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Sign correlations and the mechanism for parity violation

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The problem of possible sign correlations of P-odd effects due to parity mixing on the compound nuclear stage is discussed. For an unambiguous test of the conventional model of parity violation on the compound nuclear stage, it is not enough only to measure the sign correlation of P-odd effects on some resonances, but it is necessary also to know the neutron decay amplitudes for each resonance.

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I. INTRODUCTION

This paper is devoted to the problem of the statistical properties of P-violating effects in neutron-induced reactions on complex nuclei. The latest experimental result [1] on the parity-violating longitudinal analyzing power P in ^{232}Th was interpreted as being inconsistent with the conventional model of parity mixing on the compound nuclear stage. We will show that this conclusion may not be correct and that a more complete experimental investigation is required to test the conventional model.

First, we shall recall some theoretical results related to this problem. Let us consider the total cross-section ($\sigma_{\pm}$) difference for opposite neutron helicities for the neutron transmission through the target. This difference can be expressed through the amplitudes $f_{\pm}$ for the forward scattering of neutrons with positive and negative helicities as

$$\Delta_{\rm tot}=\sigma_- -\sigma_+ =\frac{4\pi}{k}{\rm Im}(f_- -f_+) \tag{1.1}$$

(where k is the neutron momentum) and has the obvious relation to the longitudinal analyzing power [2] of

$$P=\frac{\Delta_{\rm tot}}{\sigma_- +\sigma_+}=\frac{\sigma_- -\sigma_+}{\sigma_- +\sigma_+} . \tag{1.2}$$

The general formalism for calculating the P-violating difference of amplitudes in Eq. (1.1) is described in Refs. [3–5]. Following these papers, one can obtain the P-violating part of the reaction matrix (using the distorted-wave Born approximation in the weak interaction) as

$$T_{PNC}=\langle \Psi_f^- |W|\Psi_i^+ \rangle , \tag{1.3}$$

where W is the weak-interaction operator. According to the microscopic theory of nuclear reactions [6], the initial and final wave functions are

$$\Psi_{i,f}^{\pm}=\sum_k a_{k(i,f)}^{\pm}(E)\phi_k+\sum_m \int b_{m(i,f)}^{\pm}(E,E')\chi_m^{\pm}(E')dE' . \tag{1.4}$$

Here ϕ_k and χ_m are the wave functions of the kth nuclear compound resonance and the potential scattering in the channel m. The first coefficient in Eq. (1.4) is

$$a_{k(i,f)}^{\pm}(E)=\frac{\exp(\pm i\delta_{i,f})}{(2\pi)^{1/2}}\frac{(\Gamma_k^{i,f})^{1/2}}{E-E_k\pm(i/2)\Gamma_k} , \tag{1.5}$$

where E_k, Γ_k, and Γ_k^i are the energy, total width, and partial width in the ith channel of the kth nuclear compound resonance, E is the neutron energy, and δ_i is the potential scattering phase,

$$(\Gamma_k^i)^{1/2}=(2\pi)^{1/2}\langle \chi_i(E)|V|\phi_k \rangle , \tag{1.6}$$

where V is a residual interaction operator,

$$b_{m(\alpha)}^{\pm}=\exp(\pm i\delta_\alpha)\delta(E-E')\delta_{m,\alpha}+a_{k(\alpha)}^{\pm}\frac{\langle \phi_k|V|\chi_m(E')\rangle}{(E-E'\pm i\varepsilon)} . \tag{1.7}$$

Taking into account, for simplicity, only two compound resonances, we can express the P-violating matrix (1.3) as a sum of terms corresponding to the various P-violating mechanisms

$$\langle f|T|i\rangle=a_{i(\alpha)}^{+}a_{f(\beta)}^{+}\langle \phi_\beta|W|\phi_\alpha\rangle+a_{i(\alpha)}^{+}\exp i\delta_\beta\langle \chi_\beta|W|\phi_\alpha\rangle+a_{f(\beta)}^{+}\exp i\delta_\alpha\langle \phi_\beta|W|\chi_\alpha\rangle+\exp i(\delta_\alpha+\delta_\beta)\langle \chi_\beta|W|\chi_\alpha\rangle+\cdots . \tag{1.8}$$

The first term in (1.8) describes the parity mixing of the compound nuclear states. The second and third terms describe the P-violating decay and capture of a compound resonance, respectively. The fourth term corresponds to the direct (potential scattering) process caused by the weak P-odd interaction. The expression is more complicated for the valence mechanism of P violation (see, e.g., Refs. [7,8]). The relative contributions of these mechanisms were discussed in Refs. [4,9], and it was shown that the first one should be dominant for low-energy neutron scattering. For this case, we obtain

$$\Delta_{\rm tot}=-\frac{2\pi}{k^2}v(\Gamma_s^n\Gamma_p^n)^{1/2}\times\frac{[(E-E_s)\Gamma_p+(E-E_p)\Gamma_s]}{[(E-E_s)^2+\Gamma_s^2/4][(E-E_p)^2+\Gamma_p^2/4]} , \tag{1.9}$$

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where $v=-\int \phi_s W \phi_p d\tau$ is the weak matrix element (real for T-invariant interactions). Because of the statistical nature of the weak matrix element, we should expect a fluctuation in the sign of P for different resonances and, as a consequence, the vanishing of the contribution from this mechanism after averaging over a wide energy region [4].

The experimental result in Ref. [1] found that all the observed nonzero values of P on the ^{232}Th sample for neutron energy from 8 to 170 eV had the same (positive) sign. Using Eqs. (1.2) and (1.9) and suggesting that weak matrix elements and neutron widths have independent random Gaussian distributions, the authors of Ref. [1] conclude that the result they obtain is in contradiction with the mechanism of parity mixing at the compound nuclear stage. To explain this experimental result, a new mechanism of parity mixing in terms of a wave function at the nuclear surface with nonrandom wave-function behavior due to boundary conditions has been suggested in Ref. [10].

It may be worth noting that this experimental result [1] is unexpected. We must therefore look closely to determine whether or not this result is a true test of parity-violating mechanism.

We will not discuss the simple source of a sign change due to the energy-dependent term $[(E-E_s)\Gamma_p+(E-E_p)\Gamma_s]$ in Eq. (1.9). It can be easily calculated using detailed experimental information for the identification of the mixing of p- and s-wave resonances. However, for a small energy region, there is the possibility of having a few s-wave resonances whose energies are systematically more (or less) than the energies of admixed p-wave resonances. In this case, the energy-dependent term in Eq. (1.9) has a definite sign for all resonances. It should be noted that such a picture is very natural for the experimental results [1] on ^{232}Th where there are two strong s-wave resonances [11] with energies 69.19 ± 0.06 and 170.39 ± 0.17 eV.

II. STATISTICAL DESCRIPTION OF P-ODD EFFECTS

The product of the three random parameters in Eq. (1.9) can be rewritten as

$$(\Gamma_s^n)^{1/2} v (\Gamma_p^n)^{1/2} = -2\pi \langle \chi_s | V | \phi_s \rangle \langle \phi_s | W | \phi_p \rangle \langle \phi_p | V | \chi^p \rangle . \tag{2.1}$$

It is known that the three matrix elements in Eq. (2.1) can be described as Gaussian-distributed random variables with a mean of zero. The reason for their randomness is connected with the complex structure of compound resonances or, in other words, with a randomness of the wave functions ϕ for the compound resonances [12]. In principle, an additional randomness may arise from the operator of a residual interaction. However, for a small variation of the neutron energy ($\sim$100 eV) compared with the characteristic scale of the residual interaction ($\sim$10–100 keV), we can consider this operator as energy independent. (This is a good approximation in our discussion because we are interested only in the signs of the matrix elements.)

For simplicity, we start from the consideration of the statistical properties of expression (2.1) for the case of one s-wave resonance and many p-wave resonances. Then the first matrix element is the same for all p-wave resonances and the random behavior of the product (2.1) is connected with the random wave functions ϕ_p for different p-wave resonances in the last two terms:

$$M = \langle \phi_s | W | \phi_p \rangle \langle \phi_p | V | \chi_p \rangle . \tag{2.2}$$

In Ref. [1] these matrix elements are considered as independent random parameters. However, this assumption is incorrect because of the same source for the randomness of them both. To analyze the product M, we consider two random parameters

$$\begin{aligned} a_i &= \int \hat{A}(x)\phi_i(x)dx , \\ b_i &= \int \hat{B}(x)\phi_i(x)dx , \end{aligned} \tag{2.3}$$

where $\hat{A}$ and $\hat{B}$ are some operators and $\phi_i(x)$ is a random function. To simplify the mathematical procedure, we assume properties for these operators and functions appropriate for our discussion. $\hat{A}$ and $\hat{B}$ are differential operators with the power of the derivative (or momentum) not larger than 1; $\phi_i(x)$ are the radial parts of the eigenfunctions corresponding to the compound resonances, and $\phi_i(0)=\phi_i(R)=0$ (here R is the nuclear radius). Then, using partial integration in Eq. (2.3), one can obtain

$$\begin{aligned} a_i &= \int K_a(x)\phi_i(x)dx , \\ b_i &= \int K_b(x)\phi_i(x)dx , \end{aligned} \tag{2.4}$$

where $K_a(x)$ and $K_b(x)$ are some functions. From Eq. (2.4), one can see that for the random oscillating functions $\phi_i(x)$, the parameters a_i and b_i have zero mean value:

$$\langle a_i \rangle = \langle b_i \rangle = 0 . \tag{2.5}$$

According to Eq. (2.4), each function ϕ_i leads to the defined parameters a_i and b_i. One can expand the compound resonance wave function $\phi_i(x)$ in terms of simple-configuration wave functions ψ_ν which are admixed to compound resonances by strong interactions:

$$\phi_i(x) = \sum_\nu c_i(\nu)\psi_\nu(x) . \tag{2.6}$$

Here $c_i(\nu)$ are random coefficients and

$$\langle c_i(\nu)c_i(\mu) \rangle \sim \delta_{\nu\mu} . \tag{2.7}$$

Using the notation $\tilde{K}_a(\nu)=\int K_a(x)\psi_i(x)dx$ and Eq. (2.4), one can estimate

$$\begin{aligned} \langle a_i b_i \rangle &= \sum_{\nu\mu} \langle \tilde{K}_a(\nu)\tilde{K}_b(\mu)c_i(\nu)c_i(\mu) \rangle \\ &\sim \sum_\nu \tilde{K}_a(\nu)\tilde{K}_b(\nu) , \end{aligned} \tag{2.8}$$

which is not equal to zero in general because the right-hand side of Eq. (2.8) does not contain random values for a small energy interval.

To illustrate this point, let us consider a simple model which leads to Eq. (2.8). One supposes that each parameter a_i corresponds to one and only one parameter b_i. (In other words, we should consider separately different regions of the parameter manifold where such a condition is valid.) This will permit us to define b as a function of the argument a:

$$b=f(a) . \tag{2.9}$$

For a real physical system, the wave functions and kernels $K_{a,b}$ in Eq. (2.4) are smooth; therefore, one can expect a smooth behavior for the function (2.9).

Now we can write down the probability density for the product of the a_i and b_i parameters as a product of the probability density for one parameter (p_a or p_b) and the conditional probability for the other one:

$$p_{ab}=p_a\delta(b-f(a))\frac{\partial f(a)}{\partial a} . \tag{2.10}$$

The mean value of this product is

$$\langle a_i b_i\rangle=\sum a_i b_i p_{ab}$$

$$=\sum a_i f(a_i)\frac{\partial f}{\partial a}\bigg|_{a=a_i} p_a . \tag{2.11}$$

For the simple linear functional dependence $b=\alpha a$, the relation (2.11) leads to

$$\langle a_i b_i\rangle=\alpha\sum a_i^2 p_a=\alpha\sigma_a^2 , \tag{2.12}$$

where σ_a^2 is the variance of the a_i distribution. In other words, the product of two random (dependent) parameters with the zero mean values is a random parameter with nonzero mean value.

Using this result, we can conclude that in general the parameter M in Eq. (2.2) should have a nonzero mean value, and as a consequence, the product (2.1) for different s-wave resonances should also have a nonzero mean value. This fact is in good agreement with the experimental data of Ref. [1]. It should be noted that this result is obtained for a narrow energy region of neutrons compared to the characteristic scale of the residual interaction.

III. MODEL WITH SIGN CORRELATIONS

Now we would like to point out a nonstatistical possibility for the sign correlation of P-odd effects in the framework of the conventional model. This example may be interesting because the strong correlation for the product of the three parameters in Eq. (2.1) naturally arises only for heavy nuclei, where the experimental measurements have been performed.

We can see that the three-term product in Eq. (2.1) should have a constant sign for any compound resonance (in the small neutron energy region) if the weak and residual interaction operators have the same δ-shape radial behavior:

$$W\sim V\sim\delta(r-R) . \tag{3.1}$$

This is a correct result in spite of the fact that each matrix element has a random sign due to the randomness of ϕ wave functions. This conclusion is a consequence of the quasiquadratic form of Eq. (2.1) with respect to the compound resonance wave functions. Therefore, in the case of the realization of such a structure for the operators, the conventional mechanism of parity violation at the compound nuclear stage can give the sign correlation for the P parameter [here we omit the possible nonstatistical changes in the sign of P due to the energy-dependent part of Eq. (1.9)].

Now we should ask the following question: Is it possible to find a situation which actually corresponds to the conditions of Eq. (3.1)?

First, let us remind ourselves that, in the one-particle representation, the nuclear operator of the weak interaction can be written as [13]

$$W=\text{const}\{(\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}),\rho(r)\}_+ , \tag{3.2}$$

where $\boldsymbol{\sigma}$ is the nucleon spin and ρ is the nuclear density. Therefore the weak matrix element has significant a contribution from the region with maximum change of nuclear density, i.e., from the surface region. This is a remarkable fact because the matrix elements corresponding to neutron decay amplitudes have a dominant contribution from the surface region in models with surface-type residual interactions. It seems natural that the main contribution to the neutron decay width $(\Gamma^n)^{1/2}$ due to the operator V will be near the nuclear surface within the nucleon radius range ($r_0\sim 1.2$ fm). The corresponding range for the parity-violating operator obtained from Eq. (3.2) is about the diffuseness of the nuclear potential, $a\sim 0.7$ fm. Because of the fact that matrix elements in Eq. (2.1) have significant contributions from the narrow region near the nuclear surface, the corresponding operators can be considered as δ-function-like. This approximation is reasonable when the surface region is smaller than the nuclear radius R (in this case there are no wave-function oscillations in the surface region):

$$\frac{R}{r_0}\sim A^{1/3}\gg 1 . \tag{3.3}$$

Here A is the number of nucleons. Therefore, for heavy nuclei, this condition may lead to the sign correlation of the product of the chaotic parameters in Eq. (2.1), which appears natural from the physical point of view.

IV. CONCLUSIONS

We can conclude that, for an unambiguous test of the conventional model of parity violation at the compound nuclear stage, it is not enough only to measure the sign correlation of P-odd effects on some resonances in heavy nuclei for narrow regions of the neutron energy. The prediction of this model is the randomness of weak matrix elements, but this may not automatically provide for random signs in P. However, to extract the value of v (whose sign distribution is crucial for the test of the conventional model) instead of the product (2.1), it is necessary to know the neutron decay amplitudes for each resonance. These amplitudes can be obtained from complete

correlation measurements of different P-odd and P-even correlations because the different correlations are proportional to the different combinations of the amplitudes. This gives the opportunity to obtain all amplitudes in an unambiguous way from the experimental data. It should be pointed out that for the extraction of neutron decay amplitudes from such experiments, as a rule, it is necessary to use the multiresonance approach (see, e.g., Ref. [14]).

Note added. While this paper was being considered for publication, the referee kindly brought the paper of Bowman *et al.* [15] to my attention. In this paper [15] the idea of the correlation of the weak matrix element and neutron widths has been used to explain the experimental result in the framework of parity mixing at the compound nuclear stage. However, the statistical approach used in that paper led to a very small average asymmetry and could not explain the experimental result without contributions from "distant states."

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