpha_s}{\pi} GG \right\rangle_{\rm vac} + \rho \left\langle N \left| \frac{\alpha_s}{\pi} GG \right| N \right\rangle \ . \tag{2.5}$$

It should be noted that we assume that the quark condensates for the u and d quarks have the same magnitudes.

Let us compare the results of Eqs. (2.1) and (2.2) with Eqs. (2.4) and (2.5). Firstly, taking into account the well known relation [10] between the nucleon σ term and nucleon matrix element in Eq. (2.4),

$$\sigma_N = \tfrac{1}{2}(m_u + m_d) \langle N|\bar{u}u + \bar{d}d|N\rangle \ , \tag{2.6}$$

we can see that Eqs. (2.1) and (2.4) are the same. It is easy to show that the two expressions for gluon condensates also coincide. Indeed, the calculation of the gluonic nucleon matrix element gives [11,12]

$$\left\langle N \left| \frac{\alpha_s}{\pi} GG \right| N \right\rangle = -\frac{8}{9}(M - \sigma_N - m_s \langle N|\bar{s}s|N\rangle) \ , \tag{2.7}$$

where the last term in the angular brackets is equal to S in Eq. (2.2) (see definitions in Refs. [7,12]). Therefore, one can see that the results which have been obtained in paper [7] to the first power in nucleon density correspond to the free gas approximation. This is a natural conclusion because Eqs. (2.1) and (2.2) have been calculated by neglecting the contribution from the nucleon kinetic energy and nucleon-nucleon interactions which lead to the $\rho^{4/3}$ or higher corrections (see, e.g., Ref. [8]).

Now we can stress two important points. Firstly, taking into account the analysis given in Ref. [7] for the accuracy of the linear density dependence for the quark condensates, we assume that this approximation is good enough for condensates at a matter density up to nuclear saturation density. This leads to the following relation between the in-medium and vacuum expectation values for an operator $\hat{O}$

$$\langle \hat{O} \rangle_\rho \simeq \langle \hat{O} \rangle_{\rm vac} + \rho \langle N|\hat{O}|N\rangle \ . \tag{2.8}$$

Secondly, using Eqs. (2.4) and (2.5) we can explain why the quark condensate changes more significantly than the gluon one in nuclear matter. In the linear density approximation, the relative variation of the expectation value of an operator $\hat{O}$ is

$$\delta(\hat{O}) = \frac{\langle \hat{O} \rangle_\rho - \langle \hat{O} \rangle_{\rm vac}}{\langle \hat{O} \rangle_{\rm vac}} = \rho \frac{\langle N|\hat{O}|N\rangle}{\langle 0|\hat{O}|0\rangle} \ . \tag{2.9}$$

In accordance with the calculations on the nucleon matrix elements [11–14] the absolute value of the numerator in Eq. (2.9) is larger for the gluon operator (2.7) than for the quark one

$$\langle N|m_q \bar{q}q|N\rangle = \frac{2}{27} M \tag{2.10}$$

due to the triangle anomaly of the energy-momentum tensor. (In Eq. (2.7) we use the renormalization-group invariant expression for the quark operator [15].) The vacuum expectation values of the quark condensates are smaller than that for the gluon condensate [16]

$$\langle m_q \bar{q}q \rangle_{\rm vac} \simeq -m_q (225 \pm 25 \ {\rm MeV})^3 \ , \tag{2.11}$$

$$\left\langle \frac{\alpha_s}{\pi} GG \right\rangle_{\rm vac} \simeq (360 \pm 20 \ {\rm MeV})^4 \ . \tag{2.12}$$

Therefore, the ratio of the variations for the corresponding expectation values is

$$\frac{\delta(m_q \bar{q}q)}{\delta((\alpha_s/\pi)GG)} = \frac{\langle N|m_q \bar{q}q|N\rangle}{\langle N|(\alpha_s/\pi)GG|N\rangle} \frac{\langle 0|(\alpha_s/\pi)GG|0\rangle}{\langle 0|m_q \bar{q}q|0\rangle} \simeq 7 \ . \tag{2.13}$$

This means that, in spite of the larger density dependent coefficient for the gluon condensate than for the quark one, the relative value of the former changes slowly when compared with the latter for low nuclear density.

III. EXPECTATION VALUE $\langle G\tilde{G} \rangle_\rho$

Now we will apply the gas approximation to estimate the density dependence for the $(\alpha_s/\pi)G\tilde{G}$ operator up to the nuclear saturation density $\rho_{\rm sat} \simeq (110 \ {\rm MeV})^3$. Using Eq. (2.8) for this operator, one obtains

$$\left\langle \frac{\alpha_s}{\pi} G\tilde{G} \right\rangle_\rho \simeq \left\langle \frac{\alpha_s}{\pi} G\tilde{G} \right\rangle_{\rm vac} + \rho \left\langle N \left| \frac{\alpha_s}{\pi} G\tilde{G} \right| N \right\rangle \ . \tag{3.1}$$

It is well known [17] that a nonzero vacuum expectation value of this operator is a direct consequence of the θ term in the QCD Lagrangian, and to first order in θ

$$\left\langle \frac{\alpha_s}{\pi} G\tilde{G} \right\rangle_{\rm vac} \simeq -\theta f_\pi^2 m_\pi^2 \frac{8 m_u m_d}{(m_u + m_d)^2} \ , \tag{3.2}$$

where m_π and f_π are the π-meson mass and decay constant, $f_\pi \simeq 93$ MeV. (In this section for all expressions we will use the limit $m_s/m_{u,d} \gg 1$.) To calculate the nucleon matrix element in Eq. (3.1) we will use the formalism of Ref. [17]. Then, to the lowest power of quark masses, we have

$$\left\langle N \left| \frac{\alpha_s}{\pi} G\tilde{G} \right| N \right\rangle \simeq \frac{1}{2}\theta K_N , \tag{3.3}$$

where K_N is the function

$$K_N = \left\{ i \int dx\, e^{iqx} \left\langle N \left| T \left[\frac{\alpha_s}{2\pi} G^a_{\mu\nu}(x)\tilde{G}^a_{\mu\nu}(x) , \frac{\alpha_s}{2\pi} G^b_{\chi\lambda}(0)\tilde{G}^b_{\chi\lambda}(0) \right] \right| N \right\rangle \right\}_{q=0} . \tag{3.4}$$

Following the method of calculation for $\langle (\alpha_s/\pi) G\tilde{G} \rangle_{\rm vac}$ in Ref. [17], we obtain

$$K_N \simeq \langle N | 4m_u \bar{u}u + 4m_d \bar{d}d | N \rangle + i \int dx \langle N | T[(2im_u \bar{u}\gamma_5 u + 2im_d \bar{d}\gamma_5 d)_x , (2im_u \bar{u}\gamma_5 u + 2im_d \bar{d}\gamma_5 d)_0] | N \rangle . \tag{3.5}$$

The first term on the right side of Eq. (3.5) is proportional to the nucleon σ term. The second one is proportional to $m^2_{u,d}$ and therefore we will neglect it. It should be noted that for the vacuum correlator [17] this term has the same order of magnitude as the first one in Eq. (3.5). This is a result of the π-meson intermediate-state contribution since the squared π-meson mass is proportional to the quark mass. In our case, there is no intermediate state with such quark mass dependence. Therefore, taking into account only the first term in Eq. (3.5) we have

$$\left\langle N \left| \frac{\alpha_s}{\pi} G\tilde{G} \right| N \right\rangle \simeq 2\theta\sigma_N . \tag{3.6}$$

Inserting this result into Eq. (3.1), we obtain the density dependence of the operator $(\alpha_s/\pi)G\tilde{G}$ as

$$\left\langle \frac{\alpha_2}{\pi} G\tilde{G} \right\rangle_\rho \simeq \left\langle \frac{\alpha_s}{\pi} G\tilde{G} \right\rangle_{\rm vac} + \rho 2\theta\sigma_N , \tag{3.7}$$

and for the relative change in the condensate

$$\frac{\langle (\alpha_s/\pi) G\tilde{G} \rangle_\rho}{\langle (\alpha_s/\pi) G\tilde{G} \rangle_{\rm vac}} \simeq 1 + \frac{\rho}{\langle \bar{q}q \rangle_{\rm vac}} \frac{\sigma_N}{m_u + m_d} \frac{(m_u + m_d)^2}{4m_u m_d} . \tag{3.8}$$

To obtain Eq. (3.8), we used Eq. (3.2) and the well known relation $2f^2_\pi m^2_\pi = -(m_u + m_d)\langle 0 | \bar{u}u + \bar{d}d | 0 \rangle$. Let us rewrite Eq. (2.1) in the same manner:

$$\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_{\rm vac}} \simeq 1 + \frac{\rho}{\langle \bar{q}q \rangle_{\rm vac}} \frac{\sigma_N}{m_u + m_d} . \tag{3.9}$$

Comparing these two expressions [(3.8) and (3.9)], we can see that they have the same density dependence [the additional multiplier in Eq. (3.8) is not significant: $(m_u + m_d)^2/(4m_u m_d) = 1.08$]. Therefore, in accordance with Ref. [7], we can conclude that the expression (3.8) leads to a reduction in $\langle (\alpha_s/\pi) G\tilde{G} \rangle_\rho$ by about 25%–50% at the nuclear saturation density.

IV. CONCLUSIONS

As is well known (see, e.g., Ref. [17]), the ratio

$$\kappa_{\rm vac} = \frac{\langle (\alpha_s/\pi) G\tilde{G} \rangle_{\rm vac}}{\langle (\alpha_s/\pi) GG \rangle_{\rm vac}} \simeq -0.02\theta \tag{4.1}$$

can serve as a measure of the CP violation due to the θ term in QCD. Therefore, we can consider as a measure of the CP violation in nuclear matter the following ratio:

$$\kappa_\rho = \frac{\langle (\alpha_s/\pi) G\tilde{G} \rangle_\rho}{\langle (\alpha_s/\pi) GG \rangle_\rho} . \tag{4.2}$$

As was mentioned in Sec. II, in accordance with the result of Ref. [7], the gluon condensate slightly changes in nuclear matter up to the saturation density. Consequently, the measure of the CP violation due to the θ term, κ_ρ, has almost the same density dependence as the quark condensates: it reduces in value at the nuclear saturation density by about 25%–50%.

Using the parameter κ_ρ we can estimate the density dependence of the parameter $\lambda = g_{CP}/g_P$ in Eq. (1.2), which is the measure of the CP violation in neutron scattering. In the one-π-meson exchange approximation for CP- and P-violating nucleon-nucleon interactions, we obtain that the parameter $\lambda = g^\pi_{CP}/g^\pi_P$, where g^π_{CP} and g^π_P are CP-odd and P-odd π-meson nucleon coupling constants. Since we are interested in CP violation in nuclear matter, the CP-odd coupling constant g^π_{CP} is proportional to the measure of CP violation κ_ρ. The P-odd coupling constant g^π_P is proportional to the quark condensate value (see, e.g., Ref. [18]). Taking into account almost the same density dependence for the numerator in Eq. (4.2) and the quark condensates [see Eqs. (3.8) and (3.9)] we can conclude that the parameter λ has negligible density dependence, the same as the gluon condensate. It should be emphasized that, as was shown in Ref. [7], the approximation used for the quark condensates has an accuracy of about 10% up to the nuclear saturation density. Therefore, our conclusion is valid to the same accuracy.

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