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# New enhanced tunneling in nuclear processes

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The small sub-barrier tunneling probability of nuclear processes can be dramatically enhanced by collision with incident charged particles. Semiclassical methods of theory of complex trajectories have been applied to nuclear tunneling, and conditions for the effects have been obtained. We demonstrate the enhancement of $\alpha$ particle decay by incident proton with energy of about 0.25 MeV. We show that the general features of this process are common for other sub-barrier nuclear processes and can be applied to nuclear fission.

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Tunneling in nuclear processes has been a subject of study for many years since this is a substantial mechanism of nuclear decay and nuclear reactions, including nuclear fission and fusion (see, for example, Refs. [1,2], and references therein). The recent interest in understanding the processes of underbarrier tunneling [3–11] has been stimulated by the calculation of bremsstrahlung radiation in $\alpha$ decay when the $\alpha$ particle is moving under the barrier [12]. In this paper we consider another feature of quantum tunneling: the possible enhancement of nuclear decay due to interactions with low energy charged particles. This enhancement has the same origin as the tunneling enhancement in nonstationary fields recently discovered in condensed matter physics [13,14], and can manifest itself in different underbarrier processes.

For the sake of simplicity, we consider nuclear $\alpha$ decay. According to the theory of Gamov, the probability of the transition of $\alpha$ particle through the nuclear Coulomb barrier is mainly ruled by the exponential factor [15,16]

$$W \sim \exp[-A_\alpha(E)], \tag{1}$$

where

$$A_\alpha(E) = \frac{\sqrt{8M}}{\hbar}\int_{R_0}^{R_\alpha} dR\sqrt{\frac{2Ze^2}{R} - E} \tag{2}$$

is the classical action measured in units of $\hbar$ [17]. $M$ is the mass of $\alpha$ particle, $Z$ is the charge of the daughter nucleus, $R_0$ is the nuclear radius, and the classical exit point $R_\alpha$ is determined by zero of the square root. Let us study how this probability changes when the decayed nuclei are placed in the beam of protons with the energy less than the barrier height $2Ze^2/R_0$. This situation resembles the process of quantum tunneling of particles controlled by a weak and varying in time electromagnetic field considered in Refs. [13,14,18–20], where the specific tunneling enhancement mechanisms have been studied. The difference between our case and these processes is mainly in the nature of the external electromagnetic field (the beam of protons, in our case). The low energy projectile protons can be treated as a source of a pulsed electromagnetic fields interacting with the $\alpha$ decayed nuclear target.

According to the results of papers [13,14], two different regimes of tunneling are possible if the proton energy and its time of the underbarrier motion satisfy the necessary conditions. The first regime is the assistance of tunneling, when the $\alpha$ particle gains a part of proton energy, which can be called the positive assistance. The second one, which occurs when the $\alpha$ particle transfers a part of its energy to the proton, is called the negative assistance of tunneling. Under conditions of the negative assistance, the $\alpha$ particle tunnels at lower energy where the barrier is less transparent. Nevertheless, contrary to any expectation, the regime of negative assistance of tunneling is unusual, since, under certain conditions, tunneling probability does not become exponentially small even for a barrier which is normally not very transparent. This phenomenon is called Euclidean resonance. Both mechanisms of positive and negative assistance are connected with the coherent multiquanta interference in the underbarrier motion. We know that the enhancement of tunneling occurs when a singularity of the nonstationary field coincides in position with a singularity of the classical Newtonian trajectory of the particle on the complex time plane. To study these processes in nuclei, we apply the semiclassical approach (based on the method of trajectories in complex time) developed for tunneling in nonstationary fields.

Consider the assistance of $\alpha$ decay by the Coulomb field of incident protons when its energy is less than the height of the Coulomb barrier. In the absence of a proton, one can calculate the decay probability using formulas (1) and (2). A low energy incoming proton interacts with the nucleus only electromagnetically, since it is stopped by the nuclear Coulomb field at a distance much larger than the radius of strong nuclear forces. The time of interactions of the proton with the decayed $\alpha$ particle (underbarrier motion time) is about of a characteristic nuclear time $\Delta t \sim 10^{-21}$ s. The proton can excite the nucleus to increase the energy of emitted $\alpha$ particle $E$ by the amount of $\Delta E$ during the interaction time $\Delta t$.

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This leads to the increase of the action in Eq. (1) by the amount $2\Delta t\Delta E/\hbar$ (the rigorous definition of $\Delta t$ is given below). At the same time, the energy gain by the excited $\alpha$ particle makes its tunneling easier due to the reduction of the action $A_\alpha(E)\rightarrow A_\alpha(E+\Delta E)$. Then, the resulting action for the proton induced $\alpha$ decay is

$$A = \frac{2\Delta t\Delta E}{\hbar} + A_\alpha(E+\Delta E). \tag{3}$$

In condensed matter physics, this equation would describe the process of the positive photon-assisted tunneling with the probability $\exp(-A)$ [13]. The first term in Eq. (3) results in a reduction of the probability due to quanta absorption and the second one describes tunneling in a more transparent (higher energy) part of the barrier. Since the flux of tunneling particles in a nonstationary field is also a nonstationary one, the expression $\exp(-A)$ relates to the maximal value of the tunneling probability. This maximal transition probability is determined by a finite value of the energy transfer $\Delta E$, which provides a minimum of the action $A$, and, hence, it is defined by the condition $\partial A(E+\Delta E)/\partial\Delta E=0$.

Equation (3) describes the tunneling assisted by a given external nonstationary field. In our case, the role of this field is played by the proton not with a "given" but a rather with a "flexible" motion affected by both the nucleus and the $\alpha$ particle. The increase of the value of $\alpha$ particle energy must be accompanied by the corresponding decrease of the proton energy value [from its initial value $\varepsilon$ down to $(\varepsilon-\Delta E)$]. In the language of trajectories, both the particles should finish the underbarrier motion at the same moment $t=0$ with the zero velocity just to proceed in real time outside the barrier. This means that the $\alpha$ particle and the proton participate in the cooperative motion from the nuclear surface $(t=i\tau_0)$ to outside of the barrier $(t=0)$. This cooperative motion can be described by the joint action $A_{\alpha p}$ which accounts the interparticle Coulomb interaction. In contrast to the $\alpha$ particle, the proton does not start its full motion at the nuclear surface but it starts outside the barrier. Therefore, the true action should be corrected as

$$A = A_{\alpha p} - \frac{2i\sigma_p}{\hbar}, \tag{4}$$

where the second term (being real and negative) accounts for the phase shift of the process of the artificial move of the proton from outside of the barrier to the nuclear surface. The classical imaginary action of the proton $\sigma_p$ can be determined from the corresponding Hamilton-Jacobi equation. At the limit of a weak $\alpha$-proton Coulomb coupling, one can estimate $2i\sigma_p/\hbar \simeq A_p(\varepsilon)$, where $A_p(\varepsilon)$ is the proton analog of the action (2),

$$A_p(\varepsilon) = \frac{\sqrt{8m}}{\hbar}\int_{R_0}^{r_e} dr\sqrt{\frac{Z_0e^2}{r}-\varepsilon}. \tag{5}$$

Here, $m$ is the proton mass and $Z_0$ is the charge of decayed nucleus. The action $A_{\alpha p}$ is defined on the joint $\alpha$-proton classical trajectory in imaginary time when both particles, with initial energies $E$ ($\alpha$ particle) and $\varepsilon$ (proton), meet on the nuclear surface. The energy exchange between the $\alpha$ particle and the proton occurs fast and weakly contributes to the joint action $A_{\alpha p}$ which can be written as $A_{\alpha p}\simeq A_\alpha(E+\Delta E)+A_p(\varepsilon-\Delta E)$. Then, the total action takes the form

$$A = A_\alpha(E+\Delta E) + A_p(\varepsilon-\Delta E) - A_p(\varepsilon). \tag{6}$$

The classical time $\tau_0$ of the underbarrier motion, which is proportional to the derivative of the action with respect to energy, has the same value both for the $\alpha$ particle and for the proton since they move together

$$\frac{2\tau_0}{\hbar} = -\frac{\partial A_\alpha(E+\Delta E)}{\partial\Delta E} = \frac{\partial A_p(\varepsilon-\Delta E)}{\partial\Delta E}. \tag{7}$$

Therefore, Eq. (7) determines the certain energy transfer $\Delta E$ which provides an extreme of the action (6). At the limit of a very small energy transfer $(\Delta E\ll\varepsilon)$, the proton motion becomes nonflexible. This corresponds to the action of Eq. (3), with $\Delta t=\tau_0$, which can be obtained from Eq. (6) after expansion on $\Delta E$. In this case, the tunneling motion of the $\alpha$ particle is affected by the nonstationary field

$$V_{int}(\vec{R},i\tau) = \frac{2e^2}{|\vec{R}-\vec{r}(i\tau)|}, \tag{8}$$

where $\vec{r}(i\tau)$ approximately describes the classical trajectory of the free proton. The total action (6) can be written in the explicit form

$$A = \frac{2\pi Ze^2}{\hbar}\sqrt{\frac{2M}{E+\Delta E}} - 4\sqrt{\frac{R_0}{\hbar^2/(4MZe^2)}} + \frac{\pi Z_0e^2\sqrt{2m}}{\hbar}\left(\frac{1}{\sqrt{\varepsilon-\Delta E}}-\frac{1}{\sqrt{\varepsilon}}\right). \tag{9}$$

Then, the relation between the optimum energy transfer $\Delta E$ and the energy of the incident proton $\varepsilon$ is given by Eq. (7):

$$\frac{\varepsilon-\Delta E}{E+\Delta E} = \left(\frac{m}{4M}\right)^{1/3}, \tag{10}$$

where we disregard the small difference between charges of the parent and daughter nuclei. It should be noted that generally the energy transfer $\Delta E$ is determined by Newtonian equations for the $\alpha$ particle and the proton in imaginary time, which are coupled by the interaction (8). As a consequence, the value of $\Delta E$ depends on the angle $\phi$ between directions of radial motions of these two particles (we consider both particles to have zero angular momenta). For example, $\Delta E$ is positive for $\phi=180°$ and negative for $\phi=0°$ (parallel motion). This means that the condition (10) of the optimum energy transfer is fulfilled for a certain angle between directions of classical motion of two particles.

One should emphasize that a finite angle between escaping $\alpha$ particle and proton corresponds only to the language of trajectories in imaginary time. The formalism of imaginary time, which allows us to find a maximal value of effect, provides a trajectory for the action minimization rather than describes real particle motion. For example, real particles (in real time) with zero angular momenta are distributed isotropically in our case despite the final angle in imaginary time description.

One can see that the energy transfer $\Delta E$ during the tunneling process can be either positive (positive assistance of tunneling) or negative (negative assistance of tunneling). The latter case, as mentioned above, is unusual, since the action can tend to zero and tunneling probability does not become exponentially small (Euclidean resonance) even for a barrier which is normally hardly transparent. Indeed, by substituting expression (10) into Eq. (9) we obtain

$$A = \frac{2\pi Z_0 e^2}{\hbar}\sqrt{\frac{2M}{E+\varepsilon}}\left[1+\left(\frac{m}{4M}\right)^{1/3}\right]^{3/2} - \frac{8}{\hbar}\sqrt{MZ_0e^2R_0} - \frac{\pi Z_0 e^2}{\hbar}\sqrt{\frac{2m}{\varepsilon}}. \tag{11}$$

If $\Delta E$ is negative, the proton energy $\varepsilon$ can be chosen small [see Eq. (10)] and the last term in Eq. (11) may reduce the action $A$ down to a zero value. It should be noted that the above equation is correct if $\exp(-A) \ll 1$. When $A$ becomes of the order of unity or less, one should use a generic formalism with the multiinstanton approach, which leads to the similar estimate of the action $A$. As follows, the proton energy $\varepsilon$ in Eq. (11) cannot be sufficiently small to keep $A$ to be finite.

Let us give an example of the calculation of the energy transfer $\Delta E$ using the method of complex trajectories. Consider a classical parallel motion of the $\alpha$ particle and the proton (the angle between particles $\phi=0$) when only $x$ components are involved and are determined by the Newton equations

$$M\frac{\partial^2 R_x}{\partial\tau^2} = -\frac{2Ze^2}{R_x^2} + \frac{2e^2}{(r_x-R_x)^2}, \quad m\frac{\partial^2 r_x}{\partial\tau^2} = -\frac{Z_0e^2}{r_x^2} - \frac{2e^2}{(r_x-R_x)^2}. \tag{12}$$

In the vicinity of the complex time $\tau_0$, when particles meet each other, the solutions of these equations have the form

$$\frac{R_x(i\tau)}{R_s} = \frac{r_x(i\tau)}{r_s} = \left(\frac{\tau_0-\tau}{\tau_0}\right)^{2/3}, \tag{13}$$

where $R_s$ and $r_s$ are some constants to be defined. The energy $\Delta E$, gained by the $\alpha$ particle,

$$\Delta E = 2e^2\int_0^{\tau_0}\frac{d\tau}{(R_x-r_x)^2}\frac{\partial R_x}{\partial\tau}, \tag{14}$$

diverges close to $\tau_0$ and should be regularized by the condition $R_s(1-\tau/\tau_0)^{2/3} > R_0$. (It should be noted that in contrast to the large contribution of the diverged interaction $V_{int}$ to $\Delta E$, its contribution to the action $A$ is not divergent and even rather small.) Then,

$$|\Delta E| = \frac{2e^2}{R_0}\left(\frac{r_s}{R_s}-1\right)^{-2}, \tag{15}$$

where the ratio $R_s/r_s$ satisfies the relation

$$\frac{M}{2m}\left(\frac{R_s}{r_s}\right)^3\left[\left(1-\frac{R_s}{r_s}\right)^2+\frac{2}{Z_0}\right] = \left(1-\frac{R_s}{r_s}\right)^2 - \frac{1}{Z_0}\left(\frac{R_s}{r_s}\right)^2. \tag{16}$$

Considering the uranium $\alpha$ decay as an example

$${}^{235}_{92}\mathrm{U} + p \to {}^{231}_{90}\mathrm{Th} + \alpha + p, \tag{17}$$

with the energy of $\alpha$ particle $E=4.678$ MeV, we can fix the ratio $M/m=4$, and obtain the parameter $R_s/r_s \simeq 0.715$. These lead to the energy transfer $|\Delta E| \simeq 1.89$ MeV and nonphysical (negative) value of $\varepsilon$. This means that in the case of the classical trajectory with $\phi=0$, the energy transfer is larger than the optimum value [see Eq. (10)]. The optimum $\Delta E$ (which leads to an extreme value of the action $A$) corresponds to a finite $\phi$ of the classical trajectory and can be found numerically using the above scheme.

The analysis of the expression for the action (11) shows that at $\varepsilon=\varepsilon_0$, where

$$\varepsilon_0 = E\left(\frac{m}{4M}\right)^{1/3} \simeq 1.85 \text{ MeV}, \tag{18}$$

the energy transfer $\Delta E=0$, when the angle $\phi \simeq 30°$. Under those conditions, the action (11) coincides with the action of the conventional $\alpha$ decay (2), resulting in the tunneling probability

$$W \sim \exp[-A_\alpha(4.678 \text{ MeV})] \simeq e^{-80.75} \simeq 10^{-35}. \tag{19}$$

The probability (19), being normalized by the nuclear attempt frequency $10^{21}$ s$^{-1}$, describes experimental data reasonably well [15,16].

At $\varepsilon < \varepsilon_0$, the optimum energy transfer $\Delta E$ becomes finite and negative, the optimum angle $\phi$ decreases, and the action (11) reduces in comparison with $A_\alpha(E)$. Upon reduction of $\varepsilon$, the action (11) turns to zero at a certain proton energy $\varepsilon_R$, which relates to Euclidean resonance. For the reaction (17) $\varepsilon_R=0.25$ MeV, the accompanied energy transfer is $\Delta E=-1.15$ MeV, and the angle between the incident proton and the emitted $\alpha$ particle is $\phi \simeq 11°$ (when, formally, $\phi=0$ the $\alpha$ particle always moves between the nucleus and the proton). In other words, when the energy of the incident proton is about $\varepsilon=0.25$ MeV, the energy of the emitted $\alpha$ particle becomes $E-|\Delta E|=3.53$ MeV (instead of $E=4.678$ MeV) and the energy of outgoing proton becomes $\varepsilon+|\Delta E|=1.40$ MeV. The cross section of the reaction (17) is not exponentially small at $\varepsilon=\varepsilon_R$ and has a very sharp peak in the vicinity of the proton energy. The typical behavior of the tunneling probability as a function of the proton energy $\varepsilon$ is shown in Fig. 1.

Positive assistance of tunneling ($\Delta E > 0$) corresponds to the domain $\varepsilon > 1.85$ MeV and Euclidean resonance at $\varepsilon=0.25$ MeV occurs at the region of negative assistance $\varepsilon < 1.85$ MeV. At $\varepsilon > \varepsilon_R$, the shape of the peak is proportional to $\exp[-(\varepsilon-\varepsilon_R)/\Delta\varepsilon]$, where $\Delta\varepsilon \simeq 3\times 10^{-3}$ MeV.

The estimation of the resonance proton energy as $\varepsilon_R=0.25$ MeV is made for zero angular momenta of $\alpha$ particle and proton, because the contribution from proton momenta less than $L_{max}(L_{max}^2/2m_0^2 \sim e^2Z_0/R_0)$ results simply in smear-

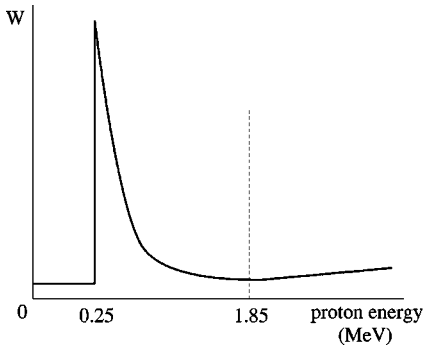


FIG. 1. The right side of the energy dependence (smooth enhancement) of the tunneling probability corresponds to the domain $\varepsilon > 1.85$ MeV and is related to positive assistance of tunneling. The Euclidean resonance occurs at $\varepsilon = 0.25$ MeV at the region of negative assistance tunneling $\varepsilon < 1.85$ MeV.

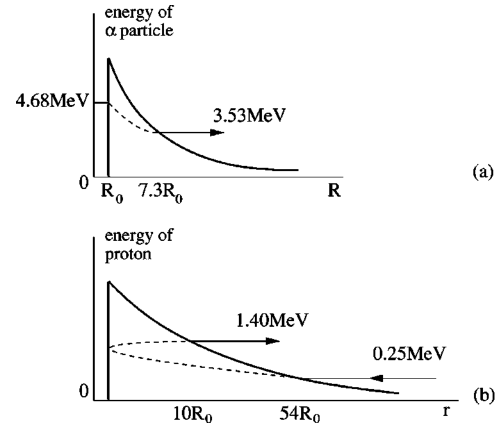


FIG. 2. (a) The trajectory of the $\alpha$ particle; (b) The trajectory of the proton. These classical trajectories in imaginary time are only a convenient way to describe the effect. In real time the proton does not approach the nucleus and interacts with it solely via the Coulomb field.

ing of $\varepsilon_R$ for about a few percents. This also leads to the estimate of the geometrical cross section of this reaction $\sigma_g = \pi b^2$ where the impact parameter $b = L_{max}/\sqrt{2m\varepsilon}$. The cross section for the considered process is the product of the geometrical cross section $\sigma_g \sim 100$ b and the probability of the enhanced tunneling.

It should be noted that at $\varepsilon = \varepsilon_R$, the exit point of $\alpha$ particle is $7.3R_0$ and of the outgoing proton is $10R_0$. The incident proton is stopped by the nuclear Coulomb field at the distance of about $54R_0$, as shown in Fig. 2.

Therefore, the trajectories in imaginary time are only a convenient language to describe the effect. In real (physical) time, the proton does not approach the nucleus and interacts with it solely via the Coulomb field.

We use the method of complex trajectories for calculations of underbarrier processes instead of numerical solution of the Schrödinger equation mostly because of an insufficiency of the existing algorithms to solve this type of Schrödinger equation in a reasonable amount of time (see, also Ref. [6]).

The idea of stimulation of nuclear tunneling processes by incident charged particles can be applied in the similar way to nuclear fission processes on the basis of models with nuclear fragments tunneling under the action of the external varying Coulomb field. However, the validity of this approach for fusion reactions is less obvious and requires a further study.

In summary, protons, approaching $\alpha$ decaying nuclei, create the nonstationary Coulomb field, acting on the tunneling $\alpha$ particle. Due to these interactions the Euclidean resonance can appear at low proton energy and the Coulomb barrier becomes practically transparent for the passage of the $\alpha$ particle. For example, normally, ${}^{235}_{92}$U emits $\alpha$ particle with the energy of 4.678 MeV. When the energy of the incident proton is close to its resonant value of 0.25 MeV, the energy of outgoing protons becomes 1.40 MeV and the energy of emitted $\alpha$ particles becomes 3.53 MeV.

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