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Interdependence of Market Risk Measures

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The proper definition and measurement of risk in asset valuation have been subject to important theoretical developments in recent years.[1] One segment of this work has focused on the behavior of common-stock price volatility, both cross-sectionally and longitudinally.[2] The present study presents further empirical evidence on the redundancy of certain risk surrogates. More specifically, using correlation and cluster analysis, we investigate the degree of informational overlap between 11 commonly used risk measures. Six of the measures are found to produce similar rankings of common stock at considerably high levels of significance.

A sample of 943 firms having month-end price data from January 1966 to January 1974 was selected from the Quarterly Compustat Industrial tape. Since this time period was characterized by wide upward and downward swings in the market, biases from examining a single market phase are reduced. Additionally, two subperiods exist—1966–69 and 1970–73—which allow analysis of intertemporal stability between rankings by risk measures.

After adjustment for stock splits and stock dividends, percentage changes in prices were computed for each of the 943 firms as follows: $(P_{t+1} - P_t)/P_t$, where P_t and P_{t+1} represent prices of a firm's stock at the beginning and end of a month. Using these percentages as inputs, we then calculated the 11 risk measures shown in table 1 for each firm during each subperiod and the overall period. This list is indicative of the wide range of risk surrogates suggested in the literature on security analysis and portfolio management.

CORRELATION AND CLUSTERING OF RISK MEASURES

Spearman rank correlation coefficients between each pair of the 11 risk measures for the three periods are contained in table 2.[3] Each correlation

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1. Eugene F. Fama, "The Behavior of Stock Market Prices," *Journal of Business* 38, no. 1 (1965): 34–105; Benjamin F. King, "Market and Industry Factors in Stock Price Behavior," *Journal of Business* 39, no. 1 (1966): 139–70; and William F. Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," *Journal of Finance* 19, no. 3 (1966): 425–42.

2. George E. Pinches and William R. Kinney, Jr., "The Measurement of the Volatility of Common Stock Prices," *Journal of Finance* 26, no. 1 (1971): 119–25; and Edward I. Altman and Robert A. Schwartz, "Common Stock Price Volatility Measures and Patterns," *Journal of Financial and Quantitative Analysis* 4, no. 5 (1970): 603–25.

3. Since nonlinearities may exist between the risk measures, the Spearman rank correlation is used rather than the Pearsonian product-moment correlation. See Sidney Siegal, *Nonparametric Statistics for the Behavioral Sciences* (New York: McGraw-Hill Book Co., 1959).

Table 1
Definition of Risk Measures

Risk Measure	Symbol	Formula*
Range	R	$r_{\max}-r_{\min}$
Semi-interquartile deviation	$SIQD$	$(Q_3-Q_1)/2$
Mean absolute deviation	MAD	$\Sigma\vert r-\bar{r}\vert/N$
Standard deviation	σ	$[\Sigma(r-\bar{r})^2/N]^{1/2}$
Semivariance	SV	$\Sigma(BAR-\bar{r})^2/N$
Lower confidence limit	LCL	$\bar{r}-3\sigma$
Coefficient of variation†	CV	$\vert\sigma/\bar{r}\vert$
Coefficient of quartile variation†	CQV	$\vert(Q_3-Q_1)/Q_2\vert$
Skewness	S	$[\Sigma(r-\bar{r})^3/N]/\sigma^3$
Kurtosis	K	$[\Sigma(r-\bar{r})^4/N]/\sigma^4$
Beta	β	$\mathrm{Cov}(r, r_{sp})/\sigma^2_{sp}$

* Symbols are as follows: r = monthly return; $\bar{r}$ = mean monthly return for a firm; Q_1, Q_2, Q_3 = first, second, and third quartile returns, respectively; N = number of observations per firm; BAR = below average returns for a firm, i.e., $r < \bar{r}$; r_{sp} = monthly return on Standard & Poor's Composite (500) Index; σ^2_{sp} = variance of r_{sp}.

† To preserve ordinal properties, the formulae used for coefficient of variation and coefficient of quartile variation are slightly different from those used conventionally.

in the table represents degree of similarity of rankings produced by risk measures. A correlation of +1.00 indicates perfectly equivalent rankings by two risk measures; −1.00 indicates perfectly inverse rankings. Although table 2 shows most correlations differing significantly from zero, in-depth analysis requires comment on the relative homogeneity of the measures.

Natural arrangements of the risk measures, with each arrangement being relatively homogeneous, can be uncovered by cluster analysis using the correlation coefficients as inputs. To some extent, clustering can be achieved by a visual inspection of the correlations themselves. However, with a large number of correlation coefficients the underlying structure is not obvious by inspection alone. Hence a clustering procedure is helpful that provides a pictorial representation as well as a consistent decision criterion for combining risk measures into homogeneous groups.[4] The procedure uses an array of similarity measures (e.g., correlation coefficients) to construct a hierarchical system of homogeneous groups of objects (e.g., risk measures).[5]

Table 3 graphically reveals the hierarchy of clusters obtained for each time period and the corresponding index of similarity for each group of clusters.[6] An analysis of table 3 shows that for all three periods semi-interquartile deviation ($SIQD$), range (R), semivariance (SV), mean absolute

4. Stephen C. Johnson, "Hierarchical Clustering Schemes," *Psychometrica* 32, no. 3 (1967): 241–54.

5. The "minimum method" or "connectedness method" of clustering criteria is used. It computes the smallest chain distance between all clusters and minimizes this distance at each stage. The chain distance measures a kind of connectedness between the clusters. For this criterion, the minimum-distance value is assigned as the index of similarity at each stage.

6. We should note that the correlation coefficients for the lower confidence limit, skewness, and kurtosis were multiplied by −1 before clustering because a larger number for these measures presumably implies less risk, whereas in all other cases a large number implies more risk. Hence changing the sign of the correlation coefficients for these measures enables their proper interpretation in the clustering output.

Table 2

Spearman Rank Correlation Coefficients between Risk Measures

	R	*SIQD*	*MAD*	σ	*SV*	*LCL*	*CV*	*CQV*	*S*	*K*	β
Period 1, 1966–69:											
R	1.00	...	...	...	...	...	...	...	...	...	...
SIQD	.79	1.00	...	...	...	...	...	...	...	...	...
MAD	.90	.94	1.00	...	...	...	...	...	...	...	...
σ	.95	.90	.99	1.00	...	...	...	...	...	...	...
SV	.92	.92	.98	.98	1.00	...	...	...	...	...	...
LCL	−.94	−.89	−.98	−.99	−.97	1.00	...	...	...	...	...
CV	−.02*	−.07*	−.05*	−.04*	−.06*	−.03*	1.00	...	...	...	...
CQV	.10	.16	.13	.12	.14	−.13	.32	1.00	...	...	...
S	.28	.02*	.14	.21	.04*	−.22	.04*	−.08	1.00	...	...
K	.36	−.16	.01*	.13	.03*	−.13	.06*	−.07	.63	1.00	...
β	.37	.37	.39	.39	.39	−.41	.12	.06*	.03*	.01*	1.00
Period 2, 1970–73:											
R	1.00	...	...	...	...	...	...	...	...	...	...
SIQD	.79	1.00	...	...	...	...	...	...	...	...	...
MAD	.91	.94	1.00	...	...	...	...	...	...	...	...
σ	.95	.90	.99	1.00	...	...	...	...	...	...	...
SV	.93	.89	.97	.98	1.00	...	...	...	...	...	...
LCL	−.95	−.89	−.98	−.99	−.97	1.00	...	...	...	...	...
CV	.04*	−.01*	.02*	.03*	.01*	−.06*	1.00	...	...	...	...
CQV	−.09	−.02*	−.08	−.09	−.07*	.08	.37	1.00	...	...	...
S	.25	.17	.24	.27	.10	−.27	.14	−.11	1.00	...	...
K	.36	−.18	−.00*	.11	.08	−.12	.07*	−.12	.26	1.00	...
β	.40	.39	.41	.40	.42	−.40	.06*	.05*	−.01*	.03*	1.00

* Not significantly different from zero at the .025 level using a two-tail test.

Table 2 (Continued)

	R	$SIQD$	MAD	σ	SV	LCL	CV	CQV	S	K	β
Period 3, 1966–73:											
R	1.00	...	...	...	...	...	...	...	...	...	...
$SIQD$	.82	1.00	...	...	...	...	...	...	...	...	...
MAD	.90	.97	1.00	...	...	...	...	...	...	...	...
σ	.94	.93	.99	1.00	...	...	...	...	...	...	...
SV	.92	.94	.98	.98	1.00	...	...	...	...	...	...
LCL	− .94	− .93	− .98	−1.00	− .98	1.00	...	...	...	...	...
CV	.07*	.06*	.06*	.07*	.05*	− .12	1.00	...	...	...	...
CQV	.13	.20	.17	.15	.18	− .16	.21	1.00	...	...	...
S	.29	.15	.22	.27	.12	− .28	.09	− .15	1.00	...	...
K	.43	− .05*	.08	.17	.12	− .18	.02*	− .13	.48	1.00	...
β	.36	.40	.40	.39	.40	− .40	.13	.07*	.00*	− .01*	1.00

Table 3

Clusters of Risk Measures

<table>
<tr><th>Index of Similarity</th><th>CV</th><th>CQV</th><th>S</th><th>K</th><th>SIQD</th><th>R</th><th>SV</th><th>MAD</th><th>σ</th><th>LCL</th><th>β</th></tr>
<tr><td colspan="12">Period 1, 1966–69:</td></tr>
<tr><td>1.00</td><td>*</td><td>*</td><td>*</td><td>*</td><td>*</td><td>*</td><td>*</td><td>*</td><td>*</td><td>*</td><td>*</td></tr>
<tr><td>.99</td><td>*</td><td>*</td><td>*</td><td>*</td><td>*</td><td>*</td><td>*</td><td colspan="3">XXXXXXXXXXXXX</td><td>*</td></tr>
<tr><td>.98</td><td>*</td><td>*</td><td>*</td><td>*</td><td>*</td><td>*</td><td colspan="4">XXXXXXXXXXXXXXXXXX</td><td>*</td></tr>
<tr><td>.95</td><td>*</td><td>*</td><td>*</td><td>*</td><td>*</td><td colspan="5">XXXXXXXXXXXXXXXXXXXXXXX</td><td>*</td></tr>
<tr><td>.94</td><td>*</td><td>*</td><td>*</td><td>*</td><td colspan="6">XXXXXXXXXXXXXXXXXXXXXXXXXXXX</td><td>*</td></tr>
<tr><td>.63</td><td>*</td><td>*</td><td colspan="2">XXXXXXXX</td><td colspan="6">XXXXXXXXXXXXXXXXXXXXXXXXXXXX</td><td>*</td></tr>
<tr><td>.41</td><td>*</td><td>*</td><td colspan="2">XXXXXXXX</td><td colspan="7">XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX</td></tr>
<tr><td>.32</td><td colspan="2">XXXXXXXX</td><td colspan="2">XXXXXXXX</td><td colspan="7">XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX</td></tr>
<tr><td>.16</td><td colspan="11">XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX</td></tr>
<tr><td colspan="12">Period 2, 1970–73:</td></tr>
<tr><td>1.00</td><td>*</td><td>*</td><td>*</td><td>*</td><td>*</td><td>*</td><td>*</td><td>*</td><td>*</td><td>*</td><td>*</td></tr>
<tr><td>.99</td><td>*</td><td>*</td><td>*</td><td>*</td><td>*</td><td>*</td><td>*</td><td colspan="3">XXXXXXXXXXXXX</td><td>*</td></tr>
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<tr><td>.94</td><td>*</td><td>*</td><td>*</td><td>*</td><td colspan="6">XXXXXXXXXXXXXXXXXXXXXXXXXXXX</td><td>*</td></tr>
<tr><td>.42</td><td>*</td><td>*</td><td>*</td><td>*</td><td colspan="7">XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX</td></tr>
<tr><td>.37</td><td colspan="2">XXXXXXXX</td><td>*</td><td>*</td><td colspan="7">XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX</td></tr>
<tr><td>.26</td><td colspan="2">XXXXXXXX</td><td colspan="2">XXXXXXXX</td><td colspan="7">XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX</td></tr>
<tr><td>.18</td><td colspan="2">XXXXXXXX</td><td colspan="9">XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX</td></tr>
<tr><td>.12</td><td colspan="11">XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX</td></tr>
</table>

NOTE.—* indicates a cluster of an individual risk measure; XXXXX indicates a cluster formed by combination of more than one risk measure.

Table 3 (Continued)

Index of Similarity	*S*	*K*	*CV*	*CQV*	*R*	*SIQD*	*SV*	*MAD*	*σ*	*LCL*	*β*
Period 3, 1966–73:											
1.00	*	*	*	*	*	*	*	*	XXXX	XXXX	*
.99	*	*	*	*	*	*	*	XXXX	XXXX	XXXX	*
.98	*	*	*	*	*	*	XXXX	XXXX	XXXX	XXXX	*
.97	*	*	*	*	*	XXXX	XXXX	XXXX	XXXX	XXXX	*
.94	*	*	*	*	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	*
.48	XXXX	XXXX	*	*	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	*
.40	XXXX	XXXX	*	*	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX
.21	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX
.20	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX
.15	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX

deviation (MAD), standard deviation (σ), and lower confidence limit (LCL) join to form a single homogeneous cluster at levels of similarity equaling or exceeding 0.94. Table 2 shows that this group of six risk measures exhibits high correlations (exceeding .79) among themselves in all three periods.[7] These six risk measures apparently capture similar facets of risk, although small differences between risk surrogates might admittedly lead to different portfolio selection.

Close association between standard deviation and lower confidence limit is not unexpected, since standard deviation is essentially being correlated with a linear transformation of itself. High correlation between standard deviation and semivariance suggests relatively symmetrical distributions. High correlation among standard deviation, mean absolute deviation, and semi-interquartile deviation points to relative stability in standard deviation and absence of extreme variances. The group of six risk measures also has low correlations with coefficient of quartile variation (−.16 to .20), skewness (−.28 to .29), and kurtosis (−.18 to .43).

Table 3 shows skewness (S) and kurtosis (K) forming a homogeneous cluster at levels of similarity equaling .63, .26, and .48. This corresponds to significant correlations between these two measures and their generally low or nonsignificant correlations with other measures in each time period. In like manner, coefficient of variation (CV) and coefficient of quartile variation (CQV) form a cluster at levels of similarity of .32, .37, and .21. This reflects their higher correlations with each other than with other risk measures.

Beta (β) consistently joins the cluster of six measures at levels of similarity between .42 and .40. This results from its low significant correlations with the group of six measures. Beta also exhibits nonsignificant correlations with coefficient of quartile variation, skewness, and kurtosis in each of the three periods.

Finally, clusters (S,K) and (CV, CQV) join the enlarged cluster of seven risk measures ($SIQD$, R, SV, MAD, σ, LCL, β) at considerably low levels ranging from .20 to .12, which relates to either nonsignificant or low correlations of S, K, CV, and CQV with the group of six risk measures and beta.

Table 3 shows that the number of relatively homogeneous risk-measure groups identified depends on what value of index of similarity is acceptable. For example, if a high degree of homogeneity is required, such as the .94 level for 1966–69, six relatively homogeneous clusters prevail: $SIQD$, R, SV, MAD, σ, and LCL form the largest cluster, whereas CV, CQV, S, K, and β form clusters by themselves, indicating that they are dissimilar to the measures in the enlarged cluster as well as among themselves. A lesser homogeneity requirement at the .63 level results in only five clusters as skewness and kurtosis join to form a cluster (S,K). Similarly, at the low level of .16

7. Since a larger value for the lower confidence limit implies less risk, whereas a larger value for the other five measures implies more risk, high negative correlations (−.89 to −1.00) between lower confidence limit and the other five measures indicates a close association.

all 11 measures enter into a single cluster, which of course is not very homogeneous.

Further examination of table 3 reveals that for each of the three time periods the pattern of cluster formation is basically similar. Additionally, examining the rank correlations for all three periods shows that 35 of the significant correlations and nine of the nonsignificant correlations consistently fall within the same ranges and relate to the same pairs of risk measures. This indicates that the interrelationships between risk measures tend to be stable over time. Note that this does not address the question of whether a risk measure is stationary over time; only the relative position among risk measures is evaluated.

CONCLUSIONS

Potentially risk has many facets, and various risk proxies are purported to measure one or more of them. The preceding analysis indicates that six of the 11 risk measures studied, namely range, semi-interquartile deviation, mean absolute deviation, standard deviation, semivariance, and lower confidence limit, form a homogeneous group at a significantly high level of association. This indicates a fairly high degree of substitutability among these measures for the purpose of assessing relative riskiness of assets. The remaining five measures form clusters by themselves at this level and thus may provide additional risk information by capturing other dimensions of risk.

Skewness and kurtosis join as a separate group at much lower levels of association. To the extent that they are behaviorally important, each one has the potential of providing additional risk information. Previous work by Alderfer and Bierman and by Arditti indicates that higher-order moments may be behaviorally important to investors for decision making.[8] Beta also forms a relatively independent group by itself and thus may possess unique risk information. Due to their association with other risk measures at consistently low levels, coefficient of variation and coefficient of quartile variation appear to capture distributional information different from that provided by other risk measures.

Depending on the purpose at hand, selection of a risk proxy would be aided by the knowledge about degree of substitutability among various risk measures. If an indication of all dimensions of risk is desired, a judicious combination of risk measures from different homogeneous clusters would be needed. Since there is disagreement concerning the definition of risk and extent of diversification in the market, behavioral research into the valuation process, risk perception, and diversification would aid in determining which combination of risk measures would be optimum.

8. Clayton P. Alderfer and Harold Bierman, Jr., "Choices with Risk: Beyond the Mean and Variance," *Journal of Business* 43, no. 3 (1970): 341–53; and Fred D. Arditti, "Risk and the Required Rate on Equity," *Journal of Finance* 22, no. 1 (1967): 19–36.