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**Parity violation in low-energy neutron-deuteron scattering**Young-Ho Song,<sup>1,\*</sup> Rimantas Lazauskas,<sup>2,†</sup> and Vladimir Gudkov<sup>1,‡</sup><sup>1</sup>*Department of Physics and Astronomy, University of South Carolina, Columbia, South Carolina 29208, USA*<sup>2</sup>*IPHC, IN2P3-CNRS/Université Louis Pasteur, BP 28, F-67037 Strasbourg Cedex 2, France*

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Parity-violating effects for low-energy elastic neutron deuteron scattering are calculated for Desplanques, Donoghue, and Holstein (DDH) and effective field theory types of weak potentials in a distorted-wave Born approximation, using realistic hadronic strong interaction wave functions, obtained by solving three-body Faddeev equations in configuration space. The resulting relation between physical observables and low-energy constants can be used to fix low-energy constants from experiments. Potential model dependencies of parity-violating effects are discussed.

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**I. INTRODUCTION**

The study of parity-violating (PV) effects in low-energy physics is a very sensitive tool to test methods of calculations of both weak and strong interactions in the Standard model. This also can be a way to search for possible manifestations of new physics resulting from deviations from unambiguous and precise calculations of PV effects and experimental measurements. However, to use this approach, it is crucial to prove that implemented theoretical techniques are sufficient to describe experimental data with high accuracy, exceeding experimental accuracy. There is a large amount of experimental data for different PV effects in nuclear physics, each of which in general agrees with theoretical predictions. However, in the past few years it became clear (see, for example, [1–4] and references therein) that the traditional DDH [5] method for calculation of PV effects cannot reliably describe the whole available set of experimental data within the same set of parameters. If this is not a manifestation of new physics, which is very unlikely given the current accuracy of experimental measurements and theoretical calculations, then this discrepancy could be blamed on systematic errors in experimental data or theoretical uncertainties in calculations of strong interactions at low energy, or it might be that the DDH approach is not adequate for the description of a set of precise experimental data because it is based on a number of models and assumptions. To resolve this discrepancy and to eliminate nuclear-model-dependent factors in calculations, it is necessary to focus on the analysis of new and existing experimental data for different PV parameters in few-body systems, where calculations of nuclear-related effects can be done with high precision. Recently, an approach based on effective field theory (EFT) has been introduced for a model-independent parametrization of PV effects (see [1,4] and references therein), and some calculations for two-body systems have been done [6]. The power of the EFT approach for parametrization of all PV effects in terms of

a small number of constants could be utilized if we can analyze a large enough number of PV effects to be able to constrain all free parameters of the theory [usually called low-energy constants (LECs)]. Thus, one can guarantee the adequate description (parametrization) of the strong interaction hadronic parts and weak interaction constants for symmetry-violating observables. Unfortunately, the number of experimentally measured (and independent in terms of unknown LECs) PV effects in two-body systems is not enough to constrain all LECs. In spite of the fact that five independent observable parameters in a two-body system could fix five unknown PV LECs [7–10], it is impossible to measure all of them using existing experimental techniques. Therefore, one has to incorporate into analysis few-body systems and even heavier nuclei, the latter of which are actually preferable from the experimental point of view, because, as a rule, the measured effects in nuclei are much larger than in a nucleon-nucleon system due to nuclear enhancement factors [11–13].

The natural and unambiguous way to verify the applicability of the EFT for the calculation of symmetry-violating effects in nuclear reactions requires development of a regular and self-consistent approach for calculation of PV amplitudes in three-body (few-body) systems [14], with a hope of extending the formalism for the description of many-body systems. This systematic approach for the solution of the three-body PV scattering problem in the EFT framework [14] requires additional numerical efforts and will be presented elsewhere. As a first step for the clarification of the possible difference in contributions to PV effects from DDH and EFT-type potentials, one can use a “hybrid” method (similar to the method used in [15]) for the simplest process of neutron-deuteron scattering. We calculate three-body wave functions with realistic Hamiltonians of the strong interaction using the exact Faddeev equations in configuration space, and then we calculate PV effects in the first order of perturbation with the DDH potential and potentials derived in the EFT formalism. In the next section, we present our formalism for the calculation PV effects for elastic neutron-deuteron scattering with different sets of nucleon weak potentials and with DDH and weak potentials obtained from pionless and pionful EFTs.

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Then, we present results of numerical calculations and provide a discussion.

## II. FORMALISM

We treat weak nucleon interactions as a perturbation and calculate three-body wave functions exactly using the Faddeev equations with phenomenological potentials for strong interactions. A similar hybrid approach has been successfully applied to the weak and electromagnetic processes involving three-body and four-body hadronic systems [16–21]. We consider three types of parity-violating potentials. The first one is the standard DDH potential, which is based on the meson exchange mechanism of nucleon-nucleon interactions. The second and third potentials are derived from pionless and pionful versions of effective field theory with parity-violating hadronic interactions. Instead of calculating parity-violating amplitudes by summing PV diagrams in EFT, we use these potentials to calculate PV effects. This is a simplification, which we call a “hybrid” approach.

### A. Observables

Since PV effects in the neutron-deuteron system are very small, we consider only coherent processes which are related to the propagation of neutrons through the unpolarized deuteron target and, therefore, do not have an additional suppression in low-energy region. Then, two PV observable parameters are the angle  $\phi$  of rotation of neutron polarization around the neutron momentum and the relative difference of total cross sections  $P = (\sigma_+ - \sigma_-)/(\sigma_+ + \sigma_-)$  for neutrons with opposite helicities. The value of the angle of neutron spin rotation per unit length of the target sample can be expressed in terms of elastic scattering amplitudes at zero angle for opposite helicities  $f_+$  and  $f_-$  as

$$\frac{d\phi}{dz} = -\frac{2\pi N}{p} \text{Re}(f_+ - f_-), \quad (1)$$

where  $N$  is a number of target nuclei per unit volume and  $p$  is a relative neutron momentum. Using an optical theorem, one can write the relative difference of total cross sections  $P$  in terms of these amplitudes as

$$P = \frac{\text{Im}(f_+ - f_-)}{\text{Im}(f_+ + f_-)}. \quad (2)$$

It is convenient to represent the amplitudes in terms of a matrix  $\hat{R}$ , which is related to the scattering matrix  $\hat{S}$  as  $\hat{R} = \hat{1} - \hat{S}$ . With partial wave decomposition for the case of neutron-deuteron scattering,

$$|\mathbf{p}, m_n, m_d\rangle = \sum_{l_y l_y^z} \sum_{SM, J J^z} |\mathbf{p}, (l_y \mathcal{S}) J J^z\rangle \langle J J^z | l_y l_y^z, SM\rangle \times \left\langle SM \left| \frac{1}{2} m_n, 1 m_d \right. \right\rangle Y_{l_y l_y^z}^*(\hat{p}), \quad (3)$$

where  $l_y$  is an orbital angular momentum between the neutron and the deuteron,  $\mathcal{S}$  is a sum of neutron spin and deuteron total angular momentum, and  $J$  is the total angular momentum of the neutron-deuteron system, these equations can be written at

low energies as

$$\frac{1}{N} \frac{d\phi}{dz} = \frac{2\pi}{9p^2} \text{Im} \left[ R_{1\frac{1}{2}, 0\frac{1}{2}}^{\frac{1}{2}} + R_{0\frac{1}{2}, 1\frac{1}{2}}^{\frac{1}{2}} - 2\sqrt{2} R_{1\frac{3}{2}, 0\frac{1}{2}}^{\frac{1}{2}} - 2\sqrt{2} R_{0\frac{1}{2}, 1\frac{3}{2}}^{\frac{1}{2}} + 4R_{1\frac{1}{2}, 0\frac{3}{2}}^{\frac{3}{2}} + 4R_{0\frac{3}{2}, 1\frac{1}{2}}^{\frac{3}{2}} - 2\sqrt{5} R_{1\frac{3}{2}, 0\frac{3}{2}}^{\frac{3}{2}} - 2\sqrt{5} R_{0\frac{3}{2}, 1\frac{3}{2}}^{\frac{3}{2}} \right] \quad (4)$$

and

$$P = \frac{1}{3} \text{Re} \left[ R_{1\frac{1}{2}, 0\frac{1}{2}}^{\frac{1}{2}} + R_{0\frac{1}{2}, 1\frac{1}{2}}^{\frac{1}{2}} - 2\sqrt{2} R_{1\frac{3}{2}, 0\frac{1}{2}}^{\frac{1}{2}} - 2\sqrt{2} R_{0\frac{1}{2}, 1\frac{3}{2}}^{\frac{1}{2}} + 4R_{1\frac{1}{2}, 0\frac{3}{2}}^{\frac{3}{2}} + 4R_{0\frac{3}{2}, 1\frac{1}{2}}^{\frac{3}{2}} - 2\sqrt{5} R_{1\frac{3}{2}, 0\frac{3}{2}}^{\frac{3}{2}} - 2\sqrt{5} R_{0\frac{3}{2}, 1\frac{3}{2}}^{\frac{3}{2}} \right] / \text{Re} \left[ R_{0\frac{1}{2}, 0\frac{1}{2}}^{\frac{1}{2}} + 2R_{0\frac{3}{2}, 0\frac{3}{2}}^{\frac{3}{2}} \right], \quad (5)$$

where  $R_{l'S', lS}^J = \langle l'S' | R^J | lS \rangle$ , and unprimed and primed parameters correspond to initial and final states, respectively. Since we are interested in low-energy neutron scattering, it would be sufficient to include only  $s$ - and  $p$ -wave contributions to parity-violating amplitudes; for the total cross section (the denominator in the last equation), we keep only dominant contributions from  $s$ -wave neutrons. It should be noted that time-reversal invariance leads to the relation  $\langle l'S' | R^J | lS \rangle = \langle 0S' | R^J | 1S' \rangle$  between matrix elements; therefore, only half of parity-violating amplitudes are independent.

The nucleon-nucleon interaction can be written as a sum  $V = V_{\text{pc}} + V_{\text{pv}}$  of the parity-conserving ( $V_{\text{pc}}$ ) and weak parity-violating ( $V_{\text{pv}}$ ) terms. Due to the weakness of the parity-violating interaction, one can use the distorted-wave Born approximation (DWBA) to calculate PV amplitudes with a high level of accuracy as

$$R_{l_y' S', l_y S}^J \simeq 4i^{-l_y' + l_y + 1} \mu p_{\text{pc}}^{(-)} \langle \Psi, (l_y' S') J J^z | V_{\text{pv}} | \Psi, (l_y S) J J^z \rangle_{\text{pc}}^{(+)}, \quad (6)$$

where  $\mu$  is a neutron-deuteron reduced mass and  $|\Psi, (l_y' S') J J^z \rangle_{\text{pc}}^{(\pm)}$  are solutions of the three-body Faddeev equations in configuration space for the parity-conserving strong interaction Hamiltonian, defined by  $V_{\text{pc}}$  and normalized as described in Sec. II C. The factor  $i^{-l_y' + l_y}$  in this expression is introduced to match the  $R$ -matrix definition in the modified spherical harmonics convention [22] with the wave functions which are calculated in this paper using the spherical harmonics convention.

In the rest of the paper, we use only wave functions calculated for parity-conserving potentials and, therefore, will omit the subscript PC.

As will be explained in Sec. II C, we use a  $jj$ -coupling scheme (with basis states  $|l_y j_y\rangle$ ) when solving the Faddeev equations. One can transform  $jj$ -basis states into the  $l_y \mathcal{S}$  basis by means of

$$|[l_y \otimes (s_k \otimes j_x)]_{J J^z}\rangle = \sum_{j_y} |[j_x \otimes (l_y \otimes s_k)]_{j_y J J^z}\rangle \times (-1)^{j_x + j_y - J} (-1)^{l_y + s_k + j_x + J} \times [(2j_y + 1)(2S + 1)]^{\frac{1}{2}} \begin{Bmatrix} l_y & s_k & j_y \\ j_x & J & S \end{Bmatrix}. \quad (7)$$

One interesting observation is that the neutron spin rotation, as well as parameter  $P$ , in the  $|l_y j_y\rangle$  basis involves potential matrix elements only between  $j_y = \frac{1}{2}$  states.

It should be noted that at low energy  $\text{Im}(R_{l_y S', l_y S}^J) \sim p^{l_y + l_y + 1}$ , and thus the expression (4) for the angle  $\phi$  of neutron spin rotation is finite and well defined in the zero-energy limit of the  $n$ - $d$  scattering. Numerically, it is calculated by evaluating the expression  $\text{Im}(R_{l_y S', l_y S}^J)/p^{l_y + l_y + 1}$  at zero energy. On the other hand,  $\text{Re}(R_{l_y S', l_y S}^J) \sim p \text{Im}(R_{l_y S', l_y S}^J)$  at low energy, and thus the real part of this quantity vanishes in the zero-energy limit. Therefore, the parameter  $P$  is calculated at 15-keV neutron kinetic energy in the laboratory system, where both imaginary and real parts of the  $R$ -matrix elements become comparable in magnitude and thus can be discerned numerically.

### B. The parity-violating potentials

To understand the possible difference in the description of parity-violating effects by DDH and EFT-type potentials, we compare calculations with the DDH potential [5] and two different choices of EFT potentials: the potential derived from the pionless EFT Lagrangian [1] and the potential derived from the pionful EFT Lagrangian [1]. It was shown [15] that all these three potentials can be expanded in terms of a set of  $O_{ij}^{(n)}$  operators as

$$v_{ij}^\alpha = \sum_n c_n^\alpha O_{ij}^{(n)},$$

$\alpha = \text{DDH or pionless EFT or pionful EFT}$  (8)

TABLE I. Parameters and operators of parity-violating potentials.  $\pi NN$  coupling  $g_{\pi NN}$  can be represented by  $g_A$  by using the Goldberger-Treiman relation,  $g_\pi = g_A m_N / F_\pi$  with  $F_\pi = 92.4$  MeV.  $T_{ij} \equiv (3\tau_i^z \tau_j^z - \tau_i \cdot \tau_j)$ . The scalar function  $\tilde{L}_\Lambda(r) \equiv 3L_\Lambda(r) - H_\Lambda(r)$ .

$n$	$c_n^{\text{DDH}}$	$f_n^{\text{DDH}}(r)$	$c_n^\pi$	$f_n^\pi(r)$	$c_n^\pi$	$f_n^\pi(r)$	$O_{ij}^{(n)}$
1	$+\frac{g_\pi}{2\sqrt{2}m_N} h_\pi^1$	$f_\pi(r)$	$\frac{2\mu^2}{\Lambda_\chi^3} C_6^\pi$	$f_\mu^\pi(r)$	$+\frac{g_\pi}{2\sqrt{2}m_N} h_\pi^1$	$f_\pi(r)$	$(\tau_i \times \tau_j)^z (\sigma_i + \sigma_j) \cdot \mathbf{X}_{ij,-}^{(1)}$
2	$-\frac{g_\rho}{m_N} h_\rho^0$	$f_\rho(r)$	0	0	0	0	$(\tau_i \cdot \tau_j) (\sigma_i - \sigma_j) \cdot \mathbf{X}_{ij,+}^{(2)}$
3	$-\frac{g_\rho(1+\kappa_\rho)}{m_N} h_\rho^0$	$f_\rho(r)$	0	0	0	0	$(\tau_i \cdot \tau_j) (\sigma_i \times \sigma_j) \cdot \mathbf{X}_{ij,-}^{(3)}$
4	$-\frac{g_\rho}{2m_N} h_\rho^1$	$f_\rho(r)$	$\frac{\mu^2}{\Lambda_\chi^3} (C_2^\pi + C_4^\pi)$	$f_\mu^\pi(r)$	$\frac{\Lambda_\chi^2}{\Lambda_\chi^3} (C_2^\pi + C_4^\pi)$	$f_\Lambda(r)$	$(\tau_i + \tau_j)^z (\sigma_i - \sigma_j) \cdot \mathbf{X}_{ij,+}^{(4)}$
5	$-\frac{g_\rho(1+\kappa_\rho)}{2m_N} h_\rho^1$	$f_\rho(r)$	0	0	$\frac{2\sqrt{2}\pi g_A^3 \Lambda^2}{\Lambda_\chi^3} h_\pi^1$	$L_\Lambda(r)$	$(\tau_i + \tau_j)^z (\sigma_i \times \sigma_j) \cdot \mathbf{X}_{ij,-}^{(5)}$
6	$-\frac{g_\rho}{2\sqrt{6}m_N} h_\rho^2$	$f_\rho(r)$	$-\frac{2\mu^2}{\Lambda_\chi^3} C_5^\pi$	$f_\mu^\pi(r)$	$-\frac{2\Lambda_\chi^2}{\Lambda_\chi^3} C_5^\pi$	$f_\Lambda(r)$	$T_{ij} (\sigma_i - \sigma_j) \cdot \mathbf{X}_{ij,+}^{(6)}$
7	$-\frac{g_\rho(1+\kappa_\rho)}{2\sqrt{6}m_N} h_\rho^2$	$f_\rho(r)$	0	0	0	0	$T_{ij} (\sigma_i \times \sigma_j) \cdot \mathbf{X}_{ij,-}^{(7)}$
8	$-\frac{g_\omega}{m_N} h_\omega^0$	$f_\omega(r)$	$\frac{2\mu^2}{\Lambda_\chi^3} C_1^\pi$	$f_\mu^\pi(r)$	$\frac{2\Lambda_\chi^2}{\Lambda_\chi^3} C_1^\pi$	$f_\Lambda(r)$	$(\sigma_i - \sigma_j) \cdot \mathbf{X}_{ij,+}^{(8)}$
9	$-\frac{g_\omega(1+\kappa_\omega)}{m_N} h_\omega^0$	$f_\omega(r)$	$\frac{2\mu^2}{\Lambda_\chi^3} \tilde{C}_1^\pi$	$f_\mu^\pi(r)$	$\frac{2\Lambda_\chi^2}{\Lambda_\chi^3} \tilde{C}_1^\pi$	$f_\Lambda(r)$	$(\sigma_i \times \sigma_j) \cdot \mathbf{X}_{ij,-}^{(9)}$
10	$-\frac{g_\omega}{2m_N} h_\omega^1$	$f_\omega(r)$	0	0	0	0	$(\tau_i + \tau_j)^z (\sigma_i - \sigma_j) \cdot \mathbf{X}_{ij,+}^{(10)}$
11	$-\frac{g_\omega(1+\kappa_\omega)}{2m_N} h_\omega^1$	$f_\omega(r)$	0	0	0	0	$(\tau_i + \tau_j)^z (\sigma_i \times \sigma_j) \cdot \mathbf{X}_{ij,-}^{(11)}$
12	$-\frac{g_\omega h_\omega^1 - g_\rho h_\rho^1}{2m_N}$	$f_\rho(r)$	0	0	0	0	$(\tau_i - \tau_j)^z (\sigma_i + \sigma_j) \cdot \mathbf{X}_{ij,+}^{(12)}$
13	$-\frac{g_\rho}{2m_N} h_\rho^1$	$f_\rho(r)$	0	0	$-\frac{\sqrt{2}\pi g_A \Lambda^2}{\Lambda_\chi^3} h_\pi^1$	$L_\Lambda(r)$	$(\tau_i \times \tau_j)^z (\sigma_i + \sigma_j) \cdot \mathbf{X}_{ij,-}^{(13)}$
14	0	0	0	0	$\frac{2\Lambda_\chi^2}{\Lambda_\chi^3} C_6^\pi$	$f_\Lambda(r)$	$(\tau_i \times \tau_j)^z (\sigma_i + \sigma_j) \cdot \mathbf{X}_{ij,-}^{(14)}$
15	0	0	0	0	$\frac{\sqrt{2}\pi g_A^3 \Lambda^2}{\Lambda_\chi^3} h_\pi^1$	$\tilde{L}_\Lambda(r)$	$(\tau_i \times \tau_j)^z (\sigma_i + \sigma_j) \cdot \mathbf{X}_{ij,-}^{(15)}$

with parameters  $c_n^\alpha$  and operators  $O_{ij}^{(n)}$  given in Table I.

One can see that operators  $O_{ij}^{(n)}$  are products of isospin, spin, and vector operators  $\mathbf{X}_{ij,\pm}^{(n)}$  defined as

$$\begin{aligned} \mathbf{X}_{ij,+}^{(n)} &\equiv [\mathbf{p}_{ij}, f_n(r_{ij})]_+, \\ \mathbf{X}_{ij,-}^{(n)} &\equiv i[\mathbf{p}_{ij}, f_n(r_{ij})]_-, \end{aligned} \quad (9)$$

where  $\mathbf{p}_{ij} \equiv \frac{(\mathbf{p}_i - \mathbf{p}_j)}{2}$ .

For the DDH potential, radial functions  $f_x(r)$ ,  $x = \pi, \rho$ , and  $\omega$  are modified Yukawa functions,

$$f_x(r) = \frac{1}{4\pi r} \left\{ e^{-m_x r} - e^{-\Lambda_x r} \left[ 1 + \frac{\Lambda_x r}{2} \left( 1 - \frac{m_x^2}{\Lambda_x^2} \right) \right] \right\}. \quad (10)$$

For pionless EFT ( $\not{\pi}$ EFT), the functions  $f_n(r)$  are described by the single function  $f_\mu(r)$ ,

$$f_\mu(r) = \frac{1}{4\pi r} e^{-\mu r}, \quad (11)$$

with  $\mu \simeq m_\pi$ .

For the case of the pionful EFT model ( $\pi$ EFT), there are long-range interactions from one-pion exchange ( $V_{-1,\text{LR}}$ ) and from their corrections ( $V_{1,\text{LR}}$ ), middle-range interactions due to two-pion exchange ( $V_{1,\text{MR}}$ ), and short-range interactions ( $V_{1,\text{SR}}$ ) due to nucleon contact terms. The radial part of the leading term of the long-range one-pion exchange,  $V_{-1,\text{LR}}$ , is described by the function  $f_\pi(r)$ . Since the one-pion exchange contribution is dominated by the long-range part, we do not use a regulator for it; i.e., we assume that the long-range

interactions have the same radial functions  $f_\pi(r)$  as the DDH potential with infinite cutoff. The short-range interaction  $V_{1,\text{SR}}$  in pionful theory has the same structure as for pionless EFT; however, in spite of the structural similarity, their meanings are rather different. One can ignore the higher order corrections of long-range interactions,  $V_{1,\text{LR}}$ , because they can either be absorbed by renormalization of low-energy constants [6] or suppressed. The middle-range interactions  $V_{1,\text{MR}}$  are described by functions  $L(q)$  and  $H(q)$  in momentum space:

$$L(q) \equiv \frac{\sqrt{4m_\pi^2 + \mathbf{q}^2}}{|\mathbf{q}|} \ln \left( \frac{\sqrt{4m_\pi^2 + \mathbf{q}^2} + |\mathbf{q}|}{2m_\pi} \right),$$

$$H(q) \equiv \frac{4m_\pi^2}{4m_\pi^2 + \mathbf{q}^2} L(q), \quad (12)$$

where  $q^\mu = (q^0, \mathbf{q}) = p_1^\mu - p_1'^\mu = p_2'^\mu - p_2^\mu$ . To calculate the two-pion exchange functions (divergent at large  $q$ ) in the spatial representation, we use regulators  $\frac{(\Lambda^2 - 4m_\pi^2)^2}{(\Lambda^2 + \mathbf{q}^2)^2}$ . For the sake of simplicity, we use only one cutoff parameter with the same regulator, for both middle-range and for short-range interactions. Then, one can write

$$\{L_\Lambda(r), H_\Lambda(r), f_\Lambda(r)\} = \frac{1}{\Lambda^2} \int \frac{d^3q}{(2\pi)^3} e^{-iq \cdot r} \frac{(\Lambda^2 - 4m_\pi^2)^2}{(\Lambda^2 + \mathbf{q}^2)^2} \times \{L(q), H(q), 1\}. \quad (13)$$

In the given representation, the coefficients  $c_n^\alpha$  have femtometer dimension and scalar functions  $f_n^\alpha(r)$  are in  $\text{fm}^{-1}$ . One can see that only the new operator structure, which is not included in the DDH or pionless EFT, is due to  $V_{1,\text{LR}}^{\text{PV}}$ . Therefore, pionful EFT does not introduce a new operator structure, provided we neglect the  $V_{1,\text{LR}}^{\text{PV}}$  term [6,23].

To see the sensitivity to the choice of cutoff for parity-violating potentials, we used two sets of cutoff parameters for each model; these are listed in Table II.

Using these three potentials, one can represent parity-violating amplitudes as a linear expansion in terms of a given set of matrix elements for corresponding operators  $O_{ij}^{(n)}$ . Thus, the angle of neutron spin rotation can be written as

$$\frac{1}{N} \frac{d\phi}{dz} = \sum_{n=1}^{13} c_n^\alpha I_n^\alpha, \quad (14)$$

and the parameter  $P$  as

$$P = \sum_{n=1}^{13} c_n^\alpha \tilde{I}_n^\alpha, \quad (15)$$

TABLE II. Parameter of parity-violating potentials in GeV. We used meson masses  $m_\pi$ ,  $m_\rho$ , and  $m_\omega$  of, respectively, 0.138, 0.771, and 0.783 in the DDH potential.

	$\Lambda_\pi$	$\Lambda_\rho$	$\Lambda_\omega$		$\mu$	$\Lambda$
DDH-I	1.72	1.31	1.50	$\not\propto$ EFT-I	0.138	$\pi$ EFT-I
DDH-II	$\infty$	$\infty$	$\infty$	$\not\propto$ EFT-II	1.0	$\pi$ EFT-II

in terms of coefficients  $I_n^\alpha$  and  $\tilde{I}_n^\alpha$  with  $\alpha = \text{DDH-I,II}, \not\propto\text{EFT-I,II}, \pi\text{EFT-I,II}$  for different potentials and cutoff parameters.

### C. Faddeev wave function

To obtain three-body wave functions for neutron-deuteron scattering with parity-conserving interactions, we solve the Faddeev equations (also often called the Kowalski-Noyes equations) in configuration space [24,25]. For isospin-invariant interactions (with nucleon masses fixed to  $\hbar^2/m = 41.471 \text{ MeV fm}$ ), the three Faddeev equations become formally identical, having the form

$$(E - H_0 - V_{ij})\psi_k = V_{ij}(\psi_i + \psi_j), \quad (16)$$

where  $(ijk)$  are particle indices,  $H_0$  is the kinetic energy operator,  $V_{ij}$  is the two-body force between particles  $i$ , and  $j$ , and  $\psi_k = \psi_{ij,k}$  is the Faddeev component.

The wave function in the Faddeev formalism is the sum of three Faddeev components,

$$\Psi(\mathbf{x}, \mathbf{y}) = \psi_1(\mathbf{x}_1, \mathbf{y}_1) + \psi_2(\mathbf{x}_2, \mathbf{y}_2) + \psi_3(\mathbf{x}_3, \mathbf{y}_3). \quad (17)$$

Using relative Jacobi coordinates  $\mathbf{x}_k = (\mathbf{r}_j - \mathbf{r}_i)$  and  $\mathbf{y}_k = \frac{2}{\sqrt{3}}(\mathbf{r}_k - \frac{\mathbf{r}_i + \mathbf{r}_j}{2})$ , one can expand these Faddeev components in a bipolar harmonic basis:

$$\psi_k = \sum_{\alpha} \frac{F_{\alpha}(\mathbf{x}_k, \mathbf{y}_k)}{x_k y_k} |(l_x s_i s_j)_{s_x})_{j_x} (l_y s_k)_{j_y})_{JM} \otimes |(t_i t_j)_{t_x} t_k)_{TT_z}, \quad (18)$$

where index  $\alpha$  represents all allowed combinations of the quantum numbers presented in the brackets:  $l_x$  and  $l_y$  are the partial angular momenta associated with respective Jacobi coordinates, and  $s_i$  and  $t_i$  are the spins and isospins of the individual particles. The functions  $F_{\alpha}(x_k, y_k)$  are called partial Faddeev amplitudes. It should be noted that the total angular momentum  $J$  as well as its projection  $M$  are conserved, but the total isospin  $T$  of the system is not conserved due to the presence of charge-dependent terms in nuclear interactions.

The boundary conditions for Eq. (16) can be written in the Dirichlet form. Thus, the Faddeev amplitudes satisfy the regularity conditions:

$$F_{\alpha}(0, y_k) = F_{\alpha}(x_k, 0) = 0. \quad (19)$$

For neutron-deuteron scattering with energies below the breakup threshold, the Faddeev components vanish for  $\mathbf{x}_k \rightarrow \infty$ . If  $\mathbf{y}_k \rightarrow \infty$ , then interactions between particle  $k$  and cluster  $ij$  are negligible, and the Faddeev components  $\psi_i$  and  $\psi_j$  vanish. Then, for the component  $\psi_k$ , which describes the plane wave of the particle  $k$  with respect to the bound particle pair  $ij$ ,

$$\begin{aligned} & \lim_{y_k \rightarrow \infty} \psi_k(\mathbf{x}_k, \mathbf{y}_k)_{l_n j_n} \\ &= \frac{1}{\sqrt{3}} \sum_{j'_i l'_i} \{ \phi_d(\mathbf{x}_k) \}_{j_d} \otimes \{ Y_{l'_i}(\hat{\mathbf{y}}_k) \otimes s_k \}_{j'_i} \}_{JM} \\ & \otimes |(t_i t_j)_{t_d} t_k)_{\frac{1}{2}, -\frac{1}{2}} \left[ \delta_{l'_i j'_i, l_n j_n} h_{l'_i}^-(prnd) \right. \\ & \left. - S_{l'_i j'_i, l_n j_n} h_{l'_i}^+(prnd) \right], \end{aligned} \quad (20)$$

where the deuteron, being formed from nucleons  $i$  and  $j$ , has quantum numbers  $s_d = 1$ ,  $j_d = 1$ , and  $t_d = 0$ , and its wave function  $\phi_d(\mathbf{x}_k)$  is normalized to unity. Here,  $r_{nd} = (\sqrt{3}/2)y_k$  is the relative distance between neutron and deuteron target, and  $h_n^\pm$  are the spherical Hankel functions. The expression (20) is normalized to satisfy a condition of unit flux for the  $n$ - $d$  scattering wave function.

For the cases where an Urbana type three-nucleon interaction (TNI) is included, we modify the Faddeev equation (16) into

$$(E - H_0 - V_{ij})\psi_k = V_{ij}(\psi_i + \psi_j) + \frac{1}{2}(V_{jk}^i + V_{ki}^j)\Psi \quad (21)$$

by noting that the TNI among particles  $ijk$  can be written as the sum of three terms:  $V_{ijk} = V_{ij}^k + V_{jk}^i + V_{ki}^j$ .

#### D. Evaluation of matrix elements

Due to antisymmetry of the total wave function in the isospin basis, one has  $\langle \Psi | V_{12} + V_{23} + V_{31} | \Psi \rangle = 3 \langle \Psi | V_{ij} | \Psi \rangle$  for any pair  $i \neq j$ .

Using decomposition of momentum  $\mathbf{p}$ ,

$$\mathbf{p} = -i\nabla_x = -i \left( \hat{x} \frac{\partial}{\partial x} + \frac{1}{x} \hat{\nabla}_\Omega \right), \quad (22)$$

we can represent the general matrix elements of the local two-body parity-violating potential operators as

$$\begin{aligned} {}^{(-)}\langle \Psi_f | O | \Psi_i \rangle^{(+)} &= \left( \frac{\sqrt{3}}{2} \right)^3 \sum_{\alpha\beta} \left[ \int dx x^2 dy y^2 \left( \frac{\tilde{F}_{f,\alpha}^{(+)}(x, y)}{xy} \right) \right. \\ &\quad \left. \times \hat{X}(x) \left( \frac{\tilde{F}_{i,\beta}^{(+)}(x, y)}{xy} \right) \right] \langle \alpha | \hat{O}(\hat{x}) | \beta \rangle, \end{aligned} \quad (23)$$

where  $(\pm)$  means outgoing and incoming boundary conditions and  $\hat{X}(x)$  is the derivative of the scalar function or the derivative of the wave function with respect to  $x$ . [Note that we have used the fact that  $(\tilde{F}^{(-)})^* = \tilde{F}^{(+)}$ .] The partial amplitudes  $\tilde{F}_{i(f),\alpha}(x, y)$  represent the total systems wave function in one selected basis set among three possible angular momentum

coupling sequences for three particle angular momenta:

$$\begin{aligned} \Psi_{i(f)}(x, y) &= \sum_{\alpha} \frac{\tilde{F}_{i(f),\alpha}(x, y)}{xy} |(l_x(s_i s_j)_{s_x})_{j_x} (l_y s_k)_{j_y} \rangle_{JM} \\ &\quad \otimes |(t_i t_j)_{t_x t_k} \rangle_{TT_z}. \end{aligned} \quad (24)$$

The ‘‘angular’’ part of the matrix element is

$$\langle \alpha | \hat{O}(\hat{x}) | \beta \rangle \equiv \int d\hat{x} \int d\hat{y} \mathcal{Y}_\alpha^\dagger(\hat{x}, \hat{y}) \hat{O}(\hat{x}) \mathcal{Y}_\beta(\hat{x}, \hat{y}), \quad (25)$$

where  $\mathcal{Y}_\alpha(\hat{x}, \hat{y})$  is a tensor bipolar spherical harmonic with quantum number  $\alpha$ . One can see that operators for ‘‘angular’’ matrix elements have the following structure:

$$\hat{O}(\hat{x}) = (\tau_2 \odot \tau_3)(\sigma_2 \otimes \sigma_3) \cdot \hat{x}, \text{ or } (\tau_2 \odot \tau_3)(\sigma_2 \otimes \sigma_3) \cdot \nabla_\Omega, \quad (26)$$

where  $\odot, \otimes = \pm, \times$ . The explicit values of these matrix elements are summarized in the Appendix.

### III. RESULTS AND DISCUSSION

As mentioned in the previous section, because of the low-energy property of  $R_{\alpha'\alpha}^J$ , it is convenient to present results for elements  $R_{l'_y, l_y}^J$  in terms of a ratio,

$$\begin{aligned} &\frac{R_{\alpha'\alpha}^J(p)}{4\mu i^{-l'_y+l_y+1} p^{l'_y+l_y+1}} \\ &= \frac{1}{p^{l'_y+l_y}} {}^{(-)}\langle \Psi, (l'_y S') J J_z | V_n^{\text{PV}} | \Psi, (l_y S) J J_z \rangle^{(+)}. \end{aligned} \quad (27)$$

For the case of parity violation, we fix  $l'_y = 1$  and  $l_y = 0$ . To obtain the observable parameters when neutron energies are larger than thermal ones (which correspond to the zero-energy limit for neutron spin rotation), one can use a simple extrapolation based on this representation with a good accuracy up to hundreds of keV.

The contributions to parity-violating matrix elements  $\frac{2}{\pi} \frac{1}{c_n} \text{Im}[R_{\alpha'\alpha}^J(p)/4\mu p^2]$  from different terms of parity-violating potentials (see Table I) are presented in Table III. These matrix elements were calculated using strong AV18 +

TABLE III. Contributions to  $\frac{2}{\pi} \text{Im}[R_{1S',0J}^J(p)/4\mu p^2]$  at very low energy in  $\text{fm}^2$ . We chose AV18 + UIX as the strong potential and DDH-II as the parity-violating potential. Matrix elements of  $n = 6, 7$  are zero due to isospin structure.

$n$	$S' = \frac{1}{2}, J = \frac{1}{2}$	$S' = \frac{3}{2}, J = \frac{1}{2}$	$S' = \frac{1}{2}, J = \frac{3}{2}$	$S' = \frac{3}{2}, J = \frac{3}{2}$
1	0.253	0.131	$-0.151 \times 10^{-1}$	-0.522
2	$-0.182 \times 10^{-1}$	$-0.105 \times 10^{-1}$	$0.882 \times 10^{-2}$	$0.480 \times 10^{-3}$
3	$0.339 \times 10^{-2}$	$0.231 \times 10^{-1}$	$-0.428 \times 10^{-2}$	$-0.284 \times 10^{-3}$
4	$0.410 \times 10^{-2}$	$-0.154 \times 10^{-1}$	$0.221 \times 10^{-3}$	$0.797 \times 10^{-4}$
5	$0.475 \times 10^{-2}$	$-0.178 \times 10^{-1}$	$0.313 \times 10^{-3}$	$0.664 \times 10^{-4}$
8	$0.190 \times 10^{-2}$	$0.180 \times 10^{-1}$	$-0.301 \times 10^{-2}$	$-0.228 \times 10^{-3}$
9	$-0.562 \times 10^{-2}$	$0.960 \times 10^{-2}$	$0.107 \times 10^{-2}$	$0.278 \times 10^{-4}$
10	$0.388 \times 10^{-2}$	$-0.146 \times 10^{-1}$	$0.209 \times 10^{-3}$	$0.755 \times 10^{-4}$
11	$0.453 \times 10^{-2}$	$-0.170 \times 10^{-1}$	$0.298 \times 10^{-3}$	$0.631 \times 10^{-4}$
12	$0.452 \times 10^{-2}$	$0.165 \times 10^{-3}$	$-0.223 \times 10^{-3}$	$-0.105 \times 10^{-1}$
13	$0.725 \times 10^{-2}$	$0.113 \times 10^{-2}$	$-0.377 \times 10^{-3}$	$-0.175 \times 10^{-1}$

TABLE IV. Coefficients  $I_n^{\text{DDH}}$  for AV18 and AV18 + UX strong potentials and DDH-I and DDH-II parameter sets for parity-violating potentials.  $I_{6,7}^{\text{DDH}} = 0$ .

$n$	DDH-I/AV18	DDH-I/AV18 + UX	DDH-II/AV18	DDH-II/AV18 + UX
1	$0.612 \times 10^2$	$0.596 \times 10^2$	$0.616 \times 10^2$	$0.600 \times 10^2$
2	0.666	0.726	$0.114 \times 10^1$	$0.124 \times 10^1$
3	$-0.130 \times 10^1$	$-0.133 \times 10^1$	$-0.212 \times 10^1$	$-0.217 \times 10^1$
4	0.911	0.934	$0.131 \times 10^1$	$0.134 \times 10^1$
5	0.980	0.992	$0.153 \times 10^1$	$0.156 \times 10^1$
8	$-0.125 \times 10^1$	$-0.130 \times 10^1$	$-0.160 \times 10^1$	$-0.167 \times 10^1$
9	-0.615	-0.622	-0.786	-0.796
10	0.998	$0.102 \times 10^1$	$0.124 \times 10^1$	$0.127 \times 10^1$
11	$0.111 \times 10^1$	$0.113 \times 10^1$	$0.146 \times 10^1$	$0.149 \times 10^1$
12	0.991	0.983	$0.141 \times 10^1$	$0.140 \times 10^1$
13	$0.144 \times 10^1$	$0.144 \times 10^1$	$0.226 \times 10^1$	$0.225 \times 10^1$

UX and weak DDH-II parity-violating potentials for the case of low neutron energies (up to thermal ones). From this table, one can see that the main contribution to PV effects comes from the  $J = 3/2$  channel for the “best values” of DDH coupling constants.

Our results for the angle of neutron spin rotation for DDH, pionless EFT, and pionful EFT weak interaction potentials with different sets of parameters are summarized in Tables IV, V, and VI. For these calculations, we used two types of strong interacting potentials: the Argonne two-nucleon interaction AV18 and AV18 including the Urbana IX three-nucleon interaction, AV18 + UX. One can see that these results are practically independent of the choice of strong interaction potential. Also, it is clear that the matrix element related to pion exchange ( $n = 1$ ) is dominant for the DDH potential, slightly enhanced for the pionful potential, and about equal to other ones for the pionless potential.

The neutron spin asymmetry  $P$  was calculated for laboratory neutron energy  $E = 15$  keV. The results are summarized in Tables VII, VIII, and IX for DDH, pionless EFT, and pionful EFT weak interaction potentials with different sets of parameters, correspondingly. These results provide a pattern similar to that of the results for the angle of neutron spin rotation. The parameter  $J_n$  in these tables is defined as

$$J_n \equiv \frac{1}{c_n} \frac{2}{\pi} \text{Re} \left[ \frac{1}{4\mu p^2} \left( R_{1\frac{1}{2}, 0\frac{1}{2}}^{\frac{1}{2}} - 2\sqrt{2} R_{1\frac{3}{2}, 0\frac{1}{2}}^{\frac{1}{2}} + 4R_{1\frac{1}{2}, 0\frac{3}{2}}^{\frac{3}{2}} - 2\sqrt{5} R_{1\frac{3}{2}, 0\frac{3}{2}}^{\frac{3}{2}} \right) \right] \quad (28)$$

and is related to the parameter  $\tilde{I}_n$  in the expression  $P = \sum c_n \tilde{I}_n$  by

$$\tilde{I}_n = \frac{\frac{2}{3}(2\pi\mu p^2)J_n}{\text{Re} \left[ R_{0\frac{1}{2}, 0\frac{1}{2}}^{\frac{1}{2}} + 2R_{0\frac{3}{2}, 0\frac{3}{2}}^{\frac{3}{2}} \right]} = \frac{8\pi^2\mu}{9} \frac{J_n}{\sigma_{\text{tot}}}, \quad (29)$$

where  $\sigma_{\text{tot}}$  is the total  $n$ - $d$  cross section. The total cross section  $\sigma_{\text{tot}}$  can be calculated, or one can use its known experimental value.

From the presented data, one can see that the results of our calculations are only slightly different for the cases when

we use AV18 and AV18 + UX strong Hamiltonians. This indicates stability of the results with respect to three-nucleon forces. Indeed, by analyzing the DDH one-pion exchange matrix element (see Table III), one can see that, for DDH-I with potentials AV18 and AV18 + UX, the contributions to  $I_{n=1}$  are  $-0.180 \times 10^1$  and  $-0.333 \times 10^1$  for the doublet channel ( $J = 1/2$ ), and for the quartet channel ( $J = 3/2$ ) they are  $0.630 \times 10^2$  and  $0.630 \times 10^2$ , correspondingly. The quartet channel is dominated by the repulsive and long-range part of the strong interactions, but the doublet channel is defined by the attractive part. Therefore the quartet channel is less sensitive to the off-energy shell structure of the strong interactions compared to the doublet channel. Then, due to the dominant contribution from the quartet channel, the net result turns out to be rather independent of the contribution from three-nucleon forces. This fact demonstrates the independence of our results on models of the strong interaction. However, further investigations with different strong interaction potentials are desirable.

It should be noted that the dependence on cutoff parameter for the contributions from potentials with short- and middle-range interactions, even though it appears significant, does not lead to cutoff dependence for the observable parameters. Indeed, the renormalization of low-energy constants would cancel those cutoff dependencies by the cutoff dependencies of LECs. Therefore, as a result, calculated PV observables are practically cutoff independent.

All these tables present information about contributions of different PV operators to PV effects, provided we know the corresponding weak coupling constants. Then, to calculate parity-violating effects, we can use either the DDH potential or one of the considered EFT potentials. However, for the case of EFT potentials, we need to know a set of LECs, which cannot be calculated in the given theoretical framework but must be obtained from a number of independent experiments. Unfortunately, currently available experimental data are insufficient to define the LECs with the required precision. Even for the pionless EFT, the estimated LECs [1] have large uncertainties, preventing us from predicting the values of PV effects. For the pionful EFT, the situation for determining the LECs is even worse. Therefore, it is impossible to make reliable predictions for PV effects using EFT-type potentials at this time, and the

TABLE V. Coefficients  $I_n^\pi$  for AV18 and AV18 + UIX strong potentials and  $\not{n}$ EFT-I and  $\not{n}$ EFT-II parameter sets for parity-violating potentials.  $I_{2,3,5,6,7,10,11,12,13}^\pi = 0$ .

$n$	$\not{n}$ EFT-I/AV18	$\not{n}$ EFT-I/AV18 + UIX	$\not{n}$ EFT-II/AV18	$\not{n}$ EFT-II/AV18 + UIX
1	$0.616 \times 10^2$	$0.600 \times 10^2$	0.969	0.969
4	$0.606 \times 10^2$	$0.588 \times 10^2$	0.499	0.515
8	$-0.761 \times 10^2$	$-0.757 \times 10^2$	-0.677	-0.708
9	$-0.946 \times 10^1$	$-0.662 \times 10^1$	-0.341	-0.348

 TABLE VI. Coefficients  $I_n^\pi$  for AV18 and AV18 + UIX strong potentials and  $\pi$ EFT-I and  $\pi$ EFT-II parameter sets for parity-violating potentials.  $I_{2,3,6,7,10,11,12}^\pi = 0$ .

$n$	$\pi$ EFT-I/AV18	$\pi$ EFT-I/AV18 + UIX	$\pi$ EFT-II/AV18	$\pi$ EFT-II/AV18 + UIX
1	$0.616 \times 10^2$	$0.600 \times 10^2$	$0.616 \times 10^2$	$0.600 \times 10^2$
4	$0.152 \times 10^1$	$0.142 \times 10^1$	0.549	0.488
5	$0.435 \times 10^1$	$0.185 \times 10^1$	$0.123 \times 10^1$	$0.664 \times 10^{-1}$
8	$-0.184 \times 10^1$	$-0.179 \times 10^1$	-0.782	-0.748
9	-0.820	-0.730	-0.340	-0.288
13	$0.226 \times 10^2$	$0.218 \times 10^2$	$0.970 \times 10^1$	$0.936 \times 10^1$
14	$0.339 \times 10^1$	$0.333 \times 10^1$	$0.177 \times 10^1$	$0.174 \times 10^1$
15	$0.654 \times 10^2$	$0.631 \times 10^2$	$0.273 \times 10^2$	$0.264 \times 10^2$

 TABLE VII. Coefficients  $J_n^{\text{DDH}}$  for AV18 and AV18 + UIX strong potentials and DDH-I and DDH-II parameter sets for parity-violating potentials at  $E = 15$  keV in the laboratory frame.  $J_{6,7}^{\text{DDH}} = 0$ .

$n$	DDH-I/AV18	DDH-I/AV18 + UIX	DDH-II/AV18	DDH-II/AV18 + UIX
1	0.253	0.253	0.254	0.254
2	$0.246 \times 10^{-2}$	$0.245 \times 10^{-2}$	$0.390 \times 10^{-2}$	$0.384 \times 10^{-2}$
3	$-0.190 \times 10^{-2}$	$-0.147 \times 10^{-2}$	$-0.313 \times 10^{-2}$	$-0.243 \times 10^{-2}$
4	$0.769 \times 10^{-3}$	$0.393 \times 10^{-3}$	$0.110 \times 10^{-2}$	$0.563 \times 10^{-3}$
5	$0.846 \times 10^{-3}$	$0.442 \times 10^{-3}$	$0.132 \times 10^{-2}$	$0.689 \times 10^{-3}$
8	$-0.176 \times 10^{-2}$	$-0.134 \times 10^{-2}$	$-0.228 \times 10^{-2}$	$-0.175 \times 10^{-2}$
9	$-0.235 \times 10^{-3}$	$0.567 \times 10^{-4}$	$-0.259 \times 10^{-3}$	$0.118 \times 10^{-3}$
10	$0.842 \times 10^{-3}$	$0.430 \times 10^{-3}$	$0.104 \times 10^{-2}$	$0.534 \times 10^{-3}$
11	$0.957 \times 10^{-3}$	$0.500 \times 10^{-3}$	$0.126 \times 10^{-2}$	$0.657 \times 10^{-3}$
12	$0.374 \times 10^{-2}$	$0.370 \times 10^{-2}$	$0.528 \times 10^{-2}$	$0.522 \times 10^{-2}$
13	$0.563 \times 10^{-2}$	$0.559 \times 10^{-2}$	$0.874 \times 10^{-2}$	$0.868 \times 10^{-2}$

 TABLE VIII. Coefficients  $J_n^\pi$  for AV18 and AV18 + UIX strong potentials and  $\not{n}$ EFT-I and  $\not{n}$ EFT-II parameter sets for parity-violating potentials.  $J_{2,3,5,6,7,10,11,12,13}^\pi = 0$ .

$n$	$\not{n}$ EFT-I/AV18	$\not{n}$ EFT-I/AV18 + UIX	$\not{n}$ EFT-II/AV18	$\not{n}$ EFT-II/AV18 + UIX
1	0.254	0.254	$0.372 \times 10^{-2}$	$0.369 \times 10^{-2}$
4	$0.503 \times 10^{-1}$	$0.240 \times 10^{-1}$	$0.421 \times 10^{-3}$	$0.215 \times 10^{-3}$
8	-0.111	$-0.854 \times 10^{-1}$	$-0.984 \times 10^{-3}$	$-0.763 \times 10^{-3}$
9	$-0.241 \times 10^{-2}$	$0.338 \times 10^{-2}$	$-0.904 \times 10^{-4}$	$0.750 \times 10^{-4}$



TABLE IX. Coefficients  $J_n^\pi$  for AV18 and AV18 + UIX strong potentials and  $\pi$ EFT-I and  $\pi$ EFT-II parameter sets for parity-violating potentials.  $J_{2,3,6,7,10,11,12}^\pi = 0$ .

$n$	$\pi$ EFT-I/AV18	$\pi$ EFT-I/AV18 + UIX	$\pi$ EFT-II/AV18	$\pi$ EFT-II/AV18 + UIX
1	0.254	0.254	0.254	0.254
4	$0.106 \times 10^{-2}$	$0.352 \times 10^{-3}$	$0.309 \times 10^{-3}$	$0.333 \times 10^{-4}$
5	$0.741 \times 10^{-2}$	$0.512 \times 10^{-2}$	$0.292 \times 10^{-2}$	$0.221 \times 10^{-2}$
8	$-0.276 \times 10^{-2}$	$-0.212 \times 10^{-2}$	$-0.127 \times 10^{-2}$	$-0.100 \times 10^{-2}$
9	$-0.148 \times 10^{-3}$	$0.301 \times 10^{-3}$	$-0.278 \times 10^{-4}$	$0.168 \times 10^{-3}$
13	$0.976 \times 10^{-1}$	$0.981 \times 10^{-1}$	$0.421 \times 10^{-1}$	$0.423 \times 10^{-1}$
14	$0.137 \times 10^{-1}$	$0.136 \times 10^{-1}$	$0.714 \times 10^{-2}$	$0.712 \times 10^{-2}$
15	0.283	0.284	0.119	0.120

TABLE X. DDH PV coupling constants in units of  $10^{-7}$ . Strong couplings are  $\frac{g_\pi^2}{4\pi} = 13.9$ ,  $\frac{g_\rho^2}{4\pi} = 0.84$ ,  $\frac{g_\omega^2}{4\pi} = 20$ ,  $\kappa_\rho = 3.7$ , and  $\kappa_\omega = 0$ ;  $h'_\rho$  contribution is neglected. The four-parameter and three-parameter fits use the same  $h_\rho^1$  and  $h_\omega^1$  with DDH “best.”

DDH coupling	DDH “best”	Four-parameter fit [26]	Three-parameter fit [26]
$h_\pi^1$	+4.56	-0.456	-0.5
$h_\rho^0$	-11.4	-43.3	-33
$h_\rho^2$	-9.5	37.1	41
$h_\omega^0$	-1.9	13.7	0
$h_\rho^1$	-0.19	-0.19	-0.19
$h_\omega^1$	-1.14	-1.14	-1.14

TABLE XI. Neutron spin rotation in  $10^{-7}$  rad  $\text{cm}^{-1}$  for the case of a DDH-II potential with an AV18 + UIX strong potential for a liquid deuteron density  $N = 0.4 \times 10^{23}$  atoms/ $\text{cm}^3$ .

	DDH “best”	Four-parameter fit [26]	Three-parameter fit [26]
1	0.108	$-0.108 \times 10^{-1}$	$-0.118 \times 10^{-1}$
2	$0.386 \times 10^{-2}$	$0.147 \times 10^{-1}$	$0.112 \times 10^{-1}$
3	$-0.317 \times 10^{-1}$	-0.120	$-0.918 \times 10^{-1}$
4	$0.349 \times 10^{-4}$	$0.349 \times 10^{-4}$	$0.349 \times 10^{-4}$
5	$0.150 \times 10^{-3}$	$0.150 \times 10^{-3}$	$0.150 \times 10^{-3}$
8	$-0.423 \times 10^{-2}$	$0.305 \times 10^{-1}$	0.000
9	$-0.202 \times 10^{-2}$	$0.146 \times 10^{-1}$	0.000
10	$0.967 \times 10^{-3}$	$0.967 \times 10^{-3}$	$0.967 \times 10^{-3}$
11	$0.113 \times 10^{-2}$	$0.113 \times 10^{-2}$	$0.113 \times 10^{-2}$
12	$0.102 \times 10^{-2}$	$0.102 \times 10^{-2}$	$0.102 \times 10^{-2}$
total	$0.768 \times 10^{-1}$	$-0.682 \times 10^{-1}$	$-0.891 \times 10^{-1}$

TABLE XII. Neutron spin asymmetry for the case of a DDH-II potential with an AV18 + UIX strong potential (with a total cross section  $\sigma_{\text{tot}} = 3.35$  b at  $E = 15$  keV).

	DDH “best”	Four-parameter fit [26]	Three-parameter fit [26]
1	$0.947 \times 10^{-8}$	$-0.947 \times 10^{-9}$	$-0.104 \times 10^{-8}$
2	$0.248 \times 10^{-9}$	$0.943 \times 10^{-9}$	$0.719 \times 10^{-9}$
3	$-0.740 \times 10^{-9}$	$-0.281 \times 10^{-8}$	$-0.214 \times 10^{-8}$
4	$0.304 \times 10^{-12}$	$0.304 \times 10^{-12}$	$0.304 \times 10^{-12}$
5	$0.138 \times 10^{-11}$	$0.138 \times 10^{-11}$	$0.138 \times 10^{-11}$
8	$-0.922 \times 10^{-10}$	$0.665 \times 10^{-9}$	0.000
9	$0.620 \times 10^{-11}$	$-0.447 \times 10^{-10}$	0.000
10	$0.843 \times 10^{-11}$	$0.843 \times 10^{-11}$	$0.843 \times 10^{-11}$
11	$0.104 \times 10^{-10}$	$0.104 \times 10^{-10}$	$0.104 \times 10^{-10}$
12	$0.797 \times 10^{-10}$	$0.797 \times 10^{-10}$	$0.797 \times 10^{-10}$
total	$0.899 \times 10^{-8}$	$-0.209 \times 10^{-8}$	$-0.236 \times 10^{-8}$

only reasonable way to estimate magnitudes of PV effects is to use the DDH potential. Taking into account the difficulty of the systematic description of PV effects using “standard” DDH potentials (see the discussion in Sec. I), we estimate PV effects using the DDH potential for different sets of weak coupling constants: for the “best value” coupling constants and for two possible sets of values of the coupling constants recently obtained by Bowman [26] from the fit of reliable existing experimental data (see Table X). The results for these three sets of weak coupling constants are summarized in Tables XI and XII for the angle of spin rotation and for neutron spin asymmetry, correspondingly. One can see that, in contrast to the fact that the one-pion exchange dominates in the DDH-“best” coupling parameter set, the  $\rho$ -meson exchange dominates in the case of Bowman’s coupling parameter set. One can see that the angle of neutron spin rotation has almost the same magnitude for all three sets of parameters, but it has opposite signs for the “best value” set and for the Bowman fits. Not only does the neutron spin asymmetry have opposite signs but also essentially different values for these two choices of parameters. This allows one to choose between two possible sets of DDH parameters and, as a consequence, to test the dominance of the  $\pi$ -meson contribution in PV effects in  $n$ - $d$  scattering.

Finally, we would like to mention that our results are quite different from the results obtained in [15]. For example, in [15], the values of  $I_n$  for  $J = \frac{1}{2}$  and  $J = \frac{3}{2}$  have the same signs for operator with  $n = 1$ , but our results show opposite signs for these matrix elements. Another discrepancy is related to the systematic difference between the values of matrix elements calculated [15] for AV18 and AV18 + UIX potentials, which indicates a large wave function difference for AV18 and AV18 + UIX potentials. In contrast to those, our results show that these matrix elements are insensitive to the presence of the three-nucleon force.<sup>1</sup>

#### IV. CONCLUSION

We have calculated the parity-violating angle of neutron spin rotation and asymmetry in the transmission of neutrons with opposite helicities for low-energy neutron-deuteron scattering. Using the distorted-wave Born approximation for weak interactions with realistic three-nucleon wave functions from the Faddeev equations in configuration space, we have parameterized PV observables in terms of matrix elements presented in the DDH weak potential and in weak potentials derived from pionless and pionful EFTs. It is shown that our results are practically independent of the choice of strong interaction potentials or cutoff parameters.

Based on the given analysis, one can see that for DDH potential, the dominant contribution to observable PV effects, comes from the pion-exchange matrix element with  $n = 1$ . However, for a pionless EFT potential, all types of matrix elements contribute almost equally, and for a pionful EFT

potential the pion-exchange matrix element is slightly enhanced as compared to the other ones. Therefore, it would be interesting to compare the estimation of observable PV effects using appropriate LECs and coupling constants for DDH. Unfortunately, due to insufficient data for LECs this is impossible at this time. However, a comparison of PV effects for two different sets of coupling constants shows that  $n$ - $d$  scattering experimental results can be used to distinguish among different sets of DDH coupling constants and to help in clarification of the issue about the importance of the contribution of the pion-exchange weak potential.

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#### APPENDIX: EXPLICIT RESULTS OF THE ANGULAR PART OF THE MATRIX ELEMENTS

Explicit values of the matrix elements of isospin operators for two-body states are

$$\begin{aligned}
 \langle T' T'_z | \tau_1 \cdot \tau_2 | T T_z \rangle &= \delta_{T'_z, T_z} \delta_{T', T} [1 \delta_{T, 1} - 3 \delta_{T, 0}], \\
 \langle T' T'_z | (\tau_1 + \tau_2)^z | T T_z \rangle &= \delta_{T', T} \delta_{T'_z, T_z} [2 T_z], \\
 \langle T' T'_z | (\tau_1 - \tau_2)^z | T T_z \rangle &= \delta_{T', T \pm 1} \delta_{T'_z, T_z} \delta_{T_z, 0} [2], \\
 \langle T' T'_z | i(\tau_1 \times \tau_2)^z | T T_z \rangle &= \delta_{T'_z, T_z} \delta_{T, 0} \delta_{T', T \pm 1} [\pm 2], \\
 \langle T' T'_z | T_{12}^z | T T_z \rangle &= \delta_{T', 1} \delta_{T, 1} \delta_{T'_z, T_z} [2 \delta_{T_z, 1} - 4 \delta_{T_z, 0} \\
 &\quad + 2 \delta_{T_z, -1}], \tag{A1}
 \end{aligned}$$

and matrix elements of orbital and spin operators for two-body states  $|(l_x s_x) j_x j_x^z\rangle$  are

$$\begin{aligned}
 \langle (j_x \pm 1, 1) j_x j_x^z | (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \hat{\boldsymbol{x}} | (j_x, 1) j_x j_x^z \rangle \\
 = \langle (j_x, 1) j_x j_x^z | (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \hat{\boldsymbol{x}} | (j_x \pm 1, 1) j_x j_x^z \rangle \\
 = -2 \sqrt{\frac{j_x + 1/2 \mp 1/2}{2j_x + 1}}, \tag{A2}
 \end{aligned}$$

$$\begin{aligned}
 \langle (j_x \pm 1, 1) j_x j_x^z | (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \cdot \hat{\boldsymbol{x}} | (j_x, 0) j_x j_x^z \rangle \\
 = \langle (j_x, 0) j_x j_x^z | (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \cdot \hat{\boldsymbol{x}} | (j_x \pm 1, 1) j_x j_x^z \rangle \\
 = \mp 2 \sqrt{\frac{j_x + 1/2 \pm 1/2}{2j_x + 1}}, \tag{A3}
 \end{aligned}$$

$$\begin{aligned}
 \langle (j_x, 0) j_x j_x^z | i(\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2) \cdot \hat{\boldsymbol{x}} | (j_x \pm 1, 1) j_x j_x^z \rangle \\
 = (-) \langle (j_x \pm 1, 1) j_x j_x^z | i(\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2) \cdot \hat{\boldsymbol{x}} | (j_x, 0) j_x j_x^z \rangle \\
 = \pm 2 \sqrt{\frac{j_x + 1/2 \pm 1/2}{2j_x + 1}}, \tag{A4}
 \end{aligned}$$

$$\begin{aligned}
 \langle (j_x \pm 11) j_x j_x^z | (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \hat{\nabla}_{\Omega_x} | (j_x, 1) j_x j_x^z \rangle \\
 = \pm 2 \frac{(j_x + 1/2 \mp 1/2) \sqrt{j_x + 1/2 \mp 1/2}}{\sqrt{2j_x + 1}}, \\
 \langle (j_x, 1) j_x j_x^z | (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \hat{\nabla}_{\Omega_x} | (j_x \pm 11) j_x j_x^z \rangle \\
 = \mp 2 \frac{(j_x + 1/2 \pm 3/2) \sqrt{j_x + 1/2 \mp 1/2}}{\sqrt{2j_x + 1}}, \tag{A5}
 \end{aligned}$$

<sup>1</sup>We thank R. Schiavilla and M. Viviani for discussions which clarified that the reason for these discrepancies is related to numerical errors in Ref. [15].

$$\begin{aligned}
& \langle (j_x \pm 11)j_x j_x^z | (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \cdot \hat{V}_{\Omega_x} | (j_x 0)j_x j_x^z \rangle \\
&= 2 \frac{(j_x + 1/2 \mp 1/2)\sqrt{j_x + 1/2 \pm 1/2}}{\sqrt{2j_x + 1}}, \\
& \langle (j_x 0)j_x j_x^z | (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \cdot \hat{V}_{\Omega_x} | (j_x \pm 11)j_x j_x^z \rangle \\
&= -2 \frac{(j_x + 1/2 \pm 3/2)\sqrt{j_x + 1/2 \pm 1/2}}{\sqrt{2j_x + 1}}. \quad (\text{A6})
\end{aligned}$$

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