Computer Vision Based, Non-Contacting Deformation and Shape Measurements: A Revolution in Progress

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The rapid expansion of computer technology in the past thirty years has impacted society in innumerable ways. Of interest for this discussion is the breath-taking change in measurement sciences that has occurred as the digital age dawned and has now become part of our daily lives. In particular, the effect on image-based, non-contacting measurements will be described and the continuing revolutionary impact that the resulting technology has engendered will be reviewed. Additionally, the unprecedented expansion of capabilities will be highlighted by presenting a set of illustrative example studies.

Introduction

Making measurements during physical observations is a natural part of human inquiry and dates back to the early days of recorded history. Most of us recall the remarkable engineering feats of the Egyptians when building the pyramids, the Great Wall in China and the aquaduct system developed by the Romans, all requiring basic measurements to complete their engineering marvels of the time. In modern times, a range of measurement methods have been developed and employed by engineers and scientists to improve understanding of physical phenomena and complete complex engineering feats.

In the field of solid mechanics, the engineering measurement methods that were developed and used in practical applications generally were point-wise. That is, each measurement device was used to obtain data at a specific location and the decision regarding the location of the measurement device was oftentimes based on experience; if something unexpected occurred at a location, then measurement devices would be placed in this area. Typical measurement devices that have been used over the past century include

- strain gages to measure change in length/unit length
- caliper/ruler to measure distance between points
- dial indicator to measure displacement of a point
- thermocouple to measure temperature at a point

In physics, advanced optical methods have been used to measure extremely small quantities for over a century. One of the most famous is the Michelson inteferometer, an optical system used to measure changes in the path length of light that are a fraction of the wavelength of light. In its earliest form, the method gave a single measurement and became part of the experimental effort to see whether the speed of light was constant. The measurement landscape began to change in the 1950's as the laser (Light Amplification through Stimulation of Emission of Radiation) was developed and became part of the toolbox used by physicists. Since laser light beams could be expanded and used to illuminate entire surfaces, it soon became apparent that the coherent nature of laser light allowed investigators to measure the change in optical path for all object points on a specimen. A typical shearing type interferogram is shown in Figure 1. To obtain the fringes that are shown in Fig. 1, a 15milliwatt Helium Neon laser was split into multiple beams and a circular flat plate was illuminated. The scattered light from the specimen was then recorded on a photographic glass plate. After bending the specimen out of plane with a central point load, the illumination process was repeated so that an double exposure interferogram was recorded on the plate. The plate was then processed in a dark room to obtain the whole specimen fringe pattern shown above[1].

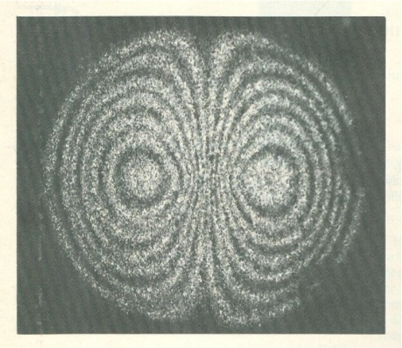

Figure 1: Laser shearing interferometry fringe pattern[1] with fringes defining lines of constant out of plane slope, $\partial w/\partial x$. Image circa 1979 obtained in T.A.M. Department at the University of Illinois in Champaign-Urbana.

Though an improvement over previous pointwise techniques, the process simply was too complex to be usable in a typical laboratory setting. Issues such as the following eventually led investigators to seek alternatives;

- Experiments must be performed in the dark
- Experiments require a laser and specialized optical components, including expensive glass plates
- Experiments must be performed in vibration isolation for optimal results due to the extreme sensitivity of these methods (limited to very small changes/motions)
- Limited range of measurements due to sensitivity and inability to extract dense data from images
- Film recording is highly nonlinear and noisy, so that it was difficult to determine spatial center of fringes, much less quantitative data between fringes.

Fortunately, the 1960s saw the beginning of a revolution in the scientific fields, the digital age, and with it the development of digital image recording devices that were the predecessors to our modern digital cameras. Of course, with

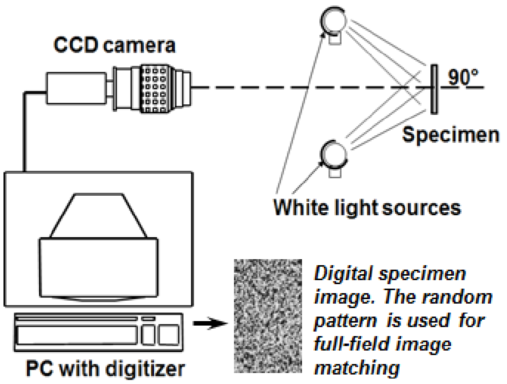

Figure 2: Schematic of an early 2D-DIC system including (a) digital camera with lens to image specimen, (b) storage system (PC with digitizer), (c) lighting system to illuminate specimen. Surface coated with high contrast speckle pattern.

the development of new image recording devices, photomechanics researchers began to ask basic questions directed at the best way to utilize this technology and improve the accuracy, repeatability and usability of full-field measurement methods. Typical questions of that time were; Do we need to have fringe patterns at all? Do we really need lasers for illumination? Can we compare images in the computer (instead of usign double exposure on film) to extract the information we need? Can we obtain images of curved structures (e.g., airplane fuselage, golf balls, turbine blades) and extract accurate measurements everywhere on the specimen? Can we make measurements where the motions are either small or large, not just when they are extremely small? As it turned out, these questions were answered over a twenty year time frame (1980-2000) at the University of South Carolina[b] with the development and application of two closely connected measurement systems. Known today as two dimensional digital image correlation (2D-DIC) and three-dimensional digital image correlation (3D-DIC), the methods are now used worldwide and are generally regarded as the most effective measurement methods available.

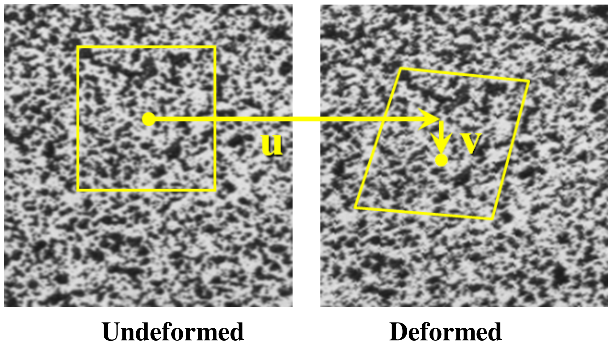

Figure 3: Image matching process to obtain specimen displacements. Image subsets in undeformed image are individually compared to deformed subsets to identify optimal results. Subset size determined by speckle pattern to ensure good contrast so that matching process is accurate.

Two-Dimensional Digital Image Correlation

Figure 2 shows a schematic of a typical 2D-DIC[2-10] measurement system that was developed at the University of South Carolina in the early-to-mid 1980s. A camera with a charge coupled device (CCD) sensor array[c] is oriented approximately perpendicular to a planar specimen. The planar specimen surface is "patterned" to obtain a random variation in contrast with a matte finish (no reflections); such a pattern is known as a "speckle" pattern. The specimen surface is illuminated by standard white lights and an image of the speicmen without any loading is acquired by the digital camera. Designated the "reference" image, the digitized intensity pattern is stored in memory. Then, in-plane loading is applied to the specimen and another image of the specimen is obtained in the "deformed" state and stored in memory. To determine how each area on the specimen deformed, digital subregion (known as subsets) in the first image are identified and the deformed position and shape of the matching subset is determined by optimally matching the "underformed" subset to "deformed" subsets.

The process is shown schematically in Figure 3, where the digital subregion (called a subset) in the undeformed image on the left is translated and distorted to match the same subset in the deformed image on the right. The optimal matching process is done mathematically for each image subset using an equation of the following form;

$$\Phi = \sum_{\text{subset}} [\, I'(x+u(x,y),\, y+v(x,y)) - I(x,y)]^2$$

where Φ is the metric used to determine how well the subsets match; I' and I are the digital intensity data in the deformed and undeformed image subsets, respectively, within the selected region of interest (see yellow square in left image of Fig 2) and u, v are the components of displacement in the horizontal and vertical directions, respectively. The displacements u(x,y) and v(x,y) are varied until Φ is minimized (patterns are matched optimally). By repeating the process for subsets across the entire image, the resulting full-field displacement data is output as the measurements.

In the early years (1982-1986), the digital matching process was very slow on the computers of that time (30 minutes to match one subset!), so that post processing to obtain a modest number of data points for a single experiment took several days. Today, modern computers and efficient coding allow investigators to obtain 10,000 matches in one second, so that data analysis is near real time and results can be analyzed and appropriate improvements implemented in a very short time.

It is also worth noting that images such as those shown in Figs 2 and 3 can be obtained with other types of imaging systems. For example, excellent high contrast images have been acquired at extremely high magnification (20,000X) using a scanning electron microscope (SEM).[d] By comparing subsets from SEM images, specimen deformations on the scale of a microstructural grain in metals or a fiber bundle in a composite have been measured in recent years.[10-13]

Advantages of 2D-DIC

The method offers many advantages over film-based methods employing laser illumination;

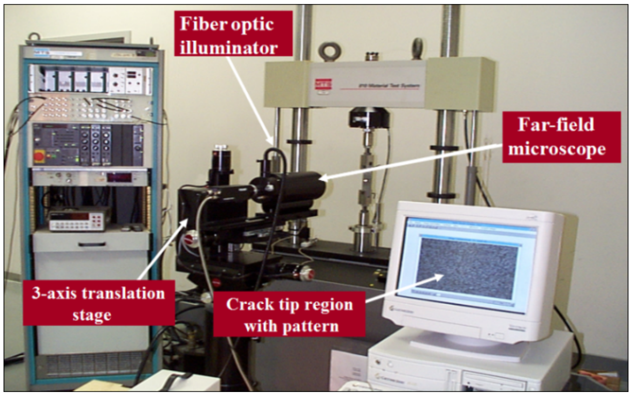

Figure 4: Photograph of original 2D-DIC system including (a) far field microscope lens with CCD camera, (b) PC for image storage and image viewing, (c) 3-axis translation system for camera positioning and (d) specimen in fatigue loading system.

- Simple setup that is relatively unaffected by standard laboratory noise sources
- White light sources, eliminating need for lasers
- Ability to quantify both small and large motions, without difficulty
- Elimination of film as a recording medium
- Digital recording of information allows for near-permanent storage of images and unlimited post-processing of the data
- Advances in computer technology result in increasingly fast processing time and improved post-processing capabilities
- Dramatic improvements in digital cameras have increased both the spatial resolution (number of pixels in a scientific quality sensor array up to 5000 x 5000) and the temporal resolution (some high speed cameras can acquire excellent images every 5 nanoseconds!), allowing investigators to make *quantitative measurements* with the method that were previously impossible.

Typical Example

Figure 4 shows the original photograph of a special 2D-DIC system that was developed for NASA Langley in 1998; the system is still in use today.[17] The goal of the NASA investigators was to understand how cyclic loading of specimens with a flaw, which is known as fatigue, affects the response of the material in the crack tip region.[18] The system incorporates several unique features

- High magnification optics (Questar Far Field Microscope Lens) to allow investigators to make high magnification measurements over a 0.7mm by 0.5mm area surrounding the crack tip as it was undergoing fatigue loading.
- Three dimensional translation stage with autofocus software to (a) keep surface in focus as the experiment progresses and (b) move the lens and follow the growing crack tip region.
- Novel patterning method using toner powder to produce a high contrast image on the surface of the specimen
- Figure 5 shows the type of data obtained by the investigators.

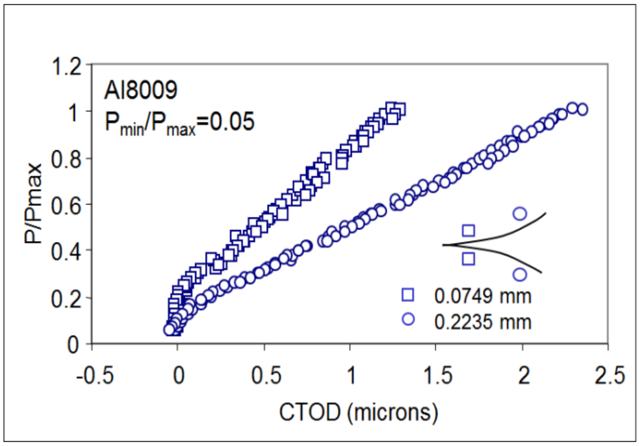

Figure 5: Measured Crack Tip Opening Displacement (CTOD) results at two locations behind tip for several load levels.[17, 18] 2D-DIC results have incredibly small variability, on the order of 40 nanometers (200 atoms), delineating clear trends in the data.

Here, the investigators wanted to know how the crack surfaces opened up as the loading was increased, so they measured the relative displacement across the crack line. Their results showed that the crack opened in a peeling mode, with points far from the crack tip opening first at a small load and points near the crack tip separating at a higher load.

Three-Dimensional Digital Image Correlation

Though 2D-DIC has many signficant advantages, the method suffers from two major drawbacks that limit its applicability in many situations;

- Specimen must be planar
- Loading process must deform and move the specimen in the original plane of the specimen; no motions are allowed in the direction perpendicular to the specimen.

Clearly, many significant applications do not meet these requirements. In fact, in almost every application there is some motion "out-of-plane", even if this is unintended, since it is difficult to load any material in a single plane and maintain only in-plane motions. Furthermore, the vast majority of applications

Figure 6: Modern stereo-vision system. Light source is located between the cameras. Cameras are mounted to a rigid cross-bar and can be rotated and translated along bar. Bar is mounted to a sturdy tripod so that the height can be adjusted as needed.

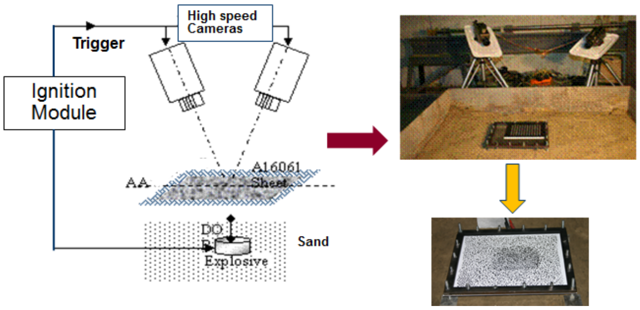

Figure 7: Schematic with photo of the experimental setup:: Sand filled pit used as blast medium. Two high speed cameras mounted outside pit and aligned to view specimen. One gram of PETN explosive is buried below specimen and detonated. Rectangular Al-6061 specimen bolted into steel frame for experiments.

include non-planar sections of the component that cannot be measured by 2D-DIC.

To overcome this problem and measure the complete three-dimensional motion of curved or planar specimens, between 1992 and 1996 investigators at USC used concepts from the imaging community and developed a modified non-contacting measurement system[19-22] based on the use of stereo vision. Processing of the images utilized modified image correlation concepts and perspective projection to obtain highly accurate 3D deformation measurements.[e] Figure 6 shows a modern stereo-vision system that is used throughout the world. As shown in Fig. 6, two cameras are mounted firmly to a common rigid bar. The bar is then attached to a sturdy tripod to adjust both the height and orientation of the bar for the application of interest.

To use a stereo vision system and acquire 3D deformation measurements, the following process is performed.

- Construct system and focus cameras on specimen position.
- Remove specimen and take images of a calibration target in several orientations to calibrate the stereo vision system. Complete calibration process. Replace specimen and begin experiment, taking pairs of images of the patterned specimen surface before loading and during loading process. Complete experiment.
- Analyse all image pairs using 3D-DIC software to extract the complete 3D motion and deformation.
- Similar to 2D-DIC, subsets in the undeformed state are selected in one camera as the reference and compared to subsets in the deformed image pairs to extract the matching image positions.
- Triangulation is performed using the calibrated camera parameters to locate the 3D spatial position of the object point.

Advantages of 3D-DIC

The method has all of the advantages identified earlier for a 2D-DIC system, as well as the following additional advantages;

- Measures the complete, three-dimensional shape and motion of a specimen surface, so that out-of-plane motion does not affect the ability to make measurements.

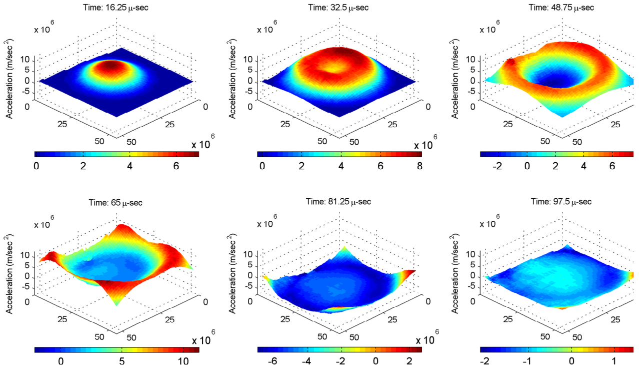

Figure 8: Measured out of plane accelerations during the blast loading process. L-R on top row: t=16.25µs, 32.5 µs, 48.75 µs. L-R on bottom row: t =65µs, 81.25µs, 97.5µs. Full field data of this type provides clear trends in the specimen's dynamic response that is used to improve understanding of high rate events and validate simulation model predictions.

- Capable of making measurements on curved or planar surfaces for large[23, 24] or extremely small objects[25], as long as both cameras can image the same region.
- Can make measurements even when the specimen experiences large displacements and strains, as long as it remains in focus.

Because of the broad applicability of the 3D-DIC method and the relative simplicity of constructing and using modern systems, it has replaced the two-dimensional approach and is now the method of choice in research laboratories and industrial facilities worldwide.

3D-DIC Application in Blast Loading Event

To demonstrate the power of the method, results obtained from a series of blast loading experiments[26-30] are presented. To make these measurements, modern high speed cameras were employed to acquire images of the specimen as a buried explosive is detonated and the blast debris impacted the specimen.

Figure 7 shows a schematic of the arrangement, as well as a photo of the actual camera setup prior to detonation of the explosive. In these studies, two very high speed digital cameras (Phantom V12.1, manufactured by Vision Research) were used to acquire 2000 images at a rate of ~62,000 frames per second with a reduced camera resolution of 128 by 128 pixels.

After analysing the images with 3D-DIC, the three-dimensional displacement field is determined at each time increment. Using the displacement measurements, the accelerations are computed at each point in the field of view. Figure 8 shows the corresponding measured out of plane (vertical) acceleration field, $\partial^2 w/\partial t^2$, for the specimen at six time intervals after detonation of the explosive. As shown in Figure 8, the measurement method is capable of quantifying the complex spatial and temporal evolution of material response, even at the extremely high rates of loading that are applied to the specimen during a blast loading event. Of particular interest in the data is the presence and propagation of an "acceleration wave" in the

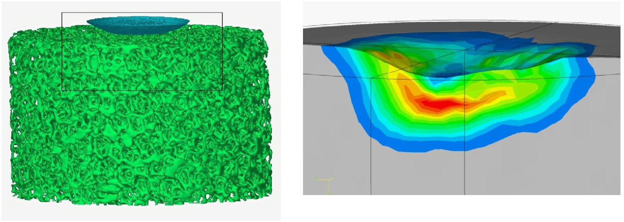

Figure 9: Left: Spherically indented cylindrical aluminum foam specimen imaged by micro-CT scanning. Right: Maximum compressive strains obtained using Volumetric DIC. Results show maximum compressive strain is below the top surface. (Courtesy Prof. B.K. Bay, Oregon State University

plate, which appears around 32µs after detonation and quickly propagates across the entire plate to the boundary.

Recent Measurement Developments: Volumetric DIC

In the late 1990s, Prof. Brian Bay and his colleagues[31-33] showed that the concepts embodied in 2D-DIC could be directly extended to make ***internal*** deformation measurements within materials that have sufficient contrast in the reconstructed images for accurate matching of sub-volumes. Using Computed Image Tomography (CT) to acquire internal images of a bone specimen before and during mechanical loading, he was able to show that DIC could be used to digitally match sub-volumes and obtain internal three-dimensional deformations to improve understanding of the behaviour of this biological material.

Since that time, the method known as Volumetric DIC has grown steadily within the research community. Since the method requires good contrast to accurately match sub-volumes, it is limited to those applications where achieving high contrast is possible. When using CT scanning, image contrast is achievable when there are differences in density within the material. Examples include concrete curing, particle reinforced composites and the behaviour of porous metallic materials undergoing mechanical loading, all of which have naturally occurring material differences that provide contrast in the digitized images. Interestingly, the method is also being used in the medical community with other volumetric imaging methods, such Confocal Image Microscopy, Positron Emission Tomography, and Magnetic Resonance Imaging. In each case, an understanding of how the images are formed is necessary to develop approaches that ensure sufficient contrast in the images.

Figure 9 shows both the CT image (left) and the contours for the measured maximum compressive normal strain (right) for a 5cm diameter, open-foam aluminum specimen undergoing compressive loading by a spherical indenter. Of particular note is the observation that the maximum compressive strain region (in red) occurs slightly below the indentation surface, confirming that internal measurements can provide additional insight into the behaviour of complex material systems. To obtain the results shown, the sub-volume size used in the DIC analysis encompassed at least three internal void regions in each direction so that there was sufficient contrast to identify the correct matching sub-volume in the deformed image.

Remarks Regarding Volumetric DIC

An obvious advantage of the method is the ability to obtain internal deformation measurements during mechanical or environmental loading. However, there are also several disadvantages of the method that have limited its applicability when attempting to use DIC and acquire internal motion and strain data.

- Reasonable local contrast is required in the images to identify the corresponding deformed positions, perform the matching and extract reliable measurements.
- Slow 3D scanning process limits applications to static conditions where scans up to several minutes or even hours may be necessary to obtain high quality images.
- Reconstruction process may lead to variations in the images that degrade the accuracy of the measurements.
- Volumetric imaging system cost can be quite high, necessitating use of existing systems that may not be designed to obtain needed image quality.

Even with these issues, the method continues to grow in popularity and it is expected to expand in use for those applications where such measurements are desired.

Future Trends: A Design Paradigm Shift through Integration of Simulation and Experimentation

As DIC methods have become the preferred measurement technology for a broad range of engineering and science studies, investigators have begun to consider how this unique, digitally-based measurement method can be combined with modern simulation software to obtain an integrated suite of analysis and measurement tools. In particular, Profs. Francois Hild and Stephane Roux[34-36] at ENS-Cachan in Paris, France have led this effort in the European community, with all three of the DIC tools noted previously as the primary measurement systems in their developments.

In principle, the general concept is as follows. In many fields, the design process is now computer based (Computer-Aided Engineering and Computer Aided Manufacturing are denoted by CAE and CAM, respectively). Using CT imaging or even 3D-DIC to obtain the surface geometry for simple specimen shapes, the initial size and shape of a specimen can be measured. These data can be input into CAD models to define the initial shape of the component. Once this is completed, the CAD model is transferred to modern computer analysis programs (Finite Element and Boundary Element are denoted FE and BE) for detailed mechanical, thermal and environmental effects analysis.

To determine whether the FE/BE models are adequate, a series of experiments for the specimen geometry are performed and the response of the specimen to known mechanical, thermal and/or environmental effects is measured. These response measurements are then compared to the FE/BE predictions, with iteration on the assumed material properties and/or boundary conditions performed until adequate agreement is reached between the measurements and predictions.

Once satisfactory agreement is reached, the FE/BE predictions can be performed for other conditions (e.g., different applied loads or environmental conditions) to quickly assess whether the design meets requirements in a broad range of potential conditions, effectively removing excessive conservatism when performing simplified design analysis while also reducing to a minimum the requirement for additional experimental studies by using the complete full-field response of the component in the design analysis.

In many ways, the approach outlined above is a true paradigm shift in the design process, away from “standardized” methods and towards a rich, data-driven environment that offers the designer an improved understanding of the actual situation and a much better feel for where design factors are most relevant. As such, the concepts will require considerable effort to develop an effective overall approach that will work in a broad range of design scenarios. Even so, the potential impact of such an approach is breath-taking in scope and as such will almost certainly continue to be the long-term focus of forward thinking investigators and designers worldwide.

Conclusions

The development of a set of digitally-based, non-contacting measurement methods known as 2D-DIC, 3D-DIC and Volumetric-DIC has led to their worldwide acceptance as the preferred measurement technique for a broad range of scientific, engineering, and industrial applications. With the ability to acquire data on a time scale from 10^{-9} sec to 10^{5} sec or larger, and for characteristic lengths in each direction from 10^{-6}m to 10^{2}m (or larger), the methodology is unprecedented in its breadth of applicability. Furthermore, the natural synergy of these digitally based measurement approaches with modern design and analysis software provides a platform for combining measurement science with simulation techniques to obtain an integrated suite of design, analysis and measurement tools. Together, they provide a unique platform that has the potential to become a paradigm shift in design and analysis.

Notes and references

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[b] *Technology developed was spun off to form in 1996 the only US-based corporation focused on development and sales of DIC-based measurement systems, Correlated Solutions, Incorporated;121 Dutchman Blvd, Columbia, SC 29063;* www.correlatedsolutions.com

[c] *In the early years, analog cameras were all that was available, with PC digitizing boards used to convert the analog image signal into digital intensity data. Modern CCD and Complementary Metal-Oxide Semiconductor (CMOS) arrays have replaced analog processes in cameras, with on-camera digitization of the individual sensor data so that data transfer to memor y is a direct digital transfer without requiring a separate digitizing board on a PC or memory storage device.*

[d] *Recently, scientists have used 2D-DIC with Atomic Force Microscope images*[14-16] *and demonstrated that these can also be used to quantify deformations, though the noise levels are higher due to the AFM scanning process currently in use.*

[e] *An original impetus for these studies came from a critical need identified by Dr. Charles Harris at NASA Langley circa 1990. Dr. Harris indicated that a method was needed to make quantitative, full-field measurements on aircraft fuselages as part of on-going Aging Aircraft research focused on understanding how flaws propagate and link up along a series of rivet holes. At that time, no method existed to make such measurements.*

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