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## *CP*-odd nucleon potential

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The *P*-odd and *CP*-odd nucleon potential for different models of *CP* violation in the one-meson exchange approximation is studied. It is shown that the main contribution is due to the  $\pi$ -meson exchange which leads to a simple one-parameter *CP*-odd nucleon potential.

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### I. INTRODUCTION

There has been a lot of activity in looking for *CP* violation in nuclear [1] and atomic systems [2] in the last few years due to possible large enhancement factors for *CP* violating effects in these systems. The theoretical calculations are very complicated. To calculate various *CP* violating effects in nuclear and atomic systems we need to go through different levels of theoretical models. Firstly one needs to obtain the effective low energy *CP* violating Lagrangian at the quark level for particular models of *CP* violation. The second step is to calculate the *CP* violating nucleon-nucleon interaction using the low energy Lagrangian obtained from the first step. The third step is to calculate nuclear *CP* violating effects using the calculated *CP* violating nucleon-nucleon interaction and particular nuclear models. And in the case of atomic systems, one should use particular atomic models to calculate atomic parameters and take into account nuclear *CP*-odd moments. Moreover in atomic systems, *CP*-odd effects involving electrons must also be considered. Each step of these calculations is model dependent. Even if there are methods to exclude or minimize the dependence on models [3], the calculations are still complicated.

The purpose of this paper is to consider the calculation of the *P* and *CP* violating nucleon-nucleon interaction using the one-meson-exchange approximation. It is well known that in order to describe the standard *P*-odd and *CP*-even nucleon-nucleon interaction it is necessary to calculate at least six different meson-nucleon coupling constants [4]. Due to the model dependent nature (QCD at long distances) of the calculation for each constant and the different isospin properties and masses of the mesons under consideration, the six-parameter *P*-odd and *CP*-even nucleon potential leads to difficulties in the calculations of nuclear effects and a sensitive dependence on nuclear models.

We will show that for the case of *CP* violation, the structure of the *CP*-odd nucleon potential is much simpler. With good accuracy it is enough to take into account only the  $\pi$ -meson contribution. In other words, we can use only one parameter (as a consequence a simple structure of the potential) to calculate the *CP* violating effects in nuclei and atoms.

This paper is arranged as follows. In Sec. II, we will

discuss the reason for the enhancement of the *CP* violating  $\pi$ -meson nucleon interaction for different gauge models of *CP* violation. The structure of the *CP*-odd nucleon potential is discussed in Sec. III. In Sec. IV, we give our conclusions.

### II. *CP*-ODD MESON-NUCLEON COUPLING CONSTRAINTS

The consideration of different renormalizable *CP* violating models leads to the obvious conclusion that there are several different sources of *CP* violation. (a) Complex quark mass matrices. In the mass eigenstate basis, there will be *CP* violation in the charged current due to exchange of gauge particles. One of the best known examples is the Kobayashi-Maskawa model [5] (the standard model). The phase of the determinant of the quark mass matrices is also related to the *CP* violating  $\theta$  term in QCD. (b) Complex mixing angles for gauge bosons. An example is the left-right symmetric model [6]. (c) Complex vacuum expectation values of Higgs bosons, for example, the Weinberg model [7]. (d) *CP*-odd pure gluonic interaction, such as the  $\theta$  term in QCD [8]. In a specific process, some or all of these *CP* violating sources will contribute. In this section we will discuss *CP*-odd meson-nucleon coupling constants which would arise from all possible contributions.

*CP*-odd effects in nuclear (low energy) physics are model dependent in general. To estimate these effects, the low energy effective Lagrangian for different models should be used. These effective Lagrangians will include *CP*-odd pure quark, quark-gluon, and pure gluonic operators. We will consider operators up to dimension six. The pure quark operators will appear in the form current $\times$ current due to gauge boson exchange, or pseudoscalar $\times$ scalar structure due to scalar boson exchange. The most important feature of these Lagrangians is the presence of the right-current $\times$ left-current or the pseudoscalar $\times$ scalar structures. These operators have enhanced contributions to the *CP*-odd pseudoscalar meson-nucleon couplings. This is a principal difference compared to the structure of the *P*-odd and *CP*-even effective Lagrangian. To describe the *P*-odd and *CP*-even nucleon-nucleon interaction, it is necessary to calculate at least six meson-nucleon coupling constants [4]. We will show in the following that, due to

the enhanced  $CP$ -odd pseudoscalar meson-nucleon coupling mentioned above, it is possible to describe the  $CP$ -odd nucleon-nucleon interaction using only one meson-nucleon coupling constant.

Let us consider the low energy effective Lagrangian involving only  $u$  and  $d$  quarks. Exchanging gauge bosons at the tree level in the  $a$  and  $b$  types of models will produce the following structure of the Lagrangian:

$$\mathcal{L} \sim L \times L + L \times R + R \times L + R \times R \\ = C_{LL}O_{LL} + C_{LR}O_{LR} + C_{RL}O_{RL} + C_{RR}O_{RR}, \quad (1)$$

where  $O_{LL} = \bar{u}_L \gamma_\mu d_L \bar{d}_L \gamma_\mu u_L$  and other operators are defined in a similar way. Note that only  $L \times R$  and  $R \times L$  have a  $CP$  violating interaction. In the standard model (SM)  $C_{LR}$  and  $C_{RL}$  are zero at the tree and one-loop levels. However at higher orders they can be generated.

At the tree level the  $c$  type of model will lead to the  $CP$  violating effective Lagrangian

$$\mathcal{L} \sim S \times P = C_{SP} \bar{q}_1 q_2 \bar{q}_3 \gamma_5 q_4 + \text{H.c.}, \quad (2)$$

where  $q_i$  can be  $u$  and  $d$  quarks depending whether a charged or neutral scalar is exchanged to produce the effective Lagrangian. Again in the SM  $C_{SP}$  can only be generated at higher orders.

The  $L \times R$  term also contains a term proportional to  $S \times P$ . This can be seen by making a Fierz transformation on  $O_{LR}$ . We have

$$O_{LR}^F = -2[\frac{1}{3} \bar{u}_L u_R \bar{d}_R d_L + \frac{1}{2} \bar{u}_L \lambda^a u_R \bar{d}_R \lambda^a d_L] + \text{H.c.} \quad (3)$$

We can now calculate the  $CP$ -odd meson-nucleon coupling constants for  $\pi$  and  $\rho$  mesons from the  $L \times R$  term. Using the factorization approximation and the vector meson dominance hypothesis, we have

$$\bar{g}_{\pi^- NN} \approx \langle \pi^- | p | L \times R | n \rangle_{CP} = i \frac{\text{Im} C_{LR}}{2} \langle \pi^- | \bar{d} \gamma_\mu \gamma_5 u | 0 \rangle \langle p | \bar{u} \gamma_\mu d | n \rangle \\ = i \frac{\text{Im} C_{LR}}{2} \frac{m_d^2 - m_u^2}{m_\pi^2} \langle \pi^- | \bar{d} \gamma_5 u | 0 \rangle \langle p | \bar{u} d | n \rangle, \quad (4) \\ \bar{g}_{\rho NN} \approx \langle \rho^- | p | L \times R | n \rangle_{CP} = \frac{\text{Im} C_{LR}}{2} \frac{m_\rho^2}{f_\rho} g_A,$$

where  $m_\rho$  and  $f_\rho$  are the mass and strong form factor for  $\rho$  meson with  $f_\rho^2/4\pi \approx 2$ , and  $g_A$  is the nucleon axial form factor.

Using the Fierz transformed operator  $O_{LR}^F$  and the factorization approximation, we obtain

$$\bar{g}_{\pi^0 NN} \approx \frac{1}{3} \text{Im} C_{LR} (\langle \pi^0 | \bar{d} \gamma_5 d | 0 \rangle \langle N | \bar{u} u | N \rangle - \langle \pi^0 | \bar{u} \gamma_5 u | 0 \rangle \langle N | \bar{d} d | N \rangle). \quad (5)$$

From the above equation we clearly see that there is a suppression factor  $(m_d^2 - m_u^2)/m_\pi^2$  for  $\bar{g}_{\pi^- NN}$  compared with  $\bar{g}_{\pi^0 NN}$ .

To compare the contributions from the  $\pi$  and  $\rho$  meson exchanges to the  $CP$ -odd nucleon potential we remind the readers that for the standard  $P$ -odd and  $CP$ -even interactions  $L \times L$ , we have the same structure of the corresponding coupling constants

$$g_{\pi^- NN}^p \approx \frac{C_{LL}}{2} \frac{m_d^2 - m_u^2}{m_\pi^2} \langle \pi^- | \bar{d} \gamma_5 u | 0 \rangle \langle p | \bar{u} d | n \rangle, \quad (6)$$

$$g_{\rho NN}^p \approx \frac{C_{LL}}{2} \frac{m_\rho^2}{f_\rho} g_A.$$

It is well known that for the  $P$ -odd and  $CP$ -even nucleon potentials the contributions from the  $\pi$  and  $\rho$  mesons have the same order of magnitude if the relative strength of the couplings is given by Eq. (6) [4]. It is expected that the same thing should happen for the  $CP$ -odd nucleon potential. From Eq. (4) we expect that the contributions from the  $\rho$  and  $\pi^-$  meson exchanges to the  $CP$ -odd potential will have the same order of magnitude. Therefore we conclude that the dominant contribution to the  $CP$ -odd nucleon potential is from the  $\pi^0$  meson exchange.

We have similar results for the  $c$  type of model. In this type of model, the  $\rho$ -meson nucleon coupling will

be much smaller than  $\pi$ -meson nucleon couplings for the same reason given above. However unlike the situation in the  $a$  and  $b$  types of models where the  $\pi^0$  meson-nucleon coupling is much larger than the  $\pi^-$  meson-nucleon coupling, the charged and neutral pion-nucleon couplings can be the same order of magnitude. Therefore  $\pi^\pm$  and  $\pi^0$  exchange can all make significant contributions to the  $CP$ -odd nucleon potential. The reason for this enhancement on  $\bar{g}_{\rho NN}$  is due to the large contribution of the pseudoscalar and scalar quark densities in the local approximation. A similar enhancement factor for the strange quark current has been found in penguin induced  $K$ -meson decays [9]. We conclude that the dominant  $CP$  violating nucleon-nucleon interaction in the one-meson-exchange approximation is from the  $\pi$ -meson exchange. The situation here is quite different from the  $P$ -odd and  $CP$ -even nucleon potentials where  $\pi$ ,  $\rho$ , and  $\omega$  all contribute significantly [4].

$CP$ -odd pure gluonic operators ( $J^{PC} = 0^{-+}$ ) can be generated in many models [10], in particular the  $c$  and  $d$  types of models. It is interesting to note that because of the pseudoscalar nature of the operators, the pseudoscalar meson-nucleon coupling constants are much bigger than the vector meson-nucleon coupling constants, just as they are for the pure quark operators. To estimate the ratio of the coupling constants for pseudoscalar and vector mesons we will use two different methods: (1) us-

ing reduction formulas, and (2) using naive dimensional analysis. For the first method, in the low energy limit, we obtain for the meson-nucleon matrix elements

$$\langle \rho N | \hat{O} | N \rangle \approx im_\rho^2 \int dx e^{-iqx} \langle N | T \{ \hat{O}(0), \rho(x) \} | N \rangle, \quad (7)$$

$$\langle \pi N | \hat{O} | N \rangle \approx \frac{i}{f_\pi} \int dx e^{-iqx} \langle N | T \{ \hat{O}(0), \partial_\mu A_\mu \} | N \rangle,$$

where  $\hat{O}(x)$  is a  $CP$ -odd pseudoscalar gluonic operator, and  $f_\pi$  is the pion decay constant. In the last formula we have used the partially conserved axial vector current (PCAC) hypothesis to express the pseudoscalar field in terms of an axial vector current. Using the vector meson dominance hypothesis, one has  $\rho_\mu = (m_\rho^2/f_\rho)V_\mu$  with  $V_\mu$  being a vector current. We now use the factorization method to estimate the nucleon matrix elements

$$\langle N | T \{ \hat{O}, \hat{A} \} | N \rangle \approx \langle 0 | \hat{O} | 0 \rangle \langle N | \hat{A} | N \rangle. \quad (8)$$

From this we obtain

$$\frac{\langle \pi N | \hat{O} | N \rangle}{\langle \rho_\mu N | \hat{O} | N \rangle} \approx \frac{M_N}{f_\pi g_\rho} \gg 1, \quad (9)$$

where  $g_\rho = f_\rho^2/4\pi$ .

The naive dimensional analysis gives

$$\frac{\langle \pi N | \hat{O} | N \rangle}{\langle \rho_\mu N | \hat{O} | N \rangle} \approx \frac{g_{\pi NN}}{g_\rho}, \quad (10)$$

where  $g_{\pi NN}$  is the strong  $\pi NN$  coupling constant. Using the Goldberger-Treiman relation, we can see that this result corresponds to the estimate of Eq. (9). Therefore, we conclude that for gluonic  $CP$ -odd operators the coupling constant of the pseudoscalar meson to nucleon is larger by about 1 order of magnitude than vector meson-nucleon coupling constant.

Let us now discuss operators with quarks and gluons. The lowest-order  $CP$ -odd quark-gluon operator is the color-electric dipole moment

$$\hat{O} = \bar{q} \sigma_{\mu\nu} \gamma_5 \frac{\lambda^a}{2} q G^{a\mu\nu}. \quad (11)$$

We can apply the same argument as for the pure gluonic operators to conclude that one will obtain a large  $\bar{g}_{\pi NN}/\bar{g}_{\rho NN}$  ratio from this operator. Alternatively, we can use the extended relation

$$\langle g_s \bar{q} \sigma_{\mu\nu} \gamma_5 (\lambda^a/2) q G_{\mu\nu}^a \rangle \approx m_{st}^2 \langle i \bar{q} \gamma_5 q \rangle, \quad (12)$$

in analogy to the well-known relation  $\langle i g_s \bar{q} \sigma_{\mu\nu} \frac{\lambda^a}{2} q G_{\mu\nu}^a \rangle = m_0^2 \langle \bar{q} q \rangle$  [11]. Here  $m_0$  and  $m_{st}$  are characteristic masses about 0.9 GeV. Together with the use of the PCAC hypothesis we obtain similar results to Eq. (10). The analysis for the  $\rho$ -meson contribution can also be applied to the  $\omega$  meson.

Now we can give the main result of this section. For all types of models of  $CP$  violation, in the one-meson-exchange approximation the contributions to the  $CP$ -violating nucleon-nucleon interaction from pseudoscalar mesons are larger than the contributions from the vector meson by about 1 order of magnitude. Therefore, to calculate  $CP$ -odd effects in nuclei with a reasonable accuracy, we need only consider pseudoscalar meson exchange.

### III. CP-ODD NUCLEON POTENTIAL

To describe the  $CP$ -odd nucleon potential due to one-meson exchange we restrict ourselves by mesons with masses less than the  $\rho$ -meson mass, since contributions from heavy mesons are suppressed in the low energy limit ( $E \leq E_{\text{Fermi}}$ ) due to the repulsive force (due to the nucleon core). We will need to consider contributions from the  $\pi$ ,  $K$ ,  $\eta$ ,  $\rho$ , and  $\omega$  mesons. We have argued in the previous section that the  $\rho$  and  $\omega$  contributions are much smaller than that of the pseudoscalar mesons. We can safely neglect their contributions and only consider the pseudo-scalar contributions. In the  $b$ ,  $c$ , and  $d$  types of models, the  $CP$ -odd meson-nucleon coupling will be generated at the first order in weak interaction. If only nucleon-nucleon interactions between the neutron and proton are concerned, the  $K$ -meson exchange will not contribute to the lowest order. For these types of models we only need to consider the  $\pi$  and  $\eta$  contributions. It is obvious that the nucleon potentials for both mesons have the same spatial behaviors. We have [12]

$$V_{CP}^\pi = -\frac{m_\pi^2}{8\pi m_N} g_{\pi NN} \bar{g}_{\pi NN} (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \cdot \mathbf{r} \frac{e^{-m_\pi r}}{m_\pi r^2} \left[ 1 + \frac{1}{m_\pi r} \right], \quad (13)$$

$$V_{CP}^\eta = -\frac{m_\eta^2}{2\pi m_N} g_{\eta NN} \bar{g}_{\eta NN} (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \cdot \mathbf{r} \frac{e^{-m_\eta r}}{m_\eta r^2} \left[ 1 + \frac{1}{m_\eta r} \right],$$

where  $g$  and  $\bar{g}$  are  $CP$ -even and  $CP$ -odd coupling constants,  $\boldsymbol{\sigma}$  and  $\boldsymbol{\tau}$  are the spin and isospin of the nucleon, respectively.

Using the expressions for the  $CP$ -odd nucleon potentials, one can estimate the relative contributions of them to  $CP$ -odd nucleon interactions. It should be mentioned that such an estimation is natural for systems with a

large number of nucleons, but may not be valid for a few nucleon systems where some  $CP$ -odd effects are forbidden by isospin selection rules. The ratio of  $\bar{g}_{\pi NN}/\bar{g}_{\eta NN}$  is usually less than  $m_\eta^2/m_\pi^2$ , therefore one obtains for a short distance interaction ( $r \approx 1/m_\rho$ )

$$\frac{V_{CP}^\pi}{V_{CP}^\eta} \sim 1, \quad (14)$$

and for a large distance interaction ( $r \approx 1/m_\pi$ )

$$\frac{V_{CP}^\pi}{V_{CP}^\eta} \sim 10. \quad (15)$$

Since almost all nuclear effects have the main contribution from the long distance region ( $r \sim 1/m_\pi$ ), we conclude that for the calculation of  $CP$ -odd nuclear effects with a reasonable accuracy (10%) it is enough to know only one  $CP$  violating constant: the  $\pi$ -nucleon coupling constant.

In the SM  $CP$ -odd meson-nucleon couplings can be generated only at the second or higher orders in a weak interaction. In this case, besides the contributions to the  $CP$ -odd nucleon potential from the  $\pi$  mesons, the  $K$  mesons also have significant contributions. This has been discussed in detail in Ref. [13].

The coupling constants of  $\pi$  and  $\eta$  to nucleons for other types of models of  $CP$  violation have been calculated by several authors [3, 13, 14]. Using these results, it is possible to calculate the electric dipole moments of nuclei [2] and  $CP$ -odd effects in nucleon scattering [1]. The simple structure of the  $CP$ -odd nucleon potential gives the

unique opportunity to calculate  $CP$  violating effects in complicated nuclei.

#### IV. CONCLUSIONS

The nucleon  $CP$ -odd potential has a main contribution from one  $\pi$ -meson exchange for various models of  $CP$  violation. Therefore for the estimation of different  $CP$  violating effects in nuclear physics it is necessary to calculate only the  $\pi$ -meson nucleon coupling constant. This fact leads to the simple parametrization of all  $CP$ -odd effects using only one parameter, and provides an opportunity to test different models of  $CP$  violation.

Using one-parameter  $CP$ -odd  $\pi$ -nucleon coupling, it is possible to obtain direct relation between  $CP$ -odd nuclear effects and the value of the neutron electric dipole moment if the one-meson loop gives the main contribution [8, 13, 15].

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