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Solar neutrinos, SNO and neutrino–deuteron reactions

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Abstract
The standard nuclear physics approach and effective field theory approach for calculations of neutrino–deuteron cross sections for the solar neutrino energies are considered. Their main features, the level of accuracy and problems to be addressed for further developments are discussed.

1. Introduction

The neutrino–deuteron reactions are extremely important in connection with the highly consequential experiments at the Sudbury Neutrino Observatory (SNO). The ν–d reactions are also important in that they provide basic information that is useful for studying other neutrino–nucleus reactions involving more complex targets. The ν–d reactions, which can be calculated with much higher precision than neutrino reactions on complex nuclei, can serve as a benchmark case for neutrino–nucleus reactions in general.

The recent SNO experiments [1], which measure simultaneously the charge current and neutral current ν–d reaction, and pure-leptonic neutrino–electron scattering have established that the total solar neutrino flux (summed over all flavours) agrees with the prediction of the standard solar model [2], whereas the electron neutrino flux from the sun is significantly smaller than the total solar neutrino flux. The amount of deficit in the electron neutrino flux is consistent with what used to be known as the solar neutrino problem. These results of the SNO experiments have given firm evidence for the transmutation of solar electron neutrinos into neutrinos of other flavours. It is obvious that a precise knowledge of the ν–d reaction cross sections is of primary importance in interpreting the existing and future SNO data. We describe here some of salient features of the recent developments in the calculation of the ν–d reactions. The existing approaches can be classified into two categories: the standard nuclear physics approach and the effective field theory (EFT) approach.

The standard nuclear physics approach uses the phenomenological potential which has been highly successful in describing many different kinds of nuclear phenomena. In this approach an A-nucleon system is described by a Hamiltonian of the form

$$H = \sum_{i}^{A} t_i + \sum_{i<j}^{A} V_{ij} + \sum_{i<j<k}^{A} V_{ijk} + \cdots, \tag{1}$$

where t_i is the kinetic energy of the ith nucleon, V_{ij} is a phenomenological two-body potential between the ith and jth nucleons, V_{ijk} is a phenomelogical three-body potential, and so on. Since the interactions involving three or more nucleons are known to play much less important roles than the two-body interactions, we shall concentrate on V_{ij}. Once the Hamiltonian H is specified, the nuclear wavefunction $|\Psi\rangle$ is obtained by solving the Shrödinger equation

$$H|\Psi\rangle = E|\Psi\rangle. \tag{2}$$

There is large freedom in choosing possible forms of V_{ij} apart from a well-established requirement that, as the inter-nucleon distance r_{ij} becomes sufficiently large, V_{ij} should approach the one-pion exchange Yukawa potential. For the model-dependent short-range part of V_{ij}, one needs to assume certain functional forms and fix the parameters appearing therein by demanding that the solutions of equation (2) for the $A = 2$ case reproduce the nucleon–nucleon scattering data (typically up to the pion-production threshold energy) as well as some of the deuteron properties. There are by now several so-called *modern high-precision phenomenological* N–N potential that can reproduce all the existing two-nucleon data with normalized χ^2 values close to 1. These potentials differ significantly in the ways they parametrize short-range physics, and, as a consequence, they exhibit substantial difference in their off-shell behaviour.

In normal circumstances, nuclear responses to external electroweak probes are given, to good approximation, by one-body terms; these are also called the impulse approximation terms. To obtain higher accuracy, however, one must also consider exchange current terms, which represent the contributions of nuclear responses involving two or more nucleons. These exchange currents (usually taken to be two-body operators) are derived from one-boson exchange diagrams, and the vertices featuring in the relevant diagrams are determined to satisfy the low-energy theorems and current algebra [3, 4]. We refer to a formalism based on this picture as the *standard nuclear physics approach* (SNPA). (This is also called a potential model in the literature.)

Although SNPA has been scoring great successes in correlating and explaining a vast variety of data, it is still important from a formal point of view to raise the following issues. First, since hadrons and hadronic systems (including nuclei) are governed by quantum chromodynamics (QCD), one should ultimately be able to relate this approach with QCD, but this relation has not been established. In particular, while chiral symmetry is known to be a fundamental symmetry of QCD, the formulation of the standard approach is largely disjoint from this symmetry. Secondly, in the standard approach, even for describing low-energy phenomena, we start with a 'realistic' phenomenological potential which is tailored to encode short-range (high-momentum) and long-range (low-momentum) physics simultaneously. This mixing of the two different scales seems theoretically dissatisfactory and can be pragmatically inconvenient. Thirdly, in writing down a phenomenological Lagrangian for describing the nuclear interaction and nuclear responses to the electroweak currents, this approach is not equipped with a clear guiding principle; it is not clear whether there is any identifiable expansion parameter that helps us to control the possible forms of terms in the Lagrangian and that provides a general measure of errors in our calculation. To address these and other related issues, a new approach based on EFT was proposed [5] and it has been studied with great intensity (for reviews, see [6, 7]).

The general idea of EFT is in fact rather simple. In describing phenomena characterized by a typical energy–momentum scale Q, we expect that we need not include in our Lagrangian those degrees of freedom that pertain to energy–momentum scales much higher than Q. This expectation motivates us to introduce a cut-off scale Λ that is sufficiently larger than Q and we classify our fields (to be generically represented by ϕ) into two groups: high-frequency fields ϕ_H and low-frequency fields ϕ_L. By eliminating (or *integrating out*) ϕ_H, we arrive at an *effective* Lagrangian that only involves ϕ_L as explicit dynamical variables. Using the notion of path integrals, the effective Lagrangian $\mathcal{L}_{\rm eff}$ is related to the original Lagrangian $\mathcal{L}$ as

$$\int [\mathrm{d}\phi]\, \mathrm{e}^{\mathrm{i}\int \mathrm{d}^4x \mathcal{L}(\phi)} = \int (\mathrm{d}\phi_L)\, \mathrm{e}^{\mathrm{i}\int d^4x \mathcal{L}_{\rm eff}(\phi_L)}. \tag{3}$$

One can show that $\mathcal{L}_{\rm eff}$ defined by equation (3) inherits the symmetries (and the patterns of symmetry breaking, if there are any) of the original Lagrangian $\mathcal{L}$. It also follows that $\mathcal{L}_{\rm eff}$ should be the sum of all possible monomials of ϕ_L and their derivatives that are consistent with the symmetry requirements dictated by $\mathcal{L}$. Since a term involving n derivatives scales such as $(Q/\Lambda)^n$, the terms in $\mathcal{L}_{\rm eff}$ can be organized into a perturbative series in which Q/Λ serves as an expansion parameter. The coefficients of terms in this expansion scheme are called the low-energy constants (LECs). Provided all the LECs up to a specified order n can be fixed either from theory or from fitting to the experimental values of the relevant observables, $\mathcal{L}_{\rm eff}$ serves as a complete (and hence model-independent) Lagrangian to the given order of expansion.

Having specified the basic idea of EFT, we now discuss specific aspects of EFT as applied to nuclear physics. The underlying Lagrangian $\mathcal{L}$ in this case is the QCD Lagrangian $\mathcal{L}_{\rm QCD}$, whereas, for the typical nuclear physics energy–momentum scale $Q \ll \Lambda_\chi \sim 1$ GeV, the effective degrees of freedom that would feature in $\mathcal{L}_{\rm eff}$ are hadrons rather than the quarks and gluons. It is a non-trivial task to apply the formal definition in equation (3) to derive $\mathcal{L}_{\rm eff}$ written in terms of hadrons starting from $\mathcal{L}_{\rm QCD}$, because the hadrons cannot be straightforwardly identified with the low-frequency field, ϕ_L in equation (3), in the original Lagrangian. The best one could do at present is to resort to symmetry considerations and the momentum expansion scheme. Here chiral symmetry plays an important role. We know that chiral symmetry is spontaneously broken, resulting in the generation of the pions as Nambu–Goldstone bosons, i.e., chiral symmetry is realized in the Goldstone mode. This feature can be incorporated by assigning suitable chiral transformation properties to the Goldstone bosons and writing down all possible chiral-invariant terms up to a specified chiral order [8]. The above consideration presupposes exact chiral symmetry in $\mathcal{L}_{\rm QCD}$. In reality, $\mathcal{L}_{\rm QCD}$ contains small but finite quark mass terms, which explicitly violate chiral symmetry and lead to a non-vanishing value of the pion mass m_π. Again, there is a well-defined method to determine what terms are needed in the Goldstone boson sector to represent the effect of explicit chiral symmetry breaking [8]. These considerations lead to an EFT called chiral perturbation theory (χPT) [9, 10].

A problem we encounter in extending χPT to the nucleon sector is that, as the nucleon mass m_N is comparable to the cut-off scale Λ_χ, a simple application of expansion in Q/Λ does not work. This problem can be circumvented by employing heavy-baryon chiral perturbation theory (HBχPT), which essentially consists in shifting the reference point of the nucleon energy from 0 to m_N and in integrating out the small component of the nucleon field as well as the anti-nucleonic degrees of freedom. An effective Lagrangian in HBχPT therefore involves as explicit degrees of freedom the pions and the large components of the redefined nucleon field. HBχPT has as expansion parameters Q/Λ_χ, m_π/Λ_χ and Q/m_N. HBχPT has been used with great success to the one-nucleon sector [6].

It is to be noted, however, that HBχPT cannot be applied in a straightforward manner to nuclei that contain more than one nucleon. The reason is that nuclei involve very low-lying excited states, and the existence of this small energy scale upsets the original counting rule [5]. Weinberg's idea to avoid this difficulty is as follows. Classify Feynman diagrams into two groups, irreducible and reducible diagrams. Those diagrams in which every intermediate state has at least one meson in flight are called irreducible diagrams; all others are classified as reducible diagrams. We then apply the above-mentioned chiral counting rules only to irreducible diagrams, and the contribution of all the irreducible diagrams (up to a specified chiral order) is treated as an effective potential acting on nuclear wavefunctions. By summing up the geometric series of irreducible diagrams (by solving either the Schrödinger equation or the Lippman–Schwinger equation), we can incorporate the contributions of reducible diagrams [5]. We refer to this two-step procedure as *nuclear* χPT (this is often called the Λ-counting scheme [11]).

To apply nuclear χPT to a process that involves (an) external current(s), we derive a nuclear transition operator $\mathcal{T}$ by evaluating the complete set of all the irreducible diagrams (up to a given chiral order ν) involving the relevant external current(s). To preserve consistency in chiral counting, the nuclear matrix element of $\mathcal{T}$ must be calculated with the use of nuclear wavefunctions which are governed by nuclear interactions that represent all the irreducible A-nucleon diagrams up to νth order. Thus, a transition matrix in nuclear EFT is given by

$$\mathcal{M}_{fi}^{\mathrm{EFT}} = \left\langle \Psi_f^{\mathrm{EFT}} \right| \sum_{\ell}^{A} \mathcal{O}_{\ell}^{\mathrm{EFT}} + \sum_{\ell<m}^{A} \mathcal{O}_{\ell m}^{\mathrm{EFT}} \left| \Psi_i^{\mathrm{EFT}} \right\rangle, \tag{4}$$

where the superscript, 'EFT', means that the relevant quantities are obtained according to EFT as described above. If this programme is carried out exactly, it would constitute an *ab initio* calculation. We note that in EFT we know exactly at what chiral order three-body operators start to contribute to $\mathcal{T}$, and that, to chiral orders relevant to the applications described below, there is no need for three-body operators. With this understanding, we have retained only one- and two-body operators in equation (4). This unambiguous classification of transition operators according to their chiral orders is a great advantage of EFT.

2. Standard nuclear physics approach

We now describe in some detail the latest calculations of the ν–d cross sections based on SNPA [12, 13]. We consider four processes contributing to the total and differential cross sections for the CC and NC reactions of neutrinos and anti-neutrinos with the deuteron:

$$\nu_e + d \to e^- + p + p \qquad \text{(CC)} \tag{5}$$

$$\nu_x + d \to \nu_x + n + p \qquad \text{(NC)} \tag{6}$$

$$\bar{\nu}_e + d \to e^+ + n + n \qquad (\bar{\nu}\text{-CC}) \tag{7}$$

$$\bar{\nu}_x + d \to \bar{\nu}_x + n + p \qquad (\bar{\nu}\text{-NC}), \tag{8}$$

where $x = e, \mu$ or τ.

The four-momenta of the participating particles are labelled as

$$\nu/\bar{\nu}(k) + d(P) \to \ell(k') + N_1(p_1') + N_2(p_2'), \tag{9}$$

where ℓ corresponds to $e^{\pm}$ for the CC reactions (equations (5) and (7)), and to ν or $\bar{\nu}$ for the NC reactions (equations (6) and (8)). The energy–momentum conservation reads: $k + P = k' + P'$ with $P' \equiv p_1' + p_2'$. A momentum transferred from the lepton to the two-nucleon system is denoted by $q^\mu = k^\mu - k'^\mu = P'^\mu - P^\mu$. In the laboratory system, which we use throughout this work, we write

$$\begin{aligned} &k^\mu = (E_\nu, \mathrm{k}), \qquad k'^\mu = (E'_\ell, \mathrm{k}'), \qquad P^\mu = (M_d, 0), \\ &P'^\mu = (P'^0, \mathrm{P}'), \qquad q^\mu = (\omega, \mathrm{q}). \end{aligned} \tag{10}$$

The interaction Hamiltonian for semileptonic weak processes is given by the product of the hadron current (J_λ) and the lepton current (L^λ) as

$$H_W^{CC} = \frac{G'_F V_{ud}}{\sqrt{2}} \int \mathrm{dx}\big[J_\lambda^{CC}(\mathrm{x}) L^{CC,\lambda}(\mathrm{x}) + \mathrm{h.c.}\big] \tag{11}$$

for the CC process and

$$H_W^{NC} = \frac{G'_F}{\sqrt{2}} \int \mathrm{dx}\big[J_\lambda^{NC}(\mathrm{x}) L^{NC,\lambda}(\mathrm{x}) + \mathrm{h.c.}\big] \tag{12}$$

for the NC process. Here G'_F is the weak coupling constant, and V_{ud} is the K–M matrix element. For the weak coupling constant, instead of:

The lepton current is given by

$$L^\lambda(\boldsymbol{x}) = \bar{\psi}_l(\boldsymbol{x})\gamma^\lambda(1-\gamma^5)\psi_\nu(\boldsymbol{x}), \tag{13}$$

and its matrix element is written as

$$\begin{aligned} l^\lambda \equiv \langle k'|L^\lambda(0)|k\rangle &= \bar{u}_l(k')\gamma^\lambda(1-\gamma^5)u_\nu(k) \qquad \text{for } \nu\text{-reaction}, \\ &= \bar{v}_{\bar{\nu}}(k)\gamma^\lambda(1-\gamma^5)v_{\bar{l}}(k') \qquad \text{for } \bar{\nu}\text{-reaction}. \end{aligned} \tag{14}$$

The hadronic charged current has the form

$$J_\lambda^{CC}(\mathrm{x}) = V_\lambda^{\pm}(\mathrm{x}) + A_\lambda^{\pm}(\mathrm{x}), \tag{15}$$

where V_λ and A_λ denote the vector and axial-vector currents, respectively. The superscript ($\pm$) denotes the isospin-raising (-lowering) operator for the $\bar{\nu}(\nu)$-reaction. The hadronic neutral current is given by the standard model as

$$J_\lambda^{NC}(\mathrm{x}) = (1 - 2\sin^2\theta_W)V_\lambda^3 + A_\lambda^3 - 2\sin^2\theta_W V_\lambda^s, \tag{16}$$

where θ_W is the Weinberg angle. V_λ^s is the isoscalar part of the vector current, and the superscript '3' denotes the third component of the isovector current. The hadron current consists of one-nucleon impulse approximation (IA) terms and two-body exchange current (EXC) terms.

Since the incident neutrino energy of the solar neutrinos in the lab-frame is rather low ($E_\nu \lesssim 20$ MeV), the dominant contribution to the cross section is defined by the space component, A, of the axial current (A_μ). Therefore, the theoretical precision of $\sigma_{\nu d}$ is controlled essentially by the accuracy with which one can calculate the nuclear matrix element of A.

We decompose A as $\mathrm{A} = \mathrm{A_{IA}} + \mathrm{A_{EXC}}$, where $\mathrm{A_{IA}}$ and $\mathrm{A_{EXC}}$ are the impulse approximation and exchange-current contributions, respectively. Since $\mathrm{A_{IA}}$ is well known [14–18], the theoretical uncertainty is confined to $\mathrm{A_{EXC}}$. Among the various terms contributing to $\mathrm{A_{EXC}}$, the Δ-excitation current (A_Δ) gives the most important contribution, and A_Δ involves the coupling constants for the $A_\mu N\Delta$ vertex, the $\pi N\Delta$ vertex and the $\rho N\Delta$ vertex, and the corresponding form factors. Although the quark model is believed to provide reasonable estimates for these coupling constants, it is at present impossible to test their individual values empirically; only the overall strength of the Δ-excitation current can be monitored with electroweak processes in a few-nucleon system. Carlson *et al* [19] used the tritium β-decay rate, Γ_t^β, and to fix the strength of the Δ-excitation current, and the same strength was used in [13].

2.1. Impulse approximation current

The IA current is determined by the single-nucleon matrix elements of J_λ. The nucleon matrix elements of the currents are written as

$$\langle N(p')|V_\lambda^{\pm}(0)|N(p)\rangle = \bar{u}(p')\left[f_V\gamma_\lambda + \mathrm{i}\frac{f_M}{2M_N}\sigma_{\lambda\rho}q^\rho\right]\tau^{\pm}u(p), \tag{17}$$

$$\langle N(p')|A_\lambda^{\pm}(0)|N(p)\rangle = \bar{u}(p')[f_A\gamma_\lambda\gamma^5 + f_P\gamma^5 q_\lambda]\tau^{\pm}u(p), \tag{18}$$

where M_N is the average of the masses of the final two nucleons. For the third component of the isovector current, we simply replace $\tau^{\pm}$ with $\frac{\tau^3}{2}$. For the isoscalar current

$$\langle N(p')|V_\lambda^{s}(0)|N(p)\rangle = \bar{u}(p')\left[f_V\gamma_\lambda + \mathrm{i}\frac{f_M^s}{2M_N}\sigma_{\lambda\rho}q^\rho\right]\frac{1}{2}u(p). \tag{19}$$

The non-relativistic forms of the IA currents are given by

$$V_{IA,0}^{\pm}(\boldsymbol{x}) = \sum_i f_V\tau_i^{\pm}\delta(\boldsymbol{x}-\boldsymbol{r}_i), \tag{20}$$

$$\boldsymbol{V}_{IA}^{\pm}(\boldsymbol{x}) = \sum_i\left[f_V\frac{\boldsymbol{p}_i'+\boldsymbol{p}_i}{2M_N} + \frac{f_V+f_M}{2M_N}\nabla\times\boldsymbol{\sigma}_i\right]\tau_i^{\pm}\delta(\boldsymbol{x}-\boldsymbol{r}_i), \tag{21}$$

$$A_{IA,0}^{\pm}(\boldsymbol{x}) = \sum_i\left[\frac{f_A}{2M_N}\boldsymbol{\sigma}_i\cdot(\boldsymbol{p}_i'+\boldsymbol{p}_i) - \frac{if_P\omega}{2M_N}\boldsymbol{\sigma}_i\cdot\nabla\right]\tau_i^{\pm}\delta(\boldsymbol{x}-\boldsymbol{r}_i), \tag{22}$$

$$\boldsymbol{A}_{IA}^{\pm}(\boldsymbol{x}) = \sum_i\left[f_A\boldsymbol{\sigma}_i + \frac{f_P}{2M_N}\nabla(\nabla\cdot\boldsymbol{\sigma}_i)\right]\tau_i^{\pm}\delta(\boldsymbol{x}-\boldsymbol{r}_i), \tag{23}$$

$$V_{IA,0}^{s}(\boldsymbol{x}) = \sum_i f_V\frac{1}{2}\delta(\boldsymbol{x}-\boldsymbol{r}_i), \tag{24}$$

$$\boldsymbol{V}_{IA}^{s}(\boldsymbol{x}) = \sum_i\left[f_V\frac{\boldsymbol{p}_i'+\boldsymbol{p}_i}{2M_N} + \frac{f_V+f_M^s}{2M_N}\nabla\times\boldsymbol{\sigma}_i\right]\frac{1}{2}\delta(\boldsymbol{x}-\boldsymbol{r}_i). \tag{25}$$

It is useful to rewrite $\boldsymbol{p}_i + \boldsymbol{p}_i' = \boldsymbol{q} + \boldsymbol{P} \pm 2\boldsymbol{p}_N$, where the ($\pm$) sign corresponds to $i = 1$ ($i = 2$), and the derivative operator $\boldsymbol{p}_N$ should act on the deuteron wavefunction; in the laboratory system we are working in, we have $\boldsymbol{P} = \boldsymbol{0}$.

As for the q_μ^2 dependence of the form factors we can use [39, 20]:

$$f_V(q_\mu^2) = G_D(q_\mu^2)(1+\mu_p\eta)(1+\eta)^{-1}, \tag{26}$$

$$f_M(q_\mu^2) = G_D(q_\mu^2)(\mu_p-\mu_n-1-\mu_n\eta)(1+\eta)^{-1}, \tag{27}$$

$$f_A(q_\mu^2) = -g_A G_A(q_\mu^2), \tag{28}$$

$$f_P(q_\mu^2) = \frac{2m}{m_\pi^2-q_\mu^2}f_A(q_\mu^2), \tag{29}$$

$$f_M^s(q_\mu^2) = G_D(q_\mu^2)(\mu_p+\mu_n-1+\mu_n\eta)(1+\eta)^{-1}, \tag{30}$$

with

$$G_D(q_\mu^2) = \left(1-\frac{q_\mu^2}{0.71\ \mathrm{GeV}^2}\right)^{-2}, \tag{31}$$

$$G_A(q_\mu^2) = \left(1 - \frac{q_\mu^2}{1.04\ \mathrm{GeV}^2}\right)^{-2}, \tag{32}$$

where $\mu_p = 2.793$, $\mu_n = -1.913$, $\eta = -\frac{q_\mu^2}{4m^2}$ and m_π is the pion mass.

2.2. Exchange currents

The axial-vector EXC, $A^\mu_{\rm EXC}$, consists of a pion-pole term and a non-pole term, $\bar{A}^\mu_{\rm EXC}$. Using the PCAC hypothesis, however, we can express $A^\mu_{\rm EXC}$ in terms of the non-pole contribution alone:

$$A^\mu_{\rm EXC} = \bar{A}^\mu_{\rm EXC} - \frac{q^\mu}{m_\pi^2 - q_\mu^2}(\mathrm{q} \cdot \bar{\boldsymbol{A}}_{\rm EXC} - \omega \bar{A}_{\rm EXC,0}). \tag{33}$$

We therefore need only specify a model for the non-pole terms; the total contribution of $A^\mu_{\rm EXC}$ can be obtained with the use of equation (33). Regarding the space component of the axial-vector current, as mentioned earlier, we employ $\boldsymbol{A}_{\rm EXC}$ adjusted in such a manner that the experimental value of Γ^β_t be reproduced. Thus, following Schiavilla *et al* [21], we consider the π-pair current (denoted by πS), ρ-pair current (ρS), π-exchange Δ-excitation current ($\Delta\pi$), ρ-exchange Δ-excitation current ($\Delta\rho$) and $\pi\rho$-exchange current ($\pi\rho$). The explicit expressions of these two-body currents (acting on the ith and jth nucleons) are as follows:

$$\begin{aligned}
\bar{\boldsymbol{A}}^\pm_{ij}(\boldsymbol{q};\pi\mathrm{S}) &= -\frac{f_A}{m}\frac{f^2_{\pi NN}}{m_\pi^2}\frac{\boldsymbol{\sigma}_j\cdot\boldsymbol{k}_j}{m_\pi^2+\boldsymbol{k}_j^2}f_\pi^2(\boldsymbol{k}_j) \\
&\quad\times\left\{(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^\pm\boldsymbol{\sigma}_i\times\boldsymbol{k}_j - \tau_j^\pm[\boldsymbol{q}+\mathrm{i}\boldsymbol{\sigma}_i\times(\boldsymbol{p}_i+\boldsymbol{p}_i')]\right\} + (i \rightleftharpoons j), \\
\bar{\boldsymbol{A}}^\pm_{ij}(\boldsymbol{q};\rho\mathrm{S}) &= f_A\frac{g_\rho^2(1+\kappa_\rho)^2}{4m^3}\frac{f_\rho^2(\boldsymbol{k}_j)}{m_\rho^2+\boldsymbol{k}_j^2}\big(\tau_j^\pm\{(\boldsymbol{\sigma}_j\times\boldsymbol{k}_j)\times\boldsymbol{k}_j \\
&\quad - \mathrm{i}[\boldsymbol{\sigma}_i\times(\boldsymbol{\sigma}_j\times\boldsymbol{k}_j)]\times(\boldsymbol{p}_i+\boldsymbol{p}_i')\} + (\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^\pm\{\boldsymbol{q}\boldsymbol{\sigma}_i\cdot(\boldsymbol{\sigma}_j\times\boldsymbol{k}_j) \\
&\quad + \mathrm{i}(\boldsymbol{\sigma}_j\times\boldsymbol{k}_j)\times(\boldsymbol{p}_i+\boldsymbol{p}_i') - [\boldsymbol{\sigma}_i\times(\boldsymbol{\sigma}_j\times\boldsymbol{k}_j)]\times\boldsymbol{k}_j\}\big) + (i \rightleftharpoons j), \\
\bar{\boldsymbol{A}}^\pm_{ij}(\boldsymbol{q};\Delta\pi) &= \frac{16}{25}f_A\frac{f^2_{\pi NN}}{m_\pi^2(m_\Delta-m)}\frac{\boldsymbol{\sigma}_j\cdot\boldsymbol{k}_j}{m_\pi^2+\boldsymbol{k}_j^2}f_\pi^2(\boldsymbol{k}_j) \\
&\quad\times\left[4\tau_j^\pm\boldsymbol{k}_j - (\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^\pm\boldsymbol{\sigma}_i\times\boldsymbol{k}_j\right] + (i \rightleftharpoons j), \\
\bar{\boldsymbol{A}}^\pm_{ij}(\boldsymbol{q};\Delta\rho) &= -\frac{4}{25}f_A\frac{g_\rho^2(1+\kappa_\rho)^2}{m^2(m_\Delta-m)}\frac{f_\rho^2(\boldsymbol{k}_j)}{m_\rho^2+\boldsymbol{k}_j^2} \\
&\quad\times\left\{4\tau_j^\pm(\boldsymbol{\sigma}_j\times\boldsymbol{k}_j)\times\boldsymbol{k}_j - (\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^\pm\boldsymbol{\sigma}_i\times[(\boldsymbol{\sigma}_j\times\boldsymbol{k}_j)\times\boldsymbol{k}_j]\right\} + (i \rightleftharpoons j), \\
\bar{\boldsymbol{A}}^\pm_{ij}(\boldsymbol{q};\pi\rho) &= 2f_A\frac{g_\rho^2}{m}\frac{\boldsymbol{\sigma}_j\cdot\boldsymbol{k}_j}{(m_\rho^2+\boldsymbol{k}_i^2)(m_\pi^2+\boldsymbol{k}_j^2)}f_\rho(\boldsymbol{k}_i)f_\pi(\boldsymbol{k}_j)(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^\pm \\
&\quad\times[(1+\kappa_\rho)\boldsymbol{\sigma}_i\times\boldsymbol{k}_i - \mathrm{i}(\boldsymbol{p}_i+\boldsymbol{p}_i')] + (i \rightleftharpoons j),
\end{aligned} \tag{34}$$

here m_ρ and m_Δ are the masses of the ρ-meson and Δ-particle, respectively; f_A is the axial form factor given in equation (28). The total three-momentum transfer is $\boldsymbol{q} \equiv \boldsymbol{k}_i + \boldsymbol{k}_j$, with $\boldsymbol{k}_{i(j)}$ being the momentum transferred to the ith (jth) nucleon; $\boldsymbol{p}_i$ and $\boldsymbol{p}_i'$ are the initial and final momenta of the ith nucleon. The form factors, $f_\pi(\boldsymbol{k})$ and $f_\rho(\boldsymbol{k})$, for the pion-nucleon and ρ-nucleon vertices are parametrized as

$$f_\pi(\boldsymbol{k}) = \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 + \boldsymbol{k}^2}, \qquad f_\rho(\boldsymbol{k}) = \frac{\Lambda_\rho^2 - m_\rho^2}{\Lambda_\rho^2 + \boldsymbol{k}^2} \tag{35}$$

with $\Lambda_\pi = 4.8$ fm^{-1} and $\Lambda_\rho = 6.8$ fm^{-1}. The quark model has been used to relate the coupling constants of the $\pi N\Delta$, $\rho N\Delta$ and $A_\mu N\Delta$ vertices to the πNN, ρNN and $A_\mu NN$ vertices, respectively. Schiavilla *et al* [21] have pointed out that the experimental value of Γ_t^β can be reproduced if the strengths of $\bar{\boldsymbol{A}}(\Delta\pi)$ in equation (34) and $\bar{\boldsymbol{A}}(\Delta\rho)$ in equation (34) are reduced by a common factor of 0.8. We employ here the same adjustment of $\bar{\boldsymbol{A}}(\Delta\pi)$ and $\bar{\boldsymbol{A}}(\Delta\rho)$. For the third component of the isovector current, we simply replace $\tau_i^\pm$ and $(\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)^\pm$ with $\tau_i^3/2$ and $(\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)^3/2$, respectively. (The same prescription is applied to the other exchange currents as well.) For the time component we use the one-pion exchange current (the so-called KDR current [22]), which gives the dominant exchange current to $\bar{A}_{0ij}^\pm$. The explicit form of the KDR current, with a vertex form factor supplemented[1], reads

$$\bar{A}_{0ij}^\pm(\boldsymbol{q};KDR) = \frac{2}{\mathrm{i}f_A}\left(\frac{f}{m_\pi}\right)^2 f_\pi^2(\boldsymbol{k}_j)\frac{\boldsymbol{\sigma}_j\cdot\boldsymbol{k}_j}{m_\pi^2+\boldsymbol{k}_j^2}(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^\pm + (i \rightleftharpoons j). \tag{36}$$

Regarding the vector exchange currents, we first note that the time component should be negligibly small since its contribution vanishes in the static limit. For the space component, $\boldsymbol{V}$, we take account of the pair, pionic and isobar currents. As in [12], we adopt the one-pion exchange model for the pair and pionic currents and the one-pion and one-ρ-meson exchange model for the isobar current. Their explicit expressions are

$$\boldsymbol{V}_{ij}^\pm(\boldsymbol{q};pair) = -2\mathrm{i}f_V\left(\frac{f}{m_\pi}\right)^2 f_\pi^2(\boldsymbol{k}_j)\frac{\boldsymbol{\sigma}_i(\boldsymbol{\sigma}_j\cdot\boldsymbol{k}_j)}{m_\pi^2+\boldsymbol{k}_j^2}(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^\pm + (i \rightleftharpoons j), \tag{37}$$

$$\boldsymbol{V}_{ij}^\pm(\boldsymbol{q};\pi) = 2\mathrm{i}\left(\frac{f}{m_\pi}\right)^2 f_\pi(\boldsymbol{k}_i)f_\pi(\boldsymbol{k}_j)\frac{(\boldsymbol{\sigma}_i\cdot\boldsymbol{k}_i)(\boldsymbol{\sigma}_j\cdot\boldsymbol{k}_j)(\boldsymbol{k}_i-\boldsymbol{k}_j)}{\left(m_\pi^2+\boldsymbol{k}_i^2\right)\left(m_\pi^2+\boldsymbol{k}_j^2\right)}(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^\pm + (i \rightleftharpoons j), \tag{38}$$

$$\begin{aligned}\boldsymbol{V}_{ij}^\pm(\boldsymbol{q};\Delta) = -\mathrm{i}4\pi\frac{f_V+f_M}{2m}\Bigg[&\frac{f_\pi^2(\boldsymbol{k}_j)}{m_\pi^2+\boldsymbol{k}_j^2}\boldsymbol{q}\times\left\{c_0\boldsymbol{k}_j\tau_j^\pm + d_1(\boldsymbol{\sigma}_i\times\boldsymbol{k}_j)(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^\pm\right\}(\boldsymbol{\sigma}_j\cdot\boldsymbol{k}_j)\\ &+\frac{f_\rho^2(\boldsymbol{k}_j)}{m_\rho^2+\boldsymbol{k}_j^2}\boldsymbol{q}\times\left\{c_\rho\boldsymbol{k}_j\times(\boldsymbol{\sigma}_j\times\boldsymbol{k}_j)\tau_j^\pm + d_\rho\boldsymbol{\sigma}_i\times(\boldsymbol{k}_j\times(\boldsymbol{\sigma}_j\times\boldsymbol{k}_j))(\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j)^\pm\right\}\Bigg]\\ &+(i \rightleftharpoons j).\end{aligned} \tag{39}$$

The numerical values of the various parameters are

$$\begin{aligned}&\frac{f^2}{4\pi} = 0.08, \qquad c_0 m_\pi^3 = 0.188, \qquad d_1 m_\pi^3 = -0.044,\\ &c_\rho m_\rho^3 = 36.2, \qquad d_\rho = -\tfrac{1}{4}c_\rho.\end{aligned} \tag{40}$$

These values lead to $np \to d\gamma$ cross sections that agree with the experimental values.

2.3. Multipole expansion of hadron current

To evaluate the two-nucleon matrix element of the hadron current, we first separate the centre-of-mass and relative wavefunctions,

$$\begin{aligned}\langle \boldsymbol{r}_1,\boldsymbol{r}_2 \mid d(P)\rangle &= \mathrm{e}^{\mathrm{i}\boldsymbol{P}\cdot\boldsymbol{R}}\psi_d(\boldsymbol{r})\\ \langle \boldsymbol{r}_1,\boldsymbol{r}_2 \mid NN(P')\rangle &= \mathrm{e}^{\mathrm{i}\boldsymbol{P}'\cdot\boldsymbol{R}}\psi_{\boldsymbol{p}'}(\boldsymbol{r}),\end{aligned} \tag{41}$$

[1] For A_0 and the vector currents, we use the same form factors as in [12]. They are parametrized as in equation (35), but the numerical values of Λ_π and Λ_ρ are: $\Lambda_\pi = 6.0$ fm^{-1}, $\Lambda_\rho = 7.3$ fm^{-1}.

where $\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$ and $\boldsymbol{R} = \frac{\boldsymbol{r}_1+\boldsymbol{r}_2}{2}$, and ψ_d and $\psi_{\boldsymbol{p}'}$ represent, respectively, the deuteron wavefunction and a scattering-state wavefunction with asymptotic relative momentum $\boldsymbol{p}'$. Then the matrix element of the hadron current for charged-current reaction is given by

$$\begin{aligned} j_\lambda^{CC} &\equiv \langle NN(P)'|J_\lambda^{CC}(0)|d(P)\rangle \\ &= \int \mathrm{d}\boldsymbol{r}\, \psi_{\boldsymbol{p}'}^*(\boldsymbol{r}) \left[\int \mathrm{d}\boldsymbol{R}\, \mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{R}} J_\lambda^{CC}(0)\right] \psi_d(\boldsymbol{r}). \end{aligned} \tag{42}$$

As for the neutral-current reaction, we just replace J_λ^{CC} with J_λ^{NC}. In the following equations, J_λ without superscript applies for both NC and CC. Eliminating the dependence of the current $J_\lambda(\boldsymbol{x})$ on the centre-of-mass coordinate, $\boldsymbol{R}$, we can write

$$j_\lambda = \langle \psi_{p'}| \int \mathrm{d}\boldsymbol{x}\, \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{x}} \mathcal{J}_\lambda(\boldsymbol{x})|\psi_d\rangle, \tag{43}$$

where $\mathcal{J}_\lambda(\boldsymbol{x}) \equiv J_\lambda(\boldsymbol{x})|_{\boldsymbol{R}=0}$. Similarly, we define $\mathcal{V}_\lambda(\boldsymbol{x}) \equiv V_\lambda(\boldsymbol{x})|_{\boldsymbol{R}=0}$ and $\mathcal{A}_\lambda(\boldsymbol{x}) \equiv A_\lambda(\boldsymbol{x})|_{\boldsymbol{R}=0}$. We now introduce the standard multipole expansion of the nuclear currents [23]. The multipole operator for the time component of a current is defined by

$$T_C^{JM}(\mathcal{J}) = \int \mathrm{d}\boldsymbol{x}\, j_J(qx) Y_{JM}(\hat{\boldsymbol{x}}) \mathcal{J}_0(\boldsymbol{x}), \tag{44}$$

where $j_J(qx)$ is the spherical Bessel function of order J, $q \equiv |\boldsymbol{q}|$, and $\hat{\boldsymbol{x}} \equiv \boldsymbol{x}/|\boldsymbol{x}|$. The electric and magnetic multipole operators are defined by

$$T_E^{JM}(\mathcal{J}) = \frac{1}{q} \int \mathrm{d}\boldsymbol{x}\, \nabla \times [j_J(qx) \boldsymbol{Y}_{JJM}(\hat{\boldsymbol{x}})] \cdot \boldsymbol{\mathcal{J}}(\boldsymbol{x}), \tag{45}$$

$$T_M^{JM}(\mathcal{J}) = \int \mathrm{d}\boldsymbol{x}\, j_J(qx) \boldsymbol{Y}_{JJM}(\hat{\boldsymbol{x}}) \cdot \boldsymbol{\mathcal{J}}(\boldsymbol{x}), \tag{46}$$

where $\boldsymbol{Y}_{JJM}(\hat{\boldsymbol{x}})$ are vector spherical harmonics. The longitudinal multipole operator is defined by

$$T_L^{JM}(\mathcal{J}) = \frac{\mathrm{i}}{q} \int \mathrm{d}\boldsymbol{x}\, \nabla[j_J(qx) Y_{JM}(\hat{\boldsymbol{x}})] \cdot \boldsymbol{\mathcal{J}}(\boldsymbol{x}). \tag{47}$$

Using the conservation of the vector current, the longitudinal multipole operator of the vector current can be related to the charge density operator as

$$T_L^{J_o}(\mathcal{V}) = -\frac{\omega}{q} T_C^{J_o}(\mathcal{V}). \tag{48}$$

An explicit form of the electric multipole operator for the vector current is given by

$$\begin{aligned} T_E^{JM}(\mathcal{V}) = -\mathrm{i}\sqrt{\frac{J}{2J+1}} &\int \mathrm{d}\boldsymbol{x}\, j_{J+1}(qx) \boldsymbol{Y}_{JJ+1M}(\hat{\boldsymbol{x}}) \cdot \boldsymbol{\mathcal{V}}(\boldsymbol{x}) \\ &+ \mathrm{i}\sqrt{\frac{J+1}{2J+1}} \int \mathrm{d}\boldsymbol{x}\, j_{J-1}(qx) \boldsymbol{Y}_{JJ-1M}(\hat{\boldsymbol{x}}) \cdot \boldsymbol{\mathcal{V}}(\boldsymbol{x}). \end{aligned} \tag{49}$$

Here again we can use the current conservation to rewrite equation (49) into a form that has the correct long wavelength limit of an electric multipole operator:

$$T_E^{JM}(\mathcal{V}) = -\sqrt{\frac{J+1}{J}} \frac{\omega}{q} T_C^{JM}(\mathcal{V}) - \mathrm{i}\sqrt{\frac{2J+1}{J}} \int \mathrm{d}\boldsymbol{x}\, j_{J+1}(qx) \boldsymbol{Y}_{JJ+1M}(\hat{\boldsymbol{x}}) \cdot \boldsymbol{\mathcal{V}}(\boldsymbol{x}). \tag{50}$$

2.4. Cross sections

Following the standard procedure, we obtain the cross section for the CC reaction as

$$\mathrm{d}\sigma = \sum_{\bar{i},f} \frac{\delta^4(k+P-k'-P')}{(2\pi)^5} \frac{G_F^2 \cos^2\theta_C}{2} F(Z, E'_\ell) \left| l^\lambda j_\lambda^{CC} \right|^2 \mathrm{d}\boldsymbol{k}' \, \mathrm{d}\boldsymbol{p}'_1 \, \mathrm{d}\boldsymbol{p}'_2, \tag{51}$$

and the cross section for the NC reaction as

$$\mathrm{d}\sigma = \sum_{\bar{i},f} \frac{\delta^4(k+P-k'-P')}{(2\pi)^5} \frac{G_F^2}{2} \left| l^\lambda j_\lambda^{NC} \right|^2 \mathrm{d}\boldsymbol{k}' \, \mathrm{d}\boldsymbol{p}'_1 \, \mathrm{d}\boldsymbol{p}'_2. \tag{52}$$

The matrix elements, l^λ and j_λ, have been defined in equations (14) and (42), respectively. In equation (51), we have included the Fermi function $F(Z, E'_\ell)$ to take into account the Coulomb interaction between the electron and the nucleons. In fact, this factor is relevant only to the $\nu_e + d \to e^- + p + p$ reaction, for which we should use $F(Z=2, E'_\ell)$; for the $\bar{\nu}_e + d \to e^+ + n + n$ reaction we have $F(Z=0, E'_\ell) \equiv 1$.

Substitution of the multipole operators defined in equations (44)–(47) leads to

$$l^\lambda j_\lambda = \sum_{J_o M_o} 4\pi i^{J_o} (-1)^{M_o} \langle \psi_{\boldsymbol{p}'} | \times \left[T_C^{J_o M_o} \ell_C^{J_o - M_o} + T_E^{J_o M_o} \ell_E^{J_o - M_o} + T_L^{J_o M_o} \ell_L^{J_o - M_o} + T_M^{J_o M_o} \ell_M^{J_o - M_o} \right] | \psi_d \rangle, \tag{53}$$

where the lepton matrix elements are given as

$$\ell_C^{JM} = Y_{JM}(\hat{\boldsymbol{q}}) l^0, \tag{54}$$

$$\ell_E^{JM} = \left(\sqrt{\frac{J+1}{2J+1}} \boldsymbol{Y}_{J-1JM}(\hat{\boldsymbol{q}}) + \sqrt{\frac{J}{2J+1}} \boldsymbol{Y}_{J+1JM}(\hat{\boldsymbol{q}}) \right) \cdot \boldsymbol{l}, \tag{55}$$

$$\ell_M^{JM} = \boldsymbol{Y}_{JJM}(\hat{\boldsymbol{q}}) \cdot \boldsymbol{l}, \tag{56}$$

$$\ell_L^{JM} = \left(\sqrt{\frac{J}{2J+1}} \boldsymbol{Y}_{J-1JM}(\hat{\boldsymbol{q}}) - \sqrt{\frac{J+1}{2J+1}} \boldsymbol{Y}_{J+1JM}(\hat{\boldsymbol{q}}) \right) \cdot \boldsymbol{l}. \tag{57}$$

To proceed, we use a scattering wavefunction of the following form:

$$\psi_{\boldsymbol{p}'}(\boldsymbol{r}) = \sum_{L,S,J,T} 4\pi (1/2, s_1, 1/2, s_2 | S\mu)(1/2, \tau_1, 1/2, \tau_2 | T, T_z)(LmS\mu | JM) \times \mathrm{i}^L Y_{L,m}^*(\hat{\boldsymbol{p}}') \psi_{LSJT}(\boldsymbol{r}) \tag{58}$$

with

$$\psi_{LSJT}(\boldsymbol{r}) = \frac{1-(-1)^{L+S+T}}{\sqrt{2}} \sum_{L'} \mathcal{Y}_{L'SJ}(\hat{\boldsymbol{r}}) R^J_{L',L;S}(r) \eta_{T,T_z}, \tag{59}$$

$$\mathcal{Y}_{LSJ}(\hat{\boldsymbol{r}}) = [Y_L(\hat{\boldsymbol{r}}) \otimes \chi_S]_{(J)}, \tag{60}$$

where χ_S (η_T) is the two-nucleon spin (isospin) wavefunction with total spin S (isospin T). The above wavefunction is normalized in such a manner that, in the plane wave limit, it satisfies

$$R^J_{L',L;S}(r) \to j_L(p'r) \delta_{L,L'}. \tag{61}$$

The partial wave expansion of the scattering wavefunction (equation (58)) gives

$$l^\lambda j_\lambda = \sum_{L,S,J,T,m} \sum_{J_o,M_o} (-1)^{M_o} \mathrm{i}^{J_o - L} \frac{(4\pi)^2}{\sqrt{2J+1}} (1/2, s_1, 1/2, s_2|S\mu) \times (1/2, \tau_1, 1/2, \tau_2|T, T_z)(1m_d J_o M_o|JM)(LmS\mu|JM) Y_{L,m}(\hat{\boldsymbol{p}}') \times \sum_{X=C,E,L,M} \langle T_X^{J_o} \rangle \ell_X^{J_o - M_o}, \tag{62}$$

where m_d is the z-component of the deuteron angular momentum. We have used here a simplified notation

$$\langle O^{J_o} \rangle = \langle \psi_{LSJT} \| O^{J_o} \| \psi_d \rangle \tag{63}$$

for the reduced matrix element defined by

$$\langle J'M'|O^{J_o M_o}|JM\rangle = \frac{1}{\sqrt{2J'+1}} (J, M, J_o, M_o|J', M')\langle J' \| O^{J_o} \| J\rangle, \tag{64}$$

where $O^{J_o M_o}$ are the multipole operators that appear in equations (44)–(47).

2.4.1. Cross sections for charged-current reaction. For the CC reaction, observables of interest are the total cross section and the lepton differential cross sections. We therefore integrate equation (51) over the momenta of the final two nucleons. Then equation (51) leads to

$$\mathrm{d}\sigma = \frac{G_F^2 \cos^2\theta_C}{3\pi^2} F(Z, E'_\ell)|M|^2 \delta(M_d + k - E'_\ell - P'^0) \bar{J} p'^2 \,\mathrm{d}p' k'^2 \,\mathrm{d}k' \,\mathrm{d}\Omega_{\boldsymbol{k}'}, \tag{65}$$

where

$$\begin{aligned} |M|^2 = \sum_{LSJ,J_o} \Bigg\{ & \left|\langle T_C^{J_o}(\mathcal{V})\rangle\right|^2 \left(1 + \hat{\boldsymbol{k}}\cdot\boldsymbol{\beta} + \frac{\omega^2}{\boldsymbol{q}^2}(1 - \hat{\boldsymbol{k}}\cdot\boldsymbol{\beta} + 2\hat{\boldsymbol{q}}\cdot\boldsymbol{\beta}\hat{\boldsymbol{q}}\cdot\hat{\boldsymbol{k}}) - \frac{2\omega}{q}\hat{\boldsymbol{q}}\cdot(\hat{\boldsymbol{k}} + \boldsymbol{\beta})\right) \\ & + \left|\langle T_C^{J_o}(\mathcal{A})\rangle\right|^2 (1 + \hat{\boldsymbol{k}}\cdot\boldsymbol{\beta}) + \left|\langle T_L^{J_o}(\mathcal{A})\rangle\right|^2 (1 - \hat{\boldsymbol{k}}\cdot\boldsymbol{\beta} + 2\hat{\boldsymbol{q}}\cdot\boldsymbol{\beta}\hat{\boldsymbol{q}}\cdot\hat{\boldsymbol{k}}) \\ & + 2\,\mathrm{Re}\left[\langle T_C^{J_o}(\mathcal{A})\rangle\langle T_L^{J_o}(\mathcal{A})\rangle^*\right]\hat{\boldsymbol{q}}\cdot(\hat{\boldsymbol{k}} + \boldsymbol{\beta}) + \left[\left|\langle T_M^{J_o}(\mathcal{V})\rangle\right|^2\right. \\ & + \left|\langle T_E^{J_o}(\mathcal{V})\rangle\right|^2 + \left|\langle T_M^{J_o}(\mathcal{A})\rangle\right|^2 + \left.\left|\langle T_E^{J_o}(\mathcal{A})\rangle\right|^2\right](1 - \hat{\boldsymbol{q}}\cdot\hat{\boldsymbol{k}}\,\hat{\boldsymbol{q}}\cdot\boldsymbol{\beta}) \\ & \mp 2\,\mathrm{Re}\left[\langle T_M^{J_o}(\mathcal{V})\rangle\langle T_E^{J_o}(\mathcal{A})\rangle^* + \langle T_M^{J_o}(\mathcal{A})\rangle\langle T_E^{J_o}(\mathcal{V})\rangle^*\right]\hat{\boldsymbol{q}}\cdot(\hat{\boldsymbol{k}} - \boldsymbol{\beta}) \Bigg\}. \end{aligned} \tag{66}$$

In the above, $k' \equiv |\boldsymbol{k}'|$ and $\boldsymbol{\beta} \equiv \boldsymbol{k}'/E'_\ell$; $\boldsymbol{p}'$ is the relative momentum of the final two nucleons, and $p' \equiv |\boldsymbol{p}'|$. Of the double sign in the last line of equation (66), the upper (lower) sign corresponds to the ν ($\bar{\nu}$) reaction. The appearance of the factor $\bar{J}$ in equation (65) needs an explanation; when relativistic kinematics is adopted, there arises a Jacobian, J, associated with the introduction of $\boldsymbol{p}'$ but it is a good approximation to use $\bar{J}$, the angle-averaged value of J (see [12] for details).

For the total cross section, the use of relativistic kinematics gives

$$\sigma = \int \mathrm{d}T \int \mathrm{d}(\cos\theta_\mathrm{L}) \frac{G_F^2 \cos^2\theta_C}{3\pi} \frac{\bar{J} E'_\ell \left(\sqrt{P_\mu'^2}/2\right) p' k'}{1 + E'_\ell (1 - k\cos\theta_\mathrm{L}/k')/\sqrt{P_\mu'^2 + \boldsymbol{q}^2}} F(Z, E'_\ell)|M|^2, \tag{67}$$

where T is the kinetic energy of the final NN relative motion and θ_L is the lepton scattering angle ($\cos\theta_\mathrm{L} = \hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{k}}'$) in the laboratory frame. If instead we use non-relativistic kinematics, the results would be

$$\sigma = \int \mathrm{d}T \int \mathrm{d}(\cos\theta_{\mathrm{L}}) \frac{G_F^2 \cos^2\theta_C}{3\pi} \frac{E'_\ell(2M_r)p'k'}{1 + E'_\ell(1 - k\cos\theta_{\mathrm{L}}/k')/(M_{N1} + M_{N2})} F(Z, E'_\ell)|M|^2, \tag{68}$$

where M_{Ni} is the mass of the ith nucleon, and M_r is the reduced mass of the final NN system.

Equation (65) also leads to the double differential cross sections for the $\nu_e + d \to e^- + p + p$ reaction:

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega_{\boldsymbol{k}'}\,\mathrm{d}E'_\ell} = \frac{G_F^2\cos^2\theta_C}{12\pi^2} F(Z, E'_\ell)\bar{J} p'k'E'_\ell \sqrt{P'^2_\mu + \boldsymbol{q}^2}|M|^2. \tag{69}$$

The electron energy spectrum and the electron angular distribution are obtained from equation (69) as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E'_\ell} = \int \mathrm{d}\Omega_{\boldsymbol{k}'} \left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega_{\boldsymbol{k}'}\,\mathrm{d}E'_\ell}\right)_{\mathrm{eq}\ (70)} \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_{\boldsymbol{k}'}} = \int \mathrm{d}E'_\ell \left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega_{\boldsymbol{k}'}\,\mathrm{d}E'_\ell}\right)_{\mathrm{eq}\ (70)}. \tag{70}$$

2.4.2. Cross sections for neutral-current reaction. The total cross section for the NC reaction can be calculated in essentially the same manner as above. The result is

$$\sigma = \int \mathrm{d}T \int \mathrm{d}(\cos\theta_{\mathrm{L}}) \frac{2G_F^2}{3\pi} \frac{\bar{J}E'_\ell\big(\sqrt{P'^2_\mu}/2\big)p'k'}{1 + E'_\ell(1 - k\cos\theta_{\mathrm{L}}/k')\big/\sqrt{P'^2_\mu + \boldsymbol{q}^2}}|M|^2, \tag{71}$$

where $|M|^2$ is given by equation (66) with, however, the charged current replaced by the neutral current. By contrast, in calculating neutron differential cross sections we can no longer integrate over the relative momentum of the final nucleons. We therefore work with the following expressions:

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega_{\boldsymbol{p}'_n}\,\mathrm{d}T_n} = \int \mathrm{d}\Omega_{\boldsymbol{k}'} \frac{G_F^2}{3(2\pi)^5} \frac{E_p k'^2 p'_n E_n}{E_p - \boldsymbol{p}'_p \cdot \hat{\boldsymbol{k}}'} \sum_{m_d, s_n, s_p} |j_\lambda l^\lambda|^2, \tag{72}$$

where we have indicated explicitly averaging over the initial spin and summing over the final spins. The energy and momentum of the final proton (neutron) are denoted by $(E'_\alpha, \boldsymbol{p}'_\alpha)$ with $\alpha = p$ $(\alpha = n)$; T_n is the kinetic energy of the neutron. The neutron energy spectrum and the neutron angular distribution are then evaluated as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}T_n} = \int \mathrm{d}\Omega_{\boldsymbol{p}'_n} \left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega_{\boldsymbol{p}'_n}\,\mathrm{d}T_n}\right)_{\mathrm{eq}\ (73)} \qquad \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_{\boldsymbol{p}'_n}} = \int \mathrm{d}T_n \left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega_{\boldsymbol{p}'_n}\,\mathrm{d}T_n}\right)_{\mathrm{eq}\ (73)}. \tag{73}$$

3. Effective field theory calculation

Recently the EFT approach has been successfully applied to the calculations of the ν–d cross sections by Butler, Chen and Kong [24] and by Ando *et al* [25]. The results of [24] (after one free parameter is fine-tuned) agree with those obtained in SNPA. This fact can be considered to give strong support for the basic soundness of the standard approach. Ando *et al* [25] carried out an EFT calculation of $\sigma_{\nu d}$ free from any adjustable parameter; we outline their approach here. The approach employs a formalism developed in the studies of the solar *pp* fusion reaction [26, 27] and the solar *Hep* process [28, 27] using heavy-baryon chiral perturbation theory (HBχPT). The transition operators are constructed from two-body irreducible diagrams according to Weinberg's counting scheme [5] and the nuclear matrix elements are evaluated by sandwiching the EFT-controlled transition operators between the nuclear wavefunctions that are obtained by solving the Schrödinger equation involving high-quality realistic nuclear interactions.

It is sufficient for practical purposes to consider up to next-to-next-to-next-to-leading order (N^3LO) in HBχPT. To this order there is only one unknown LEC, denoted by $\hat{d}_R$ in [27]. Like L_{1A} in [24], $\hat{d}_R$ controls the strength of the axial-current-four-nucleon contact coupling and subsumes short-distance physics that has been integrated out. An important point noted in [27] is that, since the tritium β-decay rate Γ_t^β is also sensitive to $\hat{d}_R$, the $\hat{d}_R$ can be determined from the well-known experimental value of Γ_t^β. This allows us to calculate $\sigma_{\nu d}$ without any parameter.

The ν–d reactions can lead to various values of the relative orbital angular momentum, L, of the final two nucleons. We concentrate here, however, on the $L = 0$ state ($^1S_0$), since it is this partial wave that involves the $\hat{d}^R$ term and since the contributions of higher partial waves are well understood in terms of the one-body operators. As in SNPA, the one-body currents can be obtained from the phenomenological form factors of the weak-nucleon current.

The two-body current operators are derived from the chiral lagrangian $\mathcal{L}$, which is expanded as $\mathcal{L} = \sum_{\bar{\nu}} \mathcal{L}_{\bar{\nu}} = \mathcal{L}_0 + \mathcal{L}_1 + \cdots$, where $\mathcal{L}_0$ and $\mathcal{L}_1$ are LO and NLO Lagrangians, respectively. Their explicit expressions are

$$\mathcal{L}_0 = \bar{N}\,[\mathrm{i}v\cdot D + 2\mathrm{i}g_A S\cdot\Delta]\,N + f_\pi^2\,\mathrm{Tr}\left(-\Delta\cdot\Delta + \frac{\chi_+}{4}\right), \tag{74}$$

$$\begin{aligned}\mathcal{L}_1 = \frac{1}{2m_N}\bar{N}\big[&(v\cdot D)^2 - D^2 + 2g_A\{v\cdot\Delta, S\cdot D\}\\ &-\left(8\hat{c}_2 - g_A^2\right)(v\cdot\Delta)^2 - 8\hat{c}_3\Delta\cdot\Delta - (4\hat{c}_4+1)[S^\mu, S^\nu][\Delta_\mu,\Delta_\nu]\\ &-2\mathrm{i}(1+\kappa_V)[S^\mu,S^\nu]f^+_{\mu\nu}\big]N + \frac{g_A}{m_N f_\pi^2}[-4\mathrm{i}\hat{d}_1\bar{N}S\cdot\Delta N\bar{N}N\\ &+2\mathrm{i}\hat{d}_2\epsilon^{abc}\epsilon_{\mu\nu\alpha\beta}v^\mu\Delta^{a,\nu}\bar{N}S^\alpha\tau^b N\bar{N}S^\beta\tau^c N],\end{aligned} \tag{75}$$

where v^μ is the velocity vector $v^\mu = (1, \vec{0})$ and S^μ is the spin operator $2S^\mu = (0, \vec{\sigma})$. The explicit expressions of the fields, D_μ, Δ_μ, $f^+_{\mu\nu}$ and χ_+, are given in [29], and f_π is the pion decay constant. The LECs, $\hat{c}_i$, have been determined at the tree level [30]

$$\hat{c}_2 = 1.67\pm0.09, \qquad \hat{c}_3 = -3.66\pm0.08, \qquad \hat{c}_4 = 2.11\pm0.08. \tag{76}$$

The two-body transition operators can be constructed from two-body irreducible Feynman diagrams up to N^3LO in Weinberg's counting rule [5]. Since the tree-level two-body operators are higher in chiral counting than the tree-level one-body operators by two orders, we can limit ourselves to tree diagrams for the two-body operators. In addition, since the g_P term is highly suppressed, we do not consider it in the two-body operators.

The diagrams for the two-body operators are given in figure 1. Since there are only nucleons and pions in $\mathcal{L}$, the effects involving exchange of heavier mesons such as the σ and ρ mesons are embedded in the contact term, diagram (f) in figure 1. The Λ is a momentum scale below which our nucleon–pion-only description is expected to be valid. To prevent the exchanged momentum from surpassing Λ, the cut-off function, $S_\Lambda(\vec{k}) = \mathrm{e}^{-\vec{k}^2/(2\Lambda^2)}$, has been introduced in calculating the Fourier transforms of the two-body transition operators [27]. As noted in [27], the short-range part of the two-body contributions can be lumped together into an axial-current-four-nucleon contact coupling term with the strength $\hat{d}^R$, where $\hat{d}^R = \hat{d}_1 + 2\hat{d}_2 + \frac{1}{3}\hat{c}_3 + \frac{2}{3}\hat{c}_4 + \frac{1}{6}$. Then, for a given value of Λ, the $\hat{d}^R$ has been determined from the empirical value of Γ_t^β. The results are [27]

$$\hat{d}^R = 1.00\pm0.07, \qquad 1.78\pm0.08, \qquad 3.90\pm0.10, \tag{77}$$

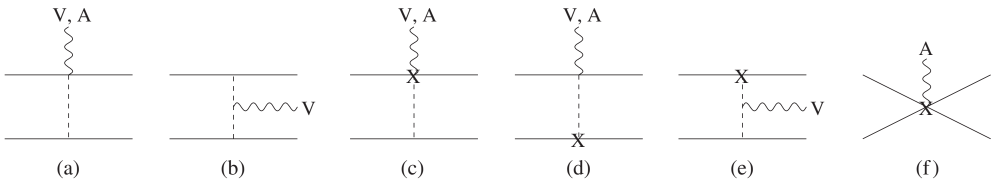

Figure 1. Diagrams for two-body current operators of order $\nu = 1$ (a, b) and $\nu = 2$ (c, d, e, f). The wavy lines with V and A attached denote the vector and axial-vector current, respectively, the dashed line denotes the pion, and vertices without (with) 'X' arise from the LO (NLO) Lagrangian.

for $\Lambda = 500$, 600, 800 MeV, respectively. The explicit expressions of the current operators for the CC reaction have been given in [29].

The total cross sections calculated using the non-relativistic formula are

$$\sigma_{\nu d}(E_\nu) = \int \mathrm{d}p \int \mathrm{d}y \frac{1}{(2\pi)^3} \frac{2p^2k'^2}{k'/E' + (k' - E_\nu y)/(2m_N)} F(Z, E') \frac{1}{3} \sum_{spin} |T|^2, \tag{78}$$

with the energy conservation relation valid up to $1/m_N$,

$$m_d + E_\nu - E' - 2m_N - \frac{1}{m_N}\left[p^2 + \frac{1}{4}\left(E_\nu^2 + k'^2 - 2E_\nu k' y\right)\right] = 0, \tag{79}$$

where E_ν (E') is the energy of the initial neutrino (final lepton), p is the magnitude of the relative three-momentum between the final two nucleons, k' is that of the outgoing lepton ($k' = |\vec{k}'|$), and y is the cosine of the angle between the incoming and outgoing leptons ($y = \hat{k}_\nu \cdot \hat{k}'$). $F(Z, E')$ is the Fermi function and m_d is the deuteron mass. The transition matrix T decomposition is $T = T_{1B} + T_{2B}$, where T_{1B} and T_{2B} are the contribution of the one-body and two-body operators, respectively. The expressions for T_{1B} and T_{2B} are presented in [31] and [25], respectively.

4. Radiative corrections

At the level of precision provided by the above-described two approaches, the radiative corrections to neutrino–deuteron interactions become relevant [32–34]. According to [34], radiative corrections increases $\sigma_{\nu d}^{CC}$ by 4% at low E_ν and by 3% at the higher end of the solar neutrino energy, while radiative corrections lead to an E_ν-independent increase of $\sigma_{\nu d}^{NC}$ by ∼1.5%. These corrections for $\sigma_{\nu d}^{CC}$ consist of the 'inner' and 'outer' corrections. The former is sensitive to hadronic dynamics but energy-independent, while the latter is largely independent of hadronic dynamics but has energy-dependence. A related question is what value should be used for the weak coupling constant. One possibility is to use the standard Fermi constant, G_F, which has been derived from μ-decay and hence does not contain any hadron-related radiative corrections. Another possibility is to employ an effective coupling constant (denoted by G_F') that includes the so-called inner radiative corrections for nuclear β-decay [35]. Since the inner corrections are established reasonably well, it seems more natural to use G_F' [35] obtained from $0^+ \to 0^+$ nuclear β-decays, since this allows one to take account of the bulk of the 'inner' corrections.

To obtain reasonable up-to-date estimates of the remaining 'outer' corrections, we may proceed as follows. For $\sigma_{\nu d}^{CC}$, we may adopt as the 'outer' correction the difference between the result of the paper [34] (4%–3%) and the estimated 'inner' corrections (2.4%). For $\sigma_{\nu d}^{NC}$, there is no 'outer' corrections at the level of precision in question. In adopting this prescription, we are leaving unaddressed a delicate issue of the possible difference between radiative corrections for the single nucleon and radiative corrections for multi-nucleon systems.

It should be mentioned that calculations of radiative corrections for neutrino–deuteron reactions can be important for deeper understanding of the strong-interaction-dependent parts of the radiative corrections for $0^+ \rightarrow 0^+$ nuclear β-decays and for neutron β-decay. The radiative corrections for *the* ν-*d* reactions have a specific feature that they belong to the axial-vector part of hadronic matrix element. Note that, for $0^+ \rightarrow 0^+$ nuclear β-decays, only the radiative corrections for the vector-current matter. Meanwhile, since neutron β-decay receives contributions from axial-vector as well as vector currents, one might think that it is sensitive to radiative corrections of both types. It turns out, however, that *effectively* neutron β-decay is only sensitive to the radiative corrections for Fermi-type transitions. There are two main reasons for this. First, the 'outer' corrections (which do not depend on nucleon structure) are naturally the identical [36, 37] for the Fermi and Gamow–Teller transitions, since they are related to the point-like (structureless) nucleon. The 'inner' corrections are different, for the two types of transitions, but they lead to the effective renormalization [36] of the ratio of axial-vector and vector nucleon coupling constants, $\lambda = f_V/g_A$. This is a crucial result since all observables in neutron β-decay can be expressed in terms of λ rather than in terms of g_A. (This is correct for lower-order approximations [36] which are sufficient for the accuracy of the existing experimental data for neutron β-decay.) It is therefore impossible to extract the non-renormalized constant, g_A, from neutron decay experiments within the current level of accuracy; i.e. until we reach the accuracy of 10^{-5} [38]. The ν–*d* reaction involves the disintegration of a deuteron from the ground-state ${}^3S_0$ $(T = 0, L = 0, S = 1)$ to continuum dominated by the state ${}^1S_0$ $(T = 1, L = 0, S = 0)$. Therefore, this process is dominated by a Gamow–Teller transition and, as a consequence, the radiative corrections are pure axial-vector corrections. This fact leads to a unique opportunity to study in details the 'inner' corrections and to obtain information about the bare axial-vector coupling constant by comparing ν–*d* reactions and neutron β-decay.

5. Summary

Within the SNPA detailed calculations of the ν–*d* cross sections have been done by Nakamura *et al* [12, 13]. As demonstrated in [19], the exchange currents in SNPA are dominated by the Δ-particle excitation diagram [19], and the reliability of estimation of this diagram depends on the precision with which the coupling constant $g_{\pi N\Delta}$ is known. In the calculations [12, 13] the $g_{\pi N\Delta}$ was fixed by fitting the experimental value of Γ^t_β, the tritium β-decay rate. Meanwhile, an EFT calculation [24] of the ν–*d* cross sections by Butler, Chen and Kong (BCK) based on the PDS scheme [40] agree with those of SNPA in the following sense. EFT calculations often involve some LECs that are not known *a priori* and hence need to be determined from empirical input. Indeed, in the calculation [24], the coefficient (denoted by $L_{1\mathrm{A}}$) of a four-nucleon axial-current coupling term appears as such an unknown parameter, although a *naturalness* argument (based on a dimensional analysis) gives an order-of-magnitude estimate, $|L_{1\mathrm{A}}| \leqslant 6\ \mathrm{fm}^3$. BCK therefore determined $L_{1\mathrm{A}}$ by requiring that the cross sections of SNPA be reproduced by their EFT calculation. With the value of $L_{1\mathrm{A}}$ fine-tuned this way, the cross sections obtained in [24] show a perfect agreement with those of the papers [12, 13] for all the four reactions (CC and NC channels for ν and $\bar{\nu}$) for the entire solar neutrino energy range, $E_\nu \lesssim 20$ MeV. Moreover, the optimal value, $L_{1\mathrm{A}} = 5.6\ \mathrm{fm}^3$, obtained in [24] is consistent with the value expected from the naturalness argument. The fact that an EFT calculation (with one parameter fine-tuned) reproduces the results of the standard approach very well strongly indicates that SNPA captures the basic physics involved.

An EFT-controlled parameter-free calculation was carried out by Ando *et al* [25], and the ν–d cross sections obtained in this calculation were found to agree within 1% with those obtained in SNPA [13]. These results show that the ν–d cross sections used in interpreting the SNO experiments [1] are reliable at the 1% precision level, and hence the evidence for neutrino oscillations reported in those experiments is robust against nuclear physics ambiguities. A recent 'self-calibrating' analysis [41] of the SNO data and Super-Kamiokande data has given a value of L_{1A} consistent with the information obtained from $\bar{\nu}$–d reaction experiments with the use of reactor-generated anti-neutrinos [42, 43]. This analysis confirms that the possible corrections to the calculated cross sections are within the claimed accuracy at the level of 1%–2%. Therefore, further detailed estimations of the radiative corrections for the ν–d reactions are highly desirable for further improvments in the descriptions of these reactions.

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